

IRSTI 14.25

*M. B. Sikymbay<sup>1</sup>*

<sup>1</sup>Suleyman Demirel University  
Kaskelen, Kazakhstan

## FEATURES OF STRUCTURE OF MATHEMATICAL ABILITIES OF HIGH SCHOOL STUDENTS

**Abstract.** The modern educational practice of the school system is aimed at creating conditions for the development of each child based on its individual characteristics. In this regard, studies to determine the qualitative uniqueness of the structure of mathematical abilities of students depending on the profile of training are of a particular interest.

The aim of study was to identify the structural features of the mathematical abilities of advanced high-school students studying computers. We relied on the structure of V.A. Krutetskiy consisting of four components: obtaining mathematical information (perceptual component); processing mathematical information (procedural component); storage mathematical information (mnemonic component); mathematical orientation of the mind (general synthetic component). The following hypothesis was proposed: the use of information technology affects the positive trend in the development of all components of mathematical abilities, and especially on general synthetic component due to the harmonious combination of two sub-components: verbal-logical and visual-figurative.

**Keywords:** mathematical abilities; giftedness; information technology; programming; structure of mathematical abilities; cognitive processes.

\*\*\*

**Аннотация.** Современная образовательная практика школьной системы направлена на создание условий для развития каждого ребенка по его индивидуальным особенностям. В этой связи особый интерес представляют исследования по определению качественной уникальности структуры математических способностей учащихся в зависимости от профиля обучения.

Целью исследования было выявить структурные особенности математических способностей одаренных учащихся средних школ, изучающих компьютеры. Мы полагались на структуру В.А. Крутецкого, которая состоит из четырех компонентов: получение математической информации (перцептивной составляющей); обработка математической информации (процедурный компонент); запоминающая математическая информация (мнемоническая составляющая); математическая ориентация ума (общая синтетическая составляющая). Предложена следующая гипотеза: использование информационных технологий влияет на

положительную тенденцию в развитии всех компонентов математических способностей, и особенно, на общем синтетическом компоненте из-за гармоничного сочетания двух подкомпонентов: вербально-логического и визуально-образного.

**Ключевые слова:** математические способности, одаренность, информационные технологии, программирование, структура математических способностей, когнитивные процессы.

\*\*\*

**Андатпа.** Мектеп жүйесінің қазіргі заманғы білім беру тәжірибесі әрбір жеке тұлғаның ерекшеліктерін ескере отырып, әр баланың дамуына жағдай жасауға бағытталған. Осыған байланысты оқушылардың бейініне байланысты оқушылардың математикалық қабілеттерінің құрылымының сапалық бірегейлігін анықтауға арналған зерттеулер ерекше қызығушылық тудырады.

Зерттеудің мақсаты компьютерлерді меңгерген алдыңғы қатарлы жоғары сынып оқушыларының математикалық қабілеттерінің құрылымдық ерекшеліктерін анықтау болды. Біз В.А. Крутецкий төрт компоненттен тұратын: математикалық ақпаратты алу (қабылдау компоненті); математикалық ақпаратты өңдеу (процедуралық компонент); математикалық ақпаратты сақтау (мнемоникалық компонент); ақылдың математикалық бағыттылығы (жалпы синтетикалық компонент). Келесі гипотеза ұсынылды: ақпараттық технологияларды қолдану математикалық қабілеттердің барлық компоненттерін дамытуда, әсіресе синтетикалық құрамдас бөліктердің дамуында жағымды үрдіске әсер етеді: екі компоненттің үйлесімді комбинациясы: сөздік-логикалық және көрнекі-фигуративті.

**Түйінді сөздер:** математикалық қабілеттер; дарындылық; ақпараттық технологиясы; бағдарламалау; математикалық қабілеттердің құрылымы; когнитивті процестер.

Modern educational practice of the system of school education is aimed at creating conditions for the development of each child, taking into account its individual characteristics. In this regard, particular interest is the research on the identification of the qualitative uniqueness of the structure of the mathematical abilities of schoolchildren, depending on the profile of training, for example, in the context of the use of information technology, in particular the programming of mathematical problems on the computer. Programming as a person's practical activity radically affects his consciousness, since he has a specific nature of operating with formal concepts and objects: allows the programmer to formalize and model the studied processes, to predict the results. This allows him to see the picture of the phenomena being studied in the process of development, i.e. in dynamics. Thus, O.G. Levina believes that

"the expressive capabilities of the programming language, the additional capabilities that programmers use: structuring, classifying, designing the data being processed and their presentation, affect the perception and processing of information encoded by the computer" [1, 21]. Indeed, on the one hand, the work of the programmer requires a high level of abstraction and logical thinking, and on the other hand, develops the ability to visualize abstract mathematical relationships and dependencies. For example, one of the first steps in solving a computer problem is the question of how and how to present the data structure, i.e. schematically depict the structure and typology of the initial, intermediate and output data. The next stages of program development, in particular, the choice of decision algorithms, directly depend on this choice. The well-known programmer N. Wirth believes that "ultimately programs are concrete based on real representation and data structure of the embodiment of abstract algorithms" [2, 8]. The process of programming tasks, thus, contributes to the development of visual-figurative thinking, as it aims at the constant use of heuristic techniques in order to find optimal solutions to new problems, the discovery of new knowledge. Perception and comprehension of the same information with the help of a computer is more versatile, in G.L. Smolyan and K.B. Shoshnikova: "... along with its symbolic representation, visual, auditory, tactile and other representations are also preserved in human memory. Due to this, the same elements of information are fixed in different contexts "[3, 42].

A specific feature of the process of teaching students using computers is also the widespread use of accompanying humanitarian technology, which is to promote the self-expression of learners and self-realization of their intellectual qualities. This is indicated by AA Semenova, studying the features of self-awareness of the individual in intellectually gifted adolescents [4, 203]. In this connection, studies of a high degree of specificity in the structure of the mathematical abilities of schoolchildren are of special interest, depending on the profile of instruction associated with the programming of mathematical problems on the computer.

The purpose of the study was to identify the features of the structure of the mathematical abilities of high school students in the information profile of education. We were also interested in the age dynamics of the development of the structure of mathematical abilities during the period of study in the basic and senior schools. The following hypothesis was put forward: there are features of the development of the structure of mathematical abilities in schoolchildren, expressed in the heterogeneity of trends towards development; the application of information technology affects the positive dynamics in the development of all components of mathematical abilities, especially the general synthetic component (the mathematical direction of the mind), due to a harmonious combination of two components: verbal-logical and visual-figurative.

The main thing in the study of mathematical abilities is the question of the essence of this complex psychological education. Problems to be studied: the specificity of mathematical abilities; structure of mathematical abilities; typology of differences in mathematical abilities. This classification of problems is in good agreement with the model of human characteristics according to B.G. Ananyev: 1) the subject of activity - operational mechanisms (structural ability); 2) individual - functional mechanisms (specificity of abilities); 3) personality - motivational mechanisms (typology of differences in abilities) [5, 15]. In domestic psychology, a great contribution to the study of the structure of abilities was made by many researchers. Thus, V.D. Shadrikov in the structure of abilities identifies two important components: functional and operational. The latter is viewed through the prism of activity, is an adaptation to the requirements of reality [6, 38]. V.N. Druzhinin represents a three-component structure of general abilities: intelligence (the ability to solve problems on the basis of existing knowledge), learning (the ability to acquire knowledge) and creativity (the ability to transform knowledge with imagination and fantasy) [7, 12]. M.A. Cold in its concept of intellect as a form of organization of mental experience broadens the structure of general abilities to four components: learning, creativity, cognitive styles and convergent abilities, while convergent abilities mean the following components: verbal intelligence, nonverbal intelligence, combinatorial properties of intelligence, [8, 13].

In my opinion, attention should be paid to the procedural properties of the intellect, as they include the processing of information, as well as operations (analysis, synthesis, generalization, comparison, abstraction, specification, classification, classification), techniques (knowledge and skills) and strategy of intellectual activity. When studying the question of how modern technologies of teaching schoolchildren affect the specificity of the formation of special abilities, in particular mathematical ones, let us turn to the studies carried out within the framework of the cognitive approach. Согласно Г.В. Zalevsky "cognitive processes, or cognitions, are, most likely, those processes that accumulate, store, process, transform, evoke, then use, and so on. Mental activities or functions, such as: perception, representation, memory, thinking, problem solving and action are understood as possible steps in the process of processing information" [9, 37].

Studying mathematical abilities, V.A. Krutetsky singled out in their structure four main components: "... a) obtaining mathematical information (formalized perception of the problem); b) processing or, in other words, the procedural component (logic of reasoning, generalization of mathematical material, coarseness of mathematical thinking, flexibility of thought processes, desire for the elegance of solutions); c) storage of mathematical information, i.e. mathematical memory and d) the general synthetic component (the mathematical direction of the mind, which determines the typology of

mathematical abilities) "[10, 375]. For comparison, we give the structure of mathematical abilities, proposed A.N. Kolmogorov.

The famous mathematician believed that mathematical abilities consist of three main components: ". logical reasoning, algorithmic computing abilities and geometric imagination "[11, 227]. We in our work adhered to the structure of mathematical abilities, developed by VA Krutetskii, who developed it from the point of view of obtaining, processing and storing mathematical information, which brings the author closer to representatives of the cognitive approach in psychology. It should be especially noted that the scientist associates such a structural component as the mathematical direction of the mind, not with the motivational mechanisms of the individual, but with the degree of development of the two components: verbal-logical and visual-figurative subcomponents [10, 345].

To reveal the structure of mathematical abilities, we developed a methodology [12, 38], which was based on separate series of mathematical problems used by VA. Krutetsky in [10, 186]. The methodology for identifying the structure of mathematical abilities is a shortened version of the tasks (5 series of 26) according to VA Krutetsky's classification: «Задачи с постепенной трансформацией из конкретного в абстрактный план» на получение математической информации (назовем это перцептивным компонентом А), а также на хранение математической информации (будем называть это мнемоническим компонентом В);

- "Heuristic tasks" and "Tasks of general mathematical and logical" for the processing of mathematical information (procedural component B);

- "Problems with different degrees of clarity of the solution" to determine the general synthetic component (the general synthetic component of D).

In total, 9 test batteries of tasks were compiled, the solution of each problem in all series was evaluated on a dichotomous scale, then the result was expressed as a percentage of the number of correctly solved tasks in the series. The series, aimed at studying the general synthetic component, contained equally tasks for the use of verbal-logical and visual-figurative approaches to the solution. The technique is described in detail in the monograph [12, 38] and the paper [13, 134] and was tested on a general sample of more than 300 students with different levels of mathematical ability at the age of 11 to 18 years. Of the total sample, 140 students were selected from different classes (from the 5th to the 11th) with in-depth study of mathematics (7 groups of 20 people each with  $IQ > 120$ ) to study the age dynamics of mathematical abilities. Two groups were also drawn from the general sample: high-school students of the information profile of training - "programmers" (participants in subject Olympiads in informatics at the regional level) and high school students-"non-programmers" (school students of a mathematical profile who do not study the fundamentals of programming in depth). The sample size for each

group was 21 people. At the same time, the conditions of internal validity by gender, age and academic achievement in mathematics were met.

The study showed that all four structural components of mathematical abilities undergo qualitative and quantitative changes: they tend to increase with age of schoolchildren, the most uniform development occurs only after 7th grade. The P-level of significance of the revealed differences according to the Kruskal-Wallis criterion for solving test tasks by different groups was no more than 0.001 for the empirical criterion values from 43.8 to 75.8. At the same time, the qualitative leap in the levels of all four components falls on the period of schoolchildren's education in the 6th and 7th grades: the p-level of significance of the observed differences by the Student's test was from 0.001 to 0.022, with observed gradations of the empirical value of the criterion from 5.66 to 2, 39. Thus, the conclusion of BA is confirmed. Krutetskogo [10, 364] that in the middle age (in our case, when students moved from the 6th to the 7th grade) under the influence of school instruction, the tendency to formalize perceptions, isolate the formal structure, generalize the mathematical material, and qualitatively change other parameters of the procedural component and mathematical memory acquire a broader character among students. Analysis of the tests showed that the schoolchildren-programmers are dominated by higher indicators for all the structural components: A, B, C and G. At the same time, significantly significant differences according to the Student's test are observed in the general synthetic ( $t = 3.44$  at  $p = 0.001$ ) and ( $t = 2.003$  for  $p = 0.05$ ) of the components. A high percentage of solved tasks for the determination of the general synthetic component were received by programmers for the successful solution of problems involving support for both verbal and logical reasoning and for visual representations and schemes. Senior students-non-programmers had difficulties in solving both types of problems. This fact allows us to assume that well-developed visual-figurative thinking of programmers contributed to the increase in the procedural component of their mathematical abilities.

The obtained results allow to conclude that the conducted research has confirmed the hypothesis and initial assumptions, and also gives grounds for continuing further study of the structure of mathematical abilities. In the conditions of applying computer technologies, high school students more successfully solve problems that rely on visual representations and schemes, as in the process of programming students gain more discrete perception of the world, visual representations grow, which in turn helps them to develop the parameters of the procedural component: flexibility and high degree the curtailment of mathematical thinking, the desire for the elegance of solutions to problems and the generalization of mathematical material. Therefore, the use of programming in the conditions of school education will allow to develop creatively creative potential, heuristic abilities, flexibility of thinking and

competence of students, which is especially significant in the light of the reforms that the modern Russian education is exposed to.

### **References:**

1 Levina, O.G. Vzaimodeystvie komp'yutera i cheloveka kak sotsial'noe yavlenie [The interaction of computer and human as a social phenomenon]. // *Pedagogicheskiy vestnik*. – 1998. – no. 2. – Pp. 21-27.

2 Virt N. Algoritmy i struktury dannykh [Algorithms and data structures]. – Dossa: Khamarayan, 1997. – 360 p.

3 Smolyan G.L., Shoshnikov K.B. Fenomen personal'noy EVM: filosofsko-metodologicheskiy aspekt [The phenomenon of a personal computer: the philosophical and methodological aspect]. // *Voprosy filosofii*. – 1986. – no. 6. – P. 42-55.

4 Semenova A.A. Samosoznanie lichnosti i ego osobennosti u intellektual'no oda-rennykh podrostkov [Self-consciousness and its features possessed by the intellectually gifted adolescents]. *Izvestiya Rossiyskogo gosudarstvennogo pedagogicheskogo universiteta imeni. Aspirantskie tetrad. // Pedagogika i psikhologiya, teoriya i metodika obucheniya*. – 2007. – no. 17 (43). – Pt. II. – P. 203-207.

5 Anan'ev B.G. O sootnoshenii sposobnostey i odarenosti [On the relation between abilities and talents]. – Myasishchev V.N. (ed.) *Problemy sposobnostey [Problems of abilities]*. Moscow: Akademiya Publ., 1962. – P. 15-32.

6 Shadrikov V.D. O strukture poznavatel'nykh sposobnostey [On the structure of cognitive abilities]. // *Psikhologicheskiy zhurnal*. – 1985. – no. 3. – P. 38-47.

7 Druzhinin V.N. Psikhologiya obshchikh sposobnostey [The psychology of general abilities]. – St. Petersburg: Piter Publ. – 1999. – 368 p.

8 Kholodnaya M.A. Osnovnye napravleniya izucheniya psikhologii sposobnostey v Institute psikhologii RAN [The main directions in the study of psychological faculties at the Institute of Psychology RAS]. // *Psikhologicheskiy zhurnal*. – 2002. – vol. 23. – no. 3. – P. 13-22.

9 Zalevskiy G.V. Osnovy bikheviortal'no-kognitivnoy terapii i konsul'tirovaniya [The fundamentals of behavioral-cognitive therapy and counseling]. – Tomsk: Tomsk State University Publ., 2006. – 50 p.

10 Krutetskiy V.A. Psikhologiya matematicheskikh sposobnostey shkol'nikov [Psychology of mathematical abilities of school students]. – Moscow: Institute of Applied Psychology; Voronezh: NPO MODEK Publ., 1998. – 416 p.