Пусть выполняются предположения 1 и 4, функции $u(t) \in U$, $v(t) \in V$ определены на $[0, \theta]$, причем $u(t)$ – измерима, а $v(t)$ – кусочно–постоянная. Тогда, если в (5) подставить вместо параметров u и v указанные функции, то решение $z(t)$ уравнения (5) существует и единственное на всем отрезке $[0, \theta]$.

В рассматриваемой модели догоняющий игрок P, как и выше, распоряжается параметром u и его допустимым управлением является измеримая функция $u(t)$ со значением в U.

Параметр v является случайно величиной, ее реализации изменяются в конечные моменты времени и управляется игроком P. Игрок P играет в ε-стратегиях.

Пусть $\Phi : E^n \to E^1$ – непрерывное отображение, $\theta > 0$ – фиксированный момент времени. Величина $\Phi(z(\theta))$ является случайной величиной и цель игрока P – минимизировать ее математическое ожидание.

Поскольку заранее не известны моменты изменения реализаций помехи, то выбор этих моментов предоставляется игроку-противнику E.

В игровых моделях исследуются задачи сближения-уклонения, которые описываются терминальным множеством, множеством фазовых ограничений или терминальным функционалом.

Список литературы:

[1] Гусятников, П.Б. К вопросу об информированности игроков в дифференциальной игре // Прикладная математика и механика. - 1972. -Т.36, № 5. - С. 917-924.
[2] Пшеничный Б.Н., Остапенко В.В. Дифференциальные игры. - Киев, 1992. - 260 с.
[3] Остапенко В.В., Амиргалиева С.Н., Остапенко Е.В. Выпуклый анализ и дифференциальные игры. – Алматы, 2005. – 392 с.

УДК 51

Erdogan A.S.[1]
[1] Assoc. Prof., *Suleyman Demirel University, Almaty, Kazakhstan*
e-mail: abdullah.erdogan@sdu.edu.kz

ON A RIGHT HAND SIDE IDENTIFICATION PROBLEM OF A PARABOLIC EQUATION

Abstract. In many physical phenomena, especially in temperature over-specification partial differential equations with an unknown source function appears. The present paper is devoted to the study of the well-posedness of the approximate solution of a right-hand side identification problem for a parabolic equation.

Key words: Identification problem, stability estimates

Introduction

Inverse parabolic problems is of significant importance in mathematical sciences, applied sciences and engineering. In many physical phenomena, for instance in the process of transportation, diffusion and conduction of natural materials, the parabolic partial differential equation is induced (see [1] and the references therein). In inverse problems, the optimal overdetermination conditions are analyzed in some classical boundary conditions or/and similar conditions given at a point. The problem of determining the temperature at one end of a rod from temperature measurements at an interior point is an example of an inverse heat conduction problem (IHCP) which has been extensively studied [2].

Considerable efforts have been expanded in formulating numerical solution methods that are both accurate and efficient. Methods of numerical solutions of parabolic problems with parameters have been studied by many researchers (see, [3-6]). One usually focuses himself on the uniqueness and the stability of the inverse problem. For the uniqueness, we refer [7]. The discussion of the stability is preparatory to the numerical implementation for the inverse problem in the theoretical respect. During the last decades, some numerical techniques have been proposed to solve a one-dimensional IHCP. Determination of a control function in three-dimensional parabolic equation and in polar coordinate system are also investigated. Among them the finite difference method and the finite element method are so far the principal numerical tool of choice for the modeling and simulation of the IHCP.

Problem formulation

One application of inverse heat conduction problem in engineering and science is to predict the thermal conductivity from the measured temperature profiles. The inverse estimation of thermal conductivity by the measured temperature profiles has been studied by many researchers. This work is devoted to the study of the well-posedness of the approximate solution of the right-hand side identification problem

$$\begin{cases} u_t(t,x)-a(x)u_{xx}(t,x)+\sigma u(t,x)=P(t)Q(x)+f(t,x), \\ 0<x<l, 0<t<T, \\ u(t,0)=u(t,l)=0, 0\le t\le T, \\ u(0,x)=\varphi(x), 0\le x\le l \end{cases} \tag{1}$$

where $P(t)Q(x)=p_1(t)q_1(x)+p_2(t)q_2(x)+\cdots+p_n(t)q_n(x)$. Here $u(t,x)$ and $p_i(t)\,(i=1,2,\cdots,n)$ are unknown functions, $f(t,x)$, $q_i(x)\,(i=1,2,\cdots,n)$, $\varphi(x)$, and $a(x)$ are given sufficiently smooth functions, $a(x)\ge\delta>0$ and $\sigma>0$ is a sufficiently large number. For solving the parabolic inverse problem (1), the overdetermined conditions $u(t,s_1)=\rho_1(t), u(t,s_2)=\rho_2(t),\cdots,u(t,s_n)=\rho_n(t)$ where $s_1,s_2,\cdots,s_n$ are inner points, $\rho_i(t)$ $(i=1,2,\cdots,n)$, $0\le t\le T$ are sufficiently smooth functions are determined. Let us assume $q(0)=q(l)=0$ and $q(s_1),q(s_2),\cdots q(s_n)$ are different from zero. In this paper, for the clarity $n=2$ is taken.

Stability analysis of solutions $u(t,x), p_1(t)$ and $p_2(t)$ are given by the help of an auxiliary problem. To formulate the auxiliary problem, the transformation

$$u(t,x)=w(t,x)+\eta_1(t)q_1(x)+\eta_2(t)q_2(x), \tag{2}$$

where

$$\eta_1(t)=\int_0^t p_1(s)ds, \eta_1(0)=0$$

and

$$\eta_2(t)=\int_0^t p_2(s)ds, \eta_2(0)=0$$

is determined. Taking partial derivative of both sides of equation (2), we get

$$u_t(t,x)=w_t(t,x)+p_1(t)q_1(x)+p_2(t)q_2(x) \tag{3}$$

and

$$u_{xx}(t,x)=w_{xx}(t,x)+\eta_1(t)q_{1,xx}(x)+\eta_2(t)q_{2,xx}(x). \tag{4}$$

Using the overdetermined conditions, we can write

$$\rho_1(t)=u(t,s_1)=w(t,s_1)+\eta_1(t)q_1(s_1)+\eta_2(t)q_2(s_1)$$

and

$$\rho_2(t)=u(t,s_2)=w(t,s_2)+\eta_1(t)q_1(s_2)+\eta_2(t)q_2(s_2).$$

If

$$J(s_1,s_2)=\begin{vmatrix} q_1(s_1) & q_2(s_1) \\ q_1(s_2) & q_2(s_2)\end{vmatrix}\neq 0,$$

one can easily show that

$$\eta_1(t)=\frac{\begin{vmatrix} \rho_1(t)-w(t,s_1) & q_2(s_1) \\ \rho_2(t)-w(t,s_2) & q_2(s_2)\end{vmatrix}}{J(s_1,s_2)} \tag{5}$$

and

$$\eta_2(t)=\frac{\begin{vmatrix} q_1(s_1) & \rho_1(t)-w(t,s_1) \\ q_1(s_2) & \rho_2(t)-w(t,s_2)\end{vmatrix}}{J(s_1,s_2)}. \tag{6}$$

Replacing equations (3)-(6) in (1), we reach to the following auxiliary problem

$$\begin{cases} w_t(t,x)-a(x)w_{xx}(t,x)+\sigma w(t,x)=f(t,x) \\ +\dfrac{\rho_1(t)q_2(s_2)-\rho_2(t)q_2(s_1)-w(t,s_1)q_2(s_2)+w(t,s_2)q_2(s_1)}{q_1(s_1)q_2(s_2)-q_1(s_2)q_2(s_1)}\left(q_{1,xx}(x)-\sigma q_1(x)\right) \\ +\dfrac{\rho_2(t)q_1(s_1)-\rho_1(t)q_1(s_2)-w(t,s_2)q_1(s_1)+w(t,s_1)q_1(s_2)}{q_1(s_1)q_2(s_2)-q_1(s_2)q_2(s_1)}\left(q_{2,xx}(x)-\sigma q_2(x)\right) \\ 0<x<l, 0<t<T, \\ u(t,0)=u(t,l)=0, 0\le t\le T, \\ u(0,x)=\varphi(x), 0\le x\le l \end{cases} \tag{7}$$

under the same assumptions on $q(x)$.

Theoretical considerations

In this section, coercive stability estimates of problem (1) are obtained where additional condition is observed with and without noise. To formulate our results, we introduce the Banach space $\overset{\circ}{C}{}^{\alpha}[0,L]$, $\alpha\in(0,1)$, of all continuous functions $\phi(x)$ defined on $[0,L]$ with $\phi'(0)=\phi(L)=0$ satisfying a Hölder condition for which the following norm is finite

$$\|\phi\|_{\overset{\circ}{C}{}^{\alpha}[0,L]}=\max_{0\le x\le L}|\phi(x)|+\sup_{0\le x<x+h\le L}\frac{|\phi(x+h)-\phi(x)|}{h^{\alpha}}.$$

In a Banach space E, with the help of a positive operator A we introduce the fractional spaces $E_\alpha, 0<\alpha<1$, consisting of all $v\in E$ for which the following norm is finite:

$$\|v\|_{E_\alpha}=\|v\|_E+\sup_{\lambda>0}\lambda^{1-\alpha}\|A\exp\{-\lambda A\}v\|_E.$$

Positive constants will be indicated by M which can be differ in time.

Theorem 1 1 *Let $\varphi(x)\in \overset{\circ}{C}{}^{2\alpha+2}[0,L]$, $f(t,x)\in C\left([0,T],\overset{\circ}{C}{}^{2\alpha}[0,L]\right)$ and $\rho'(t)\in C[0,T]$. Then for the solution of problem (1) the following coercive stability estimates*

$$\|u_t\|_{C\left([0,T],\overset{\circ}{C}{}^{2\alpha}[0,L]\right)}+\|u\|_{C\left([0,T],\overset{\circ}{C}{}^{2\alpha+2}[0,L]\right)}\le M(x^*,q)\|\rho'\|_{C[0,T]}$$

$$+M(a,\delta,\sigma,\alpha,x^*,q,T)\left(\|\varphi\|_{\overset{\circ}{C}{}^{2\alpha+2}[0,L]}+\|f\|_{C\left([0,T],\overset{\circ}{C}{}^{2\alpha}[0,L]\right)}+\|\rho\|_{C[0,T]}\right),$$

$$\|p_1\|_{C[0,T]}\le M(x^*,q)\|\rho'\|_{C[0,T]}$$

$$+M\left(a,\delta,\sigma,\alpha,x^*,q,T\right)\left[\|\varphi\|_{\overset{\circ}{C}^{2\alpha+2}[0,L]}+\|f\|_{C\left([0,T],\overset{\circ}{C}^{2\alpha}[0,L]\right)}+\|\rho\|_{C[0,T]}\right],$$

$$\|p_2\|_{C[0,T]}\le M\left(x^*,q\right)\|\rho'\|_{C[0,T]}$$

$$+M\left(a,\delta,\sigma,\alpha,x^*,q,T\right)\left[\|\varphi\|_{\overset{\circ}{C}^{2\alpha+2}[0,L]}+\|f\|_{C\left([0,T],\overset{\circ}{C}^{2\alpha}[0,L]\right)}+\|\rho\|_{C[0,T]}\right]$$

hold.

Theorem 22 *For the solution of problem (7), the following coercive stability estimate*

$$\|w_t\|_{\overset{\circ}{C}^{2\alpha}[0,L]}\le M\left(a,\delta,\sigma,\alpha,x^*,q,T\right)$$

$$\times\left(\|\varphi\|_{\overset{\circ}{C}^{2\alpha+2}[0,L]}+\|f\|_{C\left([0,T],\overset{\circ}{C}^{2\alpha}[0,L]\right)}+\|\rho\|_{C[0,T]}\right)$$

holds.

The proof of Theorems 1 and 2 can be given in a similar manner by the proof given in [8, Theorem 2.1 and Theorem 2.2].

References:

1 L. Yan, C.L. Fu, F.L. Yang, The method of fundamental solutions for the inverse heat source problem, Eng. Anal. Bound. Elem. 32 (2008) 216–222.

2 O.M. Alifanov, Inverse Heat Transfer Problems, International Series in Heat and Mass Transfer Series, Springer, 2011.

3 A. Ashyralyev, A.S. Erdogan, O. Demirdag, On the determination of the right-hand side in a parabolic equation, Appl. Numer. Math. 62 (2012) 1672– 1683.

4 V. Borukhov, P. Vabishchevich, Numerical solution of the inverse problem of reconstructing a distributed right-hand side of a parabolic equation, Comput. Phys. Commun. 126 (2000) 32 – 36.

5 J.R. Cannon, H.M. Yin, Numerical solutions of some parabolic inverse problems, Numer. Meth. Part. D. E. 6 (1990) 177–191.

6 M. Dehghan, Determination of a control parameter in the two-dimensional diffusion equation, Appl. Numer. Math. 37 (2001) 489–502.

7 V. Isakov, Inverse Source Problems, volume 34 of Mathematical Surveys and Monographs, American Mathematical Society, 1990.

8 A.S. Erdogan, A note on the right-hand side identification problem arising in biofluid mechanics, Abstract and Applied Analysis 2012 (2012) 1–25.