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COUNTERINTUITIVE QUESTIONS IN SCIENCE AND MATHEMATICS

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Abstract

Nowadays intuition continues to be of great interest to mathematics and science educators. The use of counterintuitive questions in math lessons is one way to captivate, interest, motivate, and stimulate intellectual development of teenage students. Additional benefits to students include opportunities for: metacognition, critical thinking, discovery learning, connections to real-life applications and history. These questions make students aware of the inadequacies of their own thinking by exposing them to situations whose outcomes are inconsistent with what they would expect. We hope that by using such problems in the classroom, teachers will increase students' interest in mathematics. The purpose of this article was to develop a theoretical model for the use of counterintuitive examples in mathematics at the school students.

Keywords: Counterintuitivity, Counterintuitive examples, teaching mathematics, secondary school math, logic and intuition.

Counterintuitive examples in science and mathematics

Problem-solving exercises is one of the most important part of the learning and teaching of sciences and mathematics. Researchers offer teachers a variety of effective learning paths. This article discusses one of these methods - increasing students' knowledge through counterintuitive questions. Previously, several researchers have used counterintuitive questions in their research. Nuri Balta & Ali Eryılmaz wrote in their article “Counterintuitive Dynamics Test” that counterintuitivity in science is a very powerful way to stimulate interest, motivate students to challenge their hidden misconceptions about science and promote thinking of a higher order. Moreover, when students sees the results of counterintuitive (CI) tasks, they are often surprised, amazed, puzzled [12].

Juan Miguel Campanario, in his article “Using Counterintuitive Problems in Teaching Physics” (1998) says: “I suggest the use of in-class problems that yield solutions that challenge students’ expectations or are worded in such a manner that students obtain a wrong solution by making some standard mistakes. Problems with counterintuitive solutions force students to think before rushing ahead into calculations. Soon they become aware of discrepencies between their existing ideas and the solution they find for the problem”.

Learning new concepts is limited the ability of students to overcome conflicting information [10]. Research suggests that learning mathematics and science do not simply involve acquiring new information, also changes existing conceptual frameworks and constructs new concepts alongside them. When children come to the classroom, some new concepts seem counterintuitive as they contradict these naïve theories. For example, in mathematics, early learning that positive numbers increase in magnitude ($1 < 2 < 3$) but the same integers as negative numbers decrease in magnitude ($-1 > -2 > -3$). Behavioural studies have demonstrated that solving counterintuitive mathematics and science problems requires one to inhibit an incorrect strategy and children and adolescents will have better inhibitory control. Prefrontal brain regions have also been activated more when experts compared to novices, solve counterintuitive problems, suggesting that they are able to inhibit intuitive responses [11].

Marshall Gordon (1991) in his article “Counterintuitive Instances Encourage Mathematical Thinking” said that intuition, experience and reason are the main ways through which human understand their environment and gain knowledge. Our intuition, which senses a situation immediately, has considerable weight, of course, with regard to what we believe and so deserves the attention of teachers and text book writers involved with mathematics education. Using intuition in teaching involves presenting mathematical examples that are contrary to common sense. For it is not only examples that counterintuitive that attract the attention of students, but also what was believed to be true turns out to be wrong. Such examples also help students challenge the habits of thought and practice so that they become better thinkers. By introducing students to math issues that challenge common sense and common practice, the teacher empowers them to better understand the need for exploration, reflection, and reasoning.

Examples of Counterintuitive questions

In his article, Campanario (1998) discussed six counterintuitive examples in physics, shared his experience of using these questions in the classroom. Here are some examples from his article.

Example 1. Friction Forces. An object of 10-kg mass is laid on a flat surface. A force F = 4 N is exerted on the object at a positive angle of 30^o with the horizontal surface. Assume = 0.5. Find the resultant force acting on the object. Compare this result with the resultant force of the object when F forms a negative angle of 30^o with the horizontal surface [9].

Apparent answer: Resultant force is directed to (pointing toward) the left.

Correct answer: Resultant force is equal to 0 (friction force is equal to applied force).

Students are so accustomed to careless methodology and mechanical calculations that they obtain wrong solutions to this problem. Given that the problem provides a friction coefficient, many take the wrong step of computing the friction force first. The rule they use states that Ff = N, where N is the normal force exerted by the surface on the block. However, N depends both on the weight and the vertical component of F. After doing the calculations, students realize (much to their surprise) that the horizontal component of the net force is directed toward the left. Thus, the net force is always against F and the block will move contrary to the applied force. Who can deny a very counterintuitive result? This example illustrates how students can discover on their own that there is something wrong with their ways of solving problems [9].

Example 2. Falling Bodies. A man is standing on a balance whose scale indicates a given weight P. Suddenly he squats with acceleration a. What value of weight, P' will the scale indicate while the man is squatting [9]?

Apparent answer: The P' weight is greater because while squatting the man is pushing down on the balance.

Correct answer: The P' weight is lower because while the man is squatting the center of gravity is being accelerated downwards.

A classic problem in almost every textbook involves computing the apparent weight of objects placed on balances in moving elevators. Most new physics students believe that while the man is squatting the scale will reflect a greater weight because he is "pushing down" on the balance. It may be necessary to encourage students to try the experience, because they may be hard to convince otherwise. However, because the squatting person is accelerated downward, the normal force (and thus, reaction or apparent weight) is of smaller magnitude [9].

Marshall (1991) in his article offers several examples in mathematics. For example, if teachers are using the following exercise while studying the topic of volume, the counterintuitive solution to the following problem can help them understand that the interface of algebraic and geometric elements can enrich their mathematical understanding.

Example 3. The problem is to determine the change in volume of a 15 x 18 x 22 (width by height by length) rectangular parallelepiped, when one of the dimensions is doubled [5].

Students' intuition generally leads them to claim initially that doubling the length (which has the greatest magnitude) has the greatest effect on the volume, whereas doubling the width has the least. They appear concerned when they determine that the volumes resulting from doubling any of the dimensions are all the same. Suggesting that they focus on the elements in the product they used to determine the volumes helps them to note that 2 can be factored out from each of the three products, so doubling any one dimension doubles the volume regardless of the relative sizes of each dimension:

$$(2x15)\ x18\ x\ 22 = 15\ x\ (2\ x\ 18)x\ 22 = 15x18\ (2\ X\ 22) = 2\ x\ (15\ x\ 18\ x\ 22).$$

This numerical explanation seems to be quite convincingly, but more fully understanding arises from the fact that the shape of the parallelepiped changes in each a business; namely, discussing what doubling each of the dimensions actually corresponds to Figure 1:

a) The actual doubling of the height creates second parallelepiped of the same size on top first.

b) In essence, doubling the width results in the second box next to the first.

c) Doubling the length of the effect puts the second box is behind the first.

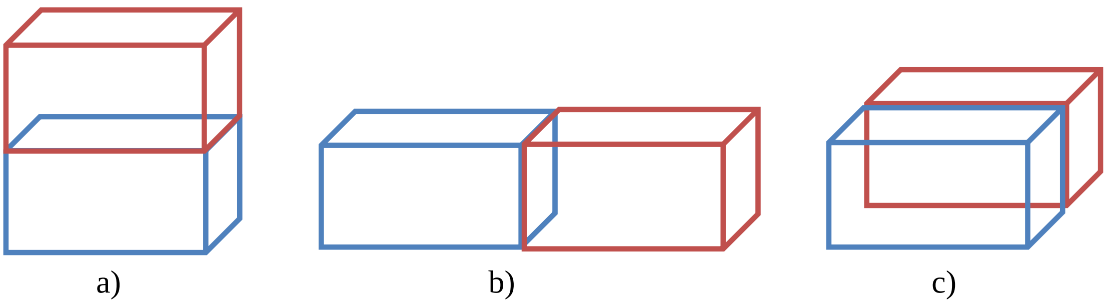

Figure 1. Doubling different sizes of a parallelepiped.

Wolfson G., in his article "Logic vs. Intuition" (2018) asks “Why do many people dislike mathematics?”. Because mathematics in any "obvious" situation forces you to prove something! This article provides several examples that show that our intuition can be wrong. Below are examples that show that proof is important!

Example 4. The watermelon peel is three times thinner than pulp (if there is a line from the center of the watermelon to its surface, three-fourth line will come to the pulp, and the fourth to the peel). How many times more pulp volume than peel [2]? Figure 3.

The intuition of students speaks three times, nine, etc. Now let's try to solve this problem. We assume that the watermelon has the shape of a perfect ball, inside it is a smaller ball. Let's say that the radius of a large ball is 4x, inside which is a ball with a radius of 3x.

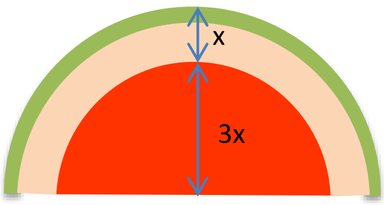

Figure 3. The size of the pulp and the peel of the watermelon

By condition, the thickness of the peel is exactly three times less than the thickness of the pulp. 4x-3x = x;

According to the formula for the volume of a sphere, we obtain that a sphere with a radius of 4x is equal to: $V_1 = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(4x)^3 = \frac{4}{3}\pi \cdot 64x^3$;

The volume of a sphere with a radius of 3x is: $V_2 = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(3x)^3 = \frac{4}{3}\pi \cdot 27x^3$;

The volume of watermelon peel is $V = V_1 - V_2 = \frac{4}{3}\pi \cdot 64x^3 - \frac{4}{3}\pi \cdot 27x^3 = \frac{4}{3}\pi \cdot 37x^3$

$$V = \frac{4}{3}\pi \cdot 37x^3$$

The volume of watermelon pulp is $V_2 = \frac{4}{3}\pi \cdot 27x^3$

Thus, the volume of the watermelon peel is not just less (3 or 9 times), but almost one and a half times more than the volume of the pulp.

Example 5. The globe was encircled with a ribbon tightly attached to the ground. Then the tape was increased by 1 meter and evenly distributed along the surface of the planet. Find the distance between the ground and the tape? Can we put a ball with a diameter of 15 cm in the resulting gap? (The distance around the Earth at the Equator, its circumference, is 40 000 kilometers) [2].

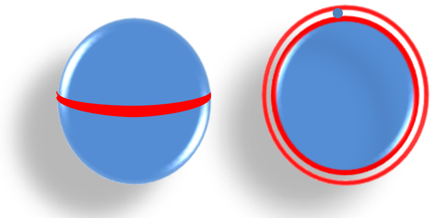

Figure5. The globe encircled with tape

Intuition tells that the difference will be very small and imperceptible. The length of the tape will be about 40,000 kilometers, so an increase of 1 meter is unlikely to be noticeable. And if further we evenly move the tape away from the ground, we get a microscopic gap.

Now let's start the calculations.

The length of the first tape $L_1 = 2\pi R_1$

R_1-the radius of the Earth. $R_1 = \frac{L_1}{2\pi}$

The length of the second tape is $L_1 + 1$. So, new radius $R_2 = \frac{L_1+1}{2\pi} = \frac{L_1}{2\pi} + \frac{1}{2\pi} = R_1 + \frac{1}{2\pi}$

The new radius is $\frac{1}{2\pi}$ meter larger than the old one. Since π ≈ 3.14, then $\frac{1}{2\pi} \approx 0.16$. This means that the gap is approximately 0.16 meters, or 16 centimeters.

If the resulting gap is 16 cm, then yes, we can place a ball with a radius of 15 cm.

In article “Using counterintuitive problems” by Maylone (2000), recommends using counterintuitive math problems to help maintain active student participation in their mathematics education. Counterintuitive problems are not necessarily trick questions; rather, prior to discussion, their answers seem to defy common sense. This is one of their main advantages: these problems cause short-term confusion; then discussion; and finally understanding. The hallmark of counterintuitive problems is that even after the solution has been verified, it is difficult to ignore this quiet voice that continues to whisper, "But this answer cannot be correct!" This dissonance is part of the enjoyment of using counterintuitive problems, as teens tend to defend what they think is right. Even restrained students will be drawn into these discussions. Motivated students not only will pay attention, but will think and reflect, and perhaps most importantly, will continue to be willing to facilitate even after they have being proved wrong.

Any middle school teacher can confirm that adolescents enjoy debating. Discussion and debate engage students and promote new thinking, reinforce learning, and offer important social erudition opportunities. Maylone (2000) In his article, shared the following examples with high school math teachers. These examples will help students become more active during the lesson. In this way, students can improve their mathematical skills. The following examples can motivate a basic belief in geometry and probability, spark class discussion, and thereby open students' interest in mathematics.

Example 6. Workers of an ancient civilization attempted to roll a stone block along a series of logs. The figure below illustrates the situation. The diameter of each log is 25 cm. If each of the underlying logs makes one complete roll forward, how many centimeters in relation to the ground will the block move [6]? (Use 3.14 for pi.)

Students are likely to propose an answer of 78.5 cm, the circumference of one log. Students should be given credit for knowing and applying the circumference formula and for demonstrating an understanding of the meaning of pi. They should also be tactfully informed that they are incorrect. An explanation follows.

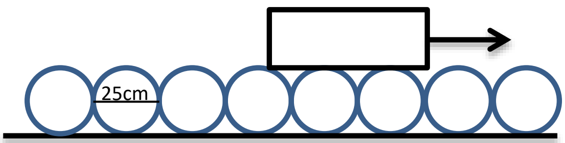

Figure 6. The Rolling the Block.

Rolling over the tops of 25 cm diameter logs as they make one turn will move the block $L = 2\pi R = 25\pi = 78.5\ cm$. This situation fails to take into account that the logs themselves also move forward by 78.5 cm. The forward progress of the block is combined with the forward motion of the logs themselves, so the block will roll 78.5 + 78.5 =157 cm for each log.

Example 7. At noon (12:00), a careless cat falls into a six meter deep hole, which has very slippery sides. Immediately, the cat tries to climb out and makes progress at this rate: it climbs up two meter per minute, and then slips back one meter over the next minute. What time will the cat emerge from the hole [6]?

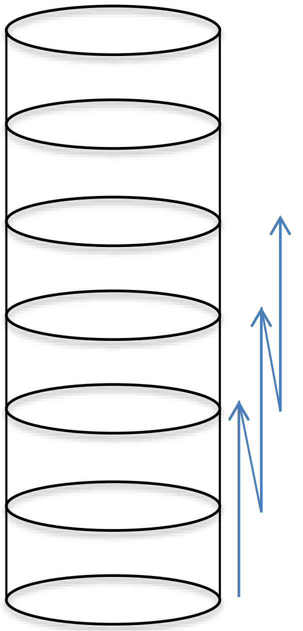

Figure 7. The Careless Cat

Encourage this tactic, as it is an appropriate problem-solving strategy. Students' sketches may look something like figure 6. The zigzags represent the cat's upward, then downward, progress and regression. One eighth grader who was working on this problem stated, "Going up two feet in a minute, then going down one foot the next is like going up one foot every two minutes." At that rate, the cat would emerge from the six-foot hole at 12:12 p.m. However, once the cat reaches the top of the hole at 12:09 it no longer falls back. Continuing with the zigzag pattern suggested in figure 6 may clarify this problem for skeptical students.

Such examples are hotly debated not only in mathematics lesson, but also among Internet users. One such example is the “Potato paradox” and the “Perfect parallelogram”.

Example 8. Fred brings home 100 kg of potatoes, which (being purely mathematical potatoes) consist of 99% water. He then leaves them outside overnight so that they consist of 98% water. What is their new weight [1]?

When solving this problem, students immediately think that if a potato loses 1% of its weight, then 100 kg of potatoes will be 90-95 kg. But the answer is quite different

The weight of water in the fresh potatoes is $\mathbf{0.99 \cdot 100}$. If $\boldsymbol{x}$ is the weight of water lost from the potatoes when they dehydrate then $\mathbf{0.98 \cdot (100 - \boldsymbol{x})}$ is the weight of water in the dehydrated potatoes. Figure2. Therefore:

$\mathbf{0.99 \cdot 100 - 0.98(100 - \boldsymbol{x}) = \boldsymbol{x}}$

Expanding brackets and simplifying

$\mathbf{99 - (98 - 0.98\boldsymbol{x}) = \boldsymbol{x}}$

$\mathbf{99 - 98 + 0.98\boldsymbol{x} = \boldsymbol{x}}$

$\mathbf{1 + 0.98\boldsymbol{x} = \boldsymbol{x}}$

Subtracting the smaller $\boldsymbol{x}$ term from each side

$\mathbf{1 + 0.98\boldsymbol{x} - 0.98\boldsymbol{x} = \boldsymbol{x} - 0.98\boldsymbol{x}}$

$\mathbf{1 = 0.02\boldsymbol{x}}$

$\mathbf{50 = \boldsymbol{x}}$

And the dehydrated weight of the potatoes as: 50kg.

$\mathbf{100 - \boldsymbol{x} = 100 - 50 = 50}$

Figure 2. Potato paradox.

In 100 kg of potatoes, 99% water (by weight) means that there is 99 kg of water, and 1 kg of solids. This is a 1:99 ratio.

If the percentage decreases to 98%, then the solids must now account for 2% of the weight: a ratio of 2:98, or 1:49. Since the solids still weigh 1 kg, the water must weigh 49 kg to produce a total of 50 kg.

Example 9. What we get if we draw any a four-sided polygon and connect midpoint on each side of the polygon together. Figure 9. (It can be of any size, irregular shape, concave, convex, etc. The main thing is that it has four corners and straight sides.) [4].

Below are a few of the possible choices for quadrangles. Students look at the squares in the picture and immediately think different shapes come out from different quadrangles. But the answer will come as a surprise to the students.

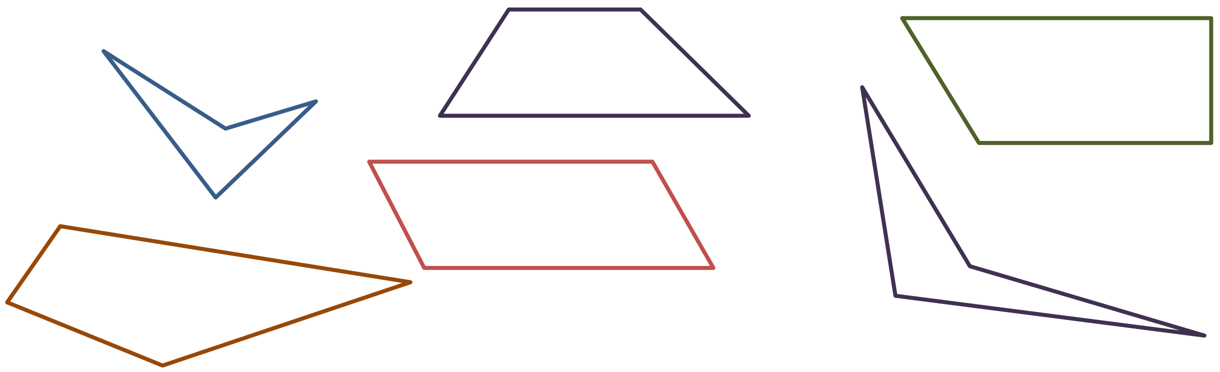

Figure 9. The four-sided polygon

You will end up with a perfect parallelogram every time.

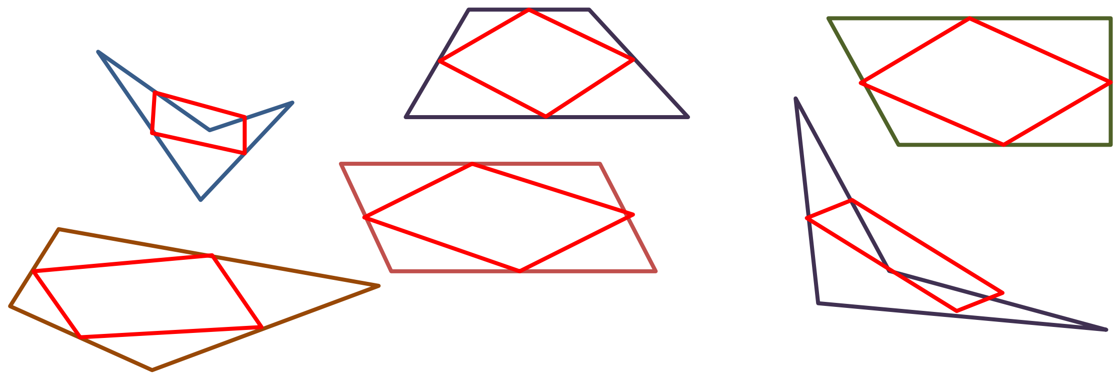

Figure 10. Perfect parallelogram.

Conclusion

The article explains the use of counterintuitive examples in mathematics, geometry, and physics lessons. By the results of other studies above, it can be seen that when such questions are applied to middle school students in the classroom, students' understanding of the lesson is higher. It also helps students develop a passion for math and science by having a lively discussion during class. When teachers use such examples, it is observed that students look for a complete way to prove some problems, even after finding a solution to the problem. Use the counterintuitive examples in the classroom, as shown in this article, to contribute to middle-school students' meaningful thoughts and provide a framework or substantive, intellectual discussion

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