Methodology of solving complex problems associated with an isosceles triangle

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List of Abbreviations

STEM - Science, Technology, Engineering, and Mathematics
IBL - Injure-Based Learning
ICT - Information and Communication Technologies
EFA - Exploratory factor analysis

ABSTRACT

This study examines the impact of solving complex problems related to isosceles triangles on the development of students' geometric thinking and academic achievements. The research project included a performance test and an attitude test. The study was conducted with 24 students of the seventh grade, who were divided into two groups: experimental and control. The performance test, which included a pre-test and a follow-up test, measured the students' performance before and after the intervention.

In the control group, students received traditional instruction, while the experimental group studied problem-based learning (inquiry-based learning). Analysis of the results of the test on academic achievement showed that students in the experimental group demonstrated significantly higher performance in mathematics compared to the control group. This demonstrates the effectiveness of the IBL methodology in improving learning outcomes. A survey was used to assess attitudes towards mathematics. Analysis of the survey showed a significant improvement in attitude to the subject after the intervention.

The results showed a significant improvement in students' ability to solve geometric problems in the experimental group. Conclusions emphasize the effectiveness of structured problem solving and conceptual understanding in improving logical reasoning and academic outcomes.

Key words: isosceles triangle, geometric thinking, complex tasks, teaching methodology.

АҢДАТПА

Бұл зерттеу жұмысы теңбүйірлі үшбұрыштарға байланысты күрделі есептерді шешудің оқушылардың геометриялық ойлау қабілеттері мен академиялық жетістіктеріне әсерін қарастырады. Зерттеу барысында оқушылардың жетістігін анықтауға арналған тест және математикаға деген қатынасын бағалауға арналған сауалнама қолданылды. Зерттеу 7-сыныптың 24 оқушысы арасында жүргізілді, олар екі топқа бөлінді: эксперименттік және бақылау топтары. Жетістік тесті (алдын ала және соңғы тесттер) интервенцияға дейін және кейін оқушылардың білім деңгейін бағалады.

Бақылау тобындағы оқушылар дәстүрлі оқыту әдісі бойынша білім алса, эксперименттік топ сұраққа негізделген оқыту әдістемесі (inquiry-based learning) арқылы білім алды. Академиялық жетістіктерге арналған тест нәтижелерін талдау эксперименттік топтағы оқушылардың математикадан жоғары нәтижелер көрсеткенін көрсетті. Бұл зерттеу IBL әдістемесінің оқу нәтижелерін жақсартуда тиімді екенін дәлелдейді. Математикаға деген қатынасты бағалау үшін сауалнама қолданылды. Сауалнама нәтижелері интервенциядан кейін пәнге деген қатынастың едәуір жақсарғанын көрсетті.

Зерттеу нәтижелері эксперименттік топтағы оқушылардың геометриялық есептерді шешу қабілетінің айтарлықтай артқанын көрсетті. Қорытындылар құрылымдалған есеп шығару мен ұғымдық түсінудің логикалық ойлау мен оқу жетістіктерін жақсартудағы маңыздылығын атап көрсетеді.

Кілт сөздер: теңбүйірлі үшбұрыш, геометриялық ойлау, күрделі тапсырмалар, оқыту әдістемесі.

АННОТАЦИЯ

В данном исследовании изучается влияние решения сложных проблем, связанных с равнобедренными треугольниками на развитие геометрического мышления студентов и академические достижения. Проект исследования включал в себя тест на успеваемость и тест на отношение к предмету. Исследование было проведено с участием 24 учеников седьмого класса, которые были разделены на две группы: экспериментальную и контрольную. Тест на успеваемость, включающий, в себя предварительное тестирование и последующее тестирование, измерял успеваемость учащихся до и после вмешательстава.

В контрольной группе учащиеся получали традиционное обучение, в то время как экспериментальная группа занимались методикой основанный на решении проблем (inquiry-based learning). Анализ результатов теста на академическую успеваемость показал, что учащиеся экспериментальной группы продемонстрировали значительно более высокие показатели по математике по сравнению с контрольной группой. Это свидетельствует об эффективности методики IBL в улучшении учебных результатов. Для оценки отношения к математике использоваться опрос. Анализ опроса показал существенное улучшение отношения к предмету после проведённого вмешательства.

Результаты продемонстрировали значительное улучшение способности студентов решать геометрические проблемы в экспериментальной группе. Выводы подчеркивают эффективность структурированного решения проблем и концептуального понимания в улучшении логического рассуждения и академических результатов.

Ключевые слова: равнобедренный треугольник, геометрическое мышление, сложные задачи, методика обучения.

INTRODUCTION

Made up of two equal length sides, isosceles triangles provide a unique mix of geometric characteristics and challenges in mathematical problem-solving. Often, these challenges are connected to trigonometry, as students battle algebraic transformations, particularly with unknowns in numerators or denominators (Ngu & Phan, 2020). Algebraic transformation skills are crucial for handling such problems; one can observe the similarity and difference between trigonometric functions (Ngu & Phan, 2020). Moreover, the intrinsic symmetry of isosceles triangles can sometimes conceal the application of traditional geometric theorems and trigonometric identities, hence requiring a thorough understanding of their features. Sometimes isosceles triangles are found in more complicated geometric shapes as well, including compound shapes, quadrilaterals, and other triangles. Therefore, the right computation of areas, perimeters, and other basic geometric values in these challenging activities relies on how exactly they are specified and applied (Retnowati & Fadlila, 2023).

The requirement to combine several geometric ideas and theorems causes many of the difficulty in isosceles triangle issue solution. For example, issues could call for the concurrent use of the Pythagorean theorem, angle bisector theorem, and characteristics of related triangles, hence requiring a thorough knowledge of geometric ideas. Moreover, slanted coordinate systems can aggravate these challenges, especially when examining vector components linked to forces or displacements inside the triangle (Mikula & Heckler, 2014). The combination of coordinate geometry with Euclidean geometry in the framework of isosceles triangles calls for a solid basis in both fields.

In many engineering and physics applications, isosceles triangles are a basic building block; knowledge of their characteristics is crucial for structural analysis, statics, and dynamics. For instance, in structural engineering, truss designs often employ isosceles triangle shapes since the distribution of loads and stresses depends quite much on the triangle's form. The desire to discover different ways to teach mathematics to raise student performance motivated the study (Mosese & Ogbonnaya, 2021). Geometrical forms seem practically everywhere, so geometry helps education for the growth of critical thinking and problem solving (Altakhyneh, 2018; Serin, 2018). Architects need to be good at geometry (Liapi, 2002; Mifetu, 2023). A mathematical subject called geometry evaluates mathematical creative thinking capacity and it develops logical and methodical thinking (Sahliawati & Nurlaelah, 2020). Students are exposed to geometric ideas like lines, planes, and spaces before they even start school, so Geometry is a branch of mathematics they are more likely to grasp than other branches.

In geometry lessons in secondary school, it is important to teach students how to solve complex problems. In particular, tasks related to an isosceles triangle contribute to the development of logical thinking among students and demonstrate the possibility of applying their mathematical knowledge in real life. Geometry is not only a theoretical knowledge, but also one of the practical disciplines aimed at solving problems that we face in real life. In this context, President of the Republic of

Kazakhstan Kassym-Jomart Tokayev has repeatedly spoken in his messages about improving the quality of the education system and its viability.

So, in his speech, the President noted: "Our main task is to create an economy based on advanced knowledge and high technologies. To achieve this goal, first of all, high-quality education is needed." This requires special attention to the field of education, especially to subjects of mathematical and technical orientation.

In addition, in the 2021 Message, Kassym-Jomart Tokayev noted: "Modernization of technical and vocational education is our main task. We must give priority to mathematics, engineering education, and new technologies," she stressed the relevance of teaching mathematics and technical sciences. In this regard, the use of complex problems related to an isosceles triangle in geometry lessons taught in schools encourages students to apply theoretical knowledge in life, develops their analytical and logical thinking.

Overcoming Challenges in Isosceles Triangle Problems. Effectively tackling the difficulties isosceles triangle problems provide calls for a multi-faceted strategy emphasizing increasing basic knowledge, creating problem-solving techniques, and promoting critical thinking abilities (Ayan & Isiksal-Bostan, 2019). Injure Based Learning motivates students to approach critical thinking and problem-solving practically (Chau et al., 2021). Emphasizing the need of visualization and geometric intuition helps students to create correct diagrams and mentally move forms to better grasp the issue. Promoting the use of algebraic methods to solve geometric problems is another important factor (Ayan & Isiksal-Bostan, 2019). Including practical uses into math education is based on various fundamental educational ideas stressing IBL, student involvement, and the growth of critical thinking abilities (Chau et al., 2021). This means learning to use trigonometric identities, solve systems of equations, and manipulate equations such that students may convert geometric relationships into algebraic representations and the other way around. Injure Based Learning is a successful approach whereby students work on long-term projects applying mathematical ideas to address practical issues (Chau et al., 2021).

One cannot understate the significance technology plays in improving geometry education and learning, especially in relation to isosceles triangles. Interactive geometry tools like GeoGebra let students dynamically investigate geometric characteristics, create precise diagrams, and see transformations, hence promoting a better knowledge of ideas (Daulay et al., 2021). Such technologies allow students to test hypotheses, see the consequences of parameter changes, and control geometric objects, hence enabling a more interesting and intuitive learning environment. Moreover, educational technology can enable cooperative learning by letting students collaborate on problem-solving assignments, exchange ideas, and benefit from one another (Santos-Trigo et al., 2021). Studies are required that not only contrast student involvement in light of particular teaching practices with other instructional strategies, but also investigate the relationship between these teaching practices and student problem solving under instruction stressing "integrating connective components of teacher–student relationships with connective elements of instruction" (Yusuf et al., 2021). Active involvement and prior math performance are highlighted in this paper as factors improving problem-solving skills (Yusuf et al., 2021).

When solving problems related to isosceles triangles, many students make mistakes due to a lack of understanding of the relationships between the elements of the triangle and the inability to apply the knowledge gained in practice. The teacher's task is to teach students how to correctly analyze the conditions of a problem, use the properties of an isosceles triangle, and apply theorems to find unknown elements.

Real-world examples and uses of isosceles triangles can be helpfully used by the study to increase student involvement and drive. Students are more likely to value the relevance and use of their studies by linking mathematical ideas to daily life (Chau et al., 2021). For example, drawings from architecture, engineering, and art can show how isosceles triangles are employed in structural designs, bridge building, and artistic compositions (Mikula & Heckler, 2014). Future studies could also look at how well various teaching strategies help students solve isosceles triangle issues. This could mean contrasting conventional lecture-based techniques with more engaging, student-centered ones like problem-based or inquiry-based learning. Word issues relate to the evolution of abstract formal structures and mathematics (Yusuf et al., 2021). Included in curricular papers and assessment tools, word problems are part of formal mathematics instruction (Chau et al., 2021). A common method to show kids how to apply math to solve practical problems is through mathematical word problems, which is also a vital skill for them.

Important measures of success are student motivation and involvement (Chau et al., 2021). Research has demonstrated that more student involvement results in more academic success (Schukajlow et al., 2023; Serin, 2018). In the twenty-first century, problem-solving teaching methods have changed to an insight-based problem-solving approach (Acquandoh et al., 2022). A key field of mathematics education has become students' ability to solve problems. Problem-solving has been a major component of teaching and studying mathematics during the last few decades (Hafizi & Kamarudin, 2020). By means of careful consideration of these elements and use of suitable pedagogical approaches, educators can create a more interesting and efficient learning environment, hence reducing disengagement and promoting a better knowledge of mathematical ideas (Schukajlow et al., 2023). Teachers can assist students build confidence in their problem-solving skills, lower anxiety, and promote a growth mindset by establishing a supportive and motivating classroom environment (Mailisman et al., 2020). This means commending effort, offering helpful criticism, and designing chances for students to present their answers and benefit from their errors (Chau et al., 2021; Rahmah, 2017).

Controversy in Isosceles Triangle Problem Solving. One source of debate in resolving isosceles triangle problems is the suitable degree of formalism and rigor in mathematical proofs. Emphasizing conceptual knowledge and geometric reasoning over rigorous following of formal proof methods, some teachers support a more intuitive and casual approach. Younger pupils or those who find abstract ideas challenging may find this strategy especially useful since it lets them grasp the fundamental ideas more deeply without being mired in technicalities. Some teachers, nevertheless, contend that cultivating pupils' logical reasoning abilities and equipping them for more advanced mathematics courses depend on a strict attitude to mathematical proofs. Active discovery produces questions and answers that closely fit real-world applications since students are urged to find issues, theorize answers, and examine data in ways that reflect professional practices (Chau et al., 2021). Generally speaking, problem-solving has been acknowledged as a way to advance thinking skills and an important learning process in the mathematics curriculum (Yusuf et al., 2021). Active participation in the mathematical problem-solving process helps students acquire mathematical ideas, techniques, and skills as well as alternative modes of thought and action.

Another subject of continuous discussion is mathematics instruction using technology. Some teachers fear that although technology might improve visualization, investigation, and problem-solving, if not handled correctly it could also result in a shallow grasp of ideas. To guarantee that it supports and improves conventional teaching approaches, not replaces them entirely, effective integration of technology calls for deliberate planning and educational considerations (Santos-Trigo et al., 2021). Because they provide pupils chances to grow their own knowledge of mathematics, mathematical activities are rather essential for mathematics education. Mathematical challenges allow teachers to either introduce new subjects to students or provide them chances to use what they have already acquired. Through problem-solving, teachers can assess and inspire students' capacity to apply mathematical ideas in original and interesting ways (Salim & Tiawa, 2014). Through problem-solving, teachers can assess and support students' capacity to apply mathematical ideas in creative and interesting ways (Salim & Tiawa, 2014). In mathematics, problem-solving is fundamental, (Alfayez et al., 2022). Mathematics depends on the capacity to answer problems (Alfayez et al., 2022). It is managing challenging or unknown circumstances utilizing mathematics knowledge, skills, and understanding. Mathematics educators have employed several strategies in their teaching to enable students to improve as problem solvers (Isnawati et al., 2021).

Mathematical knowledge acquisition could change from one class of pupils to another (Febrilia & Nissa, 2019). Given the various teaching techniques, it is therefore essential to find whether or not secondary school pupils' mathematics performance varies. The results of this study showed that students exposed to problem-solving strategy outperformed their peers exposed to traditional approach (Syaiful et al., 2020). Students should be able to actively pursue new mathematical information and employ the skills they presently have. Establishing a learning environment that minimizes any possible tension or anxiety and encourages interest and excitement for mathematics is another aim (Febrilia & Nissa, 2019). Teachers can make learning more pertinent and interesting by using real-world examples, hands-on activities, and group projects (Febrilia & Nissa, 2019). Teachers should also ensure they provide chances for students to collaborate, discuss their ideas, and benefit from each other's knowledge (Yerizon et al., 2019). Teachers are supposed to provide pupils a lot of practice in contextual issues (Ningrum et al., 2019).

Significance of Isosceles Triangle Problem Teaching. Isosceles triangle problems are taught in mathematics education with utmost importance since they help students' cognitive growth and problem-solving skills in many ways (Ezeddine et al., 2023). More sophisticated geometric ideas and applications are built on a foundation of knowledge of the characteristics and relationships inside isosceles triangles. Furthermore, mastery of isosceles triangle problem solving develops critical thinking abilities like logical reasoning, spatial perception, and deductive analysis. This covers the capacity to evaluate and consider the problem-solving process itself as well as to adapt and apply various techniques to address challenges (Hobri et al., 2018). Problems that come up in actual situations should be identified and addressed using mathematical modeling (Chau et al., 2021). A basic talent that is much appreciated in many spheres of life is the capacity to solve mathematical challenges (Santos-Trigo et al., 2021). Success in STEM disciplines as well as in daily decision-making and critical thinking depend on mathematical problem-solving abilities (Yuristia & Musdi, 2020). Students' ability to solve mathematical problems is a key predictor of their academic and professional performance, hence this ability must be developed and strengthened (Zulfa & Andriyani, 2023).

Students who work on isosceles triangle issues learn to apply mathematical ideas to actual scenarios, hence promoting a better respect of the relevance and utility of mathematics. Inherently symmetrical and special, isosceles triangles offer a rich background for investigating geometric relationships and honing problem-solving abilities. A basic talent that is quite appreciated in many areas of life is the capacity to solve mathematical problems (Kharomah & Abduh, 2023). Given that students' ability to solve mathematical problems is a key predictor of their academic and professional performance, this ability must be developed and strengthened ("Analysis of Mathematical Problem Solving Ability Viewed from Student Learning Style," 2020; Battista et al., 2018). Moreover, the study of isosceles triangles improves students' spatial thinking skills and geometric intuition, which are crucial for success in many sectors including engineering, architecture, and computer graphics. Students encounter changes to their information in the learning process by adding parts, enhancing, evolving, or altering the old knowledge (Setyaningsih et al., 2019).

Some have argued that identifying patterns is a key strategy for addressing mathematical problems and can help foster critical thinking (Fadiana et al., 2018). Effectively solving mathematical problems depends on the ability to think critically, which is also quite important (Herawati & Amelia, 2021; Yanto, 2019). A person who can think critically will always investigate and question whether the encounter he meets relates to what he already knows (Maulidiya & Nurlaelah, 2019). Furthermore, when students are instructed on how to understand issues, build mathematical models, offer solutions, and analyze the results, their problem-solving skills are strengthened (Rahmah, 2017). Students' mathematical creative thinking capacity is a habit that may be developed (Istiqomah et al., 2018). Based on intuition but yet in awareness, mathematical creative thinking is a mix of logical and divergent thinking (Rochmad et al., 2019). Thus, we require a learning approach that can energize students, so they can help others to simplify the implementation of spatial mathematics connected to three-dimensional material, and expand students' mathematical knowledge, as well as have abilities in learning autonomy (Isyrofinnisak et al., 2020; Rochmad et al., 2019; Saputri et al., 2020). Mastering isosceles triangle problems helps students build a strong basis in geometry and hone necessary problem-solving abilities that will help them in future mathematics projects and outside (Ayan & Isiksal-Bostan, 2019). The National Mathematics Advisory Panel study underlined the need of not just computational fluency but also conceptual knowledge and problem-solving abilities (Yusuf et al., 2021). Consequently, good teaching techniques have to include chances for students to interact with difficult issues calling for original and innovative application of their knowledge (Chau et al., 2021).

The study of triangles, especially geometry, provides a rich environment for developing mathematical creativity and problem-solving abilities (Sahliawati & Nurlaelah, 2020). Geometry, which includes spatial reasoning, visualization, and the investigation of links and patterns, provides a natural stage for the growth of mathematical creativity. Mathematics is good for encouraging creativity (Nadjafikhah & Yaftian, 2013). Moreover, even if math is not usually linked with it, mathematical education should be seen as a chance for growing creativity (Švecová et al., 2014). Geometric challenges inspire pupils to think creatively, investigate many angles, and create original answers. Creative thinking is seeing things from various angles, hence mathematical creativity guarantees general mathematical development (Sahliawati & Nurlaelah, 2020). Teachers can create a classroom atmosphere that respects innovation and intellectual curiosity by motivating students to investigate several possibilities and defend their logic. Through creativity, students can create new ideas connected to work or daily life and address challenges in unconventional ways (Isyrofinnisak et al., 2020; Nadjafikhah & Yaftian, 2013). Moreover, including technology into the instruction of geometry can give pupils interactive tools and simulations that improve their knowledge and encourage innovative investigation.

Actively including pupils in mathematical exercises will provide them a chance to think creatively and inventively (Zakeri et al., 2023). Active student participation in mathematics increases their chances of developing a deeper knowledge of the subject and a higher respect of its beauty and usefulness (Mann, 2006). Actively including pupils in mathematical exercises can improve their problem-solving abilities, increase their confidence, and help them to have a good attitude towards mathematics. Teachers in the classroom can foster creativity by motivating students to question, investigate other answers, and defend their rationale. Though teachers have to overcome obstacles to guarantee proper integration, including real-world applications into math instruction has notable advantages (Chau et al., 2021). Using addition, subtraction, multiplication, and division, teachers used real-life situations like budgeting a weekly allowance or creating a shopping list (Chau et al., 2021). This strategy not only improves students' problem-solving skills but also increases their love of mathematics in everyday life.

Teachers can assist students to recognize the relevance and usefulness of mathematics by motivating them to apply mathematical ideas to actual circumstances (Ayan & Isiksal-Bostan, 2019).

All of these factors support students' creative performance in a creative educational environment, which includes difficult tasks, freedom, learning-oriented motivation, trust, and debate. Students are more likely to acquire a deeper knowledge of the content and a higher respect for its beauty and usefulness when they are confronted with challenges requiring innovative thinking and application of their knowledge in novel ways (Katz & Stupel, 2015). Teachers should support pupils' logical skills, acknowledge and develop their innovative ideas (Er, 2023). Emphasizing difficult assignments, supporting independence, stressing learning-oriented motivation, establishing trust, and supporting discussion help teachers to inspire mathematical creativity (Rodrigues et al., 2018; Silver, 1997). Teachers may build a classroom atmosphere that respects creativity and intellectual curiosity by allowing kids the opportunity to investigate various techniques and motivating them to learn from their failures.

Their enthusiasm and performance can be greatly enhanced by actively including children in the learning process and linking mathematical ideas to practical problems (Herges et al., 2017). While those who are intrinsically driven study because they are personally pushed by the content or just enjoy it, those who are extrinsically motivated sometimes study for a test to obtain a good mark (Herges et al., 2017). Taking demanding math classes helped students develop intrinsic motivation; completing a rigorous assignment helped them believe in their math skills, which in turn drove their desire to learn more (Buket Özüm Bülbül, 2021). Teaching and teachers shape student drive towards learning (Schukajlow et al., 2023). By creating a welcoming and inclusive classroom, giving their students chances to achieve, and providing them consistent feedback and assistance, teachers can excite and inspire their pupils (Herges et al., 2017; Isyrofinnisak et al., 2020). Furthermore, educators ought to provide chances for pupils to collaborate, exchange ideas, and benefit from one another's knowledge (Gordon, 2019; Haylock, 1985).

To thoroughly investigate the complexities of isosceles triangles, emphasizing their special characteristics and the difficulties they pose in mathematical problem-solving, therefore improving mathematical skills among pupils. The study of isosceles triangles, with their natural symmetry and special qualities, provide a rich background for investigating geometric ideas and honing problem-solving abilities (Hafizi & Kamarudin, 2020). Practicing in class calls for quality time set aside (Febrilia & Nissa, 2019). The teacher's job is to run the class, ask questions, offer tools, and help the students build answers so the teacher can track their involvement and offer direction as required. Dealing with these issues calls for a multifaceted strategy combining creative teaching methods, focused interventions, and a supportive learning environment.

Moreover, the study of isosceles triangles offers a great chance to improve students' geometric intuition, spatial thinking, and problem-solving skills, therefore equipping them for more complex mathematical ideas and practical uses (Chau et al., 2021). Understanding more complicated geometric shapes and relationships is strongly founded on mastering the characteristics of isosceles triangles.

Teachers can assist children in overcoming obstacles and growing a better respect for the beauty and usefulness of geometry by means of well planned exercises and teaching. Students can deepen their knowledge of geometric ideas and improve their problem-solving abilities by investigating the links between angles, sides, and areas inside isosceles triangles. Solving different mathematical problems depends on knowledge of angle and side relationships inside triangles (Zubainur et al., 2020). Teachers should motivate kids to investigate several ways to solve isosceles triangle-related issues, hence promoting innovation and critical thinking.

Exploring geometric relationships inside isosceles triangles helps students to develop their spatial reasoning and visualization abilities, which are required for advanced problem-solving.

Effective teaching and learning in geometry education depend on knowledge of prevalent misunderstandings (Salifu, 2021). Dealing with widespread misunderstandings about isosceles triangles calls for focused teaching and exercises emphasizing the particular areas of student difficulty (Ayan & Isiksal-Bostan, 2019). Teachers can assist students in developing a more correct and strong knowledge of isosceles triangles by means of direct attention to these problems and by giving them chances to rectify their misconceptions (Prehatiningsih & Suparno, 2019).

The natural intricacy of the topic causes students in geometry to find abstract ideas difficult (Battista et al., 2018). Many would-be preschool instructors lack fundamental knowledge in geometry, thereby stressing the need of improved geometrical knowledge (Markovits & Patkin, 2020). Proportional thinking is used in the vital learning fields of geometry and measurement (Ayan & Isiksal-Bostan, 2019). By establishing a learning environment where knowledge of structures and mathematical skills prevails, teachers can help students with low mathematical beliefs (Skilling et al., 2021).

The main focus of research is on resolving mathematical issues connected to isosceles triangles, particularly those in proportional reasoning within geometry and measurement (Ayan & Isiksal-Bostan, 2019). Students' struggles with proportional reasoning issues in geometry and measurement underscore the necessity for thorough research (Ayan & Isiksal-Bostan, 2019). Daily life is directly tied to the study of geometry, which helps pupils to better grasp geometric ideas (Anista & Marsigit, 2020).

Relevance of the study. The study of isosceles triangles included in the 7th grade geometry course plays an important role in the formation of students' spatial thinking, logical analysis skills and problem solving skills. The isosceles triangle, with its unique properties, serves as an element that serves as a basis for developing ideas about symmetry, the connections between walls and corners, and preparing for the study of more complex topics.

The subject for research. The methodology for solving complex problems related to an isosceles triangle in geometry lessons in high school.

The object for research.The process of developing mathematical thinking and creative abilities of high school students in the process of solving complex problems related to an isosceles triangle in geometry lessons.

Aim of the research. Improving students ' skills in solving complex problems related to isosceles triangles.

Research question. How does solving complex problems related to isosceles triangles affect the formation of geometric thinking of students and their academic performance?

Hypothesis. If in high school the methodology of solving complex problems related to an isosceles triangle is used, then the geometric and analytical thinking of students develops, and the skills of applying mathematical knowledge in life increase.

Objectives of the study

1. Determination of the theoretical basis for solving complex problems related to isosceles triangles

2. Development of a new methodology for isosceles triangles

3. Experimental verification of the effectiveness of the methodology, processing the results

Special tasks that develop proportional thinking and the ability to represent objects in space, especially when combined with creative teaching methods, help students better cope with complex challenges on isosceles triangles. This in turn strengthens their mathematical skills and increases their self-confidence.

The premise is that modern methods in class would improve the learning experience of the student and produce general better academic outcome (Bakar & Ismail, 2020). Student-centered projects link math to reality via Injure Based Learning. This approach promotes teamwork, critical thinking, and problem-solving, hence improving knowledge and pleasure (Ayan & Isiksal-Bostan, 2019). Students who grasp geometry can properly grasp real-world scenarios, examine relationships, and solve difficulties.

Mathematical problem solving is based on proportional reasoning (Yusuf et al., 2021). Completing practice-based training helped future instructors to increase their proportional thinking (Pişkin Tunç & Çakıroğlu, 2022). They usually used a small number of techniques to solve issues and applied algebraic processes before the instruction without linking significance (Pişkin Tunç & Çakıroğlu, 2022). Prospective instructors are said to have a highly vital competence in problem solving.

For teachers to develop methods in helping students, they must grasp the techniques of problem-solving. Teachers can use it in real life using problem-solving techniques (Bahri et al., 2021). Teachers using efficient teaching techniques should guide students for better knowledge (Hoth et al., 2022). Learning about ratios and proportions might be difficult for many students (Andini & Jupri, 2017). Single ratio, step-by-step increase and cross multiplication are some of the most common ways to solve problems on relation and proportion (Setyaningsih et al., 2019). Although cross-multiplication allows for a quick answer, students often use it mechanically, not understanding how the proportion itself works (Öztürk et al., 2021). According to observations of Akara (Setyaningsih et al., 2019), pupils often solve such problems from textbooks by cross-multiplication. Focusing on the multiplicative links between values helps one to teach proportional thinking. It means helping pupils to identify and articulate the consistent ratio between two values.

Proportional thinking is the mental comparison of quantities multiplicatively and the resolution of proportional problems in many practical settings and activities (Ayan &

Isiksal-Bostan, 2019). It also underlines the need of understanding the several facets of proportional situations. Students who battle ratio and proportion are said to be battling multiplicative thinking. Even if they encounter proportions in daily life, students can find it difficult to grasp the proportional link of a situation (Mudrika et al., 2024). In one of the studies, it was found that students find it difficult to understand how many pairs in proportion should be multiplied or divided by the same number without any changes in the equation (Kohen & Orenstein, 2021). Errors in the proportional reasoning problems are the result of teaching students processes without understanding the concepts on which they were based.

Real situational problems often involve proportional thinking in geometrical and measuring problems or their properties. Functional reasoning helps to understand and apply the natural relationships in tasks, whereas proportional reasoning is based on arithmetic operations for solving problems where an equal relationship is necessary (Akatugba & Wallace, 2009). Understanding complex mathematical subjects and scientific ideas, including volume, speed and temperature, requires proportional and logical thinking (Miragliotta & Baccaglini-Frank, 2021). Research has revealed a correlation between students' proportional reasoning skills and their performance in algebra and other higher-level mathematics classes (Mudrika et al., 2024).

1. LITERATURE REVIEW

1.1 Theoretical foundations of solving problems related to isosceles triangles

Deeply rooted in the history of mathematics, from ancient civilizations such as the Egyptians and Greeks, the idea of an isosceles triangle characterized by at least two sides of the same length has great resonance. Early mathematicians, including those in ancient Greece, understood the special characteristics of these triangles, such as the coincidence of the basic angles opposing equal sides, which led to the emergence of many geometric theories and constructions (Ezeddine et al., 2023). A basic work in geometry, Euclid's Elements contains several ideas and hypotheses about triangles, especially isosceles triangles, which set the stage for later studies of their characteristics (Ponte et al., 2023).

Historically, architecture, topography, and astronomy (Sunzuma & Maharaj, 2020), as well as other disciplines, have used the internal symmetry and geometric features of isoscales. For a long time, geometry has been the preferred tool for describing and understanding the environment (Ponte et al., 2023). These ideas help to understand the relationships between the sides and angles of isosceles triangles, thus allowing us to solve a wide range of geometric problems (Altakhyneh, 2018).

Modern research covers a wide range of subjects from theoretical mathematics to pragmatic applications, thus expanding the traditional knowledge of isosceles triangles. In engineering and design environments, research on the use of computer-aided design and dynamic geometry has studied and optimized the geometric characteristics of isosceles triangles (Ayan & Isiksal-Bostan, 2019). In addition, the extension of the meaning of isosceles triangles beyond the limits of classical geometry is the study of non-Euclidean spaces.

Modern mathematical education makes use of isosceles triangles as required instruments to expose pupils to basic geometric concepts including symmetry, congruence, and angular relations. Often part of instructional strategies, practical exercises, geometric creations, and problem solving inspiring students to explore and capture the features of these triangles on their own - all part of geometry improves mathematical thinking and creative problem solving (Kusno et al., 2021; Mosese & Ogbonnaya, 2021).

Geometry as a subject, especially triangles including isosceles triangles play a key role in the development of students' skills of spatial and critical thinking, from include in the national curriculum of mathematics in Kazakhstan. The growing emphasis on creative educational strategies that use the characteristics of isosceles triangles to improve spatial reasoning is evident in recent educational studies conducted by Kazakh scientists (Popova et al., 2022). These approaches align with international trends that promote STEM education to foster higher-order and creative thinking abilities (Zakeri et al., 2023).

Beyond traditional classroom instruction, the use of technology has become a critical factor in teaching geometry. Tools that go beyond mentally representing geometric solids during lessons, such as GeoGebra, allow students to visualize and manipulate shapes, improving understanding and engagement. Current research emphasizes the use of such tools to create culturally relevant and accessible educational materials that meet the needs of diverse learners (Verner et al., 2019).

Isosceles triangles also allow one to teach more intricate geometries including trigonometry, coordinate geometry, and geometric transformations. Knowing and referencing various geometric forms including circles, squares, polygons, and shapes in space helps one to have a more whole knowledge of the spatial interactions of geometric shapes - triangles offers a point of reference. Scientists studying geometry have discovered a link with the isosceles triangles and the Fibonacci gold ratio, thus proving its significance not only in mathematics but also aesthetically and constructively significant role in such areas as art, architecture and natural forms.

The continuous study of isosceles triangles is still important today, as a new view on them reveals more and more interrelations with the surrounding world, and this shows its relevance in mathematics. For the study of theory, as well as practical application of knowledge about isosceles trumpeters their simplicity is emphasized, symmetry making them close to perfection. Knowledge of the qualities and properties of isosceles triangles gives students a solid foundation for understanding the significance of geometry and solving complex mathematical problems in real life.

The study of problems and tasks related to isosceles triangles from the time of Euclid and Pythagoras until our times proves their special importance not only in developing mathematical skills, but also help students in expanding other useful qualities, Including real-life problem solving, logical and creative thinking. Their unmatched value in terms of developing mathematical literacy and knowledge about mathematics demonstrates their constant presence in courses and studies.

Isoseles triangles are academically significant for the development of logical thinking abilities and geometric thinking in secondary school, according to recent studies and observations of Kazakhstani math teachers (Musaibekov et al., 2023). These materials show that the properties and qualities of isosceles triangles are fundamental in the teaching of geometry and serve as an important foundation, as well as help to pave the way for further in-depth study of topics in geometry.

Analysis of problems of the republican and international mathematical olympiads has shown that triangles, including properties and characteristics of isosceles triangles, are often used in complex geometric tasks, which allows students to solve complex spatial problems (Sarsenov, 2020). The curriculum built on this basis will make Kazakh schoolchildren competitive and strengthen their status for greater success in the future.

Efforts to incorporate ethnomathematical elements into geometry instruction in Kazakh-language schools have also used isosceles triangle patterns found in traditional crafts and architecture, demonstrating their cultural and pedagogical relevance (Tleubergenova, 2019). This proposition laid the foundation for connecting formal geometric reasoning with national identity and heritage in classroom practice.

The development of a problem-centered approach to teaching geometry in Kazakhstan was significantly shaped by the work of Abylkasymova, who emphasized the use of triangle-based tasks to foster logical reasoning and independent thinking (Abylkasymova, 2004).

The integration of visual and constructive strategies into the teaching of triangle properties, including isosceles triangles, was advanced through Dalinger's cognitive-visual methodology, which highlighted the role of analogical thinking and dynamic constructions in geometric reasoning (Dalinger, 2001).

The psychological and logical complexity of triangle-based proof tasks was systematically analyzed through Friedman's structural-task approach, providing a theoretical basis for evaluating the cognitive demands of geometric problems in classroom settings (Friedman, 1983).

The inclusion of culturally relevant elements, such as traditional Kazakh patterns based on triangle symmetries, laid the foundation for ethnomathematical approaches that connect formal geometric instruction with national identity and visual heritage (Temirbekova, 2016).

The use of isosceles triangle tasks in mathematical Olympiads and national assessments contributed to the development of higher-order reasoning skills and demonstrated their central role in Kazakhstan's geometry curriculum (Baimukhanov & Smirnov, 2016).

1.2 Key properties and theorems related to isosceles triangles

Isosceles triangles possess a unique set of properties and theorems that distinguish them from other types of triangles. One of the defining characteristics of an isosceles triangle is that it has two sides of equal length, which are referred to as the legs, while the third side is called the base. The angles opposite the equal sides, known as the base angles, are congruent, meaning they have the same measure. This property, known as the base angle theorem, is fundamental in solving problems involving isosceles triangles. The axis of symmetry, which bisects the base and the vertex angle opposite the base, divides the triangle into two congruent right triangles, which is another key characteristic.

Another important theorem related to isosceles triangles is the angle bisector theorem, which states that the angle bisector of the vertex angle in an isosceles triangle bisects the base and is perpendicular to it. This theorem provides a powerful tool for solving problems involving the lengths of the sides and the measures of the angles in isosceles triangles. The angle bisector theorem is crucial for solving various geometric problems. Conversely, if a median is also an altitude, then the triangle is isosceles (Heydari & Muroi, 2023).

The properties and theorems associated with isosceles triangles serve as essential tools for geometric problem-solving. In addition to the base angle theorem and the angle bisector theorem, other important properties of isosceles triangles include the fact that the altitude from the vertex angle to the base bisects the base and the vertex angle. These properties and theorems are not only valuable for solving mathematical problems but also have practical applications in various fields, such as engineering, architecture, and design.

The study of isosceles triangles also extends to more advanced topics, such as trigonometry and analytic geometry, where their properties can be used to derive trigonometric identities and solve geometric equations. By understanding the key properties and theorems related to isosceles triangles, students and professionals alike can develop a deeper appreciation for the beauty and utility of this fundamental geometric shape.

1.3 Common challenges in solving problems related to isosceles triangles

Despite the well-established properties and theorems associated with isosceles triangles, students often encounter several challenges when attempting to solve related problems. One common challenge lies in correctly identifying the given information and determining which properties and theorems are applicable to the specific problem. Students may struggle to distinguish between the legs and the base of the isosceles triangle, or they may fail to recognize the relationships between the sides, angles, and altitudes.

Another challenge involves applying the appropriate algebraic techniques to solve for unknown quantities.Students need to develop proficiency in using these techniques to solve for unknown side lengths, angle measures, or other geometric elements. Students sometimes have difficulties when solving trigonometric problems (Ngu & Phan, 2020).

Additionally, students may struggle with visualizing the geometric relationships within isosceles triangles and drawing accurate diagrams. This is further compounded by an inability to see the proportional relationships (Ayan & Isiksal-Bostan, 2019). A well-drawn diagram can often provide valuable insights into the problem and help students identify the appropriate solution strategies.

These challenges can be addressed through targeted instruction, practice, and problem-solving strategies that emphasize the importance of understanding the underlying concepts, developing strong algebraic skills, and visualizing the geometric relationships within isosceles triangles. Teachers will support students in mastering geometric contents, so that students can identify and discuss the relationships between the properties of different shapes (Kusno et al., 2021).

Additionally, it is important to note that many students think that mathematics is a very difficult subject (Sulistyaningsih et al., 2021). It is possible that some students have a strong understanding of math principles, but when they are asked to apply those concepts in the real world and write them down, they find it more challenging (Benedicto & Andrade, 2022).

1.4 Pedagogical, didactic, and cognitive conditions for solving complex problems related to isosceles

Pedagogical, didactic, and cognitive approaches to teaching isosceles triangle problems have been significantly shaped by Kazakhstani mathematics educators. Recent contributions emphasize the integration of structured problem-solving methods, visual reasoning, and context-based strategies to enhance students' ability to approach complex geometric tasks involving isosceles triangles.

Effective problem-solving in mathematics, particularly with complex isosceles triangle problems, necessitates careful consideration of pedagogical, didactic, and cognitive conditions to foster students' understanding and skills. Instruction should be student-centered, beginning with concrete examples and gradually progressing to abstract concepts, allowing students to construct their knowledge actively (Rahmah, 2017). This pedagogical approach aligns with constructivist learning theories, which emphasize the importance of prior knowledge and experiences in shaping understanding (Jahnke et al., 2022). Teachers should cultivate student's mindset to cooperate, so that they find mathematics an interesting subject (Setyaningsih et al., 2019).

The foundational work of Abylkasymova (2004) established a framework for task-based learning in mathematics education, where problems related to isosceles triangles are used to develop formal proof skills and spatial understanding. Her methodological guidelines promote a gradual progression from basic classification tasks to advanced problems requiring deductive argumentation and construction-based reasoning. This framework laid the foundation for building mathematical thinking through problem complexity and scaffolding techniques (Abylkasymova, 2004).

Dalinger (2001) further advanced the instructional methodology by focusing on visual-cognitive strategies. His approach encourages the use of diagrams, dynamic geometry software, and analogical modeling to explore properties of isosceles triangles—such as the coincidence of median, altitude, and angle bisector from the apex. This method fosters deep structural understanding and supports the transition from empirical observation to formal proof in geometry education.

Didactically, teachers should employ varied instructional strategies such as cooperative learning, problem-based learning, and inquiry-based learning to cater to diverse learning styles and preferences (Klang et al., 2021). Cooperative learning, for instance, promotes peer interaction and collaboration, enabling students to learn from each other and develop their problem-solving skills collectively (Klang et al., 2021). Problem-based learning, on the other hand, presents students with real-world scenarios that require them to apply their knowledge of isosceles triangles to find solutions (Alvin, 2022; Rivai et al., 2021). Inquiry-based learning encourages students to explore and investigate mathematical concepts through experimentation and discovery (Friedman, 1983; Vale & Barbosa, 2023).

Friedman’s structural-task analysis, though originally developed in the Soviet pedagogical tradition, remains widely applied in Kazakhstan. His model provides a system for evaluating the cognitive load of geometric problems, highlighting the importance of internal structure and logical chains in tasks involving isosceles triangle constructions or proofs (Friedman, 1983).

Temirbekova (2019) contributed to culturally responsive pedagogy by incorporating traditional Kazakh patterns and architectural forms that often rely on isosceles triangle symmetries. This approach not only contextualizes abstract geometric concepts but also enhances student engagement through meaningful and locally relevant content. Such ethnomathematical strategies have been shown to increase motivation and comprehension among students in Kazakh-language schools. Moreover, Kazakhstan mathematics teachers and Olympiad coaches have played a critical role in formalizing instructional sequences for advanced triangle problems. Through national competitions and enrichment programs, they developed systematic approaches to teaching geometric construction, loci, and concurrency problems involving isosceles triangles—often serving as benchmark cases for more complex reasoning tasks (Baimukhanov & Smirnov, 2016).

Cognitively, students need to develop strong spatial reasoning skills to visualize and manipulate geometric shapes effectively. Cognitive load theory suggests that the amount of information that students can process at one time is limited, and instructional design should aim to minimize extraneous cognitive load to optimize learning outcomes (Mangarin & Caballes, 2024). The use of visual aids, such as diagrams and animations, can help reduce cognitive load by presenting information in a clear and concise manner. Scaffolding, where teachers provide temporary support to students as they learn new concepts, can also help reduce cognitive load and promote mastery. Motivational factors, such as interest, relevance, and self-efficacy, play a significant role in students' engagement and achievement in mathematics. Students who are interested in the topic are more likely to be motivated to learn and persist in the face of challenges. Teachers should emphasize the development of metacognitive skills, such as self-monitoring and self-regulation, to help students become aware of their thinking processes and monitor their progress towards problem-solving goals. Students should be mindful of the issues pertaining to social, cultural, political and ethical dimensions, which can impact the solution process (English, 2023).

Furthermore, assessment practices should align with instructional goals and provide students with opportunities to demonstrate their understanding and problem-solving skills in various ways, such as through written explanations, diagrams, and presentations. Teachers should teach using appropriate methods for effective learning (Ezeddine et al., 2023).

Collectively, these contributions have established a pedagogical foundation in Kazakhstan that balances formal rigor with accessibility, promotes visual reasoning, and embraces both cultural identity and global standards in geometry education. By addressing these pedagogical, didactic, and cognitive conditions, educators can create a supportive and stimulating learning environment that enables students to develop the knowledge, skills, and attitudes necessary to tackle complex problems related to isosceles triangles and other mathematical concepts (Darmawan et al., 2020; Gordon, 2019).

Incorporating real-world applications and interdisciplinary connections into the study of isosceles triangles can significantly enhance students' understanding, engagement, and appreciation for the subject matter. By demonstrating the practical relevance of isosceles triangles in various fields, such as architecture, engineering, and design, educators can motivate students to learn and see the value of mathematics in their daily lives (Chau et al., 2021). For instance, students can explore how isosceles triangles are used in the construction of bridges, buildings, and other structures, or they can investigate how they are applied in the design of furniture, clothing, and artwork (Serin, 2018).

Furthermore, interdisciplinary connections can enrich students' learning experience by integrating mathematical concepts with other subjects, such as science, technology, engineering, and the arts. Students' interest can be stimulated, and an active environment can be created by strengthening the application of modeling ideas (Segura & Ferrando, 2023). For example, students can investigate the relationship between isosceles triangles and the golden ratio in art and architecture or explore the use of isosceles triangles in the design of musical instruments (Chau et al., 2021). Such interdisciplinary explorations can help students develop a deeper understanding of mathematical concepts and their connections to the broader world.

Moreover, real-world applications and interdisciplinary connections can foster students' critical thinking, problem-solving, and creativity skills. By engaging in authentic tasks and projects that require them to apply their knowledge of isosceles triangles to solve real-world problems, students can develop their ability to analyze complex situations, identify relevant information, and generate innovative solutions. Teachers should be able to incorporate real-world problems into their lessons, as they may feel pressured to adhere to rigid guidelines and prepare students for exams that focus on procedural skills rather than critical thinking (Chau et al., 2021). For instance, students can design a sustainable building using isosceles triangles or create a mathematical model to estimate the amount of material needed to construct a triangular structure. This approach not only makes the course content more relatable but also fosters a sense of mastery, as students apply classroom material to real-world problems (Mebert et al., 2020).

Integrating real-world applications into math education is grounded in educational theories that emphasize active learning, student engagement, and the development of critical thinking skills (Chau et al., 2021). The messy, open-ended nature of real-life situations contrasts with the tidier hypothetical situations many students are accustomed to in the classroom (Mebert et al., 2020). Incorporating real-world applications and interdisciplinary connections into the study of isosceles triangles requires careful planning and implementation.

Teachers need to identify relevant and engaging real-world examples and design activities and projects that align with the curriculum and learning objectives. They also need to provide students with the necessary resources and support to successfully complete these tasks. In addition, teachers need to assess students' understanding and problem-solving skills in authentic contexts and provide them with feedback on their performance. By implementing these strategies, educators can create a learning environment that enables students to develop a deep understanding of isosceles triangles and their applications in the real world (Rochmad et al., 2019; Rodrigues et al., 2018). Meaningful learning can be achieved by applying mathematics to real-world situations (Bahri et al., 2021). Such opportunities can also assist students in appreciating that mathematics is not merely a means of calculating answers but is also a vehicle for social justice, where critical thinking plays a key role (English, 2023).

In mathematics education within a STEM setting, there is a dual challenge: developing a mathematical literacy perspective that encompasses a rich view of mathematical epistemic practices, and representing the diverse professional settings in which mathematics is created and used (Herawati & Amelia, 2021). This approach enables them to appreciate the relevance and applicability of mathematics in various contexts (Wardono et al., 2021). In light of these considerations, research on pedagogical psychology in solving complex problems associated with isosceles triangles should focus on the development and evaluation of instructional interventions that incorporate cognitive principles, metacognitive strategies, and motivational techniques. Investigating the effects of different instructional approaches on students' problem-solving skills, spatial reasoning abilities, and attitudes towards mathematics can provide valuable insights into effective teaching practices. It is essential to foster the ability to identify and utilize mathematical concepts in real-world contexts, thereby enhancing problem-solving abilities and deepening understanding (Abdul-Basir et al., 2021). Academic engagement is undeniably critical to successful problem-solving, influencing student learning outcomes (Yusuf et al., 2021). Students who are more engaged tend to be more academically successful and satisfied with their educational experiences (Mebert et al., 2020). By emphasizing the connections between mathematical concepts and real-life scenarios, educators can make mathematics more accessible and engaging for students (Astuti et al., 2021). Actively involving students in applying their knowledge to novel situations is crucial. Collaboration among students is beneficial (Mebert et al., 2020). Integrating math with real-world applications through active learning not only helps to meet diverse students' needs but also maintains academic rigor (Chau et al., 2021).

1.5 Structural model for working with isosceles triangles

One must have a disciplined model including analytical approaches, spatial reasoning tactics, and problem-solving procedures if one wants to properly negotiate the complexity related with isosceles triangles.

The model should first stress the need of knowing the basic features of isosceles triangles, such congruence of base angles and the link between side lengths and angles. While learning mathematics, students must grasp real settings (Segura & Ferrando, 2023). Mathematical problem-solving becomes a very significant element to be ingrained in kids since it helps them to strengthen their thinking ability (Aggarwal, 2020).

Laying the foundation for task-taxonomy-based work with isosceles triangle problems, Ardabaeva (2023) suggested a methodical approach for geometry instruction in middle schools comprising sequenced levels of difficulty, integration of intersubject links, and varied approaches of student activity. From identification to formal proof and building, this offers a structural framework for task organization, therefore facilitating student development across cognitive processes.

Second, the model should include a range of problem-solving techniques including geometric transformations to examine congruence and similarity, Pythagorean theorem to find missing side lengths, and trigonometric ratios to estimate angles (Chau et al., 2021). Another approach for students to participate in activities demanding them to solve real-world problems is project-based learning (Bakar & Ismail, 2020; Cunha et al., 2024). In this sense, students tackle real-world problems using mathematical ideas and techniques.

An esteemed teacher from Nur-Sultan, Dybyspayev (2021), underlined in task-based courses the development of creative thinking in geometry education. His efforts on innovative problem-solving help students to produce nonstandard answers to isosceles triangle constructions and proofs, therefore strengthening the creative aspect of the structural model.

Students should be urged to employ these flexible and imaginative techniques, customizing them to fit the particular setting of every challenge. Teachers should help pupils to grasp the issue and organize the way of remedy. Scaffolding can be given by teachers to help pupils with first attempts at solving problems (Daulay et al., 2021). Hints, leading questions, or deconstruction of the problem into smaller, more doable steps can all fit this structure.

Furthermore stressing the need of reviewing solutions and considering the process of problem-solving should be the model. Students should be urged to assess the rationality of their responses and to confirm them using several techniques (Barana et al., 2022). Developing injure based questions can help students to have chances to strengthen their ability to solve problems (Zulfa & Andriyani, 2023).

Recent research on educating pre-service mathematics teachers at ENU exposed methodological difficulties and geometric figure drawing solutions. It shown that employing dynamic digital environments such GeoGebra and 3D-modeling tools enables precise construction and mental comprehension of isosceles triangles, thereby improving the structural model with modern ICT tools (Manganyana et al., 2020).

Third, the model should encourage the acquisition of spatial reasoning abilities including mentally rotating and reflecting shapes, building precise diagrams and representations, and perceiving and manipulating geometric figures in three dimensions, mentally. Encouragement of students to create several answers to a problem would help teachers also promote innovative problem-solving. This method not only improves their ability to solve problems but also advances a closer knowledge of mathematical ideas. Mathematical education in both elementary and high schools has included problem-solving (Eisenmann et al., 2022). The aim is to equip students with the ability to apply mathematical knowledge and abilities to solve real-world situations, make informed judgments, and so properly help society (Astuti et al., 2021).

Emphasizing expert-based criteria and the efficacy of software like AutoCAD for applied geometric abilities, Orazali & Mekebaev (2024) created a computer-assisted geometry teaching model For complicated triangle building, their technique uses digital check-and-feedback loops to improve structural fluency and correctness.

Fourth, the model should encourage analytical thinking abilities like generalizations, pattern recognition, and logical conclusions grounded on geometric ideas. Teachers should help pupils to learn lifetime and to change with the times. Furthermore, students can investigate several ways to approach a problem, therefore improving their ability to solve it as well as their knowledge of mathematical ideas. Good cooperation can help to raise the performance in solving problems.

Furthermore underlined by Zhaguparev (2021) the application of "method of the key problem" key-problem approaches in geometry courses. This method arranges learning around central isosceles triangle problems serving as gates to related geometric ideas and techniques.

Moreover, the model should have a feedback system that lets students evaluate their approaches of solving problems, spot areas needing work, and polish their plans. Teachers can provide their pupils the tools and knowledge they need to boldly address challenging problems involving isosceles triangles by using such a disciplined approach. Changing the educational system will help to raise the caliber of human resources (Mailisman et al., 2020). Among the mathematical ability needed of pupils are those related to problem-solving (Mailisman et al., 2020). Students should be able to use their mathematics knowledge and skills to address practical challenges.

This methodology fosters advanced thinking and curriculum requirements by skillfully merging Kazakhstani research contributions with worldwide instructional techniques. It strikes a mix of digital fluency, strategic focus on important issues in geometry instruction, creative inquiry, and orderly development.

Given the incorporation of technology, dynamic geometry software presents a revolutionary method of teaching and learning about isosceles triangles thereby allowing students to investigate geometric ideas dynamically. This incorporation of technology emphasizes its possibilities in enhancing mathematical education, developing critical thinking, and so strengthening a deeper knowledge of mathematical concepts (Cunha et al., 2024; Rholey R. Picaza, 2023). An interesting mathematics learning environment depends on the technological competency of teachers (Öztürk et al., 2021).

Dynamic geometry tools let students test hypotheses in real-time, adjust geometric figures, and see relationships (Miragliotta & Baccaglini-Frank, 2021). This practical experience can improve students' knowledge of the characteristics of isosceles triangles, including the congruence of base angles and the relationship between side lengths and angles (Daulay et al., 2021; *Global Education Monitoring Report 2023: Technology in Education: A Tool on Whose Terms?*, 2023). By letting students see and modify geometric figures in three dimensions (Cirneanu & Moldoveanu, 2024), dynamic geometry tools can also assist students build spatial reasoning skills. Moreover, the application of technology in the classroom helps to improve the teaching and learning geometry concepts' efficacy using diagrams (Retnowati & Fadlila, 2023). Teachers can build a more interesting and successful learning environment that supports students' problem-solving skills, spatial reasoning ability, and attitudes toward mathematics by including technology into mathematics instruction (Cirneanu & Moldoveanu, 2024). Digital tools' capability to increase efficiency, provide visual representations, eliminate errors, and enable complex problem-solving ability magnifies their efficacy in mathematics instruction ((Cirneanu & Moldoveanu, 2024; Miragliotta & Baccaglini-Frank, 2021). Thus, increasing the quality of education depends on including technology and digital mathematics into learning environments (Atteh et al., 2020; Cirneanu & Moldoveanu, 2024).

Customizing courses to meet various learning styles, interests, and degrees of knowledge is essential so that teachers may guarantee that every student may connect with and gain from practical applications (Chau et al., 2021). This approach not only improves the outcomes of instruction but also promotes a more inclusive and motivating classroom.

Including technology into mathematics education offers absolutely essential help for the learning process (Atteh et al., 2020; Cirneanu & Moldoveanu, 2024). Acting as a beneficial tool for acquiring, storing, and retrieving knowledge, it motivates students to search out the most recent information from many sources (Aggarwal, 2020). Technology can help to raise understanding of specific student learning needs and support student-centered teaching approaches, therefore improving student involvement with mathematics (Attard & Holmes, 2020). The integration of technology in classroom instruction enhances motivation, elevates self-esteem and confidence, refines questioning skills, encourages initiative and independent learning, enhances presentation of information, cultivates problem-solving capabilities, improves information handling skills, extends focus time on tasks, boosts attendance rates, and nurtures positive attitudes towards both technology and education (Abdullah & Shin, 2019; Aggarwal, 2020; Manganyana et al., 2020; Ponte et al., 2023). Technology in mathematics classrooms thus enables students to concentrate more on tactics and response analysis rather than laborious computations (Atteh et al., 2020; Cevikbas et al., 2023). In secondary schools, digital tools can improve mathematics and scientific knowledge (Hillmayr et al., 2020). Effective addressing of students' different needs and interests can help teachers to create a more interesting, inclusive, and successful learning environment, so influencing the educational experiences of the students (Mebert et al., 2020). Digital learning environments can produce data on learning agents that can subsequently be used to enhance learning in a variety of ways (Barana et al., 2021). These developments entail not just the application of digital tools but also the design of tailored, interactive, flexible learning opportunities.

A structured model can be used to solve the difficulties in solving complicated problems including isosceles triangles by means of structure. This kind of model will give pupils a methodical way to solve problems, helping them through the required actions and techniques to get at correct answers. Starting with a firm awareness of the characteristics of isosceles triangles, including the congruence of base angles and the relationship between side lengths and angles, the model should Visual tools include interactive software and illustrations help pupils grasp these qualities even more. Students must make good use of technology to assist in the promotion of mathematics education (Kim et al., 2022). Technology in mathematics teaching allegedly enables pupils to concentrate more on tactics and response analysis rather than laborious computations (Silver, 1997).

The model should also provide a methodical approach to problem-solving, guiding students through the process of spotting pertinent material, choosing suitable tactics, and completing required computations. Moreover, the model should inspire students to review their answers and consider their approach of addressing their problems, thereby fostering metacognitive abilities and self-regulation (Jahnke et al., 2022). Interactive tutorials help students to reach the right response, therefore encouraging self-learning and problem-solving abilities (Mikula & Heckler, 2014). By actively participating and helping the teacher to plan activities, students can develop their ability to solve problems (Ezeddine et al., 2023).

Furthermore, the model should underline the need of mathematical thinking and communication. Students should be urged to enhance their capacity to clearly express mathematical ideas by justifying their solutions and reasoning. Moreover, the model must to include chances for students to work together and grow personally from each other. This is in line with the need of teachers using efficient teaching strategies to guarantee that their students get mastery in addressing problems (Isnawati et al., 2021).

Teachers that want to teach problem-solving have to take into account certain facets of the teaching and learning process (Hafizi & Kamarudin, 2020). These cover the classroom, teacher effectiveness, pedagogogy, and approaches to problem-solving. The classroom should be fit for solving problems, giving students chances to investigate, try out ideas, and work on projects (Barana et al., 2022). Teachers have to be content knowledge, pedagogical knowledge, and problem-solving capable if they want to support effective instruction (Chau et al., 2021). The need of instructors' pedagogical subject knowledge in improving mathematics teaching quality and student performance underlined in this paper. Good teaching strategies including inquiry-based learning and Injure Based Learning can involve students in significant tasks including addressing problems.

Using such a structural model can help students grasp isosceles triangles more deeply and increase their ability to solve problems. Overcoming obstacles in reaching objectives is considered as the process of solving problems (Mudrika et al., 2024; Syaiful et al., 2020). Critical thinking, analysis, and imagination are all part of problem-solving (Bakar & Ismail, 2020). Long-standing recognition of the benefit of raising problem-solving capacity as a focus of mathematics instruction. Teachers or educational institutions stress problem solving in learning mathematics both in the classroom and outside of it, in both form as questions given and learning models (Yusuf et al., 2021). Teachers can encourage pupils to deepen their knowledge of mathematics and enhance their problem-solving abilities by tackling these problems and concentrating on these domains (Chau et al., 2021).

1.6 Content and classification of complex problems involving isosceles triangles

Their substance, the query, and the applications of the properties and theorems let one classify hard situations involving isosceles triangles. Considered harder are problems requiring sophisticated solution strategies, where several properties or theorems regarding the isosceles triangle need to be used, or when algebraic approaches are needed (Santos-Trigo et al., 2021).

The content of such problems typically involves various geometric concepts, such as triangle similarity, side-angle ratios, triangle midpoints, finding area and perimeter, and transformations. Classification of problems with increased complexity can also take into account the level of representation and abstraction.

For example, problems that require students to visualize and represent three-dimensional objects, or tasks that include abstract geometric concepts. In addition, the division of complex problems related to isosceles triangles can also take into account their description and the context in which the problem is shown. Problems based on reality, interdisciplinary connections, and open-ended questions that require independent research can increase the complexity and challenge of these problems. Complex problems are like a new opponent in a game that challenges students and improves their skills and abilities in solving new problems (Benedicto & Andrade, 2022).

As in the investigations we do during classes, so in practical mathematics, complicated issues (or challenging tasks) play a vital part and are a necessary instrument for the development of logical, critical and analytical thinking, reasoning about them prepares students for real-life problems. Modern constructivist and Injure Based Learning paradigms center these activities (Lili Supardi et al., 2021; Segura & Ferrando, 2023).

A complex mathematical problem is one that is characterized by multiple interrelated components, a nonlinear solution, and a high degree of ambiguity, where solutions may not be immediately identifiable. Typically, such problems:

- Include multiple subproblems with different solution paths or steps that interact dynamically, requiring problem solving and strategic planning (Zulfa & Andriyani, 2023).
- Do not have a single, clear path to solution, requiring critical thinking, flexible reasoning, rather than the mechanical application of algorithms and a single formula or theorem.
- Require iterative processes, including guessing, testing, revising, and generalizing, to move toward a solution (Lili Supardi et al., 2021).
- They are often related to or based on real-life situations, making learning more understandable, useful, and applicable in practice .

In Kazakhstan, complex problems are usually classified based on their cognitive complexity and the required procedural steps and actions. According to Turganbaeva B.A., a complex problem is a problem that develops students' thinking, requires the use of several mathematical concepts, and is solved using more than one formula, but requires non-traditional strategies (Turganbaeva, 2002).

Koyanbaev Zh.B. and Koyanbaev R.M. also note: Complex problems require the application of fundamental knowledge from previous topics in new contexts, including the ability to analyze, synthesize, and draw conclusions based on facts (Koyanbaev & Koyanbaev, 2004).

According to Abylkasymova A.E.: Complex problems include subtasks that cover several mathematical topics simultaneously, require logical reasoning, and form the basis for developing subject competence (Abylkasymova, 2004).

Complex geometric problems are a special class of mathematical problems that distinguish them from others by the need for spatial reasoning, visual representation, and fundamental knowledge of geometric proof strategies.

Their defining traits include many geometric relationships - likeness, similitude, parallelism, symmetry, and parallelism - interacting in one context.

Often requiring auxiliary elements (e.g., angle bisectors, medians, heights, etc.), including diagrams and formal proofs into constructs either via transformation geometry or otherwise.

Step-by-step planning helps to gradually solve complexities, especially in construction problems (e.g. compass and straightedge) where visual precision and logical consistency are vital (Cunha et al., 2024; Miragliotta & Baccaglini-Frank, 2021).

Emphasis on visual-spatial modeling, especially when enhanced by geometry-building software such as GeoGebra, which supports hypothesis testing, precision in parameter assignment, and visualization (Atteh et al., 2020; Rholey R. Picaza, 2023).

Complex problems that students encounter can be a tool for developing mental skills such as analytics, visualization, and identifying cause-and-effect relationships for a number of reasons:

- analysis of the problem statement and complexity assessment;
- hypotheses generation and testing;
- planning the sequence of solution steps through transformations and construction.

Kazakh researchers such as Ardabaeva (2023) and Zhaguparev (2021) emphasize methods such as key problem modeling and problem sequencing in geometry education. These approaches classify problems related to isosceles triangles around basic geometric theorems and properties, planning a step-by-step logical chain from conceptual understanding to the ability to solve and prove and apply knowledge (Ardabaeva, 2023; Zhaguparev, 2021).

Complex problems related to isosceles triangles require different methods for solving, including not only geometric formulas, theorems and properties, but also algebraic approaches and computational foundations.

Geometric methods include the properties of similarity of triangles, the ratio of angles to sides, theorems that help to derive unknown quantities (English, 2023). The algebraic approach includes constructing equations and solving them based on geometric properties, making it easy to find the length of the sides, angles or areas (Klang et al., 2021).

Computational methods involve the use of information technology for geometric construction and dynamic manipulation, which allows for experimental research and verification of geometric relationships (Konnova et al., 2020).

In addition to the above methods, trigonometric functions cannot be excluded, which can be used to relate angles and side lengths, especially in problems related to non-right-angled isosceles triangles. Coordinate geometry offers another robust method, where a triangle is placed on a coordinate plane and algebraic methods are used to analyze geometric properties. The choice of the most appropriate method often depends on the specific details of the problem, and in many cases a hybrid approach combining several methods is most effective.

1.7 Integration of isosceles triangle concepts into the mathematics curriculum in Kazakhstan

Complex problems involving isosceles triangles can be included in the middle school mathematics curriculum for a number of reasons. For example, they promote critical thinking and mathematical reasoning, the tasks require logical connections, and improve skills in solving complex and heterogeneous problems.

Mathematical modeling makes it possible to relate real-world problems to isosceles triangles (Kohen & Orenstein, 2021). By incorporating problems involving isosceles triangles into other disciplines such as physics, chemistry, and engineering, students can appreciate the importance and application of mathematical knowledge in real life (Serin, 2018). The integration of technology can enhance students' exploration and understanding of complex problems involving isosceles triangles (Buket Özüm Bülbül, 2021).

The introduction of visualization technologies and figure plotting programs in geometry, as well as computer algebra systems, can enable students to manipulate geometric shapes, test hypotheses, and visualize mathematical relationships (Chau et al., 2021). The curriculum can emphasize the importance of mathematical communication and reasoning.

Students should be given opportunities to solve problems independently and formulate their own problem-solving strategies, explain their solution steps, and justify their arguments in finding solutions to problems. In addition, it is necessary to take into account the students' knowledge, problem-solving skills, each may be different, and we must not forget that when checking problems, the teacher should give the right direction for each student based on his or her level of knowledge. Complex problems open new opportunities for students to explore, this provides an opportunity to assess their knowledge by the problem-solving strategies they use. The curriculum should become a ladder for growth, complex problems should become an opportunity and a step for growth.

The study of isosceles triangles plays an important role in mathematics education because they provide a foundation for understanding complex geometric concepts and problem-solving strategies.

First, the relationship between the side lengths and angles of isosceles triangles provides insight into key concepts in trigonometry and advanced geometry.

Second, their special properties, such as the equality of two sides and two angles, provide useful experience in performing geometric constructions and proofs, helping to develop students' logical thinking.

Third, the internal symmetry of isosceles triangles introduces students to the concept of symmetry in geometry and develops their spatial awareness.

This section analyzes geometry textbooks developed by different authors and programs in our country, focusing on the extent to which they pay attention to problems on isosceles triangles. According to the reviewed literature, geometry materials usually include many properties, theorems, and corollaries that help students recognize and construct isosceles triangles. For example, calculation-based tasks contribute to the development of the ability to apply knowledge in practical situations (Abdullah & Shin, 2019). However, many geometry textbooks do not provide enough problems on isosceles triangles, especially those aimed at developing creative thinking and spatial abilities (Daulay et al., 2021). Therefore, research on the creative potential of secondary school students is relevant (Sahliawati & Nurlaelah, 2020). Students can develop creativity and spatial imagination using shape combining and dissecting techniques.

Table 1.7.1 Types of Tasks on Isosceles Triangles in Kazakhstani Geometry Textbooks

Textbook Authors	Grade	Number of Tasks	Types of Tasks
A.N.Shynybekov, D. A. Shynybekov	Grade 7	36	Analytical; Proof; Construction; Application of Theorems; Proportional Reasoning Problems
V. A. Smirnov, E. A. Tuyakov	Grade 7	38	Shape Recognition and Classification; Analytical; Proof; Construction; Application of Theorems
A.N.Shynybekov, D. A. Shynybekov, R. N. Zhumabayev	Grade 8	36	Shape Recognition and Classification; Analytical; Proof; Construction; Transformations; Coordinate Geometry Tasks; Application of Theorems
V. A. Smirnov, E. A. Tuyakov	Grade 8	34	Shape Recognition and Classification; Analytical; Proof; Construction; Coordinate Geometry Tasks; Application of Theorems
A.N.Shynybekov, D. A. Shynybekov, R. N. Zhumabayev	Grade 9	23	Shape Recognition and Classification; Analytical; Proof; Construction; Coordinate Geometry Tasks; Application of Theorems; Proportional Reasoning; Transformations; Application of Theorems and Properties
V. A. Smirnov, E. A. Tuyakov	Grade 9	19	Shape Recognition and Classification; Analytical; Proof; Construction

Main conclusions:

Geometry textbooks (Table 1.7.1) for grades 7–9 contain problems on isosceles triangles, but their level of difficulty varies. Many textbooks contain a sufficient number of tasks aimed at mastering the basic properties of an isosceles triangle. Some textbooks predominantly contain problems on proof, while others focus on calculations and constructive constructions. By grade 9, the tasks become more complex and require the use of theorems, but the total number of such problems remains small.

The following materials are taken from Kazakh textbooks of 7th class geometry and illustrate key theorems and properties related to isosceles triangles.

Theorem 1. Angles at the base of an isosceles triangle are equal.

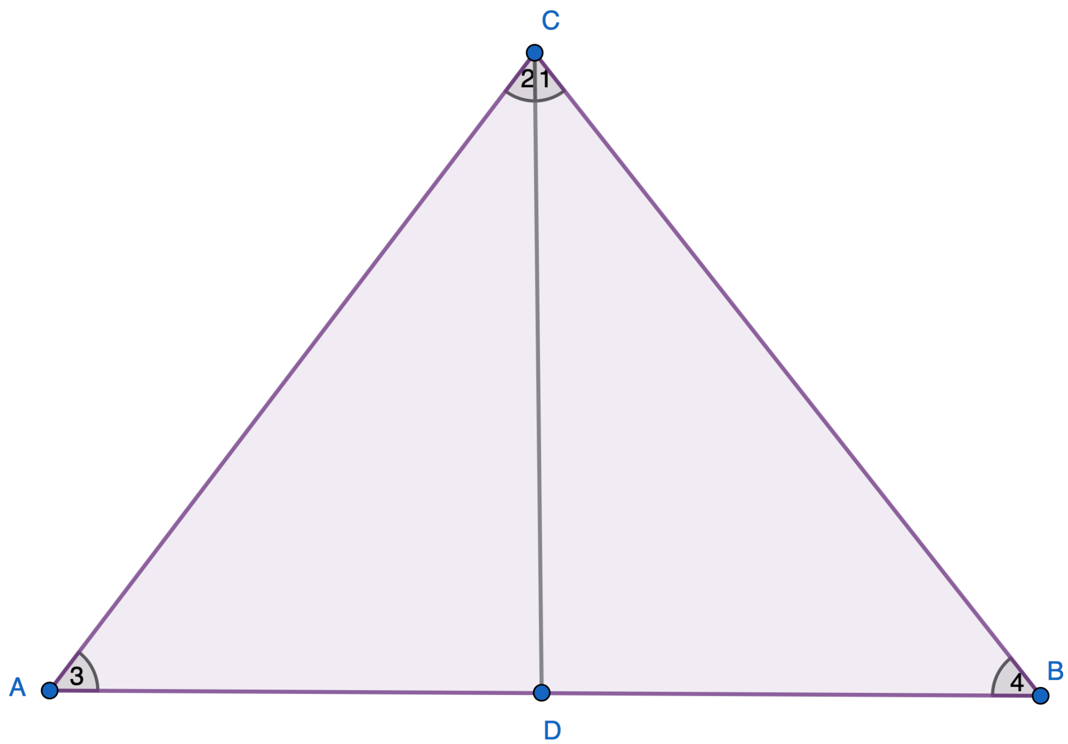
In the given triangle ABC, as illustrated in Figure 1.7.1, where AC and BC are equal sides, and AB is the base. Let $\angle 3$ and $\angle 4$ be the angles adjacent to the base. We will prove that $\angle 3 = \angle 4$.

Proof:

Run bisector CD. According to the first feature of similarity of triangles, triangles $\triangle ACD = \triangle BCD$ are similar ($AC = BC$, CD - common, and $\angle 1 = \angle 2$). Therefore, $\angle 3 = \angle 4$.

The theorem is proven.

Figure 1.7.1 Isosceles Triangle ABC with Bisector CD



Theorem 2. The Bisectrix, carried from the vertex to the base of an isosceles triangle, is also a median and height.

Proof:

According to theorem 1, from the equality of triangles $\triangle ACD = \triangle BCD$ follows that $AD = BD$, and therefore, CD is a median. Similarly, $\angle ADC = \angle BDC$.

Hence, $\angle ADC = \angle BDC = 90°$ (since ($\angle ADC$ and $\angle BDC$ are half of the extended angle). Therefore, CD is the height of the triangle.

The theorem is proven.

Theorem 3. (Inverse Theorem 1)

If the two angles of the triangle are equal, then the triangle is equilateral.

Let give a triangle ABC, as illustrated in Figure 1.7.1, with $\angle 3 = \angle 4$. Let us prove that $AC = BC$.

Proof:

Run the bisector of the angle CD. Then $\angle 2 = \angle 1$. According to theorem 1, in triangles ACD and BDC have $\angle ADC = \angle BDC$. For these triangles CD is the common side, and the adjacent angles are equal. On the second feature of similarity of triangles $\triangle ACD = \triangle BCD$. Therefore, $AC = BC$.

The theorem is proven.

Consequence of theorems 1 and 2:

In a triangle, the sides opposite equal angles are themselves equal and vice versa, the angles opposite equal sides are also equal.

In order to effectively teach students to solve problems involving isosceles triangles, they must first learn to analyze the problem statement by identifying the given data, the required quantities, and the relevant geometric theorems (e.g., "the base angles of an isosceles triangle are equal" or "the sides opposite equal angles are also equal"). Understanding the context of the problem is a critical first step in finding a solution.

In the process of teaching how to solve geometric problems involving isosceles triangles, it is important to follow a structured and purposeful sequence of actions. Based on the methodology of A.E. Abylkasymova "Theory and Methodology of Teaching Mathematics: Didactic and Methodological Foundations", the process of solving problems can be divided into four main stages:

1. Analysis of the problem statement.

At this stage, students study the subject content of the problem, determine what is given and what needs to be found, and identify which theorems and properties are applicable. For isosceles triangles, students should ask leading questions such as: "Which angles are equal?", "Is the triangle really isosceles?", or "What conclusions can be drawn from these elements?"

2. Selecting a solution method and developing a plan.

After completing the analysis, students decide which theorems to apply - for example, Theorem 1 (base angles are equal), Theorem 2 (angle bisector drawn from the vertex also acts as the median and altitude), or the converse theorem (equal angles imply equal sides).

3. Implementing the solution.

Following the plan, students carry out the solution steps, justify their reasoning, construct geometric diagrams, and clearly write down the solution.

4. Checking the result and drawing conclusions.

Students should check the correctness of their solutions, evaluate the logic of their reasoning, and confirm that the theorems and constructions applied are correct.

This structured model helps students develop a systematic approach to solving problems involving isosceles triangles, improves their logical reasoning and proof skills, and builds confidence in the effective use of geometric language and diagrams.

According to V. M. Bradis, a solution can only be considered correct if it is:

1. error-free;
2. logically sound;
3. complete.

Therefore, students must review their reasoning, ensure that each step was correct, and that the conclusion follows logically from the given conditions.

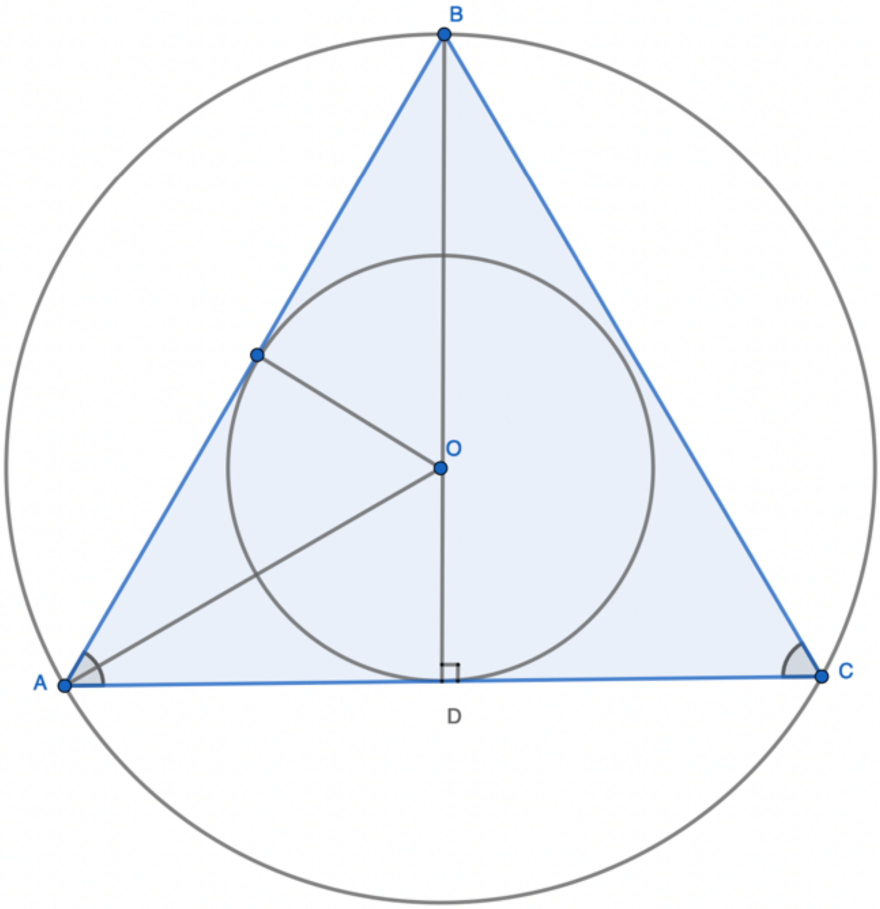
For example, if a student comes to the conclusion that $\triangle ACD = \triangle BCD$, he or she must clearly indicate which criterion for similarity of triangles was used and confirm that the resulting property (such as $\angle 3 = \angle 4$ or $AC = BC$) is correctly derived.

The four steps described above are closely interrelated and act as a structured guide for solving problems involving isosceles triangles. If the problem becomes more complex or one theorem is not enough, the student may need to return to the previous stage, demonstrating the development of reflection and self-control skills. Moreover, the complete completion of each stage prepares the student not only to solve a separate problem, but also to assimilate the culture of geometric reasoning and justification. This structured methodology promotes logical, consistent and mathematically sound thinking and deepens the students' understanding of geometry as a discipline.In the area of mastering geometric knowledge, the study of isosceles triangles plays a vital role for students of different academic levels. First, the isosceles triangle, which has two sides and two angles that are equal, is a basic geometric figure. Although angles are not a basic geometric concept, they are important because a figure without angles is simply a collection of elements inside a circle. Angles are ubiquitous in everyday life, reinforcing their importance. Second, the isosceles triangle is the basis for various rectangles and solids, forming the basic structure when constructing the diagonals of polygons.

In the context of isosceles triangles, ideal lines play a major role. The law of sines, elements of trigonometry, application of the properties of the tangent to a circle, and the properties of the inner and outer circle provide effective tools for solving various problems involving isosceles triangles. Although school mathematics mainly deals with problems of a simple level, capable students expand their knowledge through individual research and practical projects and apply them at later stages. Helps you learn how to effectively use the listed elements when solving problems involving isosceles triangles. This approach not only improves problem solving skills, but also expands the application of geometric principles beyond traditional methods.

Below are some examples of complex problems related to isosceles triangles, and what methods and solution strategies have been applied to each of these tasks. A different approach was applied to each task and applied from the basic concepts of geometry, mathematical calculation, figure construction, as well as trigonometric identities.

Example 1. The radius of the circumcircle of an isosceles triangle is 25 cm, and the radius of the incircle is 12 cm. Find the side lengths of the triangle.



Given:
$\triangle ABC$ is an isosceles triangle
R = 25 cm (circumradius)
r = 12 cm (inradius)
To find:
AB, AC – ?

Solution:

We use two formulas for the area of a triangle:

$$S_{\triangle} = \mathrm{pr} \ = \frac{2AB + AC}{2} \cdot 12 = 12\mathrm{AB} + 6\mathrm{AC}$$

$$S_{\triangle} = \frac{abc}{4R} = \frac{AB^2 \cdot AC}{100}$$

By equating the two expressions for the area, we obtain:

$$12\mathrm{AB} + 6\mathrm{AC} = \frac{AB^2 \cdot AC}{100} \qquad (1)$$

Let $\angle ACB = \alpha$. Then, by the Law of Sines in triangle ABC:

$$\frac{AB}{\sin\alpha} = 2R$$

$$AB = 2R \cdot \sin\alpha \qquad (2)$$

Now, using the identity from trigonometry and equation (2):

$$\cos\alpha = \frac{AD}{AB}$$

$$AD = AB \cdot \cos\alpha = 2R \cdot \sin\alpha \cdot \cos\alpha$$

$$AC = 2 \cdot AD = 4R \cdot \sin\alpha \cdot \cos\alpha \ \ (AD = DC) \qquad (3)$$

Substitute equations (2) and (3) into equation (1):

$$12\mathrm{AB} + 6\mathrm{AC} = \frac{AB^2 \cdot AC}{100}$$

$$24R \cdot \sin\alpha + 24R \cdot \sin\alpha \cdot \cos\alpha = \frac{4R^2 \cdot \sin^2\alpha \cdot 4R \cdot \sin\alpha \cdot \cos\alpha}{100}$$

Simplifying:

$$150(1 + \cos\alpha) = R^2(1 - \cos^2\alpha) \cdot \cos\alpha$$

Now solve the equation:

$$25\cos^2\alpha - 25\cos\alpha + 6 = 0$$

Solving the quadratic equation, we get two solutions:

1) $\cos\alpha = \frac{2}{5}; \sin\alpha = \frac{\sqrt{21}}{5}$
2) $\cos\alpha = \frac{3}{5}; \sin\alpha = \frac{4}{5}$

Now substitute the values back to find the side lengths:

Case 1:

$$1) AB = 2 \cdot 25 \cdot \frac{\sqrt{21}}{5} = 10\sqrt{21}$$

$$AC = 100 \cdot \frac{\sqrt{21}}{5} \cdot \frac{2}{5} = 8\sqrt{21}$$

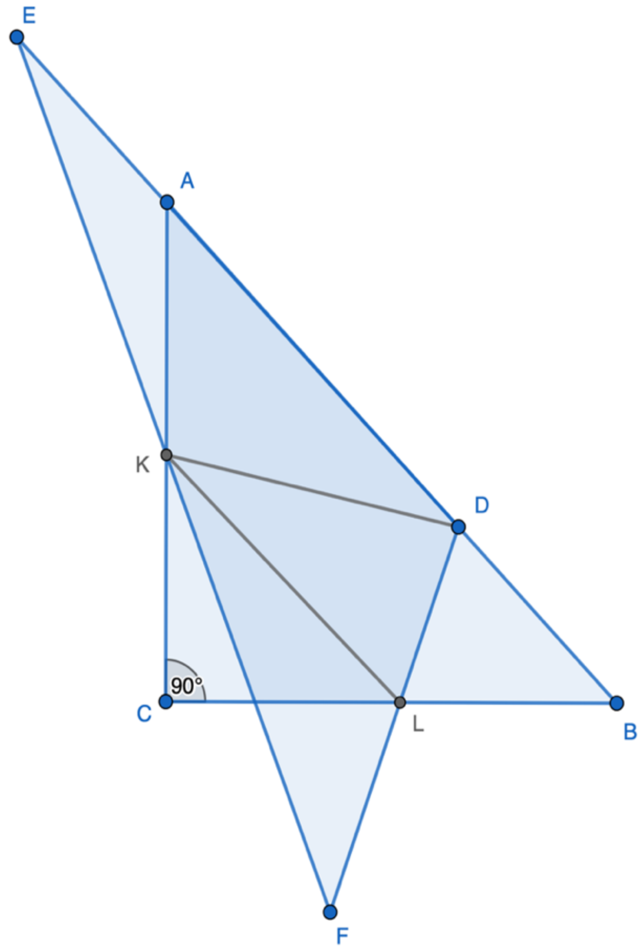
Case 2:

$$2)\ AB = 2 \cdot 25 \cdot \frac{4}{5} = 40$$

$$AC = 100 \cdot \frac{4}{5} \cdot \frac{3}{5} = 48$$

Answer: $10\sqrt{21}, 8\sqrt{21};\ 40, 80$

Example 2. Triangle ABC is an isosceles triangle with ∠C = 90°, and triangle DEF is placed such that point D lies on side AB, and point E lies on the extension of AB beyond point A. Segment KL is the midline (median line) common to both triangles. The area of quadrilateral DKLB is equal to 5/8 of the area of triangle ABC. Find the angle ∠DEF.



Given:
△ABC – isosceles triangle, ∠C = 90°
△DEF – triangle
D ∈ AB
KL – midline
$S_{DKLB} = \frac{5}{8} S_{ABC}$
To find:
∠DEF – ?

Solution:

Let us denote:

Let side $AB = a$, and since triangle ABC is isosceles with a right angle at C, then:

$$AC = BC = a$$
$$AB = FD = a\sqrt{2}$$

From the problem:

$$KL = \frac{1}{2} AB = \frac{1}{2} ED$$

Step 1: Area relationship

We are told that:

$$S_{DKLB} = S_{ABC} - S_{CKL} - S_{AKD}$$

From the problem:

$$S_{DKLB} = \frac{5}{8} S_{ABC}$$

Since triangle CKL is formed from half the base and height of triangle ABC, it has:

$$S_{CKL} = \frac{1}{4} S_{ABC}$$

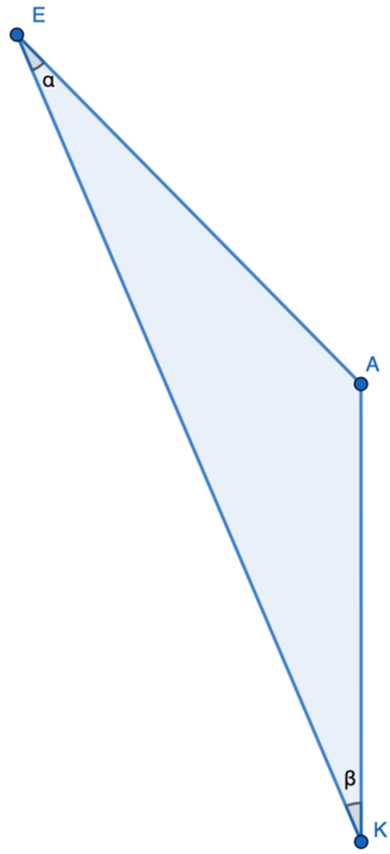
So plugging into the equation:

$$\frac{5}{8} S_{ABC} = S_{ABC} - \frac{1}{4} S_{ABC} - S_{AKD}$$

Solving for S_{AKD}:

$$S_{AKD} = \frac{1}{8} S_{ABC} \Longrightarrow S_{ABC} = \frac{a^2}{2} \Longrightarrow S_{AKD} = \frac{1}{8} \cdot \frac{a^2}{2} = \frac{a^2}{16}$$

We now calculate the area S_{AKD} using trigonometry.



Step 2: Expressing the Area of Triangle AKD

Let's denote the length from point D to point F as x.
Then we compute the area of triangle AKD using the sine formula:

$$S_{AKD} = \frac{1}{2} \cdot \frac{a}{2} \left(a\sqrt{2} - x\right) \cdot \sin 45^{\circ}$$

$$= \frac{a}{4} \left(a\sqrt{2} - x\right) \cdot \frac{\sqrt{2}}{2}$$

Using $\sin 45^{\circ} = \frac{\sqrt{2}}{2}$:

$$S_{AKD} = \frac{a}{4} \left(a\sqrt{2} - x\right) \cdot \frac{\sqrt{2}}{2} = \frac{a^2}{4} - \frac{a\sqrt{2}}{8} x$$

We already know:

$$S_{AKD} = \frac{1}{8} \cdot \frac{a^2}{2} = \frac{a^2}{16}$$

Now we equate the two expressions for S_{AKD}:

$$\frac{a^2}{4} - \frac{a\sqrt{2}}{8} x = \frac{a^2}{16}$$

$$1{,}5a^2 - \sqrt{2}ax = 0 \Longrightarrow \sqrt{2}ax = \frac{3}{2} a^2 \Longrightarrow$$

$$x = \frac{3a^2}{2\sqrt{2}a} = \frac{3\sqrt{2}a}{4}$$

Step 3: Use of Law of Sines in Triangle DEF

Let angle $\angle DEF = \beta$, angle $\angle DFE = \alpha$, and angle at point K is $135° + \beta$.

Then:

$$\beta + 135° + \alpha = 180° \Longrightarrow \beta = 45° - \alpha$$

Now use the Law of Sines in triangle DEF:

$$\frac{\frac{a}{2}}{\sin\alpha} = \frac{\frac{3\sqrt{2}a}{4}}{\sin(45°-\alpha)} \Rightarrow \frac{a}{2\sin\alpha} = \frac{3\sqrt{2}a}{4\sin(45°-\alpha)} \Rightarrow \frac{1}{\sin\alpha} = \frac{3\sqrt{2}}{2\sin(45°-\alpha)}$$

Use identity for $\sin(45°-\alpha) = \sin 45° \cos\alpha - \sin\alpha\cos 45° = \frac{\sqrt{2}}{2}(\cos\alpha - \sin\alpha)$

Then:

$$2\left(\frac{\sqrt{2}}{2}(\cos\alpha - \sin\alpha)\right) = 3\sqrt{2}\sin\alpha$$

$$\cos\alpha - \sin\alpha = 3\sin\alpha$$

$$\cos\alpha = 4\sin\alpha$$

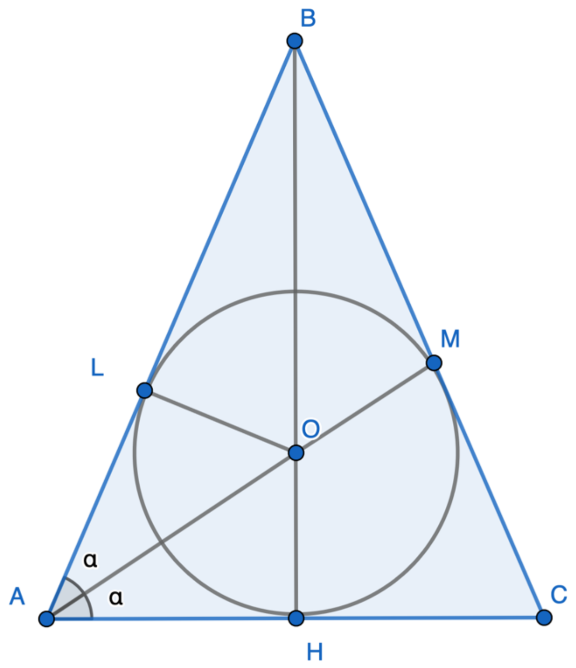
Divide both sides by $\cos\alpha$:

$$tg\ \alpha = \frac{1}{4} \Rightarrow \alpha = arctg\left(\frac{1}{4}\right)$$

Answer: $\angle DEF = arctg\left(\frac{1}{4}\right)$

Example 3. Point O is the center of the incircle of an isosceles triangle (with AB=BC). The line AO intersects segment BC at point M. If AO=3 and OM=2711, find the angles of triangle ABC and its area.

Given:
$\triangle ABC$ is an isosceles triangle
O is the incenter of triangle $\triangle ABC$
$AB = BC$
$AO = 3, OM = \frac{27}{11}$
To find:
$\angle A, \angle B, \angle C$ – ?
S_{ABC}– ?



Solution:

Step 1: Use trigonometry to express cosine of angle α

Let $\angle A = \angle C = \alpha$ (since the triangle is isosceles with $AB = BC$). Given $AO = 3$, and point M lies on BC, with $OM = \frac{27}{11}$.

From the triangle's incenter properties, we know:

$$\cos\alpha = \frac{b}{3}$$

Apply the double-angle identity:

$$\cos 2\alpha = 2\cos^2\alpha - 1$$

We also know from triangle geometry that:

$$\cos 2\alpha = \frac{AH}{AB}$$

So substituting:

$$\frac{b}{a} = 2\cdot\left(\frac{b}{3}\right)^2 - 1 = \frac{2b^2}{9} - 1 = \frac{2b^2 - 9}{9}$$

$$a = \frac{9b}{2b^2 - 9}$$

Step 2: Use expressions for $a + b$ and $a + 2b$

Now calculate:

$$a + b = \frac{9b}{2b^2 - 9} + b$$

$$a + b = \frac{2b^3}{2b^2 - 9}$$

And:

$$a + 2b = \frac{2b^3}{2b^2 - 9} + b \Longrightarrow a + 2b = \frac{4b^3 - 9b}{2b^2 - 9}$$

Step 3: Use formula for the length of segment AM

The squared length of median (or in this case, the segment from the incenter) is:

$$AM^2 = l^2 = \frac{a\cdot b\cdot(2a + 2b)\cdot(a + 2b - a)}{a + 2b} = \frac{a\cdot b\cdot(2a + 2b)\cdot 2b}{a + 2b}$$

Substitute the values:

$$a = \frac{9b}{2b^2 - 9}, a + b = \frac{2b^3}{2b^2 - 9}, a + 2b = \frac{4b^3 - 9b}{2b^2 - 9}$$

$$l^2 = \frac{2\cdot\left(\frac{9b}{2b^2 - 9}\right)\cdot b\cdot\left(\frac{2b^3}{2b^2 - 9}\right)\cdot 2b}{\frac{4b^3 - 9b}{2b^2 - 9}} = \frac{8\cdot 9\cdot b^5\cdot 2b}{(4b^3 - 9b)^2} = \left(\frac{60}{11}\right)^2$$

From here, solve for b:

$$b = \sqrt{5};\ a = 9\sqrt{5}$$

Step 4: Calculate triangle height and area

$$BH = \sqrt{(9\sqrt{5})^2 - (\sqrt{5})^2} = 20$$

$$S_{ABC} = \frac{2\sqrt{5} \cdot 20}{2} = 20\sqrt{5}$$

Step 5: Find angles

We use the double-angle identity again:

$$\cos 2\alpha = \frac{b}{a} = \frac{\sqrt{5}}{9\sqrt{5}} = \frac{1}{9} \Longrightarrow \alpha = \frac{1}{2} arccos\left(\frac{1}{9}\right)$$

$$\angle A = \angle C = 2\alpha = arccos\left(\frac{1}{9}\right); \ \angle B = \pi - 2\alpha = \pi - 2arccos\left(\frac{1}{9}\right)$$

Answer: $\angle A = \angle C = arccos\left(\frac{1}{9}\right)$; $\angle B = \pi - 2arccos\frac{1}{9}$; $S_{ABC} = 20\sqrt{5}$

Example 4. The distance between the intersection points of the medians and angle bisectors of an isosceles triangle is equal to 2. If the length of the incircle of the triangle is 20π, find the perimeter of the triangle.

Given:

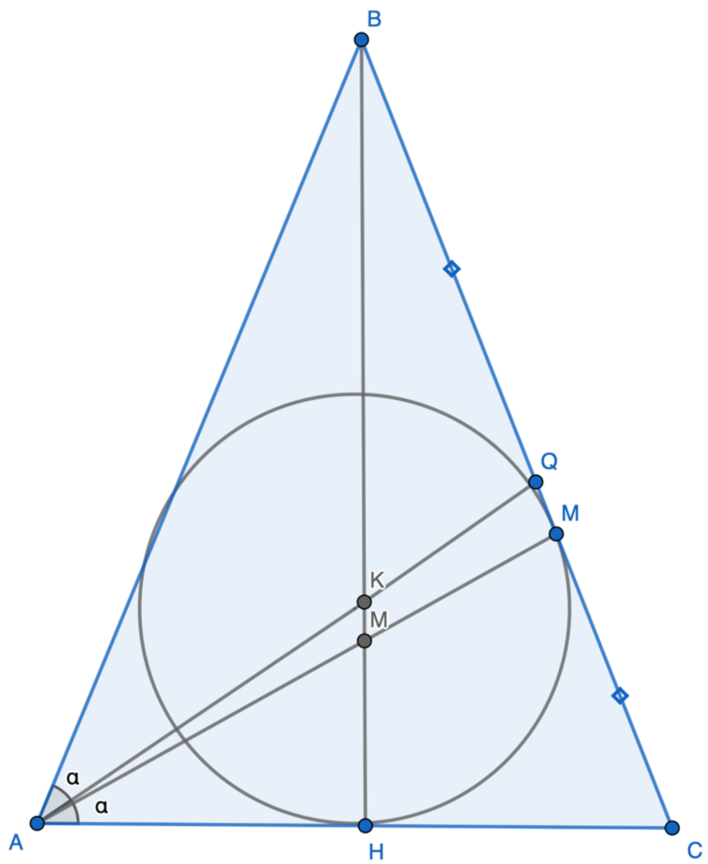
$\triangle ABC$ is an isosceles triangle

$O_1O_2 = 2$ — the distance between the centroid and the incenter

$C_{\text{incircle}} = 20\pi$ — the circumference of the incircle

To find:

$P_{ABC} - ?$



Solution:

Using properties of centroid and triangle height:

In an isosceles triangle, the centroid O_2 lies one third of the way along the median from the vertex to the base. So:

$$O_2H = \frac{1}{3}BH = \frac{a \sin \alpha}{3}$$

Since $O_1O_2 = 2$, and the incenter lies at distance $O_1H =$

10 along the altitude, the total altitude:

$$a \cdot \sin\alpha = 24 \Rightarrow a = \frac{24}{\sin\alpha}$$

Use cosine rule to find side AC:

Since triangle is isosceles, $AC\ = 2a \cdot cos\alpha$, and:

$$HC = AH = \frac{AC}{2} = \frac{2a \cdot \cos\alpha}{2} = a \cdot \cos\alpha,\, BH = \ a \cdot \sin\alpha$$

Using trigonometry to relate incenter location and triangle sides:

$$tg\frac{\alpha}{2} = \frac{O_1H}{AH} = \frac{10}{a \cdot \cos\alpha}$$

Substituting $a = \frac{24}{\sin\alpha}$, we get:

$$tg\frac{\alpha}{2} = \frac{10 \cdot \sin\alpha}{24 \cdot \cos\alpha}$$

Simplifying using trigonometric identities:

Use the identity $\sin\alpha = 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$, and let $\cos^2\frac{\alpha}{2} = x$, resulting in:

$$6 \cdot \cos\alpha = 5 \cdot \cos^2\frac{\alpha}{2} \Rightarrow \cos\alpha = \frac{5}{6} \cdot \cos^2\frac{\alpha}{2}$$

$$\cos^2\frac{\alpha}{2} - \left(1 - \cos^2\frac{\alpha}{2}\right) = \frac{5}{6} \cdot \cos^2\frac{\alpha}{2}$$

$$\cos^2\frac{\alpha}{2} - \cos^4\frac{\alpha}{2} = \frac{5}{6} \cdot \cos^2\frac{\alpha}{2}$$

$$\frac{7}{6}\cos^2\frac{\alpha}{2} = \cos^4\frac{\alpha}{2}$$

$$\cos^2\frac{\alpha}{2}\left(\frac{7}{6} - \cos^2\frac{\alpha}{2}\right) = 0$$

$$\frac{7}{6} \cdot \cos^2\frac{\alpha}{2} = 1 \Rightarrow \cos^2\frac{\alpha}{2} = \frac{6}{7}$$

$$\frac{1 + \cos\alpha}{2} = \frac{6}{7}$$

$$7(1 + \cos\alpha) = 12$$

$$7\cos\alpha = 5$$

$$\cos\alpha = \frac{5}{7};\ \sin\alpha = \frac{2\sqrt{6}}{7}$$

Calculate side lengths and perimeter:

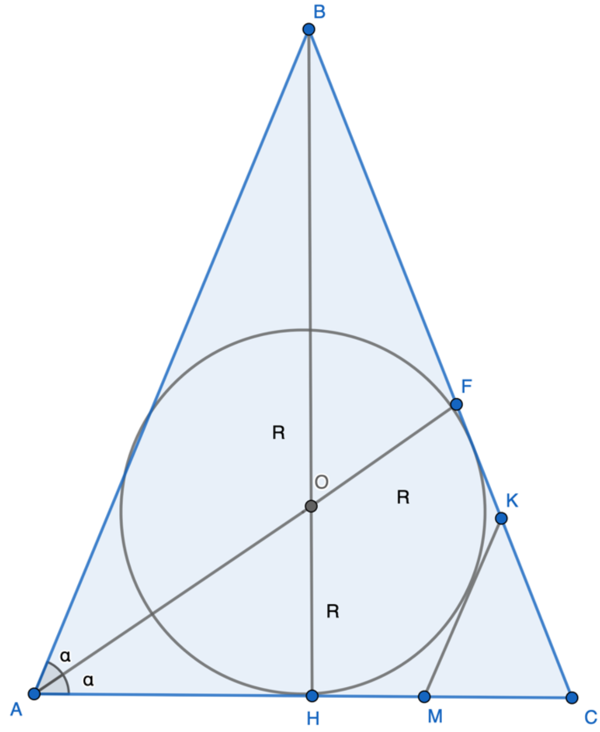
$$AB = a = \frac{24}{\frac{2\sqrt{6}}{7}} = 14\sqrt{6}$$

$$AC = 2 \cdot 14\sqrt{6} \cdot \frac{5}{7} = 20\sqrt{6}$$

$$P = 14\sqrt{6} + 14\sqrt{6} + 20\sqrt{6} = 48\sqrt{6}$$

Answer: $P = 48\sqrt{6}$

Example 5. An incircle is drawn inside the isosceles triangle ∆ABC (with AB=BC). A line parallel to side AB and tangent to the incircle intersects side AC at point M, such that MC=25·AC. If the perimeter of triangle ABC is 20, find the radius of the incircle.



Given:
$\triangle ABC$ is an isosceles triangle,
$AB = BC$
$P_{ABC} = 20$
Line $MK \parallel AB$ and is tangent to the incircle,
$MC = \frac{2}{5}AC$
To find:
$r - ?$

Solution:

Step 1: Let the length of side $AC = a$

Since triangle ABC is isosceles and symmetric about its height from vertex B, point H (the foot of the height from B) divides base AC equally:

$$AH = HC = \frac{a}{2}$$

We are told the line tangent to the incircle intersects AC at point M, and $MC = \frac{2}{5}a$.

Let x be the distance from A to the point where the tangent touches the incircle on side AC. Then:

$$x + \frac{2}{5}a = \frac{a}{2} \Rightarrow x = \frac{1}{10} \cdot a$$

Step 2: Use perimeter condition

Let side $AB = BC = b$, since the triangle is isosceles. We are told the perimeter is 20:

$$a + 2b = 20$$

Step 3: Use geometric relationships to connect a and b

From the figure (implied):

- The height from B to base AC is perpendicular, so $\triangle BHC$ is a right triangle.
- The point where the tangent intersects is on a line parallel to AB, meaning it also creates similar triangles or symmetrical segments.

Use geometric expressions:

$$HC = CF = \frac{a}{2}$$

Let $FK = \frac{a}{2} - \frac{2}{5}b$, and $x = \frac{1}{10} \cdot a$

Then, from symmetry and proportion:

$$\frac{a}{2} - \frac{2}{5}b + \frac{a}{10} = \frac{2}{5}b \implies a = \frac{4}{3}b$$

Substitute into the perimeter equation:

$$\frac{4}{3}b + 2b = 20 \implies b = 6;\ a = 8$$

Step 4: Use Pythagorean theorem to find triangle height

In right triangle $\triangle BHC$:

$$BH^2 = 36 - 16 = 20 \implies BH = \sqrt{20}$$

$$AC = 8$$

Step 5: Find triangle area

Use the formula for the area of a triangle:

$$s_{ABC} = \frac{1}{2} \cdot \text{base} \cdot \text{height} = \frac{1}{2} \cdot 8 \cdot \sqrt{20} = 8\sqrt{5}$$

Step 6: Find inradius

Use the formula:

$$r = \frac{S}{p} = \frac{2 \cdot 8\sqrt{5}}{20} = \frac{4\sqrt{5}}{5}$$

Answer: $r = \frac{4\sqrt{5}}{5}$

Example 6. In the isosceles triangle $\triangle ABC$, side AC is a chord of a circle, and the center of the circle lies inside triangle ABC. A line passing through point B intersects the circle at points D and E. Given that $AB = BC = 2$, $\angle ABC = 2\, arcsin \frac{1}{\sqrt{5}}$, and the radius of the circle is 1, find the area of triangle $\triangle DBE$.

Given:

$AB = BC = 2$

$\angle ABC = 2\, arcsin \frac{1}{\sqrt{5}}$

$R = 1$ (the radius of the circle)

AC is a chord of the circle

To find:

S_{BDE} – ?

Solution:

Let's assume the center of the circle is point OO, and since the radius is 1, we know:

$$OE = AO = OC = OD = 1$$

Step 1: Determine coordinates and trigonometric values

We are given:

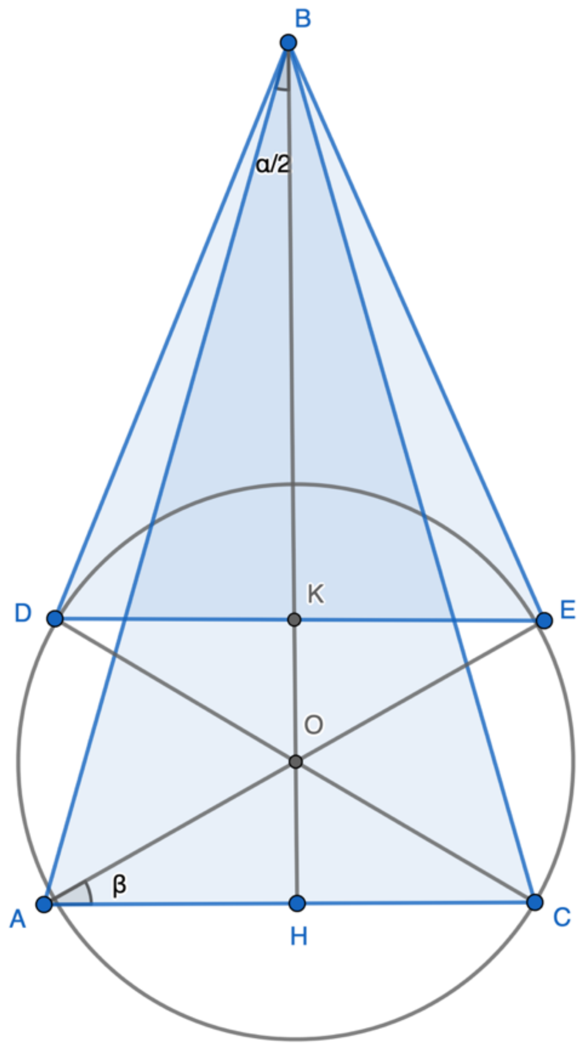
$\angle ABC = 2\, arcsin\left(\frac{1}{\sqrt{5}}\right)$, so half of that angle is:

$$\beta = \ arcsin\left(\frac{1}{\sqrt{5}}\right)$$

This implies:

$\cos\beta = \frac{AH}{AO} = \frac{\frac{2}{\sqrt{5}}}{1} = \frac{2}{\sqrt{5}}$, and $\sin\beta = \sqrt{1-\left(\frac{2}{\sqrt{5}}\right)^2} = \frac{1}{\sqrt{5}}$

$$\sin\beta = \frac{OH}{AO} = \frac{1}{\sqrt{5}} \Rightarrow OH = AO \cdot \sin\beta = \frac{1}{\sqrt{5}}$$



Step 2: Use triangle properties

Since the triangle is isosceles, the height splits the base, and:

$$\sin\frac{\alpha}{2} = \frac{AH}{AB} = \frac{AH}{2} \Rightarrow AH = 2 \cdot \sin\left(arcsin\left(\frac{1}{\sqrt{5}}\right)\right) = \frac{2}{\sqrt{5}}$$

Then:

$$AC = AH \cdot 2 = \frac{4}{\sqrt{5}}$$

Step 3: Use Pythagoras to find segment lengths

$$BH^2 = AB^2 - AH^2 = 4 - \frac{4}{5} = \frac{16}{5} \Rightarrow BH = \frac{4}{\sqrt{5}}$$

$$OB = BH - OH = \frac{4}{\sqrt{5}} - \frac{1}{\sqrt{5}} = \frac{3}{\sqrt{5}}$$

$$BD^2 = BO^2 - OD^2 \ \Rightarrow BD = BE = \sqrt{\frac{9}{5} - 1} = \frac{2}{\sqrt{5}}$$

Step 4: Use triangle area formula for $\triangle\, BDE$

Let angle φ be such that:

$$\sin\varphi = \frac{BD}{BO} = \frac{\frac{2}{\sqrt{5}}}{\frac{3}{\sqrt{5}}} = \frac{2}{3}$$

Then the height DK from point D to side BE is:

$$DK = \sin\varphi \cdot 1 = \frac{2}{3}$$

Using Pythagoras:

$$OK^2 = DO^2 - DK^2 \Rightarrow OK = \sqrt{1 - \frac{4}{9}} = \frac{\sqrt{5}}{3}$$

$$BK = BO - OK = \frac{3}{\sqrt{5}} - \frac{\sqrt{5}}{3} = \frac{4}{3\sqrt{5}}$$

Also:

$$DE = 2 \cdot DK = 2 \cdot \frac{2}{3} = \frac{4}{3}$$

Now calculate area:

$$S_{BDE} = \frac{1}{2} \cdot BK \cdot DE = \frac{1}{2} \cdot \frac{4}{3\sqrt{5}} \cdot \frac{4}{3} = \frac{8}{9\sqrt{5}}$$

Answer: $S_{BDE} = \frac{8}{9\sqrt{5}}$

2. METHODOLOGY

2.1 Instrument

In our study, two main tools were developed to assess the impact of solving complex problems related to isosceles triangles on the development of students' geometric thinking and their academic performance. The first tool was a thematic test on the topic "Application of the Features and Properties of an Isosceles Triangle" aimed at identifying the level of academic achievement. The second tool was a survey determining the level of geometric thinking and problem solving skills on the topic "Isosceles Triangle".

2.1.1 Achievement test

The achievement test was developed to assess the level of students' geometric thinking, their ability to apply the properties of an isosceles triangle when solving problems, make calculations, draw conclusions, and interpret geometric situations. The test consists of 20 multiple-choice tasks corresponding to the secondary school level. Each task is aimed at testing one or more of the following skills: understanding the features of an isosceles triangle; application of theorems on the equality of angles and sides; calculation of perimeter, height, area; analysis of geometric conditions; interpretation of diagrams and drawings; solving problems with indirect conditions; application of knowledge in compound (contextual) tasks. Closed questions were compiled, with five answer options (A-E), with one correct answer.

The content of the tasks includes: Numerical tasks (calculation of perimeter, height, sides); Geometric reasoning (angles, properties of the median, bisector, height); Tasks with drawings; Proportions and relationships in triangles; Contextual tasks on the application of the properties of an isosceles triangle.

The difficulty level of the tasks varies from basic to advanced and meets the requirements of the 7th grade school curriculum. Some tasks are suitable for preparing for Olympiads or diagnostic tests.

The reliability of the test content was confirmed by expert assessments of experienced mathematics teachers, who provided feedback on the compliance of the test tasks with the learning objectives.

When conducting the study, we developed a plan for conducting a test on the topic "Application of the features and properties of an isosceles triangle" to ensure the accuracy and reliability of the collected data. This process includes several key stages:

- The researcher worked with experts in the field of mathematics in education to develop a test that determines the level of knowledge. All questions were checked by experts and an analysis was made of the specificity of the questions and their level of difficulty.
- A pilot test was conducted with participants who were not included in the list of main participants. This check showed which questions needed to be replaced or corrected. Based on this check, adjustments were made.
- The organization of the main testing included the preparation of test materials in printed form, proctors were instructed on the correctness during testing.

- On the day of the main testing, all participants were instructed. During the testing, proctors monitored the situation to avoid any violations and to maintain academic integrity, and answered questions that participants might have.
- To ensure the reliability of the data obtained, it was important that the testing conditions were the same for both groups.
- Calculation of the scores took place. Every student's result were examined and compiled into a report.

Ensuring the validity of the results and their possible influence on educational practice and policy depended mostly on following ethical norms, openness and methodological rigidity throughout this procedure.

2.1.2 Attitude Test

To comply with the topic guidelines and assess problem solving levels, a survey was developed where the responses were based on a Likert scale. Since the survey was not actually tested, pilot tests were conducted with a sample of materials that were not participants in the study to establish reliability. An internal agreement survey was conducted using Cronbach's alpha coefficient.

The survey consists of 18 questions and includes 4 subscales: general attitude towards mathematics, geometric thinking, problem solving skills, and the use of technology in mathematics. Respondents rated their understanding of all questions using a five-point Likert scale.

The following indicators will be used in the future:

General attitude to mathematics: This school of research gives the level of interest and how the student gets acquainted with mathematics in general. The questions determine whether students consider mathematics interesting and necessary for representing life.

Geometric Thinking: measures students' ability to visualize, spatially represent, logically analyze, and understand geometric texts. Particular attention was paid to the purpose of the experiment on isosceles triangles.

Problem Solving Skills: This assessment evaluates students' ability to approach mathematical problems strategically and systematically. Does the student know how to choose the right solution path and know where and what formula to use.

Use of Technology in Mathematics: The scale measures digital literacy and assumes that modern technologies improve the process of learning mathematics, such as calculators, GeoGebra, educational platforms, etc. The perception of technology as a resource that facilitates understanding of the material is also assessed. This scale reflects the digital literacy of students and their readiness to use modern teaching tools in mathematics.

In the reverse survey, the Cronbach's alpha coefficient was 0.979, which is an indicator of low internal consistency. For all questions, the indicator of good correlation between them (above 0.80) except for one, question Q3 showed the lowest low correlation ($r = 0.300$), and the indicator in the restriction question alpha would have been up to 0.986.

Table 2.1.1 Reliability Statistics of the Questionnaire Scale (based on Cronbach's α coefficient)

	Mean	SD	Cronbach's α
scale	3.76	0.805	0.979

When an item is eliminated for every question in the questionnaire, the Table 2.1.2 displays the values of standard deviation (SD), item-rest correlation, and Cronbach's α coefficient. For most items, the item-rest correlation values show high consistency with the general scale—that is, values exceeding 0.8. About 0.96, elements Q1, Q5, and Q10 show the strongest association value. With a clearly poor correlation with the others (0.300), Question Q3 indicates its weak link with the general scale's construct. Deleting Q3 would raise Cronbach's α to 0.986, far higher than that of the other questions, suggesting perhaps the necessity of its revision or elimination.

Table 2.1.2 Item Reliability Statistics and Cronbach's Alpha if Item Deleted

			If item dropped
	SD	**Item-rest correlation**	**Cronbach's α**
Q1	1.084	0.966	0.977
Q2	0.754	0.803	0.979
Q3	1.215	0.300	0.986
Q4	0.754	0.803	0.979
Q5	1.084	0.966	0.977
Q6	0.853	0.885	0.978
Q7	0.793	0.875	0.978
Q8	0.985	0.905	0.978
Q9	0.577	0.806	0.979
Q10	0.739	0.964	0.977
Q11	0.900	0.806	0.979
Q12	0.900	0.899	0.978
Q13	0.888	0.900	0.978
Q14	1.115	0.933	0.977
Q15	0.996	0.916	0.977
Q16	1.073	0.945	0.977

	SD	Item-rest correlation	If item dropped
			Cronbach's α
Q17	0.953	0.917	0.977
Q18	0.937	0.939	0.977

Exploratory factor analysis (EFA) was performed to determine the suitability of the data for factor analysis. The analysis showed that all questions except Q3 loaded strongly on one factor (from 0.78 to 0.98), Q3 had a low value (0.306), which indicates a weak connection with the survey as a whole. In the Table 2.1.3 we show Bartlett's test of sphericity was significant ($\chi^2 = \infty$, $p < 0.001$), which indicates variability of the data for factor analysis. We came to this conclusion, the question Q3 was changed: "For the safety and reliability of our survey."

Table 2.1.3 Bartlett's Test of Sphericity for Factorability Assessment

χ^2	df	p
Inf	153	<.001

The correlation analysis between the variables was tested using the Pearson correlation coefficients. Table 2.1.4 shows the correlation between the following variables: General attitude to mathematics, Geometric Thinking, Problem Solving Skills, Use of Technology in Mathematics, Academic Achievement. The results of the correlation analysis showed that:

The correlation between General attitude to mathematics and Academic Achievement is 0.654. This indicates that there is a moderately strong positive relationship between these variables. Students with a more positive attitude to mathematics have higher learning outcomes.

The correlation between Problem Solving Skills and Academic Achievement is 0.518. This indicates that there is a positive correlation between these variables. This shows that good problem solving skills contribute to academic success in mathematics.

The correlation between Geometric Thinking and Academic Achievement is 0.514. This indicates that there is a moderately positive correlation between these variables. This shows that geometric thinking directly affects academic achievement in mathematics.

The correlation between Use of Technology in Mathematics and Academic Achievement is 0.125. This indicates that there is a very weak correlation between these variables, or no correlation at all. This means that the use of technology in teaching does not have a significant impact on academic achievement in mathematics in this study.

The correlation between General attitude to mathematics and Geometric Thinking is 0.447. This indicates that there is a positive correlation between these variables. Students with a more positive attitude to mathematics have more developed geometric thinking.

The correlation between General attitude to mathematics and Problem Solving Skills is 0.403. This indicates that there is a positive correlation between these variables. Students with a positive attitude to mathematics often have better problem solving skills.

Table 2.1.4 Correlation coefficients for main variables

	Attitude	**Geometry**	**Problem-Solving**	**Technology**	**Academic Achievement**
Attitude	—	0.447	0.403	0.060	0.654
Geometry	0.447	—	0.462	0.233	0.514
Problem-Solving	0.403	0.462	—	0.402	0.518
Technology	0.060	0.233	0.402	—	0.125
Academic Achievement	0.654	0.514	0.518	0.125	—

To summarize the data presented above, the results of the correlation analysis showed that there is a statistically significant positive relationship between attitudes toward mathematics, geometric thinking, problem solving skills and academic performance. However, the level of connection with the use of technology was weak ($r = 0.125$), which requires further research and detailed analysis.

2.2. Participants

To foster effective learning, teaching should adapt teaching methods to bring about positive changes in students (Ezeddine et al., 2023). One must search for ways to teach and learn that align with a constructivist approach (Popova et al., 2022). Integrating real-world applications into math education is based on educational theories that emphasize active learning, student engagement, and critical thinking skills (Chau et al., 2021). Project-based learning, case studies, and Injure Based Learning are effective in making the learning process relevant, meaningful, and challenging (Mebert et al., 2020).

Effective teaching strategies include using group work, encouraging students to ask questions, and giving students problems related to daily life (Rivai et al., 2021). Group work encourages collaborative learning, which allows students to learn from one another, share ideas, and develop teamwork skills (Chau et al., 2021).

This professional development can include workshops, mentoring, and peer observation. Teachers must be given the time and resources they need to plan and implement engaging lessons, so that they can work together to share best practices and reflect on their teaching.

The results of the pedagogical experiment were published in 2025 by the Ministry of Education and Science of the Republic of Kazakhstan “Dostyq” (Almaty). This study used a quasi-experimental design with whole-group sampling. Two groups with 7th-grade students were selected for the experiment; these groups already existed before the experiment. Each group consisted of 12 students, and in total, 24 participants participated in our study.

2.3 Data Collection

The experimental part of our work began with the preparatory part, where each participant in the experiment was informed about the study. This procedure was done to comply with ethical standards for obtaining information, the age of the participants was taken into account. The study was anonymous, to avoid leaking personal data and information. Since the participants were minors, consent was obtained from the head teacher of the school to conduct the experiment. The purpose of the study was to develop a methodology for solving complex problems related to isosceles triangles. Two tests were used: a test for academic performance, a survey. The tests were conducted twice for each group to determine the impact of our methodology on the experimental group. The duration of the study was nine weeks, which allowed for enough time to observe significant changes or trends arising as a result of educational interventions. This period was considered sufficient for the implementation of the developed methodology. The experiment was conducted on a voluntary basis, which simplified the process, the participants felt comfortable, due to which one could not doubt the honesty of the respondents' answers. The data were collected using a paper survey. Students were given instructions on how the testing would be conducted and how to fill out the survey. There was enough time for the survey, which ensured maximum honesty from the participants. Careful preparation of the participants and a planned process of implementation provided us with high reliability of the data and the result.

2.4 Process

During the teaching process, the teacher integrated a new teaching method to better understand new topics and increase student engagement. The method that was used for our study is called Inquiry-Based Learning. This is an approach to learning where students make discoveries themselves, actively explore, ask questions, based on which they make their own hypotheses and analyze the collected data, and then make reasonable conclusions. The advantage of this method is that the student himself becomes a researcher and discovers new things for himself, instead of receiving ready-made information from the teacher. The main participant in this method is the student, and the teacher is a person who only gives direction. The IBL method is based on the constructivist theory of learning, which states that students understand and remember a topic better if they explore it on their own, using their experiences and the knowledge they have acquired. The student develops critical thinking, intelligence, the ability to rely on data and fundamental understanding.

An example lesson plan is provided below to show how the IBL approach is actually implemented. This course of instruction took place during the experimental stage of the research and concentrated on the use of geometric reasoning to solve isosceles triangle problems.

<table>
<tr><td>Тема долгосрочного плана: Треугольники</td><td colspan="2">Школа: Образовательный центр Достық</td></tr>
<tr><td>День:</td><td colspan="2">ФИО учителя: Дүйсеке А.Қ.</td></tr>
<tr><td>Класс: 7</td><td>Присутствовали:</td><td>Отсутствовали:</td></tr>
<tr><td>Тема урока</td><td colspan="2">Равнобедренный треугольник, его свойства и признаки</td></tr>
<tr><td>Цели обучения, достигаемые на этом уроке (ссылка на учебную программу)</td><td colspan="2">7.1.1.23
Применение признаков и свойств равнобедренного треугольника.</td></tr>
<tr><td>Цели обучения</td><td colspan="2">Все учащиеся: знают определение равнобедренного треугольника, его свойства и признаки.
Большинство учащихся: умеют применять свойства и признаки равнобедренных треугольников при решении задач, а также обладают навыками построения, используя их элементы.
Некоторые учащиеся: умеют анализировать и доказывать свойства и признаки равнобедренного треугольника и применять их при решении сложных задач.</td></tr>
<tr><td>Критерии оценивания</td><td colspan="2">Различает равнобедренный треугольник и его элементы.
Может решать задачи, используя свойства и признаки равнобедренного треугольника.
Применяет свойства и признаки равнобедренного треугольника при решении задач повышенного уровня с доказательствами.</td></tr>
<tr><td>Предметная лексика и термины:</td><td colspan="2">Учащиеся: объясняют признаки и свойства равностороннего треугольника письменно и устно.</td></tr>
</table>

	Предметная лексика и терминология: треугольник, равнобедренный, равносторонний, разносторонний, элементы треугольника, высота, биссектриса, медиана, признаки равенства треугольников.
Связь с историей, культурой и языком:	История (пирамиды), география (геометрические формы в окружающей среде и архитектуре).
Межпредметные связи	История, изобразительное искусство, информатика, география.
Используемые цифровые ресурсы:	Интерактивная доска, GeoGebra, Kahoot, Blooket.
Предварительные знания:	Учащиеся знают: треугольник, признаки равенства треугольников, понятия высоты, биссектрисы, медианы.

Ход урока

Этап	Деятельность учителя	Деятельность учащихся	Методы и ресурсы
I. Мотивация и постановка проблемы 5 мин	Преподносит ситуацию: “Почему архитекторы часто используют равнобедренные треугольники в постройках?” Показывает фото (мост, юрта, памятник). Ставит проблемный вопрос: *Что делает равнобедренный треугольник особенным?*	Отвечают на наводящие вопросы, формулируют гипотезы Дескрипторы: – Отвечает на вопрос, используя собственные примеры или наблюдения – Формулирует хотя бы одну гипотезу о свойствах равнобедренного треугольника – Участвует в обсуждении предложенной проблемы	Метапредметный подход, визуализация
II. Исследование (Inquiry)	Делит класс на группы, даёт задания:	Строят равнобедренный	IBL, работа в группах,

15 мин	1) Постройте треугольник с двумя равными сторонами. 2) Измерьте углы. 3) Сформулируйте гипотезу. Помогает с инструментами (линейка, транспортир, GeoGebra)	треугольник, проводят измерения, обсуждают закономерности, записывают выводы Дескрипторы: Верно строит равнобедренный треугольник (с соблюдением условий) Правильно измеряет углы и стороны Делает вывод о свойствах на основе наблюдений Работает в команде, вносит вклад в общее задание	активное исследование
III. Объяснение и обобщение 10 мин	Направляет дискуссию: учащиеся защищают свои гипотезы, а учитель подводит к теореме о равных углах при основании	Представляют результаты, обобщают свойства Дескрипторы: Участвует в обсуждении и защите гипотезы Чётко формулирует одно или несколько свойств равнобедренного треугольника	Работа с доской, коллективный анализ
IV. Применение знаний 10 мин	Раздаёт индивидуальные задания на применение свойств (расчёт углов, доказательства). Проверка с помощью дескрипторов	Решают задачи, аргументируют ход решения Дескрипторы: Верно применяет свойства при решении задачи Аргументирует свой выбор решения или хода построения	Самостоятельная работа, критериальное оценивание

		Решает задачу без ошибок / с минимальными ошибками	
V. Рефлексия и домашнее задание 5 мин	Просит заполнить карточку «Я узнал / Мне было сложно / Я хочу узнать» + даёт домашнее задание: «Найти равнобедренный треугольник в жизни и объяснить его свойства»	Делятся впечатлениями, задают вопросы Дескриптор: Осмысленно заполняет рефлексивную форму: «Я узнал / Мне было сложно / Я хочу узнать» Формулирует, какие знания получил на уроке Делится впечатлениями и задаёт вопросы Понимает суть домашнего задания и может кратко объяснить его цель	Рефлексия, метакогнитивные навыки

The use of this methodology and technology has improved the learning environment, where everyone understands and learns new information. In addition to academic success, students have developed their social skills, such as communication, public speaking, and negotiation, which are necessary for their personal growth and adaptation to the outside world.

2.5 Achievement test analysis

The descriptive data presented in Table 2.5.1 provide a complete statistical understanding of the pre- and post-test data in the control and experimental groups. There were 12 participants in both groups. The control group participants had a mean score of 6.92 before the experiment, indicating a low level of knowledge regarding isosceles triangles. After the experimental period, the mean post-test score increased to 10.92, indicating an improvement in the level of achievement. The standard deviation of the pre-test scores was 2.15, indicating how scattered the participants' responses were around the mean, which slightly increased to 2.75 in the post-test. Now let's analyze the experimental group, the mean pre-test score was 7.42, and after the introduction of the new methodology, the post-test score increased and was 17.75 points, indicating significant improvements in the level of knowledge of the participants. The standard deviation of the pre-test is 1.78, and after 1.76, indicating little improvement. These results show that the implementation of the new teaching method had a high positive impact on the level of students' knowledge in geometry in the experimental group, which is proven by a significant improvement in the post-test grades compared to the control group.

Overall, a detailed analysis of Table 2.5.1 provides valuable information about the effectiveness of the implementation of Inquiry-Based Learning in improving the level of students' knowledge and emphasizes the importance of improving the methods, which is proven by the result after implementation.

Also, Table 2.5.1 shows the results of the Shapiro-Wilk test conducted to assess the normality of the pre- and post-test grades of both groups. The results showed that the distribution of the performance data in both groups, at the pre- and post-test stages, does not deviate from normal ($p > 0.05$).

Table 2.5.1 Descriptive data and Shapiro–Wilk normality test for control and experimental group

					Shapiro-Wilk	
	Group	**N**	**Mean**	**SD**	**W**	**p**
Pre-Test (Academic Achievement)	Control	12	6.92	2.15	0.930	0.375
	Experimental	12	7.42	2.75	0.976	0.964
Post-Test (Academic Achievement)	Control	12	10.92	1.78	0.872	0.070
	Experimental	12	17.75	1.76	0.918	0.267

To test whether there was a statistically significant difference in the students' performance before and after the study period, a paired sample t-test was conducted. In Table 2.5.2 you can see the results, this analysis compared the mean scores of the pre-test and post-test in the control group of 12 people. The result of the analysis showed little statistical improvement in the performance of the control group ($p < .001$). This suggests that the differences between the pre-test and post-test were very unlikely to be the result of chance.

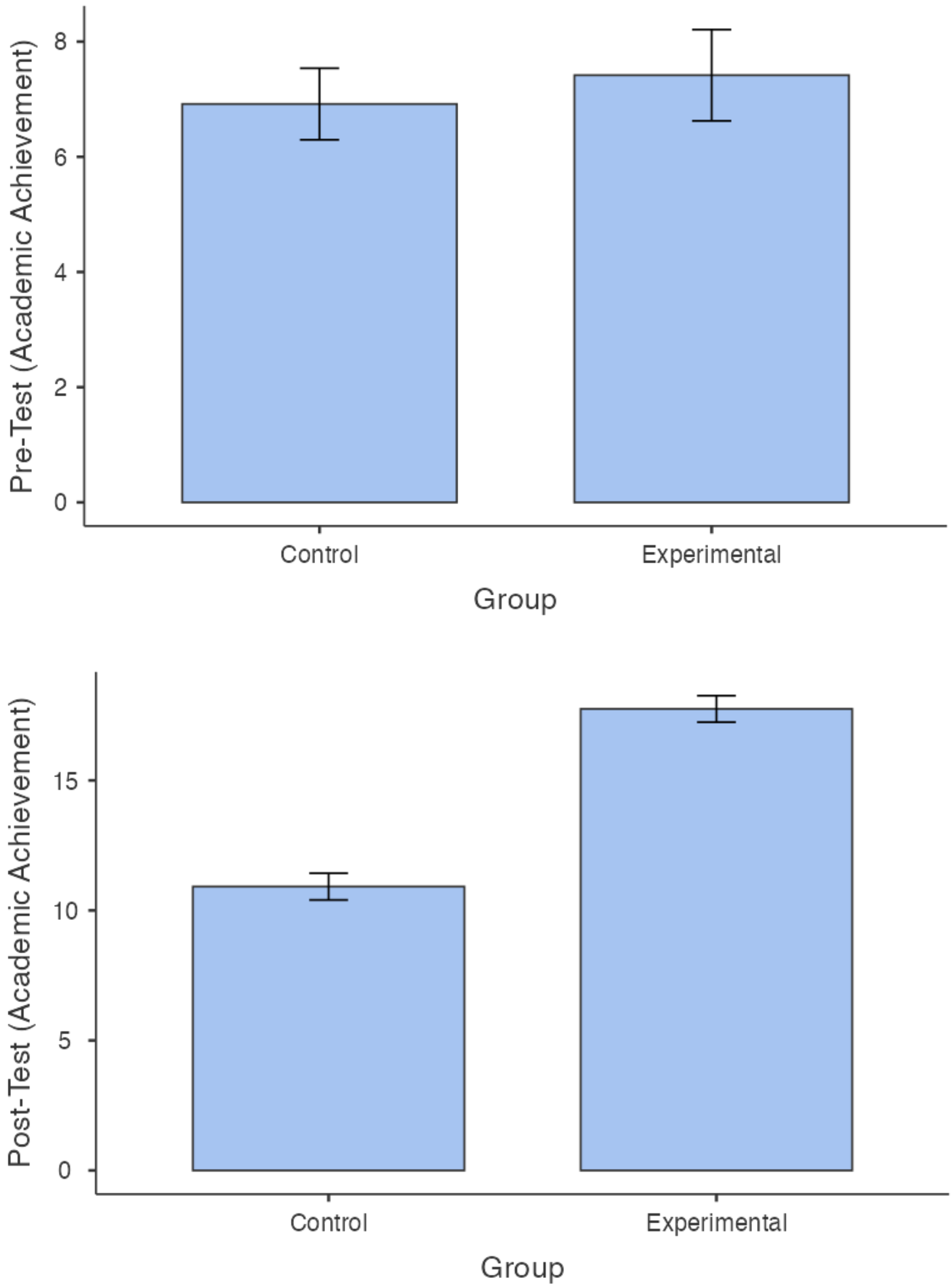
Table 2.5.2 Paired samples t-test for pre-test and post-test of control group

			statistic	**df**	**p**
Pre-Test (Academic Achievement)	Post-Test (Academic Achievement)	Student's t	-8.39	11.0	<.001

Note. H_a $\mu_{\text{Measure 1 - Measure 2}} \neq 0$

Figure 2.5.2 shows a bar chart illustrating the average performance of students in the control and experimental groups at the pre-test and post-test stages. Although the experimental group scored slightly higher before the intervention, the difference was not that big. The average scores in both groups were similar before the experiment (pre-test: experimental group 7.42, indicating no significant differences in the initial level of preparation; the control group showed an average score of about 6.92).

Figure 2.5.2 Comparison of Control and Experimental Group Performance on Pre- and Post-Test (Academic Achievement)



After the intervention, which included the use of an exploratory approach (IBL) in the experimental group, the Post-Test results demonstrate a noticeable difference. The experimental group achieved significantly higher results (an average of 17.75 points), compared to the control group (an average of 10.92 points).

Thus, the visualized data confirm the effectiveness of the proposed methodology aimed at developing mathematical thinking through active research and practical application of geometric concepts, such as the properties of an isosceles triangle.

To check whether there was a statistically significant difference in the students' performance before and after the introduction of the new methodology into the curriculum, a Paired samples t-Test was conducted. In Table 2.5.3 you can see the results, this analysis compared the mean scores of the pre-test and post-test in two groups of 12 people. The result of the analysis showed a highly significant increase in the performance of the experimental group ($p < .001$). This suggests that the variations between the pre-test and the post-test were quite improbable to have resulted from chance. A negative t-value denotes that the post-test results exceeded the pre-test ones.

Table 2.5.3 Paired samples t-test for pre-test and post-test of experimental group

			statistic	**df**	**p**
Pre-Test (Academic Achievement)	Post-Test (Academic Achievement)	**Student's t**	-13.4	11.0	<.001

Note. H_a $\mu_{\text{Measure 1 - Measure 2}} \neq 0$

Table 2.5.4 shows the test of the level of growth in academic performance and the comparison was made using the Independent Mann–Whitney U-test since the t-test showed inequality of dispersions and required further research. The Mann–Whitney criterion ($U = 0.500$, $p < .001$) indicates a statistically significant difference between the two control and experimental groups. This shows that the growth in the experimental group was significantly higher than in the control group. The data confirm the effectiveness of the implemented methodology.

Table 2.5.4 Independent samples t-test for Academic Achievement

		Statistic	**p**
Academic Achievement	Mann-Whitney U	0.500	<.001

Note. H_a $\mu_{\text{Control}} \neq \mu_{\text{Experimental}}$

Summarizing our analyses, we can say that according to the Paired samples t-Test, the results before and after the control group improved over time, but not as much as the experimental group. On the contrary, the experimental group showed a highly significant increase in the results after. This shows that the introduction of the new methodology into teaching had a great positive impact on students.

2.6 Attitude test analysis

In Table 2.6.1 you can see the descriptive statistics and the Shapiro-Wilk test of normal distribution of data, where you can see the general attitude of students towards mathematics. The analysis was conducted for the control and experimental groups before and after the interventions. The mean value for the control group pre-test was 2.95, and for the experimental 2.93. This shows similar results with a low standard deviation in both groups. And the Shapiro-Wilk test indicates normal distribution of data ($p > 0.05$). The results of the post-test of the control group, which was taught in the traditional format, the mean value was 3.43, and for the experimental group, where a new methodology was introduced into the learning process, the mean value was 4.27. This shows an increase in the result in both groups, but we see a significant difference in the results of the experimental group. The Shapiro-Wilk test analysis also indicates that the data is normally distributed ($p > 0.05$), which allows us to do parametric tests for analysis, such as Paired samples t-Test or Independent Samples T-Test.

Table 2.6.1 Descriptive data and Shapiro–Wilk normality test for pre-test and post-test of General attitude to mathematics

					Shapiro-Wilk	
	Group	**N**	**Mean**	**SD**	**W**	**p**
Pre-Test (General attitude)	Control	12	2.95	0.258	0.931	0.394
	Experimental	12	2.93	0.477	0.959	0.774
Post-Test (General attitude)	Control	12	3.43	0.239	0.929	0.372
	Experimental	12	4.27	0.299	0.953	0.682

Table 2.6.2 shows us the descriptive statistics and the test for normality of data distribution. The mean of the pre-test of the control group, which is 2.82 ($SD = 0.508$), while the result of the experimental group with a mean of 3.05 ($SD = 0.392$), does not show us a significant difference in the two groups. The Shapiro-Wilk value also shows the normality of data distribution in both groups ($p > 0.05$). The results of the post-test show us a significant increase in the experimental group, whose mean increased ($M = 4.25$). The control group also improved its average score ($M = 3.37$). However, the experimental group showed a significantly higher increase. And here the distribution of data was in the normality mark, which allows using parametric tests to analyze the experimental data.

Table 2.6.2 Descriptive data and Shapiro–Wilk normality test for pre-test and post-test of Geometric thinking

					Shapiro-Wilk	
	Group	**N**	**Mean**	**SD**	**W**	**p**
Pre-Test (Geometry)	Control	12	2.82	0.508	0.910	0.216
	Experimental	12	3.05	0.392	0.925	0.327
Post-Test (Geometry)	Control	12	3.37	0.239	0.905	0.182
	Experimental	12	4.25	0.243	0.940	0.495

Table 2.6.3 shows us the descriptive statistics and the Shapiro-Wilk test for normality of data distribution in two groups. The mean value of the control group before 2.82, and after 3.37, also for the experimental group before 3.05 and after 4.25. We can conclude that initially the results did not have a big difference, and the results after say that both groups showed growth, but statistically significant for the experimental group. The values of the Shapiro-Wilk test are also normal.

Table 2.6.3 Descriptive data and Shapiro–Wilk normality test for pre-test and post-test of Problem solving skills

					Shapiro-Wilk	
	Group	**N**	**Mean**	**SD**	**W**	**p**
Pre-Test (Geometry)	Control	12	2.82	0.508	0.910	0.216
	Experimental	12	3.05	0.392	0.925	0.327
Post-Test (Geometry)	Control	12	3.37	0.239	0.905	0.182
	Experimental	12	4.25	0.243	0.940	0.495

Table 2.6.4 shows us the descriptive statistics and the Shapiro-Wilk test for normality of data distribution in two groups. The mean value of the control group before 2.78, and after 3.00, also for the experimental group before 3.72 and after 4.28. It can be concluded that the experimental group initially had higher results, and the results after show that for both groups, the introduction of technologies does not play a big role, which proves a small increase. The Shapiro-Wilk test analysis also indicates that the data is normally distributed ($p > 0.05$), which allows us to do parametric tests for analysis, such as Paired samples t-Test or Independent Samples T-Test.

Table 2.6.4 Descriptive data and Shapiro–Wilk normality test for pre-test and post-test of Use of technology in mathematics

					Shapiro-Wilk	
	Group	**N**	**Mean**	**SD**	**W**	**p**
Pre-Test (Application of technology)	Control	12	2.78	0.538	0.947	0.600
	Experimental	12	3.00	0.586	0.898	0.148
Post-Test (Application of technology)	Control	12	3.72	0.489	0.911	0.223
	Experimental	12	4.28	0.278	0.843	0.030

To assess the significance of the differences between the results before and after the experiment, a Paired Samples T-Test was conducted within the experimental group. The test shows whether the introduction of new methods in teaching related to isosceles triangles affects statistically significant changes on four scales. According to the test results, it can be seen that the study showed that the difference between the pretest and posttest is statistically significant ($p < .001$) on all scales under study. This fact confirms the effectiveness of the technique.

Table 2.6.5 Paired t-test for pre-test and post-test for Attitude test

			statistic	**df**	**p**
Pre-Test (General attitude)	Post-Test (General attitude)	Student's t	-7.13	23.0	<.001
Pre-Test (Geometry)	Post-Test (Geometry)	Student's t	-7.67	23.0	<.001
Pre-Test (Problem-solving skills)	Post-Test (Problem-solving skills)	Student's t	-5.16	23.0	<.001
Pre-Test (Application of technology)	Post-Test (Application of technology)	Student's t	-7.63	23.0	<.001

Note. H_a $\mu_{\text{Measure 1 - Measure 2}} \neq 0$

Table 2.6.6 shows the test of the effectiveness of the pedagogical intervention between the groups, the Independent Mann-Whitney U test was used. On the scale of general attitude toward mathematics, the difference is statistically significant ($U = 12.5$, $p < .001$), the experimental group has a higher result than the control group. The scale of geometric thinking and problem solving skills also shows significant improvements. And on the scale of use of technology ($U = 54.5$, $p = .319$), the differences between the groups are not so significant ($p > .05$), which indicates the absence of influence, which requires further research.

Table 2.6.6 Independent samples t-test for Attitude test

		Statistic	p
General attitude	Mann-Whitney U	12.5	<.001
Geometry	Mann-Whitney U	23.5	0.005
Problem-solving skills	Mann-Whitney U	24.0	0.006
Application of technology	Mann-Whitney U	54.5	0.319

Note. H_a $\mu_{Control} \neq \mu_{Experimental}$

3. RESULTS

The results of this study provided important indicators of the interaction between students' attitudes towards geometry, and more specifically towards isosceles triangles. Descriptive statistics, relevance testing, correlation analysis between factors, paired sample t-tests and independent sample t-tests were used to achieve the objective of the study, which is to determine how solving complex problems related to isosceles triangles affects students' results and their academic performance.

The internal consistency of the survey showed a high rate (Cronbach's $\alpha = 0.986$), and this confirms the validity of this instrument, which is used to assess students' opinions and academic abilities.

In Pearson's correlation analysis, most of the indicators showed a positive relationship ($r = 0.514 - 0.654$), which proves their influence on each other and are important during lessons. But the use of technology showed no significant correlation ($r = 0.125$), which requires further research and analysis. With p-values $<.001$ in each case, the results of paired sample t-tests showed statistically significant increases in all four assessed areas (attitude, geometry, problem solving, and technology use) following the instructional intervention. These results imply that the Injure-Based Learning intervention, which focused on solving complex problems using isosceles triangles, was successful in improving both the affective and cognitive elements of student engagement.

Further analysis using independent samples t-tests compared delta scores between the control and experimental groups. Statistically significant differences were observed for:

Attitude toward mathematics ($t(22) = –4.54$, $p < .001$);

Geometric thinking ($t(22) = –3.46$, $p = .002$);

Problem-solving skills ($t(22) = –3.31$, $p = .003$).

These findings show that in several important areas the experimental group showed better progress than the control group. As in the previous analysis, there was no significant variation in technology use ($t(22) = –1.02$, $p = .318$), indicating a weak relationship with academic achievement. This suggests that the technology component may need to be revised or more tightly integrated to ensure significant educational gains.

Furthermore confirmed in non-parametric terms were these results using a Mann-Whitney U test. Apart from the technology use scale, the results matched those of the independent t-tests, indicating notable variations in the delta scores of all scales once more underlining the stability of this trend across statistical techniques.

Taken together, these results imply that the experimental strategy—based on solving real-life and geometrically rich tasks linked to isosceles triangles—was beneficial in enhancing students' conceptual development, engagement, and achievement. The method proved very effective in improving attitudes and reasoning abilities, which are prerequisites for ongoing mathematical achievement. Since the study involved 24 students from one school, there may be limitations. Therefore, the results may not be valid for a larger population and require further larger-scale research in the future.

Further research should also look at how technology can be used in mathematics instruction to highlight the potential for more successful integration and barriers.

Ultimately, the study supports the need to incorporate Injure Based Learning approaches — especially those that focus on challenging geometry tasks — into the middle school mathematics curriculum. These strategies not only help students achieve academic success, but also strengthen their deeper connection to the topic, making mathematics more relevant, interesting, and motivating.

CONCLUSION

The aim of our study was to improve students' skills in solving complex problems involving isosceles triangles. Thus, in the course of the work performed, all the main objectives of the study were achieved and solutions to the following problems were found:

1. The theoretical foundations for solving complex problems involving isosceles triangles were described;
2. The methodology for solving complex problems involving isosceles triangles was improved;
3. The effectiveness of the proposed methodology was experimentally tested and the results obtained were processed.

Based on an exploratory approach, culturally relevant teaching strategies, and a comparative analysis of Kazakhstani textbooks, this quasi-experimental study provided evidence of the positive impact of real and complex problems involving triangles on students' cognitive development.

An experimental group of 24 seventh-grade students from a Kazakhstani school demonstrated significant improvements in both academic performance and engagement levels, especially in areas such as geometric thinking, problem-solving strategies, and application of isosceles triangle properties. Quantitative results showed statistically significant improvements in post-test scores, with descriptive statistics, paired t-tests, and Shapiro-Wilk tests confirming the effectiveness of the intervention. In particular, the use of contextually based problems facilitated deeper understanding and transfer of knowledge to unfamiliar settings.

Analysis also revealed that students became more confident in identifying and applying properties of isosceles triangles, including base angle congruence, leg equality, and alignment of medians, altitudes, and angle bisectors from the vertex. Furthermore, students' increased ability to construct logical proofs and engage in higher-order reasoning reflected their development of geometric thinking skills. This was further supported by their improvement in responses on the attitude scale.

The pedagogical model used in this study emphasized the integration of culturally relevant motifs, historical references, and technological tools (e.g., GeoGebra, interactive whiteboards), which contributed to increased student motivation and engagement. The inclusion of Kazakh traditional ornaments and architectural patterns contributed to the development of national identity while grounding mathematical concepts in students' life experiences.

Furthermore, the analysis of textbooks revealed that although Kazakh textbooks contain a sufficient number of isosceles triangle problems, they often lack the variety and complexity needed to develop spatial thinking and creativity. This points to the need for future textbook reform and the integration of higher-level problems in line with international standards.

From a theoretical perspective, the study is consistent with cognitive load theory and supports the use of scaffolding and step-by-step modeling to help students manage complex information. The study also supports the usefulness of the "key problem method," in which instruction is structured around a central geometric problem that introduces and relates key properties.

In conclusion, the results of this study provide compelling evidence for the value of integrating challenging isosceles triangle problems into secondary geometry lessons. This approach not only improves academic achievement, but also promotes critical mathematical habits of mind, including visual thinking, logical deduction, and the ability to connect abstract concepts with real-world applications.

Key Findings:

- Geometry instruction should include more challenging, culturally relevant problems that allow for exploration, construction, and proof.
- Teacher professional development should emphasize methods to support geometric thinking, including the use of visual, technological, and real-world contexts.
- Curriculum developers should consider balancing computational problems with open-ended, creative, and evidence-based problems to develop deeper understanding.

Recommendations for Future Research:

- Expand the study to include a larger sample from a variety of regions and school types in Kazakhstan.
- Investigate the long-term retention of geometric concepts following research-based interventions.
- To examine the impact of culturally responsive mathematics education on other areas of mathematical literacy, such as algebraic thinking or data interpretation.

Ultimately, this study contributes to a growing body of research advocating for innovative, student-centered, and contextually relevant methods of teaching mathematics. It highlights the role of geometric thinking in shaping not only students' academic trajectories, but also their sense of identity, curiosity, and problem-solving confidence beyond the classroom.

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APPENDIX

Appendix 1. Attitude test

Данная анкета является анонимной и не будет показана никому, кроме исследовательской группы. Вся информация хранится в конфиденциальной форме и используется только для научно-исследовательской работы.

1 – полностью не согласен, 2 – не согласен, 3 – нейтрально, 4 – согласен, 5 – полностью согласен

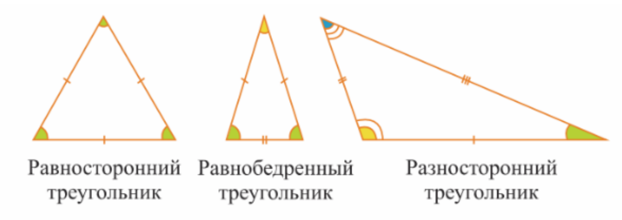
	Вопросы	1	2	3	4	5
1	Мне нравится урок математики.					
2	Я думаю, что математика пригодится мне в будущей жизни.					
3	Мне легко решать математические задачи.					
4	Для меня важно получать хорошие оценки по математике.					
5	Мне нравится решать задачи самостоятельно.					
6	На уроке геометрии мне интересно чертить фигуры.					
7	Я умею различать виды треугольников.					
8	Я знаю свойства равнобедренного треугольника.					
9	Задачи, связанные с равнобедренным треугольником, даются мне легко.					
10	Я умею правильно представлять себе геометрические фигуры.					
11	При решении задач я всегда составляю план.					
12	Если задача сложная, я разбиваю её на части и решаю поэтапно.					
13	Я могу объяснить решение геометрических задач.					
14	Я умею решать задачи, используя чертёж.					
15	Я хочу научиться приводить доказательства при решении задач.					

16	На уроке геометрии интересно использовать такие программы, как GeoGebra.					
17	Компьютерные модели помогают мне понять задачу.					
18	Манипулируя фигурой на экране, я лучше понимаю материал.					

ВИДЫ ТРЕУГОЛЬНИКОВ. РАВНОБЕДРЕННЫЙ ТРЕУГОЛЬНИК.

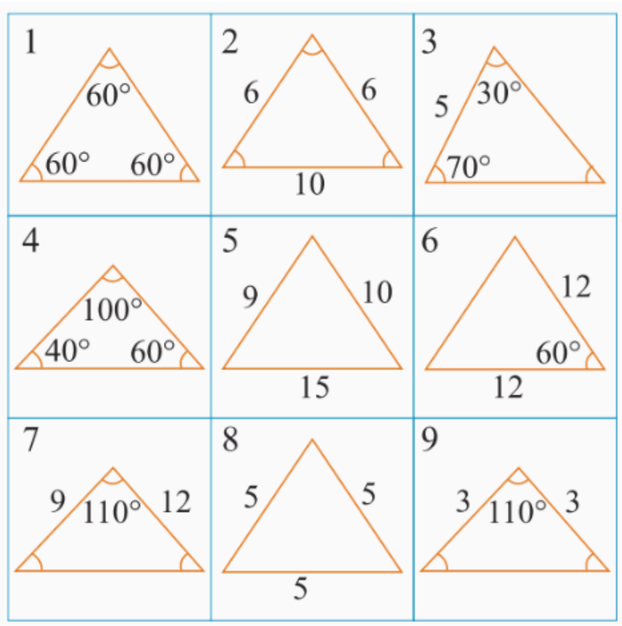
Запомните

Виды треугольников по длине его сторон:
Равнобедренный треугольник - треугольник, у которого две стороны равны.
Равносторонний треугольник - треугольник, у которого все стороны равны.
Разносторонний треугольник - треугольник, у которого стороны имеют разные длины.



- **Пример 1**

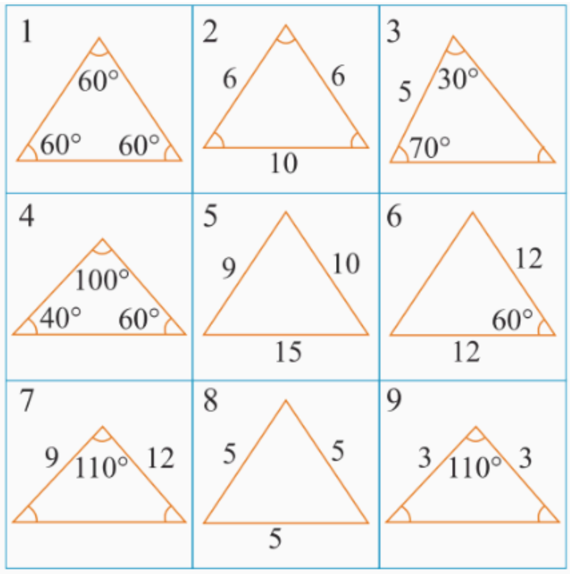
Запишите равнобедренные треугольники по таблице.



Решение:
Равнобедренные треугольники: 2, 4, 9

- **Теперь попробуйте сами**

Запишите равносторонние и разносторонние треугольники по таблице.

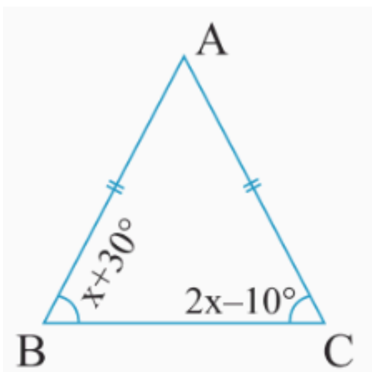


Запомните

Теорема: Углы при основании равнобедренного треугольника равны.
Обратная теорема: Если два угла треугольника равны, то это равнобедренный треугольник.

- **Пример 2**

По ниже данному рисунку найдите значение x.

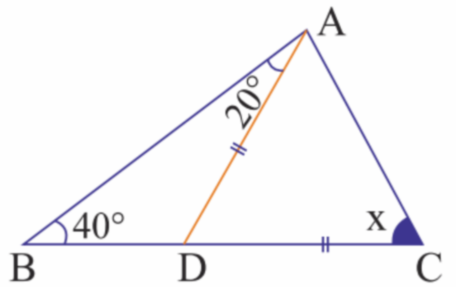


Решение:
В треугольнике ABC стороны AB = AC , поэтому это равнобедренный треугольник. Тогда $\angle B = \angle C$:

$$x + 30° = 2x - 10°$$
$$x = 40°$$

- **Пример 3**

По нижеданному рисунку найдите значение x.

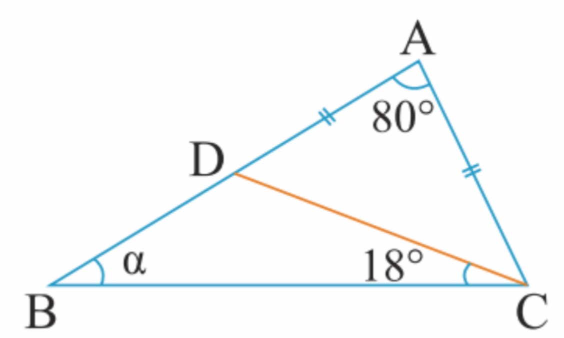


Решение:
Так как стороны AD = DC, то угол ∠DAC также равен x.
Составим уравнение:

$$x + (x + 20°) + 40° = 180°$$
$$2x + 60° = 180° \Rightarrow x = 60°$$

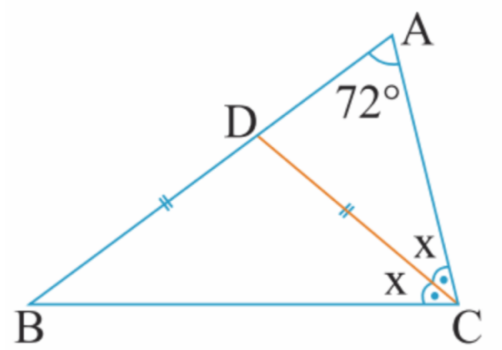
♦ **Теперь попробуйте сами**

По нижеданному рисунку найдите значение α.



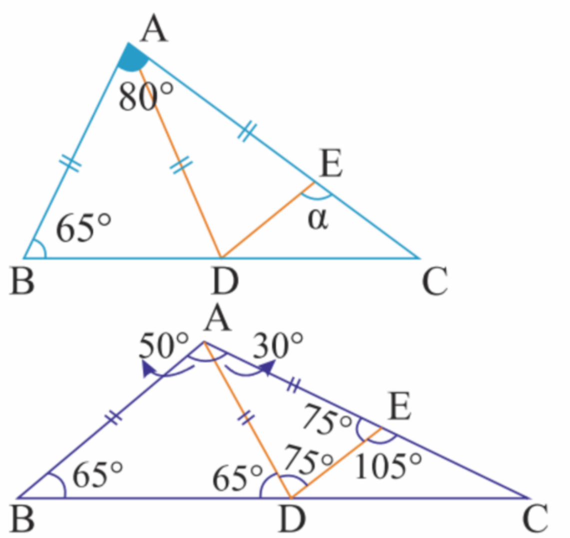
♦ **Теперь попробуйте сами**

По нижеданному рисунку найдите значение x.



- **Пример 4**

В треугольнике ABC угол ∠BAC = 80°. Найдите значение угла α.



Стороны AB = AD, углы ∠ABC = ∠BDA = 65°.
Рассмотрим треугольник ABD:

$$\angle ABD + \angle BDA + \angle DAB = 180°$$
$$65° + 65° + \angle DAB = 180° \Rightarrow \angle DAB = 50°$$

Так как угол ∠BAC = ∠DAB + ∠DAE, то:

$$80° = 50° + \angle DAE \Rightarrow \angle DAE = 30°$$

Рассмотрим треугольник ADE:

$$\angle DAE + \angle AED + \angle EDA = 180°$$

Поскольку AD = AE, углы при основании равны:

$$\angle ADE = \angle AED = x$$

$$30° + x + x = 180° \Rightarrow 2x = 150° \Rightarrow x = 75°$$

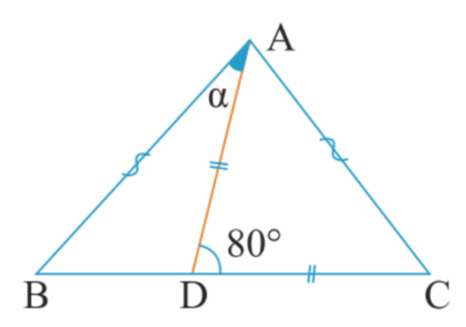
Теперь найдем угол ∠DEC, который смежный с ∠AED:

$$\angle AED + \angle DEC = 180° \Rightarrow$$
$$75° + \angle DEC = 180° \Rightarrow \angle DEC = 105°$$

Следовательно, $\alpha = \angle DEC = 105°$

- **Теперь попробуйте сами**

По нижеданному рисунку найдите значение α.

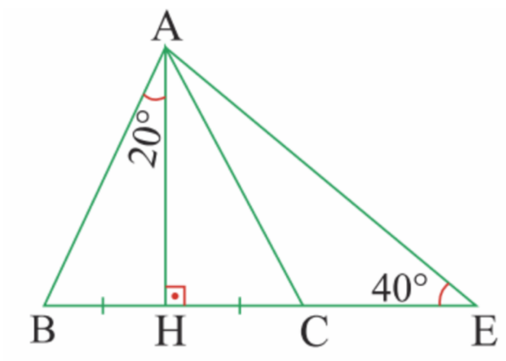


<u>Запомните</u>

Следствие: Биссектриса, опущенная на основание равнобедренного треугольника, является одновременно и *медианой*, и *высотой*.

- **Теперь попробуйте сами**

В треугольнике ABE отрезок AH⊥BE, BH = HC, ∠BAH = 20°, ∠AEB = 40°. Найдите значение угла ∠CAE.



Задача 1

На изображении представлены традиционные казахские украшения в виде равнобедренных треугольников.

Вычислите периметр каждого украшения.

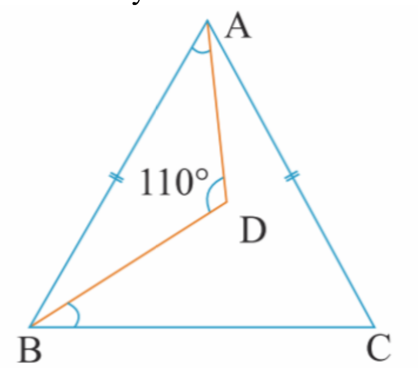
У первого украшения стороны треугольника равны: 7 см, 7 см, 10 см.

У второго украшения стороны составляют: 8 см, 8 см, 12 см.

У третьего украшения стороны составляют:
9 см, 9 см, 13 см.

Задача 2

Если в треугольнике AB = AC, углы ∠BAD = ∠DBC, и угол ∠ADB = 110°, то найдите значение угла ∠BAC.

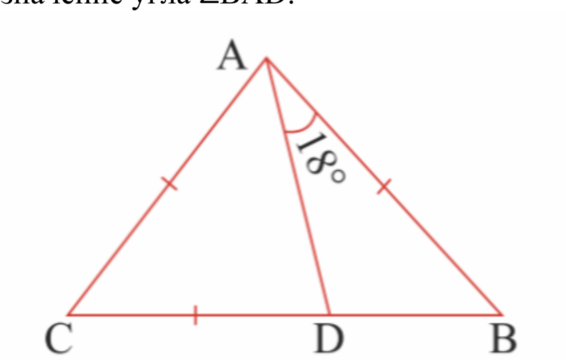


Задача 3

Углы A, B и C треугольника ABC равны 72°, 72° и 36° соответственно. Если сумма длин биссектрисы AK и
отрезка KC равна 8 см, то найдите длину стороны AB.

Задача 4

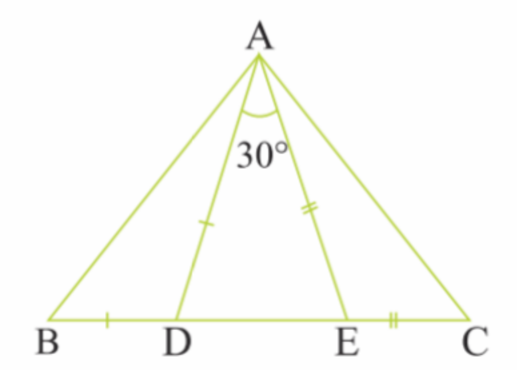
В треугольнике ABC даны равенства: AC = AB = BD, а угол ∠DAC = 18°. Найдите значение угла ∠BAD.



A) 48°
B) 56°
C) 66°
D) 72°
E) 78°

Задача 5

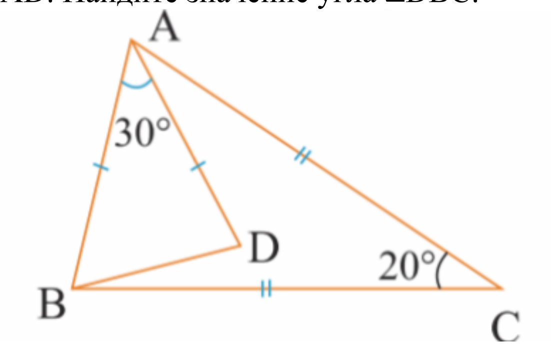
В треугольнике ABC даны равенства: AD = BD, AE = EC, и угол ∠DAE = 30°. Найдите значение угла ∠BAC.



A) 75°
B) 90°
C) 95°
D) 100°
E) 105°

Задача 6

В треугольнике ABC угол ∠BAD = 30°, угол ∠ACB = 20°, стороны AC = BC, AB = AD. Найдите значение угла ∠DBC.



A) 5°
B) 10°
C) 15°
D) 20°
E) 25°

Задача 6
Стороны равнобедренного треугольника равны 3 см и 5 см. Вычислите периметр треугольника.

A) 11 см
B) 13 см
C) 11 см или 13 см
D) 16 см
E) 13 см или 16 см

Methodology of solving complex problems associated with an isosceles triangle

Akbota Duiseke

A thesis submitted to the Faculty of Education and Humanities

in partial fulfillment of the requirements for the degree of

MASTER OF PEDAGOGICAL SCIENCES

in Mathematics

«SDU University»

Department of Pedagogy of Natural Sciences

Thesis Advisor: PhD, Associate
Professor Dzhanbulat Kayinbaev

«SDU University»

Faculty of Education and Humanities
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Akbota Duiseke

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Kaskelen, 2025

AUTHOR AGREEMENT

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Author Signature

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Date

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DECLARATION

I, the undersigned, hereby declare that this submission is entirely my work, in my own words, and that all sources used in researching it are fully acknowledged and all quotations properly identified to my best extent. It has not been submitted, in whole or in part, by me or another person, to obtain any other credit, except where due acknowledgement is made in the thesis. I understand the ethical implications of my research, and this work meets the requirements of the Faculty of Education and Humanities Research Ethics Policy.

Name: Akbota Duiseke

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Signature

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Date

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List of Abbreviations

STEM - Science, Technology, Engineering, and Mathematics
IBL - Injure-Based Learning
ICT - Information and Communication Technologies
EFA - Exploratory factor analysis

ABSTRACT

This study examines the impact of solving complex problems related to isosceles triangles on the development of students' geometric thinking and academic achievements. The research project included a performance test and an attitude test. The study was conducted with 24 students of the seventh grade, who were divided into two groups: experimental and control. The performance test, which included a pre-test and a follow-up test, measured the students' performance before and after the intervention.

In the control group, students received traditional instruction, while the experimental group studied problem-based learning (inquiry-based learning). Analysis of the results of the test on academic achievement showed that students in the experimental group demonstrated significantly higher performance in mathematics compared to the control group. This demonstrates the effectiveness of the IBL methodology in improving learning outcomes. A survey was used to assess attitudes towards mathematics. Analysis of the survey showed a significant improvement in attitude to the subject after the intervention.

The results showed a significant improvement in students' ability to solve geometric problems in the experimental group. Conclusions emphasize the effectiveness of structured problem solving and conceptual understanding in improving logical reasoning and academic outcomes.

Key words: isosceles triangle, geometric thinking, complex tasks, teaching methodology.

АҢДАТПА

Бұл зерттеу жұмысы теңбүйірлі үшбұрыштарға байланысты күрделі есептерді шешудің оқушылардың геометриялық ойлау қабілеттері мен академиялық жетістіктеріне әсерін қарастырады. Зерттеу барысында оқушылардың жетістігін анықтауға арналған тест және математикаға деген қатынасын бағалауға арналған сауалнама қолданылды. Зерттеу 7-сыныптың 24 оқушысы арасында жүргізілді, олар екі топқа бөлінді: эксперименттік және бақылау топтары. Жетістік тесті (алдын ала және соңғы тесттер) интервенцияға дейін және кейін оқушылардың білім деңгейін бағалады.

Бақылау тобындағы оқушылар дәстүрлі оқыту әдісі бойынша білім алса, эксперименттік топ сұраққа негізделген оқыту әдістемесі (inquiry-based learning) арқылы білім алды. Академиялық жетістіктерге арналған тест нәтижелерін талдау эксперименттік топтағы оқушылардың математикадан жоғары нәтижелер көрсеткенін көрсетті. Бұл зерттеу IBL әдістемесінің оқу нәтижелерін жақсартуда тиімді екенін дәлелдейді. Математикаға деген қатынасты бағалау үшін сауалнама қолданылды. Сауалнама нәтижелері интервенциядан кейін пәнге деген қатынастың едәуір жақсарғанын көрсетті.

Зерттеу нәтижелері эксперименттік топтағы оқушылардың геометриялық есептерді шешу қабілетінің айтарлықтай артқанын көрсетті. Қорытындылар құрылымдалған есеп шығару мен ұғымдық түсінудің логикалық ойлау мен оқу жетістіктерін жақсартудағы маңыздылығын атап көрсетеді.

Кілт сөздер: теңбүйірлі үшбұрыш, геометриялық ойлау, күрделі тапсырмалар, оқыту әдістемесі.

АННОТАЦИЯ

В данном исследовании изучается влияние решения сложных проблем, связанных с равнобедренными треугольниками на развитие геометрического мышления студентов и академические достижения. Проект исследования включал в себя тест на успеваемость и тест на отношение к предмету. Исследование было проведено с участием 24 учеников седьмого класса, которые были разделены на две группы: экспериментальную и контрольную. Тест на успеваемость, включающий, в себя предварительное тестирование и последующее тестирование, измерял успеваемость учащихся до и после вмешательстава.

В контрольной группе учащиеся получали традиционное обучение, в то время как экспериментальная группа занимались методикой основанный на решении проблем (inquiry-based learning). Анализ результатов теста на академическую успеваемость показал, что учащиеся экспериментальной группы продемонстрировали значительно более высокие показатели по математике по сравнению с контрольной группой. Это свидетельствует об эффективности методики IBL в улучшении учебных результатов. Для оценки отношения к математике использовалься опрос. Анализ опроса показал существенное улучшение отношения к предмету после проведённого вмешательства.

Результаты продемонстрировали значительное улучшение способности студентов решать геометрические проблемы в экспериментальной группе. Выводы подчеркивают эффективность структурированного решения проблем и концептуального понимания в улучшении логического рассуждения и академических результатов.

Ключевые слова: равнобедренный треугольник, геометрическое мышление, сложные задачи, методика обучения.

INTRODUCTION

Made up of two equal length sides, isosceles triangles provide a unique mix of geometric characteristics and challenges in mathematical problem-solving. Often, these challenges are connected to trigonometry, as students battle algebraic transformations, particularly with unknowns in numerators or denominators (Ngu & Phan, 2020). Algebraic transformation skills are crucial for handling such problems; one can observe the similarity and difference between trigonometric functions (Ngu & Phan, 2020). Moreover, the intrinsic symmetry of isosceles triangles can sometimes conceal the application of traditional geometric theorems and trigonometric identities, hence requiring a thorough understanding of their features. Sometimes isosceles triangles are found in more complicated geometric shapes as well, including compound shapes, quadrilaterals, and other triangles. Therefore, the right computation of areas, perimeters, and other basic geometric values in these challenging activities relies on how exactly they are specified and applied (Retnowati & Fadlila, 2023).

The requirement to combine several geometric ideas and theorems causes many of the difficulty in isosceles triangle issue solution. For example, issues could call for the concurrent use of the Pythagorean theorem, angle bisector theorem, and characteristics of related triangles, hence requiring a thorough knowledge of geometric ideas. Moreover, slanted coordinate systems can aggravate these challenges, especially when examining vector components linked to forces or displacements inside the triangle (Mikula & Heckler, 2014). The combination of coordinate geometry with Euclidean geometry in the framework of isosceles triangles calls for a solid basis in both fields.

In many engineering and physics applications, isosceles triangles are a basic building block; knowledge of their characteristics is crucial for structural analysis, statics, and dynamics. For instance, in structural engineering, truss designs often employ isosceles triangle shapes since the distribution of loads and stresses depends quite much on the triangle's form. The desire to discover different ways to teach mathematics to raise student performance motivated the study (Mosese & Ogbonnaya, 2021). Geometrical forms seem practically everywhere, so geometry helps education for the growth of critical thinking and problem solving (Altakhyneh, 2018; Serin, 2018). Architects need to be good at geometry (Liapi, 2002; Mifetu, 2023). A mathematical subject called geometry evaluates mathematical creative thinking capacity and it develops logical and methodical thinking (Sahliawati & Nurlaelah, 2020). Students are exposed to geometric ideas like lines, planes, and spaces before they even start school, so Geometry is a branch of mathematics they are more likely to grasp than other branches.

In geometry lessons in secondary school, it is important to teach students how to solve complex problems. In particular, tasks related to an isosceles triangle contribute to the development of logical thinking among students and demonstrate the possibility of applying their mathematical knowledge in real life. Geometry is not only a theoretical knowledge, but also one of the practical disciplines aimed at solving problems that we face in real life. In this context, President of the Republic of

Kazakhstan Kassym-Jomart Tokayev has repeatedly spoken in his messages about improving the quality of the education system and its viability.

So, in his speech, the President noted: "Our main task is to create an economy based on advanced knowledge and high technologies. To achieve this goal, first of all, high-quality education is needed." This requires special attention to the field of education, especially to subjects of mathematical and technical orientation.

In addition, in the 2021 Message, Kassym-Jomart Tokayev noted: "Modernization of technical and vocational education is our main task. We must give priority to mathematics, engineering education, and new technologies," she stressed the relevance of teaching mathematics and technical sciences. In this regard, the use of complex problems related to an isosceles triangle in geometry lessons taught in schools encourages students to apply theoretical knowledge in life, develops their analytical and logical thinking.

Overcoming Challenges in Isosceles Triangle Problems. Effectively tackling the difficulties isosceles triangle problems provide calls for a multi-faceted strategy emphasizing increasing basic knowledge, creating problem-solving techniques, and promoting critical thinking abilities (Ayan & Isiksal-Bostan, 2019). Injure Based Learning motivates students to approach critical thinking and problem-solving practically (Chau et al., 2021). Emphasizing the need of visualization and geometric intuition helps students to create correct diagrams and mentally move forms to better grasp the issue. Promoting the use of algebraic methods to solve geometric problems is another important factor (Ayan & Isiksal-Bostan, 2019). Including practical uses into math education is based on various fundamental educational ideas stressing IBL, student involvement, and the growth of critical thinking abilities (Chau et al., 2021). This means learning to use trigonometric identities, solve systems of equations, and manipulate equations such that students may convert geometric relationships into algebraic representations and the other way around. Injure Based Learning is a successful approach whereby students work on long-term projects applying mathematical ideas to address practical issues (Chau et al., 2021).

One cannot understate the significance technology plays in improving geometry education and learning, especially in relation to isosceles triangles. Interactive geometry tools like GeoGebra let students dynamically investigate geometric characteristics, create precise diagrams, and see transformations, hence promoting a better knowledge of ideas (Daulay et al., 2021). Such technologies allow students to test hypotheses, see the consequences of parameter changes, and control geometric objects, hence enabling a more interesting and intuitive learning environment. Moreover, educational technology can enable cooperative learning by letting students collaborate on problem-solving assignments, exchange ideas, and benefit from one another (Santos-Trigo et al., 2021). Studies are required that not only contrast student involvement in light of particular teaching practices with other instructional strategies, but also investigate the relationship between these teaching practices and student problem solving under instruction stressing "integrating connective components of teacher–student relationships with connective elements of instruction" (Yusuf et al., 2021). Active involvement and prior math performance are highlighted in this paper as factors improving problem-solving skills (Yusuf et al., 2021).

When solving problems related to isosceles triangles, many students make mistakes due to a lack of understanding of the relationships between the elements of the triangle and the inability to apply the knowledge gained in practice. The teacher's task is to teach students how to correctly analyze the conditions of a problem, use the properties of an isosceles triangle, and apply theorems to find unknown elements.

Real-world examples and uses of isosceles triangles can be helpfully used by the study to increase student involvement and drive. Students are more likely to value the relevance and use of their studies by linking mathematical ideas to daily life (Chau et al., 2021). For example, drawings from architecture, engineering, and art can show how isosceles triangles are employed in structural designs, bridge building, and artistic compositions (Mikula & Heckler, 2014). Future studies could also look at how well various teaching strategies help students solve isosceles triangle issues. This could mean contrasting conventional lecture-based techniques with more engaging, student-centered ones like problem-based or inquiry-based learning. Word issues relate to the evolution of abstract formal structures and mathematics (Yusuf et al., 2021). Included in curricular papers and assessment tools, word problems are part of formal mathematics instruction (Chau et al., 2021). A common method to show kids how to apply math to solve practical problems is through mathematical word problems, which is also a vital skill for them.

Important measures of success are student motivation and involvement (Chau et al., 2021). Research has demonstrated that more student involvement results in more academic success (Schukajlow et al., 2023; Serin, 2018). In the twenty-first century, problem-solving teaching methods have changed to an insight-based problem-solving approach (Acquandoh et al., 2022). A key field of mathematics education has become students' ability to solve problems. Problem-solving has been a major component of teaching and studying mathematics during the last few decades (Hafizi & Kamarudin, 2020). By means of careful consideration of these elements and use of suitable pedagogical approaches, educators can create a more interesting and efficient learning environment, hence reducing disengagement and promoting a better knowledge of mathematical ideas (Schukajlow et al., 2023). Teachers can assist students build confidence in their problem-solving skills, lower anxiety, and promote a growth mindset by establishing a supportive and motivating classroom environment (Mailisman et al., 2020). This means commending effort, offering helpful criticism, and designing chances for students to present their answers and benefit from their errors (Chau et al., 2021; Rahmah, 2017).

Controversy in Isosceles Triangle Problem Solving. One source of debate in resolving isosceles triangle problems is the suitable degree of formalism and rigor in mathematical proofs. Emphasizing conceptual knowledge and geometric reasoning over rigorous following of formal proof methods, some teachers support a more intuitive and casual approach. Younger pupils or those who find abstract ideas challenging may find this strategy especially useful since it lets them grasp the fundamental ideas more deeply without being mired in technicalities. Some teachers, nevertheless, contend that cultivating pupils' logical reasoning abilities and equipping them for more advanced mathematics courses depend on a strict attitude to mathematical proofs. Active discovery produces questions and answers that closely fit real-world applications since students are urged to find issues, theorize answers, and examine data in ways that reflect professional practices (Chau et al., 2021). Generally speaking, problem-solving has been acknowledged as a way to advance thinking skills and an important learning process in the mathematics curriculum (Yusuf et al., 2021). Active participation in the mathematical problem-solving process helps students acquire mathematical ideas, techniques, and skills as well as alternative modes of thought and action.

Another subject of continuous discussion is mathematics instruction using technology. Some teachers fear that although technology might improve visualization, investigation, and problem-solving, if not handled correctly it could also result in a shallow grasp of ideas. To guarantee that it supports and improves conventional teaching approaches, not replaces them entirely, effective integration of technology calls for deliberate planning and educational considerations (Santos-Trigo et al., 2021). Because they provide pupils chances to grow their own knowledge of mathematics, mathematical activities are rather essential for mathematics education. Mathematical challenges allow teachers to either introduce new subjects to students or provide them chances to use what they have already acquired. Through problem-solving, teachers can assess and inspire students' capacity to apply mathematical ideas in original and interesting ways (Salim & Tiawa, 2014). Through problem-solving, teachers can assess and support students' capacity to apply mathematical ideas in creative and interesting ways (Salim & Tiawa, 2014). In mathematics, problem-solving is fundamental, (Alfayez et al., 2022). Mathematics depends on the capacity to answer problems (Alfayez et al., 2022). It is managing challenging or unknown circumstances utilizing mathematics knowledge, skills, and understanding. Mathematics educators have employed several strategies in their teaching to enable students to improve as problem solvers (Isnawati et al., 2021).

Mathematical knowledge acquisition could change from one class of pupils to another (Febrilia & Nissa, 2019). Given the various teaching techniques, it is therefore essential to find whether or not secondary school pupils' mathematics performance varies. The results of this study showed that students exposed to problem-solving strategy outperformed their peers exposed to traditional approach (Syaiful et al., 2020). Students should be able to actively pursue new mathematical information and employ the skills they presently have. Establishing a learning environment that minimizes any possible tension or anxiety and encourages interest and excitement for mathematics is another aim (Febrilia & Nissa, 2019). Teachers can make learning more pertinent and interesting by using real-world examples, hands-on activities, and group projects (Febrilia & Nissa, 2019). Teachers should also ensure they provide chances for students to collaborate, discuss their ideas, and benefit from each other's knowledge (Yerizon et al., 2019). Teachers are supposed to provide pupils a lot of practice in contextual issues (Ningrum et al., 2019).

Significance of Isosceles Triangle Problem Teaching. Isosceles triangle problems are taught in mathematics education with utmost importance since they help students' cognitive growth and problem-solving skills in many ways (Ezeddine et al., 2023). More sophisticated geometric ideas and applications are built on a foundation of knowledge of the characteristics and relationships inside isosceles triangles. Furthermore, mastery of isosceles triangle problem solving develops critical thinking abilities like logical reasoning, spatial perception, and deductive analysis. This covers the capacity to evaluate and consider the problem-solving process itself as well as to adapt and apply various techniques to address challenges (Hobri et al., 2018). Problems that come up in actual situations should be identified and addressed using mathematical modeling (Chau et al., 2021). A basic talent that is much appreciated in many spheres of life is the capacity to solve mathematical challenges (Santos-Trigo et al., 2021). Success in STEM disciplines as well as in daily decision-making and critical thinking depend on mathematical problem-solving abilities (Yuristia & Musdi, 2020). Students' ability to solve mathematical problems is a key predictor of their academic and professional performance, hence this ability must be developed and strengthened (Zulfa & Andriyani, 2023).

Students who work on isosceles triangle issues learn to apply mathematical ideas to actual scenarios, hence promoting a better respect of the relevance and utility of mathematics. Inherently symmetrical and special, isosceles triangles offer a rich background for investigating geometric relationships and honing problem-solving abilities. A basic talent that is quite appreciated in many areas of life is the capacity to solve mathematical problems (Kharomah & Abduh, 2023). Given that students' ability to solve mathematical problems is a key predictor of their academic and professional performance, this ability must be developed and strengthened ("Analysis of Mathematical Problem Solving Ability Viewed from Student Learning Style," 2020; Battista et al., 2018). Moreover, the study of isosceles triangles improves students' spatial thinking skills and geometric intuition, which are crucial for success in many sectors including engineering, architecture, and computer graphics. Students encounter changes to their information in the learning process by adding parts, enhancing, evolving, or altering the old knowledge (Setyaningsih et al., 2019).

Some have argued that identifying patterns is a key strategy for addressing mathematical problems and can help foster critical thinking (Fadiana et al., 2018). Effectively solving mathematical problems depends on the ability to think critically, which is also quite important (Herawati & Amelia, 2021; Yanto, 2019). A person who can think critically will always investigate and question whether the encounter he meets relates to what he already knows (Maulidiya & Nurlaelah, 2019). Furthermore, when students are instructed on how to understand issues, build mathematical models, offer solutions, and analyze the results, their problem-solving skills are strengthened (Rahmah, 2017). Students' mathematical creative thinking capacity is a habit that may be developed (Istiqomah et al., 2018). Based on intuition but yet in awareness, mathematical creative thinking is a mix of logical and divergent thinking (Rochmad et al., 2019). Thus, we require a learning approach that can energize students, so they can help others to simplify the implementation of spatial mathematics connected to three-dimensional material, and expand students' mathematical knowledge, as well as have abilities in learning autonomy (Isyrofinnisak et al., 2020; Rochmad et al., 2019; Saputri et al., 2020). Mastering isosceles triangle problems helps students build a strong basis in geometry and hone necessary problem-solving abilities that will help them in future mathematics projects and outside (Ayan & Isiksal-Bostan, 2019). The National Mathematics Advisory Panel study underlined the need of not just computational fluency but also conceptual knowledge and problem-solving abilities (Yusuf et al., 2021). Consequently, good teaching techniques have to include chances for students to interact with difficult issues calling for original and innovative application of their knowledge (Chau et al., 2021).

The study of triangles, especially geometry, provides a rich environment for developing mathematical creativity and problem-solving abilities (Sahliawati & Nurlaelah, 2020). Geometry, which includes spatial reasoning, visualization, and the investigation of links and patterns, provides a natural stage for the growth of mathematical creativity. Mathematics is good for encouraging creativity (Nadjafikhah & Yaftian, 2013). Moreover, even if math is not usually linked with it, mathematical education should be seen as a chance for growing creativity (Švecová et al., 2014). Geometric challenges inspire pupils to think creatively, investigate many angles, and create original answers. Creative thinking is seeing things from various angles, hence mathematical creativity guarantees general mathematical development (Sahliawati & Nurlaelah, 2020). Teachers can create a classroom atmosphere that respects innovation and intellectual curiosity by motivating students to investigate several possibilities and defend their logic. Through creativity, students can create new ideas connected to work or daily life and address challenges in unconventional ways (Isyrofinnisak et al., 2020; Nadjafikhah & Yaftian, 2013). Moreover, including technology into the instruction of geometry can give pupils interactive tools and simulations that improve their knowledge and encourage innovative investigation.

Actively including pupils in mathematical exercises will provide them a chance to think creatively and inventively (Zakeri et al., 2023). Active student participation in mathematics increases their chances of developing a deeper knowledge of the subject and a higher respect of its beauty and usefulness (Mann, 2006). Actively including pupils in mathematical exercises can improve their problem-solving abilities, increase their confidence, and help them to have a good attitude towards mathematics. Teachers in the classroom can foster creativity by motivating students to question, investigate other answers, and defend their rationale. Though teachers have to overcome obstacles to guarantee proper integration, including real-world applications into math instruction has notable advantages (Chau et al., 2021). Using addition, subtraction, multiplication, and division, teachers used real-life situations like budgeting a weekly allowance or creating a shopping list (Chau et al., 2021). This strategy not only improves students' problem-solving skills but also increases their love of mathematics in everyday life.

Teachers can assist students to recognize the relevance and usefulness of mathematics by motivating them to apply mathematical ideas to actual circumstances (Ayan & Isiksal-Bostan, 2019).

All of these factors support students' creative performance in a creative educational environment, which includes difficult tasks, freedom, learning-oriented motivation, trust, and debate. Students are more likely to acquire a deeper knowledge of the content and a higher respect for its beauty and usefulness when they are confronted with challenges requiring innovative thinking and application of their knowledge in novel ways (Katz & Stupel, 2015). Teachers should support pupils' logical skills, acknowledge and develop their innovative ideas (Er, 2023). Emphasizing difficult assignments, supporting independence, stressing learning-oriented motivation, establishing trust, and supporting discussion help teachers to inspire mathematical creativity (Rodrigues et al., 2018; Silver, 1997). Teachers may build a classroom atmosphere that respects creativity and intellectual curiosity by allowing kids the opportunity to investigate various techniques and motivating them to learn from their failures.

Their enthusiasm and performance can be greatly enhanced by actively including children in the learning process and linking mathematical ideas to practical problems (Herges et al., 2017). While those who are intrinsically driven study because they are personally pushed by the content or just enjoy it, those who are extrinsically motivated sometimes study for a test to obtain a good mark (Herges et al., 2017). Taking demanding math classes helped students develop intrinsic motivation; completing a rigorous assignment helped them believe in their math skills, which in turn drove their desire to learn more (Buket Özüm Bülbül, 2021). Teaching and teachers shape student drive towards learning (Schukajlow et al., 2023). By creating a welcoming and inclusive classroom, giving their students chances to achieve, and providing them consistent feedback and assistance, teachers can excite and inspire their pupils (Herges et al., 2017; Isyrofinnisak et al., 2020). Furthermore, educators ought to provide chances for pupils to collaborate, exchange ideas, and benefit from one another's knowledge (Gordon, 2019; Haylock, 1985).

To thoroughly investigate the complexities of isosceles triangles, emphasizing their special characteristics and the difficulties they pose in mathematical problem-solving, therefore improving mathematical skills among pupils. The study of isosceles triangles, with their natural symmetry and special qualities, provide a rich background for investigating geometric ideas and honing problem-solving abilities (Hafizi & Kamarudin, 2020). Practicing in class calls for quality time set aside (Febrilia & Nissa, 2019). The teacher's job is to run the class, ask questions, offer tools, and help the students build answers so the teacher can track their involvement and offer direction as required. Dealing with these issues calls for a multifaceted strategy combining creative teaching methods, focused interventions, and a supportive learning environment.

Moreover, the study of isosceles triangles offers a great chance to improve students' geometric intuition, spatial thinking, and problem-solving skills, therefore equipping them for more complex mathematical ideas and practical uses (Chau et al., 2021). Understanding more complicated geometric shapes and relationships is strongly founded on mastering the characteristics of isosceles triangles.

Teachers can assist children in overcoming obstacles and growing a better respect for the beauty and usefulness of geometry by means of well planned exercises and teaching. Students can deepen their knowledge of geometric ideas and improve their problem-solving abilities by investigating the links between angles, sides, and areas inside isosceles triangles. Solving different mathematical problems depends on knowledge of angle and side relationships inside triangles (Zubainur et al., 2020). Teachers should motivate kids to investigate several ways to solve isosceles triangle-related issues, hence promoting innovation and critical thinking.

Exploring geometric relationships inside isosceles triangles helps students to develop their spatial reasoning and visualization abilities, which are required for advanced problem-solving.

Effective teaching and learning in geometry education depend on knowledge of prevalent misunderstandings (Salifu, 2021). Dealing with widespread misunderstandings about isosceles triangles calls for focused teaching and exercises emphasizing the particular areas of student difficulty (Ayan & Isiksal-Bostan, 2019). Teachers can assist students in developing a more correct and strong knowledge of isosceles triangles by means of direct attention to these problems and by giving them chances to rectify their misconceptions (Prehatiningsih & Suparno, 2019).

The natural intricacy of the topic causes students in geometry to find abstract ideas difficult (Battista et al., 2018). Many would-be preschool instructors lack fundamental knowledge in geometry, thereby stressing the need of improved geometrical knowledge (Markovits & Patkin, 2020). Proportional thinking is used in the vital learning fields of geometry and measurement (Ayan & Isiksal-Bostan, 2019). By establishing a learning environment where knowledge of structures and mathematical skills prevails, teachers can help students with low mathematical beliefs (Skilling et al., 2021).

The main focus of research is on resolving mathematical issues connected to isosceles triangles, particularly those in proportional reasoning within geometry and measurement (Ayan & Isiksal-Bostan, 2019). Students' struggles with proportional reasoning issues in geometry and measurement underscore the necessity for thorough research (Ayan & Isiksal-Bostan, 2019). Daily life is directly tied to the study of geometry, which helps pupils to better grasp geometric ideas (Anista & Marsigit, 2020).

Relevance of the study. The study of isosceles triangles included in the 7th grade geometry course plays an important role in the formation of students' spatial thinking, logical analysis skills and problem solving skills. The isosceles triangle, with its unique properties, serves as an element that serves as a basis for developing ideas about symmetry, the connections between walls and corners, and preparing for the study of more complex topics.

The subject for research. The methodology for solving complex problems related to an isosceles triangle in geometry lessons in high school.

The object for research.The process of developing mathematical thinking and creative abilities of high school students in the process of solving complex problems related to an isosceles triangle in geometry lessons.

Aim of the research. Improving students ' skills in solving complex problems related to isosceles triangles.

Research question. How does solving complex problems related to isosceles triangles affect the formation of geometric thinking of students and their academic performance?

Hypothesis. If in high school the methodology of solving complex problems related to an isosceles triangle is used, then the geometric and analytical thinking of students develops, and the skills of applying mathematical knowledge in life increase.

Objectives of the study

1. Determination of the theoretical basis for solving complex problems related to isosceles triangles

2. Development of a new methodology for isosceles triangles

3. Experimental verification of the effectiveness of the methodology, processing the results

Special tasks that develop proportional thinking and the ability to represent objects in space, especially when combined with creative teaching methods, help students better cope with complex challenges on isosceles triangles. This in turn strengthens their mathematical skills and increases their self-confidence.

The premise is that modern methods in class would improve the learning experience of the student and produce general better academic outcome (Bakar & Ismail, 2020). Student-centered projects link math to reality via Injure Based Learning. This approach promotes teamwork, critical thinking, and problem-solving, hence improving knowledge and pleasure (Ayan & Isiksal-Bostan, 2019). Students who grasp geometry can properly grasp real-world scenarios, examine relationships, and solve difficulties.

Mathematical problem solving is based on proportional reasoning (Yusuf et al., 2021). Completing practice-based training helped future instructors to increase their proportional thinking (Pişkin Tunç & Çakıroğlu, 2022). They usually used a small number of techniques to solve issues and applied algebraic processes before the instruction without linking significance (Pişkin Tunç & Çakıroğlu, 2022). Prospective instructors are said to have a highly vital competence in problem solving.

For teachers to develop methods in helping students, they must grasp the techniques of problem-solving. Teachers can use it in real life using problem-solving techniques (Bahri et al., 2021). Teachers using efficient teaching techniques should guide students for better knowledge (Hoth et al., 2022). Learning about ratios and proportions might be difficult for many students (Andini & Jupri, 2017). Single ratio, step-by-step increase and cross multiplication are some of the most common ways to solve problems on relation and proportion (Setyaningsih et al., 2019). Although cross-multiplication allows for a quick answer, students often use it mechanically, not understanding how the proportion itself works (Öztürk et al., 2021). According to observations of Akara (Setyaningsih et al., 2019), pupils often solve such problems from textbooks by cross-multiplication. Focusing on the multiplicative links between values helps one to teach proportional thinking. It means helping pupils to identify and articulate the consistent ratio between two values.

Proportional thinking is the mental comparison of quantities multiplicatively and the resolution of proportional problems in many practical settings and activities (Ayan &

Isiksal-Bostan, 2019). It also underlines the need of understanding the several facets of proportional situations. Students who battle ratio and proportion are said to be battling multiplicative thinking. Even if they encounter proportions in daily life, students can find it difficult to grasp the proportional link of a situation (Mudrika et al., 2024). In one of the studies, it was found that students find it difficult to understand how many pairs in proportion should be multiplied or divided by the same number without any changes in the equation (Kohen & Orenstein, 2021). Errors in the proportional reasoning problems are the result of teaching students processes without understanding the concepts on which they were based.

Real situational problems often involve proportional thinking in geometrical and measuring problems or their properties. Functional reasoning helps to understand and apply the natural relationships in tasks, whereas proportional reasoning is based on arithmetic operations for solving problems where an equal relationship is necessary (Akatugba & Wallace, 2009). Understanding complex mathematical subjects and scientific ideas, including volume, speed and temperature, requires proportional and logical thinking (Miragliotta & Baccaglini-Frank, 2021). Research has revealed a correlation between students' proportional reasoning skills and their performance in algebra and other higher-level mathematics classes (Mudrika et al., 2024).

1. LITERATURE REVIEW

1.1 Theoretical foundations of solving problems related to isosceles triangles

Deeply rooted in the history of mathematics, from ancient civilizations such as the Egyptians and Greeks, the idea of an isosceles triangle characterized by at least two sides of the same length has great resonance. Early mathematicians, including those in ancient Greece, understood the special characteristics of these triangles, such as the coincidence of the basic angles opposing equal sides, which led to the emergence of many geometric theories and constructions (Ezeddine et al., 2023). A basic work in geometry, Euclid's Elements contains several ideas and hypotheses about triangles, especially isosceles triangles, which set the stage for later studies of their characteristics (Ponte et al., 2023).

Historically, architecture, topography, and astronomy (Sunzuma & Maharaj, 2020), as well as other disciplines, have used the internal symmetry and geometric features of isoscales. For a long time, geometry has been the preferred tool for describing and understanding the environment (Ponte et al., 2023). These ideas help to understand the relationships between the sides and angles of isosceles triangles, thus allowing us to solve a wide range of geometric problems (Altakhyneh, 2018).

Modern research covers a wide range of subjects from theoretical mathematics to pragmatic applications, thus expanding the traditional knowledge of isosceles triangles. In engineering and design environments, research on the use of computer-aided design and dynamic geometry has studied and optimized the geometric characteristics of isosceles triangles (Ayan & Isiksal-Bostan, 2019). In addition, the extension of the meaning of isosceles triangles beyond the limits of classical geometry is the study of non-Euclidean spaces.

Modern mathematical education makes use of isosceles triangles as required instruments to expose pupils to basic geometric concepts including symmetry, congruence, and angular relations. Often part of instructional strategies, practical exercises, geometric creations, and problem solving inspiring students to explore and capture the features of these triangles on their own - all part of geometry improves mathematical thinking and creative problem solving (Kusno et al., 2021; Mosese & Ogbonnaya, 2021).

Geometry as a subject, especially triangles including isosceles triangles play a key role in the development of students' skills of spatial and critical thinking, from include in the national curriculum of mathematics in Kazakhstan. The growing emphasis on creative educational strategies that use the characteristics of isosceles triangles to improve spatial reasoning is evident in recent educational studies conducted by Kazakh scientists (Popova et al., 2022). These approaches align with international trends that promote STEM education to foster higher-order and creative thinking abilities (Zakeri et al., 2023).

Beyond traditional classroom instruction, the use of technology has become a critical factor in teaching geometry. Tools that go beyond mentally representing geometric solids during lessons, such as GeoGebra, allow students to visualize and manipulate shapes, improving understanding and engagement. Current research emphasizes the use of such tools to create culturally relevant and accessible educational materials that meet the needs of diverse learners (Verner et al., 2019).

Isosceles triangles also allow one to teach more intricate geometries including trigonometry, coordinate geometry, and geometric transformations. Knowing and referencing various geometric forms including circles, squares, polygons, and shapes in space helps one to have a more whole knowledge of the spatial interactions of geometric shapes - triangles offers a point of reference. Scientists studying geometry have discovered a link with the isosceles triangles and the Fibonacci gold ratio, thus proving its significance not only in mathematics but also aesthetically and constructively significant role in such areas as art, architecture and natural forms.

The continuous study of isosceles triangles is still important today, as a new view on them reveals more and more interrelations with the surrounding world, and this shows its relevance in mathematics. For the study of theory, as well as practical application of knowledge about isosceles trumpeters their simplicity is emphasized, symmetry making them close to perfection. Knowledge of the qualities and properties of isosceles triangles gives students a solid foundation for understanding the significance of geometry and solving complex mathematical problems in real life.

The study of problems and tasks related to isosceles triangles from the time of Euclid and Pythagoras until our times proves their special importance not only in developing mathematical skills, but also help students in expanding other useful qualities, Including real-life problem solving, logical and creative thinking. Their unmatched value in terms of developing mathematical literacy and knowledge about mathematics demonstrates their constant presence in courses and studies.

Isoseles triangles are academically significant for the development of logical thinking abilities and geometric thinking in secondary school, according to recent studies and observations of Kazakhstani math teachers (Musaibekov et al., 2023). These materials show that the properties and qualities of isosceles triangles are fundamental in the teaching of geometry and serve as an important foundation, as well as help to pave the way for further in-depth study of topics in geometry.

Analysis of problems of the republican and international mathematical olympiads has shown that triangles, including properties and characteristics of isosceles triangles, are often used in complex geometric tasks, which allows students to solve complex spatial problems (Sarsenov, 2020). The curriculum built on this basis will make Kazakh schoolchildren competitive and strengthen their status for greater success in the future.

Efforts to incorporate ethnomathematical elements into geometry instruction in Kazakh-language schools have also used isosceles triangle patterns found in traditional crafts and architecture, demonstrating their cultural and pedagogical relevance (Tleubergenova, 2019). This proposition laid the foundation for connecting formal geometric reasoning with national identity and heritage in classroom practice.

The development of a problem-centered approach to teaching geometry in Kazakhstan was significantly shaped by the work of Abylkasymova, who emphasized the use of triangle-based tasks to foster logical reasoning and independent thinking (Abylkasymova, 2004).

The integration of visual and constructive strategies into the teaching of triangle properties, including isosceles triangles, was advanced through Dalinger’s cognitive-visual methodology, which highlighted the role of analogical thinking and dynamic constructions in geometric reasoning (Dalinger, 2001).

The psychological and logical complexity of triangle-based proof tasks was systematically analyzed through Friedman's structural-task approach, providing a theoretical basis for evaluating the cognitive demands of geometric problems in classroom settings (Friedman, 1983).

The inclusion of culturally relevant elements, such as traditional Kazakh patterns based on triangle symmetries, laid the foundation for ethnomathematical approaches that connect formal geometric instruction with national identity and visual heritage (Temirbekova, 2016).

The use of isosceles triangle tasks in mathematical Olympiads and national assessments contributed to the development of higher-order reasoning skills and demonstrated their central role in Kazakhstan’s geometry curriculum (Baimukhanov & Smirnov, 2016).

1.2 Key properties and theorems related to isosceles triangles

Isosceles triangles possess a unique set of properties and theorems that distinguish them from other types of triangles. One of the defining characteristics of an isosceles triangle is that it has two sides of equal length, which are referred to as the legs, while the third side is called the base. The angles opposite the equal sides, known as the base angles, are congruent, meaning they have the same measure. This property, known as the base angle theorem, is fundamental in solving problems involving isosceles triangles. The axis of symmetry, which bisects the base and the vertex angle opposite the base, divides the triangle into two congruent right triangles, which is another key characteristic.

Another important theorem related to isosceles triangles is the angle bisector theorem, which states that the angle bisector of the vertex angle in an isosceles triangle bisects the base and is perpendicular to it. This theorem provides a powerful tool for solving problems involving the lengths of the sides and the measures of the angles in isosceles triangles. The angle bisector theorem is crucial for solving various geometric problems. Conversely, if a median is also an altitude, then the triangle is isosceles (Heydari & Muroi, 2023).

The properties and theorems associated with isosceles triangles serve as essential tools for geometric problem-solving. In addition to the base angle theorem and the angle bisector theorem, other important properties of isosceles triangles include the fact that the altitude from the vertex angle to the base bisects the base and the vertex angle. These properties and theorems are not only valuable for solving mathematical problems but also have practical applications in various fields, such as engineering, architecture, and design.

The study of isosceles triangles also extends to more advanced topics, such as trigonometry and analytic geometry, where their properties can be used to derive trigonometric identities and solve geometric equations. By understanding the key properties and theorems related to isosceles triangles, students and professionals alike can develop a deeper appreciation for the beauty and utility of this fundamental geometric shape.

1.3 Common challenges in solving problems related to isosceles triangles

Despite the well-established properties and theorems associated with isosceles triangles, students often encounter several challenges when attempting to solve related problems. One common challenge lies in correctly identifying the given information and determining which properties and theorems are applicable to the specific problem. Students may struggle to distinguish between the legs and the base of the isosceles triangle, or they may fail to recognize the relationships between the sides, angles, and altitudes.

Another challenge involves applying the appropriate algebraic techniques to solve for unknown quantities.Students need to develop proficiency in using these techniques to solve for unknown side lengths, angle measures, or other geometric elements. Students sometimes have difficulties when solving trigonometric problems (Ngu & Phan, 2020).

Additionally, students may struggle with visualizing the geometric relationships within isosceles triangles and drawing accurate diagrams. This is further compounded by an inability to see the proportional relationships (Ayan & Isiksal-Bostan, 2019). A well-drawn diagram can often provide valuable insights into the problem and help students identify the appropriate solution strategies.

These challenges can be addressed through targeted instruction, practice, and problem-solving strategies that emphasize the importance of understanding the underlying concepts, developing strong algebraic skills, and visualizing the geometric relationships within isosceles triangles. Teachers will support students in mastering geometric contents, so that students can identify and discuss the relationships between the properties of different shapes (Kusno et al., 2021).

Additionally, it is important to note that many students think that mathematics is a very difficult subject (Sulistyaningsih et al., 2021). It is possible that some students have a strong understanding of math principles, but when they are asked to apply those concepts in the real world and write them down, they find it more challenging (Benedicto & Andrade, 2022).

1.4 Pedagogical, didactic, and cognitive conditions for solving complex problems related to isosceles

Pedagogical, didactic, and cognitive approaches to teaching isosceles triangle problems have been significantly shaped by Kazakhstani mathematics educators. Recent contributions emphasize the integration of structured problem-solving methods, visual reasoning, and context-based strategies to enhance students' ability to approach complex geometric tasks involving isosceles triangles.

Effective problem-solving in mathematics, particularly with complex isosceles triangle problems, necessitates careful consideration of pedagogical, didactic, and cognitive conditions to foster students' understanding and skills. Instruction should be student-centered, beginning with concrete examples and gradually progressing to abstract concepts, allowing students to construct their knowledge actively (Rahmah, 2017). This pedagogical approach aligns with constructivist learning theories, which emphasize the importance of prior knowledge and experiences in shaping understanding (Jahnke et al., 2022). Teachers should cultivate student's mindset to cooperate, so that they find mathematics an interesting subject (Setyaningsih et al., 2019).

The foundational work of Abylkasymova (2004) established a framework for task-based learning in mathematics education, where problems related to isosceles triangles are used to develop formal proof skills and spatial understanding. Her methodological guidelines promote a gradual progression from basic classification tasks to advanced problems requiring deductive argumentation and construction-based reasoning. This framework laid the foundation for building mathematical thinking through problem complexity and scaffolding techniques (Abylkasymova, 2004).

Dalinger (2001) further advanced the instructional methodology by focusing on visual-cognitive strategies. His approach encourages the use of diagrams, dynamic geometry software, and analogical modeling to explore properties of isosceles triangles—such as the coincidence of median, altitude, and angle bisector from the apex. This method fosters deep structural understanding and supports the transition from empirical observation to formal proof in geometry education.

Didactically, teachers should employ varied instructional strategies such as cooperative learning, problem-based learning, and inquiry-based learning to cater to diverse learning styles and preferences (Klang et al., 2021). Cooperative learning, for instance, promotes peer interaction and collaboration, enabling students to learn from each other and develop their problem-solving skills collectively (Klang et al., 2021). Problem-based learning, on the other hand, presents students with real-world scenarios that require them to apply their knowledge of isosceles triangles to find solutions (Alvin, 2022; Rivai et al., 2021). Inquiry-based learning encourages students to explore and investigate mathematical concepts through experimentation and discovery (Friedman, 1983; Vale & Barbosa, 2023).

Friedman’s structural-task analysis, though originally developed in the Soviet pedagogical tradition, remains widely applied in Kazakhstan. His model provides a system for evaluating the cognitive load of geometric problems, highlighting the importance of internal structure and logical chains in tasks involving isosceles triangle constructions or proofs (Friedman, 1983).

Temirbekova (2019) contributed to culturally responsive pedagogy by incorporating traditional Kazakh patterns and architectural forms that often rely on isosceles triangle symmetries. This approach not only contextualizes abstract geometric concepts but also enhances student engagement through meaningful and locally relevant content. Such ethnomathematical strategies have been shown to increase motivation and comprehension among students in Kazakh-language schools. Moreover, Kazakhstan mathematics teachers and Olympiad coaches have played a critical role in formalizing instructional sequences for advanced triangle problems. Through national competitions and enrichment programs, they developed systematic approaches to teaching geometric construction, loci, and concurrency problems involving isosceles triangles—often serving as benchmark cases for more complex reasoning tasks (Baimukhanov & Smirnov, 2016).

Cognitively, students need to develop strong spatial reasoning skills to visualize and manipulate geometric shapes effectively. Cognitive load theory suggests that the amount of information that students can process at one time is limited, and instructional design should aim to minimize extraneous cognitive load to optimize learning outcomes (Mangarin & Caballes, 2024). The use of visual aids, such as diagrams and animations, can help reduce cognitive load by presenting information in a clear and concise manner. Scaffolding, where teachers provide temporary support to students as they learn new concepts, can also help reduce cognitive load and promote mastery. Motivational factors, such as interest, relevance, and self-efficacy, play a significant role in students' engagement and achievement in mathematics. Students who are interested in the topic are more likely to be motivated to learn and persist in the face of challenges. Teachers should emphasize the development of metacognitive skills, such as self-monitoring and self-regulation, to help students become aware of their thinking processes and monitor their progress towards problem-solving goals. Students should be mindful of the issues pertaining to social, cultural, political and ethical dimensions, which can impact the solution process (English, 2023).

Furthermore, assessment practices should align with instructional goals and provide students with opportunities to demonstrate their understanding and problem-solving skills in various ways, such as through written explanations, diagrams, and presentations. Teachers should teach using appropriate methods for effective learning (Ezeddine et al., 2023).

Collectively, these contributions have established a pedagogical foundation in Kazakhstan that balances formal rigor with accessibility, promotes visual reasoning, and embraces both cultural identity and global standards in geometry education. By addressing these pedagogical, didactic, and cognitive conditions, educators can create a supportive and stimulating learning environment that enables students to develop the knowledge, skills, and attitudes necessary to tackle complex problems related to isosceles triangles and other mathematical concepts (Darmawan et al., 2020; Gordon, 2019).

Incorporating real-world applications and interdisciplinary connections into the study of isosceles triangles can significantly enhance students' understanding, engagement, and appreciation for the subject matter. By demonstrating the practical relevance of isosceles triangles in various fields, such as architecture, engineering, and design, educators can motivate students to learn and see the value of mathematics in their daily lives (Chau et al., 2021). For instance, students can explore how isosceles triangles are used in the construction of bridges, buildings, and other structures, or they can investigate how they are applied in the design of furniture, clothing, and artwork (Serin, 2018).

Furthermore, interdisciplinary connections can enrich students' learning experience by integrating mathematical concepts with other subjects, such as science, technology, engineering, and the arts. Students' interest can be stimulated, and an active environment can be created by strengthening the application of modeling ideas (Segura & Ferrando, 2023). For example, students can investigate the relationship between isosceles triangles and the golden ratio in art and architecture or explore the use of isosceles triangles in the design of musical instruments (Chau et al., 2021). Such interdisciplinary explorations can help students develop a deeper understanding of mathematical concepts and their connections to the broader world.

Moreover, real-world applications and interdisciplinary connections can foster students' critical thinking, problem-solving, and creativity skills. By engaging in authentic tasks and projects that require them to apply their knowledge of isosceles triangles to solve real-world problems, students can develop their ability to analyze complex situations, identify relevant information, and generate innovative solutions. Teachers should be able to incorporate real-world problems into their lessons, as they may feel pressured to adhere to rigid guidelines and prepare students for exams that focus on procedural skills rather than critical thinking (Chau et al., 2021). For instance, students can design a sustainable building using isosceles triangles or create a mathematical model to estimate the amount of material needed to construct a triangular structure. This approach not only makes the course content more relatable but also fosters a sense of mastery, as students apply classroom material to real-world problems (Mebert et al., 2020).

Integrating real-world applications into math education is grounded in educational theories that emphasize active learning, student engagement, and the development of critical thinking skills (Chau et al., 2021). The messy, open-ended nature of real-life situations contrasts with the tidier hypothetical situations many students are accustomed to in the classroom (Mebert et al., 2020). Incorporating real-world applications and interdisciplinary connections into the study of isosceles triangles requires careful planning and implementation.

Teachers need to identify relevant and engaging real-world examples and design activities and projects that align with the curriculum and learning objectives. They also need to provide students with the necessary resources and support to successfully complete these tasks. In addition, teachers need to assess students' understanding and problem-solving skills in authentic contexts and provide them with feedback on their performance. By implementing these strategies, educators can create a learning environment that enables students to develop a deep understanding of isosceles triangles and their applications in the real world (Rochmad et al., 2019; Rodrigues et al., 2018). Meaningful learning can be achieved by applying mathematics to real-world situations (Bahri et al., 2021). Such opportunities can also assist students in appreciating that mathematics is not merely a means of calculating answers but is also a vehicle for social justice, where critical thinking plays a key role (English, 2023).

In mathematics education within a STEM setting, there is a dual challenge: developing a mathematical literacy perspective that encompasses a rich view of mathematical epistemic practices, and representing the diverse professional settings in which mathematics is created and used (Herawati & Amelia, 2021). This approach enables them to appreciate the relevance and applicability of mathematics in various contexts (Wardono et al., 2021). In light of these considerations, research on pedagogical psychology in solving complex problems associated with isosceles triangles should focus on the development and evaluation of instructional interventions that incorporate cognitive principles, metacognitive strategies, and motivational techniques. Investigating the effects of different instructional approaches on students' problem-solving skills, spatial reasoning abilities, and attitudes towards mathematics can provide valuable insights into effective teaching practices. It is essential to foster the ability to identify and utilize mathematical concepts in real-world contexts, thereby enhancing problem-solving abilities and deepening understanding (Abdul-Basir et al., 2021). Academic engagement is undeniably critical to successful problem-solving, influencing student learning outcomes (Yusuf et al., 2021). Students who are more engaged tend to be more academically successful and satisfied with their educational experiences (Mebert et al., 2020). By emphasizing the connections between mathematical concepts and real-life scenarios, educators can make mathematics more accessible and engaging for students (Astuti et al., 2021). Actively involving students in applying their knowledge to novel situations is crucial. Collaboration among students is beneficial (Mebert et al., 2020). Integrating math with real-world applications through active learning not only helps to meet diverse students' needs but also maintains academic rigor (Chau et al., 2021).

1.5 Structural model for working with isosceles triangles

One must have a disciplined model including analytical approaches, spatial reasoning tactics, and problem-solving procedures if one wants to properly negotiate the complexity related with isosceles triangles.

The model should first stress the need of knowing the basic features of isosceles triangles, such congruence of base angles and the link between side lengths and angles. While learning mathematics, students must grasp real settings (Segura & Ferrando, 2023). Mathematical problem-solving becomes a very significant element to be ingrained in kids since it helps them to strengthen their thinking ability (Aggarwal, 2020).

Laying the foundation for task-taxonomy-based work with isosceles triangle problems, Ardabaeva (2023) suggested a methodical approach for geometry instruction in middle schools comprising sequenced levels of difficulty, integration of intersubject links, and varied approaches of student activity. From identification to formal proof and building, this offers a structural framework for task organization, therefore facilitating student development across cognitive processes.

Second, the model should include a range of problem-solving techniques including geometric transformations to examine congruence and similarity, Pythagorean theorem to find missing side lengths, and trigonometric ratios to estimate angles (Chau et al., 2021). Another approach for students to participate in activities demanding them to solve real-world problems is project-based learning (Bakar & Ismail, 2020; Cunha et al., 2024). In this sense, students tackle real-world problems using mathematical ideas and techniques.

An esteemed teacher from Nur-Sultan, Dybyspayev (2021), underlined in task-based courses the development of creative thinking in geometry education. His efforts on innovative problem-solving help students to produce nonstandard answers to isosceles triangle constructions and proofs, therefore strengthening the creative aspect of the structural model.

Students should be urged to employ these flexible and imaginative techniques, customizing them to fit the particular setting of every challenge. Teachers should help pupils to grasp the issue and organize the way of remedy. Scaffolding can be given by teachers to help pupils with first attempts at solving problems (Daulay et al., 2021). Hints, leading questions, or deconstruction of the problem into smaller, more doable steps can all fit this structure.

Furthermore stressing the need of reviewing solutions and considering the process of problem-solving should be the model. Students should be urged to assess the rationality of their responses and to confirm them using several techniques (Barana et al., 2022). Developing injure based questions can help students to have chances to strengthen their ability to solve problems (Zulfa & Andriyani, 2023).

Recent research on educating pre-service mathematics teachers at ENU exposed methodological difficulties and geometric figure drawing solutions. It shown that employing dynamic digital environments such GeoGebra and 3D-modeling tools enables precise construction and mental comprehension of isosceles triangles, thereby improving the structural model with modern ICT tools (Manganyana et al., 2020).

Third, the model should encourage the acquisition of spatial reasoning abilities including mentally rotating and reflecting shapes, building precise diagrams and representations, and perceiving and manipulating geometric figures in three dimensions, mentally. Encouragement of students to create several answers to a problem would help teachers also promote innovative problem-solving. This method not only improves their ability to solve problems but also advances a closer knowledge of mathematical ideas. Mathematical education in both elementary and high schools has included problem-solving (Eisenmann et al., 2022). The aim is to equip students with the ability to apply mathematical knowledge and abilities to solve real-world situations, make informed judgments, and so properly help society (Astuti et al., 2021).

Emphasizing expert-based criteria and the efficacy of software like AutoCAD for applied geometric abilities, Orazali & Mekebaev (2024) created a computer-assisted geometry teaching model For complicated triangle building, their technique uses digital check-and-feedback loops to improve structural fluency and correctness.

Fourth, the model should encourage analytical thinking abilities like generalizations, pattern recognition, and logical conclusions grounded on geometric ideas. Teachers should help pupils to learn lifetime and to change with the times. Furthermore, students can investigate several ways to approach a problem, therefore improving their ability to solve it as well as their knowledge of mathematical ideas. Good cooperation can help to raise the performance in solving problems.

Furthermore underlined by Zhaguparev (2021) the application of "method of the key problem" key-problem approaches in geometry courses. This method arranges learning around central isosceles triangle problems serving as gates to related geometric ideas and techniques.

Moreover, the model should have a feedback system that lets students evaluate their approaches of solving problems, spot areas needing work, and polish their plans. Teachers can provide their pupils the tools and knowledge they need to boldly address challenging problems involving isosceles triangles by using such a disciplined approach. Changing the educational system will help to raise the caliber of human resources (Mailisman et al., 2020). Among the mathematical ability needed of pupils are those related to problem-solving (Mailisman et al., 2020). Students should be able to use their mathematics knowledge and skills to address practical challenges.

This methodology fosters advanced thinking and curriculum requirements by skillfully merging Kazakhstani research contributions with worldwide instructional techniques. It strikes a mix of digital fluency, strategic focus on important issues in geometry instruction, creative inquiry, and orderly development.

Given the incorporation of technology, dynamic geometry software presents a revolutionary method of teaching and learning about isosceles triangles thereby allowing students to investigate geometric ideas dynamically. This incorporation of technology emphasizes its possibilities in enhancing mathematical education, developing critical thinking, and so strengthening a deeper knowledge of mathematical concepts (Cunha et al., 2024; Rholey R. Picaza, 2023). An interesting mathematics learning environment depends on the technological competency of teachers (Öztürk et al., 2021).

Dynamic geometry tools let students test hypotheses in real-time, adjust geometric figures, and see relationships (Miragliotta & Baccaglini-Frank, 2021). This practical experience can improve students' knowledge of the characteristics of isosceles triangles, including the congruence of base angles and the relationship between side lengths and angles (Daulay et al., 2021; *Global Education Monitoring Report 2023: Technology in Education: A Tool on Whose Terms?*, 2023). By letting students see and modify geometric figures in three dimensions (Cirneanu & Moldoveanu, 2024), dynamic geometry tools can also assist students build spatial reasoning skills. Moreover, the application of technology in the classroom helps to improve the teaching and learning geometry concepts' efficacy using diagrams (Retnowati & Fadlila, 2023). Teachers can build a more interesting and successful learning environment that supports students' problem-solving skills, spatial reasoning ability, and attitudes toward mathematics by including technology into mathematics instruction (Cirneanu & Moldoveanu, 2024). Digital tools' capability to increase efficiency, provide visual representations, eliminate errors, and enable complex problem-solving ability magnifies their efficacy in mathematics instruction ((Cirneanu & Moldoveanu, 2024; Miragliotta & Baccaglini-Frank, 2021). Thus, increasing the quality of education depends on including technology and digital mathematics into learning environments (Atteh et al., 2020; Cirneanu & Moldoveanu, 2024).

Customizing courses to meet various learning styles, interests, and degrees of knowledge is essential so that teachers may guarantee that every student may connect with and gain from practical applications (Chau et al., 2021). This approach not only improves the outcomes of instruction but also promotes a more inclusive and motivating classroom.

Including technology into mathematics education offers absolutely essential help for the learning process (Atteh et al., 2020; Cirneanu & Moldoveanu, 2024). Acting as a beneficial tool for acquiring, storing, and retrieving knowledge, it motivates students to search out the most recent information from many sources (Aggarwal, 2020). Technology can help to raise understanding of specific student learning needs and support student-centered teaching approaches, therefore improving student involvement with mathematics (Attard & Holmes, 2020). The integration of technology in classroom instruction enhances motivation, elevates self-esteem and confidence, refines questioning skills, encourages initiative and independent learning, enhances presentation of information, cultivates problem-solving capabilities, improves information handling skills, extends focus time on tasks, boosts attendance rates, and nurtures positive attitudes towards both technology and education (Abdullah & Shin, 2019; Aggarwal, 2020; Manganyana et al., 2020; Ponte et al., 2023). Technology in mathematics classrooms thus enables students to concentrate more on tactics and response analysis rather than laborious computations (Atteh et al., 2020; Cevikbas et al., 2023). In secondary schools, digital tools can improve mathematics and scientific knowledge (Hillmayr et al., 2020). Effective addressing of students' different needs and interests can help teachers to create a more interesting, inclusive, and successful learning environment, so influencing the educational experiences of the students (Mebert et al., 2020). Digital learning environments can produce data on learning agents that can subsequently be used to enhance learning in a variety of ways (Barana et al., 2021). These developments entail not just the application of digital tools but also the design of tailored, interactive, flexible learning opportunities.

A structured model can be used to solve the difficulties in solving complicated problems including isosceles triangles by means of structure. This kind of model will give pupils a methodical way to solve problems, helping them through the required actions and techniques to get at correct answers. Starting with a firm awareness of the characteristics of isosceles triangles, including the congruence of base angles and the relationship between side lengths and angles, the model should Visual tools include interactive software and illustrations help pupils grasp these qualities even more. Students must make good use of technology to assist in the promotion of mathematics education (Kim et al., 2022). Technology in mathematics teaching allegedly enables pupils to concentrate more on tactics and response analysis rather than laborious computations (Silver, 1997).

The model should also provide a methodical approach to problem-solving, guiding students through the process of spotting pertinent material, choosing suitable tactics, and completing required computations. Moreover, the model should inspire students to review their answers and consider their approach of addressing their problems, thereby fostering metacognitive abilities and self-regulation (Jahnke et al., 2022). Interactive tutorials help students to reach the right response, therefore encouraging self-learning and problem-solving abilities (Mikula & Heckler, 2014). By actively participating and helping the teacher to plan activities, students can develop their ability to solve problems (Ezeddine et al., 2023).

Furthermore, the model should underline the need of mathematical thinking and communication. Students should be urged to enhance their capacity to clearly express mathematical ideas by justifying their solutions and reasoning. Moreover, the model must to include chances for students to work together and grow personally from each other. This is in line with the need of teachers using efficient teaching strategies to guarantee that their students get mastery in addressing problems (Isnawati et al., 2021).

Teachers that want to teach problem-solving have to take into account certain facets of the teaching and learning process (Hafizi & Kamarudin, 2020). These cover the classroom, teacher effectiveness, pedagogogy, and approaches to problem-solving. The classroom should be fit for solving problems, giving students chances to investigate, try out ideas, and work on projects (Barana et al., 2022). Teachers have to be content knowledge, pedagogical knowledge, and problem-solving capable if they want to support effective instruction (Chau et al., 2021). The need of instructors' pedagogical subject knowledge in improving mathematics teaching quality and student performance underlined in this paper. Good teaching strategies including inquiry-based learning and Injure Based Learning can involve students in significant tasks including addressing problems.

Using such a structural model can help students grasp isosceles triangles more deeply and increase their ability to solve problems. Overcoming obstacles in reaching objectives is considered as the process of solving problems (Mudrika et al., 2024; Syaiful et al., 2020). Critical thinking, analysis, and imagination are all part of problem-solving (Bakar & Ismail, 2020). Long-standing recognition of the benefit of raising problem-solving capacity as a focus of mathematics instruction. Teachers or educational institutions stress problem solving in learning mathematics both in the classroom and outside of it, in both form as questions given and learning models (Yusuf et al., 2021). Teachers can encourage pupils to deepen their knowledge of mathematics and enhance their problem-solving abilities by tackling these problems and concentrating on these domains (Chau et al., 2021).

1.6 Content and classification of complex problems involving isosceles triangles

Their substance, the query, and the applications of the properties and theorems let one classify hard situations involving isosceles triangles. Considered harder are problems requiring sophisticated solution strategies, where several properties or theorems regarding the isosceles triangle need to be used, or when algebraic approaches are needed (Santos-Trigo et al., 2021).

The content of such problems typically involves various geometric concepts, such as triangle similarity, side-angle ratios, triangle midpoints, finding area and perimeter, and transformations. Classification of problems with increased complexity can also take into account the level of representation and abstraction.

For example, problems that require students to visualize and represent three-dimensional objects, or tasks that include abstract geometric concepts. In addition, the division of complex problems related to isosceles triangles can also take into account their description and the context in which the problem is shown. Problems based on reality, interdisciplinary connections, and open-ended questions that require independent research can increase the complexity and challenge of these problems. Complex problems are like a new opponent in a game that challenges students and improves their skills and abilities in solving new problems (Benedicto & Andrade, 2022).

As in the investigations we do during classes, so in practical mathematics, complicated issues (or challenging tasks) play a vital part and are a necessary instrument for the development of logical, critical and analytical thinking, reasoning about them prepares students for real-life problems. Modern constructivist and Injure Based Learning paradigms center these activities (Lili Supardi et al., 2021; Segura & Ferrando, 2023).

A complex mathematical problem is one that is characterized by multiple interrelated components, a nonlinear solution, and a high degree of ambiguity, where solutions may not be immediately identifiable. Typically, such problems:

- Include multiple subproblems with different solution paths or steps that interact dynamically, requiring problem solving and strategic planning (Zulfa & Andriyani, 2023).
- Do not have a single, clear path to solution, requiring critical thinking, flexible reasoning, rather than the mechanical application of algorithms and a single formula or theorem.
- Require iterative processes, including guessing, testing, revising, and generalizing, to move toward a solution (Lili Supardi et al., 2021).
- They are often related to or based on real-life situations, making learning more understandable, useful, and applicable in practice .

In Kazakhstan, complex problems are usually classified based on their cognitive complexity and the required procedural steps and actions. According to Turganbaeva B.A., a complex problem is a problem that develops students' thinking, requires the use of several mathematical concepts, and is solved using more than one formula, but requires non-traditional strategies (Turganbaeva, 2002).

Koyanbaev Zh.B. and Koyanbaev R.M. also note: Complex problems require the application of fundamental knowledge from previous topics in new contexts, including the ability to analyze, synthesize, and draw conclusions based on facts (Koyanbaev & Koyanbaev, 2004).

According to Abylkasymova A.E.: Complex problems include subtasks that cover several mathematical topics simultaneously, require logical reasoning, and form the basis for developing subject competence (Abylkasymova, 2004).

Complex geometric problems are a special class of mathematical problems that distinguish them from others by the need for spatial reasoning, visual representation, and fundamental knowledge of geometric proof strategies.

Their defining traits include many geometric relationships - likeness, similitude, parallelism, symmetry, and parallelism - interacting in one context.

Often requiring auxiliary elements (e.g., angle bisectors, medians, heights, etc.), including diagrams and formal proofs into constructs either via transformation geometry or otherwise.

Step-by-step planning helps to gradually solve complexities, especially in construction problems (e.g. compass and straightedge) where visual precision and logical consistency are vital (Cunha et al., 2024; Miragliotta & Baccaglini-Frank, 2021).

Emphasis on visual-spatial modeling, especially when enhanced by geometry-building software such as GeoGebra, which supports hypothesis testing, precision in parameter assignment, and visualization (Atteh et al., 2020; Rholey R. Picaza, 2023).

Complex problems that students encounter can be a tool for developing mental skills such as analytics, visualization, and identifying cause-and-effect relationships for a number of reasons:

- analysis of the problem statement and complexity assessment;
- hypotheses generation and testing;
- planning the sequence of solution steps through transformations and construction.

Kazakh researchers such as Ardabaeva (2023) and Zhaguparev (2021) emphasize methods such as key problem modeling and problem sequencing in geometry education. These approaches classify problems related to isosceles triangles around basic geometric theorems and properties, planning a step-by-step logical chain from conceptual understanding to the ability to solve and prove and apply knowledge (Ardabaeva, 2023; Zhaguparev, 2021).

Complex problems related to isosceles triangles require different methods for solving, including not only geometric formulas, theorems and properties, but also algebraic approaches and computational foundations.

Geometric methods include the properties of similarity of triangles, the ratio of angles to sides, theorems that help to derive unknown quantities (English, 2023). The algebraic approach includes constructing equations and solving them based on geometric properties, making it easy to find the length of the sides, angles or areas (Klang et al., 2021).

Computational methods involve the use of information technology for geometric construction and dynamic manipulation, which allows for experimental research and verification of geometric relationships (Konnova et al., 2020).

In addition to the above methods, trigonometric functions cannot be excluded, which can be used to relate angles and side lengths, especially in problems related to non-right-angled isosceles triangles. Coordinate geometry offers another robust method, where a triangle is placed on a coordinate plane and algebraic methods are used to analyze geometric properties. The choice of the most appropriate method often depends on the specific details of the problem, and in many cases a hybrid approach combining several methods is most effective.

1.7 Integration of isosceles triangle concepts into the mathematics curriculum in Kazakhstan

Complex problems involving isosceles triangles can be included in the middle school mathematics curriculum for a number of reasons. For example, they promote critical thinking and mathematical reasoning, the tasks require logical connections, and improve skills in solving complex and heterogeneous problems.

Mathematical modeling makes it possible to relate real-world problems to isosceles triangles (Kohen & Orenstein, 2021). By incorporating problems involving isosceles triangles into other disciplines such as physics, chemistry, and engineering, students can appreciate the importance and application of mathematical knowledge in real life (Serin, 2018). The integration of technology can enhance students' exploration and understanding of complex problems involving isosceles triangles (Buket Özüm Bülbül, 2021).

The introduction of visualization technologies and figure plotting programs in geometry, as well as computer algebra systems, can enable students to manipulate geometric shapes, test hypotheses, and visualize mathematical relationships (Chau et al., 2021). The curriculum can emphasize the importance of mathematical communication and reasoning.

Students should be given opportunities to solve problems independently and formulate their own problem-solving strategies, explain their solution steps, and justify their arguments in finding solutions to problems. In addition, it is necessary to take into account the students' knowledge, problem-solving skills, each may be different, and we must not forget that when checking problems, the teacher should give the right direction for each student based on his or her level of knowledge. Complex problems open new opportunities for students to explore, this provides an opportunity to assess their knowledge by the problem-solving strategies they use. The curriculum should become a ladder for growth, complex problems should become an opportunity and a step for growth.

The study of isosceles triangles plays an important role in mathematics education because they provide a foundation for understanding complex geometric concepts and problem-solving strategies.

First, the relationship between the side lengths and angles of isosceles triangles provides insight into key concepts in trigonometry and advanced geometry.

Second, their special properties, such as the equality of two sides and two angles, provide useful experience in performing geometric constructions and proofs, helping to develop students' logical thinking.

Third, the internal symmetry of isosceles triangles introduces students to the concept of symmetry in geometry and develops their spatial awareness.

This section analyzes geometry textbooks developed by different authors and programs in our country, focusing on the extent to which they pay attention to problems on isosceles triangles. According to the reviewed literature, geometry materials usually include many properties, theorems, and corollaries that help students recognize and construct isosceles triangles. For example, calculation-based tasks contribute to the development of the ability to apply knowledge in practical situations (Abdullah & Shin, 2019). However, many geometry textbooks do not provide enough problems on isosceles triangles, especially those aimed at developing creative thinking and spatial abilities (Daulay et al., 2021). Therefore, research on the creative potential of secondary school students is relevant (Sahliawati & Nurlaelah, 2020). Students can develop creativity and spatial imagination using shape combining and dissecting techniques.

Table 1.7.1 Types of Tasks on Isosceles Triangles in Kazakhstani Geometry Textbooks

Textbook Authors	**Grade**	**Number of Tasks**	**Types of Tasks**
A.N.Shynybekov, D. A. Shynybekov	Grade 7	36	Analytical; Proof; Construction; Application of Theorems; Proportional Reasoning Problems
V. A. Smirnov, E. A. Tuyakov	Grade 7	38	Shape Recognition and Classification; Analytical; Proof; Construction; Application of Theorems
A.N.Shynybekov, D. A. Shynybekov, R. N. Zhumabayev	Grade 8	36	Shape Recognition and Classification; Analytical; Proof; Construction; Transformations; Coordinate Geometry Tasks; Application of Theorems
V. A. Smirnov, E. A. Tuyakov	Grade 8	34	Shape Recognition and Classification; Analytical; Proof; Construction; Coordinate Geometry Tasks; Application of Theorems
A.N.Shynybekov, D. A. Shynybekov, R. N. Zhumabayev	Grade 9	23	Shape Recognition and Classification; Analytical; Proof; Construction; Coordinate Geometry Tasks; Application of Theorems; Proportional Reasoning; Transformations; Application of Theorems and Properties
V. A. Smirnov, E. A. Tuyakov	Grade 9	19	Shape Recognition and Classification; Analytical; Proof; Construction

Main conclusions:

Geometry textbooks (Table 1.7.1) for grades 7–9 contain problems on isosceles triangles, but their level of difficulty varies. Many textbooks contain a sufficient number of tasks aimed at mastering the basic properties of an isosceles triangle. Some textbooks predominantly contain problems on proof, while others focus on calculations and constructive constructions. By grade 9, the tasks become more complex and require the use of theorems, but the total number of such problems remains small.

The following materials are taken from Kazakh textbooks of 7th class geometry and illustrate key theorems and properties related to isosceles triangles.

Theorem 1. Angles at the base of an isosceles triangle are equal.

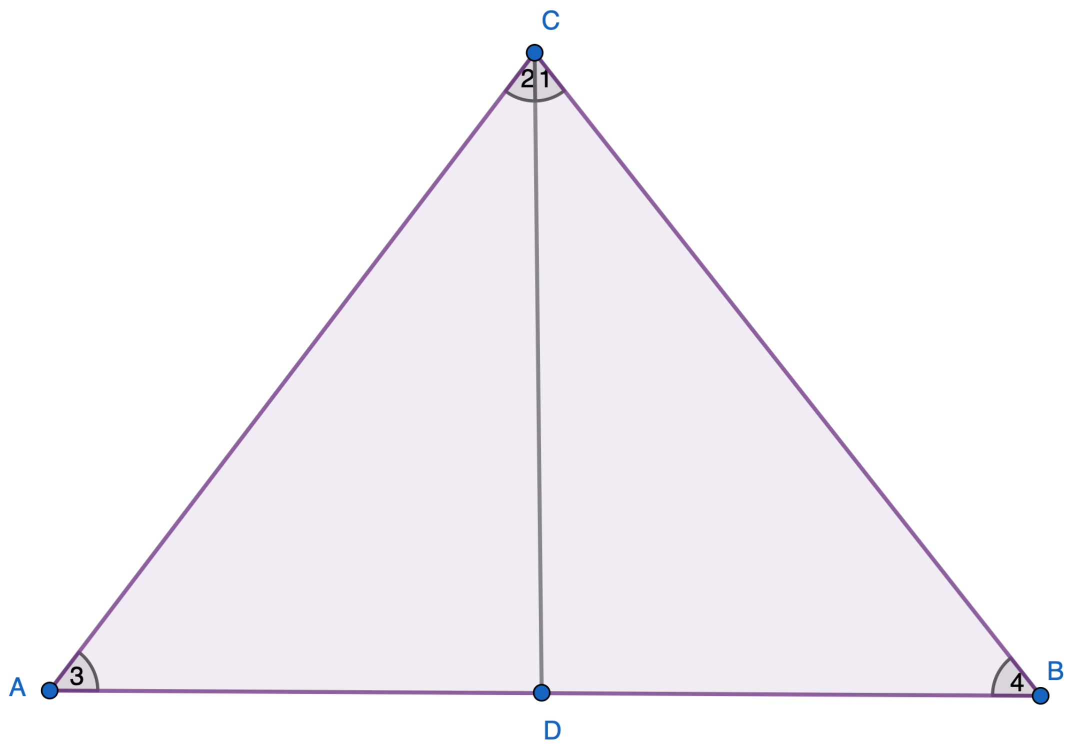
In the given triangle ABC, as illustrated in Figure 1.7.1, where AC and BC are equal sides, and AB is the base. Let $\angle 3$ and $\angle 4$ be the angles adjacent to the base. We will prove that $\angle 3 = \angle 4$.

Proof:

Run bisector CD. According to the first feature of similarity of triangles, triangles $\triangle ACD = \triangle BCD$ are similar ($AC = BC$, CD - common, and $\angle 1 = \angle 2$). Therefore, $\angle 3 = \angle 4$.

The theorem is proven.

Figure 1.7.1 Isosceles Triangle ABC with Bisector CD



Theorem 2. The Bisectrix, carried from the vertex to the base of an isosceles triangle, is also a median and height.

Proof:

According to theorem 1, from the equality of triangles $\triangle ACD = \triangle BCD$ follows that $AD = BD$, and therefore, CD is a median. Similarly, $\angle ADC = \angle BDC$.

Hence, $\angle ADC = \angle BDC = 90°$ (since ($\angle ADC$ and $\angle BDC$ are half of the extended angle). Therefore, CD is the height of the triangle.

The theorem is proven.

Theorem 3. (Inverse Theorem 1)

If the two angles of the triangle are equal, then the triangle is equilateral.

Let give a triangle ABC, as illustrated in Figure 1.7.1, with $\angle 3 = \angle 4$. Let us prove that $AC = BC$.

Proof:

Run the bisector of the angle CD. Then $\angle 2 = \angle 1$. According to theorem 1, in triangles ACD and BDC have $\angle ADC = \angle BDC$. For these triangles CD is the common side, and the adjacent angles are equal. On the second feature of similarity of triangles $\triangle ACD = \triangle BCD$. Therefore, $AC = BC$.

The theorem is proven.

Consequence of theorems 1 and 2:

In a triangle, the sides opposite equal angles are themselves equal and vice versa, the angles opposite equal sides are also equal.

In order to effectively teach students to solve problems involving isosceles triangles, they must first learn to analyze the problem statement by identifying the given data, the required quantities, and the relevant geometric theorems (e.g., "the base angles of an isosceles triangle are equal" or "the sides opposite equal angles are also equal"). Understanding the context of the problem is a critical first step in finding a solution.

In the process of teaching how to solve geometric problems involving isosceles triangles, it is important to follow a structured and purposeful sequence of actions. Based on the methodology of A.E. Abylkasymova "Theory and Methodology of Teaching Mathematics: Didactic and Methodological Foundations", the process of solving problems can be divided into four main stages:

1. Analysis of the problem statement.

At this stage, students study the subject content of the problem, determine what is given and what needs to be found, and identify which theorems and properties are applicable. For isosceles triangles, students should ask leading questions such as: "Which angles are equal?", "Is the triangle really isosceles?", or "What conclusions can be drawn from these elements?"

2. Selecting a solution method and developing a plan.

After completing the analysis, students decide which theorems to apply - for example, Theorem 1 (base angles are equal), Theorem 2 (angle bisector drawn from the vertex also acts as the median and altitude), or the converse theorem (equal angles imply equal sides).

3. Implementing the solution.

Following the plan, students carry out the solution steps, justify their reasoning, construct geometric diagrams, and clearly write down the solution.

4. Checking the result and drawing conclusions.

Students should check the correctness of their solutions, evaluate the logic of their reasoning, and confirm that the theorems and constructions applied are correct.

This structured model helps students develop a systematic approach to solving problems involving isosceles triangles, improves their logical reasoning and proof skills, and builds confidence in the effective use of geometric language and diagrams.

According to V. M. Bradis, a solution can only be considered correct if it is:

1. error-free;
2. logically sound;
3. complete.

Therefore, students must review their reasoning, ensure that each step was correct, and that the conclusion follows logically from the given conditions.

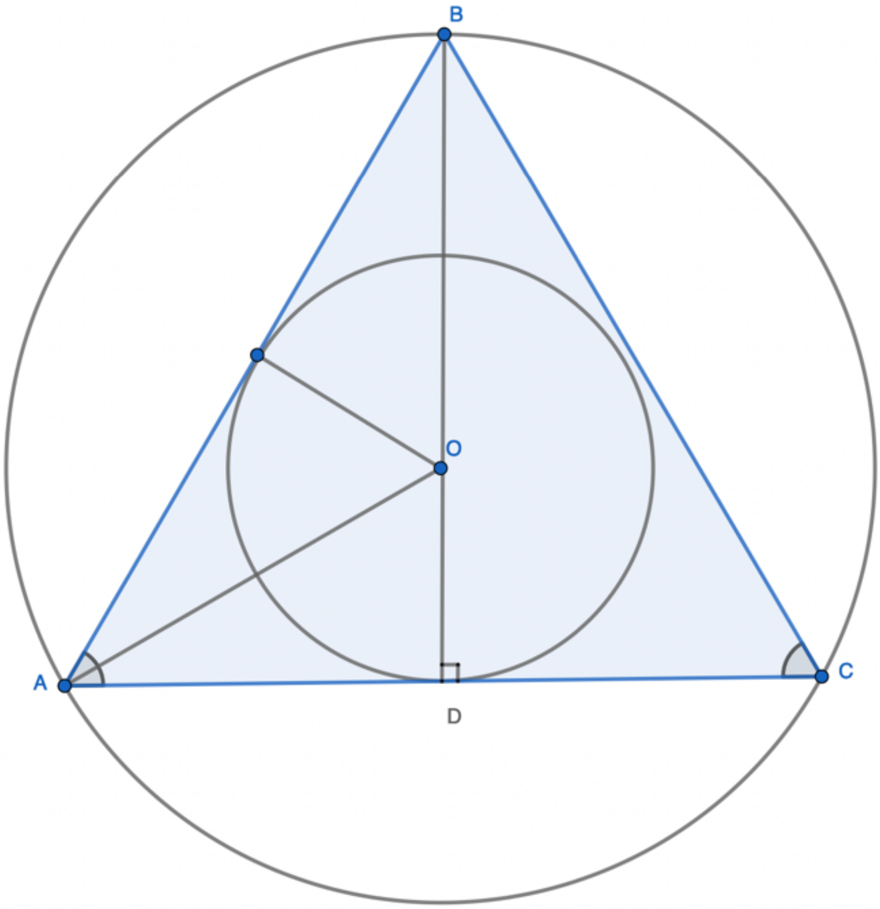
For example, if a student comes to the conclusion that $\triangle ACD = \triangle BCD$, he or she must clearly indicate which criterion for similarity of triangles was used and confirm that the resulting property (such as $\angle 3 = \angle 4$ or $AC = BC$) is correctly derived.

The four steps described above are closely interrelated and act as a structured guide for solving problems involving isosceles triangles. If the problem becomes more complex or one theorem is not enough, the student may need to return to the previous stage, demonstrating the development of reflection and self-control skills. Moreover, the complete completion of each stage prepares the student not only to solve a separate problem, but also to assimilate the culture of geometric reasoning and justification. This structured methodology promotes logical, consistent and mathematically sound thinking and deepens the students' understanding of geometry as a discipline.In the area of mastering geometric knowledge, the study of isosceles triangles plays a vital role for students of different academic levels. First, the isosceles triangle, which has two sides and two angles that are equal, is a basic geometric figure. Although angles are not a basic geometric concept, they are important because a figure without angles is simply a collection of elements inside a circle. Angles are ubiquitous in everyday life, reinforcing their importance. Second, the isosceles triangle is the basis for various rectangles and solids, forming the basic structure when constructing the diagonals of polygons.

In the context of isosceles triangles, ideal lines play a major role. The law of sines, elements of trigonometry, application of the properties of the tangent to a circle, and the properties of the inner and outer circle provide effective tools for solving various problems involving isosceles triangles. Although school mathematics mainly deals with problems of a simple level, capable students expand their knowledge through individual research and practical projects and apply them at later stages. Helps you learn how to effectively use the listed elements when solving problems involving isosceles triangles. This approach not only improves problem solving skills, but also expands the application of geometric principles beyond traditional methods.

Below are some examples of complex problems related to isosceles triangles, and what methods and solution strategies have been applied to each of these tasks. A different approach was applied to each task and applied from the basic concepts of geometry, mathematical calculation, figure construction, as well as trigonometric identities.

Example 1. The radius of the circumcircle of an isosceles triangle is 25 cm, and the radius of the incircle is 12 cm. Find the side lengths of the triangle.



Given:
$\triangle$ABC is an isosceles triangle
R = 25 cm (circumradius)
r = 12 cm (inradius)
To find:
AB, AC – ?

Solution:

We use two formulas for the area of a triangle:

$$S_{\triangle} = \mathrm{pr} \ = \frac{2AB + AC}{2} \cdot 12 = 12\mathrm{AB} + 6\mathrm{AC}$$

$$S_{\triangle} = \frac{abc}{4R} = \frac{AB^2 \cdot AC}{100}$$

By equating the two expressions for the area, we obtain:

$$12\mathrm{AB} + 6\mathrm{AC} = \frac{AB^2 \cdot AC}{100} \qquad (1)$$

Let ∠ACB = α. Then, by the Law of Sines in triangle ABC:

$$\frac{AB}{\sin\alpha} = 2R$$

$$AB = 2R \cdot \sin\alpha \qquad (2)$$

Now, using the identity from trigonometry and equation (2):

$$\cos\alpha = \frac{AD}{AB}$$

$$AD = AB \cdot \cos\alpha = 2R \cdot \sin\alpha \cdot \cos\alpha$$

$$AC = 2 \cdot AD = 4R \cdot \sin\alpha \cdot \cos\alpha \ \ (AD = DC) \qquad (3)$$

Substitute equations (2) and (3) into equation (1):

$$12\text{AB} + 6\text{AC} = \frac{AB^2 \cdot AC}{100}$$

$$24R \cdot \sin\alpha + 24R \cdot \sin\alpha \cdot \cos\alpha = \frac{4R^2 \cdot \sin^2\alpha \cdot 4R \cdot \sin\alpha \cdot \cos\alpha}{100}$$

Simplifying:

$$150(1 + \cos\alpha) = R^2(1 - \cos^2\alpha) \cdot \cos\alpha$$

Now solve the equation:

$$25\cos^2\alpha - 25\cos\alpha + 6 = 0$$

Solving the quadratic equation, we get two solutions:

1) $\cos\alpha = \frac{2}{5}; \sin\alpha = \frac{\sqrt{21}}{5}$
2) $\cos\alpha = \frac{3}{5}; \sin\alpha = \frac{4}{5}$

Now substitute the values back to find the side lengths:

Case 1:

$$1) AB = 2 \cdot 25 \cdot \frac{\sqrt{21}}{5} = 10\sqrt{21}$$

$$AC = 100 \cdot \frac{\sqrt{21}}{5} \cdot \frac{2}{5} = 8\sqrt{21}$$

Case 2:

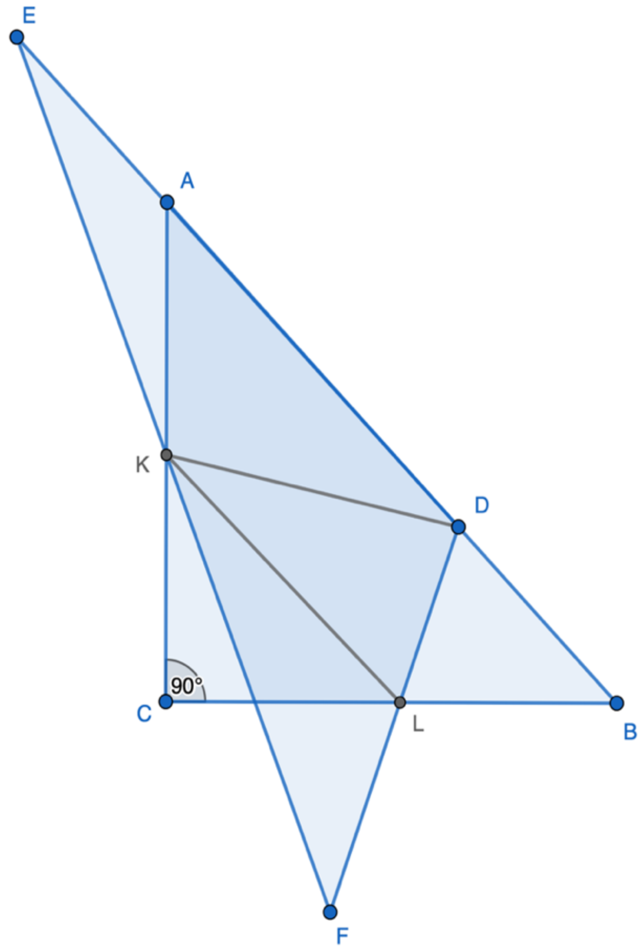
$$2) \ AB = 2 \cdot 25 \cdot \frac{4}{5} = 40$$

$$AC = 100 \cdot \frac{4}{5} \cdot \frac{3}{5} = 48$$

Answer: $10\sqrt{21}, 8\sqrt{21};\ 40, 80$

Example 2. Triangle ABC is an isosceles triangle with ∠C = 90°, and triangle DEF is placed such that point D lies on side AB, and point E lies on the extension of AB beyond point A. Segment KL is the midline (median line) common to both triangles. The area of quadrilateral DKLB is equal to 5/8 of the area of triangle ABC. Find the angle ∠DEF.

Given:
△ABC – isosceles triangle, ∠C = 90°
△DEF – triangle
D ∈ AB
KL – midline
$S_{DKLB} = \frac{5}{8} S_{ABC}$
To find:
∠DEF – ?



Solution:

Let us denote:

Let side $AB = a$, and since triangle ABC is isosceles with a right angle at C, then:

$$AC = BC = a$$

$$AB = FD = a\sqrt{2}$$

From the problem:

$$KL = \frac{1}{2} AB = \frac{1}{2} ED$$

Step 1: Area relationship

We are told that:

$$S_{DKLB} = S_{ABC} - S_{CKL} - S_{AKD}$$

From the problem:

$$S_{DKLB} = \frac{5}{8} S_{ABC}$$

Since triangle CKL is formed from half the base and height of triangle ABC, it has:

$$S_{CKL} = \frac{1}{4} S_{ABC}$$

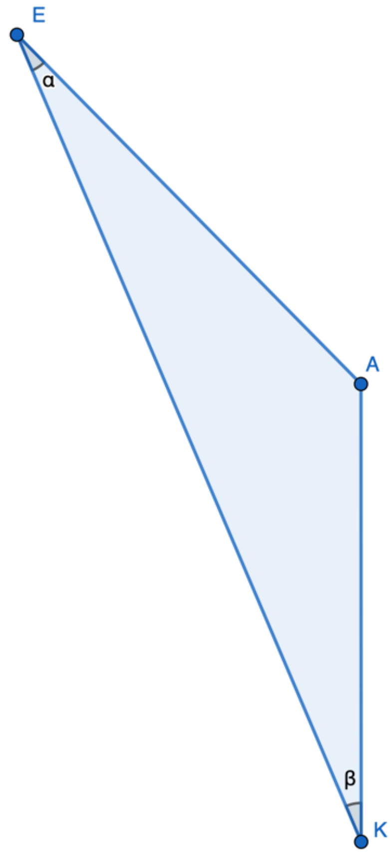
So plugging into the equation:

$$\frac{5}{8} S_{ABC} = S_{ABC} - \frac{1}{4} S_{ABC} - S_{AKD}$$

Solving for S_{AKD}:

$$S_{AKD} = \frac{1}{8} S_{ABC} \Longrightarrow S_{ABC} = \frac{a^2}{2} \Longrightarrow S_{AKD} = \frac{1}{8} \cdot \frac{a^2}{2} = \frac{a^2}{16}$$

We now calculate the area S_{AKD} using trigonometry.



Step 2: Expressing the Area of Triangle AKD

Let's denote the length from point D to point F as x.
Then we compute the area of triangle AKD using the sine formula:

$$S_{AKD} = \frac{1}{2} \cdot \frac{a}{2} \left(a\sqrt{2} - x\right) \cdot \sin 45^\circ$$

$$= \frac{a}{4} \left(a\sqrt{2} - x\right) \cdot \frac{\sqrt{2}}{2}$$

Using $\sin 45^\circ = \frac{\sqrt{2}}{2}$:

$$S_{AKD} = \frac{a}{4} \left(a\sqrt{2} - x\right) \cdot \frac{\sqrt{2}}{2} = \frac{a^2}{4} - \frac{a\sqrt{2}}{8} x$$

We already know:

$$S_{AKD} = \frac{1}{8} \cdot \frac{a^2}{2} = \frac{a^2}{16}$$

Now we equate the two expressions for S_{AKD}:

$$\frac{a^2}{4} - \frac{a\sqrt{2}}{8} x = \frac{a^2}{16}$$

$$1{,}5a^2 - \sqrt{2}ax = 0 \Longrightarrow \sqrt{2}ax = \frac{3}{2} a^2 \Longrightarrow$$

$$x = \frac{3a^2}{2\sqrt{2}a} = \frac{3\sqrt{2}a}{4}$$

Step 3: Use of Law of Sines in Triangle DEF

Let angle $\angle DEF = \beta$, angle $\angle DFE = \alpha$, and angle at point K is $135° + \beta$.

Then:

$$\beta + 135° + \alpha = 180° \Longrightarrow \beta = 45° - \alpha$$

Now use the Law of Sines in triangle DEF:

$$\frac{\frac{a}{2}}{\sin\alpha} = \frac{\frac{3\sqrt{2}a}{4}}{\sin(45°-\alpha)} \Longrightarrow \frac{a}{2\sin\alpha} = \frac{3\sqrt{2}a}{4\sin(45°-\alpha)} \Longrightarrow \frac{1}{\sin\alpha} = \frac{3\sqrt{2}}{2\sin(45°-\alpha)}$$

Use identity for $\sin(45°-\alpha) = \sin 45° \cos\alpha - \sin\alpha \cos 45° = \frac{\sqrt{2}}{2}(\cos\alpha - \sin\alpha)$

Then:

$$2\left(\frac{\sqrt{2}}{2}(\cos\alpha - \sin\alpha)\right) = 3\sqrt{2}\sin\alpha$$

$$\cos\alpha - \sin\alpha = 3\sin\alpha$$

$$\cos\alpha = 4\sin\alpha$$

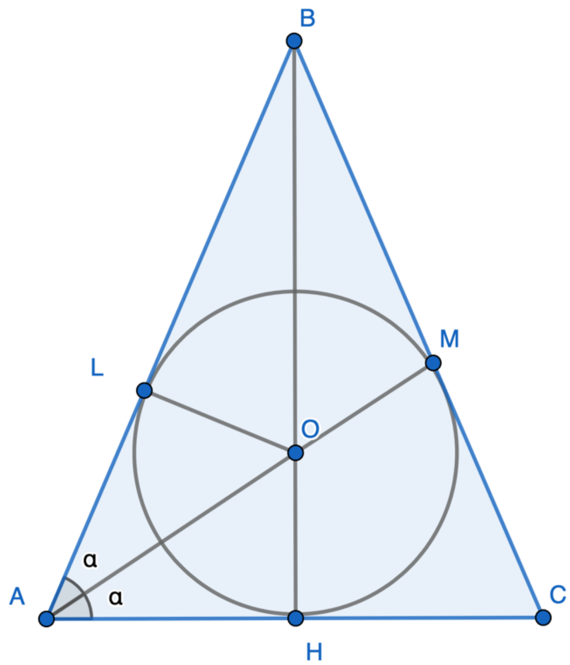
Divide both sides by $\cos\alpha$:

$$tg\ \alpha = \frac{1}{4} \Longrightarrow \alpha = arctg\left(\frac{1}{4}\right)$$

Answer: $\angle DEF = arctg\left(\frac{1}{4}\right)$

Example 3. Point O is the center of the incircle of an isosceles triangle (with AB=BC). The line AO intersects segment BC at point M. If AO=3 and OM=2711, find the angles of triangle ABC and its area.

Given:
$\triangle ABC$ is an isosceles triangle
O is the incenter of triangle $\triangle ABC$
$AB = BC$
$AO = 3, OM = \frac{27}{11}$
To find:
$\angle A, \angle B, \angle C$ – ?
S_{ABC}– ?



Solution:

Step 1: Use trigonometry to express cosine of angle α

Let $\angle A = \angle C = \alpha$ (since the triangle is isosceles with $AB = BC$). Given $AO = 3$, and point M lies on BC, with $OM = \frac{27}{11}$.

From the triangle's incenter properties, we know:

$$\cos\alpha = \frac{b}{3}$$

Apply the double-angle identity:

$$\cos 2\alpha = 2\cos^2\alpha - 1$$

We also know from triangle geometry that:

$$\cos 2\alpha = \frac{AH}{AB}$$

So substituting:

$$\frac{b}{a} = 2\cdot\left(\frac{b}{3}\right)^2 - 1 = \frac{2b^2}{9} - 1 = \frac{2b^2 - 9}{9}$$

$$a = \frac{9b}{2b^2 - 9}$$

Step 2: Use expressions for $a + b$ and $a + 2b$

Now calculate:

$$a + b = \frac{9b}{2b^2 - 9} + b$$

$$a + b = \frac{2b^3}{2b^2 - 9}$$

And:

$$a + 2b = \frac{2b^3}{2b^2 - 9} + b \Longrightarrow a + 2b = \frac{4b^3 - 9b}{2b^2 - 9}$$

Step 3: Use formula for the length of segment AM

The squared length of median (or in this case, the segment from the incenter) is:

$$AM^2 = l^2 = \frac{a\cdot b\cdot(2a + 2b)\cdot(a + 2b - a)}{a + 2b} = \frac{a\cdot b\cdot(2a + 2b)\cdot 2b}{a + 2b}$$

Substitute the values:

$$a = \frac{9b}{2b^2 - 9}, a + b = \frac{2b^3}{2b^2 - 9}, a + 2b = \frac{4b^3 - 9b}{2b^2 - 9}$$

$$l^2 = \frac{2\cdot\left(\frac{9b}{2b^2 - 9}\right)\cdot b\cdot\left(\frac{2b^3}{2b^2 - 9}\right)\cdot 2b}{\frac{4b^3 - 9b}{2b^2 - 9}} = \frac{8\cdot 9\cdot b^5\cdot 2b}{(4b^3 - 9b)^2} = \left(\frac{60}{11}\right)^2$$

From here, solve for b:

$$b = \sqrt{5};\ a = 9\sqrt{5}$$

Step 4: Calculate triangle height and area

$$BH = \sqrt{\left(9\sqrt{5}\right)^2 - \left(\sqrt{5}\right)^2} = 20$$

$$S_{ABC} = \frac{2\sqrt{5} \cdot 20}{2} = 20\sqrt{5}$$

Step 5: Find angles

We use the double-angle identity again:

$$\cos 2\alpha = \frac{b}{a} = \frac{\sqrt{5}}{9\sqrt{5}} = \frac{1}{9} \Longrightarrow \alpha = \frac{1}{2} arccos\left(\frac{1}{9}\right)$$

$$\angle A = \angle C = 2\alpha = arccos\left(\frac{1}{9}\right); \ \angle B = \pi - 2\alpha = \pi - 2arccos\left(\frac{1}{9}\right)$$

Answer: $\angle A = \angle C = arccos\left(\frac{1}{9}\right); \ \angle B = \pi - 2arccos\frac{1}{9}; S_{ABC} = 20\sqrt{5}$

Example 4. The distance between the intersection points of the medians and angle bisectors of an isosceles triangle is equal to 2. If the length of the incircle of the triangle is 20π, find the perimeter of the triangle.

Given:

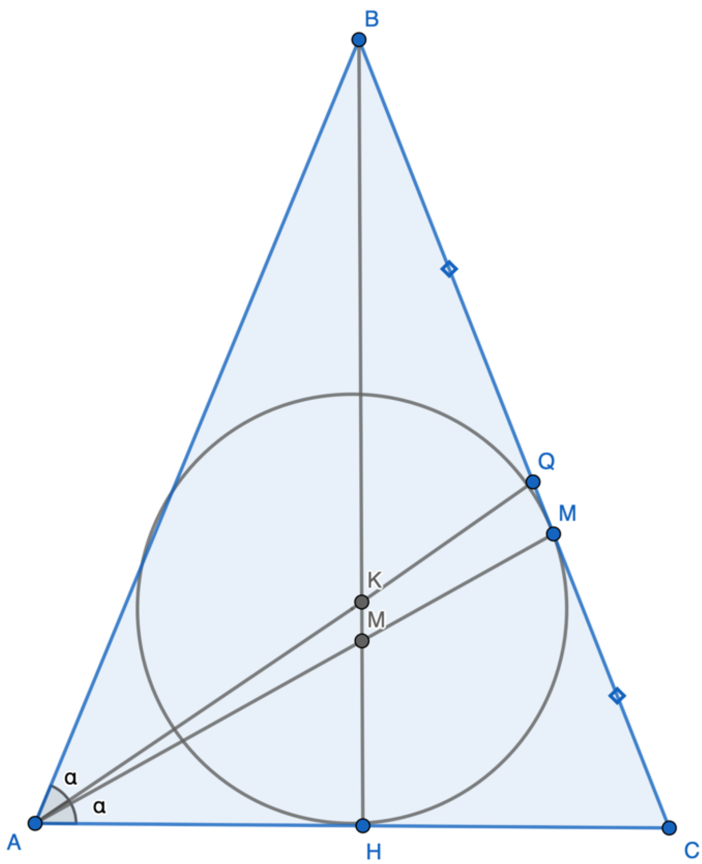
$\triangle ABC$ is an isosceles triangle

$O_1O_2 = 2$ — the distance between the centroid and the incenter

$C_{\text{incircle}} = 20\pi$ — the circumference of the incircle

To find:

$P_{ABC} - ?$



Solution:

Using properties of centroid and triangle height:

In an isosceles triangle, the centroidO_2 lies one third of the way along the median from the vertex to the base. So:

$$O_2H = \frac{1}{3}BH = \frac{a \sin \alpha}{3}$$

Since $O_1O_2 = 2$, and the incenter lies at distance $O_1H =$

10 along the altitude, the total altitude:

$$a \cdot \sin\alpha = 24 \Rightarrow a = \frac{24}{\sin\alpha}$$

Use cosine rule to find side AC:

Since triangle is isosceles, $AC\ = 2a \cdot cos\alpha$, and:

$$HC = AH = \frac{AC}{2} = \frac{2a\cdot\cos\alpha}{2} = a \cdot \cos\alpha,\ BH = \ a \cdot \sin\alpha$$

Using trigonometry to relate incenter location and triangle sides:

$$tg\frac{\alpha}{2} = \frac{O_1H}{AH} = \frac{10}{a \cdot \cos\alpha}$$

Substituting $a = \frac{24}{\sin\alpha}$, we get:

$$tg\frac{\alpha}{2} = \frac{10 \cdot \sin\alpha}{24 \cdot \cos\alpha}$$

Simplifying using trigonometric identities:

Use the identity $\sin\alpha = 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$, and let $\cos^2\frac{\alpha}{2} = x$, resulting in:

$$6 \cdot \cos\alpha = 5 \cdot \cos^2\frac{\alpha}{2} \Rightarrow \cos\alpha = \frac{5}{6} \cdot \cos^2\frac{\alpha}{2}$$

$$\cos^2\frac{\alpha}{2} - \left(1 - \cos^2\frac{\alpha}{2}\right) = \frac{5}{6} \cdot \cos^2\frac{\alpha}{2}$$

$$\cos^2\frac{\alpha}{2} - \cos^4\frac{\alpha}{2} = \frac{5}{6} \cdot \cos^2\frac{\alpha}{2}$$

$$\frac{7}{6}\cos^2\frac{\alpha}{2} = \cos^4\frac{\alpha}{2}$$

$$\cos^2\frac{\alpha}{2}\left(\frac{7}{6} - \cos^2\frac{\alpha}{2}\right) = 0$$

$$\frac{7}{6} \cdot \cos^2\frac{\alpha}{2} = 1 \Rightarrow \cos^2\frac{\alpha}{2} = \frac{6}{7}$$

$$\frac{1 + \cos\alpha}{2} = \frac{6}{7}$$

$$7(1 + \cos\alpha) = 12$$

$$7\cos\alpha = 5$$

$$\cos\alpha = \frac{5}{7};\ \sin\alpha = \frac{2\sqrt{6}}{7}$$

Calculate side lengths and perimeter:

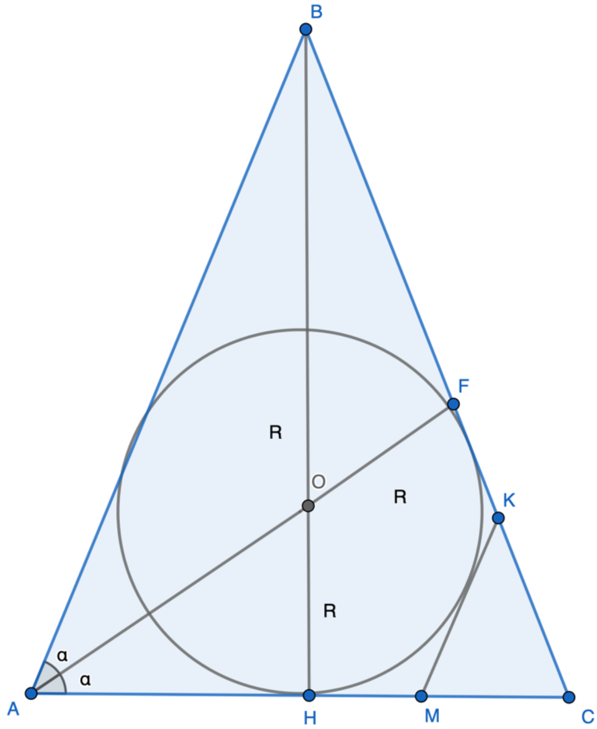
$$AB = a = \frac{24}{\frac{2\sqrt{6}}{7}} = 14\sqrt{6}$$

$$AC = 2 \cdot 14\sqrt{6} \cdot \frac{5}{7} = 20\sqrt{6}$$

$$P = 14\sqrt{6} + 14\sqrt{6} + 20\sqrt{6} = 48\sqrt{6}$$

Answer: $P = 48\sqrt{6}$

Example 5. An incircle is drawn inside the isosceles triangle △ABC (with AB=BC). A line parallel to side AB and tangent to the incircle intersects side AC at point M, such that MC=25·AC. If the perimeter of triangle ABC is 20, find the radius of the incircle.



Given:
$\triangle ABC$ is an isosceles triangle,
$AB = BC$
$P_{ABC} = 20$
Line $MK \parallel AB$ and is tangent to the incircle,
$MC = \frac{2}{5}AC$
To find:
$r - ?$

Solution:

Step 1: Let the length of side $AC = a$

Since triangle ABC is isosceles and symmetric about its height from vertex B, point H (the foot of the height from B) divides base AC equally:

$$AH = HC = \frac{a}{2}$$

We are told the line tangent to the incircle intersects AC at point M, and $MC = \frac{2}{5}a$.

Let x be the distance from A to the point where the tangent touches the incircle on side AC. Then:

$$x + \frac{2}{5}a = \frac{a}{2} \Rightarrow x = \frac{1}{10} \cdot a$$

Step 2: Use perimeter condition

Let side $AB = BC = b$, since the triangle is isosceles. We are told the perimeter is 20:

$$a + 2b = 20$$

Step 3: Use geometric relationships to connect a and b

From the figure (implied):

- The height from B to base AC is perpendicular, so $\triangle BHC$ is a right triangle.
- The point where the tangent intersects is on a line parallel to AB, meaning it also creates similar triangles or symmetrical segments.

Use geometric expressions:

$$HC = CF = \frac{a}{2}$$

Let $FK = \frac{a}{2} - \frac{2}{5}b$, and $x = \frac{1}{10} \cdot a$

Then, from symmetry and proportion:

$$\frac{a}{2} - \frac{2}{5}b + \frac{a}{10} = \frac{2}{5}b \implies a = \frac{4}{3}b$$

Substitute into the perimeter equation:

$$\frac{4}{3}b + 2b = 20 \implies b = 6;\ a = 8$$

Step 4: Use Pythagorean theorem to find triangle height

In right triangle $\triangle BHC$:

$$BH^2 = 36 - 16 = 20 \implies BH = \sqrt{20}$$

$$AC = 8$$

Step 5: Find triangle area

Use the formula for the area of a triangle:

$$s_{ABC} = \frac{1}{2} \cdot \text{base} \cdot \text{height} = \frac{1}{2} \cdot 8 \cdot \sqrt{20} = 8\sqrt{5}$$

Step 6: Find inradius

Use the formula:

$$r = \frac{S}{p} = \frac{2 \cdot 8\sqrt{5}}{20} = \frac{4\sqrt{5}}{5}$$

Answer: $r = \frac{4\sqrt{5}}{5}$

Example 6. In the isosceles triangle $\triangle ABC$, side AC is a chord of a circle, and the center of the circle lies inside triangle ABC. A line passing through point B intersects the circle at points D and E. Given that $AB = BC = 2$, $\angle ABC = 2\,arcsin\frac{1}{\sqrt{5}}$, and the radius of the circle is 1, find the area of triangle $\triangle DBE$.

Given:

$AB = BC = 2$

$\angle ABC = 2\,arcsin\frac{1}{\sqrt{5}}$

$R = 1$ (the radius of the circle)

AC is a chord of the circle

To find:

$S_{BDE}-?$

Solution:

Let's assume the center of the circle is point OO, and since the radius is 1, we know:

$$OE = AO = OC = OD = 1$$

Step 1: Determine coordinates and trigonometric values

We are given:

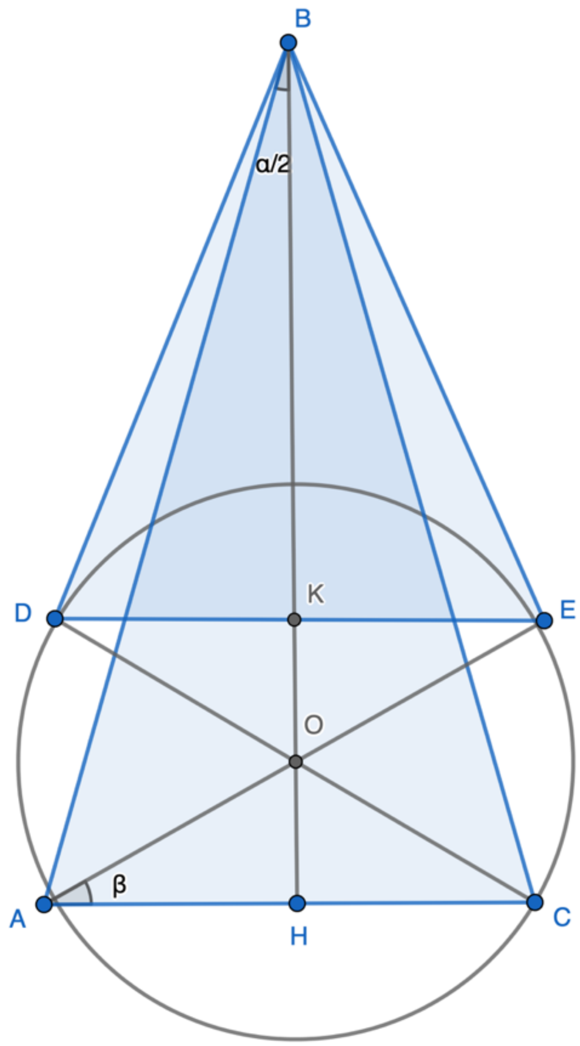
$\angle ABC = 2\, arcsin\left(\frac{1}{\sqrt{5}}\right)$, so half of that angle is:

$$\beta = \ arcsin\left(\frac{1}{\sqrt{5}}\right)$$

This implies:

$\cos\beta = \frac{AH}{AO} = \frac{\frac{2}{\sqrt{5}}}{1} = \frac{2}{\sqrt{5}}$, and $\sin\beta = \sqrt{1-\left(\frac{2}{\sqrt{5}}\right)^2} = \frac{1}{\sqrt{5}}$

$$\sin\beta = \frac{OH}{AO} = \frac{1}{\sqrt{5}} \Rightarrow OH = AO \cdot \sin\beta = \frac{1}{\sqrt{5}}$$



Step 2: Use triangle properties

Since the triangle is isosceles, the height splits the base, and:

$$\sin\frac{\alpha}{2} = \frac{AH}{AB} = \frac{AH}{2} \Rightarrow AH = 2 \cdot \sin\left(arcsin\left(\frac{1}{\sqrt{5}}\right)\right) = \frac{2}{\sqrt{5}}$$

Then:

$$AC = AH \cdot 2 = \frac{4}{\sqrt{5}}$$

Step 3: Use Pythagoras to find segment lengths

$$BH^2 = AB^2 - AH^2 = 4 - \frac{4}{5} = \frac{16}{5} \Rightarrow BH = \frac{4}{\sqrt{5}}$$

$$OB = BH - OH = \frac{4}{\sqrt{5}} - \frac{1}{\sqrt{5}} = \frac{3}{\sqrt{5}}$$

$$BD^2 = BO^2 - OD^2 \ \Rightarrow BD = BE = \sqrt{\frac{9}{5} - 1} = \frac{2}{\sqrt{5}}$$

Step 4: Use triangle area formula for $\triangle\, BDE$

Let angle φ be such that:

$$\sin\varphi = \frac{BD}{BO} = \frac{\frac{2}{\sqrt{5}}}{\frac{3}{\sqrt{5}}} = \frac{2}{3}$$

Then the height DK from point D to side BE is:

$$DK = \sin\varphi \cdot 1 = \frac{2}{3}$$

Using Pythagoras:

$$OK^2 = DO^2 - DK^2 \Rightarrow OK = \sqrt{1 - \frac{4}{9}} = \frac{\sqrt{5}}{3}$$

$$BK = BO - OK = \frac{3}{\sqrt{5}} - \frac{\sqrt{5}}{3} = \frac{4}{3\sqrt{5}}$$

Also:

$$DE = 2 \cdot DK = 2 \cdot \frac{2}{3} = \frac{4}{3}$$

Now calculate area:

$$S_{BDE} = \frac{1}{2} \cdot BK \cdot DE = \frac{1}{2} \cdot \frac{4}{3\sqrt{5}} \cdot \frac{4}{3} = \frac{8}{9\sqrt{5}}$$

Answer: $S_{BDE} = \frac{8}{9\sqrt{5}}$

2. METHODOLOGY

2.1 Instrument

In our study, two main tools were developed to assess the impact of solving complex problems related to isosceles triangles on the development of students' geometric thinking and their academic performance. The first tool was a thematic test on the topic "Application of the Features and Properties of an Isosceles Triangle" aimed at identifying the level of academic achievement. The second tool was a survey determining the level of geometric thinking and problem solving skills on the topic "Isosceles Triangle".

2.1.1 Achievement test

The achievement test was developed to assess the level of students' geometric thinking, their ability to apply the properties of an isosceles triangle when solving problems, make calculations, draw conclusions, and interpret geometric situations. The test consists of 20 multiple-choice tasks corresponding to the secondary school level. Each task is aimed at testing one or more of the following skills: understanding the features of an isosceles triangle; application of theorems on the equality of angles and sides; calculation of perimeter, height, area; analysis of geometric conditions; interpretation of diagrams and drawings; solving problems with indirect conditions; application of knowledge in compound (contextual) tasks. Closed questions were compiled, with five answer options (A-E), with one correct answer.

The content of the tasks includes: Numerical tasks (calculation of perimeter, height, sides); Geometric reasoning (angles, properties of the median, bisector, height); Tasks with drawings; Proportions and relationships in triangles; Contextual tasks on the application of the properties of an isosceles triangle.

The difficulty level of the tasks varies from basic to advanced and meets the requirements of the 7th grade school curriculum. Some tasks are suitable for preparing for Olympiads or diagnostic tests.

The reliability of the test content was confirmed by expert assessments of experienced mathematics teachers, who provided feedback on the compliance of the test tasks with the learning objectives.

When conducting the study, we developed a plan for conducting a test on the topic "Application of the features and properties of an isosceles triangle" to ensure the accuracy and reliability of the collected data. This process includes several key stages:

- The researcher worked with experts in the field of mathematics in education to develop a test that determines the level of knowledge. All questions were checked by experts and an analysis was made of the specificity of the questions and their level of difficulty.
- A pilot test was conducted with participants who were not included in the list of main participants. This check showed which questions needed to be replaced or corrected. Based on this check, adjustments were made.
- The organization of the main testing included the preparation of test materials in printed form, proctors were instructed on the correctness during testing.

- On the day of the main testing, all participants were instructed. During the testing, proctors monitored the situation to avoid any violations and to maintain academic integrity, and answered questions that participants might have.
- To ensure the reliability of the data obtained, it was important that the testing conditions were the same for both groups.
- Calculation of the scores took place. Every student's result were examined and compiled into a report.

Ensuring the validity of the results and their possible influence on educational practice and policy depended mostly on following ethical norms, openness and methodological rigidity throughout this procedure.

2.1.2 Attitude Test

To comply with the topic guidelines and assess problem solving levels, a survey was developed where the responses were based on a Likert scale. Since the survey was not actually tested, pilot tests were conducted with a sample of materials that were not participants in the study to establish reliability. An internal agreement survey was conducted using Cronbach's alpha coefficient.

The survey consists of 18 questions and includes 4 subscales: general attitude towards mathematics, geometric thinking, problem solving skills, and the use of technology in mathematics. Respondents rated their understanding of all questions using a five-point Likert scale.

The following indicators will be used in the future:

General attitude to mathematics: This school of research gives the level of interest and how the student gets acquainted with mathematics in general. The questions determine whether students consider mathematics interesting and necessary for representing life.

Geometric Thinking: measures students' ability to visualize, spatially represent, logically analyze, and understand geometric texts. Particular attention was paid to the purpose of the experiment on isosceles triangles.

Problem Solving Skills: This assessment evaluates students' ability to approach mathematical problems strategically and systematically. Does the student know how to choose the right solution path and know where and what formula to use.

Use of Technology in Mathematics: The scale measures digital literacy and assumes that modern technologies improve the process of learning mathematics, such as calculators, GeoGebra, educational platforms, etc. The perception of technology as a resource that facilitates understanding of the material is also assessed. This scale reflects the digital literacy of students and their readiness to use modern teaching tools in mathematics.

In the reverse survey, the Cronbach's alpha coefficient was 0.979, which is an indicator of low internal consistency. For all questions, the indicator of good correlation between them (above 0.80) except for one, question Q3 showed the lowest low correlation ($r = 0.300$), and the indicator in the restriction question alpha would have been up to 0.986.

Table 2.1.1 Reliability Statistics of the Questionnaire Scale (based on Cronbach's α coefficient)

	Mean	**SD**	**Cronbach's α**
scale	3.76	0.805	0.979

When an item is eliminated for every question in the questionnaire, the Table 2.1.2 displays the values of standard deviation (SD), item-rest correlation, and Cronbach's α coefficient. For most items, the item-rest correlation values show high consistency with the general scale—that is, values exceeding 0.8. About 0.96, elements Q1, Q5, and Q10 show the strongest association value. With a clearly poor correlation with the others (0.300), Question Q3 indicates its weak link with the general scale's construct. Deleting Q3 would raise Cronbach's α to 0.986, far higher than that of the other questions, suggesting perhaps the necessity of its revision or elimination.

Table 2.1.2 Item Reliability Statistics and Cronbach's Alpha if Item Deleted

			If item dropped
	SD	**Item-rest correlation**	**Cronbach's α**
Q1	1.084	0.966	0.977
Q2	0.754	0.803	0.979
Q3	1.215	0.300	0.986
Q4	0.754	0.803	0.979
Q5	1.084	0.966	0.977
Q6	0.853	0.885	0.978
Q7	0.793	0.875	0.978
Q8	0.985	0.905	0.978
Q9	0.577	0.806	0.979
Q10	0.739	0.964	0.977
Q11	0.900	0.806	0.979
Q12	0.900	0.899	0.978
Q13	0.888	0.900	0.978
Q14	1.115	0.933	0.977
Q15	0.996	0.916	0.977
Q16	1.073	0.945	0.977

			If item dropped
	SD	Item-rest correlation	Cronbach's α
Q17	0.953	0.917	0.977
Q18	0.937	0.939	0.977

Exploratory factor analysis (EFA) was performed to determine the suitability of the data for factor analysis. The analysis showed that all questions except Q3 loaded strongly on one factor (from 0.78 to 0.98), Q3 had a low value (0.306), which indicates a weak connection with the survey as a whole. In the Table 2.1.3 we show Bartlett's test of sphericity was significant ($\chi^2 = \infty$, $p < 0.001$), which indicates variability of the data for factor analysis. We came to this conclusion, the question Q3 was changed: "For the safety and reliability of our survey."

Table 2.1.3 Bartlett's Test of Sphericity for Factorability Assessment

χ^2	df	p
Inf	153	<.001

The correlation analysis between the variables was tested using the Pearson correlation coefficients. Table 2.1.4 shows the correlation between the following variables: General attitude to mathematics, Geometric Thinking, Problem Solving Skills, Use of Technology in Mathematics, Academic Achievement. The results of the correlation analysis showed that:

The correlation between General attitude to mathematics and Academic Achievement is 0.654. This indicates that there is a moderately strong positive relationship between these variables. Students with a more positive attitude to mathematics have higher learning outcomes.

The correlation between Problem Solving Skills and Academic Achievement is 0.518. This indicates that there is a positive correlation between these variables. This shows that good problem solving skills contribute to academic success in mathematics.

The correlation between Geometric Thinking and Academic Achievement is 0.514. This indicates that there is a moderately positive correlation between these variables. This shows that geometric thinking directly affects academic achievement in mathematics.

The correlation between Use of Technology in Mathematics and Academic Achievement is 0.125. This indicates that there is a very weak correlation between these variables, or no correlation at all. This means that the use of technology in teaching does not have a significant impact on academic achievement in mathematics in this study.

The correlation between General attitude to mathematics and Geometric Thinking is 0.447. This indicates that there is a positive correlation between these variables. Students with a more positive attitude to mathematics have more developed geometric thinking.

The correlation between General attitude to mathematics and Problem Solving Skills is 0.403. This indicates that there is a positive correlation between these variables. Students with a positive attitude to mathematics often have better problem solving skills.

Table 2.1.4 Correlation coefficients for main variables

	Attitude	**Geometry**	**Problem-Solving**	**Technology**	**Academic Achievement**
Attitude	—	0.447	0.403	0.060	0.654
Geometry	0.447	—	0.462	0.233	0.514
Problem-Solving	0.403	0.462	—	0.402	0.518
Technology	0.060	0.233	0.402	—	0.125
Academic Achievement	0.654	0.514	0.518	0.125	—

To summarize the data presented above, the results of the correlation analysis showed that there is a statistically significant positive relationship between attitudes toward mathematics, geometric thinking, problem solving skills and academic performance. However, the level of connection with the use of technology was weak ($r = 0.125$), which requires further research and detailed analysis.

2.2. Participants

To foster effective learning, teaching should adapt teaching methods to bring about positive changes in students (Ezeddine et al., 2023). One must search for ways to teach and learn that align with a constructivist approach (Popova et al., 2022). Integrating real-world applications into math education is based on educational theories that emphasize active learning, student engagement, and critical thinking skills (Chau et al., 2021). Project-based learning, case studies, and Injure Based Learning are effective in making the learning process relevant, meaningful, and challenging (Mebert et al., 2020).

Effective teaching strategies include using group work, encouraging students to ask questions, and giving students problems related to daily life (Rivai et al., 2021). Group work encourages collaborative learning, which allows students to learn from one another, share ideas, and develop teamwork skills (Chau et al., 2021).

This professional development can include workshops, mentoring, and peer observation. Teachers must be given the time and resources they need to plan and implement engaging lessons, so that they can work together to share best practices and reflect on their teaching.

The results of the pedagogical experiment were published in 2025 by the Ministry of Education and Science of the Republic of Kazakhstan "Dostyq" (Almaty). This study used a quasi-experimental design with whole-group sampling. Two groups with 7th-grade students were selected for the experiment; these groups already existed before the experiment. Each group consisted of 12 students, and in total, 24 participants participated in our study.

2.3 Data Collection

The experimental part of our work began with the preparatory part, where each participant in the experiment was informed about the study. This procedure was done to comply with ethical standards for obtaining information, the age of the participants was taken into account. The study was anonymous, to avoid leaking personal data and information. Since the participants were minors, consent was obtained from the head teacher of the school to conduct the experiment. The purpose of the study was to develop a methodology for solving complex problems related to isosceles triangles. Two tests were used: a test for academic performance, a survey. The tests were conducted twice for each group to determine the impact of our methodology on the experimental group. The duration of the study was nine weeks, which allowed for enough time to observe significant changes or trends arising as a result of educational interventions. This period was considered sufficient for the implementation of the developed methodology. The experiment was conducted on a voluntary basis, which simplified the process, the participants felt comfortable, due to which one could not doubt the honesty of the respondents' answers. The data were collected using a paper survey. Students were given instructions on how the testing would be conducted and how to fill out the survey. There was enough time for the survey, which ensured maximum honesty from the participants. Careful preparation of the participants and a planned process of implementation provided us with high reliability of the data and the result.

2.4 Process

During the teaching process, the teacher integrated a new teaching method to better understand new topics and increase student engagement. The method that was used for our study is called Inquiry-Based Learning. This is an approach to learning where students make discoveries themselves, actively explore, ask questions, based on which they make their own hypotheses and analyze the collected data, and then make reasonable conclusions. The advantage of this method is that the student himself becomes a researcher and discovers new things for himself, instead of receiving ready-made information from the teacher. The main participant in this method is the student, and the teacher is a person who only gives direction. The IBL method is based on the constructivist theory of learning, which states that students understand and remember a topic better if they explore it on their own, using their experiences and the knowledge they have acquired. The student develops critical thinking, intelligence, the ability to rely on data and fundamental understanding.

An example lesson plan is provided below to show how the IBL approach is actually implemented. This course of instruction took place during the experimental stage of the research and concentrated on the use of geometric reasoning to solve isosceles triangle problems.

Тема долгосрочного плана: Треугольники	Школа: Образовательный центр Достық	
День:	ФИО учителя: Дүйсеке А.Қ.	
Класс: 7	Присутствовали:	Отсутствовали:
Тема урока	Равнобедренный треугольник, его свойства и признаки	
Цели обучения, достигаемые на этом уроке (ссылка на учебную программу)	7.1.1.23 Применение признаков и свойств равнобедренного треугольника.	
Цели обучения	Все учащиеся: знают определение равнобедренного треугольника, его свойства и признаки. Большинство учащихся: умеют применять свойства и признаки равнобедренных треугольников при решении задач, а также обладают навыками построения, используя их элементы. Некоторые учащиеся: умеют анализировать и доказывать свойства и признаки равнобедренного треугольника и применять их при решении сложных задач.	
Критерии оценивания	Различает равнобедренный треугольник и его элементы. Может решать задачи, используя свойства и признаки равнобедренного треугольника. Применяет свойства и признаки равнобедренного треугольника при решении задач повышенного уровня с доказательствами.	
Предметная лексика и термины:	Учащиеся: объясняют признаки и свойства равностороннего треугольника письменно и устно.	

<table>
<tr><td></td><td colspan="3">Предметная лексика и терминология:
треугольник, равнобедренный, равносторонний, разносторонний, элементы треугольника, высота, биссектриса, медиана, признаки равенства треугольников.</td></tr>
<tr><td>Связь с историей, культурой и языком:</td><td colspan="3">История (пирамиды), география (геометрические формы в окружающей среде и архитектуре).</td></tr>
<tr><td>Межпредметные связи</td><td colspan="3">История, изобразительное искусство, информатика, география.</td></tr>
<tr><td>Используемые цифровые ресурсы:</td><td colspan="3">Интерактивная доска, GeoGebra, Kahoot, Blooket.</td></tr>
<tr><td>Предварительные знания:</td><td colspan="3">Учащиеся знают: треугольник, признаки равенства треугольников, понятия высоты, биссектрисы, медианы.</td></tr>
<tr><td colspan="4">Ход урока</td></tr>
<tr><td>Этап</td><td>Деятельность учителя</td><td>Деятельность учащихся</td><td>Методы и ресурсы</td></tr>
<tr><td>I. Мотивация и постановка проблемы

5 мин</td><td>Преподносит ситуацию: “Почему архитекторы часто используют равнобедренные треугольники в постройках?” Показывает фото (мост, юрта, памятник). Ставит проблемный вопрос: Что делает равнобедренный треугольник особенным?</td><td>Отвечают на наводящие вопросы, формулируют гипотезы
Дескрипторы:
– Отвечает на вопрос, используя собственные примеры или наблюдения
– Формулирует хотя бы одну гипотезу о свойствах равнобедренного треугольника
– Участвует в обсуждении предложенной проблемы</td><td>Метапредметный подход, визуализация</td></tr>
<tr><td>II. Исследование (Inquiry)</td><td>Делит класс на группы, даёт задания:</td><td>Строят равнобедренный</td><td>IBL, работа в группах,</td></tr>
</table>

15 мин	1) Постройте треугольник с двумя равными сторонами. 2) Измерьте углы. 3) Сформулируйте гипотезу. Помогает с инструментами (линейка, транспортир, GeoGebra)	треугольник, проводят измерения, обсуждают закономерности, записывают выводы Дескрипторы: Верно строит равнобедренный треугольник (с соблюдением условий) Правильно измеряет углы и стороны Делает вывод о свойствах на основе наблюдений Работает в команде, вносит вклад в общее задание	активное исследование
III. Объяснение и обобщение 10 мин	Направляет дискуссию: учащиеся защищают свои гипотезы, а учитель подводит к теореме о равных углах при основании	Представляют результаты, обобщают свойства Дескрипторы: Участвует в обсуждении и защите гипотезы Чётко формулирует одно или несколько свойств равнобедренного треугольника	Работа с доской, коллективный анализ
IV. Применение знаний 10 мин	Раздаёт индивидуальные задания на применение свойств (расчёт углов, доказательства). Проверка с помощью дескрипторов	Решают задачи, аргументируют ход решения Дескрипторы: Верно применяет свойства при решении задачи Аргументирует свой выбор решения или хода построения	Самостоятельная работа, критериальное оценивание

		Решает задачу без ошибок / с минимальными ошибками	
V. Рефлексия и домашнее задание 5 мин	Просит заполнить карточку «Я узнал / Мне было сложно / Я хочу узнать» + даёт домашнее задание: «Найти равнобедренный треугольник в жизни и объяснить его свойства»	Делятся впечатлениями, задают вопросы Дескриптор: Осмысленно заполняет рефлексивную форму: «Я узнал / Мне было сложно / Я хочу узнать» Формулирует, какие знания получил на уроке Делится впечатлениями и задаёт вопросы Понимает суть домашнего задания и может кратко объяснить его цель	Рефлексия, метакогнитивные навыки

The use of this methodology and technology has improved the learning environment, where everyone understands and learns new information. In addition to academic success, students have developed their social skills, such as communication, public speaking, and negotiation, which are necessary for their personal growth and adaptation to the outside world.

2.5 Achievement test analysis

The descriptive data presented in Table 2.5.1 provide a complete statistical understanding of the pre- and post-test data in the control and experimental groups. There were 12 participants in both groups. The control group participants had a mean score of 6.92 before the experiment, indicating a low level of knowledge regarding isosceles triangles. After the experimental period, the mean post-test score increased to 10.92, indicating an improvement in the level of achievement. The standard deviation of the pre-test scores was 2.15, indicating how scattered the participants' responses were around the mean, which slightly increased to 2.75 in the post-test. Now let's analyze the experimental group, the mean pre-test score was 7.42, and after the introduction of the new methodology, the post-test score increased and was 17.75 points, indicating significant improvements in the level of knowledge of the participants. The standard deviation of the pre-test is 1.78, and after 1.76, indicating little improvement. These results show that the implementation of the new teaching method had a high positive impact on the level of students' knowledge in geometry in the experimental group, which is proven by a significant improvement in the post-test grades compared to the control group.

Overall, a detailed analysis of Table 2.5.1 provides valuable information about the effectiveness of the implementation of Inquiry-Based Learning in improving the level of students' knowledge and emphasizes the importance of improving the methods, which is proven by the result after implementation.

Also, Table 2.5.1 shows the results of the Shapiro-Wilk test conducted to assess the normality of the pre- and post-test grades of both groups. The results showed that the distribution of the performance data in both groups, at the pre- and post-test stages, does not deviate from normal ($p > 0.05$).

Table 2.5.1 Descriptive data and Shapiro–Wilk normality test for control and experimental group

					Shapiro-Wilk	
	Group	**N**	**Mean**	**SD**	**W**	**p**
Pre-Test (Academic Achievement)	Control	12	6.92	2.15	0.930	0.375
	Experimental	12	7.42	2.75	0.976	0.964
Post-Test (Academic Achievement)	Control	12	10.92	1.78	0.872	0.070
	Experimental	12	17.75	1.76	0.918	0.267

To test whether there was a statistically significant difference in the students' performance before and after the study period, a paired sample t-test was conducted. In Table 2.5.2 you can see the results, this analysis compared the mean scores of the pre-test and post-test in the control group of 12 people. The result of the analysis showed little statistical improvement in the performance of the control group ($p < .001$). This suggests that the differences between the pre-test and post-test were very unlikely to be the result of chance.

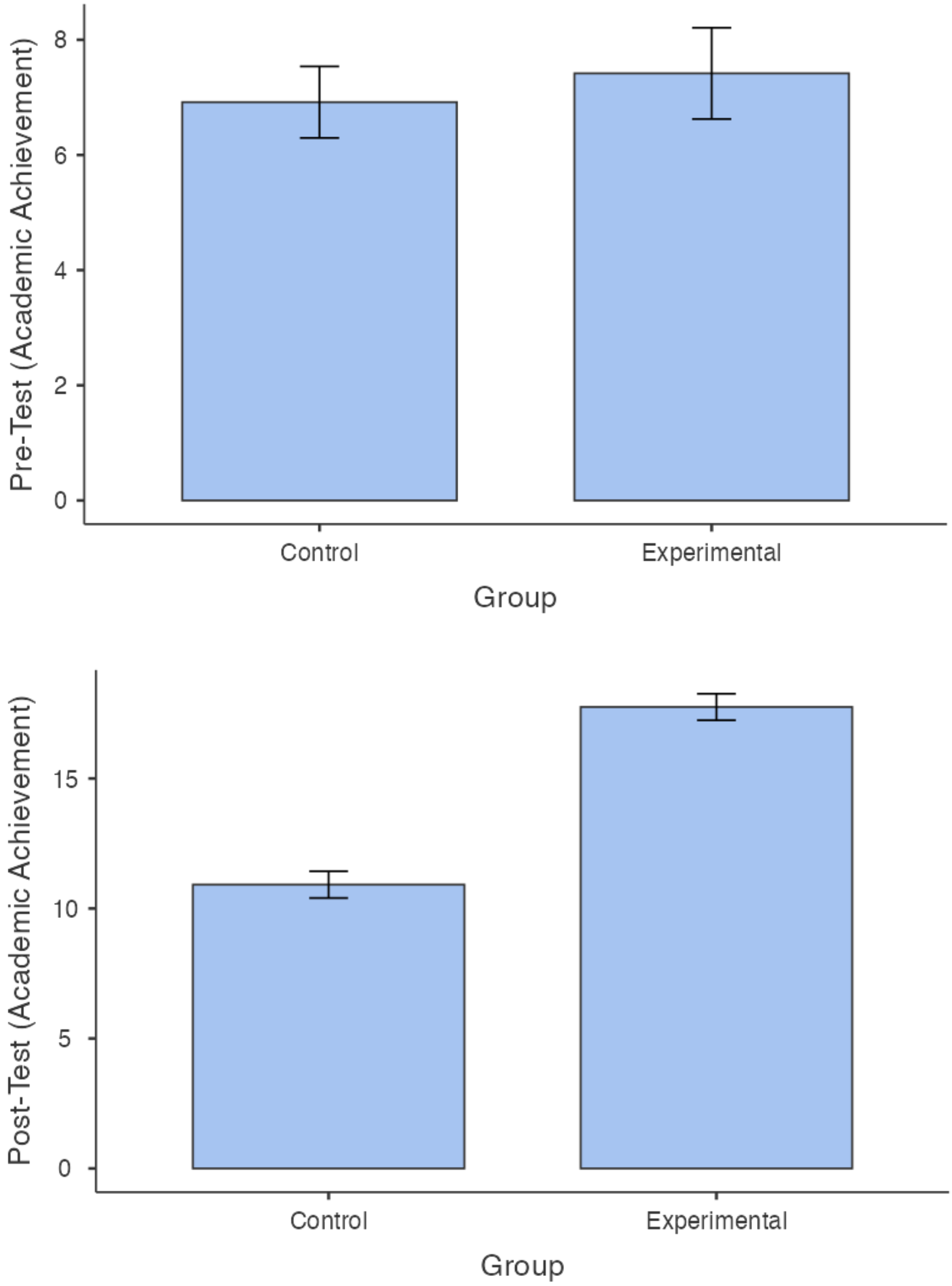
Table 2.5.2 Paired samples t-test for pre-test and post-test of control group

			statistic	**df**	**p**
Pre-Test (Academic Achievement)	Post-Test (Academic Achievement)	Student's t	-8.39	11.0	<.001

Note. H_a $\mu_{\text{Measure 1 - Measure 2}} \neq 0$

Figure 2.5.2 shows a bar chart illustrating the average performance of students in the control and experimental groups at the pre-test and post-test stages. Although the experimental group scored slightly higher before the intervention, the difference was not that big. The average scores in both groups were similar before the experiment (pre-test: experimental group 7.42, indicating no significant differences in the initial level of preparation; the control group showed an average score of about 6.92).

Figure 2.5.2 Comparison of Control and Experimental Group Performance on Pre- and Post-Test (Academic Achievement)



After the intervention, which included the use of an exploratory approach (IBL) in the experimental group, the Post-Test results demonstrate a noticeable difference. The experimental group achieved significantly higher results (an average of 17.75 points), compared to the control group (an average of 10.92 points).

Thus, the visualized data confirm the effectiveness of the proposed methodology aimed at developing mathematical thinking through active research and practical application of geometric concepts, such as the properties of an isosceles triangle.

To check whether there was a statistically significant difference in the students' performance before and after the introduction of the new methodology into the curriculum, a Paired samples t-Test was conducted. In Table 2.5.3 you can see the results, this analysis compared the mean scores of the pre-test and post-test in two groups of 12 people. The result of the analysis showed a highly significant increase in the performance of the experimental group ($p < .001$). This suggests that the variations between the pre-test and the post-test were quite improbable to have resulted from chance. A negative t-value denotes that the post-test results exceeded the pre-test ones.

Table 2.5.3 Paired samples t-test for pre-test and post-test of experimental group

			statistic	**df**	**p**
Pre-Test (Academic Achievement)	Post-Test (Academic Achievement)	**Student's t**	-13.4	11.0	<.001

Note. H_a $\mu_{\text{Measure 1 - Measure 2}} \neq 0$

Table 2.5.4 shows the test of the level of growth in academic performance and the comparison was made using the Independent Mann–Whitney U-test since the t-test showed inequality of dispersions and required further research. The Mann–Whitney criterion ($U = 0.500$, $p < .001$) indicates a statistically significant difference between the two control and experimental groups. This shows that the growth in the experimental group was significantly higher than in the control group. The data confirm the effectiveness of the implemented methodology.

Table 2.5.4 Independent samples t-test for Academic Achievement

		Statistic	**p**
Academic Achievement	Mann-Whitney U	0.500	<.001

Note. H_a $\mu_{\text{Control}} \neq \mu_{\text{Experimental}}$

Summarizing our analyses, we can say that according to the Paired samples t-Test, the results before and after the control group improved over time, but not as much as the experimental group. On the contrary, the experimental group showed a highly significant increase in the results after. This shows that the introduction of the new methodology into teaching had a great positive impact on students.

2.6 Attitude test analysis

In Table 2.6.1 you can see the descriptive statistics and the Shapiro-Wilk test of normal distribution of data, where you can see the general attitude of students towards mathematics. The analysis was conducted for the control and experimental groups before and after the interventions. The mean value for the control group pre-test was 2.95, and for the experimental 2.93. This shows similar results with a low standard deviation in both groups. And the Shapiro-Wilk test indicates normal distribution of data ($p > 0.05$). The results of the post-test of the control group, which was taught in the traditional format, the mean value was 3.43, and for the experimental group, where a new methodology was introduced into the learning process, the mean value was 4.27. This shows an increase in the result in both groups, but we see a significant difference in the results of the experimental group. The Shapiro-Wilk test analysis also indicates that the data is normally distributed ($p > 0.05$), which allows us to do parametric tests for analysis, such as Paired samples t-Test or Independent Samples T-Test.

Table 2.6.1 Descriptive data and Shapiro–Wilk normality test for pre-test and post-test of General attitude to mathematics

					Shapiro-Wilk	
	Group	**N**	**Mean**	**SD**	**W**	**p**
Pre-Test (General attitude)	Control	12	2.95	0.258	0.931	0.394
	Experimental	12	2.93	0.477	0.959	0.774
Post-Test (General attitude)	Control	12	3.43	0.239	0.929	0.372
	Experimental	12	4.27	0.299	0.953	0.682

Table 2.6.2 shows us the descriptive statistics and the test for normality of data distribution. The mean of the pre-test of the control group, which is 2.82 ($SD = 0.508$), while the result of the experimental group with a mean of 3.05 ($SD = 0.392$), does not show us a significant difference in the two groups. The Shapiro-Wilk value also shows the normality of data distribution in both groups ($p > 0.05$). The results of the post-test show us a significant increase in the experimental group, whose mean increased ($M = 4.25$). The control group also improved its average score ($M = 3.37$). However, the experimental group showed a significantly higher increase. And here the distribution of data was in the normality mark, which allows using parametric tests to analyze the experimental data.

Table 2.6.2 Descriptive data and Shapiro–Wilk normality test for pre-test and post-test of Geometric thinking

	Group	N	Mean	SD	Shapiro-Wilk	
					W	p
Pre-Test (Geometry)	Control	12	2.82	0.508	0.910	0.216
	Experimental	12	3.05	0.392	0.925	0.327
Post-Test (Geometry)	Control	12	3.37	0.239	0.905	0.182
	Experimental	12	4.25	0.243	0.940	0.495

Table 2.6.3 shows us the descriptive statistics and the Shapiro-Wilk test for normality of data distribution in two groups. The mean value of the control group before 2.82, and after 3.37, also for the experimental group before 3.05 and after 4.25. We can conclude that initially the results did not have a big difference, and the results after say that both groups showed growth, but statistically significant for the experimental group. The values of the Shapiro-Wilk test are also normal.

Table 2.6.3 Descriptive data and Shapiro–Wilk normality test for pre-test and post-test of Problem solving skills

	Group	N	Mean	SD	Shapiro-Wilk	
					W	p
Pre-Test (Geometry)	Control	12	2.82	0.508	0.910	0.216
	Experimental	12	3.05	0.392	0.925	0.327
Post-Test (Geometry)	Control	12	3.37	0.239	0.905	0.182
	Experimental	12	4.25	0.243	0.940	0.495

Table 2.6.4 shows us the descriptive statistics and the Shapiro-Wilk test for normality of data distribution in two groups. The mean value of the control group before 2.78, and after 3.00, also for the experimental group before 3.72 and after 4.28. It can be concluded that the experimental group initially had higher results, and the results after show that for both groups, the introduction of technologies does not play a big role, which proves a small increase. The Shapiro-Wilk test analysis also indicates that the data is normally distributed ($p > 0.05$), which allows us to do parametric tests for analysis, such as Paired samples t-Test or Independent Samples T-Test.

Table 2.6.4 Descriptive data and Shapiro–Wilk normality test for pre-test and post-test of Use of technology in mathematics

					Shapiro-Wilk	
	Group	**N**	**Mean**	**SD**	**W**	**p**
Pre-Test (Application of technology)	Control	12	2.78	0.538	0.947	0.600
	Experimental	12	3.00	0.586	0.898	0.148
Post-Test (Application of technology)	Control	12	3.72	0.489	0.911	0.223
	Experimental	12	4.28	0.278	0.843	0.030

To assess the significance of the differences between the results before and after the experiment, a Paired Samples T-Test was conducted within the experimental group. The test shows whether the introduction of new methods in teaching related to isosceles triangles affects statistically significant changes on four scales. According to the test results, it can be seen that the study showed that the difference between the pretest and posttest is statistically significant ($p < .001$) on all scales under study. This fact confirms the effectiveness of the technique.

Table 2.6.5 Paired t-test for pre-test and post-test for Attitude test

			statistic	**df**	**p**
Pre-Test (General attitude)	Post-Test (General attitude)	Student's t	-7.13	23.0	<.001
Pre-Test (Geometry)	Post-Test (Geometry)	Student's t	-7.67	23.0	<.001
Pre-Test (Problem-solving skills)	Post-Test (Problem-solving skills)	Student's t	-5.16	23.0	<.001
Pre-Test (Application of technology)	Post-Test (Application of technology)	Student's t	-7.63	23.0	<.001

Note. H_a $\mu_{\text{Measure 1 - Measure 2}} \neq 0$

Table 2.6.6 shows the test of the effectiveness of the pedagogical intervention between the groups, the Independent Mann-Whitney U test was used. On the scale of general attitude toward mathematics, the difference is statistically significant ($U = 12.5$, $p < .001$), the experimental group has a higher result than the control group. The scale of geometric thinking and problem solving skills also shows significant improvements. And on the scale of use of technology ($U = 54.5$, $p = .319$), the differences between the groups are not so significant ($p > .05$), which indicates the absence of influence, which requires further research.

Table 2.6.6 Independent samples t-test for Attitude test

		Statistic	**p**
General attitude	Mann-Whitney U	12.5	<.001
Geometry	Mann-Whitney U	23.5	0.005
Problem-solving skills	Mann-Whitney U	24.0	0.006
Application of technology	Mann-Whitney U	54.5	0.319

Note. H_a $\mu_{Control} \neq \mu_{Experimental}$

3. RESULTS

The results of this study provided important indicators of the interaction between students' attitudes towards geometry, and more specifically towards isosceles triangles. Descriptive statistics, relevance testing, correlation analysis between factors, paired sample t-tests and independent sample t-tests were used to achieve the objective of the study, which is to determine how solving complex problems related to isosceles triangles affects students' results and their academic performance.

The internal consistency of the survey showed a high rate (Cronbach's $\alpha = 0.986$), and this confirms the validity of this instrument, which is used to assess students' opinions and academic abilities.

In Pearson's correlation analysis, most of the indicators showed a positive relationship ($r = 0.514 - 0.654$), which proves their influence on each other and are important during lessons. But the use of technology showed no significant correlation ($r = 0.125$), which requires further research and analysis. With p-values $<.001$ in each case, the results of paired sample t-tests showed statistically significant increases in all four assessed areas (attitude, geometry, problem solving, and technology use) following the instructional intervention. These results imply that the Injure-Based Learning intervention, which focused on solving complex problems using isosceles triangles, was successful in improving both the affective and cognitive elements of student engagement.

Further analysis using independent samples t-tests compared delta scores between the control and experimental groups. Statistically significant differences were observed for:

Attitude toward mathematics ($t(22) = –4.54$, $p < .001$);

Geometric thinking ($t(22) = –3.46$, $p = .002$);

Problem-solving skills ($t(22) = –3.31$, $p = .003$).

These findings show that in several important areas the experimental group showed better progress than the control group. As in the previous analysis, there was no significant variation in technology use ($t(22) = –1.02$, $p = .318$), indicating a weak relationship with academic achievement. This suggests that the technology component may need to be revised or more tightly integrated to ensure significant educational gains.

Furthermore confirmed in non-parametric terms were these results using a Mann-Whitney U test. Apart from the technology use scale, the results matched those of the independent t-tests, indicating notable variations in the delta scores of all scales once more underlining the stability of this trend across statistical techniques.

Taken together, these results imply that the experimental strategy—based on solving real-life and geometrically rich tasks linked to isosceles triangles—was beneficial in enhancing students' conceptual development, engagement, and achievement. The method proved very effective in improving attitudes and reasoning abilities, which are prerequisites for ongoing mathematical achievement. Since the study involved 24 students from one school, there may be limitations. Therefore, the results may not be valid for a larger population and require further larger-scale research in the future.

Further research should also look at how technology can be used in mathematics instruction to highlight the potential for more successful integration and barriers.

Ultimately, the study supports the need to incorporate Injure Based Learning approaches — especially those that focus on challenging geometry tasks — into the middle school mathematics curriculum. These strategies not only help students achieve academic success, but also strengthen their deeper connection to the topic, making mathematics more relevant, interesting, and motivating.

CONCLUSION

The aim of our study was to improve students' skills in solving complex problems involving isosceles triangles. Thus, in the course of the work performed, all the main objectives of the study were achieved and solutions to the following problems were found:

1. The theoretical foundations for solving complex problems involving isosceles triangles were described;
2. The methodology for solving complex problems involving isosceles triangles was improved;
3. The effectiveness of the proposed methodology was experimentally tested and the results obtained were processed.

Based on an exploratory approach, culturally relevant teaching strategies, and a comparative analysis of Kazakhstani textbooks, this quasi-experimental study provided evidence of the positive impact of real and complex problems involving triangles on students' cognitive development.

An experimental group of 24 seventh-grade students from a Kazakhstani school demonstrated significant improvements in both academic performance and engagement levels, especially in areas such as geometric thinking, problem-solving strategies, and application of isosceles triangle properties. Quantitative results showed statistically significant improvements in post-test scores, with descriptive statistics, paired t-tests, and Shapiro-Wilk tests confirming the effectiveness of the intervention. In particular, the use of contextually based problems facilitated deeper understanding and transfer of knowledge to unfamiliar settings.

Analysis also revealed that students became more confident in identifying and applying properties of isosceles triangles, including base angle congruence, leg equality, and alignment of medians, altitudes, and angle bisectors from the vertex. Furthermore, students' increased ability to construct logical proofs and engage in higher-order reasoning reflected their development of geometric thinking skills. This was further supported by their improvement in responses on the attitude scale.

The pedagogical model used in this study emphasized the integration of culturally relevant motifs, historical references, and technological tools (e.g., GeoGebra, interactive whiteboards), which contributed to increased student motivation and engagement. The inclusion of Kazakh traditional ornaments and architectural patterns contributed to the development of national identity while grounding mathematical concepts in students' life experiences.

Furthermore, the analysis of textbooks revealed that although Kazakh textbooks contain a sufficient number of isosceles triangle problems, they often lack the variety and complexity needed to develop spatial thinking and creativity. This points to the need for future textbook reform and the integration of higher-level problems in line with international standards.

From a theoretical perspective, the study is consistent with cognitive load theory and supports the use of scaffolding and step-by-step modeling to help students manage complex information. The study also supports the usefulness of the "key problem method," in which instruction is structured around a central geometric problem that introduces and relates key properties.

In conclusion, the results of this study provide compelling evidence for the value of integrating challenging isosceles triangle problems into secondary geometry lessons. This approach not only improves academic achievement, but also promotes critical mathematical habits of mind, including visual thinking, logical deduction, and the ability to connect abstract concepts with real-world applications.

Key Findings:

- Geometry instruction should include more challenging, culturally relevant problems that allow for exploration, construction, and proof.
- Teacher professional development should emphasize methods to support geometric thinking, including the use of visual, technological, and real-world contexts.
- Curriculum developers should consider balancing computational problems with open-ended, creative, and evidence-based problems to develop deeper understanding.

Recommendations for Future Research:

- Expand the study to include a larger sample from a variety of regions and school types in Kazakhstan.
- Investigate the long-term retention of geometric concepts following research-based interventions.
- To examine the impact of culturally responsive mathematics education on other areas of mathematical literacy, such as algebraic thinking or data interpretation.

Ultimately, this study contributes to a growing body of research advocating for innovative, student-centered, and contextually relevant methods of teaching mathematics. It highlights the role of geometric thinking in shaping not only students' academic trajectories, but also their sense of identity, curiosity, and problem-solving confidence beyond the classroom.

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APPENDIX

Appendix 1. Attitude test

Данная анкета является анонимной и не будет показана никому, кроме исследовательской группы. Вся информация хранится в конфиденциальной форме и используется только для научно-исследовательской работы.

1 – полностью не согласен, 2 – не согласен, 3 – нейтрально, 4 – согласен, 5 – полностью согласен

	Вопросы	1	2	3	4	5
1	Мне нравится урок математики.					
2	Я думаю, что математика пригодится мне в будущей жизни.					
3	Мне легко решать математические задачи.					
4	Для меня важно получать хорошие оценки по математике.					
5	Мне нравится решать задачи самостоятельно.					
6	На уроке геометрии мне интересно чертить фигуры.					
7	Я умею различать виды треугольников.					
8	Я знаю свойства равнобедренного треугольника.					
9	Задачи, связанные с равнобедренным треугольником, даются мне легко.					
10	Я умею правильно представлять себе геометрические фигуры.					
11	При решении задач я всегда составляю план.					
12	Если задача сложная, я разбиваю её на части и решаю поэтапно.					
13	Я могу объяснить решение геометрических задач.					
14	Я умею решать задачи, используя чертёж.					
15	Я хочу научиться приводить доказательства при решении задач.					

16	На уроке геометрии интересно использовать такие программы, как GeoGebra.					
17	Компьютерные модели помогают мне понять задачу.					
18	Манипулируя фигурой на экране, я лучше понимаю материал.					

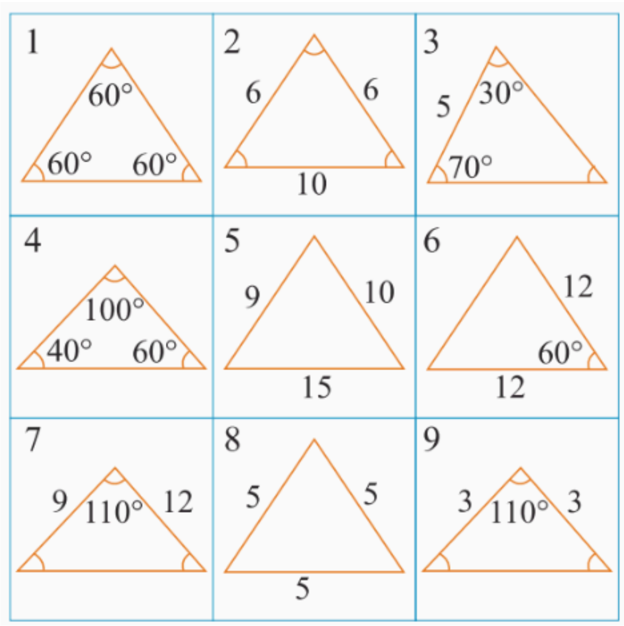
ВИДЫ ТРЕУГОЛЬНИКОВ.
РАВНОБЕДРЕННЫЙ ТРЕУГОЛЬНИК.

<u>Запомните</u>

Виды треугольников по длине его сторон:
Равнобедренный треугольник - треугольник, у которого две стороны равны.
Равносторонний треугольник - треугольник, у которого все стороны равны.
Разносторонний треугольник - треугольник, у которого стороны имеют разные длины.

- **Пример 1**

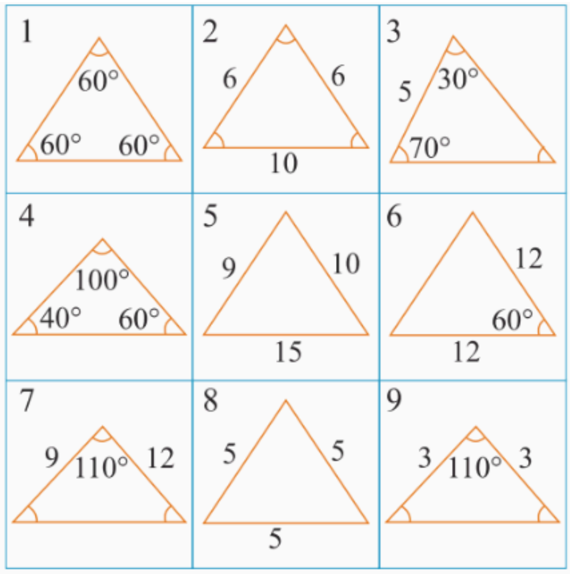
Запишите равнобедренные треугольники по таблице.



Решение:
Равнобедренные треугольники: 2, 4, 9

- **Теперь попробуйте сами**

Запишите равносторонние и разносторонние треугольники по таблице.

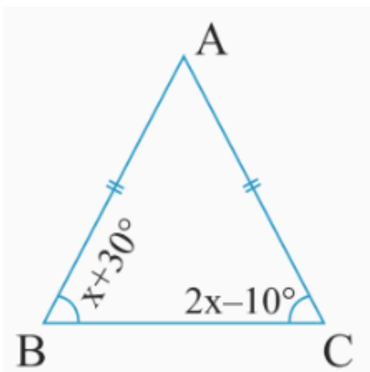


<u>Запомните</u>

Теорема: Углы при основании равнобедренного треугольника равны.
Обратная теорема: Если два угла треугольника равны, то это равнобедренный треугольник.

- **Пример 2**

По ниже данному рисунку найдите значение x.

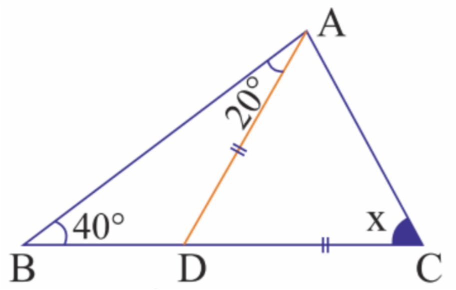


Решение:
В треугольнике ABC стороны AB = AC , поэтому это равнобедренный треугольник. Тогда $\angle B \;=\; \angle C$:

$$x + 30° = 2x - 10°$$
$$x = 40°$$

- **Пример 3**

По нижеданному рисунку найдите значение x.

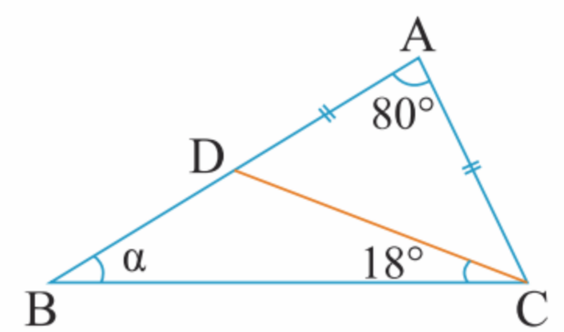


Решение:
Так как стороны AD = DC, то угол ∠DAC также равен x.
Составим уравнение:

$$x + (x + 20°) + 40° = 180°$$
$$2x + 60° = 180° \;\Rightarrow\; x = 60°$$

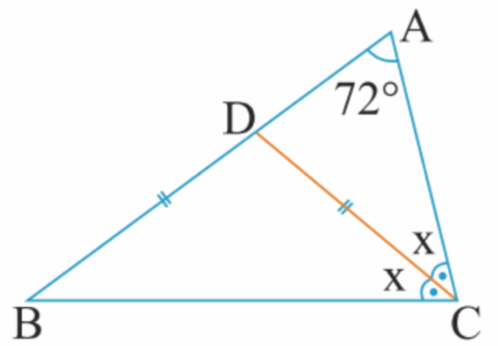
♦ **Теперь попробуйте сами**

По нижеданному рисунку найдите значение α.



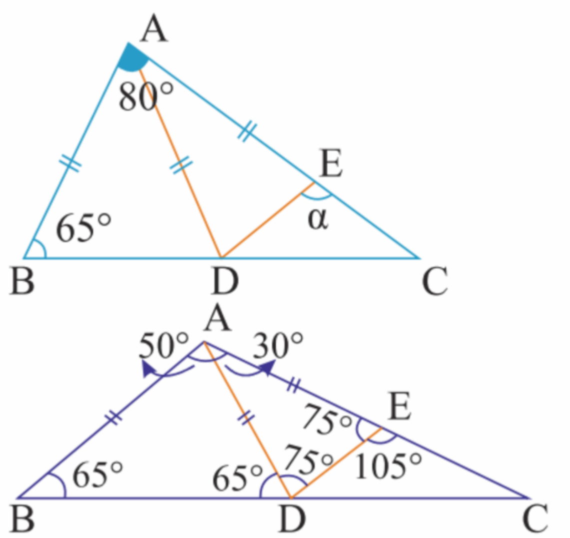
♦ **Теперь попробуйте сами**

По нижеданному рисунку найдите значение x.



- **Пример 4**

В треугольнике ABC угол ∠BAC = 80°. Найдите значение угла α.



Стороны AB = AD, углы ∠ABC = ∠BDA = 65°.
Рассмотрим треугольник ABD:

$$\angle ABD + \angle BDA + \angle DAB = 180°$$
$$65° + 65° + \angle DAB = 180° \;\Rightarrow\; \angle DAB = 50°$$

Так как угол ∠BAC = ∠DAB + ∠DAE, то:

$$80° = 50° + \angle DAE \;\Rightarrow\; \angle DAE = 30°$$

Рассмотрим треугольник ADE:

$$\angle DAE + \angle AED + \angle EDA = 180°$$

Поскольку AD = AE, углы при основании равны:

$$\angle ADE = \angle AED = x$$

$$30° + x + x = 180° \Rightarrow 2x = 150° \Rightarrow x = 75°$$

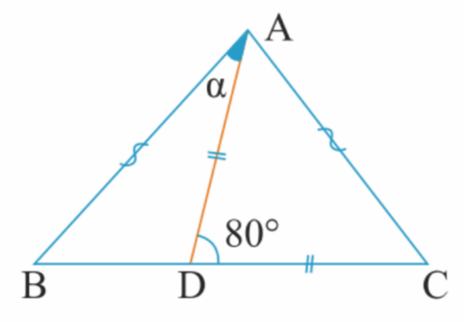
Теперь найдем угол ∠DEC, который смежный с ∠AED:

$$\angle AED + \angle DEC = 180° \Rightarrow$$
$$75° + \angle DEC = 180° \Rightarrow \angle DEC = 105°$$

Следовательно, $\alpha = \angle DEC = 105°$

- **Теперь попробуйте сами**

По нижеданному рисунку найдите значение α.

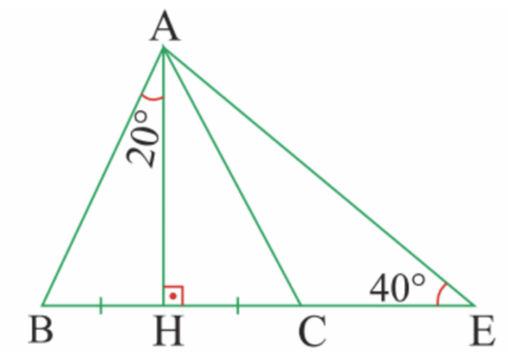


<u>Запомните</u>

Следствие: Биссектриса, опущенная на основание равнобедренного треугольника, является одновременно и *медианой*, и *высотой*.

- **Теперь попробуйте сами**

В треугольнике ABE отрезок AH⊥BE, BH = HC, ∠BAH = 20°, ∠AEB = 40°. Найдите значение угла ∠CAE.



Задача 1

На изображении представлены традиционные казахские украшения в виде равнобедренных треугольников.

Вычислите периметр каждого украшения.

У первого украшения стороны треугольника равны: 7 см, 7 см, 10 см.

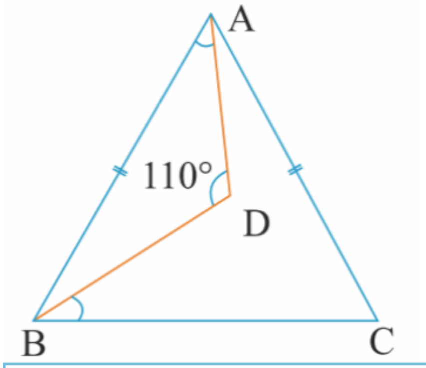
У второго украшения стороны составляют: 8 см, 8 см, 12 см.

У третьего украшения стороны составляют:

9 см, 9 см, 13 см.

Задача 2

Если в треугольнике AB = AC, углы ∠BAD = ∠DBC, и угол ∠ADB = 110°, то найдите значение угла ∠BAC.

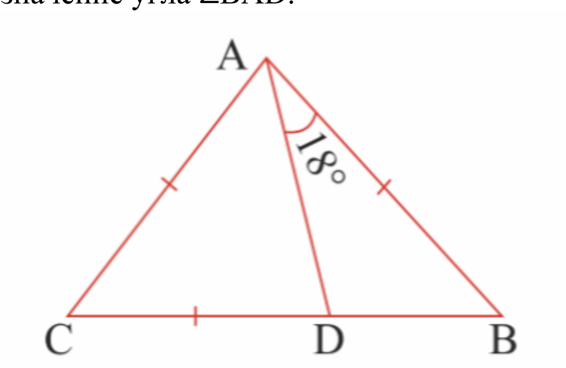


Задача 3

Углы A, B и C треугольника ABC равны 72°, 72° и 36° соответственно. Если сумма длин биссектрисы AK и
отрезка KC равна 8 см, то найдите длину стороны AB.

Задача 4

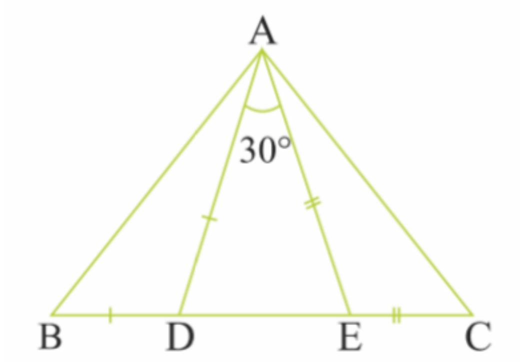
В треугольнике ABC даны равенства: AC = AB = BD, а угол ∠DAC = 18°. Найдите значение угла ∠BAD.



A) 48°
B) 56°
C) 66°
D) 72°
E) 78°

Задача 5

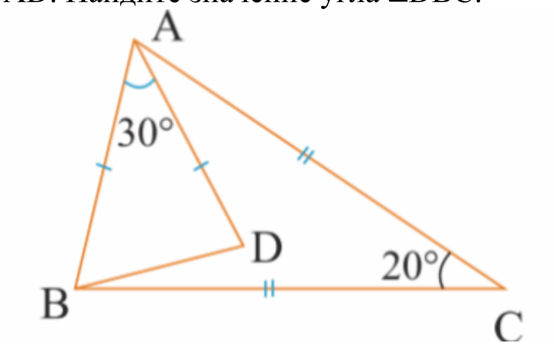
В треугольнике ABC даны равенства: AD = BD, AE = EC, и угол ∠DAE = 30°. Найдите значение угла ∠BAC.



A) 75°
B) 90°
C) 95°
D) 100°
E) 105°

Задача 6

В треугольнике ABC угол ∠BAD = 30°, угол ∠ACB = 20°, стороны AC = BC, AB = AD. Найдите значение угла ∠DBC.



A) 5°
B) 10°
C) 15°
D) 20°
E) 25°

Задача 6
Стороны равнобедренного треугольника равны 3 см и 5 см. Вычислите периметр треугольника.

A) 11 см
B) 13 см
C) 11 см или 13 см
D) 16 см
E) 13 см или 16 см