The usage of 3D modeling in teaching geometry in high school

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List of Abbreviations

3D – Three-dimensional

CAD – Computer-aided design

CG – Control group

EG – Experimental group

ICT – Information and communication technology

NCTM – National Council of Teachers of Mathematics

PSVT – Purdue Spatial Visualization Test

STEM – Science, Technology, Engineering, and Mathematics

ABSTRACT

This study examined the effect of 3D modeling tools on Grade 11 students' spatial abilities and academic achievement in geometry. The research used a quasi-experimental design with two nonequivalent groups: 25 students in the experimental group and 25 students in the control group. The experimental group studied solid geometry using Tinkercad and SketchUp Free, while the control group studied the same topics through traditional instruction.

Data were collected through the Purdue Spatial Visualization Test (PSVT), geometry grades before and after the intervention, and a questionnaire on students' attitudes toward 3D modeling. The results showed that the experimental group had a statistically significant improvement in spatial ability, while the control group did not show significant change. However, geometry grades increased only slightly and equally in both groups, so the effect on general academic achievement was not clearly demonstrated. Students generally showed a positive attitude toward 3D modeling.

The study concludes that Tinkercad and SketchUp Free can support the development of spatial visualization, mental rotation, and spatial orientation in high school geometry.

Key words: 3D modeling, spatial ability, geometry education, Tinkercad, SketchUp Free, PSVT.

АҢДАТПА

Бұл зерттеу 3D-модельдеу құралдарының 11-сынып оқушыларының кеңістіктік қабілеттеріне және геометрия пәні бойынша академиялық үлгеріміне әсерін зерттеді. Жұмыста екі тең емес топтан тұратын квазиэкспериментік дизайн қолданылды: эксперименттік топта 25 оқушы және бақылау тобында 25 оқушы болды. Эксперименттік топ стереометрияны Tinkercad және SketchUp Free құралдары арқылы оқыды, ал бақылау тобы дәстүрлі әдіспен меңгерді.

Деректер Purdue Spatial Visualization Test (PSVT), геометрия пәні бойынша араласуға дейінгі және кейінгі бағалар, сондай-ақ оқушылардың 3D-модельдеуге деген көзқарасын анықтайтын сауалнама арқылы жиналды. Нәтижелер эксперименттік топта кеңістіктік қабілеттердің статистикалық мәнді жақсарғанын көрсетті, ал бақылау тобында мәнді өзгеріс байқалмады. Алайда геометрия бағалары екі топта да аз ғана және бірдей деңгейде өсті, сондықтан жалпы академиялық үлгерімге әсері нақты дәлелденбеді. Оқушылар 3D-модельдеуді жалпы оң бағалады.

Зерттеу қорытындысы бойынша Tinkercad және SketchUp Free жоғары сыныптарда геометрияны оқытуда кеңістіктік визуализацияны, ойша айналдыру қабілетін және кеңістіктік бағдарлануды дамытуға ықпал ете алады.

Түйінді сөздер: 3D-модельдеу, кеңістіктік қабілеттер, геометрияны оқыту, Tinkercad, SketchUp Free, PSVT.

АННОТАЦИЯ

Данное исследование изучало влияние инструментов 3D-моделирования на пространственные способности и академическую успеваемость учащихся 11 класса по геометрии. В работе был использован квазиэкспериментальный дизайн с двумя неэквивалентными группами: 25 учащихся в экспериментальной группе и 25 учащихся в контрольной группе. Экспериментальная группа изучала стереометрию с использованием Tinkercad и SketchUp Free, а контрольная группа – традиционным методом.

Данные были собраны с помощью Purdue Spatial Visualization Test (PSVT), оценок по геометрии до и после вмешательства, а также анкеты об отношении учащихся к 3D-моделированию. Результаты показали статистически значимое улучшение пространственных способностей в экспериментальной группе, тогда как в контрольной группе значимых изменений не было. Однако оценки по геометрии увеличились незначительно и одинаково в обеих группах, поэтому влияние на общую академическую успеваемость не было чётко доказано. Учащиеся в целом положительно оценили использование 3D-моделирования.

Исследование позволяет сделать вывод, что Tinkercad и SketchUp Free могут способствовать развитию пространственной визуализации, мысленного вращения и пространственной ориентации при изучении геометрии в старших классах.

Ключевые слова: 3D-моделирование, пространственные способности, обучение геометрии, Tinkercad, SketchUp Free, PSVT.

1. INTRODUCTION

The relevance**.** The study of the impact of 3D modeling technologies on students' spatial abilities and academic performance in geometry in high school is one of the most relevant areas of modern pedagogical science. This research is of particular importance in the context of active digitalization of knowledge, since the pedagogical community is faced with the task of not only introducing technologies into the learning process, but also providing scientific justification for their effectiveness in relation to specific academic subjects and educational contexts.

Geometry, and especially stereometry, is traditionally considered one of the most difficult sections of the school mathematics program. The difficulties that students encounter in learning three-dimensional geometric figures are mainly associated with the underdevelopment of spatial thinking: the ability to perceive, imagine and mentally manipulate three-dimensional objects, etc. Traditional approaches to teaching based primarily on two-dimensional images of volume geometrical figures presented in books and on the blackboard cannot ensure the complete provision of an interactive experience of interaction with three-dimensional objects required for the establishment of solid spatial concepts (Sorby, 2009).

Moreover, the practical value of the study is due to the fact that the chosen three-dimensional modeling tools – Tinkercad and SketchUp Free platforms – are totally free to use, can be accessed through the web-browser, and therefore do not require additional software. Therefore, they can be effectively employed at higher educational institutions of Kazakhstan, even those that lack advanced technical facilities. Such analysis will be highly interesting for policymakers in the field of educational policy and curriculum design who want to update the technology used in the mathematics class.

Furthermore, another reason why this research is highly relevant is the fact that it helps advance professional development of pedagogues because it describes the mechanisms of influence of 3D modeling on the better understanding of mathematical concepts. These findings can be applied while developing the curricula for teachers'

professional training. It is expected that the increase in the methodological competence of teachers, in turn, will significantly improve the effectiveness of teaching and the quality of students' learning experience.

Finally, it is important to note that this study is not limited to the study of purely academic indicators. The use of interactive digital tools in the learning process creates conditions for active student participation, the development of cognitive autonomy, and the formation of positive motivation for learning geometry. Thus, this study is not just an academic task, but a very important study that is in line with the broader educational goals of preparing students for the requirements of the 21st century.

The background. 3D modeling in education is a teaching method in which students use special software platforms to create, transform, and analyze three-dimensional geometric objects. The theoretical basis of this study is the van Hiele model of levels of geometric thinking (van Hiele, 1986). According to this model, learning geometry requires purposeful practical experience working with spatial objects. It cannot be provided solely by verbal explanations or flat images. It is this theoretical formulation that underlies the educational potential of 3D modeling tools as a didactic tool in teaching geometry (Clements & Battista, 1992).

The National Council of Teachers of Mathematics (NCTM, 2000) has long advocated the inclusion of spatial thinking and three-dimensional geometry in school mathematics curricula and calls for the consideration of a variety of situations in which students explore geometric objects in one, two, and three dimensions. Similar principles are also reflected in OECD documents, which increasingly affirm spatial literacy as a key competence. In the Kazakhstani context, these guidelines are included in the State Program for the Development of Education and Science, which envisages the large-scale digitalization of the educational process.

With the widespread availability of affordable computer tools in the 1990s-2000s, researchers began to actively explore the potential of the dynamic geometry programs Cabri 3D and GeoGebra in developing students’ spatial abilities. The accumulated empirical data prove that students working with such tools show

significantly higher results in spatial visualization and geometry than those taught using traditional methods (Toptaş et al., 2012; Gecu-Parmaksiz & Delialoglu, 2019).

Recently, platforms dedicated to the creation of 3D models have gained popularity in educational settings. For instance, Autodesk released Tinkercad, which provides users with a simple graphical user interface that debuted in 2011. Students can build models using elementary geometric shapes such as cubes, cylinders, pyramids, prisms, and other types of forms. In addition, SketchUp Free, which Google produced initially in 2006, provides additional functions and is extensively used in engineering and architectural training (La Ferla et al., 2009). Both applications are free to use and available for smartphones, making them appropriate for deployment at schools in Kazakhstan.

Several quasi-experimental research projects have provided convincing support for utilizing 3D modeling in geometry lessons. Specifically, Sorby (2009) found that specific spatial tasks completed with 3D applications improve mental rotation and spatial visualization skills. Similar results were obtained by Toptaş et al. (2012) and Rafi et al. (2008), whose participants in experimental groups performed better than their counterparts in control groups in tests of spatial abilities.

The investigation of 3D modeling as an instructional approach to high school geometry lessons is relevant and critical. Geometry knowledge is viewed as one of the key prerequisites of success not only in math courses but also in many different occupations determining students' future career paths. Moreover, the importance of this study is highlighted by an increasing number of empirical findings that prove 3D modeling to be a highly effective means of improving students' spatial skills and geometry performance. This research endeavors to investigate these issues further among Kazakhstani learners enrolled in grade 11 and produce meaningful results.

Research Questions:

- How does the use of 3D modeling tools affect the spatial abilities of high school students?

- Does the use of 3D modeling in geometry instruction affect students' academic achievement compared to traditional teaching methods?

The specific objectives of this research are:

- To assess the impact of 3D modeling technologies (Tinkercad and SketchUp Free) on the spatial abilities of 11th grade students.
- To study the impact of 3D modeling on the academic performance of high school students in geometry compared to traditional teaching.
- To identify potential advantages and challenges associated with the use of 3D modeling tools in teaching high school geometry in Kazakhstani schools.

The study is based on the following hypotheses.

Null Hypothesis (H01): There is no significant difference in spatial abilities between 11th grade students taught geometry using 3D modeling tools (Tinkercad and SketchUp) and those taught through traditional methods.

Null Hypothesis (H02): There is no significant difference in the academic performance of students taught using 3D modeling tools and students taught using traditional methods in geometry.

Alternative Hypothesis (H1): There is a significant difference in spatial abilities between 11th grade students taught geometry using 3D modeling tools (Tinkercad and SketchUp) and those taught through traditional methods.

Alternative Hypothesis (H2): There is a significant difference in academic achievement in geometry between students taught using 3D modeling tools and those taught through traditional methods in geometry.

Taking into account the formulated research questions and hypotheses, this research aims to explore the role of 3D modeling in developing students' spatial skills and academic success in geometry. It is expected that this research will produce practical conclusions with regard to improving the teaching process in geometry for high school students, creating an effective learning environment, and ensuring long-term student success.

For reaching the purpose of this research with regard to the influence of 3D modeling technologies on the development of students' spatial skills and geometry proficiency, the following tasks were developed:

- Conduct an extensive review of existing literature to examine the theoretical foundations and empirical research dedicated to the application of 3D modeling technologies in the context of geometry instruction. This task involves synthesizing studies that investigate both the cognitive outcomes of using 3D tools, their impact on students' spatial abilities, and their effect on academic achievement in geometry.
- Identify and analyze various approaches to the use of 3D modeling tools in geometry instruction. This task focuses on comparing the effectiveness of different didactic strategies for integrating Tinkercad and SketchUp into the learning process, individual work with models, project-based activities, and group analysis of geometric objects, in terms of their impact on spatial thinking and students' comprehension of the material.
- Design and conduct empirical research to collect data on the academic achievement and spatial abilities of students participating in instruction incorporating 3D modeling. This involves quantitative methods: comparing pre-test and post-test results in the experimental and control groups, as well as analysis of current geometry grades. Additionally, a questionnaire survey of the experimental group students is conducted to identify their attitudes toward the use of 3D tools in the learning process.
- Analyze and synthesize the collected data to test the hypotheses that the use of 3D modeling tools improves students' spatial abilities and enhances their academic achievement in geometry. The results of the analysis will be used to clarify the understanding of how 3D modeling technologies can be effectively introduced into the educational process of Kazakhstani high schools, as well as to formulate practical recommendations for teachers and curriculum developers.

Every tasks is aimed at thoroughly testing the research hypotheses and gaining insight into ways of optimizing 3D modeling tools for improving teaching outcomes in geometry classes at the high school level.

The research aims to achieve the main goal of helping students improve their understanding and academic performance in geometry, which is a subject that places high demands on spatial imagination and abstract thinking. A study examining the impact of 3D modeling technologies on student engagement and understanding provides important information on creating a productive and engaging learning environment for geometry. In addition, the implications of the research could be applied for further improvement of pedagogical practice and curriculum design and include suggestions regarding incorporating 3D modeling techniques in teaching processes. Also, the paper will address some aspects of educational equality, suggesting how to achieve equality between students from different socioeconomic layers by providing an equal opportunity to use the latest technology in education. Lastly, the research could help change the current educational policy through the inclusion of 3D modeling technologies in educational standards of the country.

The scientific novelty of this research from a scientific perspective lies in the simultaneous use of two 3D modeling applications – Tinkercad and SketchUp. It creates an immersion that would not be achievable when conducting experiments in relation to one platform. Furthermore, the research is carried out against the backdrop of the educational system and culture of the Republic of Kazakhstan. There is very little empirical information about the use of 3D modeling in mathematics learning in the literature of this country. Therefore, the novelty of the study comes from the fact that it provides a unique geographical contribution to the field. Another novel aspect is the use of mobile devices as the main instrument for carrying out 3D modeling. Finally, the simultaneous assessment of the students' spatial ability, academic performance and survey data allows obtaining results more holistically.

2. LITERATURE REVIEW

2.1 Theoretical Framework

The theoretical foundation for this literature review is based on five interrelated theories that explain the educational reasoning behind using 3D modeling software in geometry classes in secondary schools and predict their possible impact on the spatial abilities of learners and their academic achievement. These theories include the van Hiele model of geometric thought, Sweller's theory of cognitive load, action-based learning theory, a constructivist view of math education, and spatial abilities and their elements. Each theory reveals unique aspects of the learning process in geometry, while collectively forming a complex multilevel theory for the current investigation. Before introducing specific notions from the theories mentioned, it is essential to contextualize the discussion theoretically: the existing literature on spatial abilities and math education demonstrates that geometry is especially vulnerable to changes in spatial abilities, and technology-assisted teaching that actively involves spatial systems of learners performs significantly better in geometry lessons compared to conventional textbook instruction (Herrera et al., 2024; Uttal et al., 2013).

2.1.1 The Concept of Spatial Thinking and Its Components

The study of spatial imagery has a history that dates back a hundred years of scientific progress; however, much confusion remains regarding terminology and concepts. The scientists diverge from each other when they describe spatial imagery, its subcomponents, and even when they delineate boundaries between spatial imagery and other cognitive functions like visual perception, memory, and mathematical reasoning. First and foremost, terminological misunderstanding stems from the differentiation of the notions of "ability" and "skill," which refers to inherent and relatively immutable qualities versus skills which can be easily improved upon with education and training (Sorby, 2009). This division no longer applies since there is ample evidence of malleability in spatial imagery skills and their improvement via

learning (Uttal et al., 2013). Hence, in this investigation, the terms have been used interchangeably as is common practice among recent meta-analyses.

While the former has long been perceived as an innate and more or less stable disposition, the latter has been considered to be a skill that can be consciously cultivated via training and education. One of the best-known classifications of spatial skills belongs to Linn & Petersen (1985), who described spatial skills as "the skills needed for the manipulation of symbolic and non-verbal information with regard to representation, transformation, generation, and reproduction". Three main categories have been revealed according to their meta-analysis. Spatial imagery skills involve manipulating multidimensional objects mentally, which requires complex operations, e.g., visualizing how a two-dimensional drawing may be folded into a three-dimensional object or what section will be created when cutting an object with a plane. Mental rotation involves rotating an entire image either two- or multidimensional within seconds. Spatial perception skills include the ability to define spatial relationships between two or several objects while retaining spatial awareness. It should be noted that the three types mentioned above are interconnected to some extent and positively correlated with math performance especially in geometry (Pittalis & Christou, 2010).

Carroll (1993) proposed a more precise taxonomy that includes five components in his fundamental factor analysis of hundreds of studies: visualization, spatial relationships, rate of closure, flexibility of closure, and speed of perception. Visualization in Carroll's model essentially corresponds to spatial visualization in Linn and Petersen's taxonomy, while spatial relationships are closely related to mental rotation. Maier (1998) proposed a different five-component structure. In the study, he notes that students' spatial intelligence is determined based on the results of spatial intelligence tests. The indicators of spatial intelligence used include spatial perception, visualization, mental rotation, spatial relationships, and spatial orientation. Here, spatial orientation refers to the ability to mentally change perspective while objects remain stationary. Sorby (2009) proposed a simplified two-component model that

distinguishes between spatial orientation and spatial visualization and considers mental rotation as a subset of spatial visualization. Despite these taxonomic differences, most researchers agree that spatial visualization is the most fundamental component of school geometry, as it allows for mental manipulation of three-dimensional objects, which is the primary cognitive task in learning spatial geometry (Harris, 2021).

Quantitative studies confirm a robust relationship between spatial ability and geometric skills. Guay and McDaniel (1977) reported correlations between spatial visualization and mathematical ability ranging from .47 to .86 for students in grades 2-7. Fennema and Sherman (1977) found strong correlations between spatial visualization and mathematical ability in grades 9 and 12. Pittalis and Christou (2010) demonstrated that spatial reasoning is a strong predictor of three-dimensional geometric reasoning in grades 5 and 6. Xu et al. (2025) showed that mental rotation, spatial reasoning, and spatial orientation independently predict mathematical ability across various subjects. Wai et al. (2009) found in a 50-year longitudinal study that spatial reasoning predicted career success in STEM, above and beyond verbal and mathematical ability. A large-scale twin study (Malanchini et al., 2020) found evidence for a common spatial factor underlying various subskills, while Bar-Hen-Schweiger and Henik (2024) confirmed that various spatial components independently have predictive value for mathematical ability. These results prove that spatial reasoning plays an important role in the study of geometry, which is why it is chosen as the main measurable indicator in this study.

2.1.2 Van Hiele Theory of Geometric Thinking Levels

In the late 1950s, van Hiele's theory was created by two Dutch scientists, Pierre van Hiele and Dina van Hiele-Geldof. Both scientists wrote their doctoral dissertations on this, and over time their ideas became widely known and recognized around the world, especially after other researchers, in particular Crowley in 1987, began actively developing and spreading this theory.

The main idea of the theory is as follows: when a child learns geometry, his thinking does not develop chaotically, but passes through clearly defined levels, each

of which is qualitatively different from the previous one. In other words, at first the child perceives geometric shapes simply as whole images. For example, he sees a triangle and recognizes it, but still does not understand why it is called a triangle. Then he begins to notice the properties of the shapes, then learns to build logical connections between these properties, and so on. It is important that the transition from one level to another depends not on the age or natural development of the child, but on how well and competently the training is organized. This means that if the teacher uses appropriate methods and assignments, the student can move faster to a higher level of geometric thinking. If the training is not structured correctly, the child may be stuck at the elementary level for a long time, even being old enough.

Van Hile's model identifies five successive levels of geometric thinking, each of which is qualitatively different from the previous one.

At the first level, which is called visualization, students simply recognize geometric shapes by their appearance. For example, a child sees a square and says that it is a square, but still cannot explain why it is called that and what properties it has.

At the second level, the level of analysis, students are already beginning to notice and describe the properties of shapes. For example, they understand that all corners of a rectangle are straight. However, they are not yet able to explain the logical connection between the various properties.

At the third level, the level of informal inference, students can build simple logical reasoning about geometric shapes and their properties, and also understand how some shapes relate to others.

At the fourth and fifth levels, which are called formal reasoning and rigor, students work with full-fledged mathematical proofs and various axiomatic systems. This is already a fairly high level of abstract thinking.

The key idea of the theory is that learning should correspond to the student's current level. If the teacher tries to explain the material designed for a higher level, the student simply learns the information by heart without understanding its meaning. Studies conducted in different countries show that many students, even in high school,

remain at the first or second level of geometric thinking (Naufal et al., 2021). This creates a serious problem when studying stereometry, since at least the third level is needed to understand it.

Van Hiele's theory also identifies five learning phases that promote progress at different levels: information, directed orientation, explanation, free orientation, and integration. Research has shown that instruction organized according to these phases leads to significantly greater gains in geometric reasoning than traditional deductive instruction (Gül & Sert-Çelik, 2022). The use of technology in the Van Hiele method is particularly effective: Abu et al. (2012b) demonstrated that a Google SketchUp-based module structured according to Van Hiele's phases enabled all students in the experimental group to improve their geometric reasoning by at least one level, while students in the control group showed no such improvement. Ndungo (2025) confirms that the technology-enhanced Van Hiele method consistently leads to improvements in secondary school students' geometric reasoning. The direct connection between this model and the present study is clear: Tinkercad and SketchUp Free activities in the eight-week intervention are designed to promote students to Level 2 (analysis) and support progress to Level 3 (informal reasoning) through direct interactive engagement with the properties of geometric solids.

2.1.3 Cognitive Load Theory (Sweller)

The theory of cognitive load was developed by John Sweller and his colleagues in the 1980s. It helps to understand why some learning formats work better than others, and explains how various technological tools affect the assimilation of complex material. The theory is based on the idea that human memory is divided into two types. Working memory is limited and can process only a small amount of information at a time, whereas long-term memory is practically limitless and stores knowledge in the form of organized structures called schemas (Sweller, 1988; Sweller et al., 1998).

The theory identifies three types of cognitive load, which together determine the overall load on working memory in the learning process. The internal cognitive load depends on the complexity of the learning material itself, that is, on how many elements

need to be processed simultaneously and how interconnected they are. The external cognitive load arises from the way information is presented, not from its content. If the learning materials are poorly designed and the student has to mentally put the disparate parts together on their own, the load on memory increases and learning outcomes worsen. The relevant cognitive load is a useful part of the load that is spent directly on the formation of new knowledge schemas. Optimal learning tends to maximize this type of workload, while reducing the external workload to a minimum (Ouwehand et al., 2025).

When applied to geometry teaching, this theory provides important practical insights. When three-dimensional figures are shown only in the form of static flat drawings, this creates an unnecessary load on memory, because the student has to mentally complete the three-dimensional form from a two-dimensional image. At the same time, you need to look at the figure from one fixed point of view, apply formulas and at the same time keep its three-dimensional structure in mind. All this creates a very high load on working memory, especially for those students whose spatial thinking is not yet sufficiently developed.

3D modeling programs solve this problem. They allow students to create three-dimensional models themselves, rotate them, view them from different angles, and zoom in on them in real time. All this significantly reduces unnecessary memory load and frees up working memory resources for deeper understanding of geometric content. Ouwehand et al. (2025) investigated the modern applications of this theory in geometry teaching using technology and came to the conclusion that three-dimensional computer environments consistently reduce the cognitive load when studying spatial geometry compared to traditional two-dimensional learning.

2.1.4 Activity-Based Learning Theory

The theory of learning through activity states that learning is most effective when the student does not just listen and memorize, but actively participates in meaningful practical activities, solves real problems and creates something himself. In

mathematics education, this idea has formed the basis of various approaches such as research, practical and project-based learning (Koskinen & Pitkaniemi, 2022).

In the framework of geometry lessons, the activity-based approach is especially important, since students need not only to memorize definitions and formulas, but also to interact directly with spatial objects. The study of three-dimensional geometry can be carried out by various forms of representation. A common method is to use physical models that allow students to see and touch geometric bodies. This approach can be useful at the initial stage of understanding, but physical models are often limited: they are difficult to quickly change, scale, rotate, cut or combine with other objects.

Another traditional method is to use flat drawings in a textbook or on a blackboard. This approach is widely used in geometry lessons, but it requires students to mentally reconstruct a three-dimensional object from a two-dimensional image. For students with poorly developed spatial thinking, this can be a rather difficult task, because at the same time they have to imagine the shape of the body, its hidden parts, dimensions, the location of the faces and possible transformations.

Digital 3D modeling provides a more flexible way to work with geometric objects. Unlike static drawings, 3D modeling tools allow students to independently create, rotate, resize, combine and analyze objects in a dynamic environment. Therefore, students do not only observe the finished geometric shape, but also actively work with it. This corresponds to the basic idea of activity-based learning, according to which understanding is deepened when the student participates in practical and meaningful actions, and not just receives a ready-made explanation from the teacher.

This idea is directly related to the use of Tinkercad and SketchUp Free in this study. When students create three-dimensional models of cubes, prisms, pyramids, cylinders and other geometric bodies, they carry out productive mathematical activities. They select the necessary shapes, set the dimensions, connect the individual parts of the model, rotate the object to check its structure, and relate the created model to formal geometric concepts such as faces, edges, vertices, surface and volume. 3D modeling is considered in this study not only as a visualization tool, but also as an

active learning environment. It allows students to explore geometry through the construction, transformation and analysis of objects. This is especially important when studying stereometry, because many students have difficulty imagining three-dimensional bodies based solely on flat images. Interactive digital models help to look at the same object from different angles and gradually combine visual experience with mathematical thinking.

Therefore, the integration of Tinkercad and SketchUp Free in geometry lessons complies with the principles of activity-based learning and supports the development of spatial thinking through the direct interaction of students with three-dimensional objects. Tessema et al. (2024) confirm that practical training with three-dimensional models significantly improves the conceptual understanding of spatial geometry compared to traditional training. The NCTM (2000) standards explicitly recommend the use of practical tools and technologies to develop spatial thinking from the very beginning of learning. This reflects the general view in the mathematical education community that active interaction with geometric objects creates a solid foundation for understanding geometry.

2.1.5 Constructivist Approach in Mathematics Education

Constructivism is a theoretical approach according to which knowledge is not transmitted from teacher to student in a ready-made form, but is actively built by the student himself through interaction with the environment. This approach has been one of the leading approaches in mathematics education for more than three decades. In the context of geometry teaching, constructivism manifests itself in methods that encourage students to independently explore geometric objects and the relationships between them, to put forward and test hypotheses, and to gradually form formal geometric concepts through their own experience.

The rationale for using 3D modeling tools from a constructivist perspective looks convincing. When a student creates a three-dimensional model of a prism in Tinkercad, counts its faces, edges, and vertices, resizes it, and applies the volume formula to the resulting body, they are actively constructing geometric knowledge

through purposeful interaction with an object, rather than passively memorizing definitions.

Empirical studies convincingly confirm the effectiveness of the constructivist approach in teaching geometry. Baki et al. (2009b) showed that both dynamic geometric software and physical learning materials based on constructivist principles of active student participation lead to significantly greater improvements in spatial skills compared to traditional learning. Xie et al. (2018) in a large-scale meta-analysis, they found that constructivist learning models provide significantly higher math results in a wide variety of classes and on different topics compared to approaches based on the transfer of ready-made knowledge. The effect sizes ranged from $d = 0.43$ for independent learning to $d = 0.67$ for cooperative learning. Bermejo et al. (2021) demonstrated that the constructivist mathematics intervention program led to significant and sustained improvements in mathematical results, with the greatest effect observed in students with the greatest initial difficulties. All these results confirm the constructivist basis of the present study.

2.2 Spatial Thinking and Geometry Learning

The relationship between spatial thinking and geometry learning outcomes is one of the most frequently and consistently confirmed patterns in mathematical education research. Simply put, scientists have proven over and over again that those students who are good at mentally imagining and manipulating spatial objects tend to do much better with geometry. This pattern has been confirmed in numerous studies among students of different ages and in different countries of the world, which makes it especially important for the practice of teaching geometry.

This section provides an overview of the evidence supporting this connection. In addition, it examines the specific role of spatial thinking in the learning process of geometry, analyzes the differences in the level of spatial thinking between boys and girls, and discusses methods for developing spatial thinking in school settings.

2.2.1 The Role of Spatial Abilities in Geometry Achievement

Spatial abilities play an important role in geometry learning because geometry requires students to work not only with formulas and definitions, but also with visual and spatial representations. In particular, students need to imagine geometric objects, recognize their properties, mentally rotate figures, interpret different views, and understand the relationship between two-dimensional drawings and three-dimensional solids. These skills are especially important in stereometry, where many tasks cannot be solved effectively through procedural knowledge alone. For example, when students work with prisms, pyramids, cylinders, nets, cross-sections, and rotations, they need to form accurate mental images of objects and transform these images in their minds.

According to Linn and Petersen (1985), spatial ability includes several related components, such as spatial visualization, mental rotation, and spatial perception. Spatial visualization refers to the ability to mentally transform complex figures, while mental rotation involves imagining how an object would look after being rotated in space. Spatial perception is connected with understanding spatial relations between objects or parts of an object. These components are directly related to geometry achievement because many geometry problems require students to move between visual representation and mathematical reasoning. In school geometry, weak spatial ability may become one of the reasons why students experience difficulties in understanding three-dimensional objects. When a solid figure is presented only as a flat image in a textbook or on the board, students must mentally reconstruct its actual shape, hidden parts, dimensions, and possible transformations. This process can be cognitively demanding, especially for students who have not yet developed strong spatial visualization skills. Therefore, spatial ability is not only an additional cognitive skill, but also an important condition for understanding geometric concepts more deeply. Previous studies have shown that spatial reasoning is positively related to students' achievement in geometry and their ability to solve problems involving three-dimensional figures (Pittalis & Christou, 2010; Sorby, 2009). Moreover, spatial skills are considered trainable rather than fixed. Uttal et al. (2013) showed that spatial

abilities can be improved through targeted training and practice. This is particularly relevant for the present study because 3D modeling tools create opportunities for students to actively manipulate geometric objects, observe them from different perspectives, and connect visual experience with formal mathematical concepts. For this reason, spatial ability was selected as one of the main measurable outcomes in the present study. Since the intervention focused on the use of Tinkercad and SketchUp Free in learning solid geometry, it was important to examine whether students' spatial visualization, mental rotation, and spatial orientation improved after working with interactive three-dimensional models. Thus, the focus on spatial ability is directly connected with both the theoretical foundation of the study and the practical nature of the 3D modeling intervention.

Quantitative research confirms a stable and consistent relationship between spatial thinking and geometric abilities at all levels of education. In simple terms, the better a student is able to mentally work with spatial objects, the better he copes with geometry. This pattern has been confirmed in numerous studies. Guay and McDaniel (1977) found significant correlations from .47 to .86 in students in grades 2-7. Fennema and Sherman (1977) reported strong correlations in grades 9 and 12. Pittalis and Christou (2010), in a study of 246 fifth and sixth grade students, showed that spatial reasoning is a strong predictor of three-dimensional geometric thinking, meaning the better spatial reasoning is developed, the better the student understands three-dimensional geometry. Kayhan (2005) found a strong positive correlation between spatial thinking and math performance in ninth grade students. Turgut (2010) found moderate but statistically significant positive correlations between spatial thinking and academic performance in future math teachers.

The relationship between spatial thinking and geometric abilities has been further explored in more recent research on knowledge transfer mechanisms. Lowrie and Logan (2023) identified two separate mechanisms.

- The first is the representational mechanism, in which spatial thinking training improves the ability to create and apply spatial representations of mathematical problems.
- The second is a procedural mechanism, in which such training improves the ability to perform decision procedures that require spatial manipulation.

Both mechanisms are directly related to spatial geometry, as students must simultaneously visualize three-dimensional objects and apply procedures, such as volume formulas, that require spatial understanding. Adams et al. (2022) confirmed that the effects of spatial thinking training extend directly to geometry and measurement in high school students. Wai et al. (2009), in a large-scale review of 50 years of data, demonstrated that spatial reasoning predicts career success in the fields of natural sciences, technology, engineering, and mathematics, and is also associated with verbal and mathematical abilities. Gilligan-Lee et al. (2022) argue that spatial reasoning is a missing element in mathematics curricula worldwide.

2.2.2 Gender Differences in Spatial Thinking

Gender differences in spatial abilities are one of the most studied topics in cognitive psychology and educational research. A meta-analysis by Linn and Petersen (1985), which summarized the results of dozens of studies, showed that men, on average, cope with tasks of spatial visualization and mental rotation of figures better than women. The most noticeable differences between men and women are observed precisely in tasks where you need to mentally rotate three-dimensional objects. Bar-Hen-Schweiger and Henik (2024) confirmed this pattern in later studies and noted that the differences are not uniform across all spatial components: they are most pronounced in mental rotation, moderate in spatial visualization, and minimal or absent in tasks involving spatial orientation and perspective taking. The differences are relatively small, typically d = 0.4-0.6 for mental rotation, and mask significant overlap in the distribution of spatial performance between men and women.

Of particular importance for the educational context of the present study are the results regarding the feasibility of training gender differences in spatial skills. In a

longitudinal study with engineering students, Sorby (2009) demonstrated that targeted spatial training using 3D modeling software led to significant improvements in both male and female students, with a significant reduction in gender differences in spatial performance after training. Uttal et al. (2013), in their meta-analysis of 206 spatial training studies, found that training produced similar results for men and women, with gender having no significant effect on the training effects. Mix et al. (2018) confirmed that the structure of spatial skills does not differ significantly between men and women, suggesting that mathematical abilities in both genders are based on the same spatial components. These results indicate that training using 3D modeling is at least as effective for female students as for male students and does not result in any systematic disadvantages for either group. In the present study, gender is considered as a participant characteristic and is described descriptively; gender differences are not considered a primary study variable.

2.2.3 Methods for Developing Spatial Thinking in School

Current research has provided a definitive answer to the question of whether spatial abilities can be systematically improved through learning: spatial abilities are among the most trainable cognitive abilities. The meta-analysis by Uttal et al. (2013), combining 206 spatial learning studies, showed that the average improvement was approximately 0.47 standard deviations (Hedges g), while the effects persisted over time and were transferred to tasks not included in the training. A more recent meta-analysis (Yang et al., 2020) showed an even larger average effect of 0.96 for spatial skill learning, strongly demonstrating that early spatial learning brings significant and long-term benefits. It is important to note that these effects extend not only to other spatial tasks, but also to mathematical performance in a broader sense, making spatial learning a potentially lucrative investment in mathematical education.

Several specific approaches to learning have received consistent empirical support. Gilligan et al. (2019) showed that the inclusion of spatial reasoning exercises in math classes had a significant, statistically significant impact on both spatial abilities and mathematical achievements of 8-year-olds. Hawes et al. (2017) showed that the

32-week spatial geometry program implemented by teachers led to significant improvements in many spatial and mathematical areas. Lowrie et al. (2017) during a randomized controlled trial of 6th grade students, it was found that structured visual-spatial exercises significantly improved mathematical performance by transferring them from one area to another. Moss et al. (2015) documented that the incorporation of spatial reasoning into regular math lessons through adapted Japanese language learning led to significant improvements in spatial and mathematical thinking in elementary grades.

In the context of digital tools, research has consistently confirmed the positive impact of 3D modeling and dynamic geometry software on the development of spatial abilities. Baki et al. (2009b) showed that dynamic geometry software (Cabri 3D) provides a significant increase in spatial capabilities comparable to physical manipulators, with particular advantages for spatial capabilities related to viewing. Yildiz (2009) found that a 3D virtual environment is much more effective than physical cubes for developing spatial visualization in fifth grade students. Guven and Kosa, (2008) made significant progress in all sections of the PSVT after eight weeks of 3D geometric body training. Newcombe (2017) aims to complement the curriculum with tools that promote the development of spatial thinking. Cheng and Mix (2014) demonstrated in an experimental study that spatial learning improves children's mathematical abilities, and Mix and Cheng (2012) presented a theoretical report on the impact of spatial-mathematical relationships on development and education.

2.3 Technology in Mathematics and Geometry Education

There have been significant changes in the use of technology in mathematics education over the past three decades, from early computers as computational tools to sophisticated dynamic geometry software and 3D modeling platforms. This development has been accompanied by a rapidly growing number of studies documenting the impact of various technological tools on mathematical learning and providing an increasingly rich evidence base for making decisions about integrating technology into schools and education policies.

2.3.1 Digital Tools in Mathematics Education

The National Council of Teachers of Mathematics has formulated a basic policy position on technology in mathematics education in its policy document “Principles and Standards” (NCTM, 2000), which states that technology plays an important role in teaching mathematics and that it affects both what students learn and how they They're learning. NCTM recommended that students use technology to explore mathematical concepts, visualize abstract ideas, and develop spatial thinking skills by explicitly including three-dimensional visualization among the spatial skills that technology should support. This policy position is widely used in national mathematics curricula at the international level and provides a solid institutional framework for integrating technology into geometry teaching. Previous NCTM standards (1989) also emphasized the importance of using technology for spatial development, reflecting a long-standing recognition of this need in mathematics education.

Gagnon et al. (2023) presented particularly convincing large-scale evidence of the effectiveness of integrating 3D modeling technologies into mathematical education. A multi-year study of thousands of students from different schools using a rigorous quasi-experimental approach revealed a significant positive impact of the 3D modeling and computational thinking integration program on academic performance and spatial thinking of students, and the results were comparable for different groups of students. Sebsibe and Abdella (2025) similarly describe the positive impact of dynamic mathematical software on students' understanding of mathematical concepts. Ziatdinov and Valles (2022) offer a detailed overview of how dynamic mathematical tools contribute to mathematics learning in various fields, including three-dimensional geometry. Herrera et al. (2024) came to similar conclusions based on a review of research on 3D tools, augmented and virtual reality in mathematics education, and found a sustained positive impact on spatial reasoning, mathematical understanding and student engagement with appropriate educational support.

Olkun and Altun (2003) studied the relationship between computer experience, spatial reasoning, and geometric success in elementary school students and found a

significant positive association between computer use and spatial abilities. Olkun (2003) showed that working with drawings, both on a computer and on paper, can significantly improve spatial skills, while computer training has special advantages for the development of individual components of spatial thinking. These early results helped lay the empirical foundation for subsequent research on more sophisticated digital tools in geometry teaching.

2.3.2 Dynamic Geometry Software (GeoGebra, Cabri 3D)

Dynamic geometry software is the most widely used category of educational technology in the field of geometry teaching. These programs allow users to create geometric shapes whose properties are dynamically preserved when dragging or changing elements, allowing students to study geometric patterns through direct experimentation rather than static observation. The most frequently studied systems are Cabri Geometry, Geometers Sketchpad and GeoGebra for 2D geometry and Cabri 3D for 3D geometry. Research consistently confirms the positive impact of Dynamic geometry software on students' geometric thinking, spatial abilities, and success in geometry (Baltacı et al., 2015; Guven & Kosa, 2008; Baki et al., 2009b).

Guven and Kosa (2008) investigated the impact of 3D-based learning on math teachers' spatial visualization abilities before starting pre- and post-testing as part of the study. Forty teachers trained at Cabri participated in an eight-week 3D class on prisms, cylinders, spheres and cones. The results of Purdue spatial visualization showed significant gains in all three sections: Developments (visualization of a spatial network), Rotation (three-dimensional mental rotation) and Views (viewing solids from different points of view), as well as the overall score. The authors concluded that Cabri 3D promotes the discovery of geometric relationships and the development of spatial abilities. Baki et al. (2009b) made a more accurate comparison by considering both the conditions of the dynamic geometry program and the conditions of physical manipulation, as well as the traditional training control group. Both active modes led to a significant increase in all levels of PVT, while there were no significant changes in the control group. The main difference was that the dynamic geometry software

group far surpassed the physical manipulation group in the representation section, suggesting that 3D computing environments have special advantages for developing spatial abilities related to perspective perception.

Baltacı et al. (2015) found that pre-service teachers perceived GeoGebra as highly effective for geometry instruction, particularly for tasks involving multiple representations and dynamic exploration of relationships. Sebsibe and Abdella (2025) documented significant improvements in students' understanding following GeoGebra-based instruction, with the technology enabling dynamic exploration not achievable with traditional tools. Cohen and Hegarty (2014b) investigated the effects of interactive computer animations on spatial visualization of cross-sections of three-dimensional solids, finding significant improvements in students' ability to identify and draw cross-sections following computer-based spatial training. Martín-Dorta et al. (2008) evaluated accelerated spatial ability courses including 3D modeling and found significant post-test gains on both mental rotation and spatial visualization tests, confirming that targeted digital tool use can produce substantial spatial improvements even over short intervention periods.

2.3.3 Comparison of Traditional and Technology-Enhanced Geometry Instruction

A substantial body of quasi-experimental and experimental research has compared the effects of technology-enhanced geometry instruction with traditional approaches, consistently finding advantages for the technology-enhanced conditions. Turgut (2010) found that pre-service mathematics teachers in a technology-enhanced experimental condition significantly outperformed control group peers on spatial ability tests. Yildiz (2009) found across two schools that students working with a three-dimensional virtual environment produced significantly greater spatial visualization gains than peers working with physical unit cubes. Kosa and Karakus (2010) found that Cabri 3D significantly facilitated the visualization of three-dimensional figures among prospective high school teachers, a finding consistent with the predicted

benefits of interactive 3D environments in the solid geometry content of the present study.

Koskinen and Pitkäniemi (2022), in a comprehensive research synthesis, found that active and technology-enhanced instructional approaches consistently outperform traditional transmission-based instruction on measures of mathematical understanding, particularly for content domains such as geometry requiring strong conceptual grasp of visual and spatial relationships. Boonstra et al. (2023) provides theoretical grounding for technology-enhanced geometry instruction, examining embodied abstraction in primary school geometry and arguing that hands-on and technology-based approaches are essential for developing the embodied geometric understanding that underlies formal geometric reasoning. Tessema et al. (2024) confirm that teaching solid geometry with three-dimensional models, whether physical or digital, enhances students' conceptual understanding and problem-solving skills compared to traditional approaches relying solely on textbook illustrations.

It is important to note that the comparative advantages of technology-based learning largely depend on the quality of implementation. Herrera et al. (2024) concluded that the most effective technological implementations of geometry are those that explicitly link digital actions with formal mathematical content, provide opportunities for structured thinking, and help teachers learn correctly. In this study, this problem is solved by developing 3D modeling tasks specifically designed as part of the 11th grade rigid body geometry curriculum, which complement each modeling lesson with relevant textbook tasks and ensure that students receive clear instructions on how to use their observations in modeling using formal geometric vocabulary. and formulas.

2.4 3D Modeling in Education

Three-dimensional modeling in education refers to the use of specialized computer-aided design (CAD) software to create, edit, and analyze three-dimensional digital objects in the learning process. 3D modeling, which differs from passive 3D image viewing by focusing on student independence, has rapidly moved from K-12

technical and architectural education to several disciplines. The introduction of free, browser-based, and student-friendly platforms such as Tinkercad and SketchUp Free has significantly lowered barriers to admission to educational institutions and ensured widespread adoption even with limited educational resources.

2.4.1 Definition and Types of 3D Modeling in Educational Practice

3D modeling software differs significantly in its design principles, the complexity of the user interface, and its suitability for different age groups and educational environments. The main approaches include constructive solid geometry (CSG), in which three-dimensional objects are created by combining and subtracting geometric primitives, polygonal mesh modeling, in which objects are defined by grids of vertices, edges and faces, and parametric modeling, in which three-dimensional shapes are created based on numerical parameters and constraints. CSG modeling is the most suitable approach for teaching geometry in high school: its conceptual basis is related to the content of the geometric solid course, its visual interface allows you to instantly see the relationships between operations and shapes, and it is implemented in both Tinkercad and SketchUp Free.

Hebebci and Hebebci (2021) conducted an extensive review of platforms used for educational purposes and identified Tinkercad and SketchUp as the most widely used in the K-12 education system. By the end of 2021, more than 400 million user projects have been collected in Tinkercad, which demonstrates the extraordinary reach of students and teachers around the world. Bhaduri et al. (2021) The 3DnST (3D and Spatial Thinking) framework is specifically designed to describe how Tinkercad supports the development of spatial thinking, highlighting four categories: transformation from orthogonal to isometric (transformation between 2D design specifications and 3D results), perspective change (transformation between 2D design specifications and 3D results), mental rotation (predicting and verifying the results of rotating models), as well as spatial assembly (designing and executing a combination of several primitive shapes into a single geometric unit). All four categories are directly related to the spatial capability components most relevant to Solid geometry, and offer

a theoretically sound description of the mechanisms that Tinkercad will use to improve spatial capabilities.

Ng et al. (2022) found in a 3D printing review study on mathematical education that most studies used 3D modeling or printing as tools for developing geometric concepts, with solid geometry, including volume, surface area, and properties of geometric bodies, being the most commonly used target material. The review showed that the most effective implementations combine 3D modeling with structured research, clear links with formal mathematical content, and systematic evaluation of learning outcomes. Surynková (2023) reports that secondary school students can successfully create complex geometric solid-state models in Tinkercad after a brief introductory course in a 90-minute class, and that the modeling process promotes geometrically accurate and conceptually informed thinking about solid geometry. Dilling and Witzke (2020) found that using Tinkercad in math lessons encourages students to study geometric properties and relationships more closely and generate better mathematical thinking than when doing regular textbook assignments.

2.4.2 Effects of 3D Modeling on Students' Spatial Abilities

Currently, there is an extensive research literature documenting the positive effects of 3D modeling on spatial abilities, covering various age groups, national conditions, and programming environments. Herrera et al. (2024) in the most comprehensive recent review, dozens of studies on 3D tools, augmented reality, and virtual reality in mathematics education have been analyzed and found that the integration of interactive 3D environments is always associated with improved spatial visualization, problem solving, and mathematical skills. The magnitude of the effect reported in the studies ranged from small to large, depending on the duration and quality of the intervention, while longer and more structured interventions tended to lead to larger and longer-lasting effects.

Controlled quasi-experimental studies provide compelling causal evidence for improvements in spatial abilities. Guven and Kosa (2008) noted significant progress in testing across all sections of the SVT after eight weeks of 3D training, with the largest

increases in Cabri solid geometry being achieved in the rotation and view sections. Kurtulus and Uygan documented significant progress in spatial visualization among teachers using Google SketchUp to teach polyhedra, while students developed new problem-solving strategies during the experiment. La Ferla et al. (2009) noted the significant advantages of the post-test experimental SketchUp groups compared to control groups in Turkey and the United States in the field of spatial visualization and mental rotation, providing national evidence of the advantages of SketchUp in the field of spatial visualization. Toptas and co-authors (2012) documented significant advantages in the development of spatial skills in an experimental group of eighth graders using Google SketchUp, compared with a control group teaching traditional teaching methods. Sorby and Baartmans (2000) showed that intensive 3D modeling training produced significant results after testing four separate spatial ability indicators, proving that targeted 3D modeling training can lead to significant and measurable spatial growth in several components.

Yuliardi et al. (2021) demonstrated that high school students trained in 3D geometry using Cabri 3D demonstrated significant improvements in spatial skills, reflecting progress in both visualization and spatial orientation. Passig and Eden (2001) showed that 3D tetris in virtual reality provides a significantly higher speed of rotation in space than 2D tetris, and the effect is especially noticeable for students with initially weaker spatial abilities, indicating a compensation model typical for different groups of students. De Lisi and Wolford (2002) confirmed that computer-based spatial activities produce significant and transferable spatial gains in younger students, establishing the generality of computer-based spatial training effects. Hawes et al. (2015) demonstrated through a randomized controlled study that mental rotation training produces significant improvements in children's spatial and mathematics performance.

2.4.3 Effects of 3D Modeling on Academic Achievement in Geometry

While the evidence for positive effects of 3D modeling on spatial abilities is robust and consistent, the relationship between 3D modeling instruction and academic

achievement in geometry is somewhat more variable across studies. This variability likely reflects differences in how achievement is operationalized, the duration and intensity of interventions, the alignment between modeling activities and tested content, and the quality of pedagogical design. Nevertheless, the overall pattern of evidence supports the expectation of positive achievement effects when the intervention is well-designed and sufficiently sustained.

Toptas et al. (2012) found not only significant spatial ability gains but also substantially greater improvement in geometry content knowledge in the SketchUp experimental group compared to the control group, as measured by a curriculum-referenced achievement test. The authors explain these achievements with the possibilities of dynamic SketchUp research, which allows students to study geometric relationships through direct manipulation rather than static reading. Abu et al. (2012b) found that Google SketchUp training, organized according to Van Heele stages, produced significant results both in terms of van Hiele's geometric thinking and in terms of geometry test scores based on the curriculum, with the experimental group significantly outperforming both groups on both counts. Surynková (2023) reports that working with Tinkercad to create geometric models stimulates geometrically precise and conceptually grounded use of terminology likely to translate into improved performance on curriculum-referenced geometry assessments. Gagnon et al. (2023) provide compelling and extensive evidence of the impact on school performance and document the significant positive impact of integrating 3D modeling programs on math performance of thousands of students in several schools. Herrera et al. (2024) concluded based on a broad summary that, in general, the data confirm the positive impact of 3D tools on mathematical achievements in a broad sense, while geometric content demonstrates a particular vulnerability to the integration of 3D technologies. Khurma et al. (2023) Please note that practical 3D modeling and printing classes help students better understand abstract geometric concepts, which has a positive effect on their knowledge of the content. Moru et al. (2020) clearly demonstrate that the technologically improved study of volumes of three-dimensional solids, closely related

to the content of this study, leads to significant improvements in both geometric reasoning and procedural knowledge of volume formulas.

2.4.4 Tinkercad as an Educational Tool

Tinkercad is a free browser-based 3D modeling and design platform developed by Autodesk, Inc. and launched in 2011 with the clear goal of making 3D design accessible to everyone. It works completely in a web browser without the need to install software, is fully functional on mobile phones and tablets, and is available for free to both students and teachers (Autodesk, n.d.). Architecturally, Tinkercad implements the constructive geometry of a solid body: users create 3D models by placing basic geometric shapes on a virtual working plane and combining them with logical addition and subtraction operations. Available primitives include rectangles, cylinders, spheres, cones, wedges, prisms, and other shapes. The key feature of the program is the division of shapes into two types: regular shapes that add material to the model, and shapes with holes that, when grouped with other elements, remove material from them. This allows you to create complex shapes from simple components. All shapes can be resized to within a millimeter using a measuring ruler that displays accurate numerical values in real time, which directly links the modeling process to formal geometric dimensions.

Especially for learning geometry, Tinkercad has several educationally useful features. Its three-dimensional working plane allows students to move freely through the constructed models, which contributes to the development of spatial orientation and perspective skills. His precision measurement system requires students to specify numerical dimensions for all shapes, creating an explicit link between modeling and formal geometric measurements. Real-time visual feedback clear cycle of action-response-research support Constructive learning. Its browser architecture and compatibility with mobile devices make it accessible without the need for computer lab work, allowing any student to use it on their personal devices. Bhaduri et al. (2021) conducted the most systematic empirical analysis of how Tinkercad specifically supports spatial reasoning, using the 3dnst platform to identify the four categories of spatial reasoning that the platform provides. Thom et al. (2024) identified Tinkercad

as a platform that facilitates productive dimensional switching between two-dimensional representations and three-dimensional objects, which is precisely the cognitive skill most in demand in the traditional study of solid-state geometry.

2.4.5 SketchUp Free as an Educational Tool

SketchUp is a dynamic 3D modeling software platform with an outstanding history in both the professional and educational sectors. Originally developed by Last Software and published in 2000, SketchUp was acquired by Google in 2006, which greatly expanded its user base through free distribution, and then sold to Trimble, Inc. in 2012. The browser version of SketchUp Free, available without purchase or subscription, retains the basic modeling features of the full platform and is often used for educational purposes (Hebebci & Hebebci, 2021).

The SketchUp toolkit is significantly broader than that of Tinkercad and offers a broader set of design and analysis functions required for initial geometry training. The push-pull tool allows you to vertically extrude any flat surface into a three-dimensional solid, providing a natural process of constructing geometric bodies along their surfaces and allowing you to directly explore the relationship between two-dimensional shapes and three-dimensional bodies. The Rotate tool allows you to accurately rotate objects around any given axis, which directly contributes to the development of mental abilities to rotate. The cutting plane allows users to create interactive cross-sections of three-dimensional bodies in any position and orientation, making the internal structure of geometric shapes directly visible and manageable. Cameras with the “Orbit”, “Walking” and “Inspection” functions allow you to place a virtual viewpoint anywhere in three-dimensional space, including in enclosed bodies, providing a perspective incomparable with textbooks (La Ferla et al., 2009; Kurtulus & Uygan, 2010). Fleron (2009) conducted a basic methodological analysis of the educational potential of SketchUp for teaching geometry and demonstrated five specific educational advantages: a detailed study of the geometric properties of solids in a three-dimensional environment; a comprehensive toolkit that supports geometric structures of any type in the curriculum; a camera view that displays several orthogonal

and perspective images that can be implemented in a three-dimensional environment; and a set of tools that supports geometric constructions of any type in the curriculum.; a camera view that displays multiple orthogonal and perspective images; a rotation function that facilitates geometry transformation and symmetry learning. La Ferla et al. (2009) found significant post-test advantages for SketchUp experimental groups on spatial visualization and mental rotation in Turkey and on mental rotation in the United States. Toptas et al. (2012) found significant spatial ability and geometry content knowledge gains for an eighth-grade SketchUp group. Abu et al. (2012b) demonstrated that SketchUp organized according to van Hiele phases enabled all experimental group students to progress at least one level in geometric thinking. Kurtulus and Uygan (2010) studied SketchUp for polyhedra instruction with pre-service teachers and documented both advantages and a specific limitation: the difficulty of constructing surface unfoldings (nets) in SketchUp, which informed the supplementary textbook activities in the present investigation. Carmona-Medeiro et al. (2021) confirmed that pre-service teachers perceive SketchUp as a useful and practically applicable tool for developing spatial-geometric competencies across educational levels.

2.5 Quasi-Experimental Research in Geometry Education

The overwhelming majority of empirical research on the effects of 3D modeling and dynamic geometry software on geometry learning outcomes employs quasi-experimental designs. This methodological predominance reflects the constraints of real educational contexts: random assignment of students to instructional conditions is almost never feasible in school settings, since students are taught in pre-existing class groups by their regular teachers. In order to correctly interpret the results of this study and competently compare them with existing scientific papers, it is necessary to clearly understand the methodological features and limitations of quasi-experimental research in this field.

2.5.1 Design of Quasi-Experimental Research in Education

The quasi-experimental project described in the work of Gagnon et al. (2023) includes experimental and control groups formed on the basis of pre-existing group membership rather than random distribution. In educational research, this usually means that existing school classrooms serve as experimental or control environments, the distribution of which is determined by practical considerations. The main methodological problem associated with this project is the possibility of pre-existing differences between groups, which may make it difficult to interpret the differences after examining the effects of the intervention. This problem is solved by conducting a preliminary exam, which allows you to check whether the groups are initially comparable in terms of the dependent variables of interest, as well as using statistical methods such as covariance analysis (ANCOVA) to correct for existing differences in the analysis of results after the exam.

This study uses a quasi-experimental pre- and post-test plan with two pre-existing 11th grade classes that serve as experimental and control conditions in accordance with methodological conventions adopted in the field. Armah et al. (2018b) illustrate how such systems can provide compelling evidence even without randomization: in their Van Hiele-based learning phase study with intact school classrooms, a quasi-expert diagram before and after the test provided clear evidence of the superiority of experimental conditions. Hannafin et al. (2008) similarly used a quasi-experimental approach to document the significant interaction between curriculum and spatial skills in achieving geometry results. The main methodological advantages of this project are the inclusion of pre-exams to check the basic level, the suitability of the curriculum for various conditions, as well as the general teacher, school, and period, in order to avoid many potential problems.

2.5.2 Pre-test and Post-test as Measurement Instruments

The measurement scheme before and after the test is common in research on the development of spatial abilities and geometry related to educational activities. The preliminary test performs many functions: it verifies the comparability of the initial

data of the experimental and control groups, provides baseline data for measuring changes, and allows statistical control of previous differences in the post-test analysis. The most commonly used standardized spatial ability tests in this literature include the Purdue Spatial Visualization Test (PSVT), which evaluates network visualization, three-dimensional mental rotation, and multi-angle viewing; the Santa Barbara solid-state test, which evaluates the identification of cross-sections of geometric bodies.; The Vandenberg and Coase Mental Rotation test and the DAT:SR test (Differential Ability Test: Spatial Relationships), which evaluates the ability to mentally fold two-dimensional models into three-dimensional bodies, which evaluates the ability to insert two-dimensional models into three-dimensional bodies (Sorby, 2009; Guven & Kosa, 2008; Toptas et al., 2012; Sorby & Baartmans, 2000). In this study, the measuring instruments were specifically designed with content and age group in mind, including spatial visualization tasks directly related to the geometric bodies being studied (cube, prism, pyramid, cylinder), as well as curriculum-related results that cover properties, volume formulas, and ways to apply them.

2.5.3 Overview of Related Studies

A systematic analysis of the most methodologically comparable studies shows both the empirical foundations of this study and the specific ways in which it expands and complements existing knowledge. The study by Toptas et al. (2012) is the next methodological precursor that uses quasi-experimental design before and after testing with saved grade 8 classes, uses Google SketchUp, and measures both spatial abilities and geometry skills. The present study extends this work to the 11th series. Classroom, adds Tinkercad as a second modeling tool and applies design in a Kazakh educational context. Abu et al. (2012b) provide the most direct evidence of the educational effectiveness of SketchUp, organized according to the principles of van Hiele, which is the theoretical basis of this study. Guven and Kosa (2008) and Baki et al. (2009b) offer methodological recommendations for the use of PSVT in quasi-experimental projects that consistently document a significant increase in spatial abilities using software for studying dynamic geometry. La Ferla et al. (2009) provide international

evidence for the spatial advantages of SketchUp related to the possibility of contextual generalization.

What is missing in the existing literature and what this study contributes to is a study combining two different additional 3D modeling tools (Tinkercad and SketchUp Free) that are used together as part of a coordinated intervention. A sample of 11th grade high school students for whom standard 3D modeling is not mandatory, and therefore the research that encourages it is a study combining two different complementary 3D modeling tools (Tinkercad and SketchUp Free) that are shared as part of a coordinated intervention; Solid geometry is central. a subject of the curriculum directly related to exams; simultaneous measurement of spatial abilities and academic performance as dependent variables; and its introduction into the specific educational, cultural and linguistic context of Kazakhstan. Each of these features represents a special original contribution to this field.

2.6 Research on Spatial Thinking and Geometry Education in Kazakhstan

In the Kazakhstani context, this issue is especially relevant because geometry instruction is increasingly expected to combine traditional mathematical content with digital learning tools. However, research specifically focused on the development of students' spatial abilities through 3D modeling in school geometry remains limited. Therefore, examining the use of Tinkercad and SketchUp Free in Kazakhstani high school geometry lessons can contribute to both local educational practice and the broader discussion on technology-enhanced geometry learning.

2.6.1 General overview of research in Kazakhstan

Research related to the development of spatial thinking and geometry teaching in Kazakhstan has historically been formed within the framework of engineering and teacher education, primarily through the disciplines of descriptive geometry and engineering graphics. Unlike the international scientific tradition, where spatial thinking is considered as an independent cognitive construct, in the Kazakh context it

has long been studied indirectly through the development of graphic literacy and geometric representations of students.

A systematic increase in interest in this field has been observed in recent years due to the digitalization of education and the introduction of STEM approaches. Research highlights that descriptive geometry plays a key role in shaping the ability to transition between 2D and 3D representations of objects, which is directly related to the development of spatial thinking (Zhanabayev et al., 2025). This indicates the gradual recognition of spatial thinking as an important component of engineering education.

Institutions such as L.N. Gumilyov Eurasian National University, Abai Kazakh National Pedagogical University, as well as technical universities in Kazakhstan play a leading role in the development of these studies. It is at these universities that the main research base for teaching geometry and engineering graphics is being formed.

However, despite the growing number of publications, research remains fragmented and is mainly focused on methodological recommendations rather than in-depth empirical studies of cognitive processes.

2.6.2 Research of Kazakhstani scholars

Kazakh research in the field of geometric education is mainly focused on the development of graphic and spatial skills of students of engineering and pedagogical specialties. In particular, it is emphasized that an insufficient level of spatial thinking is one of the reasons for difficulties in mastering engineering graphics and stereometry (Yessekeshova & Tuleuova, 2024). Zhanabayev et al. (2025) note that the study of descriptive geometry contributes to the development of students' ability to transform objects spatially and form visual-spatial thinking. This confirms the importance of this discipline as a basic tool for the formation of spatial perception. The works also emphasize the need to complicate geometry teaching methods, since simplified approaches reduce the level of spatial thinking development among students (Kussebayev & Tuleuova, 2023). Thus, there is a tendency to increase cognitive load through more active and visually oriented teaching methods.

2.6.3 Use of educational technologies in Kazakhstan

In recent years, Kazakhstan has been actively implementing digital technologies in geometry teaching, especially dynamic mathematical environments such as GeoGebra. Research shows that the use of GeoGebra contributes to an improved understanding of geometric concepts and the development of students' spatial thinking (Issayev & Usmanov, 2023). The effectiveness of digital tools in improving the visualization of geometric problems and developing students' analytical thinking is also noted. The use of interactive technologies makes it possible to move from static perception of geometric objects to dynamic modeling.

CAD systems (AutoCAD, Rhino, etc.) are used at universities in Kazakhstan, which make it possible to integrate theoretical knowledge with practical design of spatial objects (Tashimov, 2026). However, the level of implementation of these technologies remains uneven.

2.7 Challenges and Limitations of 3D Modeling in Geometry Education

A comprehensive review of the scientific literature should be devoted to the problems and limitations described in 3D modeling studies in geometry teaching. These problems are real and must be understood in order to develop effective measures and properly interpret research results. The problems described in the literature are divided into three main categories: technical and infrastructural barriers, methodological difficulties in pedagogical implementation, and problems arising from individual differences among students.

2.7.1 Technical and Infrastructural Barriers

The most urgent problem of introducing computer technology in schools is the accessibility and reliability of the necessary technical infrastructure. In particular, 3D modeling tools require access to devices with sufficient computing power to run CAD software, stable Internet connection for browser platforms, as well as a sufficient number of display and input devices for convenient management of 3D models. In

schools with limited resources, which include many schools in Kazakhstan and comparable learning conditions, these requirements are difficult to consistently meet (Khurma et al., 2023).

In this study, the availability of devices was solved by introducing Tinkercad and SketchUp Free on personal mobile phones of students, which are used by 11th grade students almost everywhere in Kazakhstan, and there was no need for special computer equipment. Both platforms work completely in a mobile browser without the need to install any applications and support touch controls. This approach significantly reduces the requirements for technical infrastructure compared to research, which requires access to a computer lab. However, the mobile implementation has technical limitations: the relatively small size of the smartphone screen makes precise geometric manipulation difficult compared to a desktop or laptop screen, and touch screen input differs from mouse and keyboard interaction, which can affect usability. Hebebci and Hebebci (2021), noting both the advantages of accessibility and the ergonomic limitations of mobile 3D modeling, concluded that the use of mobile devices is quite suitable for educational and research work. Internet connectivity is another infrastructure issue that is particularly relevant for browser platforms. In this study, this problem was solved by performing connection checks before the session and developing actions that can be partially performed offline, where possible.

2.7.2 Methodological Challenges of Implementation

The implementation of 3D modeling tools in geometry teaching in practice is proving to be a difficult task, and difficulties arise not only of a technical nature, but also in other aspects of the educational process. The most important of these is the requirement for teacher competence, not only in relation to the software itself, but also in the pedagogical design of activities that effectively use these tools for teaching mathematics. Baki et al. (2009b) note that the effectiveness of computer-based geometry teaching critically depends on the teacher's ability to design learning tasks that guide students' use of tools for mathematical, not just technical, research. It may

be difficult for a teacher who is not proficient in software to effectively support students in the event of technical difficulties.

Kurtulus and Uygan (2010) identified a specific and important educational limitation of SketchUp for studying the geometry of solids: drawing and processing surface sweeps (grids) of geometric bodies in SketchUp is much more difficult than direct modeling of solids. This discovery directly influenced the design of this study, in which 3D modeling courses are complemented with traditional textbook tasks after each modeling lesson to ensure that students develop skills in both the visual-spatial and formal procedural aspects of rigid body geometry. Cognitive Load Theory (Sweller et al., 1998) recommends that new tools be introduced into familiar content contexts before being applied to new content. In this study, this problem was solved by first-hand exposure to the tools in the first lesson and by conducting preliminary exams, which allowed students to develop their basic software skills before the elementary school materials were presented.

2.7.3 Individual Differences Among Students

Individual differences in initial spatial abilities, previous technology experience, and learning styles can reduce the effectiveness of 3D modeling training. De Lisi and Wolford (2002) found that students with initially higher spatial ability showed proportionally greater gains from computer-based spatial activities, suggesting a possible rich-get-richer dynamic. However, Passig and Eden (2001) found that three-dimensional Virtual Reality Tetris produced effects particularly pronounced for students with initially weaker spatial performance, suggesting a compensatory pattern. Hannafin et al. (2008) found significant interactions between students' spatial ability levels and the effectiveness of different instructional programs for geometry achievement, with lower-spatial students benefiting more from directed, structured instruction. These contrasting findings suggest that the moderating effect of initial spatial ability on 3D modeling instruction benefits is complex and context-dependent.

Prior experience with technology is a further individual difference variable. In the present study, this concern was addressed by providing all experimental group

students with a structured introductory lesson covering the basic interface and core functions of both Tinkercad and SketchUp Free before the main intervention began. Given that Grade 11 students in Kazakhstan have near-universal experience with smartphones and mobile applications, specific concerns about touch interface unfamiliarity are relatively minor. Hawes et al. (2015) demonstrated that the benefits of spatial learning are unevenly distributed and that attention to individual differences in initial spatial abilities is important to maximize the benefits of spatial learning programs. Moss et al. (2015) similarly found that the effectiveness of teaching geometry, including spatial thinking, depends on the quality of teacher assistance and the relevance of activities to a specific level of student development.

2.7.4 Level of development and limitations

An analysis of the literature shows that spatial thinking research in Kazakhstan is at a moderate stage of development. Despite the presence of works in the field of engineering graphics and methods of teaching mathematics, most of the research is descriptive in nature. One of the key limitations is the lack of quasi-experimental studies involving control and experimental groups. This limits the possibility of an objective assessment of the effectiveness of pedagogical interventions, including the use of 3D modeling. In addition, spatial reasoning is often not considered as a separate cognitive construct, but is included in broader categories such as mathematical or engineering competence. This reduces the theoretical accuracy of the research.

2.8 Summary and Gap in the Literature

The literature review presented in this chapter contains theoretical foundations, a database of empirical data and practical considerations on the use of 3D modeling tools in geometry lessons in secondary schools. This generalization allows us to draw some important conclusions and clearly identify the specific gaps that this study aims to address.

The theoretical foundations of this study are based on five complementary approaches. Van Hiele's theory establishes the levels of geometric thinking that 3D

modeling courses are aimed at, and the learning conditions that facilitate the transition between them, with direct empirical research support using SketchUp (Abu et al., 2012b) and other dynamic geometry tools. Cognitive load theory explains the scientific rationale behind 3D modeling tools: by reducing the cognitive load associated with mentally recreating three-dimensional objects using two-dimensional drawings, these tools free up working memory resources for deeper geometric processing. Activity-based learning theory and constructivist principles jointly establish the pedagogical rationale for learning through construction and investigation of geometric models rather than passive reception of information. Together, these frameworks predict that 3D modeling instruction should improve both spatial abilities and academic geometry achievement through theoretically well-specified and empirically supported mechanisms.

The empirical evidence base is substantial and consistent across studies. Spatial ability development through 3D modeling instruction has been documented in multiple quasi-experimental studies including those by Toptas et al. (2012), Guven and Kosa (2008), Baki et al. (2009b), La Ferla et al. (2009), Kurtulus and Uygan (2010), Yildiz (2009), and Sorby and Baartmans (2000). Meta-analyses and systematic reviews (Uttal et al., 2013; Yang et al., 2020; Herrera et al., 2024) confirm the overall pattern of positive effects with meaningful effect sizes. Academic achievement effects are somewhat less consistently documented but are reported by several studies (Toptas et al., 2012; Abu et al., 2012b; Gagnon et al., 2023), with the overall pattern of evidence supporting positive expectations.

The analysis of this literature reveals three significant gaps that define the scientific contribution of the present investigation.

The first gap is the underrepresentation of high school students in the 3D modeling literature, as most studies are focused on elementary, middle school, or college students. This study directly fills this gap by working with 11th grade students in the formal context of rigid body geometry, for whom the course content is directly related to the exam.

The second gap is the lack of research on the simultaneous and coordinated use of two complementary 3D modeling platforms within the same event.; A single software tool is widely used in existing educational institutions, which makes the educational potential of combining Tinkercad and SketchUp Free completely unexplored.

The third gap is the almost complete lack of 3D modeling research in geometry lessons in Kazakhstan, a country with strong educational traditions, a developing tradition of educational research, and a national education digitalization program that makes such research timely and politically relevant. Existing research in Kazakhstan demonstrates a growing interest in spatial reasoning, geometric reasoning, technical graphics, and the use of digital technologies in mathematics education. However, most of these studies focus on drawing geometry, graphic disciplines, geogebra-based learning, or the development of technical skills, rather than directly studying students' spatial thinking through 3D modeling.

In addition, the available research remains fragmented and is characterized by a predominance of theoretical and methodological discussions, while empirical studies that use rigorous quasi-experimental schemes are relatively rare. Although digital tools such as GeoGebra, CAD, and computer visualization environments are increasingly being incorporated into educational practice, their specific impact on the development of spatial thinking has not been sufficiently studied in geometry studies in high school. Thus, there is a limited amount of evidence on the effectiveness of integrating 3D modeling platforms into geometry teaching in the Kazakh educational context.

By addressing these three gaps simultaneously, this study contributes to the international literature on educational technologies in geometry teaching and provides practically useful data for mathematics teachers, curriculum developers, and educational policy makers in Kazakhstan. The results of the study will help to better understand how 3D modeling tools can be effectively integrated into geometry teaching in secondary schools in various national contexts, providing empirical

evidence for policy decisions on the integration of educational technologies in Kazakhstani schools.

3. METHODOLOGY

3.1 Research design

This study utilized a quasi-experimental design with two nonequivalent groups and pre- and post-intervention measurements. This is necessary when the study is conducted in a real-world institutional setting, and random assignment of the study subjects to groups is impossible: in this case, students are divided into existing classes.

Using existing classroom groups as experimental and control groups is not uncommon in educational research of this type (Toptas et al., 2012; Guven & Kosa, 2008).

A pre-test was administered to both groups at the beginning of the school period, and a post-test was administered at its conclusion. In the experimental group, students used 3D modeling software, Tinkercad and SketchUp Free, in geometry lessons, while in the control group, they studied the same material using the same curriculum without any 3D modeling tools, as part of a regular classroom setting. The only systematic difference between the groups was the pedagogical approach used. The data for the study was collected using three tools: the Purdue Spatial Visualization Test (PSVT), student assessments before and after the geometry experiment from school, and a questionnaire on attitudes to 3D modeling filled out by students in the experimental group at the end of the experiment.

3.2 Participants

The study involved 50 students of the eleventh grade of a secondary school in Kazakhstan. The participants were divided into two groups according to their class affiliation: an experimental group of 25 students and a control group of 25 students. A separate sampling procedure was not applied because the study included classes allocated by the school administration.

Both groups studied the same geometry curriculum for the eleventh grade, studied with the same math teacher, and attended the same school, which provided comparable learning conditions. Participation in the study was voluntary. Prior to data

collection, informed consent was obtained from all participants, as well as their parents or legal representatives.

3.3 Instruments

Three tools were used to collect the data in this study:

1. Purdue Spatial Visualization Test (PSVT).
2. Current grades before and after the geometry course experiment, obtained from official school reports.
3. A questionnaire for students of the experimental group about their attitude to 3D modeling.

3.3.1 Purdue Spatial Visualization Test (PSVT)

Spatial abilities were measured using the Purdue Spatial Visualization Test (PSVT), originally developed by Guay in 1976. PSVT is one of the most widely used and well-tested standardized tools for measuring spatial visualization in educational research. This test has been used in methodologically comparable studies on 3D modeling and geometry learning, including Guven and Kosa (2008) and Toptas et al. (2012).

The original version of the PSVT consists of 36 multiple choice questions divided into three sections of 12 questions each. For the purposes of this study, an abbreviated version of the test was used, which included 17 tasks selected from all three sections: 6 tasks from the Developments section, 5 tasks from the Rotations section, and 6 tasks from the Views section. This selection made it possible to preserve the three-factor structure of the original test and ensure the representation of all three components of spatial abilities, while reducing the testing time to one lesson lasting 15 minutes.

Each section assesses a distinct component of spatial ability:

- Developments: items present the surface unfolding (net) of a three-dimensional solid and require the student to identify which of five pictured objects

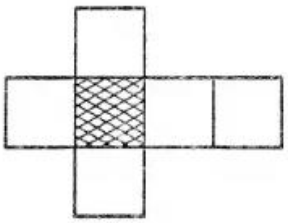
corresponds to that net. This section assesses spatial visualization, the ability to mentally fold a flat pattern into a three-dimensional solid.

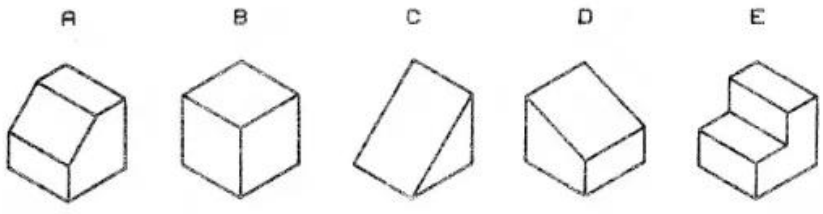
- Rotations: items show a three-dimensional object in two positions and require the student to identify the object resulting from the next step in the depicted rotation sequence. This section assesses mental rotation ability.
- Views: items place a three-dimensional object inside a transparent cube, mark one corner of the cube, and require the student to identify the appearance of the object when viewed from that corner. This section assesses spatial orientation, the ability to adopt multiple perspectives on a three-dimensional object.

Figure 3.3.1.1. Sample items from the Purdue Spatial Visualization Test

Section 1: Developments

Presented is a development and five three-dimensional objects.
The development shows the inside surfaces of a three-dimensional object.
The shaded portion of the development indicates the bottom surface of the three-dimensional object.
You are to:
picture in your mind what the development looks like when folded into a three-dimensional object;
select from among the five objects (A, B, C, D, or E) the one that looks like the folded development.
What is the correct answer to the example shown above?





The correct answer for this example is B.

Source: Adapted from the Purdue Spatial Visualization Test (PSVT), originally developed by Guay (1976)

Items were scored dichotomously: 1 point for a correct response and 0 for an incorrect response. The maximum possible score was 17. The same version of the test was administered as both the pre-test and the post-test, consistent with the recommendation of Bertoline and Miller (1990, as cited in Guven & Kosa, 2008) and the practice of analogous studies (Toptas et al., 2012).

Reliability and validity. The reliability of the original PSVT has been documented across multiple independent studies using the Kuder-Richardson Formula 20 (KR-20). The reliability of the PSVT has been supported in previous studies. For example, Clements and Battista (1992) reported acceptable internal consistency for spatial reasoning measures, and the use of PSVT-based instruments has been common in studies examining spatial visualization and geometry learning. In the present study, the shortened version of the PSVT was used to assess three components of spatial ability: spatial visualization, mental rotation, and spatial orientation. All reported values exceed the threshold of .70 considered acceptable for research purposes (Buyukozturk, 2017). The constructive validity of the test is ensured by the direct correspondence of the three sections of the test to the three components of spatial abilities identified in the scientific literature: spatial visualization, mental rotation and spatial orientation (Linn & Petersen, 1985; Maier, 1998).

3.3.2 Academic Achievement

Academic performance in geometry was assessed based on current student grades obtained from official school journals. The estimates were recorded at two points in time: seven weeks before the start of the experiment and after its completion. Grades are recorded on a 10-point scale, as used in the secondary education system of Kazakhstan.

The use of current course grades as an indicator of academic achievement is consistent with the practice adopted in comparable studies (Toptas et al., 2012; Abylkassymova et al., 2025) and reflects students' actual performance within the school's educational process. It should be noted, however, that course grades reflect

cumulative performance across the full geometry curriculum rather than exclusively the solid geometry topics covered in the intervention.

3.3.3 Student Questionnaire on Attitudes Towards 3D Modeling

Students' attitudes towards 3D modeling were assessed using a six-item questionnaire adapted from the Three-Dimensional Modeling Attitude Scale developed and validated by Benzer and Yildiz (2020). The original scale is a 30-item instrument developed for university students and validated through exploratory and confirmatory factor analysis with 204 participants. Validation yielded three factors (Importance, Interest, and Anxiety) with an overall reliability coefficient of $\alpha = 0.94$ and subscale coefficients of 0.90, 0.91, and 0.82 respectively (Benzer & Yildiz, 2020). For the present study, six items from the Interest and Importance subscales were selected as most relevant to the Grade 11 student population and the study context.

Each item was rated on a five-point Likert scale ranging from 1 (Strongly Disagree) to 5 (Strongly Agree). The questionnaire was administered only to students in the experimental group, as its purpose was to document participants' subjective experience of the 3D modeling intervention rather than to compare attitudinal outcomes across groups. Administration was anonymous.

Table 3.3.3.1. Student questionnaire on attitudes towards 3D modeling

Questionnaire: Understanding Geometry through the Use of 3D Modeling

№	Statement	Strongly agree	Agree	Not sure	Disagree	Strongly disagree
1	I enjoy observing 3D modeling and working with it through practical use.					

Source: Adapted from Benzer & Yildiz (2020)

Reliability of the instrument – pilot study. Prior to the main study, the reliability of the adapted questionnaire was examined through a pilot study. The questionnaire was administered to 22 students from a different Grade 11 class in the same school,

with the permission of the class teacher. Internal consistency reliability was assessed using Cronbach's alpha coefficient, calculated according to the following formula:

$$\alpha = \frac{k}{k-1} \times (1 - \frac{\sum \sigma_i^2}{\sigma_t^2})$$

where k is the number of items, $\sum \sigma_i^2$ is the sum of the individual item variances, and σ_t^2 is the variance of the total scale scores.

The results of the reliability calculation for the pilot sample are presented in Table 3.3.3.2.

Table 3.3.3.2. Cronbach's Alpha Calculation for the Pilot Sample (n = 22)

Item	Mean (M)	Variance (σ_i^2)	Min	Max
Q1	3.68	0.989	2	5
Q2	2.82	1.584	1	5
Q3	3.36	1.766	1	5
Q4	3.00	2.476	1	5
Q5	3.18	1.584	1	5
Q6	3.09	1.801	1	5
$\sum \sigma_i^2$ (sum of item variances)		**10.201**		
σ_t^2 (total score variance)		**33.552**		
k (number of items)	**6**			
Cronbach's Alpha (α)	**0.835**			

Note. n = 22. Variances calculated using the unbiased estimator (divided by n−1 = 21). Total score range: 8–27.

Substituting the computed values into the formula:

$$\alpha = \frac{6}{5} \times (1 - \frac{10.2013}{33.5519}) = 1.2 \times 0.6960 = 0.835$$

To verify the reliability of the questionnaire, the Cronbach alpha coefficient was calculated, which shows how consistently all the questionnaire questions measure the same thing. The resulting value was 0.835, which is in the generally accepted range from 0.70 to 0.90 and indicates good internal consistency of the instrument (Buyukozturk, 2017). In other words, all six questions in the questionnaire measure exactly what they are supposed to measure, and they do it quite accurately and consistently. This confirms that the adapted questionnaire is a reliable tool and is quite suitable for use in the main study.

3.4 Instructional Procedures

The study period lasted seven weeks, during which eight geometry lessons of 45 minutes each were conducted. There were two lessons in the first week, and one lesson in each of the remaining six weeks. All eight lessons were taught by the researcher himself in both groups. This was done to exclude a situation in which differences in results between groups could be explained not by the teaching method, but by the personal qualities or teaching style of different teachers. The lesson topics fully corresponded to the official geometry curriculum for 11th grade in Kazakhstan.

The content of the educational material in both groups was completely identical throughout all eight lessons. The only difference between the groups was the pedagogical approach used. The experimental group mastered the material using 3D modeling tools, while the control group studied using the traditional methodology.

3.4.1 Experimental Group

At the beginning of the first lesson, a short introductory session was held, during which students got acquainted with the basic interface and tools of Tinkercad (tinkercad.com) and SketchUp is Free (app.sketchup.com), after which the geometric material was introduced. Both platforms are fully browser-based, free, and compatible with mobile devices. Throughout the school period, students used personal smartphones to access the platforms, which avoided the need for a specialized computer classroom.

Each lesson in the experimental group was based on a two-stage structure. At the first stage, the researcher introduced the students to the geometric topic and explained the corresponding properties and formulas. At the second stage, students worked independently in Tinkercad or SketchUp Free, solving problems from a textbook. When solving each problem, the students first built a three-dimensional model of the geometric body described in the condition in order to visualize the object and better understand its properties. Based on the constructed model, the students then proceeded to solve the problem by applying the appropriate formulas. This approach allowed students to connect the spatial understanding developed during the modeling process with the formal mathematical procedures provided by the curriculum. The tasks offered to the students were stereometry tasks from the current geometry textbook for 11th grade. Typical examples included finding the volume of the common part of two unit cubes, calculating the volume of a cylinder based on its axial cross-section, and comparing the volumes of cylinders described around and inscribed in a regular quadrangular prism.

3.4.2 Control Group

The control group studied the same eight topics over the same seven-week period. The lessons were conducted by the same researcher, but without using any digital modeling tools, that is, as part of the usual traditional training. Each lesson was based on a standard scheme: the researcher explained the geometric material and the corresponding formulas using drawings on the blackboard, analyzed examples of problem solving, after which the students independently solved the problems from the textbook. The time distribution in the lessons and the tasks from the textbook were identical to those used in the experimental group.

3.5 Data Collection Procedures

Data collection was carried out in strict sequence throughout the entire study period, starting from the stage preceding the academic period and ending with its completion.

The pre-test was conducted in both groups in the lesson immediately preceding the school period. Students were given 15 minutes to complete 17 PSVT tasks. The test was supervised by a researcher who gave standardized instructions and ensured that all students worked independently. A similar procedure was followed during the post-test, which took place after the end of the eighth lesson. The initial grades and post-experiment grades were obtained from official school journals.

The survey was conducted only among the students of the experimental group after the final test during the final lesson. The students were given about 10 minutes to complete a six-question questionnaire. The survey was anonymous, and the students were warned in advance that their answers would not affect their grades in the subject in any way.

3.6 Ethical Considerations

This study is consistent with the ethical principles of conducting pedagogical research involving minors. Prior to the start of data collection, all necessary consents and permits were obtained from the relevant parties.

First, an official permit was obtained from the school principal, confirming the school's consent to conduct this pedagogical experiment as part of the educational process. After that, introductory meetings were held with the classroom teachers and the mathematics teacher of each class, at which the purpose and procedure for conducting the study were explained. They signed written consent forms authorizing their classes to participate in the experiment. After that, informational letters were sent to the parents or legal representatives of each student explaining the purpose of the study, the data collection procedure and confidentiality measures. Prior to the start of the study, written informed consent was obtained from all parents. The students were verbally notified of the upcoming study, its voluntary nature, and their right to withdraw from the study at any time without any consequences for their grades.

All information obtained during the study was anonymized before the analysis began. The students were identified only by their ordinal participant number. No personal data was stored in the research database. The grades were obtained from

official school journals with the permission of the school administration. Participation in the study was completely voluntary, and not a single student was subjected to any pressure.

4. ANALYSIS AND RESULTS

Data were analysed using the Python programming language (version 3.x) in the Google Colaboratory environment, with the NumPy and SciPy libraries. The following notation is used throughout this section: M = arithmetic mean; SD = standard deviation; n = sample size; p = significance level; d = Cohen's effect size coefficient; t = t-test statistic; W = Shapiro-Wilk test statistic. The significance level for all inferential tests was set at $\alpha = .05$.

4.1 Descriptive Statistics

Descriptive statistics for the pre-test and post-test scores of both groups are presented in Table 4.1.1.

Table 4.1.1. Descriptive Statistics for PSVT Scores and Geometry Grades

	CG Pre-test	**CG Post-test**	**EG Pre-test**	**EG Post-test**
PSVT (max. 17 points)				
M	10.60	10.88	11.48	13.12
Mdn	11	11	12	14
SD	3.51	3.43	3.10	3.17
Min	4	5	5	6
Max	17	17	17	17
Geometry grades (max. 10 points)				
M	7.25	7.30	7.88*	7.93
SD	1.35	1.37	1.18*	1.17

*Note. CG = control group (n = 25); EG = experimental group (n = 25 for PSVT; *n = 24 for pre-intervention grades due to missing data for one student); M = arithmetic mean; SD = standard deviation.*

Figure 4.1.1. PSVT Scores (Pre vs Post)

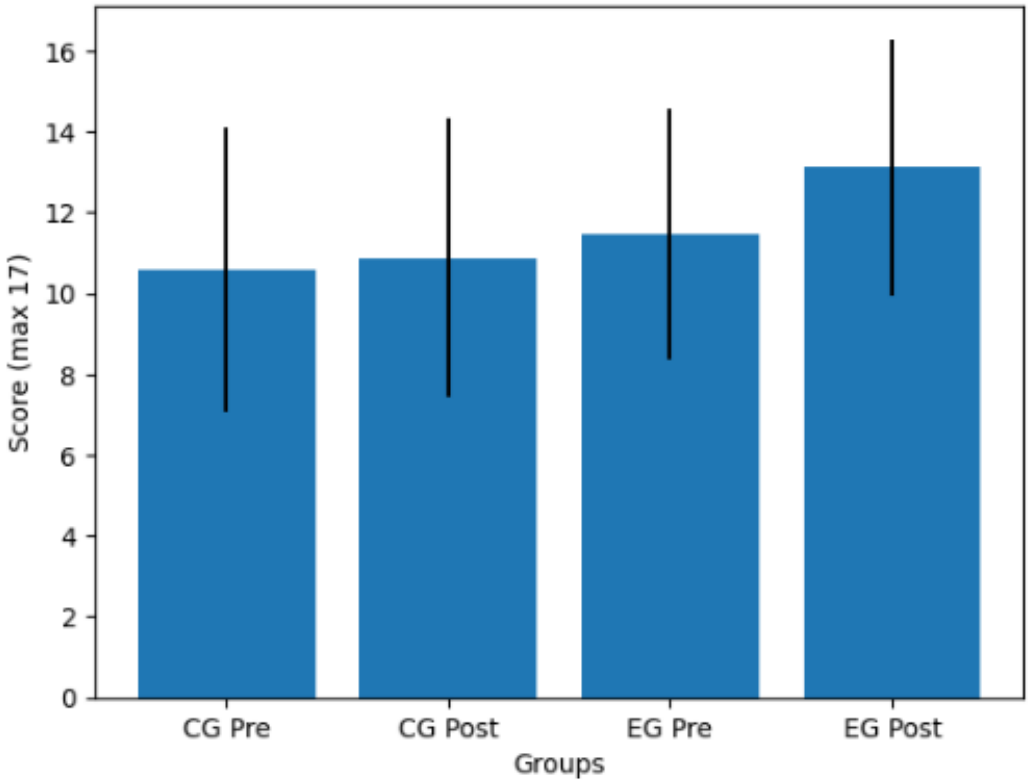


Figure 4.1.2. PSVT Scores (Pre vs Post)

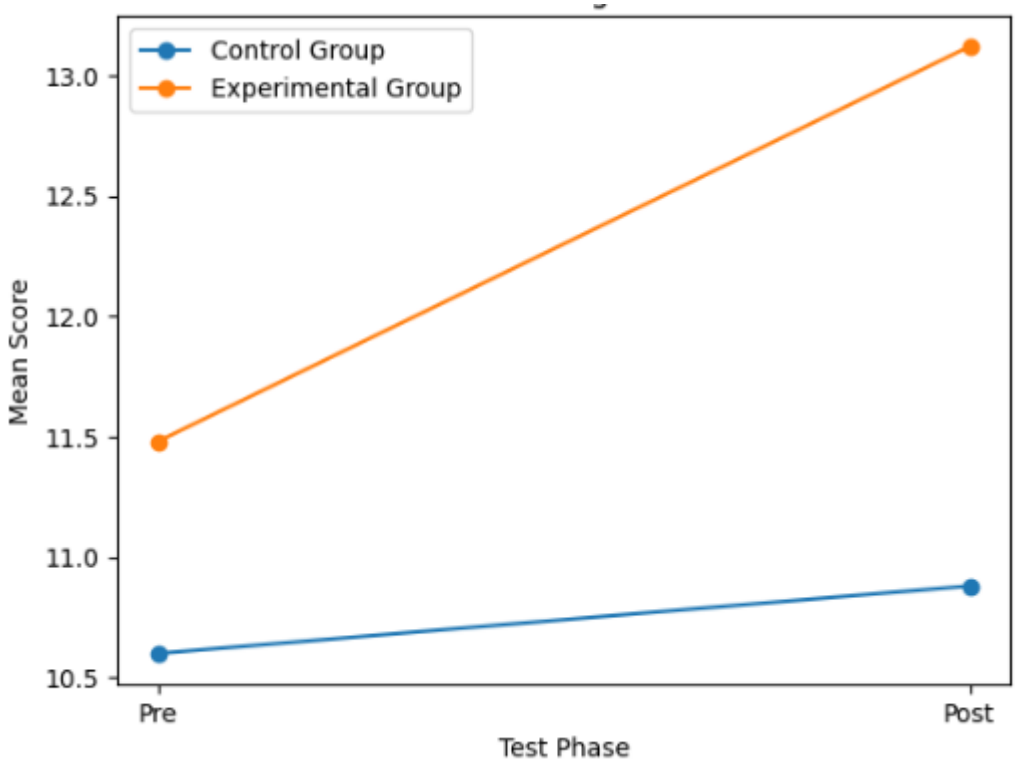


Figure 4.1.3. PSVT Progress

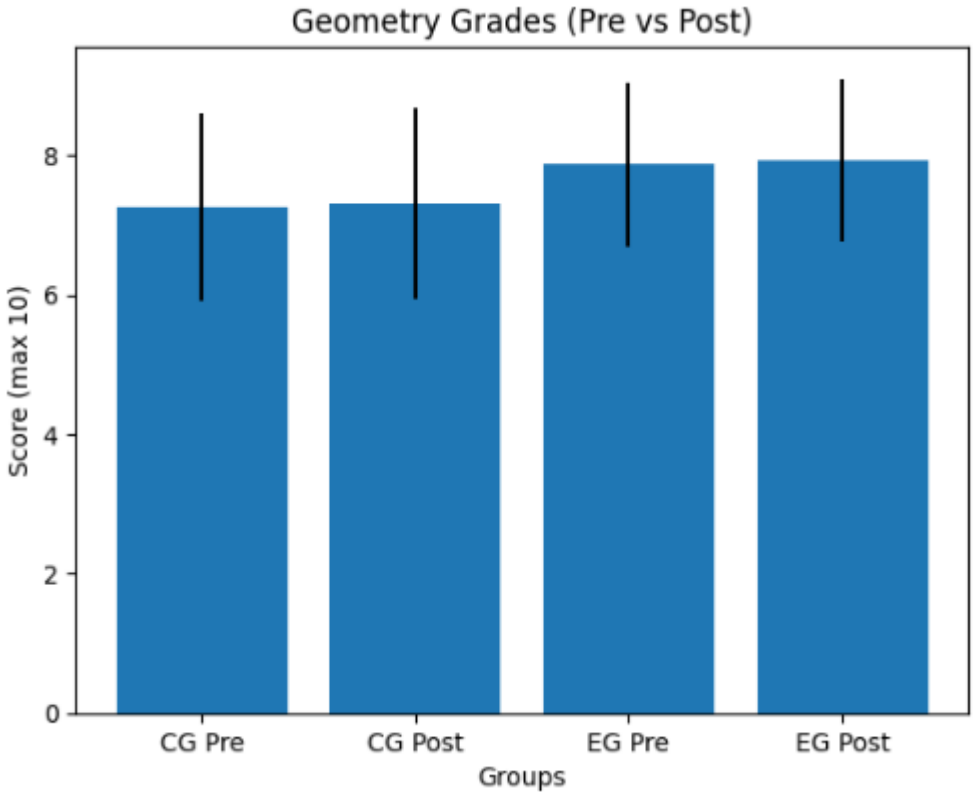
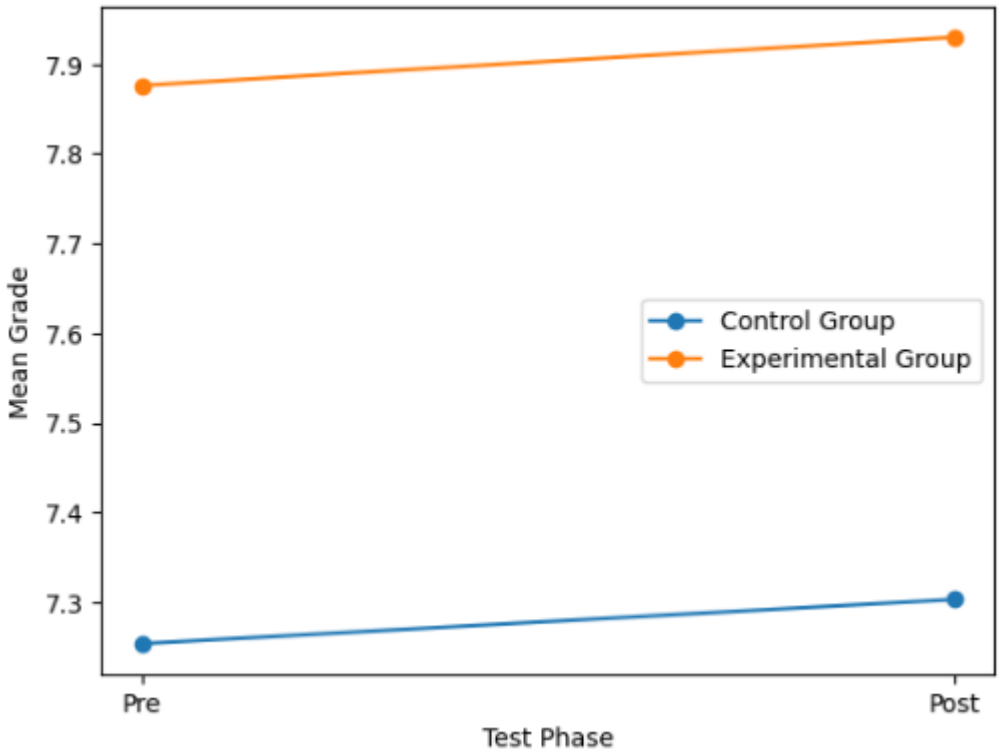


Figure 4.1.4. Geometry Grades (Pre vs Post)



4.2 Baseline Equivalence of the Groups

Before evaluating the effect of the intervention, the baseline equivalence of the control and experimental groups was examined using the PSVT pre-test scores. Since the two groups consisted of different students, an independent samples t-test was conducted to compare their initial spatial ability levels. The control group obtained a mean pre-test score of M = 10.60, SD = 3.51, while the experimental group obtained a mean pre-test score of M = 11.48, SD = 3.10. The independent samples t-test showed that this difference was not statistically significant, t(48) = -0.94, p = .352, Cohen's d = 0.27. This indicates that the two groups were statistically comparable before the intervention. Therefore, the post-test differences can be interpreted more confidently as being associated with the implementation of 3D modeling tools rather than with an initial advantage of the experimental group.

Table 4.2.1. Independent Samples t-test Results for PSVT Pre-test Scores

Compared groups	CG M	CG SD	EG M	EG SD	t	df	p	Cohen's d	Interpretation
CG pre-test vs EG pre-test	10.60	3.51	11.48	3.10	-0.94	48	0.352	0.27	Not significant

4.3 Normality Testing

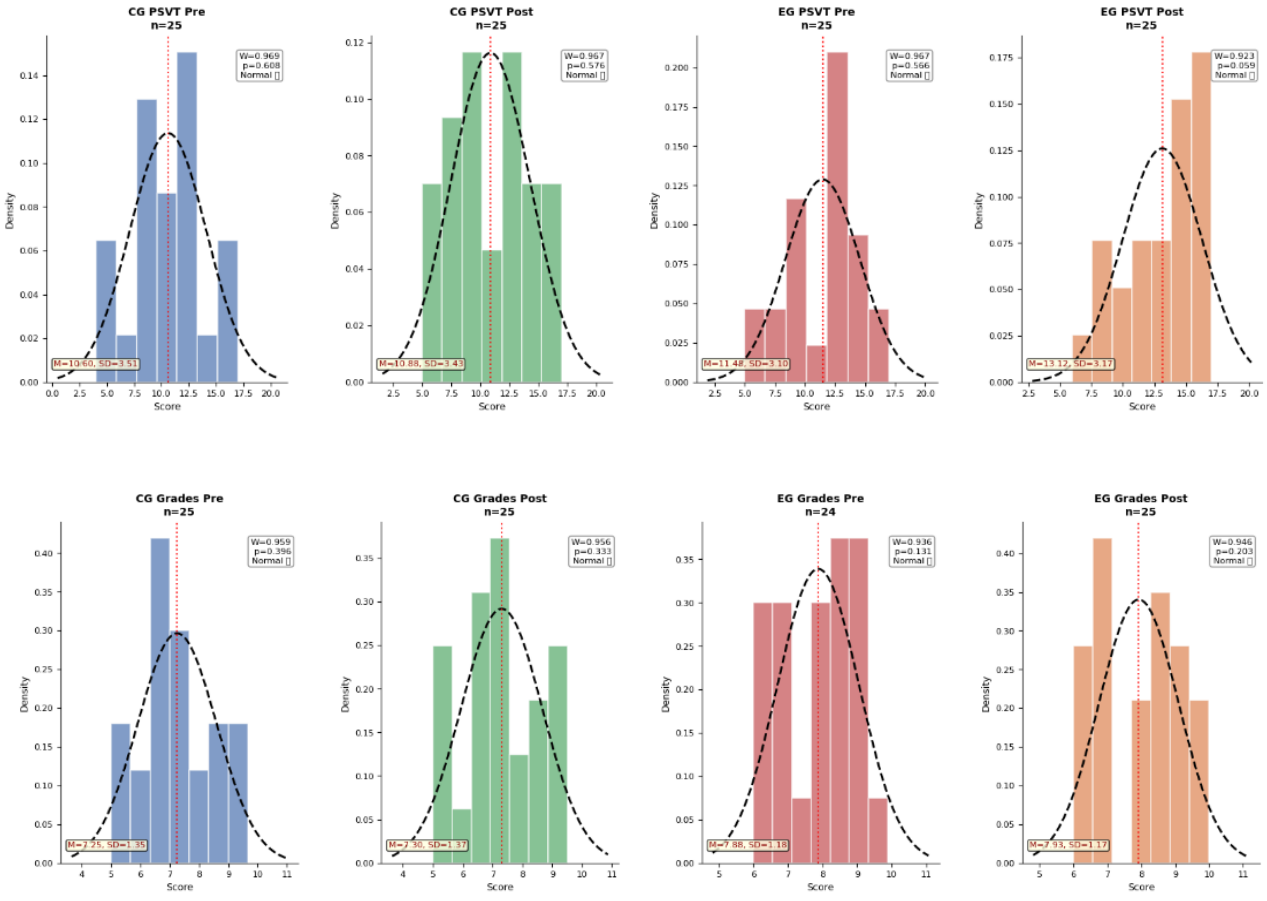
Prior to inferential testing, the normality of all score distributions was examined using the Shapiro–Wilk test. This test was selected because each group comprised 25 students: for small samples (n < 50), the Shapiro-Wilk test is more sensitive to departures from normality than the Kolmogorov-Smirnov test (Büyüköztürk, 2017). A value of p > .05 indicates that the distribution does not differ significantly from normal and that the use of parametric tests is justified. Results are presented in Table 4.3.1.

Table 4.3.1. Shapiro-Wilk Normality Test Results

Variable	W-statistic	p-value	Normality
CG Pre-test	0.969	0.6085	Yes
CG Post-test	0.967	0.5764	Yes
EG Pre-test	0.967	0.5659	Yes
EG Post-test	0.923	0.0591	Yes
CG Grade Pre	0.959	0.3959	Yes
CG Grade Post	0.956	0.3325	Yes
EG Grade Pre	0.936	0.1306	Yes
EG Grade Post	0.946	0.2033	Yes

Note. W = Shapiro-Wilk test statistic; p > .05 indicates a normal distribution.

Figure 4.3.1. Shapiro-Wilk Normality Test Results



Despite the fact that the results of the Shapiro-Wilk test indicated a normal distribution (p > 0.05), visual analysis of the histograms revealed minor deviations from a normal distribution shape.

4.4 Spatial Ability (PSVT)

Spatial ability was assessed using the Purdue Spatial Visualisation Test (PSVT, maximum score – 17 points). The normality of score distributions was examined using the Shapiro-Wilk test. Although the results indicated normal distributions across all four samples (CG pre-test: W = 0.969, p = .609; CG post-test: W = 0.967, p = .576; EG pre-test: W = 0.967, p = .566; EG post-test: W = 0.923, p = .059), visual inspection of the histograms revealed minor deviations from a normal distribution shape. Given the relatively small sample size (n = 25) and the well-documented limitation that the Shapiro–Wilk test has low statistical power with small samples, non-parametric methods were selected for subsequent analysis. The Mann–Whitney U test was used for between-group comparisons, and the Wilcoxon signed-rank test was applied for within-group comparisons (pre-test vs. post-test). The significance level was set at $\alpha = .05$. Effect size was calculated using the formula:

$$p = \frac{|Z|}{\sqrt{N}}$$

Table 4.4.1. Statistical Test Results for PSVT (Mann-Whitney U test: CG vs EG)

Stage	Mdn CG	Mdn EG	U	Z	p	r	Effect	Sig. (p < .05)
Pre-test	11.000	12.000	265.500	-0.912	0.364	0.129	small	No
Post-test	11.000	14.000	195.000	-2.280	0.023	0.322	medium	Yes

The Mann-Whitney U test results show that the control and experimental groups did not differ significantly at the pre-test stage (p = .364). This indicates that the two groups had comparable initial spatial ability levels before the intervention. However, after the intervention, the difference between the groups became statistically significant (p = .023), with a medium effect size (r = .322). This suggests that the experimental group outperformed the control group on the PSVT post-test, which may be associated with the use of Tinkercad and SketchUp Free during geometry instruction.

Figure 4.4.1. Pre-test

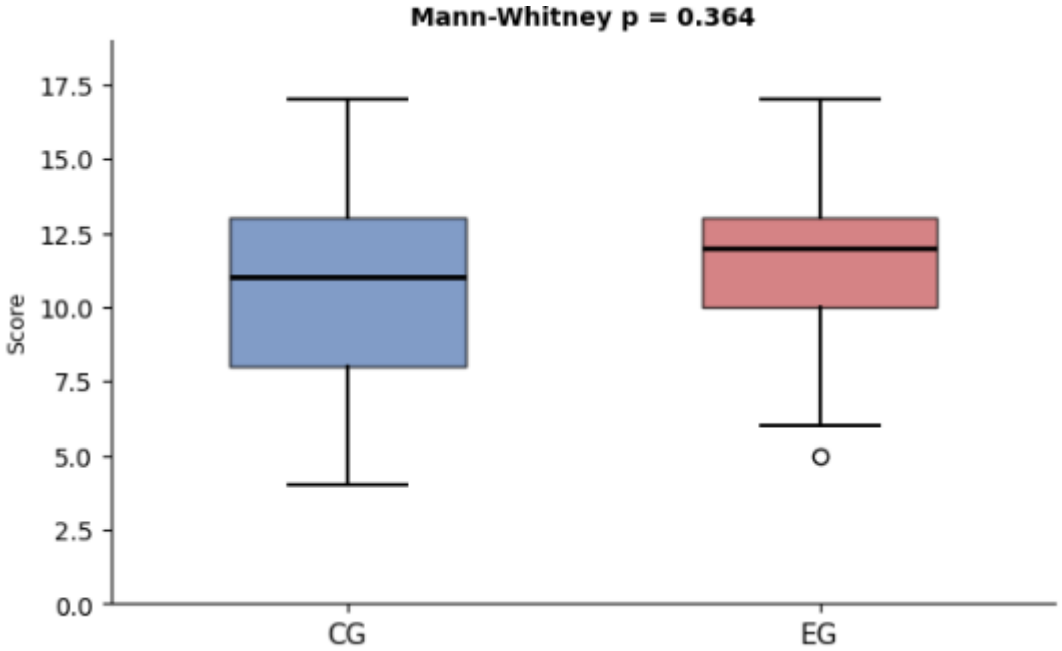


Figure 4.4.2. Pre-test

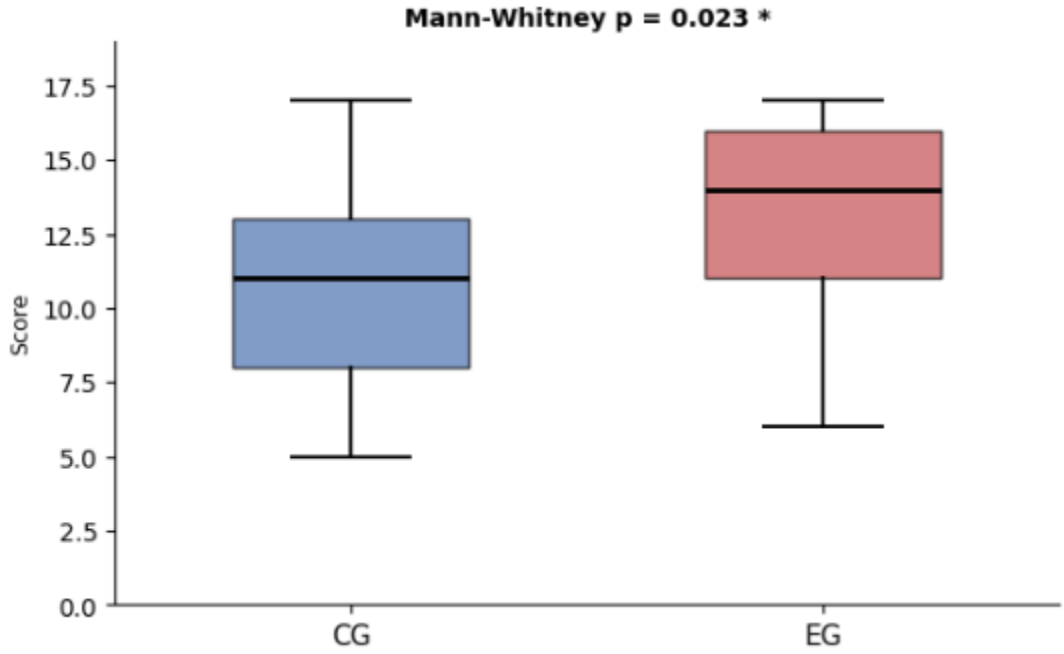
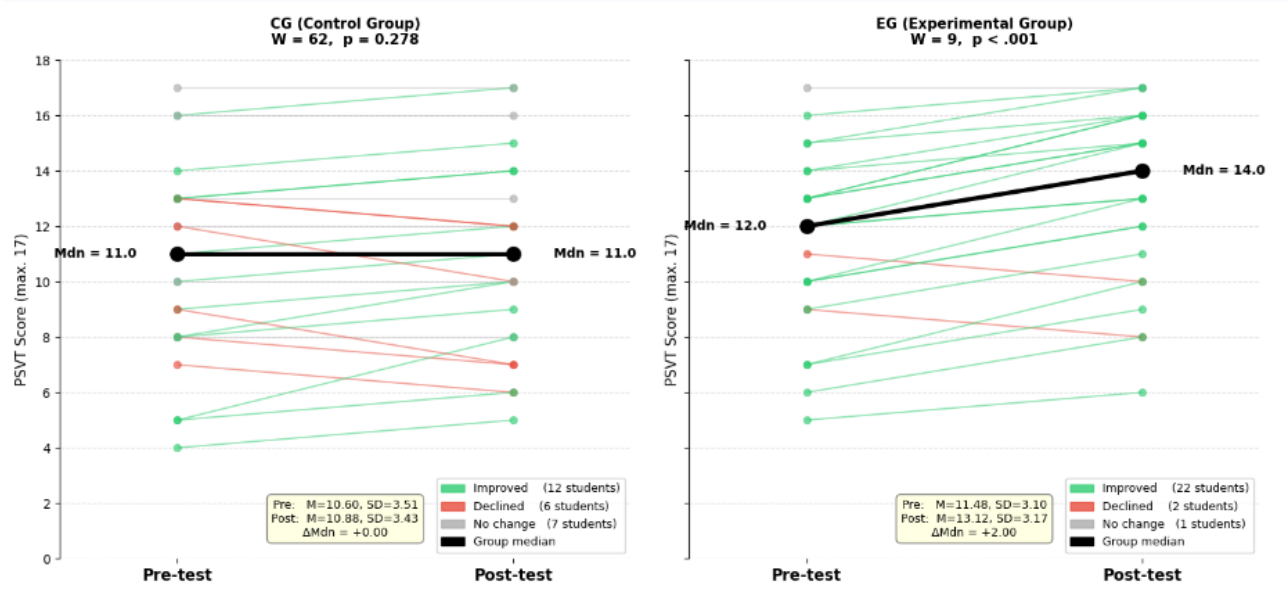


Table 4.4.2. Wilcoxon signed-rank test: Pre vs Post (within group)

Group	Mdn Pre	Mdn Post	Δ Mdn	W	p	r	Effect	Sig. (p < .05)
CG PSVT	11.000	11.000	0.000	62.000	0.278	0.217	small	No
EG PSVT	12.000	14.000	2.000	9.000	<.001	0.819	large	Yes

Figure 4.4.3. PSVT: Individual Score Changes per Student



Baseline equivalence of groups. At the pre-test stage, the median PSVT scores of the CG and EG were 11.00 and 12.00 points, respectively (CG: M = 10.60, SD = 3.51; EG: M = 11.48, SD = 3.10). The Mann–Whitney U test revealed no statistically significant difference between the groups (U = 266, Z = −0.912, p = .364, r = .13), confirming their initial comparability prior to the intervention.

Post-intervention results. Upon completion of the experimental instruction, the EG median increased to 14.00 points (M = 13.12, SD = 3.17), while the CG median remained unchanged at 11.00 points (M = 10.88, SD = 3.43). The Mann–Whitney U test detected a statistically significant difference between the groups at the post-test stage (U = 195, Z = −2.280, p = .023, r = .32), corresponding to a medium effect size according to Cohen's scale.

Within-group dynamics. Analysis of pre-to-post changes using the Wilcoxon signed-rank test showed that the EG demonstrated a statistically significant improvement (W = 9, p < .001, r = .82 – large effect): the median increased from 12.00 to 14.00 points (ΔMdn = +2.00). No significant change was observed in the CG (W = 62, p = .278, r = .22), with the median remaining stable (ΔMdn = 0.00).

Taken together, these findings indicate that the experimental intervention had a statistically significant positive effect on the spatial ability of students in the

experimental group, with a medium between-group effect size (r = .32) and a large within-group effect size (r = .82). No significant changes were observed in the control group over the same period.

4.5 Academic Achievement (Geometry Grades)

The students' academic achievement was assessed on the basis of grades in geometry on a ten-point scale. For each student, the arithmetic mean of grades was calculated for two periods: seven weeks before the start of the experiment and seven weeks during the experimental training. Since the grades reflect current academic performance within the educational process, rather than the results of a single standardized measuring instrument, the use of statistical inference methods did not seem justified. The analysis was carried out using descriptive statistics methods: the average values and standard deviations for groups for both periods were compared, and the percentage change in indicators was calculated. The results reflect a general trend in academic performance and do not imply conclusions about the statistical significance of the differences.

Figure 4.5.1. Geometry Grades: Before vs During Experiment

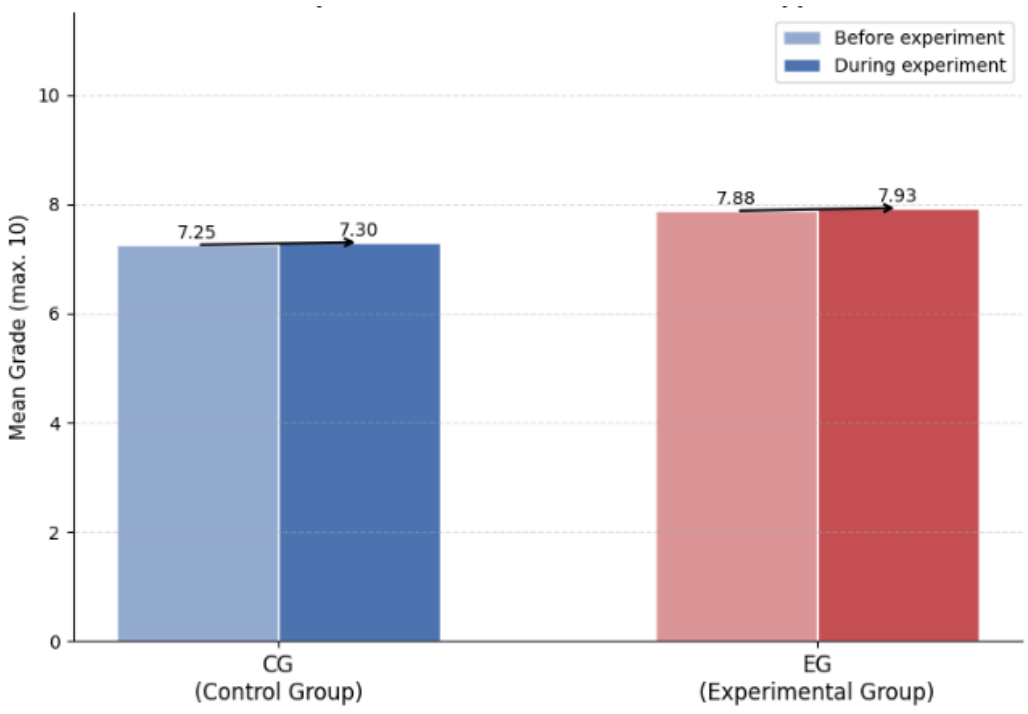
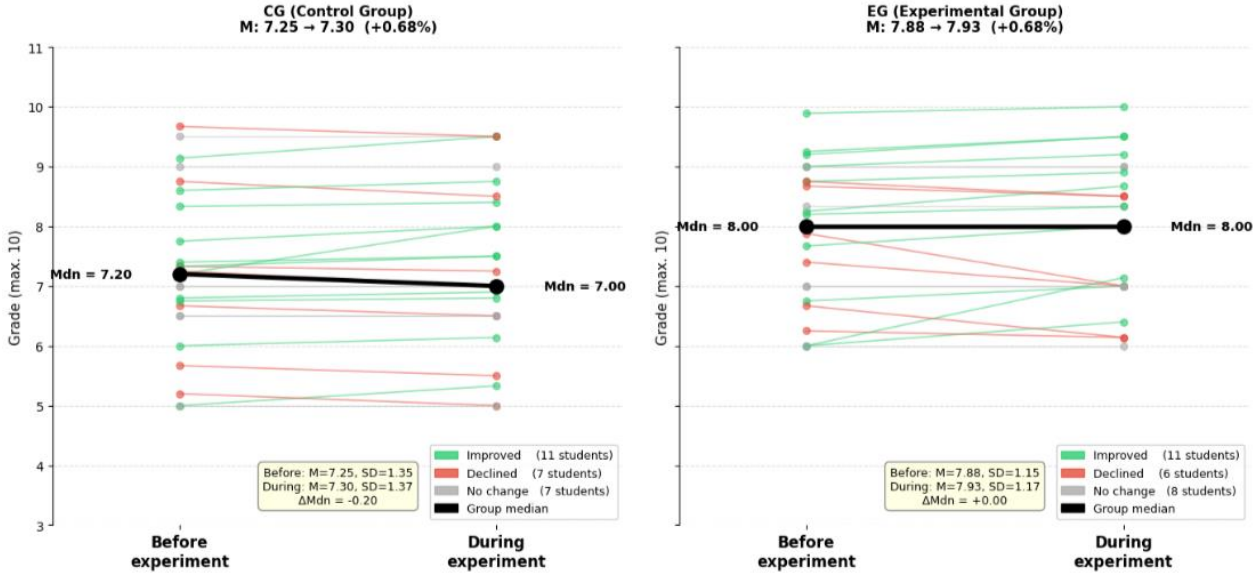


Figure 4.5.2. Geometry Grades: Before vs During Experiment



Before the intervention, the average value of geometry scores in the control group was M = 7.25 (SD = 1.35), while in the experimental group this indicator was slightly higher – M = 7.88 (SD = 1.15). During the experimental period, both groups showed a slight increase in ratings. In the control group, the average value increased to M = 7.30 (SD = 1.37), which corresponds to a change of +0.05 points (+0.68%). In the experimental group, the average value increased to M = 7.93 (SD = 1.17), which was +0.05 points (+0.68%).

Both groups showed the same percentage increase, which indicates that the experimental intervention did not have a noticeable additional effect on current academic performance in geometry compared with the control group. The results obtained should be considered as descriptive trends rather than as statistically significant differences. The moderate nature of the changes in both groups indicates that academic performance in geometry is determined by a wide range of factors beyond spatial abilities.

4.6 Questionnaire Results

Responses to the six-item questionnaire from students in the experimental group were analysed descriptively. The reliability of the scale in the main sample (n = 25)

was α = 0.978, indicating excellent internal consistency (Büyüköztürk, 2017). Descriptive statistics for each item are presented in Table 4.6.1.

Table 4.6.1. Descriptive Statistics for Questionnaire Responses (n = 25)

No.	Statement	M	SD	Min	Max
Q1	I enjoy observing and working with 3D modeling	3.40	1.32	1	5
Q2	Lessons using 3D modeling are among my favourite lessons	3.76	1.16	1	5
Q3	3D modeling lessons help my thinking and cognitive development	3.76	1.09	2	5
Q4	I want to learn more about 3D modeling	3.60	1.19	1	5
Q5	I believe learning 3D modeling will give me an advantage in the future	4.12	1.05	2	5
Q6	I believe 3D modeling will become more important in the future	4.08	1.00	2	5
	Total score	**22.72**	**6.50**	**10**	**30**

Note. n = 25; M = arithmetic mean; SD = standard deviation; scale: 1 = Strongly Disagree, 5 = Strongly Agree; α = 0.978.

The highest mean scores were recorded for Q5 (M = 4.12) and Q6 (M = 4.08), reflecting students' confidence in the practical importance of 3D modeling in the future. The lowest mean was observed for Q1 (M = 3.40), which concerned students' immediate enjoyment of working with the modeling tools. Overall, the mean total score of 22.72 out of a maximum of 30 indicates a positive attitude among experimental group students towards 3D modeling as a method of geometry instruction.

5. DISCUSSION AND CONCLUSION

The purpose of this study was to examine the effect of 3D modeling tools, namely Tinkercad and SketchUp Free, on high school students' spatial abilities and academic achievement in geometry. The study used a quasi-experimental design with two existing Grade 11 classes: one experimental group and one control group. The experimental group studied geometry using 3D modeling tools, while the control group studied the same program using traditional learning. The main results were assessed using PSVT scores, current geometry grades, and a questionnaire about students' attitudes toward 3D modeling.

The main finding of the study is that the use of 3D modeling tools had a positive effect on the spatial abilities of students. At the preliminary testing stage, the average PSVT score in the experimental group was slightly higher than in the control group. However, the difference in scores between the two groups was not statistically significant. The Mann–Whitney U test showed no significant difference between the control and experimental groups before the intervention, $U = 266$, $Z = -0.912$, $p = .364$, $r = .13$. In addition, the independent samples t-test also confirmed that the initial difference was not statistically significant, $t(48) = -0.94$, $p = .352$, Cohen's $d = 0.27$. Therefore, although the experimental group had a numerically higher average score, the two groups can be considered statistically comparable in terms of spatial abilities before the intervention.

After the intervention, the results changed more clearly. The experimental group's mean PSVT score increased from 11.48 to 13.12, while the control group's mean score increased only slightly from 10.60 to 10.88. The median score of the experimental group increased from 12.00 to 14.00, whereas the median score of the control group remained unchanged at 11.00. The Mann–Whitney U test showed a statistically significant difference between the groups at the post-test stage, $U = 195$, $Z = -2.280$, $p = .023$, $r = .32$. This indicates a medium between-group effect size. Moreover, the Wilcoxon signed-rank test showed a statistically significant improvement within the experimental group, $W = 9$, $p < .001$, $r = .82$, which

corresponds to a large effect size. In contrast, the control group did not show a statistically significant improvement, W = 62, p = .278, r = .22.

These results indicate that the use of 3D modeling tools has contributed to the development of students' spatial abilities. This result is logically related to the nature of the intervention. In traditional geometry teaching, students often work with static two-dimensional images of three-dimensional objects. This requires them to mentally recreate rigid bodies, rotate objects, and represent different views without direct manipulation. In contrast, Tinkercad and SketchUp Free allow students to create, rotate, observe, and transform three-dimensional objects directly. This interaction may have helped students develop spatial visualization, mental rotation, and spatial orientation, which are key components measured by PSVT. The structure of the PSVT used in this study included Developments, Rotations, and Views sections, which respectively evaluate the mental folding of networks, mental rotation, and viewing objects from different perspectives.

The results obtained are also consistent with the theoretical foundations of the study. From the perspective of constructivist learning, students learn more effectively when they actively construct knowledge through interaction with learning objects. In this study, the students of the experimental group not only observed the finished shapes; they manipulated the models, changed their position, studied their properties, and connected visual representations with geometric concepts. From the perspective of cognitive load theory, 3D modeling may also reduce unnecessary mental effort because students do not have to imagine all three-dimensional transformations only from flat textbook illustrations. Instead, part of the visualization process is supported by the digital environment.

However, the results related to academic achievement in geometry should be interpreted more cautiously. The analysis of geometry grades did not show a noticeable additional effect of the intervention. Before the intervention, the control group had a mean geometry grade of M = 7.25, SD = 1.35, while the experimental group had a higher mean grade of M = 7.88, SD = 1.15. During the experimental period, both

groups showed the same very small increase: the control group increased to M = 7.30, SD = 1.37, and the experimental group increased to M = 7.93, SD = 1.17. In both groups, the change was only +0.05 points, or +0.68% .

This means that the intervention did not produce a clear additional improvement in students' current geometry grades compared with traditional instruction. Therefore, the findings provide stronger evidence for the effect of 3D modeling on spatial ability than on general academic achievement in geometry. This distinction is important because school grades are broader and less controlled indicators than the PSVT. They may reflect homework completion, classroom participation, previous knowledge, teacher assessment practices, and academic performance across the broader geometry curriculum, rather than just the specific stereometry topics covered during the class. For this reason, the geometry scores in this study were analyzed descriptively and were not used as the main evidence for the effectiveness of the intervention.

The questionnaire results showed that students in the experimental group generally had a positive attitude toward the use of 3D modeling in geometry lessons. The total mean questionnaire score was 22.72 out of 30, which indicates an overall positive perception. The highest mean scores were observed for the statements that learning 3D modeling would give students an advantage in the future (M = 4.12) and that 3D modeling would become more important in the future (M = 4.08). This suggests that students perceived 3D modeling as a useful and future-oriented skill. At the same time, the lowest mean score was observed for the statement about enjoying the process of observing and working with 3D modeling (M = 3.40), which may indicate that some students still found the tools unfamiliar or technically challenging.

Overall, the results show that 3D modeling tools can be useful for developing spatial abilities in geometry teaching, but their impact on overall academic performance has not been clearly demonstrated in this study. This does not necessarily mean that 3D modeling has no academic value. Rather, it suggests that measuring academic achievement based on current school grades may not have been sensitive enough to identify changes directly related to the intervention. A more targeted

standardized geometry achievement test, focused directly on the topics taught during the intervention, may provide clearer evidence in future studies.

Several limitations should be considered when interpreting the results. First, the study used a quasi-experimental design with existing classes, so students were not randomly assigned to the control and experimental groups. Although the PSVT pre-test results showed no statistically significant difference between the groups, possible pre-existing differences between classes cannot be fully excluded. Second, the sample size was relatively small, with 25 students in each group. Third, academic achievement was measured through current school geometry grades rather than through a standardized achievement test. Fourth, one student's pre-intervention grade data was missing in the experimental group. Fifth, the study was conducted in one school and with one teacher, which limits the generalizability of the findings.

Despite these limitations, the study provides useful evidence that Tinkercad and SketchUp Free can support spatial ability development in high school geometry. Since both tools are free, browser-based, and accessible on students' personal devices, they may be practical for use in Kazakhstani schools. The results obtained are especially relevant for stereometry, where students often encounter difficulties in representing three-dimensional objects from two-dimensional representations.

The study assessed how the use of tools to create 3D models affects the spatial thinking of 11th graders and their performance in geometry. A quasi-experimental method was chosen for the study, consisting of two groups: a control and an experimental one. The students of the experimental group studied geometry using the Tinkercad and SketchUp Free programs, while the control group studied the same subjects using traditional methods.

The main conclusion of the study is that the use of 3D modeling has significantly improved the spatial abilities of students. Prior to the start of the experiment, both groups showed the same results in the preliminary spatial ability test (PSVT), although the average score was slightly higher in the experimental group, but this difference was not statistically significant. After the experiment, a statistically significant increase in

PSVT results was recorded in the experimental group, while there were no significant changes in the control group. A comparison of the results after the experiment also revealed a statistically significant superiority of the experimental group. Thus, the first research question about the impact of 3D modeling on spatial capabilities has been confirmed in practice.

The results show that Tinkercad and SketchUp Free help students develop skills in spatial visualization, mental rotation of objects, and spatial orientation. These programs allow students to work directly with 3D objects, rotate them, view them from different angles, and establish a connection between visual images and geometric concepts. This is especially useful when studying stereometry, where students often find it difficult to understand three-dimensional shapes, relying only on flat images from textbooks.

The second research question, concerning academic performance in geometry, yielded less convincing results. The analysis of current geometry scores showed only a slight improvement in both groups: the average value increased by 0.05 points, or 0.68%. Consequently, the introduction of 3D modeling did not have a noticeable additional impact on the overall academic performance of students studying geometry. For this reason, school performance was analyzed in a descriptive and thorough manner. The study more convincingly demonstrates a positive effect on spatial abilities than on overall school performance.

The results of the survey showed that the students in the experimental group generally positively assessed the use of 3D modeling. They agreed that 3D modeling could be useful in the future and that its importance would increase. This shows that students perceive 3D modeling not only as an educational activity, but also as an important skill for education and professional development.

In conclusion, the study shows that 3D modeling tools can be an effective method for developing students' spatial abilities in high school geometry. However, the study did not find clear evidence that the intervention improved students' general academic achievement as measured by current geometry grades. Therefore, the

strongest contribution of this research is the evidence that Tinkercad and SketchUp Free can support the development of spatial thinking, while their impact on academic achievement requires further investigation.

6. LIMITATIONS

When interpreting the results, it is necessary to take into account several limitations related to the design and conduct of the study.

One of the limitations of this study is the absence of a standardized test for measuring academic achievement, as well as the lack of individual student data before and after the experiment. Because of this, the analysis of this indicator was limited to descriptive statistical methods, which does not allow testing the statistical significance of the obtained results.

Secondly, since the students were not randomly assigned to groups, but were included in groups in accordance with the existing division into classes, it cannot be completely excluded that the groups initially differed in some important characteristics. For example, the level of knowledge or the desire to learn could differ markedly between classes even before the experiment began, which could affect the final results.

Thirdly, there were only 25 students in each group, which is a fairly small sample. This makes it difficult to identify small differences between the groups and does not allow the findings to be fully disseminated to other students.

Fourth, current grades in geometry were used as the main indicator of academic performance, which reflect the student's knowledge of the entire geometry course as a whole, and not just on the topics of stereometry that were studied during the experiment. This could have somewhat distorted the picture of the real impact of the experiment on academic performance.

Fifth, the study was conducted in one school with one teacher, which limits the possibility of transferring the results to other schools and educational settings.

All of these limitations are taken into account when discussing the results and determining directions for further research.

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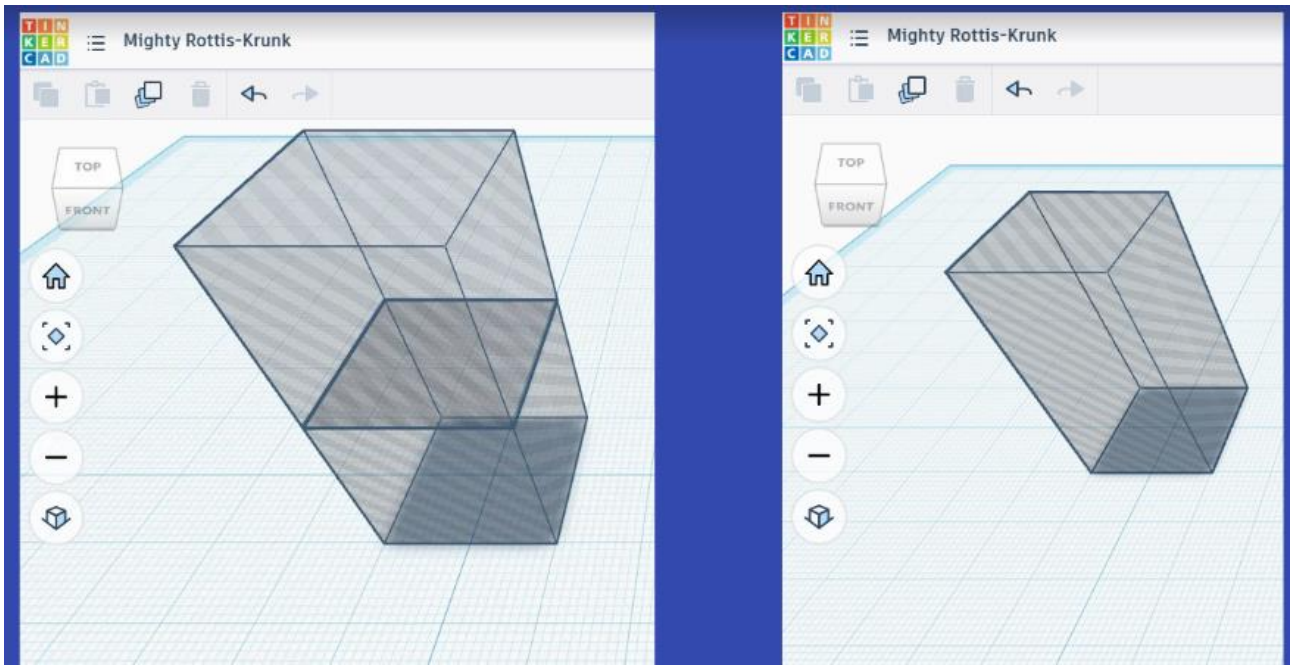
LIST OF APPENDICES

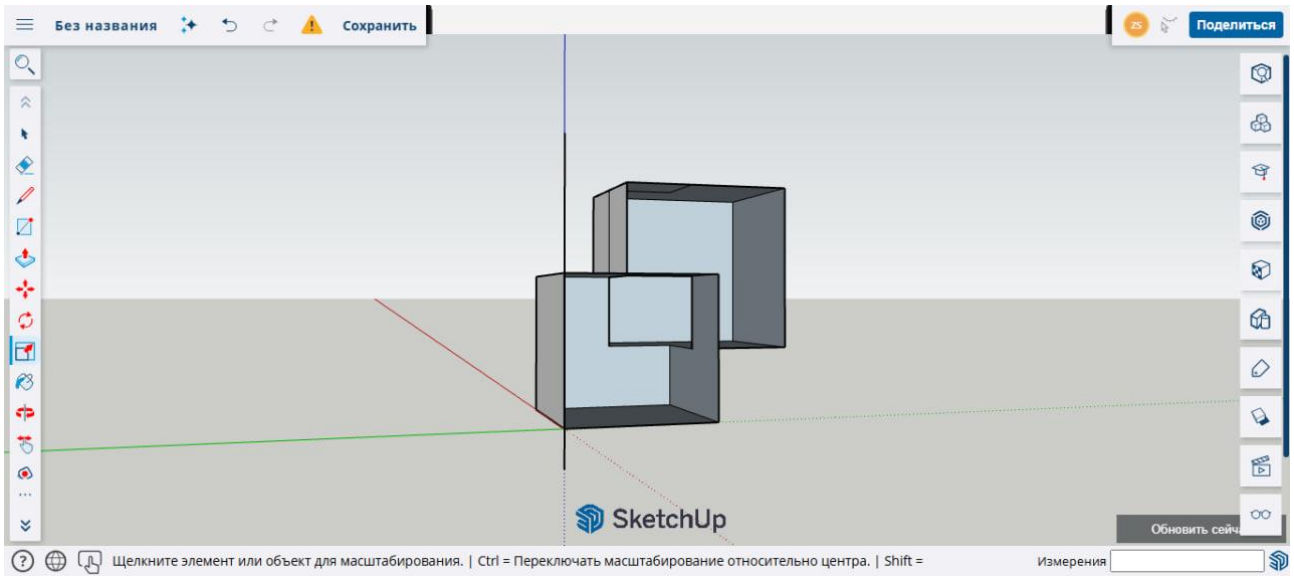
Appendix A. Schedule of Lessons and Topics Covered

Lesson	Week	Topic
1	1	Surface area of a truncated cone
2	1	Sphere and ball
3	2	Surface area of a sphere
4	3	Sections of solids of revolution with a plane
5	4	Sections of solids of revolution with a plane (continued)
6	5	General properties of volumes of solids. Volume of a prism
7	6	Volume of a pyramid and a truncated pyramid
8	7	Volume of a cylinder

Note. Each lesson was 45 minutes in duration. Two lessons were delivered in Week 1; one lesson per week in Weeks 2-7.

Appendix B. Examples of 3D modeling activities conducted during the instructional period





Appendix C. Lesson Structure for the Experimental Group

Stage	Duration	Activity	Tool
Introduction	10 min	Researcher explains topic, properties and formulas, and demonstrates 3D models	Interactive whiteboard + Tinkercad/SketchUp Free
Modelling and problem solving	30 min	Students read problem → construct 3D model → solve the problem	Tinkercad / SketchUp Free + Textbook

Note. Total lesson duration: 45 minutes.

Қысқамерзімді жоспар
№1-сабақ

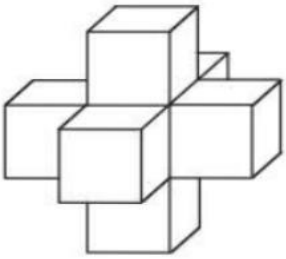
Сабақтың тақырыбы: Денелер көлемдерінің жалпы қасиеттері

Бөлім:	**Денелер көлемдері**
Педагогтің аты-жөні:	Серікбай Жанэлья Сағынқызы
Күні:	03.04.2026
Сыныбы: 11	Қатысушылар саны: Қатыспағандар саны:
Сабақтың тақырыбы:	Денелер көлемдерінің жалпы қасиеттері
Оқу бағдарламасына сәйкес оқу мақсаты:	11.3.9 - кеңістік денелерінің көлемдерінің қасиеттерін білу және қолдану.
Сабақтың мақсаты:	***Барлық оқушылар үшін:*** Білім алушылар жаңа сабақ бойынша кеңістік денелері ұғымын меңгереді. ***Көпшілік оқушылар үшін:*** Білім алушылар кеңістік денелерінің көлемдерінің қасиеттерін біледі. ***Кейбір оқушылар үшін:*** Білім алушылар кеңістік денелерінің көлемдерінің қасиеттерін қолданады.

Сабақтың барысы:

Сабақ кезеңі/Уақыты	Педагогтің іс-әрекеті	Оқушының іс-әрекеті	Бағалау	Ресурстар
Сабақтың басы Қызығушылықты ояту. 7 мин.	**(Ұ). Ұйымдастыру кезеңі:** 1.Оқушылармен амандасу, түгендеу. 2.Ынтымақтастық атмосферасын қалыптастыру 1.Мұғалім оқушыларды түгендейді. 2.Амандасады және тілектерін білдіреді.	1.Оқушылар мұғаліммен амандасады. 2. Сабаққа дайындалады. **Мақсаты:** Оқушылар бір-біріне тілек білдіреді, тыңдау дағдыларын дамытуға бағытталады, сондай-ақ барлық оқушылардың қатыстырылуы арқылы сабаққа белсенділігі артады.	Оқушылардың белсенділігі үшін *«Жарайсың!», «Керемет!»* деген сақтау сөздерімен бағалайды. **Тиімділігі:** Оқушылар бір-біріне тілек айту арқылы жақындасады, көңіл-күйін көтереді және бауырмалдығын оятады. **Саралау:** Бұл жерде саралаудың **«Жіктеу»** тәсілі көрінеді. Оқушылардың оқуға деген қызығушылығын арттыру мақсатында мүмкіндігінше оларға таңдау еркіндігі беріледі.	Оқулық, жұмыс дәптері Кітап, дәптер, қалам суреттері бейнеленген қима қағаздар топтамасы ДК экраны
Жаңа сабаққа кіріспе	**(Ұ) «Миға шабуыл»** әдісі арқылы өткен тақырыппен жаңа сабақты байланыстыру мақсатында ой қозғау сұрақтарын ұжымдық талқылау. Оқушыларға	1. Оқушылар сұрақтарға жауап беріп, өзара ұжымдық талқылау жасағаннан кейін мұғалім оқушыларға сабақтың тақырыбы, мақсатымен таныстырады. **Мақсаты:** Жылдам әрі функционалды түрде сыни ойлануды дамыту. **Тиімділігі:** оқушының танымдық дағдысы	**Саралау:** Бұл жерде саралаудың **«Диалог және қолдау көрсету»** тәсілі көрінеді. Дұрыс мағынада жауап беруге бағыттау мақсатында кейбір оқушыларға ашық сұрақтар, ал кейбір көмек қажет ететін оқушыларға жетелеуші сұрақтар қойылады.	**Өз ойын дұрыс мағынада білдіріп,** талқылауға белсенділікпен қатысқан оқушыға «Жарайсың!» деген мадақтау сөзімен ынталандыру.

	жалпылама төмендегі сұрақтар және жаттығу түрлері беріледі. **Сұрақтар:** 1.Көлем қандай шаманы сипаттайды? 2. Көлемнің өлшем бірлігі ретінде не алынады? 3.Көлемнің қасиеттерін айтыңдар. Көлемге мысал келтіріңдер.	артады. Сонымен қатар оқушыға сабақтың өмірмен байланысын көрсетеді және сабақтың тақырыбы мен мақсатын анықтауға мүмкіндік береді.	**Дескриптор:** 1. Әр оқушы өз ойымен бөліседі. 2. Өзгенің пікірін толықтырады. **Жалпы- 3 балл.**	
Сабақтың ортасы Мағынаны ашу. 31 мин.	Оқулықтағы жаңа сабақтың мәтінін оқуға тапсырма береді	1.Оқулықты оқып танысып шығады. 2.Тірек сөздермен танысып, өз дәптерлеріне жазып алады.	Тақырыпқа байланысты интернет көздерін пайдаланып есептерді шығару мүшелерімен танысады.	Слайд ДК экраны. Tinkercad платформасы
	1-тапсырманы орындатады, бақылайды, мысал, үлгі көрсетеді.	***1-тапсырма*** Егер кубтың барлық қырларын 3есе арттырсақ, онда оның көлемі неше есе артады? Оқушылар Tinkercad платформасында куб моделін құрады, оның өлшемдерін өзгертеді, модельді әртүрлі бұрыштан қарайды және көлемнің өзгеруін түсіндіреді. Оқушылар өз бақылауын формуламен байланыстырып, есептің шешімін жазады.	**Дескриптор:** 1.Кубтың бастапқы көлемін анықтайды. 2.Қыры 3 есе артқан кубтың көлемін табады. 3.Көлемнің 27 есе артатынын түсіндіреді. 4.3D модель арқылы қорытынды жасайды. **Жалпы - 4 балл.**	Tinkercad платформасы, ДК экраны, оқушылардың жеке смартфондары немесе ноутбуктері, 11-сынып геометрия оқулығы, жұмыс дәптері, слайд.
	2-тапсырманы орындатады, бақылайды, мысал, үлгі көрсетеді.	***2-тапсырма*** 12.1-суреттегі кеңістіктік фигураны құрайтын кубтардың қырлары 1см-ге тең. Берілген кеңістіктік фигураны құрайтын бірлік кубтарды Tinkercad платформасында модельдейді. Фигураны әртүрлі бұрыштан	**Дескриптор:** 1.Фигураны құрайтын бірлік кубтарды анықтайды. 2.Tinkercad-та фигураның моделін құрады. 3.Фигураның көлемін дұрыс табады. 4.Жауабын 3D модель арқылы түсіндіреді. **Жалпы - 4 балл**	Tinkercad платформасы, ДК экраны, оқушылардың жеке смартфондары немесе ноутбуктері, 11-сынып геометрия оқулығы, жұмыс дәптері, слайд.

		қарап, оның көлемін табады.  12.1-сурет		
Сабақтың соңы Ой толғаныс. Рефлексия 7 мин.	**«Еркін микрофон» әдісі.** Мұғалім сабақты қорытындылау мақсатында оқушылардың сабаққа деген көзқарасын, рефлексиясын тыңдайды. ***Мақсаты:*** Оқушы алған білімін саралай білуге дағдыланады. ***Тиімділігі:*** Тақырып бойынша оқушылардың пікірін анықтайды. Жинақталған деректердің құнды болуын қадағалайды. ***Саралау:*** Бұл кезеңде саралаудың ***«Қорытынды»*** тәсілі көрінеді.	Оқушылар бүгінгі сабақтың мақсатына жеткізетін тапсырмалар орындауына қарай, өз түсінгенін, пікірін, өз ойын айту арқылы сабаққа қорытынды жасайды. Оқушылар сабақ соңында мына сұрақтарға жауап береді: 3D модель көлемді түсінуге қалай көмектесті? Кубтың қырлары өзгергенде көлем неге жылдам өзгереді? Қайсысы түсініктірек болды: оқулықтағы сурет пе, әлде Tinkercad-тағы модель ме?	Мұғалім оқушыларды **«Жапондық бағалау»** әдісі арқылы бағалайды. Яғни *«Дұрыс келісемін», «Толықтырамын, басқа көзқарасым бар», «Менің сұрағым бар». Сонымен қатар 1-10 баллдық жүйе бойынша оқушылардың сабаққа қатысу белсенділігі бойынша бағаланады.*	 

Саралау. Сіз қандай тәсілмен көбірек қолдау көрсетпексіз? Сіз қабілетті оқушылардың алдына қандай тапсырмалар қоясыз?	Бағалау. Сіз оқушылардың материалды игеру деңгейін қалай тексеруді жоспарлап отырсыз?	Денсаулық және қауіпсіздік техникасын сақтау
«Диалог және қолдау көрсету», «Тапсырма», «Жіктеу».	«Мадақтау сөзі» әдісі	Денсаулық сақтау технологиялары. Сабақта сергіту жаттығулары мен белсенді жұмыс түрлерін қолданамын. Осы сабақта қолданылатын

Саралау тапсырмаларды іріктеуді, белгілі бір оқушыдан күтілетін нәтижені, оқушыға жеке қолдау көрсетуде, оқу материалы мен ресурстарды оқушылардың жеке қабілеттерін ескере отырып әзірлеуді қамтиды.		Қауіпсіздік техникасы ережелерінің тармақтары орындалады.

Жалпы бағалау

Сабақтың қандай екі аспектісі жақсы өтті? Оқыту туралы да, сабақ беру туралы да ойланыңыз.

1:

2:

Сабақты жақсартуға не жәрдемдесер еді? Оқыту туралы да, сабақ беру туралы да ойланыңыз.

1:

2:

Сабақ кезінде, сынып немесе жекелеген оқушылардың жетістіктері/қиыншылықтары туралы мен нені анықтадым? Келесі сабақтарда неге назар аудару керек?

1:

2:

Қысқамерзімді жоспар
№2-сабақ

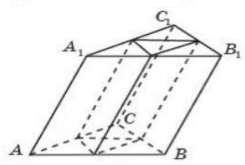
Сабақтың тақырыбы: Призма көлемі

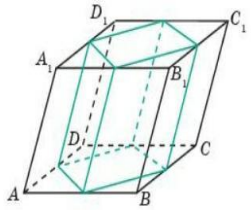
Бөлім:	**Денелер көлемдері**
Педагогтің аты-жөні:	Серікбай Жанэлья Сағынқызы
Күні:	03.04.2026
Сыныбы: 11	Қатысушылар саны: Қатыспағандар саны:
Сабақтың тақырыбы:	Призма көлемі
Оқу бағдарламасына сәйкес оқу мақсаты	11.3.10 - призма көлемін табу формуласын қолдану.
Сабақтың мақсаты:	***Барлық оқушылар үшін:*** Білім алушылар призма ұғымын меңгереді. ***Көпшілік оқушылар үшін:*** Білім алушылар призма көлемін табады. ***Кейбір оқушылар үшін:*** Білім алушылар призма көлемін табу формуласын қолданады.

Сабақтың барысы:

Сабақ кезеңі/Уақыты	Педагогтің іс-әрекеті	Оқушының іс-әрекеті	Бағалау	Ресурстар
Сабақтың басы Қызығушылықты ояту. 7 мин.	**(Ұ).** **Ұйымдастыру кезеңі:** 1.Оқушылармен амандасу, түгендеу. 2.Ынтымақтастық атмосферасын қалыптастыру	1.Оқушылар мұғалім- мен амандасады. 2. Сабаққа дайындалады. **Мақсаты:** Оқушылар бір-біріне тілек білдіреді, тыңдау дағдыларын дамытуға	Оқушылардың белсенділігі үшін *«Жарайсың!», «Керемет!»* деген сақтау сөздерімен бағалайды. **Тиімділігі:** Оқушылар бір-біріне тілек айту арқылы жақындасады, көңіл-күйін көтереді	Оқулық, жұмыс дәптері Кітап, дәптер, қалам суреттері бейнеленген қима қағаздар топтамасы ДК экраны

	1.Мұғалім оқушыларды түгендейді. 2.Амандасады және тілектерін білдіреді.	бағытталады, сондай-ақ барлық оқушылардың қатыстырылуы арқылы сабаққа белсенділігі артады.	және бауырмалдығын оятады. **Саралау:** Бұл жерде саралаудың **«Жіктеу»** тәсілі көрінеді. Оқушылардың оқуға деген қызығушылығын арттыру мақсатында мүмкіндігінше оларға таңдау еркіндігі беріледі.	
Жаңа сабаққа кіріспе	**(Ұ) «Миға шабуыл»** әдісі арқылы өткен тақырыппен жаңа сабақты байланыстыру мақсатында ой қозғау сұрақтарын ұжымдық талқылау. Оқушыларға жалпылама төмендегі сұрақтар және жаттығу түрлері беріледі. **Сұрақтар:** 1.Призманың көлемін қалай табамыз? 2.Призманың көлемін табатын формула қандай? 3.Кеңістіктегі қандай фигуралар тең шамалы деп аталады?	1. Оқушылар сұрақтарға жауап беріп, өзара ұжымдық талқылау жасағаннан кейін мұғалім оқушыларға сабақтың тақырыбы, мақсатымен таныстырады. **Мақсаты:** Жылдам әрі функционалды түрде сыни ойлануды дамыту. **Тиімділігі:** оқушының танымдық дағдысы артады. Сонымен қатар оқушыға сабақтың өмірмен байланысын көрсетеді және сабақтың тақырыбы мен мақсатын анықтауға мүмкіндік береді.	**Саралау:** Бұл жерде саралаудың **«Диалог және қолдау көрсету»** тәсілі көрінеді. Дұрыс мағынада жауап беруге бағыттау мақсатында кейбір оқушыларға ашық сұрақтар, ал кейбір көмек қажет ететін оқушыларға жетелеуші сұрақтар қойылады. **Дескриптор:** 1. Әр оқушы өз ойымен бөліседі. 2. Өзгенің пікірін толықтырады. **Жалпы- 2 балл.**	**Өз ойын дұрыс мағынада білдіріп,** талқылауға белсенділікпен қатысқан оқушыға «Жарайсың!» деген мадақтау сөзімен ынталандыру.
Сабақтың ортасы Мағынаны ашу. 31 мин.	Оқулықтағы жаңа сабақтың мәтінін оқуға тапсырма береді Мұғалім SketchUp Free платформасында үшбұрышты және төртбұрышты призманың моделін көрсетеді.	Оқулықты оқып танысып шығады. Тірек сөздермен танысып, өз дәптерлеріне жазып алады.	Тақырыпқа байланысты интернет көздерін пайдаланып бөліп шығару мүшелерімен танысады.	ДК экраны

	1- тапсырманы орындатады, бақылайды, мысал, үлгі көрсетеді.	***1-тапсырма*** Үшбұрышты призманың көлемі 12см³-ге тең. Табанының төбелері берілген призманың табанының қабырғаларының орталары болатын екінші призманың көлемін табады. Оқушылар призманың 3D моделін құрады, оның табаны мен биіктігін анықтайды, модельді бұрып қарау арқылы дененің кеңістіктік құрылымын талдайды. Кейін призманың көлемін формула арқылы есептейді және 3D модель негізінде шешімін түсіндіреді.  13.4-сурет	**Дескриптор:** 1.Призманың бастапқы көлемін анықтайды. 2.Екінші призманың табаны қалай өзгеретінін түсіндіреді. 3.Екінші призманың көлемін табады. 4.Модель арқылы шешімін дәлелдейді. **Жалпы- 5 балл.**	SketchUp Free платформасы, ДК экраны, оқушылардың жеке смартфондары немесе ноутбуктері, 11-сынып геометрия оқулығы, жұмыс дәптері, слайд.
	2- тапсырманы орындатады, бақылайды, мысал, үлгі көрсетеді.	***2-тапсырма*** Төртбұрыштардың призманың көлемі 10см³-ге тең. Табанының төбелері берілген призманың табанының қабырғаларының орталары болатын екінші призманың көлемін табады.	**Дескриптор:** 1.Есептің мазмұнын түсінеді. 2.Призманың бастапқы және жаңа табанын салыстырады. 3.Екінші призманың көлемін есептейді. 4.3D модель арқылы қорытынды жасайды. **Жалпы-3 балл.**	SketchUp Free платформасы, ДК экраны, оқушылардың жеке смартфондары немесе ноутбуктері, 11-сынып геометрия оқулығы, жұмыс дәптері, слайд.

		 13.5-сурет		
Сабақтың соңы Ой толғаныс. Рефлексия 7 мин.	**«Еркін микрофон» әдісі.** Мұғалім сабақты қорытындылау мақсатында оқушылардың сабаққа деген көзқарасын, рефлексиясын тыңдайды. ***Мақсаты:*** Оқушы алған білімін саралай білуге дағдыланады. ***Тиімділігі:*** Тақырып бойынша оқушылардың пікірін анықтайды. Жинақталған деректердің құнды болуын қадағалайды. ***Саралау:*** Бұл кезеңде саралаудың ***«Қорытынды»*** тәсілі көрінеді.	Оқушылар бүгінгі сабақтың мақсатына жеткізетін тапсырмалар орындауына қарай, өз түсінгенін, пікірін, өз ойын айту арқылы сабаққа қорытынды жасайды. Оқушылар сабақ соңында мына сұрақтарға жауап береді: Призманың көлемін табуда 3D модель қалай көмектесті? Модельді бұрып қарау есептің шартын түсінуге әсер етті ме? Призма көлемін табу формуласын қолдануда қандай қиындық болды?	Мұғалім оқушыларды **«Жапондық бағалау»** әдісі арқылы бағалайды. Яғни *«Дұрыс келісемін», «Толықтырамын, басқа көзқарасым бар», «Менің сұрағым бар». Сонымен қатар 1-10 баллдық жүйе бойынша оқушылардың сабаққа қатысу белсенділігі бойынша бағаланады.*	 

Саралау. Сіз қандай тәсілмен көбірек қолдау көрсетпексіз? Сіз қабілетті оқушылардың алдына қандай тапсырмалар қоясыз?	Бағалау. Сіз оқушылардың материалды игеру деңгейін қалай тексеруді жоспарлап отырсыз?	Денсаулық және қауіпсіздік техникасын сақтау
«Диалог және қолдау көрсету», «Тапсырма», «Жіктеу». Саралау тапсырмаларды іріктеуді, белгілі бір оқушыдан күтілетін нәтижені, оқушыға жеке қолдау көрсетуде, оқу материалы мен ресурстарды оқушылардың жеке қабілеттерін ескере отырып әзірлеуді қамтиды.	«Мадақтау сөзі» әдісі	Денсаулық сақтау технологиялары. Сабақта сергіту жаттығулары мен белсенді жұмыс түрлерін қолданамын. Осы сабақта қолданылатын Қауіпсіздік техникасы ережелерінің тармақтары орындалады.
Жалпы бағалау Сабақтың қандай екі аспектісі жақсы өтті? Оқыту туралы да, сабақ беру туралы да ойланыңыз.		

1: 2: Сабақты жақсартуға не жәрдемдесер еді? Оқыту туралы да, сабақ беру туралы да ойланыңыз. 1: 2: Сабақ кезінде, сынып немесе жекелеген оқушылардың жетістіктері/қиыншылықтары туралы мен нені анықтадым? Келесі сабақтарда неге назар аудару керек? 1: 2:

Қысқамерзімді жоспар
№3-сабақ

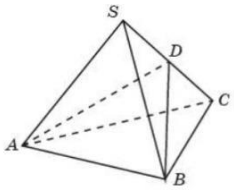
Сабақтың тақырыбы: Пирамида және қиық пирамида көлемдері

Бөлім:	**Денелер көлемдері**
Педагогтің аты-жөні:	Серікбай Жанэлья Сағынқызы
Күні:	10.04.2026
Сыныбы: 11	Қатысушылар саны: Қатыспағандар саны:
Сабақтың тақырыбы:	Пирамида және қиық пирамида көлемдері
Оқу бағдарламасына сәйкес оқу мақсаты	11.3.11 - пирамида және қиық пирамида көлемдерін табу формулаларын қолдану.
Сабақтың мақсаты:	***Барлық оқушылар үшін:*** Білім алушылар бірлесіп, пирамида және қиық пирамида көлемдерімен танысады. ***Көпшілік оқушылар үшін:*** Білім алушылар пирамида және қиық пирамида көлемдерін табады. ***Кейбір оқушылар үшін:*** Білім алушылар пирамида және қиық пирамида көлемдерін табу формулаларын қолданады.

Сабақтың барысы:

Сабақ кезеңі/Уақыты	Педагогтің іс-әрекеті	Оқушының іс-әрекеті	Бағалау	Ресурстар
Сабақтың басы Қызығушылықты ояту. 7 мин.	**(Ұ).** **Ұйымдастыру кезеңі:** 1.Оқушылармен амандасу, түгендеу. 2.Ынтымақтастық атмосферасын қалыптастыру 1.Мұғалім оқушыларды түгендейді. 2.Амандасады және тілектерін білдіреді.	1. Оқушылар сұрақтарға жауап беріп, өзара ұжымдық талқылау жасағаннан кейін мұғалім оқушыларға сабақтың тақырыбы, мақсатымен таныстырады. **Мақсаты:** Жылдам әрі функционалды түрде сыни ойлануды дамыту. **Тиімділігі:** оқушының танымдық дағдысы артады. Сонымен қатар оқушыға	Оқушылардың белсенділігі үшін «*Жарайсың!», «Керемет!»* деген сақтау сөздерімен бағалайды. **Тиімділігі:** Оқушылар бір-біріне тілек айту арқылы жақындасады, көңіл-күйін көтереді және бауырмалдығын оятады. **Саралау:** Бұл жерде саралаудың **«Жіктеу»** тәсілі көрінеді. Оқушылардың оқуға деген қызығушылығын арттыру мақсатында мүмкіндігінше оларға таңдау еркіндігі беріледі.	Оқулық, жұмыс дәптері Кітап, дәптер, қалам суреттері бейнеленген қима қағаздар топтамасы ДК экраны

		сабақтың өмірмен байланысын көрсетеді және сабақтың тақырыбы мен мақсатын анықтауға мүмкіндік береді.		
Жаңа сабаққа кіріспе	**(Ұ) «Миға шабуыл»** әдісі арқылы өткен тақырыппен жаңа сабақты байланыстыру мақсатында ой қозғау сұрақтарын ұжымдық талқылау. Оқушыларға жалпылама төмендегі сұрақтар және жаттығу түрлері беріледі. **Сұрақтар:** 1.Пирамида дегеніміз не? Қиық пирамида деген не? 2. Пирамида және қиық пирамида көлемдерін қалай табуға болады? 3.Ұқсас фигуралардың көлемдері өзара қалай байланысқан?	1. Әр оқушы өз ойымен бөліседі. 2. Өзгенің пікірін толықтырады. **Мақсаты:** Оқушылар сұрақтарға жауап беріп, өзара ұжымдық талқылау жасағаннан кейін мұғалім оқушыларға сабақтың тақырыбы, мақсатымен таныстырады.	**Саралау:** Бұл жерде саралаудың **«Диалог және қолдау көрсету»** тәсілі көрінеді. Дұрыс мағынада жауап беруге бағыттау мақсатында кейбір оқушыларға ашық сұрақтар, ал кейбір көмек қажет ететін оқушыларға жетелеуші сұрақтар қойылады. **Дескриптор:** 1. Әр оқушы өз ойымен бөліседі. 2. Өзгенің пікірін толықтырады. **Жалпы- 2 балл.**	**Өз ойын дұрыс мағынада білдіріп,** талқылауға белсенділікпен қатысқан оқушыға «Жарайсың!» деген мадақтау сөзімен ынталандыру.
Сабақтың ортасы Мағынаны ашу. 31 мин.	Оқулықтағы жаңа сабақтың мәтінін оқуға тапсырма береді	Оқулықты оқып танысып шығады. Тірек сөздермен танысып, өз дәптерлеріне жазып алады. Мұғалім Tinkercad платформасында пирамида мен қиық пирамиданың 3D моделін көрсетеді.	Тақырыпқа байланысты интернет көздерін пайдаланып бөліп шығару мүшелерімен танысады.	Слайд Кітап

	1- тапсырманы орындатады, бақылайды, мысал, үлгі көрсетеді.	***1-тапсырма*** Үшбұрышты пирамиданың табанының бір қабырғасы және оған қарсы жатқан қырының ортасы арқылы жазықтық өтеді. Бұл жазықтық пирамиданың көлемін қандай қатынаста бөледі? Tinkercad платформасында модель құрып, табаны мен биіктігін көрсет.  15.7-сурет	**Дескриптор:** 1.Пирамиданың табан ауданын табады. 2.Пирамиданың көлем формуласын дұрыс қолданады. 3.Көлемін есептейді. 4.3D модель арқылы табан мен биіктікті көрсетеді. **Жалпы- 4 балл.**	Tinkercad платформасы, ДК экраны, интерактивті тақта, оқушылардың смартфондары немесе ноутбуктері, 11-сынып геометрия оқулығы, жұмыс дәптері, слайд.
	2- тапсырманы орындатады, бақылайды, мысал, үлгі көрсетеді.	***2-тапсырма*** Нұр-Сұлтан қаласындағы Бейбітшілік пен Келісім сарайы дұрыс төртбұрышты пирамида пішіндес болып табылады. Оның биіктігі мен табанының қабырғасы 62м-ге тең. Пирамиданың көлемін табады. Модель арқылы екі табанның және биіктіктің орналасуын түсіндіреді.  15.11-сурет	**Дескриптор:** 1.Қиық пирамиданың екі табанын анықтайды. 2.Формуладағы S_1, S_2 және h мәндерін дұрыс қояды. 3.Қиық пирамиданың көлемін есептейді. 4.3D модель арқылы шешімін түсіндіреді. **Жалпы- 5 балл.**	Tinkercad платформасы, ДК экраны, интерактивті тақта, оқушылардың смартфондары немесе ноутбуктері, 11-сынып геометрия оқулығы, жұмыс дәптері, слайд.

Сабақтың соңы Ой толғаныс. Рефлексия 7 мин.	**«Еркін микрофон» әдісі.** Мұғалім сабақты қорытындылау мақсатында оқушылардың сабаққа деген көзқарасын, рефлексиясын тыңдайды. ***Мақсаты:*** Оқушы алған білімін саралай білуге дағдыланады. ***Тиімділігі:*** Тақырып бойынша оқушылардың пікірін анықтайды. Жинақталған деректердің құнды болуын қадағалайды. ***Саралау:*** Бұл кезеңде саралаудың ***«Қорытынды»*** тәсілі көрінеді.	Оқушылар бүгінгі сабақтың мақсатына жеткізетін тапсырмалар орындауына қарай, өз түсінгенін, пікірін, өз ойын айту арқылы сабаққа қорытынды жасайды. Оқушылар сабақ соңында мына сұрақтарға жауап береді: 3D модель пирамида мен қиық пирамиданың айырмашылығын түсінуге қалай көмектесті? Пирамиданың көлемі неге призманың көлемінің үштен бір бөлігі ретінде қарастырылады? Қиық пирамидада екі табанның болуы көлемді есептеуге қалай әсер етеді? Модельді бұрып қарау есептің шартын түсінуге көмектесті ме	Мұғалім оқушыларды **«Жапондық бағалау»** әдісі арқылы бағалайды. Яғни *«Дұрыс келісемін», «Толықтырамын, басқа көзқарасым бар», «Менің сұрағым бар». Сонымен қатар 1-10 баллдық жүйе бойынша оқушылардың сабаққа қатысу белсенділігі бойынша бағаланады.*	 

Саралау. Сіз қандай тәсілмен көбірек қолдау көрсетпексіз? Сіз қабілетті оқушылардың алдына қандай тапсырмалар қоясыз?	Бағалау. Сіз оқушылардың материалды игеру деңгейін қалай тексеруді жоспарлап отырсыз?	Денсаулық және қауіпсіздік техникасын сақтау
«Диалог және қолдау көрсету», «Тапсырма», «Жіктеу». Саралау тапсырмаларды іріктеуді, белгілі бір оқушыдан күтілетін нәтижені, оқушыға жеке қолдау көрсетуде, оқу материалы мен ресурстарды оқушылардың жеке қабілеттерін ескере отырып әзірлеуді қамтиды.	«Мадақтау сөзі» әдісі	Денсаулық сақтау технологиялары. Сабақта сергіту жаттығулары мен белсенді жұмыс түрлерін қолданамын. Осы сабақта қолданылатын Қауіпсіздік техникасы ережелерінің тармақтары орындалады.

Жалпы бағалау

Сабақтың қандай екі аспектісі жақсы өтті? Оқыту туралы да, сабақ беру туралы да ойланыңыз.

1:

2:

Сабақты жақсартуға не жәрдемдесер еді? Оқыту туралы да, сабақ беру туралы да ойланыңыз.
1:
2:
Сабақ кезінде, сынып немесе жекелеген оқушылардың жетістіктері/қиыншылықтары туралы мен нені анықтадым? Келесі сабақтарда неге назар аудару керек?
1:
2:

Appendix D. Sequence of Data Collection Activities

Phase	Timing	Activity	Groups
1	Before Lesson 1	Pre-test (PSVT)	Both groups
2	Before Lesson 1	Baseline geometry grades collected from school records	Both groups
3	Lessons 1–8 (7 weeks)	Instructional period: 3D modeling (EG) / standard instruction (CG)	EG and CG separately
4	After Lesson 8	Post-test (PSVT)	Both groups
5	After Lesson 8	Post-intervention geometry grades collected from school records	Both groups
6	After Lesson 8	Student questionnaire on attitudes towards 3D modeling (6 items)	EG only

Note. EG = experimental group; CG = control group.

Appendix E. PSVT test (pre-test and post-test)

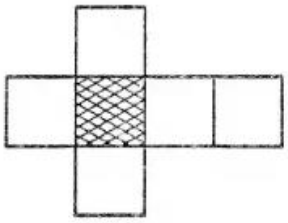
11-сынып
Геометрия

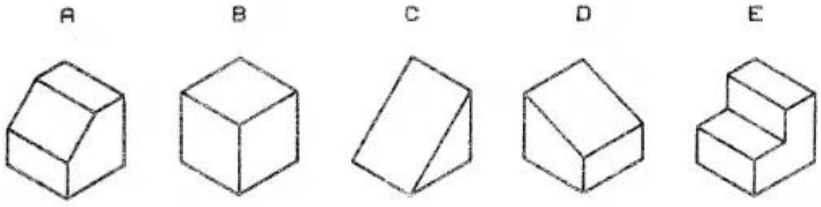
1-бөлім: Жазбалар

Берілгені – бір жазба және бес үшөлшемді дене.
Жазба үшөлшемді дененің ішкі беттерін көрсетеді.
Жазбадағы боялған бөлік үшөлшемді дененің төменгі жағын (табаны) білдіреді.
Сізге керегі:

• жазбаны ойша бүктеп, оның қандай үшөлшемді денеге айналатынын елестету;
• берілген бес дененің (A, B, C, D немесе E) ішінен бүктелген жазбаға сәйкес келетінін таңдау.

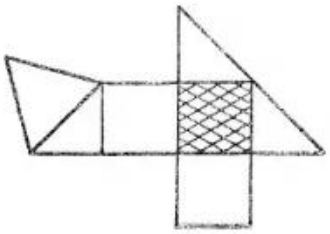
Мысалы:

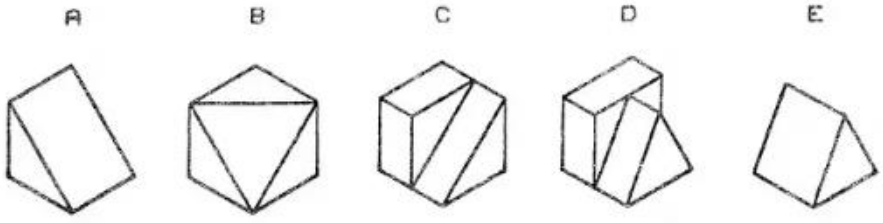




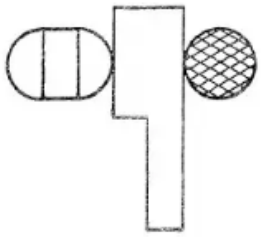
Дұрыс жауабы: B

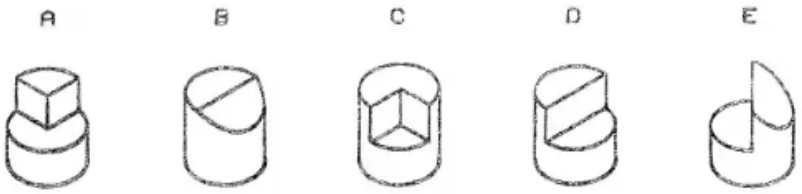
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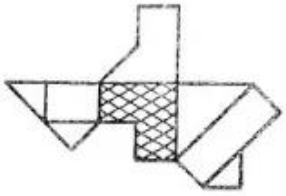


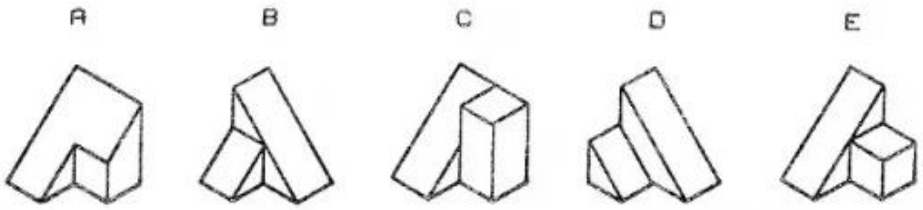
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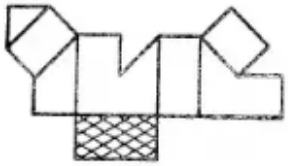


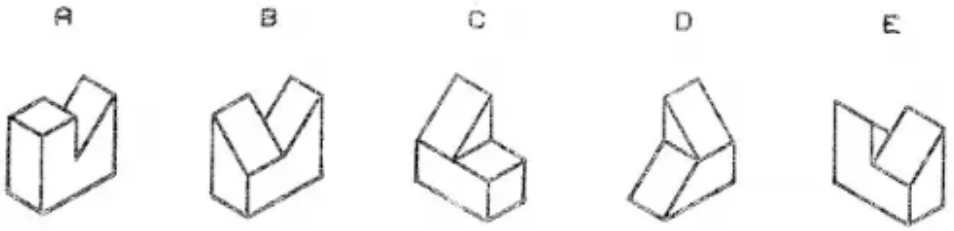
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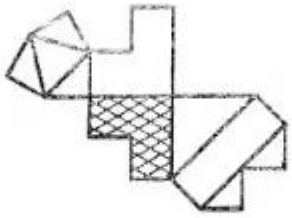


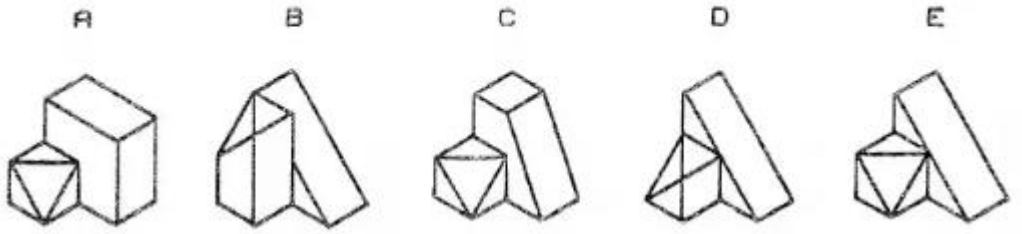
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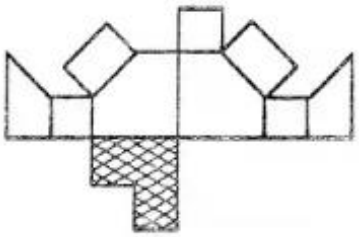


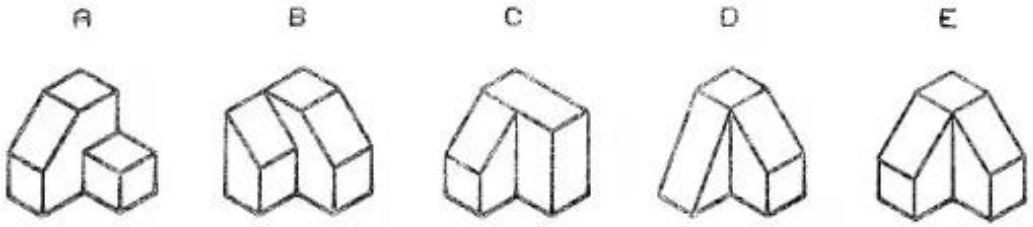
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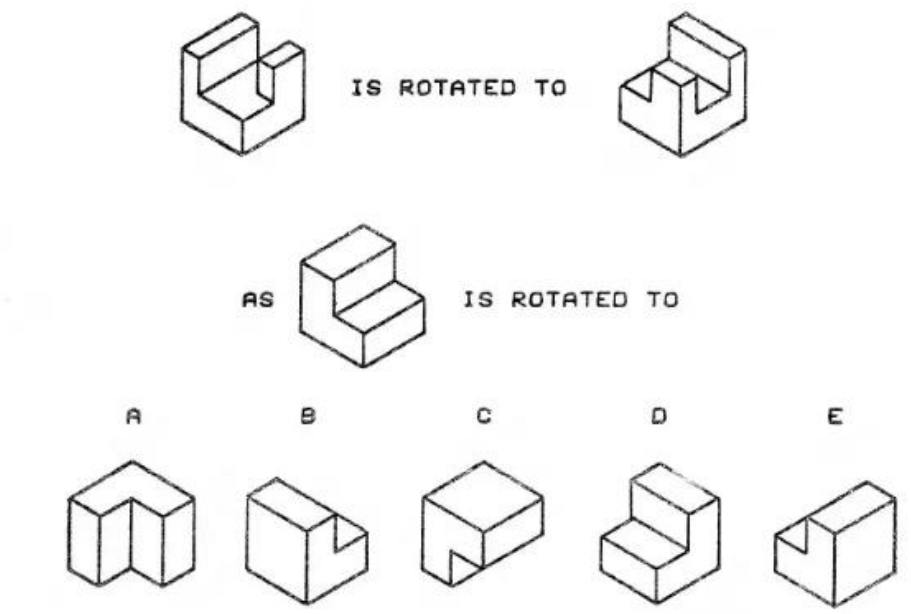
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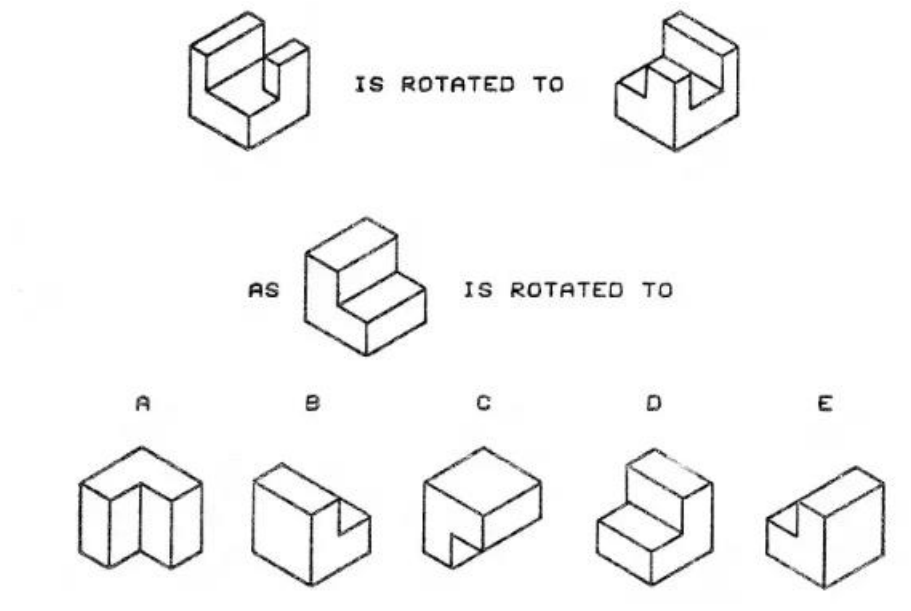
2-бөлім: Айналдырулар

Сізге керегі:
1. Сұрақтың жоғарғы қатарында берілген дененің қалай айналдырылғанын мұқият зерттеу;
2. Сұрақтың ортаңғы қатарында көрсетілген дене дәл сол тәсілмен айналдырылғанда қалай көрінетінін ойша елестету;
3. Төменгі қатарда берілген бес суреттің (A, B, C, D немесе E) ішінен дұрыс бағытта айналдырылған денеге сәйкес келетінін таңдау.
Мысалы:



Дұрыс жауабы: D

1.



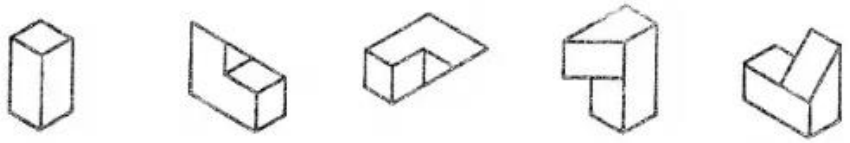
2.


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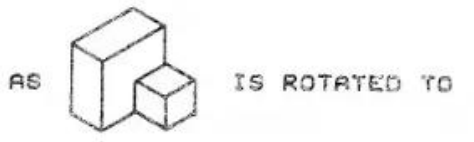

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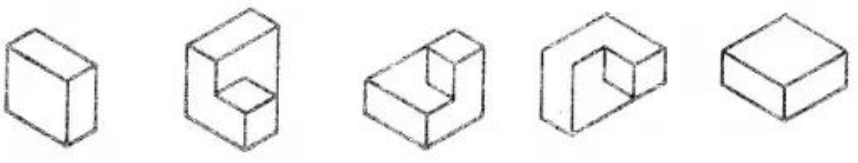
A
B
C
D
E



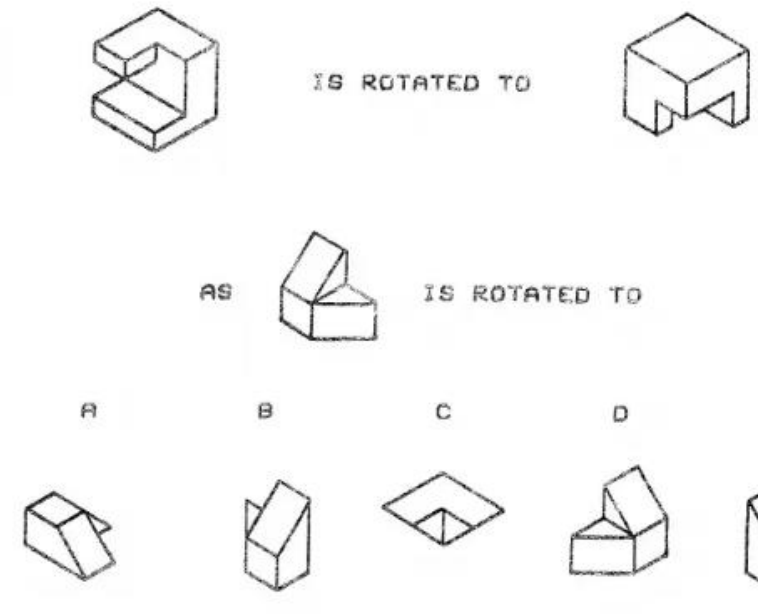

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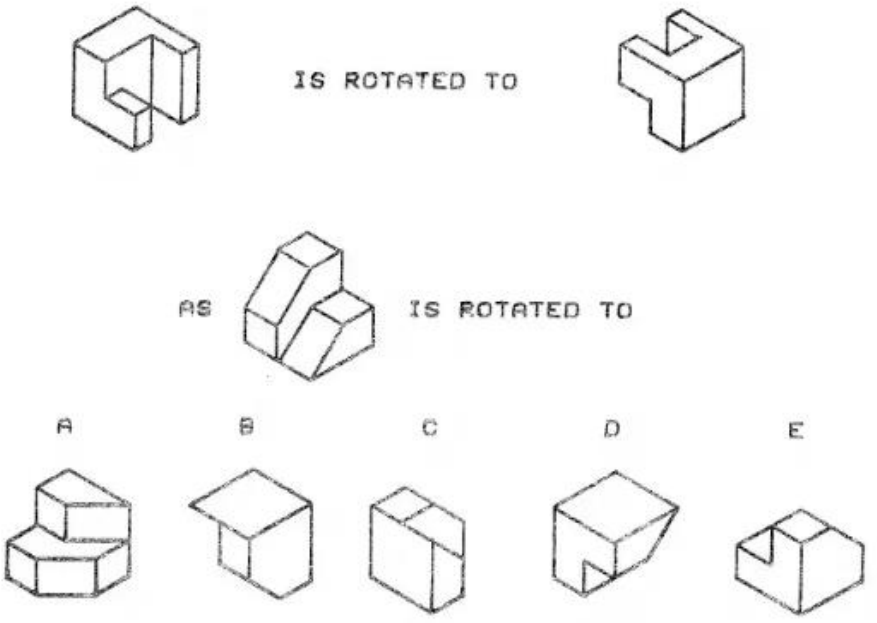
A
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4.


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3-бөлім: Көріністер

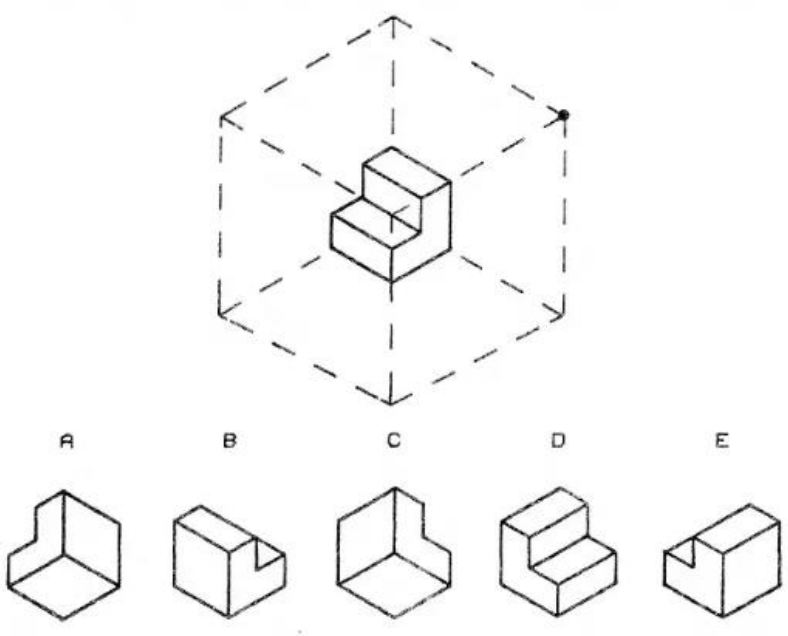
Объект “шыны қораптың” ортасында орналасқан және бес сурет берілген, олар сол объектінің әртүрлі көріністерін көрсетеді.

“Шыны қораптың” жоғарғы оң жақ бұрышындағы қара нүкте қажетті қарау бағытын белгілейді.

Сізге керегі:

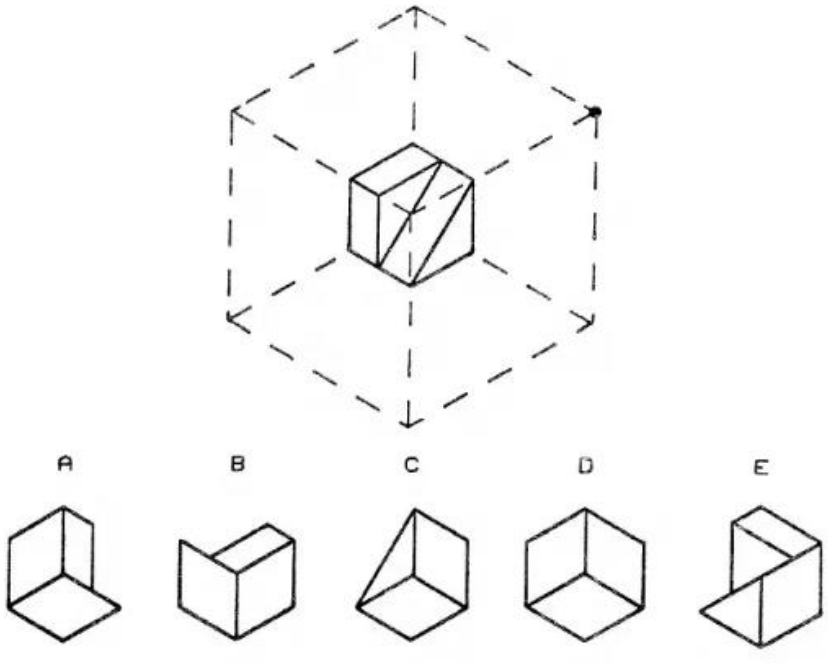
1. Өзіңізді “шыны қораптың” айналасында қозғалып жүргендей елестетіп, қара нүкте сіз бен объектінің арасында тікелей орналасқанға дейін қозғалту;
2. Осы қарау позициясынан объектінің қалай көрінетінін ойша елестету;
3. Бес суреттен (A, B, C, D немесе E) сол қарау позициясынан көрінетін объектіге сәйкес келетінін таңдау.

Мысалы:

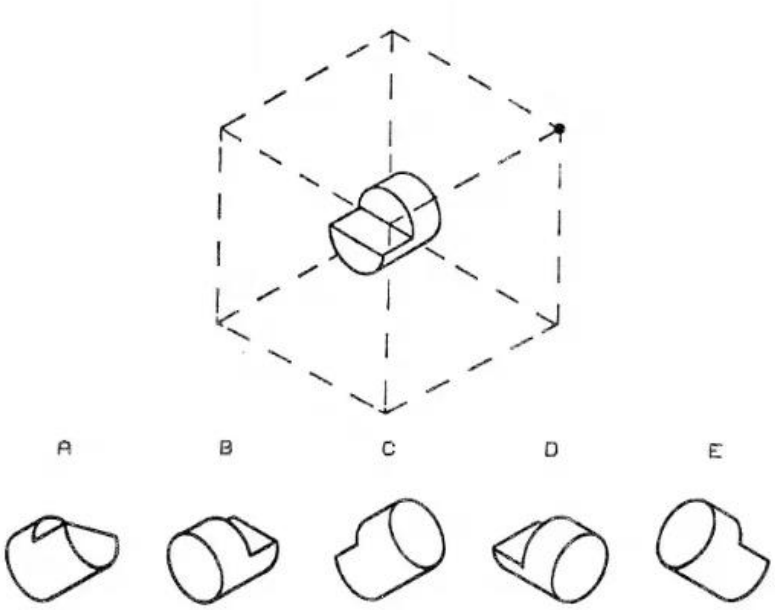


Дұрыс жауабы: E

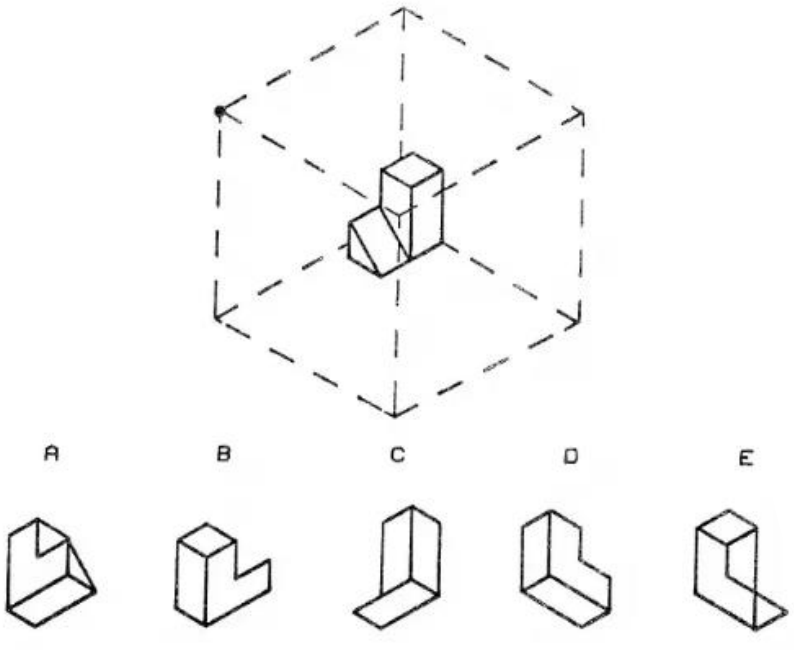
1.



2.

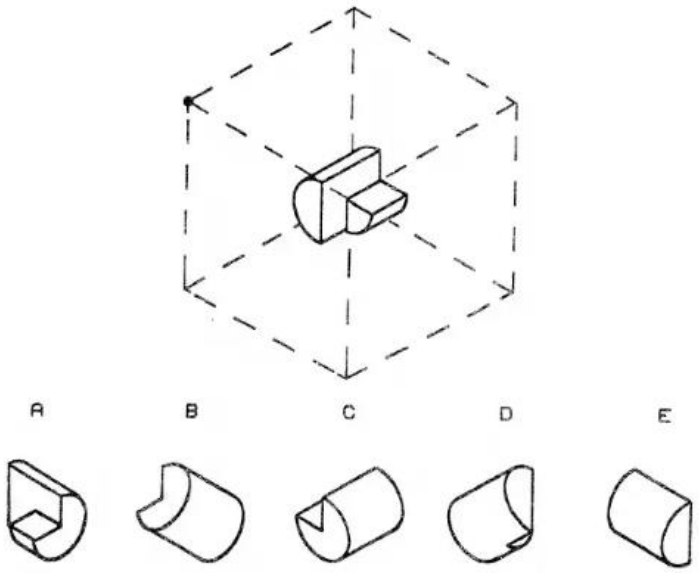


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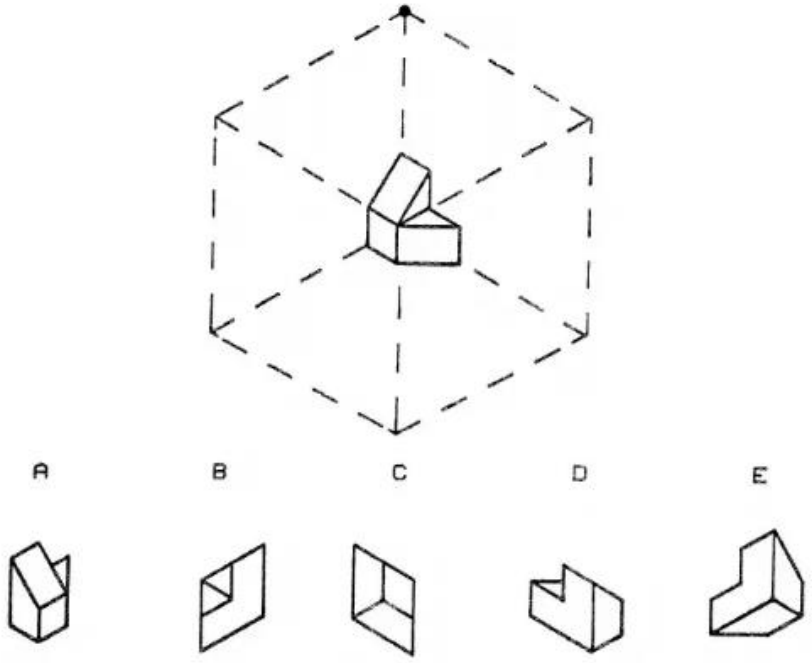


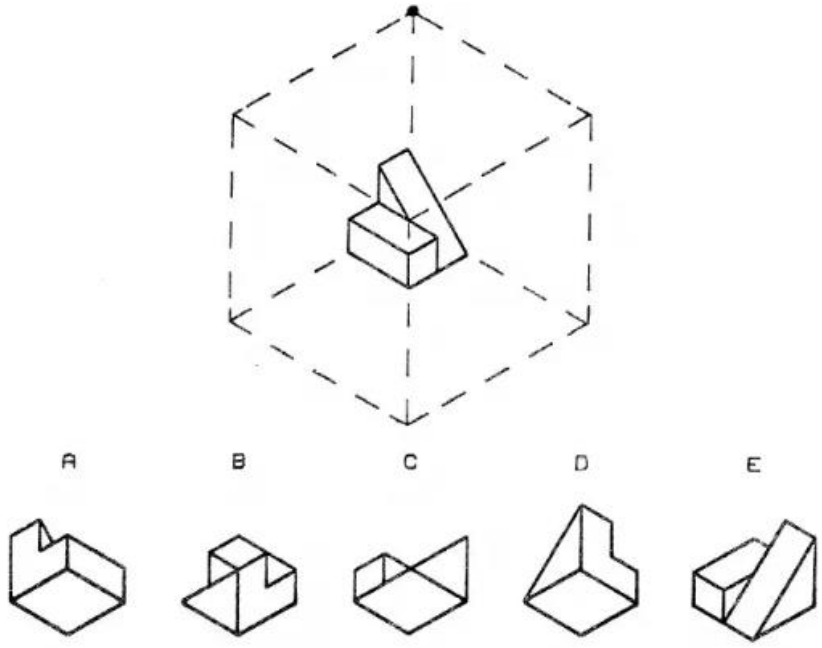
4.

5.



6.




A
B
C
D
E

Appendix F. Questionnaire

Анкета: 3D-моделдеуді қолдану арқылы геометрияны түсіну

№	Мәлімдеме	Толығымен келісемін	Келісемін	Сенімді емеспін	Келіспеймін	Мүлдем келіспеймін
1	3D-моделдеуді бақылау және оны қолдану арқылы жұмыс істеу маған ұнайды.					
2	3D-моделдеуді қолданатын сабақтар – менің ұнайтын сабағым.					
3	3D-моделдеу сабақтары менің ойлау қабілетіме және когнитивтік дамуыма көмектеседі.					
4	Мен 3D-моделдеу туралы көбірек білгім келеді.					
5	Мен 3D-моделдеуді үйренудің болашақта маған артықшылық беретініне сенемін.					
6	Мен 3D-моделдеудің болашақта маңыздырақ болатынына сенемін.					

Appendix G. Q-Q Plots for Normality Assessment of PSVT Scores and Geometry Grades

