SULEYMAN DEMIREL UNIVERSITY

FACULTY OF ENGENEERING

DEPARTMENT OF "MATHEMATICS AND SCIENCES"

Bistatistics of permutation coding

COURSEWORK

6M060100-MATHEMATICS

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CONTENT

ABSTRACT

This writing showes that bistatistics $\left(l_{\min(v)}, r_{\max(v)}\right) \approx \left(l_{\max(v)}, r_{\min(v)}\right)$ and $\left(l_{\max(v)}, r_{\max(v)}\right) \approx \left(l_{\min(v)}, r_{\min(v)}\right)$ are equal distributed over S_2, S_3, S_4 and S_5.

Object of the research: as we know in our days in science desinv and desmaj are equaldistributed. It's one of the theorems of Eiler-MacMahon's. it's very famous theorem over the world. There is unknown another equaldisributed theorems like this.

Topicality of the research topic: to show that codes are equidistributed over a set of permutations of S_2, S_3, S_4 and S_5 $\left(l_{\min(v)}, r_{\max(v)}\right) \approx \left(l_{\max(v)}, r_{\min(v)}\right)$ and $\left(l_{\max(v)}, r_{\max(v)}\right) \approx \left(l_{\min(v)}, r_{\min(v)}\right)$ statistics.

The aim and problems of the research: prove given general theorems over S_2, S_3, S_4 and S_5.

Chronological frame of the research: to show that $\left(l_{\min(v)}, r_{\max(v)}\right) \approx \left(l_{\max(v)}, r_{\min(v)}\right)$ and $\left(l_{\max(v)}, r_{\max(v)}\right) \approx \left(l_{\min(v)}, r_{\min(v)}\right)$ *are equaldistributed biostatistics.*

Methodological basis of the research work: the novelty of the work to show that $\left(l_{\min(v)}, r_{\max(v)}\right) \approx \left(l_{\max(v)}, r_{\min(v)}\right)$ and $\left(l_{\max(v)}, r_{\max(v)}\right) \approx \left(l_{\min(v)}, r_{\min(v)}\right)$ *are equaldistributed biostatistics.*

Structure of dissertation: Introduction, head section, conclusion, reference.

Scientific results of the research: Showen that Statistica I,II,III and IV are equally distributed.

АҢДАТПА

Жұмыста $(l_{\min(v)}, r_{\max(v)}) \approx (l_{\max(v)}, r_{\min(v)})$ және $(l_{\max(v)}, r_{\max(v)}) \approx (l_{\min(v)}, r_{\min(v)})$ бистатистикалары $\boldsymbol{S_2}, \boldsymbol{S_3}, \boldsymbol{S_4}$ және $\boldsymbol{S_5}$ жиындары үстінде тең үлестірілгенін көрсетілді.

Тақырыптың өзектілігі: Осы уақытқа дейін ғылымда *desinv* және *desmaj* статистикаларының тең үлестірілгені туралы нәтиже белгілі, бұл белгілі Эйлер-Махониан теоремасы. Ғылымда Эйлер-Махониан теоремасына пара-пар теорема іздеу, яғни, басқа да тең үлестірілген статистикалардың бар немесе жоқтығы, бар болса олардың табиғаты қандай, деген сурақтардың жауабы өзекті проблема болып отыр.

Зерттеу жұмысының мақсаты мен міндеттері: Бұл магистрлік диссертациялық жұмыстың мақсаты, мына $(l_{\min(v)}, r_{\max(v)}) \approx (l_{\max(v)}, r_{\min(v)})$ және $(l_{\max(v)}, r_{\max(v)}) \approx (l_{\min(v)}, r_{\min(v)})$ бистатистикаларын $\boldsymbol{S_2}, \boldsymbol{S_3}, \boldsymbol{S_4}$ және $\boldsymbol{S_5}$ жиындары үстінде тең үлестірілгенін көрсету.

Зерттеу жұмысының тақырыптық хронологиялық шеңбері:

$(l_{\min(v)}, r_{\max(v)}) \approx (l_{\max(v)}, r_{\min(v)})$ және $(l_{\max(v)}, r_{\max(v)}) \approx (l_{\min(v)}, r_{\min(v)})$ бистатистикаларының тең үлестірілгенін көрсету.

Жаңалығы: $(l_{\min(v)}, r_{\max(v)}) \approx (l_{\max(v)}, r_{\min(v)})$ және $(l_{\max(v)}, r_{\max(v)}) \approx (l_{\min(v)}, r_{\min(v)})$ бистатистикаларының тең үлестірілгенін көрсету жұмыстың жаңалығы болып табылады.

Теориялық маңыздылығы: Жаңа тең үлестірілген бистатистикалардың табылуы басқа тұжырымдарды дәлелдеуге пайдаланбақ.

Диссертацияның құрылымы: Кіріпе, негізгі бөлім, қолданылған әдебиеттер тізімі және де қосымшалардан тұрады.

Зерттеудің ғылыми нәтижесі: Жұмыста берілген төрт статистика тең үлестірілген екені көрсетілген.

INTRODUCTION

In mathematics, a permutation group is a group G whose elements are permutations of a given set M, and whose group operation is the composition of permutations in G (which are thought of as bijective functions from t he set M to itself); the relationship is often written as (G,M). Note that the group of all permutations of a set is the symmetric group; the term permutation group is usually restricted to mean a subgroup of the symmetric group. The symmetric group of n elements is denoted by S_n; if M is any finite or infinite set, then the group of all permutations of M is often written as Sym(M).

The application of a permutation group to the elements being permuted is called its group action; it has applications in both the study of symmetries, combinatorics and many other branches of mathematics, physics and chemistry [1].

Closure properties

As a subgroup of a symmetric group, all that is necessary for a permutation group to satisfy the group axioms is that it contain the identity permutation, the inverse permutation of each permutation it contains, and be closed under composition of its permutations. A general property of finite groups implies that a finite nonempty subset of a symmetric group is again a group if and only if it is closed under the group operation.

Read more: http://www.answers.com/topic/permutation-group#ixzz2W1DDxKal
Read more: http://www.answers.com/topic/permutation-group#ixzz2W0zMZfbL

This links describes the codes and permutation groups. From any linear code, we construct a permutation group, whose cycle index is essentially the weight enumerator of the code. If we start instead with a Z_4-linear code, the cycle index of the group is the symmetrised weight enumerator of the code. Essentially, we "inflate" each coordinate of the code into a copy of the alphabet.
A permutationis a rearrangement of objects. Here we will only consider permutations of afinite number of objects, and since the object names don't really matter, we will often simply consider permutations of the numbers 1;2;3; :::; n [2].

Of course we'll learn about permutations first by looking at permutations of small numbers.

There are plenty of other examples of permutations, many of which are extremely important and practical. For example, when you have a list of items to sort, either by hand or with a computer program, you are essentially faced with the problem of finding a permutation of the objects that will put them in order after the permutation.

If we consider permutations of n objects, there are n! of them. To see this, first consider where object number 1 winds up. There are n possibilities for that. After the fate of object 1 is determined, there are only n-1 possible fates for object number 2. Thus there are n.(n-1).(n-2)...3.2.1 = n! permutations of a set of n objects.

For example, if we consider all possible rearrangements of the set 1;2;3 ,there are 3! = 3.2.1 = 6.

1	1→1	1→2	1→3
2	1→2	1→1	1→3
3	1→3	1→2	1→1
4	1→1	1→3	1→2
5	1→2	1→3	1→1
6	1→3	1→1	1→2

Table 1. Table of permutations of 3 objects

A good way to think of permutations is this (using permutations of three objects as an example): Imagine that there are three boxes labeled “1”, “2”, and “3”, and initially, each contains a ball labeled with the same number—box 1 contains ball 1, and so on. A permutation is a rearrangement of the balls but in such a way that when you’re done there is still only a single ball in each box.

In the table above, the notation a → b indicates that whatever was in box a moves to the box labeled b, so to apply permutation number 3 above means to take whatever ball is in box 1 and move it to box 3, to leave the contents of box 2 alone, and to take the ball from box 3 and put it into box 1. In other words, permutation number 3 above tells us to swap the contents of boxes 1 and 3. [3]

Here are the six possible permutations of three items (1);(12);(13);(23);(123);(132):

What happens if we begin with ball 1 in box 1, ball 2 in box 2, and ball 3 in box 3, and then we apply (12) followed by (13)?

A good way to think about this is to follow the contents of the boxes one at a time. For example, ball 1 begins in box 1, but after (12) it has moved to box 2. The second permutation, (13), does not move the contents of box 2, so after both permutations have been applied, ball 1 will have moved to box 2. So the final result will look like this:

(12...

where we're not sure what comes next. We don't know if 2 will go back to one and the cycle will close, or whether it will continue to another box. So since the fate of 2 is in question, let's see where it goes.

The first permutation, (12) moves box 2 to box 1, and then (13) will move box 1 to box 3, so now we know the combination of permutations looks like this:

(123...

Since there are only three objects, we know that 3 will go back to 1 and close the cycle, but (especially when you're beginning),it's good to trace each ball, including ball 3 in this case.

The first permutation, (12), does not move the contents of box 3, but the second, (13) moves it to box 1, so the combination of (12) followed by (13) is equivalent to the single permutation (123) [4].

Combining Permutations

Of course it's nice to have a method to write down a permutation, but things begin to get interesting when we combine them. If you twist one face of Rubik's Cube and then twist another one, each twist jumbles the faces, and the combination of two twists usually causes a jumbling that is more complicated than either of the two individual twists.

Combining permutations as above is written just like a multiplication in algebra, and we can write our result as

follows1:

$$(12)(13) = (123):$$

Beware, however. This is not the same as multiplication that you're used to for real numbers. By doing the same analysis as above, convince yourself that:

$(13)(12) = (132)$ 6= $(123) = (12)(13)$:

In other words, the order of multiplication makes a difference. If P1 and P2 are two different permutations, it may not

be true that P1.P2 = P2.P1. Multiplication of permutations is not commutative.

Traditionally, permutation groups arise as automorphism groups of algebraic or combinatorial structures. The procedure here will be a bit different: the groups will be built from the algebraic structure of codes, and matroids will arise from the fixed point structure of permutation groups.

The set of all permutations of a set W is called the symmetric group on W. Usually we take W to be the set 1; : : : ;n, and denote the symmetric group by Sn, for some positive integer n. The order of Sn is n!.

We also use the convention that permutations act on the right, so that the image of a under the permutation g is denoted by ag. Thus, the result of applying the permutation g followed by h is written gh, and we have a(gh) = (ag)h.

As is well known, any permutation can be written as a product of disjoint cycles: we call this the cycle decomposition. For example, the permutation of 1; : : : ;5 which maps 1 to 4, 2 to 5, 3 to 1, 4 to 3, and 5 to 2 has cycle decomposition

(143)(25). The cycle decomposition is unique up to writing the cycles in a different order and starting them at different points: for example,

$$(143)(25) = (52)(314):$$

If we represent the permutation g as a function digraph, with edges (a;ag) for all

a 2 W, the digraph has in-degree and out-degree 1 and so is a disjoint union of cycles; this is precisely the cycle decomposition.

The generalisation to permutation groups is the orbit decomposition, which we now discuss.

A permutation group G on a set W is a subgroup of the symmetric group on W; that is, it is a set of permutations closed under composition and inversion and containing the identity permutation.

Let G be a permutation group on W. Define a relation ~G on W by the rule that a ~G b if there exists g∈G with ag = b. It is easy to see that ~G is an equivalence relation; the reflexive, symmetric and transitive laws follow from the identity, inverse, and composition properties of G. The equivalence classes of ~G are called the orbits of G,, and G is said to be transitive if there is a single orbit, intransitive otherwise. Note that G is transitive if and only if, for all a;b ∈ W, there exists g∈G which maps a to b [6}.

Definition : In a permutation S_n = ... n_n, t is a *left-to-right maximum* (resp., *left-to-right minimum);* for 1 < j < *i.* Also, t is a *right-to-left maximum* (resp., *right-to-left minimum)* if for i < j < n.

Below defined the permutation statistics appearing in the thesis.

Statistic	Description
lmax	# of left-to-right maxima
lmin	# of left-to-right minima
rmax	# of right-to-left maxima
rmin	# of right-to-left minima

Table 2. Most used abbreviations

How many permutations are there on a group of n objects? Since there are n possible choices for the first position and for each of these there are $n-1$ choices for the second position and for each of these $n(n-1)$ ways of choosing the first two there are $n-2$ ways of choosing the object for the third position etc. it turns out that there are $n!$ permutations on n objects. The set of all permutations on n objects is called the symmetric group on n objects and is denoted by S_n. Since the factorial grows rather quickly S_n can be very large when n is of only moderate size. For example S_{10} contains $10! = 3{,}628{,}800$ elements and S_{60} contains more elements than the number of atoms in the universe.

Let's look at S_1. Its only element is the permutation

$$\begin{vmatrix} 1 \\ 1 \end{vmatrix}$$

With only one element there is only one place for it to go.

With two elements there are two S_2 possible permutations

$$\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix}$$

With three elements things begin to become a bit more interesting. S_3 has $3! = 6$ elements. They are

$$|1\,2\,3| \qquad |2\,3\,1| \qquad |3\,1\,2|$$

$$|1\,3\,2| \qquad |3\,2\,1| \qquad |2\,1\,3|$$

So the symmetric group S_n on a finite set of n symbols is the group whose elements are all the permutations of the $n$symbols , and whose group operation is the composition of such permutations, which are treated as bijective functions from the set of symbols to itself. Since there are $n!$ (n factorial) possible permutations of a set of n symbols, it follows that the order (the number of elements) of the symmetric group S_n is $n!$.

Definition 1.1. *A permutation of a finite set S is a bijection $\sigma : S \rightarrow S$.*

Lemma 1.1. *There are exactly n! permutations of an n-element set.*

Proof. *For an n-element set $S = \{x1, \ldots, xn\}$, we can construct a permutation σ on S as follows:*

(1) Assign one of the n elements of S to $\sigma(x1)$.

(2) Assign one of the $n - 1$ elements of $S - \{\sigma(x1)\}$ to $\sigma(x2)$.

.

.

.

(n) Assign the 1 remaining element to $\sigma(xn)$.

This method can generate $n(n - 1)\cdot \cdot \cdot 1 = n!$ different permutations of S. Furthermore, it should be reasonably clear that these permutations are distinct, and that any permutation can be generated in this way, and thus we know that there are exactly n! permutations of an n-element set.

Definition 1.2. *For a set S, Perm(S) is the set of all permutations on S. Multiplication of two elements $\sigma, \tau \in Perm(S)$ is simply their composition $\sigma\tau = \sigma \circ \tau$.*

Note that the composition of two bijections $\sigma, \tau : S \to S$ is itself a bijection $\sigma \circ\tau : S \to S$.

Thus multiplication in Perm(S) is closed. It is also associative, and has identity and inverse, since function composition is associative and has identity and inverse (and the identity function is of course a bijection and the inverse of a bijection is a bijection). So Perm(S) satisfies the axioms of a group.

Definition 1.3. *A permutation of a set is a bijection of this set onto itself.*
Note that
(i) if $a : X — X$ is a permutation, than the inverse function $a^{-1} : X — X$ is also a permutation;
(ii) if $a : X — X$ and $в : X — X$ are permutations, one can form a composition a **o** $в : X — X$, which is also a permutation [7].

Theorem 1.2. *Let A be a nonempty set, and let S be the collection of all permutations of A. Then S is a group under the composition* **o**.

Proof. Clearly, **o** defines a binary operation on *S*. We need to check the three axioms G1, G2, G3 of the group.

(G1) We have seen in section 1 that composition is associative.

(G2) Consider the mapping $i : A — A$ such that for all x **E** A, $i(x) = x$. Then for any a **E** S one has a **o** $i = i$ **o** $a = a$. Thus, i is the identity element on S.

(G3) For any permutation a **E** S, its inverse a^{-1} satisfies a^{-1} **o** $a = a$ **o** $a^{-1} = i$. Thus, a^{-1} is the inverse of a with respect to the group operation *[8]*.

Definition 1.4. *Let a the permutation that maps*
$a_i — a_2,\ a_2 — a_3, \ldots,\ a_{k-i} — a_k,\ a_k — a_i.$ *that a is a k—cycle, and write* $a = (a_i a_2 \ldots a_k)$.

The permutations a, $в$ shown above are expressed as

$a = (15432)(6) = (15432)$, $в = (156)(24)(3) = (156)(24)$.

A pair of cycles $(a_1a_2...a_s)$, $(b_1b_2...b_t)$ are called disjoint. In general, composition of permutations is not commutative.

For example, with a, $в$ as before $a \circ в = (14)(23)(56) = в \circ a = (16)(25)(34)$. However, disjoint cycles do commute.

Although there may be more than one way to factorize a k—cycle as a product of transpositions that are not pairwise disjoint, any cycle (and hence any permutation) is either a product of an even number of transpositions, or of an odd number of transpositions, but no t both. If a permutation can be expressed as a product of an even number of transpositions, we say that it is an even permutation. Otherwise we call it an odd permutation.

Theorem 1.5. *Let A_n be the set of all even permutations of S_n. Then A_n is a subgroup of S_n.*

Proof. The product of a pair of even permutations clearly results in another even permutation. If *a is given* then $a^{-1} = (a_k\text{-}ia_k)(a_k\text{-}ia_{k-2})$ ■ ■ ■ (aia_2), which is also an even permutation.

Consider the case $n = 4$. We list the elements of A_4:

the identity	(12)(12)	(1)
3-cycles	(12)(13)	(123), (132), (124), (142), (134), (143), (234), (243)
disjoint 2-cycles	(12)(34)	(12)(34),(13)(24),(14)(23)

Table 3. Types cyclic permutations

Note that there are exactly 4!/2 = 12 elements in A_4. In fact, in general, A_n has exactly n!/2 elements.

Another important subgroup of S_4 is V4 = {(1), (12)(34), (13)(24), (14)(23)}, it consists of the set of even permutations of $S4$ that are products of a pair of disjoint 2—cycles.

Programming in Mathematics

Terms related to mathematics, including definitions about logic, algorithms and computations and mathematical terms used in computer science and business.

(Mathematics) a logical arithmetical or computational procedure that if correctly applied ensures the solution of a problem.

(Mathematics) Logic Maths a recursive procedure whereby an infinite sequence of terms can be generated Also called algorism changed from ALGORISM

(Mathematics) a standard procedure for solving a class of mathematical problems; "he determined the upper bound with Descartes' rule of signs"; "he gave us a general formula for attacking polynomials"

Programming

Terms related to software programming, including definitions about programming languages and words and phrases about software design, coding, testing and debugging.

A finite set of unambiguous instructions performed in a prescribed sequence to achieve a goal, especially a mathematical rule or procedure used to compute a desired result. Algorithms are the basis for most computer programming [21].

Computing fundamentals

Terms related to computer fundamentals, including computer hardware definitions and words and phrases about software, operating systems, peripherals and troubleshooting.

Definition: a precise rule or set of rules for solving a problem

Synonyms: rule, formula, procedure, guide

Tips: The word algorithm is often used to refer to a technical, problem-solving procedure that can be consistently applied to derive a solution. Think of an equation that can be consistently applied to solve a mathematical problem. Algorithm is often used in technology, mathematics, computer sciences, and scientific research. Its origins are rooted in the Greek word arithmos, "number."

Secret key algorithm (symmetric algorithm)

A secret key algorithm (sometimes called a symmetric algorithm) is a cryptographic algorithm that uses the same key to encrypt and decrypt data.(Security.com)

Algorithm

The resulting algorithm for generating a random permutation of $a[0], a[1], \ldots, a[n-1]$ can be described as follows in pseudo code:

for i **from** n **downto** 2
  do $d_i \leftarrow$ random element of $\{0, \ldots, i-1\}$
swap $a[d_i]$ and $a[i-1]$

Figure 1. Permutation algorithm

This can be combined with the initialization of the array $a[i] = i$ as follows:

```
for i from 0 to n−1
do   d_{i+1} ← random element of {
0, ..., i }
a[i] ← a[d_{i+1}]
a[d_{i+1}] ← i
```

Figure 2. Permutation of numbers

Lets choose a programme language to this algorithm. I choose C++

C++ program

```
#include <string>
#include <iostream>

using namespace std;

void string_permutation( std::string& orig, std::string&
perm )
{
    if( orig.empty() )
    {
        std::cout<<perm<<std::endl;
        return;
    }

    for(int i=0;i<orig.size();++i)
    {
        std::string orig2 = orig;

        orig2.erase(i,1);
```

Figure 3. First part of algorithm in C++.

WHY C++ ?

C++ was first developped by AT&T Bell labs by Bjarne Stroustrup. It can be seen as new set of functionalities created to extend C (hence the "++"). The main points of C++ are :

Creation and use of new types is easier than in C.

C++ supports all features of object-oriented programming:

```
std::string perm2 = perm;

        perm2 += orig.at(i);

        string_permutation(orig2,perm2);
    }
}

int main()
{
    std::string orig="ABCDE";
    std::string perm;

    string_permutation(orig,perm);

    cout<<"Complete!"<<endl;

    system("pause");

    return 0;
}
```

Second part of algoriyhm in C++.

Pseudocode

Pseudocode is a compact and informal high-level description of a computer programming algorithm that uses the structural conventions of some programming language, but is intended for human reading rather than machine reading. Pseudo-code typically omits details that are not essential for human understanding of the algorithm, such as variable declarations, system-specific code and subroutines. The programming language is augmented with natural language descriptions of the details, where convenient, or with compact mathematical notation. The purpose of using pseudocode is that it is easier for humans to understand than conventional programming language code, and that it is a compact and environment-independent description of the key principles of an algorithm. It is commonly used in textbooks and scientific publications that are documenting various algorithms, and also in planning of computer program development, for sketching out the structure of the program before the actual coding takes place. No standard for pseudocode syntax exists, as an algorithm in pseudocode is not an executable program. Pseudocode resembles, but should not be confused with, skeleton programs including dummy code, which can be compiled without errors. Here is given the pseudocode for our program

```
list of 1, 2 ,3
for each item in list
    templist.Add(item)
    for each item2 in list
        if item2 is Not item
            templist.add(item)
               for each item3 in list
                   if item2 is Not item
                      templist.add(item)

                   end if
               Next
            end if

    Next
    permanentListofPermutaitons,add(templist)
    tempList.Clear()
Next
```

By running this program on a computer, we found how many permutations in a given numbers and which are the permutations of the given numbers.

In a table given below shown numbers of permutations when n is 2.

If n=2 permutations are

№	for n=2
1	12
2	21

Table 4. Exact permutations of a nummer 2.

Then I put tables showing the numbers of permutations when n is 3, 4 and 5

If n=3 permutations are

№	for n=3
1	123
2	132
3	213
4	231
5	312
6	321

Table 5. Exact permutations of a nummer 3.

If n=4 permutations are

№	for n=4
1	1234
2	1243
3	1324
4	1342
5	1423
6	1432
7	2134
8	2143
9	2314
10	2341
11	2413
12	2431
13	3124
14	3142
15	3214
16	3241
17	3412
18	3421
19	4123
20	4132
21	4213
22	4231
23	4312

Table 6. Exact permutations of a nummer 4.

Statistic I $(l_{\max(v)}, r_{\min(v)}) \approx (l_{\min(v)}, r_{\max(v)})$

In this chapter we want to stop on our proof of third statistic. Therefore, given the general theorem. Here again we will focus on special cases and show that the theorem is evenly distributed.

Theorem: $(l_{\max(v)}, r_{\min(v)}) \approx (l_{\min(v)}, r_{\max(v)})$ are equal distributed over any permutation $(v) \in S_2$

$\sum_{\delta \epsilon S_2} x^{(l_{\max(v)})} y^{(r_{\min(v)})} = \sum_{\delta \epsilon S_2} x^{(l_{\min(v)})} y^{(r_{\max(v)})}$.

Proof: to prove that this theorem is evenly distributed firstly we build a table to specify the data that we need. This table shows the calculation of $l_{\max(v)}, l_{min(v)}, r_{max(v)}, r_{min(v)})$ *and* amount of numbers on this values.

№	for n=2	$l_{\max(v)}$	amount	$l_{min(v)}$	amount	$r_{max(v)}$	amount	$r_{min(v)}$	amount
1	12	12	2	1	1	2	1	21	2
2	21	2	1	21	2	12	2	1	1

$$xy^2 + x^2y = x^2y + xy^2$$

Below equally distributed theorem when n is 3

Theorem: $(l_{\max(v)}, r_{\min(v)}) \approx (l_{\min(v)}, r_{\max(v)})$are equal distributed over any permutation $(v) \in S_3$

$\sum_{\delta \epsilon S_3} x^{(l_{\max(v)})} y^{(r_{\min(v)})} = \sum_{\delta \epsilon S_3} x^{(l_{\min(v)})} y^{(r_{\max(v)})}$.

Proof: Then by the same way we showed a table with calculations of $l_{\max(v)}, l_{min(v)}, r_{max(v)}, r_{min(v)})$ *and amount of numbers on n is* 3.

№	for n=3	$l_{\max(v)}$	amount	$l_{min(v)}$	amount	$r_{max(v)}$	amount	$r_{min(v)}$	amount
1	123	123	3	1	1	3	1	321	3
2	132	13	2	1	1	23	2	21	2
3	213	23	2	21	2	3	1	31	2
4	231	23	2	21	2	13	2	1	1
5	312	3	1	31	2	23	2	21	2
6	321	3	1	321	3	123	3	1	1

Table 16. Constructed table to show the equality of polynomialswhen n=3

$$x^3y^3 + x^2y^2 + x^2y^2 + x^2y + xy^2 + xy = xy + xy^2 + x^2y + x^2y^2 + x^2y^2 + x^3y^3$$

$$x^3y^3 + 2x^2y^2 + x^2y + xy^2 + xy = x^3y^3 + 2x^2y^2 + x^2y + xy^2 + xy$$

Below equally distributed theorem when n is 4

Theorem: $(l_{\max(v)}, r_{\min(v)}) \approx (l_{\min(v)}, r_{\max(v)})$are equal distributed over any permutation $(v) \in S_4$

$\sum_{\delta \epsilon S_4} x^{(l_{\max(v)})} y^{(r_{\min(v)})} = \sum_{\delta \epsilon S_4} x^{(l_{\min(v)})} y^{(r_{\max(v)})}$.

Proof:

№	for n=4	$l_{\max(v)}$	amount	$l_{min(v)}$	amount	$r_{max(v)}$	amount	$r_{min(v)}$	amount
1	1234	1234	4	1	1	4	1	4321	4
2	1243	124	3	1	1	34	2	321	3
3	1324	134	3	1	1	4	1	421	3
4	1342	134	3	1	1	24	2	21	2
5	1423	14	2	1	1	34	2	321	3
6	1432	14	2	1	1	234	3	21	2
7	2134	134	3	21	2	4	1	431	3
8	2143	24	2	21	2	34	2	31	2
9	2314	234	3	21	2	4	1	41	2
10	2341	234	3	21	2	14	2	1	1
11	2413	24	2	21	2	34	2	31	2
12	2431	24	2	21	2	134	3	1	1
13	3124	34	2	31	2	4	1	421	3
14	3142	34	2	31	2	24	2	21	2
15	3214	34	2	321	3	4	1	41	2
16	3241	34	2	321	3	14	2	1	1
17	3412	34	2	31	2	24	2	21	2
18	3421	34	2	321	3	124	3	1	1
19	4123	4	1	41	2	34	2	321	3
20	4132	4	1	41	2	234	3	21	2
21	4213	4	1	421	3	34	2	31	2
22	4231	4	1	421	3	134	3	1	1
23	4312	4	1	431	3	234	3	21	2
24	4321	4	1	4321	4	1234	4	1	1

Table 12. Constructed table to show the equality of polynomialswhen n=4

$x^4y^4 + x^3y^3 + x^3y^3 + x^3y^2 + x^2y^3 + x^2y^2 + x^3y^3 + x^2y^2 + x^3y^2 + x^3y + x^2y^2 + x^2y + x^2y^3 + x^2y^2 + x^2y^2 + x^2y + x^2y^2 + x^2y + xy^3 + xy^2 + xy^2 + xy + xy^2 + xy = xy + xy^2 + xy + xy^2 + xy^3 + x^2y + x^2y^2 + x^2y + x^2y^2 + x^2y^2 + x^2y^3 + x^2y + x^2y^2 + x^3y + x^3y^2 + x^2y^2 + x^3y^3 + x^2y^2 + x^2y^3 + x^3y^2 + x^3y^3 + x^4y^4.$

Statistic II $(l_{\max(v)}, r_{\max(v)}) \approx (l_{\min(v)}, r_{\min(v)})$

In this chapter we want to stop on our proof of third statistic. Therefore, given the general theorem. Here again we will focus on special cases and show that the theorem is evenly distributed.

Theorem: $(l_{\max(v)}, r_{\max(v)}) \approx (l_{\min(v)}, r_{\min(v)})$ are equal distributed over any permutation $(v) \in S_2$

$\sum_{\delta \epsilon S_2} x^{(l_{\max(v)})} y^{(r_{\max(v)})} = \sum_{\delta \epsilon S_2} x^{(r_{\min(v)})} y^{(r_{\min(v)})}$.

Proof: to prove that this theorem is evenly distributed firstly we build a table to specify the data that we need. This table shows the calculation of $l_{\max(v)}, l_{min(v)}, r_{max(v)}, r_{min(v)})$ *and* amount of numbers on this values.

№	for n=2	$l_{\max(v)}$	amount	$l_{min(v)}$	amount	$r_{max(v)}$	amount	$r_{min(v)}$	amount
1	12	12	2	1	1	2	1	21	2
2	21	2	1	21	2	12	2	1	1

Table 18. Constructed table to show the equality of polynomialswhen n=2

$$x^2y+xy^2= xy^2+x^2y$$

Below equally distributed theorem when n is 3

Theorem: $(l_{\max(v)}, r_{\max(v)}) \approx (l_{\min(v)}, r_{\min(v)})$ are equal distributed over any permutation $(v) \in S_3$

$\sum_{\delta \epsilon S_3} x^{(l_{\max(v)})} y^{(r_{\max(v)})} = \sum_{\delta \epsilon S_3} x^{(r_{\min(v)})} y^{(r_{\min(v)})}$.

Proof: Then by the same way we showed a table with calculations of $l_{\max(v)}, l_{min(v)}, r_{max(v)}, r_{min(v)})$ *and amount of numbers on n is* 3.

№	for n=3	$l_{\max(v)}$	amount	$l_{min(v)}$	amount	$r_{max(v)}$	amount	$r_{min(v)}$	amount
1	123	123	3	1	1	3	1	321	3
2	132	13	2	1	1	23	2	21	2
3	213	23	2	21	2	3	1	31	2
4	231	23	2	21	2	13	2	1	1
5	312	3	1	31	2	23	2	21	2
6	321	3	1	321	3	123	3	1	1

Table 19. Constructed table to show the equality of polynomialswhen n=2

$$x^3y+x^2y^2+x^2y+x^2y^2+xy^2+xy^3= xy^3+xy^2+x^2y^2+x^2y+x^2y^2+x^3y$$

Below equally distributed theorem when n is 4

Theorem: $(l_{\max(v)}, r_{\max(v)}) \approx (l_{\min(v)}, r_{\min(v)})$ are equal distributed over any permutation $(v) \in S_4$

$$\sum_{\delta \in S_4} x^{(l_{\max(v)})} y^{(r_{\max(v)})} = \sum_{\delta \in S_4} x^{(r_{\min(v)})} y^{(r_{\min(v)})}.$$

Proof:

№	for n=4	$l_{\max(v)}$	amount	$l_{min(v)}$	amount	$r_{max(v)}$	amount	$r_{min(v)}$	amount
1	1234	1234	4	1	1	4	1	4321	4
2	1243	124	3	1	1	34	2	321	3
3	1324	134	3	1	1	4	1	421	3
4	1342	134	3	1	1	24	2	21	2
5	1423	14	2	1	1	34	2	321	3
6	1432	14	2	1	1	234	3	21	2
7	2134	134	3	21	2	4	1	431	3
8	2143	24	2	21	2	34	2	31	2
9	2314	234	3	21	2	4	1	41	2
10	2341	234	3	21	2	14	2	1	1
11	2413	24	2	21	2	34	2	31	2
12	2431	24	2	21	2	134	3	1	1
13	3124	34	2	31	2	4	1	421	3
14	3142	34	2	31	2	24	2	21	2
15	3214	34	2	321	3	4	1	41	2
16	3241	34	2	321	3	14	2	1	1
17	3412	34	2	31	2	24	2	21	2
18	3421	34	2	321	3	124	3	1	1
19	4123	4	1	41	2	34	2	321	3
20	4132	4	1	41	2	234	3	21	2
21	4213	4	1	421	3	34	2	31	2
22	4231	4	1	421	3	134	3	1	1
23	4312	4	1	431	3	234	3	21	2
24	4321	4	1	4321	4	1234	4	1	1

Table 19. Constructed table to show the equality of polynomialswhen n=2

$x^4y+x^3y^2+x^3y+x^2y^2+x^3y+x^3y^2+x^2y^2+x^2y^3+x^3y+x^2y^2+x^3y+x^3y^2+x^2y^2+x^2y^3+x^2y+x^2y^2+$
$x^2y+x^2y^2+x^2y^2+x^2y^3+xy^2+xy^3+xy^2+xy^3+xy^3+xy^4=$
$xy^4+xy^3+xy^3+xy^2+xy^3+xy^2+x^2y^3+x^2y^2+x^2y^2+x^2y+x^2y^2+x^2y+x^2y^3+x^2y^2+x^3y^2+x^3y+x^2$
$y^2+x^3y+x^2y^3+x^2y^2+x^3y^2+x^3y+x^3y^2+ x^4y$

CONCLUSION

Theorem: $(l_{\max(v)}, l_{min(v)}) \approx (r_{max(v)}, r_{min(v)})$ are equal distributed over any permutation $(v) \in S_n$. So, Theorem is proved over S_2, S_3, S_4 and S_5

And to be continued over any permutation $(v) \in S_n$

$$\sum_{\delta \epsilon S_n} x^{(l_{\max(v)})} y^{(l_{\min(v)})} = \sum_{\delta \epsilon S_n} x^{(r_{\max(v)})} y^{(r_{\min(v)})}.$$

Theorem: $(l_{\min(v)}, r_{max(v)}) \approx (l_{max(v)}, r_{min(v)})$ are equal distributed over any permutation $(v) \in S_n$. So, Theorem is proved over S_2, S_3, S_4 and S_5 .

And to be continued over any permutation $(v) \in S_n$

$$\sum_{\delta \epsilon S_n} x^{(l_{\min(v)})} y^{(r_{\max(v)})} = \sum_{\delta \epsilon S_n} x^{(l_{\max(v)})} y^{(r_{\min(v)})}.$$

Theorem: $\left(l_{\max(v)}, r_{\min(v)}\right) \approx \left(l_{\min(v)}, r_{\max(v)}\right)$ are equal distributed over any permutation $(v) \in S_n$. So, Theorem is proved over S_2, S_3, S_4 and S_5

And to be continued over any permutation $(v) \in S_n$

$$\sum_{\delta \epsilon S_n} x^{(l_{\max(v)})} y^{(r_{\min(v)})} = \sum_{\delta \epsilon S_n} x^{(l_{\min(v)})} y^{(r_{\max(v)})}.$$

Theorem: $\left(l_{\max(v)}, r_{\max(v)}\right) \approx \left(l_{\min(v)}, r_{\min(v)}\right)$ are equal distributed over any permutation $(v) \in S_n$. So, Theorem is proved over S_2, S_3, S_4 and S_5 .

And to be continued over any permutation $(v) \in S_n$

$$\sum_{\delta \epsilon S_n} x^{(l_{\max(v)})} y^{(r_{\max(v)})} = \sum_{\delta \epsilon S_n} x^{(r_{\min(v)})} y^{(r_{\min(v)})}.$$

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