Methodology for solving complex problems related to the combination of a cone and a pyramid.

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ABSTRACT

The cone and pyramid are considered in the stereometry section of general geometry.Both the pyramid and the cone have been studied more individually, but due to their combination, the information is considered less in textbooks.It is difficult for students to solve spatial problems , and the problem with a combination of figures becomes even more complicated.The article is devoted to the issue of such importance.The article discusses a systematic approach to solving complex problems related to the combination of a pyramid and a cone.The volume of geometric shapes given in the work, here are the basic principles and steps necessary to successfully solve such problems, including determining surfaces and other characteristics.The methodology also includes examples and practical tasks that help readers better understand and apply the knowledge gained in practice.

Keywords: problem, cone, combination, drawn externally on a cone, drawn internally, pyramid

АҢДАТПА

Конус пен пирамида жалпы геометрияның стереометрия бөлімінде қарастырылады. Пирамида да, конуста жекелей көп зерттелген, бірақ олардың комбинациясына байланысты мағлұматтар оқулықтарда аз қарастырылған. Оқушыларға кеңістіктегі есептерді шығару қиындыққа соғады , ал фигуралардың комбинациясымен келетін есеп болса тіптен күрделене түседі., Жобада осындай маңызы бар мәселеге арналған. пирамида мен конустың комбинациясына байланысты күрделі есептерді шешудің жүйелі тәсілі қарастырылған. Жұмыста берілген геометриялық фигуралардың көлемін, беттерін және басқа сипаттамаларын анықтауды қоса алғанда, осындай есептерді сәтті шешуге қажетті негізгі принциптер мен қадамдар берілген. Әдістеме сонымен қатар оқырмандарға алған білімдерін тәжірибеде жақсырақ түсінуге және қолдануға көмектесетін мысалдар мен практикалық тапсырмаларды қамтиды.

Түйін сөздер: есеп, конус, комбинация , конусқа сырттай сызылған , іштей сызылған , пирамида

АННОТАЦИЯ

Конус и пирамида рассматриваются в разделе стереометрии общей геометрии. Отдельно в пирамиде и конусе изучено много, но в связи с их сочетанием в учебниках мало изучены сведения. Учащимся сложно решать пространственные задачи, а задача с комбинацией фигур становится еще сложнее. Проект освящена проблеме, имеющей такое значение. В проекте рассматривается системный подход к решению сложных задач, связанных с комбинацией пирамиды и конуса. В работе представлены основные принципы и шаги, необходимые для успешного решения таких задач, включая определение объема, поверхностей и других характеристик заданных геометрических фигур. Методология также включает примеры и практические задания, которые помогают читателям лучше понять и применить полученные знания на практике.

Ключевые слова: задача (упражнение), конус, комбинация, пирамида

INTRODUCTION

Modern School Mathematics attaches special importance to the development of spatial thinking, logical and analytical abilities of students. Within the geometry course, working with spatial figures is an effective way to improve students ' spatial imagination, volumetric thinking. In particular, teaching to solve complex problems by analyzing the similarities and differences between cone – shaped and pyramid-shaped figures, their combinations-contributes to the use of students ' theoretical knowledge in practical problems. However, in the school curriculum, these topics are considered in a limited way, and methods for solving complex problems based on their combination are not systematically studied. In this regard, the development of a special methodological system aimed at increasing students ' creativity, awakening interest in mathematics is one of the most pressing problems.

One effective way to achieve this goal is to work with stereometric figures, including the study of combinations of complex shapes. One of these complex structures is the combination of a cone and a pyramid.

Cones and pyramids are bodies that are often found in spatial geometry and are widely used in real life (architecture, technology, nature). By combining these two figures, new complex shapes can be obtained, for example: Rocket head, roof structure, architectural domes, sculptural compositions. However, solving such complex problems as volume, surface area, intersection, clipping associated with such combinations in most cases causes difficulties for schoolchildren and students.

It is in this context that the ability of students and future teachers to analyze, compose, and master the methods of solving problems related to combinations of geometric shapes has a significant impact on the development of spatial thinking.

Research shows that many students and future teachers make mistakes in calculating the volume or lateral surface area of a cone and pyramid, and the ability to understand and analyze bodies formed by their combinations becomes an even more complex task. In this situation, there is a need to use special methodological approaches in the learning process, that is, methods of visualization, modeling, Graphic Imaging and step-by-step interpretation.

Complex geometric problems are problems that require analysis, consisting of many steps, containing several concepts at once. When working with figures consisting of a combination of cones and pyramids, it is necessary to be able to correctly understand not only their individual properties, but also their relationship, location in space and new geometric parameters arising from them.

However, in modern educational practice, there is not enough clear methodological system for solving such complex stereometric problems. Students often try to solve formulas by heart, and the ability to spatial imagine and geometric modeling is neglected.

In this regard, the relevance of this dissertation work is to develop an effective methodology for solving problems associated with complex geometric combinations formed by cones and pyramids, to show ways to apply it in the educational process.

Purpose of the study: The development of students ' spatial thinking, logical-analytical skills and problem-solving abilities by developing an effective methodology for solving complex problems based on combinations of cones and pyramids and applying it in the school geometry course.

Research objectives: Analysis of theoretical material related to the cone and pyramid and problems that are given by their combinations.

Systematization and classification of types of complex problems related to combinations of spatial figures.Identify effective methods for solving such problems and adapt them for use in teaching.Complation of a set of methodological tasks and reports and its application in school practice.Verification and analysis of the effectiveness of the proposed methodology through experimental work.Assessment of the influence of students on the level of spatial thinking, logical skills.

Scientific novelty of the study**:**

For the first time, a methodology for solving complex stereometric problems based on a combination of cone and pyramid figures has been systematically proposed.

This methodology is distinguished by approaches such as visualization, modeling and periodic analysis, aimed at developing the ability to think spatially.

In the course of the study, an algorithmic structure for solving multi-stage, integrated problems was developed. It was adapted to the educational content for students and future teachers.

The methods proposed in the dissertation work are effective in the development of spatial and logical thinking abilities of students and future teachers, proved by the results of a pedagogical experiment.

Theoretical and methodological basis of the work: systematic consideration of geometric construction problems in grades 10-11, contributes to improving the level of mathematical training of students on this topic.

Research methods

Theoretical analysis (literature review, modeling);

Practical method (task processing, problem analysis);

Experiment (control of the learning process);

Comparative and statistical methods (evaluation of results).

Research design: research design combines theoretical analysis and computational modeling. Such an approach ensures the mathematical accuracy and practical application of the results.Research question:

What world applications , simulations exist and how to solve them that help solve complex problems involving a combination of a cone and a pyramid ?

1. LITERATURE REVIEW

1.1 Theoretical aspects of combinations of geometric shapes

The cone and pyramid are considered in the stereometry section of general geometry. Both the pyramid and the cone have been studied more individually, but due to their combination, the information is co]]]]nsidered less in textbooks. It is difficult for students to solve spatial problems , and the problem with a combination of figures becomes even more complicated., The project is devoted to an issue of such importance. a systematic approach to solving complex problems related to the combination of a pyramid and a cone is considered. The paper provides the basic principles and steps necessary to successfully solve such problems, including determining the volume, surfaces and other characteristics of given geometric shapes. The methodology also includes examples and practical ones that help readers better understand and apply the knowledge gained in practice In various manuals for solving general geometric problems it is safe to say that there are very few tasks related to the combination of space figures . However, in modern architecture, there are many cases associated with the combination of a cone and a pyramid in the field of construction in general, in design, in archetypaltrua .The body that comes out when rotating a right triangle from its leg is called a cone.

The cross-section of the cone, which is formed when the plane passes through the cone, is called the cross-section of the cone . He is an isosceles triangle.The cone has an equilateral triangle with a plane passing through the vertex , not parallel to the apex.The resulting section is round when you cut it with a plane perpendicular to the growth of the cone.

When the cone is crossed with an inclined plane, the resulting section is an ellipse.Let's cross any cone with a plane parallel to its base. The resulting section is round and it divides the cone into two parts . One part will be a smaller cone, similar to a given cone, and the other a truncated cone . The truncated cone can often be found in archetypes. The Golden Horde Business Center in Astana is a vivid example of this .

A pyramid (ancient Greek: πυραμίς — skyscraper) is a polyhedron consisting of triangles, one of the sides of which is a Polygon (the base of a pyramid) (sometimes it can also be a triangle), and the rest of the sides (sides) have a common Vertex (the top of the pyramid).

Depending on the number of sides, the pyramid is divided into triangular, square, etc. The perpendicular lowered from the vertex to the plane of the base is called the height of the pyramid. The volume of the pyramid is calculated by the formula (where B is the area of the base, I is the height). A pyramid whose base is a correct Polygon and whose height passes through the center of its base is called a correct pyramid. The sides of the correct pyramid consist of mutually equal equilateral triangles; the height of each of these triangles is called the apothem of the correct pyramid (the apothem of the base of the pyramid will be the projection of the apothem on the plane of the base). When a pyramid is cut with a plane parallel to its base, it is divided into two parts:one is a pyramid similar to a given pyramid, and the other is a truncated pyramid. The volume of a truncated pyramid is determined by the formula [where S1, S2 are the area of the bases, and h is the height (distance between the bases)]

A combination of geometric shapes is the process of creating a new complex shape by combining, intersecting or joining two or more shapes.

What are the combinations:

Intersection (example: intersection of a cylinder and a ball);

Overlap (one on top of the other);

Fusion (considering as a single whole body, for example: a cylinder and a cone join to give the shape of a rocket);

Clipping (producing a new shape by cutting one figure with another).

Any stereometric figures can exist in combination with each other if they are:

If it has common points in space (intersects);

If placed on top of each other or inside (overlap);

Describes (merges)one whole shape;

For the combination to be:

The rules of location in space must be observed — it is necessary that the figures are located in the same plane or at a certain angle;

Size matching-it is necessary that the dimensions of figures that merge or intersect are compatible with each other;

Geometric conditions-to solve some combinations, a common axis, a common vertex, or an axis of symmetry may be required.

Cone + cylinder → geometric model of rocket;

Ball + Cube → toy or design element;

Prism + pyramid → complex construction form (e.g. roof).

Cone and pyramid

The base of the pyramid drawn externally on the cone will be a polygon drawn externally on the base of the cone , and their heights are equal among themselves.

Outwardly , only a pyramid can be drawn on a cone, in which the dihedral angles are equal (if the height of the base of the pyramid does not lie outside the Polygon at the base) . The angles at the base of the pyramid are equal in the correct pyramids and in the pyramids whose height is projected to the center of the circle .

The radius of the cone is equal to the Radus of the circle drawn inward to the base of the pyramid.

Any correct pyramid can be drawn externally on a cone . The circle at the base of the cone is drawn internally to the polygonal base of the pyramid.

Figure 1.1.1 The triangular pyramid is drawn externally on a cone

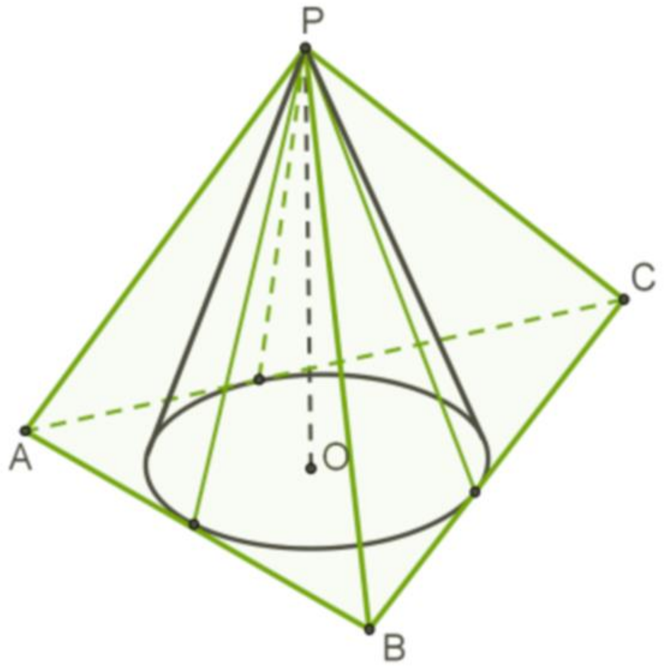
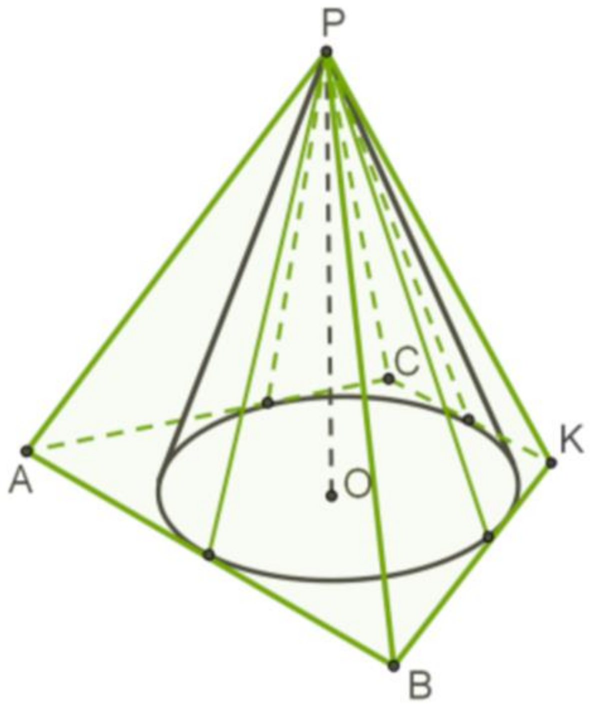


Figure 1.1.2 A square pyramid is drawn externally on a cone



The center of a circle drawn internally to a triangle is the point of intersection of its bisectors . You can draw a circle on any triangle .

Figure 1.1.3 The base of the pyramid drawn externally on a cone

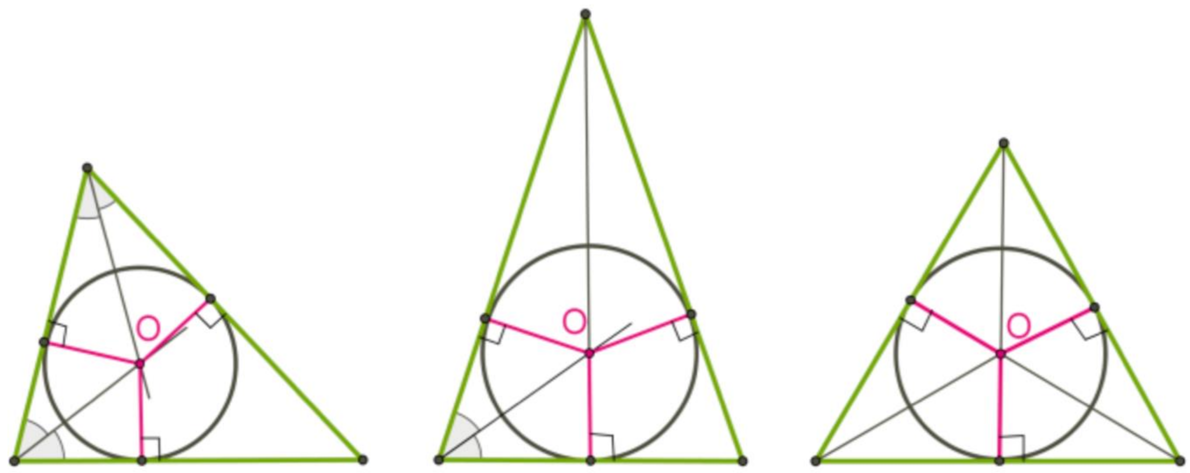
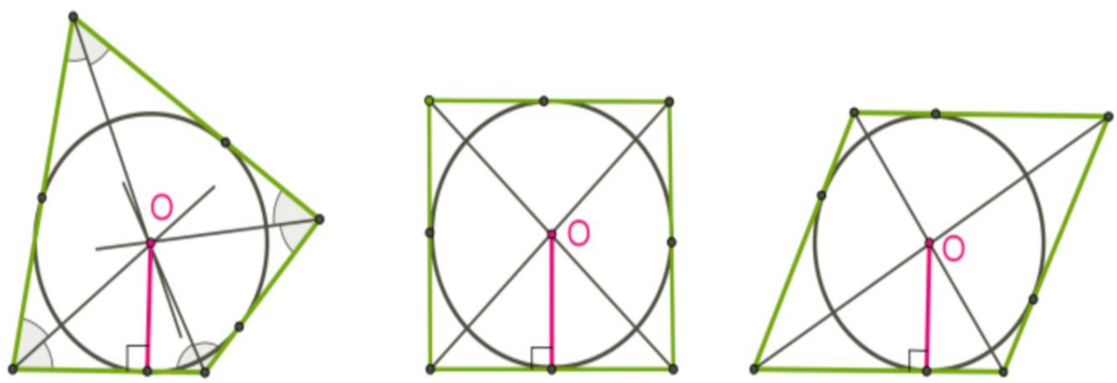


Figure 1.1.4 The base of the pyramid drawn externally on a cone



A circle that is inscribed within a rectangle has its center located at the intersection point of the rectangle's angle bisectors. Such a circle can only be inscribed if the opposite sides of the rectangle are equal in length, ensuring the figure is suitable for internal tangency.

On the other hand, a circle drawn outside a polygon—such as a square or a rhombus—has its center positioned at the intersection of the diagonals. In these cases, the diagonals are equal (in a square) or perpendicular bisectors (in a rhombus), making them natural axes of symmetry.

Similarly, when a pyramid is inscribed within a cone, the polygon forming the base of the pyramid is also inscribed within the circular base of the cone. In such constructions, the heights of the pyramid and the cone are aligned or proportional, allowing for a consistent spatial relationship between the two solids. This configuration ensures that the apex of the pyramid lies directly beneath or above the apex of the cone, preserving their shared vertical axis.

Figure 1.1.5 A cone drawn externally on a pyramid

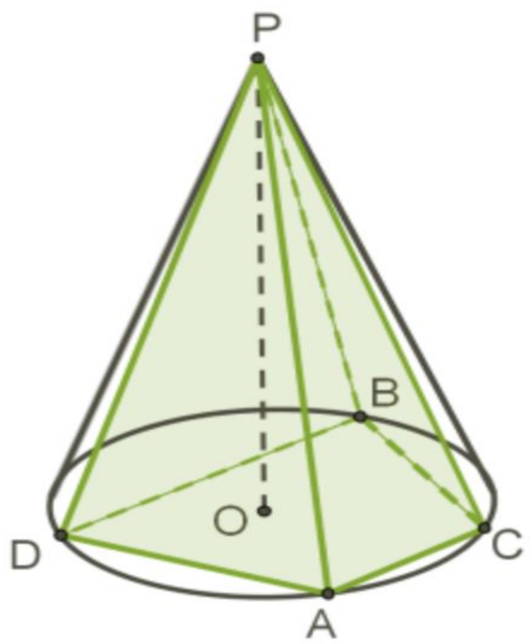
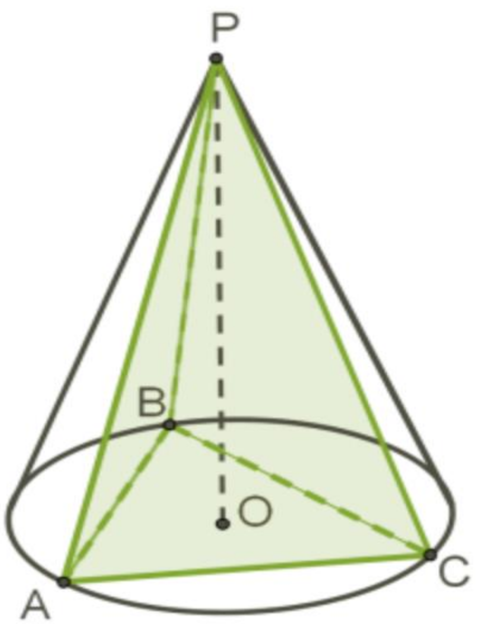


Figure 1.1.6 Pyramid drawn internally on a cone

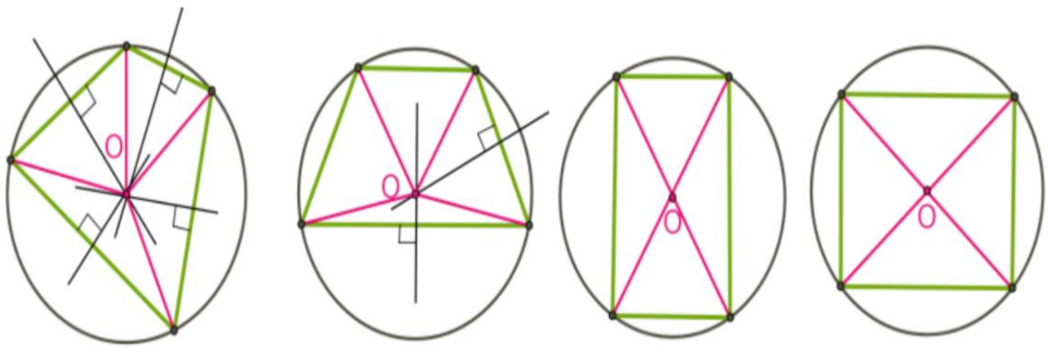


Only such pyramids can be drawn on the cone , the side sides of which are equal(correspond to the cone-forming ones).

The side walls are equal in any correct pyramid and in pyramids whose height is projected to the center of the externally drawn circle.

Pictures are drawn according to the terms of the report. Sometimes it is enough to depict only the bases of the pyramid and cone, because their heights are equal .The radius of the cone is equal to the radius of the circle drawn externally to the Polygon at the base of the pyramid

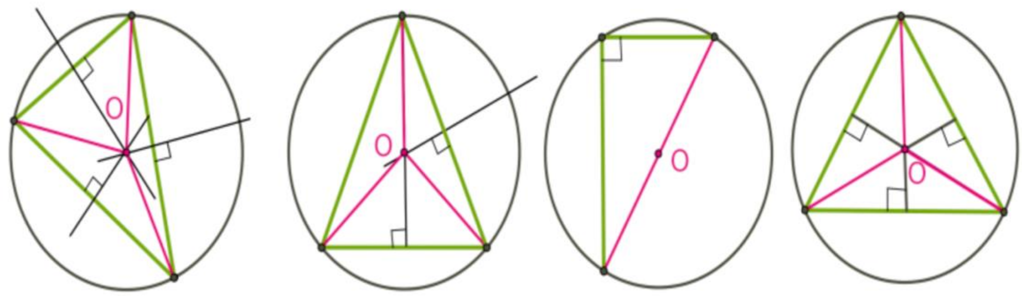
Figure 1.1.7 The base of the pyramid drawn inside the cone



The center of the circle drawn externally on the Triangle, is the point of intersection of the Perpendiculars drawn on the side walls of the Triangle.

The circle can be drawn externally on any triangle .

Figure 1.1.8 The base of the pyramid drawn inside the cone



The center of the circle drawn externally on the quadrilateral is the point of intersection of the Perpendiculars drawn on the sides of the quadrilateral. A circle can only be drawn on an equilateral trapezoid , a rectangular rectangle , and a square, with the sum of opposite angles equal to 180 degrees.

Even in the imaging of stereometric figures, free projection is used, as in the imaging of planimetric figures. That is, the relative position of the figure of space and the plane of representation, the direction of projection, is not taken into account.

The image of the polyhedron is brought to the image of its vertices and facets. Sometimes, for the visualization of the drawing, an image of its height and other lines is drawn.

To simplify the general methods of drawing an image, it is necessary that its base points are effectively selected and some individual elements are drawn in a convenient sequence.

From the original, any four points that do not lie in the same plane can be taken as basis points. But, taking polyhedral vertices as basis points allows you to get an effective scheme for constructing a polyhedral image. It is obtained from three of the vertices from one base, and the fourth from another. And even when drawing an image of a pyramid and a cone, three of the base points are taken from one base, and its Vertex as the fourth point.

Such a selection of base points allows you to effectively perform the sequence of mapping other elements of the spatial body. That is, on the basis of three basic points taken from the base, the base of the spatial body is first mapped. Then, through the fourth base point, other vertices and facets of the polyhedra are drawn, and the creators of the bodies of rotation are drawn.

When drawing an image of a polyhedron, cylinder and cone, the first is drawn an image of its base. Its mapping is performed on the basis of the previously considered theory of image construction of flat figures. That is, when the polyhedron is a right triangle, isosceles triangle, right-angled triangle, we construct its base as any Triangle based on the "first theorem of existence". And the base is a parallelogram, rectangle, Rhombus, Square, in the case of which we draw the image of its base as a parallelogram in arbitrary form on the basis of three basic points. And when the base of the polyhedron is a trapezoid, we draw its image as a trapezoid. When we draw a cone and a cylinder, we immediately draw its base as an ellipse.

In order to ensure that the requirements for drawing an image of a figure are met, it is most effective not to paint the fourth base point immediately, but to paint the base point after drawing.For example, to draw an image of a cone, mark four base points at once and draw its image. Let's analyze whether the resulting image meets the requirements for the image of the figure (image correctness, image visibility, and easy-to-perform image). To do this,we take as a base point three points from the base of the cone and one point from the vertex

We draw an ellipse, pressing points A, B, C, which lie at the base of these base points. we draw tangents from the vertex to this ellipse. The resulting drawing will give a correct, but not too visual image of the cone.

If so, it turns out that in the image of spatial bodies, obtaining the basis points in a free form can lead to obtaining a correct, but not visual image. Separately, we will focus on drawing images of some spatial bodies in order to comply with these requirements as much as possible.

Painting a triangular pyramid

The problem of mapping a triangular pyramid is solved entirely on the basis of the second theorem of existence. In fact, as an image of any triangular pyramid, you can take a square with its diagonals connected in the image plane. For example, let it be necessary to draw an image of a right triangular pyramid, the side faces of which are twice as large as the sides of the base. Although all of the pyramid shapes in Figure 1.1.5 below are correct and easy to perform, not all of them are visual.

Figure 1.1.6 is plotted with a non-vertical straight line, this given pyramid can be perceived by students as an inclined pyramid. the image of the pyramid in Figure"B "is not Visual, since the image is" incomplete " (this requirement was mentioned above), the height of the pyramid is superimposed on the edge. "B" - facets that are not indicated by a dashed line in the figure, do not allow students to correctly perceive the image of the pyramid. the pyramid image in Figure"B" gives the idea of a pyramid whose side facets are twice as large as the base facet. It is the pyramid image in "G"-figure that allows you to imagine the complete original about a given pyramid.

The segment in each pyramid image in this image will actually be an image of the height of the pyramid. But he said that since the " a "- figure does not have a vertical, and the"B" -figure faces a facet, it is not visual it is calculated. And the reason for such a non-visual image is that the "second theorem of existence" is used directly to select four basic points at once.

Figure 1.1.9 Rawing an image of a pyramid

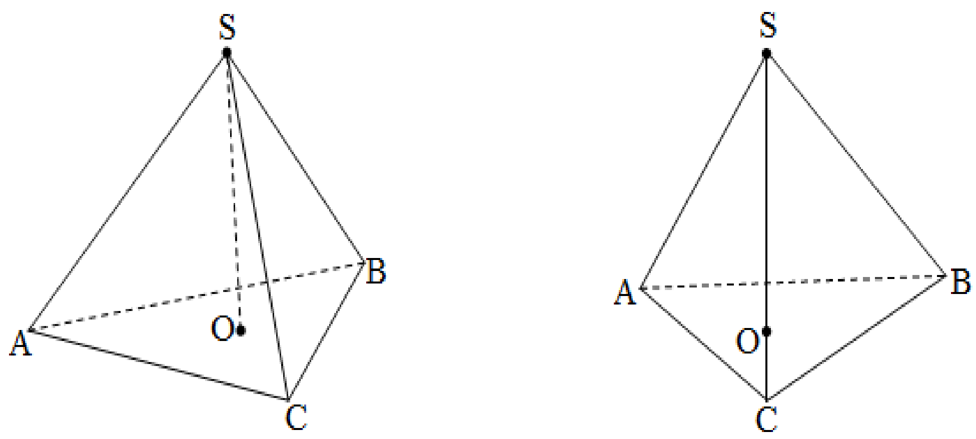
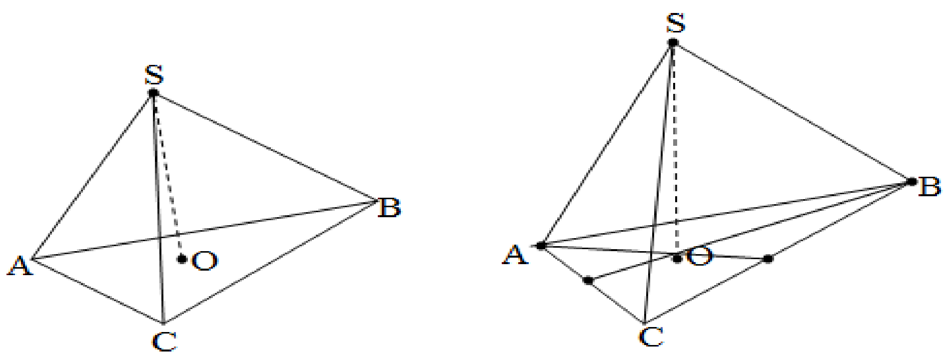


Figure 1.1.10 Rawing an image of a pyramid



If so, it turns out that in order to get a visual image, you need to choose the right pyramid shape as a drawing. Therefore,first it is necessary to draw an image of the triangle ABC at the base of the pyramid (Figure1.1.7). In this triangle,the base of the height of the pyramid is drawn point O (Figure 1.1.8). Through this point O,we construct the vertical or line (Figure 1.1.8). After that, we draw the fourth base point along the OR line.   connecting the point with points A, B, C through segments, a pyramid is built (Figure 1.1.8).

The drawing, executed in the order indicated above, allows you to get a correct, easy-to-perform and visual image.

Figure 1.1.11 Drawing an image of a polygonal pyramid

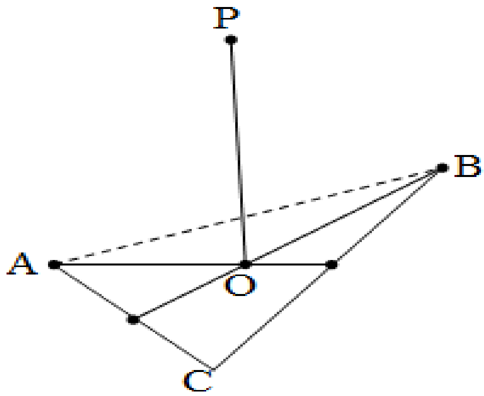
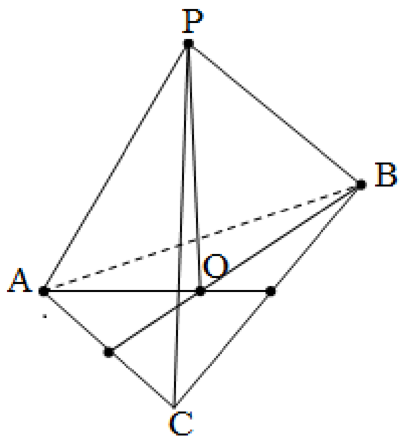


Figure 1.1.12 Drawing an image of a polygonal pyramid



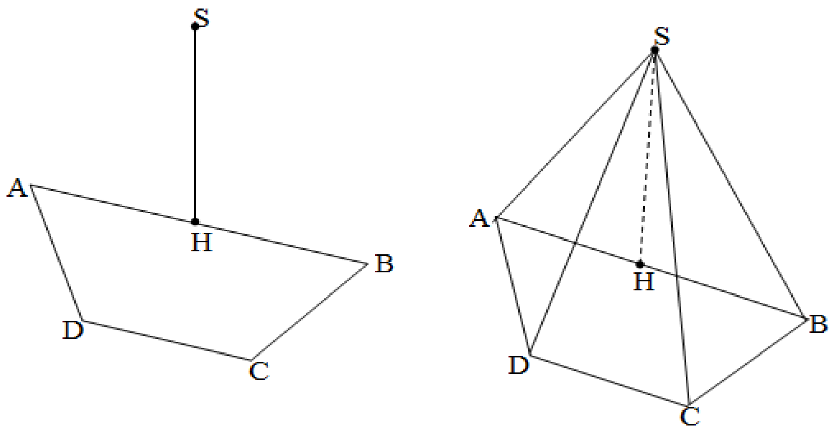
The technique of drawing the image of polygonal pyramids is also performed similarly to the method of drawing the image of a triangular pyramid. From the original pyramid, its vertices are taken as basis points. Three are taken from its base, the fourth point will be the top of the pyramid.

It will be unprofitable to immediately mark the image of the four basic points of the original on the plane of representation, as when drawing an image of a triangular pyramid. It is better to go and draw the base point, which will be the image of the top of the pyramid, the base of the pyramid, and then the height of the pyramid.

For example: given the pyramid. isosceles trapezoid. the side is an isosceles triangle and is perpendicular to the plane of the base. It is necessary to draw the image of this pyramid.

Solution: we draw an image of an equilateral trapezoid at the base as any trapezoid. In the original, the height of the pyramid falls in the middle of the AB side of the base. If so, we determine the point H, the center of the image of the side AB. We construct a vertical segment from this point. by connecting the vertices of the trapezoid at the base of the pyramid through the segments, we get the image of the sought-after pyramid. That is, this is done in the order below-in the figure.

Figure 1.1.13 Drawing an image of a pyramid whose base is a trapezoid



Drawing the Shape of a Cone: A Detailed Description of the Construction Process

Constructing the image of a cone, which is a classic solid of revolution, requires a more complex and systematic approach than the projection of regular polygonal shapes. This is primarily due to the three-dimensional nature of the cone, its circular base, and its single apex point that lies outside the base plane. Mapping bodies of rotation, such as cones, demands a strong understanding of spatial relationships, geometric projections, and knowledge of how curves transform in two-dimensional drawings. To begin, we must recognize that the base of the cone is a circle in three-dimensional space. However, when projected onto a two-dimensional drawing surface, a circle does not retain its original shape. Instead, due to the rules of perspective and projection, a circle appears as an ellipse. This is a key principle often emphasized in the geometric modeling of solids. Therefore, the first step in constructing the projection of a cone is to correctly depict its base as an ellipse.

1. Drawing the Base (Ellipse)**:** The initial step involves drawing an ellipse that represents the circular base of the cone. This ellipse must be constructed with careful consideration of its major and minor axes, which represent the diameter of the original circle from different perspectives. The more oblique the viewing angle, the more elongated the ellipse appears. The major axis lies along the horizontal projection plane, while the minor axis corresponds to the apparent "depth" of the circle.

2. Identifying the Center of the Ellipse**:** After the ellipse has been constructed, the center point of the ellipse must be determined. This point is crucial

as it will serve as a reference for constructing the axis of the cone. The center of the ellipse is found at the intersection of its major and minor axes. This center corresponds to the center of the circular base in the actual 3D object.

3. Constructing the Axis of the Cone**:** From the center of the ellipse, a vertical straight line is extended upwards. This line represents the axis of the cone and defines its height. It also helps to preserve symmetry in the cone’s construction, ensuring that the vertex is aligned properly above the base. This axis should be perpendicular to the plane of the base in an ideal right cone.

4. Locating the Vertex (Apex) of the Cone**:** The vertex of the cone is then marked on the vertical axis. The position of this point determines the height of the cone and must be placed precisely. The greater the distance from the base, the taller and narrower the cone appears; conversely, a closer apex results in a shorter, wider cone.

5. Drawing the Generators (Tangents) of the Cone: From the vertex, two straight lines are drawn tangentially to the ellipse. These lines are called the generators of the cone, and they define the slanted sides of the cone from the apex down to the base. These tangents must touch the ellipse at precisely one point each, located on opposite sides of the ellipse. The selection of these tangent points is critical, as they dictate the apparent width of the cone in the drawing. These lines visually connect the vertex to the edges of the base and form the cone's lateral surface.

6. Ensuring Symmetry and Proportion: It is essential during the construction to maintain proportional relationships between the height of the cone and the radius of the base. These proportions affect the realism and accuracy of the geometric representation. Depending on the intended application—be it educational, engineering, or artistic—the emphasis may be placed on different elements such as symmetry, shading, or structural measurements.

7. Labeling and Annotation (Referencing to Figure 1.1.11)**:** To support the reader’s comprehension, each stage of the construction should be accompanied by a labeled figure. For example, in Figure 1.1.11:

8. Subfigure (a) shows the ellipse drawn as the cone’s base;

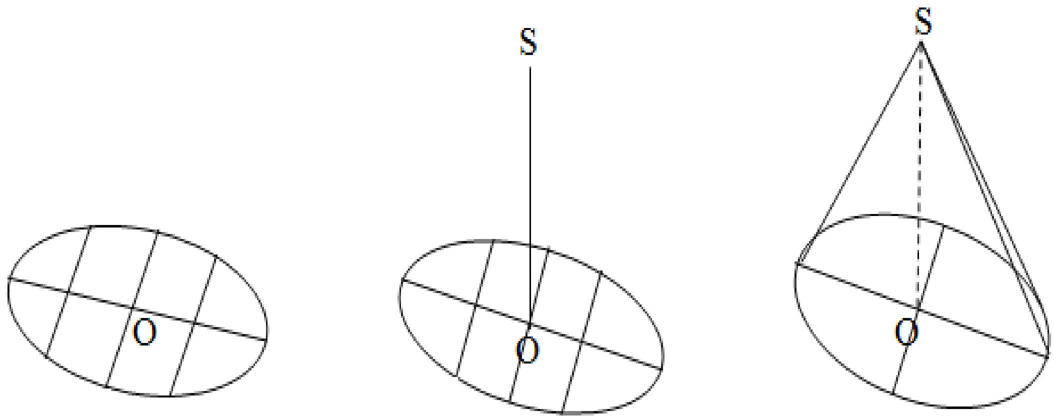
9. Subfigure (b) marks the center of the ellipse and the vertical axis from this center;

10. Subfigure (c) demonstrates the placement of the cone’s vertex along the vertical line;

11. Subfigure (d) displays the two tangents from the vertex touching the ellipse, completing the projection.

In conclusion, the construction of a cone’s projection on paper, though conceptually straightforward, requires attention to geometric accuracy and spatial understanding. This method not only teaches students to visualize three-dimensional objects but also deepens their grasp of projection geometry, which is foundational in both mathematics and technical drawing disciplines. Through repeated practice and visualization using tools such as GeoGebra or traditional instruments, students can enhance their ability to model complex solids like cones with precision.

Figure 1.1.14 Drawing a cone shape



The Study of Combinations of Geometric Shapes: Theoretical Foundations and Practical Applications

The study of combinations of geometric shapes occupies an important place in modern mathematical sciences. It stands at the intersection of spatial geometry, stereometry, and applied geometry. This field examines how different geometric solids—such as prisms, cylinders, cones, pyramids, and spheres—interact and combine with one another to form more complex structures. These combinations may involve unions, intersections, cross-sections, or compound solids created through rotation, translation, or extrusion.

The relevance of this area lies in its dual nature: on the one hand, it forms a fundamental part of the theoretical framework of geometry taught at school and university levels; on the other hand, it has wide-ranging applications in various practical disciplines. In mathematics education, the analysis of shape combinations helps develop spatial reasoning, visualization skills, and logical thinking. These are essential competencies for students preparing to engage with higher-level mathematics, computer modeling, and engineering design.

From a theoretical perspective, studying how geometric shapes combine allows learners to investigate the properties of complex solids. For example, by combining a cone and a cylinder, or intersecting a pyramid with a sphere, one can explore how volume, surface area, and symmetry behave under such transformations. Understanding these relationships lays the groundwork for more advanced studies in calculus, topology, and three-dimensional modeling.

In the field of engineering, the analysis of combined solids is indispensable. Many engineering designs—especially in mechanical and civil engineering—rely on the accurate calculation of volume, weight distribution, and structural stability of complex bodies. For instance, the fuselage of an aircraft, the nozzle of a rocket engine, or the body of a turbine often represents a combination of cones, cylinders, and spheres. Without the geometric modeling of these combinations, it would be impossible to design them accurately or to simulate their function under different conditions.

In architecture, the influence of geometric combinations is equally significant. Architects frequently utilize basic geometric forms such as cubes, pyramids, and cylinders to design buildings and structures that are both aesthetically pleasing and structurally sound. The famous glass pyramid at the Louvre Museum in Paris or the domes in Islamic architecture illustrate how combinations of shapes can result in iconic and functional structures. Geometric analysis ensures that these combinations meet both visual and physical requirements, such as load-bearing capacity, balance, and spatial harmony.

Design and industrial modeling also benefit greatly from this area of study. In product design—whether for automobiles, household appliances, or consumer electronics—geometric modeling allows designers to create ergonomic and visually appealing forms. These forms are often combinations of elementary solids that have been modified, smoothed, or optimized. The study of how geometric shapes can be combined, deconstructed, or reshaped thus becomes a core skill in 3D modeling software such as SolidWorks, AutoCAD, Rhino, and Blender.

Furthermore, in education, the topic of shape combinations enhances curriculum content in geometry and mathematics. By introducing students to real-world problems that involve combining solids—such as calculating the volume of a complex object or visualizing its cross-section—teachers help bridge the gap between abstract concepts and their practical applications. Digital tools like GeoGebra, Math3D, and 3D calculators have made it easier for students to experiment with these combinations in a virtual environment, improving both understanding and engagement.

Finally, it is important to emphasize the interdisciplinary nature of this field. Geometry, by nature, is not confined to one subject. The principles developed in the mathematical study of shapes and their combinations have applications in fields as diverse as physics (e.g., modeling particle trajectories), computer graphics (e.g., rendering 3D environments), and even biology (e.g., understanding cellular or anatomical structures).

In conclusion, the study of combinations of geometric shapes is not limited to a narrow mathematical interest but is a versatile and essential area that supports a wide range of scientific, engineering, artistic, and educational activities. As technological development continues to progress, the importance of mastering the principles of geometric combinations will only increase, making it a vital component of modern scientific literacy.

1.2 Review of works related to the combination of geometric shapes

The combination of geometric shapes is one of the most important areas of study of the interaction of shapes in mathematical space (intersection, Union, overlap, symmetry, transformation, etc.). This problem is not limited to classical geometry, but is now widely used in the fields of Applied Mathematics, Computer science, architecture, engineering modeling, robotics, computer graphics and mathematical education.

The first concepts of geometric combinations were formed in antiquity, in particular in Euclid's work "beginnings" (Elementa). This work is the beginning of a systematic logical structure based on geometric axioms. In addition, Archimedes ' methods for calculating areas and volumes were important from the point of view of analyzing combinations of geometric shapes.

During the Middle Ages and Renaissance, geometry developed in close contact with astronomy and cartography. Later, in the XX century, scientists such as Paul Erdesh, Blaise Pascal, and John Conway conducted important research in the field of discrete and combinational geometry. In this area, the problems of the c onfiguration of polygons, the theory of the arrangement of figures, and geometric cladding (tiling theory) were studied.

Currently, combinations of geometric shapes are considered as the main object of algorithmic geometry (computational geometry). Algorithms developed in this area (for example, Convex Hull, Sweep Line, Voronoi diagrams, Delaunay triangulation) allow you to effectively calculate the relationship between figures and possible combinations.

The study of the Union and intersection of figures in computer graphics influenced the development of animation, 3D modeling, design, virtual reality and game technologies. And in robotics, the planning of the movement and location of objects in space is based on this theory.

In Kazakhstan, the work of several outstanding scientists in the development of geometry and its applied aspects is especially significant.

Sarybaev N. A.-teacher-scientist who was engaged in the methodology of teaching geometry. His works focus on ways of interpreting combinations of geometric shapes through intuitive understanding and visualization in the school curriculum. Especially highly appreciated in the pedagogical community are his works on the role of the use of combinations of figures in the development of logical thinking of students.

Tulegenov A. T.-studied the study of spatial figures and ways of their application. In his scientific articles, symmetries and transformations of geometric shapes in three-dimensional space are considered and their relationship with practical problems is shown.

Kaliev S. A. is a scientist who wrote works on geometry and its Applied Problems. He studied the possibilities of using modern methods (for example, computer modeling) in building models of geometric objects.

A. H. Tursunov and K. A. Baizhumanov-considered the integration approach to teaching geometry in the field of technical and Engineering Education, analyzed the relationship between polytechnic education and geometric modeling.

Candidate of Pedagogical Sciences of Kazakhstan Musabekova zh.K. in her scientific and methodological works, she considered ways to use combinations of geometric shapes in order to develop students ' spatial thinking.

In the secondary education system in Kazakhstan, combinations of geometric shapes are also widely considered in textbooks on the subject "geometry". For example, in the textbooks of such authors as Toksanbayev, Abylkasymova, Kosanov, combinations of triangles and rectangles, their properties and ways to solve the problem are presented with specific examples.

The combination of geometric shapes is an urgent scientific direction that has not only a theoretical, but also an applied nature. This topic is not only well studied in world science, but also filled with the contribution of Kazakhstani scientists. Effective mastery of combinations of figures in the process of teaching and applying geometry opens the way to the development of spatial thinking, improving logical analysis and achieving success in scientific and technical fields.

Spatial thinking and the development of geometric knowledge

In the works of such scientists as V. A. Gusev, G. I. Sarkisov, A.V. Pogorelov, the problems of the study of spatial figures and the development of geometric thinking of students are considered.

In the textbook "Geometry" by A.V. Pogorelov, methods for the development of logical thinking of a student through the analysis of stereometric figures and their properties are presented.

The construction of new bodies by combining spatial figures is characterized as an important tool in the formation of students ' ability to imagine geometrically.

The combination and ratio of figures

The mutual intersection and combination of cone, pyramid and cylindrical bodies, for example,"geometry" edited by L. S. Atanasyan

it is considered in textbooks, but no in-depth analysis is made.

In the works of I. F. Sharygin, creative approaches to solving geometric problems and ways to develop thinking through the use of complex combinations are proposed.

The concept of a combination is most often reflected in Olympic and non-standard problems. In this direction, the tasks and reports of such authors as N. Ya.

Methodological research

In teaching problems related to the intersection and combination of geometric bodies, Kazakhstani scientists such as K. Baimukhametov, K. B. Bekturganov, A. T. Kapitbayev proposed teaching methods and noted the importance of developing creativity and logic in teaching geometry.

Some scientific and methodological journals (for example, Mathematics and Physics, School of Kazakhstan) have published models and author's methods for solving complex stereometric problems

In general, studies on combinations of geometric shapes cover a wide range, however, in a specific form, the methodological system for combinations of cones and pyramids is still insufficiently studied. This proves that this is a new and necessary direction for future research. One of the scientists who took a special place in the teaching of stereometry in the Soviet era is E. G. Gotman. His work" Stereometric problems and methods for solving them " (2006) is distinguished by a combined consideration of the theory of geometric problems and ways to solve them in practice. This book is based on a systematic methodology in the study of spatial figures and has become a valuable teaching tool for teachers and students.Specific application of the concept of combination

Gotman describes in detail the combination of spatial figures, that is, the consideration of several bodies (for example, a cone and a pyramid) as a single geometric structure. He calls such bodies "composite bodies" and proposes to use the method of classifying them into separate elements for analysis.

Application of analytical approaches Gotman proposes to use formulas for finding volume and surface area at the heart of analytical geometry. For example, when analyzing a figure connecting a pyramid and a cone, their individual volumes are first determined, and then the full volume of the combined figure is calculated:this approach opens the way to solving the problem at a logical-analytical level, rather than considering it visually.

Step-by-step solution of geometric modeling and the problem

The author offers a clear algorithm for each report, consisting of several steps:

- Figure recognition
- Marking the necessary dimensions
- Select formula
- Calculation
- Draw conclusions

This system is especially effective in solving complex combinational problems (for example, bodies with a pyramid installed on a truncated cone).

The role of visualization and drawings

E. G. Gotman considers drawings as an important tool in solving problems. It provides clear illustrations for each report and prioritizes the development of the student's spatial imagination through drawings.

With this approach, the student or student will better understand the structure of the problem, as well as get used to describing spatial figures in mathematical language.

Gotman's approaches develop students ' logical thinking, modeling ability, and spatial imagination skills. His methods, when combined with modern digital tools (for example, GeoGebra), give very effective results.

This work of is aimed at solving stereometric problems in a systematic, analytical and visual way. This work can be taken as a basis in the development of a methodology for solving complex problems for a combination of a pyramid and a cone (Gotman 2006).

The authors who have a special place in the field of geometric education are L. E. Gendenstein, A. P. Ershova and A. S. Ershova. Their textbooks and manuals for secondary school show the most effective ways to teach stereometric problems. These works are aimed at the development of spatial thinking and allow us to explain the combinations of basic figures, such as a pyramid and a cone.

The principle of modeling complex problems

The method proposed by the author is to teach figures by associating them with real life situations. For example, considering the shape of a tower or roof as a geometric combination will clarify the student's abstract understanding. This is especially effective in calculating architectural structures formed by the Union of a pyramid and a cone.

Development of spatial imagination

According to the approach of ershovas and Gendenstein, solving stereometric problems develops three-dimensional thinking in the student. They propose to analyze spatial figures through a drawing, highlighting the most important elements and breaking them down into several steps.

Step-by-step solution algorithms

The recommended method for each problem is the path "analysis → formula selection → calculation → conclusion". This structure teaches the student to think systematically and allows him not to make mistakes in complex combination problems.

- Linking geometry with Construction, Engineering, Engineering;
- Involve students in project work (for example, creating a 3D layout);
- Clarification of the mathematical description of complex bodies.

In the case when the pyramid is located inside a cone, it is necessary to find the volume of their intersecting part, the authors propose to draw a section of the figure and consider it as two separate bodies. This approach is based on logical calculation and spatial fragmentation.

Kazakh teachers-scientists Aimagambetova K. T. and Mukhambetkalieva Sh.zh. set out to combine geometric knowledge with national reality. In their works, ways are considered to teach combinations of geometric shapes in an applied context, in interdisciplinary communication, and in combination with natural sciences.An approach based on the student's practical activity.The authors propose to teach spatial figures not only of a theoretical nature, but also with life examples: the dome of the yurt (cone), historical towers (pyramid-shaped) and architectural monuments are taken as examples. Such an approach makes the student interested in the subject through national cognition. Project learning and teamwork.They introduced tasks for creating combinations of geometric bodies in the form of layouts and based this process on creativity. For example:

– "Construction of an architectural object by combining a cone and a pyramid";

– "Propose a way to use effective material by comparing volumes".

Interdisciplinary communication

The solution of complex problems by teaching geometry in connection with Physics, Technology, Computer Science increases the student's functional literacy.

Kazakh scientist A. T. Tulegenov has written scientific works on the study of spatial figures and methods of their application. His research focused mainly on geometric symmetries and transformations (transformations) in three-dimensional space.Theory of symmetry and transformations Tulegenov's scientific articles analyze in detail such transformation operations of geometric shapes in three-dimensional space as rotation, movement, reflection. These concepts play an important role in the visualization and modeling of pyramid and cone combinations. Relationship to practical problems The scientist considers geometric transformations in connection with real-life engineering problems, construction design, design elements. For example, he proved that it is possible to calculate the stability and equilibrium of geometric bodies through the properties of symmetry.

Development of spatial recognition skills.

Tulegenov's works contribute to the development of spatial imagination in schoolchildren and students. It offers new ways to describe the shape, size and other properties of three-dimensional objects through mathematical modeling.

- Determination of body symmetry in mechanics;
- Work with spatial rotations in astronomy and physics;
- Use of symmetrical structures in construction design.

S. A. Kaliev is one of the prominent representatives of the domestic field of geometry. He focused heavily on teaching geometric shapes through computer modeling and studying their relationship with real life.

The role of geometry in solving applied problems

Kaliyev identified the relationship of geometry with other branches of Mathematics (Physics, Engineering, Computer Science) and showed the close relationship of complex spatial problems with practice. This is especially important in the study of bodies formed by the Union of a pyramid and a cone.

Features of computer modeling

In his works, he considered the possibilities of creating a visual model of geometric objects using digital resources such as GeoGebra, Autodesk, 3D Max. This technique helps students to correctly understand the figures and clearly imagine their properties.Development of logical thinking through modeling

Kaliev stressed that modeling is not only a visual, but also a means of developing logical, constructive thinking. For example, separating complex combinations into parts, creating an individual model of each part, and finally combining them is an approach close to real engineering thinking.

According to the methods proposed by him, students distinguish a complex figure into parts, create a 3D model of each part individually, and finally combine it. With this approach, complex situations such as the intersection, confrontation or overlap of a pyramid and a cone can be easily explained. The works of tulegenov A. T. and Kaliev S. A. studied geometry not only theoretically, but also in combination with applied and technological methods. Their methods can be widely used in the modern educational system, especially in the context of the methodology for solving complex problems related to the combination of a pyramid and a cone. Scientists A. H. Tursunov and K. A. Baizhumanov, who dealt with the problems of teaching geometry in the applied field, in their works considered integration approaches to teaching geometry in the system of technical and engineering education. Their research reveals the relationship between polytechnic education and spatial thinking.

The authors identified the advantages of teaching by linking geometric knowledge with engineering disciplines. For example, in such disciplines as technical drawing, mechanical engineering geometry, construction design, he draws attention to the need to use spatial figures, including combinations of pyramids and cones.

Polytechnic basis of geometric modeling

Scientists have shown that through geometric modeling, students and students develop logical, engineering and creative thinking. In the course of modeling, structurally separate complex bodies (for example, the intersection, overlap or union of pyramids and cones) are proposed as an important method.

The role of geometry in solving engineering problems

Tursunov and Baizhumanov call for the use of geometry as a tool in solving engineering problems. This makes it necessary to know the exact characteristics of spatial bodies, especially in construction, technical drawing and design.

- Combination of geometry with professional skills in vocational education;
- Ensuring the execution of geometric models with engineering accuracy in design work;
- Effective learning of spatial figures through visualization tools and CAD/CAE systems.

One of the examples proposed by the authors is the modeling of a technical device consisting of the Union of a cone – shaped funnel and a support in the shape of a pyramid. In such a case, the figures are considered not only in mathematical terms, but also in engineering parameters (for example, stability, material consumption, volume efficiency).

The research of A. H. Tursunov and K. A. Baizhumanov laid the scientific and theoretical basis for the use of geometry in engineering-oriented teaching. They proposed the modeling of geometric shapes not only in a schematic or theoretical context, but as a real engineering necessity. This approach is of particular importance in solving complex problems related to the combination of a pyramid and a cone.

1.3 Depending on the combination of geometric shapes digital resources

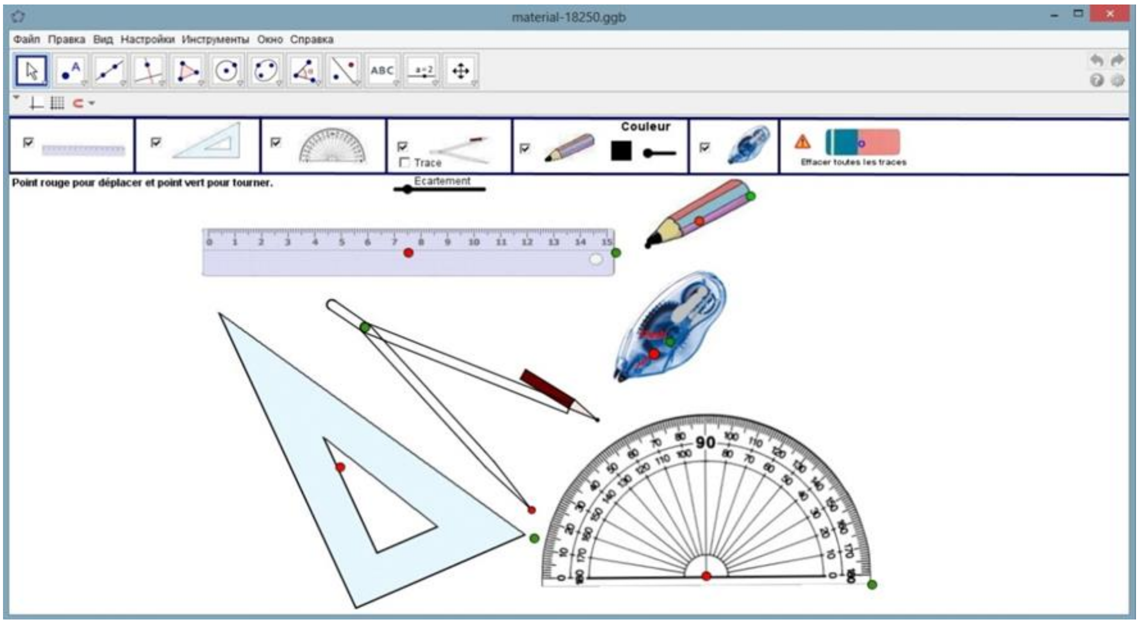
GeoGebra is a cross-platform dynamic program for all levels of mathematics, including geometry, algebra, graphs, statistics, and arithmetic.

The program has many possibilities in working with functions (graph construction, calculation of roots, extremum points and integrals).

The program was invented by Australian inventor Marcus Hohenwarter. The program is in JAVA (runs on a multi-feature operating system). Currently, Geogebra has been translated into 39 languages and is working intensively. There is a full opportunity to work in Russian.

GeoGebra is a common dynamic geometric environment. GeoGebra allows you to draw animated drawings using a compass and Ruler in planimetry.

Figure 1.3.1 The program has the following features



X = f(t); y = g(t) construction of parametric curves in Cartesian coordinate systems (postroenie);

plotting a function graph in the form y = f(x) ;

Construction of conical sections;

4. Circle: - Center and with a point on it;

with center and radius;

along three points;

Ellipse-by two foci and points;

Parabola-by Focus and directrix;

Hyperbola-with two foci and a point on the curve;

Truncated cone (free – form conics) - by five points;

Calculations

Working with The Matrix;

Add, multiply;

Transponding, inverting;

Calculation of the determinant;

Calculation of complex numbers;

Finding curves and points;

Statistical function:

Variance, calculation of mathematical expectations;

Calculation of the correlation coefficient;

Approximations of many points of a given curve;

polynomial;

exponenta;

logarithm;синусоида;

The program allows you to create a Java applet of dynamic drawings that create conditions for connecting to web pages.

Among the modern digital educational tools, the GeoGebra program stands out for its versatility and is known as a platform with widespread use. This program plays a special role in the visualization and study of geometry, and also allows students and students to develop the skills of spatial thinking, geometric modeling and analytical analysis.

GeoGebra is a free and open access program for working with two-dimensional and three-dimensional geometric objects. Especially in the study of combinations of cones and pyramids, this platform provides full conditions for the real and clear modeling of complex figures, changing their parameters and studying with visual observation.

For example, a student can build a mutual intersection of a cone and a pyramid and determine the area of their intersection. In addition, he can dynamically observe the cross-sections or volumetric changes of figures and test various mathematical patterns through experience. Such actions form not only theoretical knowledge, but also practical skills of the student.

The main advantages of the GeoGebra program include:

An interactive environment that allows you to accurately and accurately control geometric shapes;

The ability to experiment with the model, changing the parameters of the figure (height, radius, edge, angle, etc.);

The ability to automatically calculate sections, symmetry, volume and surface areas;

Visual representation of the results through animation or 3D view.

In addition, the GeoGebra platform is also a useful tool for teachers. Through it, it is possible to develop various didactic materials, compose interactive tasks and individualize the educational process.

In conclusion, GeoGebra is an affordable and effective digital resource that plays an important role in teaching and learning combinations of geometric shapes. It creates conditions for combining theory and practice in teaching mathematics and increases the cognitive activity of the student. In addition, it encourages the creative mastery of geometry and engages the student in research activities.

Figure 1.3.2 GeoGebra platform

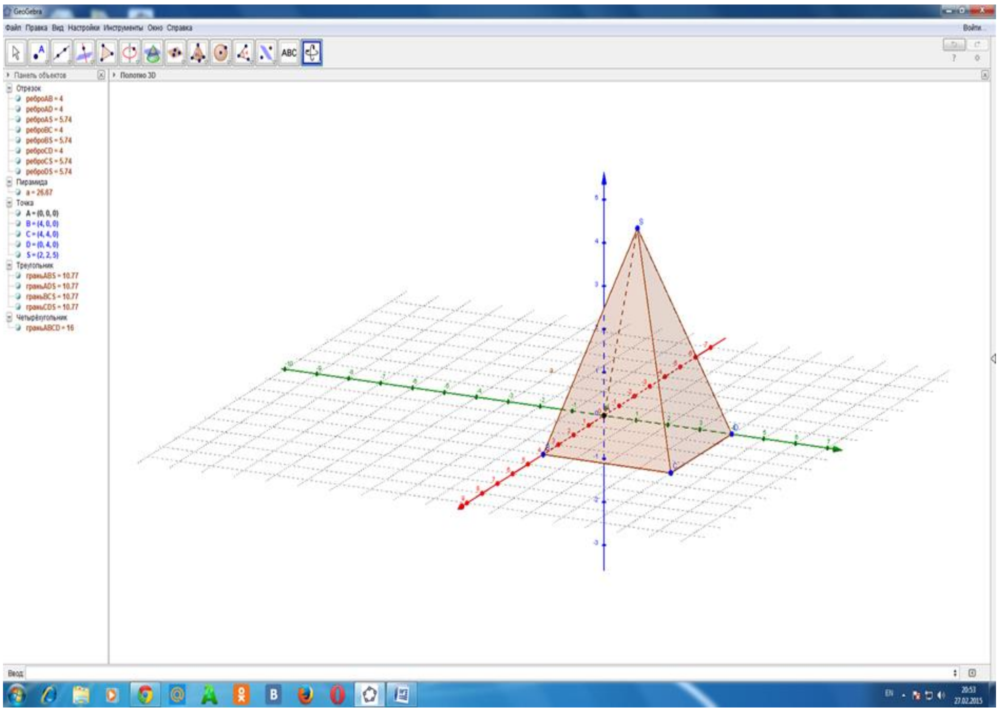
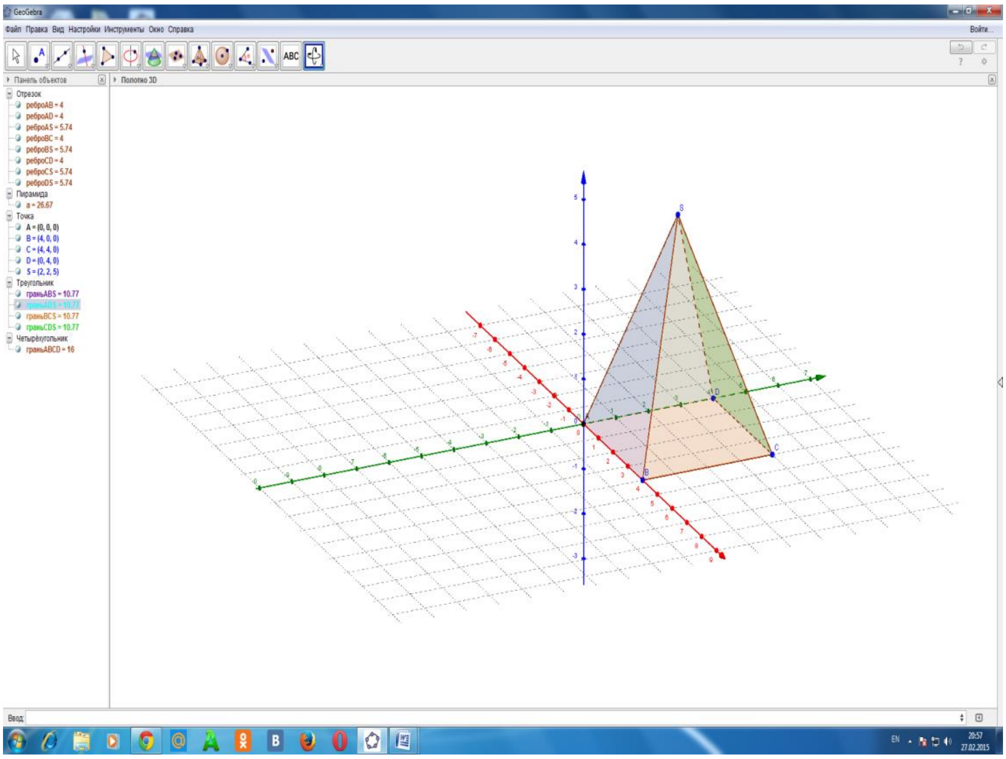


Figure 1.3.3 Parallel to the base of the pyramid divides



Now let's give some examples from stereometry problems performed with the Geogebra program:

The section of the plane parallel to the base of the pyramid divides the height in such a way as a 1:1 ratio. What is the cross-sectional area if it is the base

Figure 1.3.4 Cross-sectional area

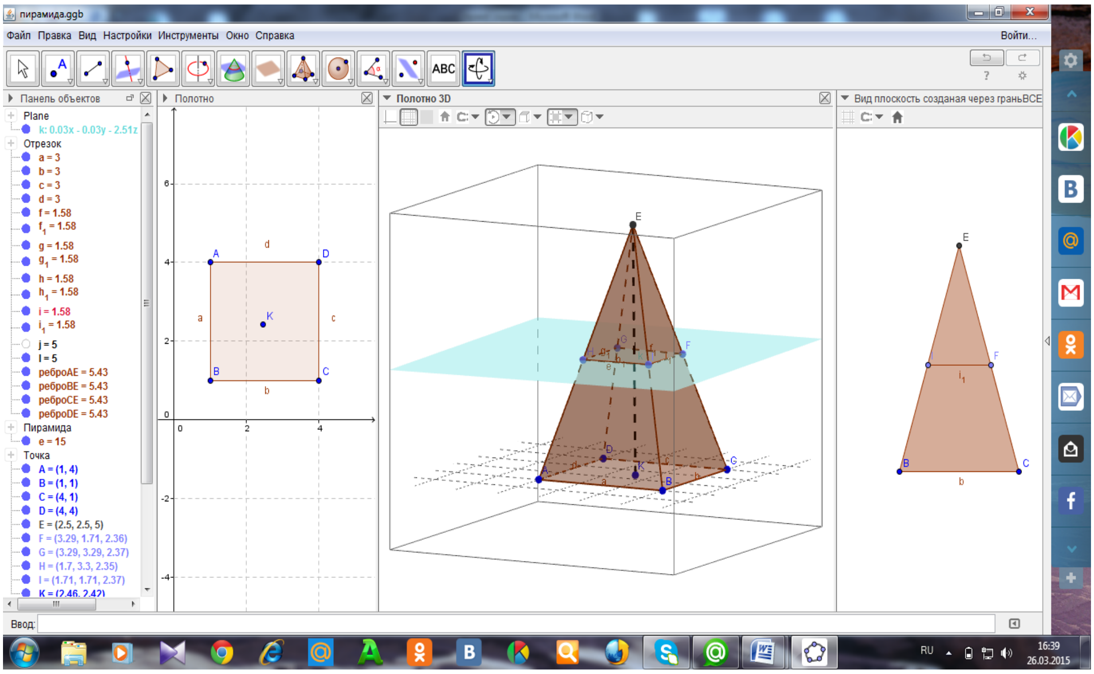
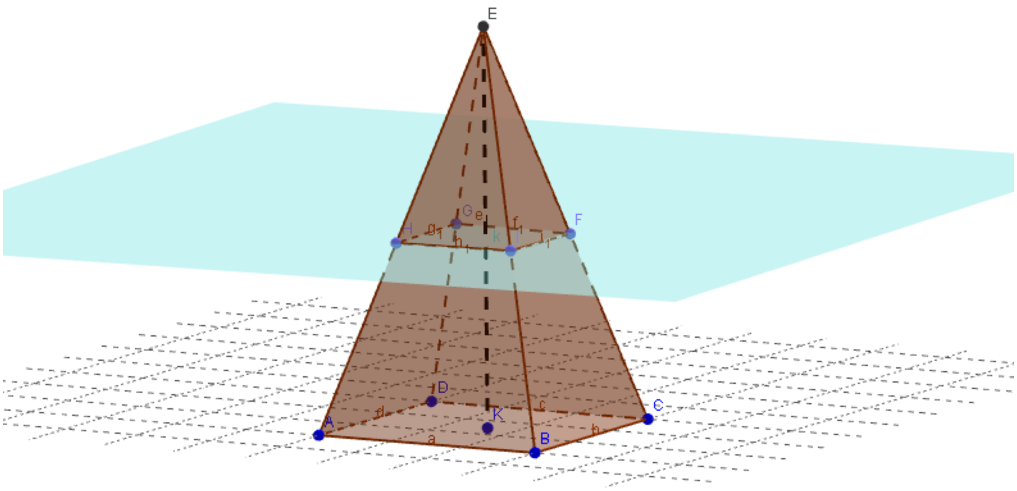


Figure 1.3.5 Similar to the base of a pyramid



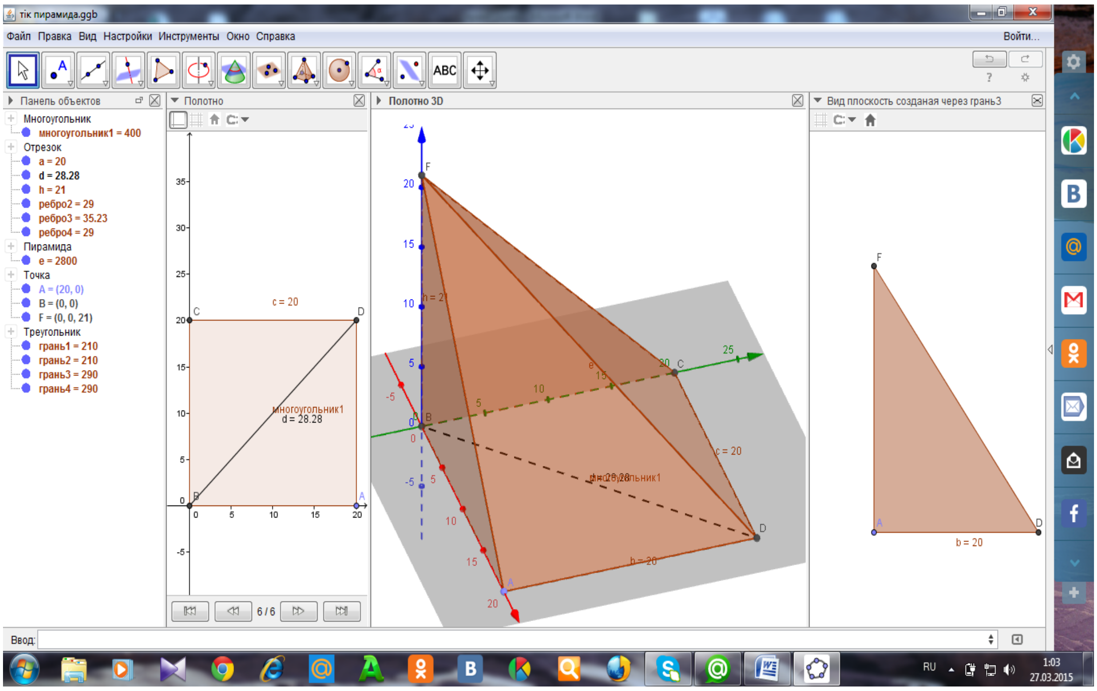
The section is similar to the base of a pyramid, the similarity coefficient, so

Answer: cross-sectional area

Report #3

The base of the pyramid is a square. Its height passes through one roof of the base. The base wall of the pyramid is 20 DM and the height is 21 DM. Find the side face of the pyramid

Figure 1.3.5 The base of the pyramid is a square



Report # 4

The volume of the cone . The height of the cone is 5 cm. A plane passes from the top of the cone at a distance of 2 cm, and parallel to its base. Find the volume of the resulting truncated cone

Figure 1.3.6 The volume of the cone

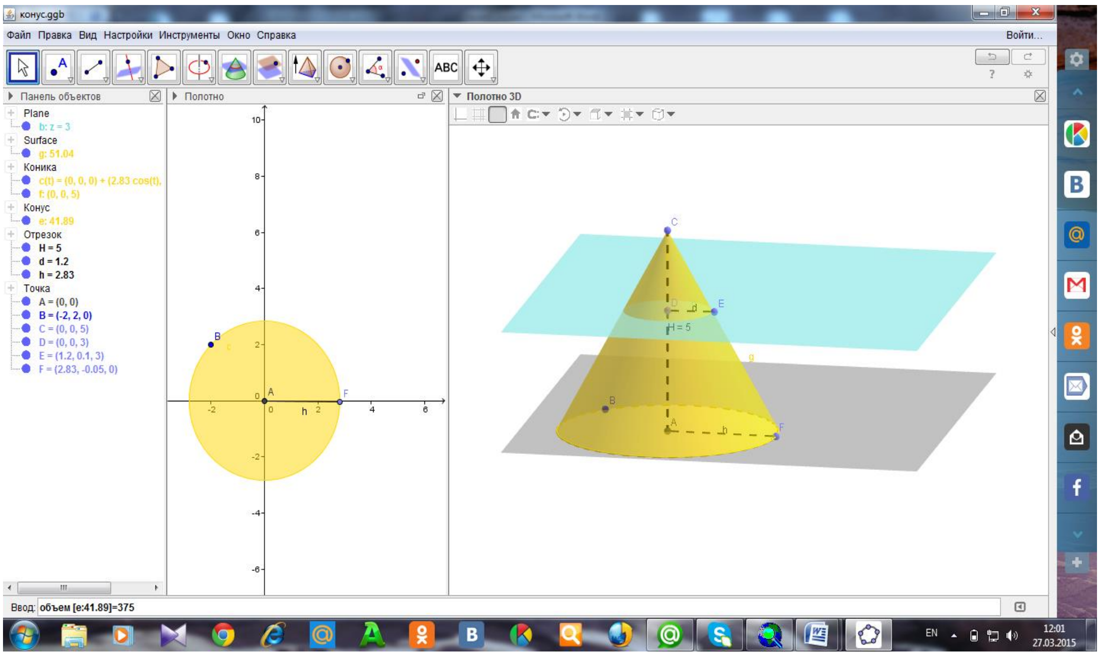
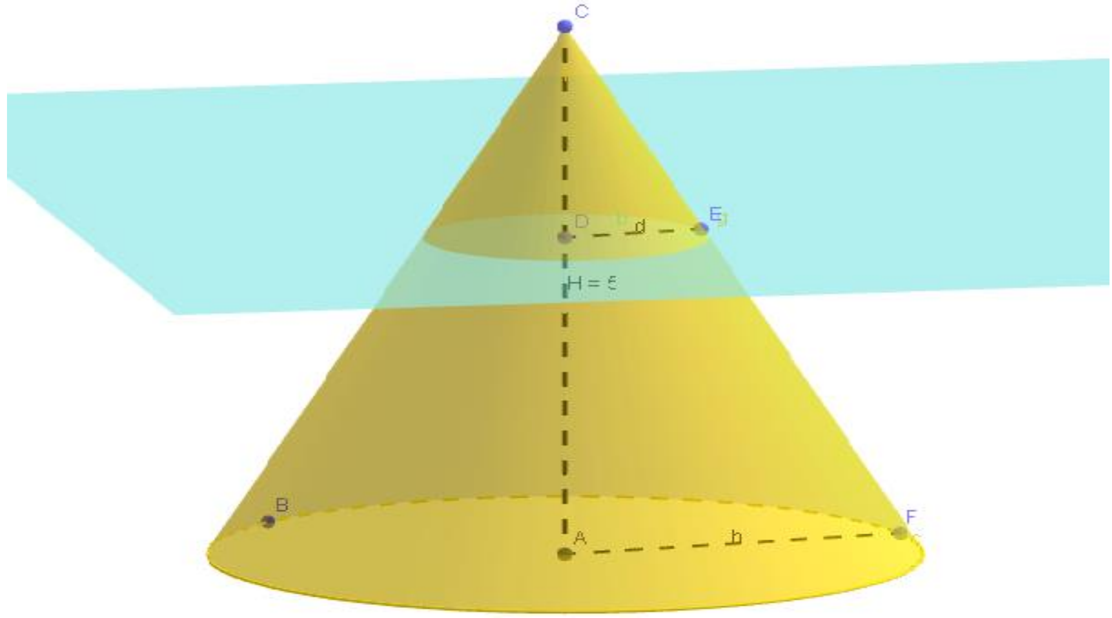


Figure 1.3.7 The height of the cone



$$V = 375cm^3 \quad V = \frac{1}{3\pi R}^{2}$$

$$H = \frac{1}{3} \cdot \pi R^2 \cdot 5\text{R=1s}$$

$$\frac{CD}{CA} = \frac{DE}{AF} \rightarrow \frac{2cm}{5cm} = \frac{r}{15cm} \rightarrow r = 6cm$$

SolidWorks is a modern professional program for three-dimensional (3D) engineering design, modeling and drawing. It is a powerful CAD (Computer-Aided Design) System for working with spatial objects. Currently, the SolidWorks program is widely used not only in production, but also in technical and mathematical education, including modeling spatial geometry, in particular, complex problems related to the combination of a pyramid and a cone.

In SolidWorks, you can build spatial bodies with precise dimensions and accurately calculate their structure, surface properties, and geometric parameters. The following features are provided in this program:

Parametric modeling: allows you to draw pyramids and cones with real dimensions (height, radius, sides of the base, angles).

Cross and fold operations: use SolidWorks to cross two or more shapes, automatically identify their common part, and calculate the volume and face area of that area.

Sketch → with Extrude/Loft/Surface tools, the figures are transformed from a drawing into a real body;

Assembly Mode-combine several figures in one space to create combinations, and you are given the opportunity to view, rotate, disassemble them at an angle;

Section View-you can visualize the substructure by cutting the crossed part in the horizontal or vertical direction.

To create a combined model of a pyramid and a cone in SolidWorks, you need to go through several main stages:

1. draw each figure separately (Sketch):

o cone by Circle + Extrude;

O formed the pyramid by Polygon (for example, a rectangle or hexagon) + Extrude.

2.o SolidWorks with the" Combine " tool can be used to cross two bodies, leaving only a common part, or to remove one body from another (Boolean operations).

o this is especially true in reports: "what is the common volume when a cone is inserted inside a pyramid?"important to answer questions such as"

3. cutting the Model (Section View):

o the possibility of cutting in the horizontal or vertical direction is provided to see the internal structure of the figure. This is especially useful for explaining to students how shapes intersect.

4. calculation of physical characteristics (Mass Properties):

o SolidWorks automatically calculates data such as volume, surface area, weight, center mass;

o it allows the student to compare the result of the calculation with the result of the program. SolidWorks can be used not only for visualization, but also for solving specific problems:

Example 1: a cone is inserted inside the pyramid. The base of the cone faces the base of the pyramid, and its tip Points towards the center of the pyramid. By modeling this combination, it is necessary to calculate their common volume.

Example 2: cut the pyramid with a cone and find the surface area of the remaining part. In this case, the remaining body model can be obtained by removing the cone part through the Subtract operation.

Example 3: position the cone according to the extreme sides of the pyramid and determine the resulting surface area using Section View.

Using the Solid Geometry program to solve stereometry problems

Solid Geometry is a program designed to accurately and quickly calculate the volume of a figure, areas and masses of various geometric bodies. They are:

Cylindrical tank

spherical segments

sphere sector

sphere, spherical sections

Pyramid, truncated pyramid

prism,

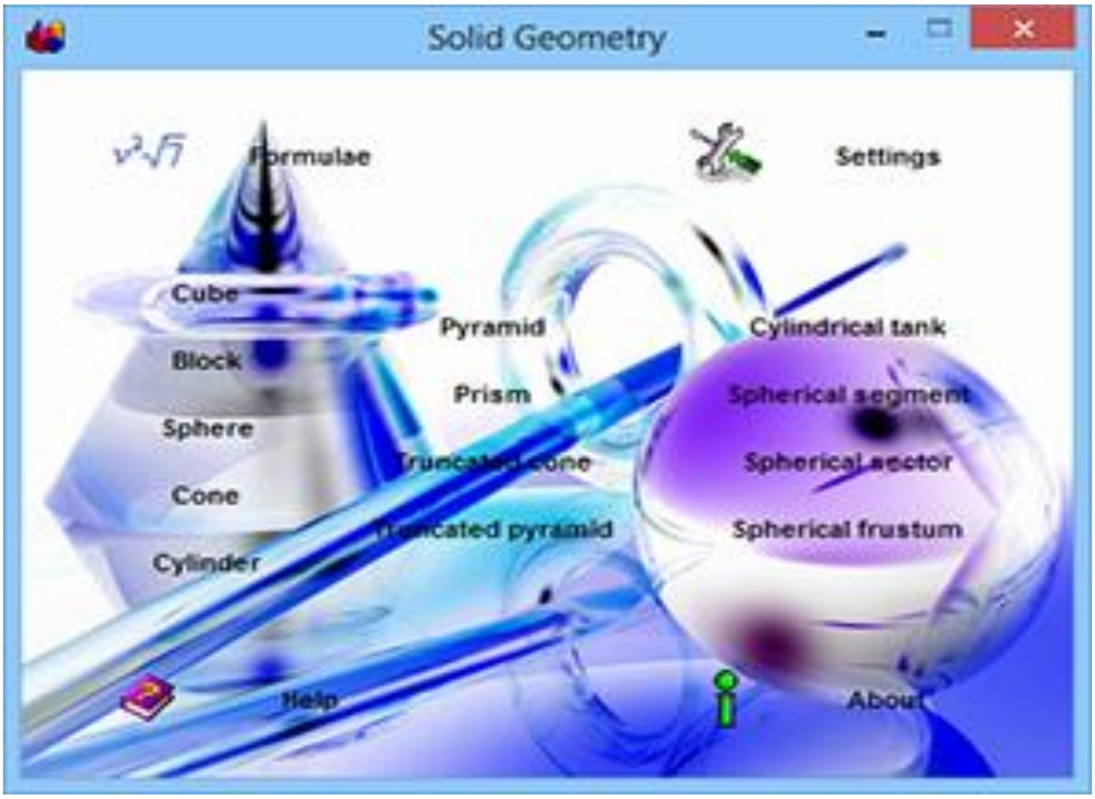
cone, beveled cone

cube, block

cylinder, etc.

The program provides for the possibility of creating an illustrative project for each geometric figure, cases of manual calculation of formulas.

Figure 1.3.8 SolidWorks



In the context of teaching spatial geometry and developing students' skills in geometric modeling, the use of modern digital tools plays an important role. One such tool is the Solid Geometry program, which allows for the visualization, construction, and measurement of three-dimensional objects. By integrating this software into classroom practice, educators can enhance learners' understanding of complex geometric properties through interactive and visual experiences. To illustrate this approach, let us consider a specific geometric problem involving a regular square pyramid. The given problem is as follows:Problem Statement**:** The base of the pyramid is a square with each side measuring 4 cm. The height from the apex (vertex of the pyramid) to the center of the base is 11 cm. All lateral sides (triangular faces) of the pyramid are equilateral. Using this data, find:

1. The lateral surface area
2. The total surface area
3. The volume of the pyramid.

The pyramid described is a regular square pyramid, which means:

The base is a square (all sides equal).

The apex is directly above the center of the base (vertical height).

Each lateral face is a congruent equilateral triangle.

Let’s denote the key elements of the pyramid:

Side of the base: a=4 cm

Height from the apex to the base (perpendicular): h=11 cm

Since the lateral faces are equilateral triangles, each slant edge is equal to the base side: l=4 cm

However, here we encounter a geometric contradiction**:** if the slant edges are 4 cm and the height is 11 cm, this cannot form a real pyramid because the slant height (hypotenuse) must be longer than the vertical height in a right triangle. Hence, either the pyramid’s height is less, or the lateral faces are not exactly equilateral.

To resolve this, we utilize the Solid Geometry program to visualize the pyramid. Using the “construct 3D shape” tool, we input:

A square base with side 4 cm.

A vertical height of 11 cm from the center of the base to the apex.

By doing this, we can observe the true length of the slant edge using the Pythagorean Theorem:

s=22+112=4+121=125≈11.18 cm

Thus, the actual slant edge is approximately 11.18 cm, and the lateral faces are isosceles triangles, not equilateral.

The area of one isosceles triangle is given by:

Atriangle=12·base·slant heigh

To find the slant height lll (height of the triangular face), we use:

l=125−4=121=11 cm

Now calculate the area of one triangle:

Atriangle=12·4·11=22 cm^2

There are 4 triangular faces:

Al=4·22=88 cm^2

Base Area

Ab=a^2=4^2=16 cm^2

Total Surface Area (A_{total})

A_{total}=88+16=104 cm^2

The volume of a pyramid is given by:

V=13·Ab·h=13·16·11=1763≈58.67 cm^3

Using the Solid Geometry program, the construction of this pyramid can be done step by step:

First, draw the square base using the specified dimensions.

From the center, erect a perpendicular segment of length 11 cm (the height).

Connect this apex to all four base vertices.

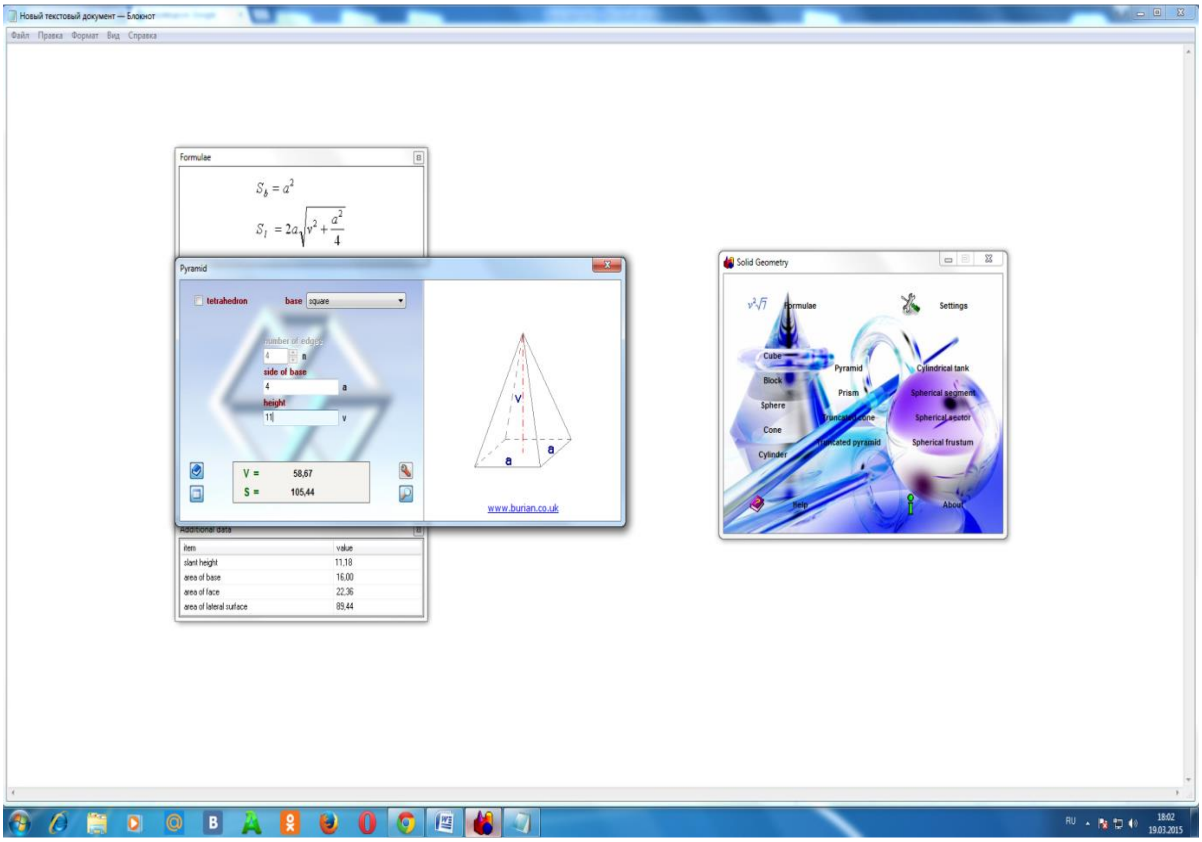
Use the measurement tools in the program to verify the slant edge and face heights.

Shade or highlight surface areas to visually demonstrate base and lateral areas.

Use volume calculation tools if available, or input formulas.

This example shows the value of combining analytical geometry with digital visualization tools**.** By using Solid Geometry software, students are not only able to verify the theoretical values but also to *see* the geometric properties in action. They develop deeper insights into spatial relationships and gain confidence in solving complex 3D geometry problems. This method is particularly effective for STEM education, where accuracy and visualization go hand in hand.

Figure 1.3.9 Solid Geometry program as an example



The height of the side surface is– $11{,}18cm$

Area of the base –$16cm^2$

One side surface area – $22{,}36\text{cm}^2$

Lateral surface area –$89{,}4\text{cm}^2$

Answer: $\text{S} = 105{,}4\text{cm}^2 \quad V = 58{,}26cm^3$

Figures drawn in Mathcad

SolidWorks is a professional tool for the visual and computational study of combinations of geometric shapes, including complex types of mergers of a pyramid and a cone. Using this program, the student will not only learn to work with spatial models, but also understand specific physical characteristics and develop engineering thinking skills.

In the modern educational system, in the process of mastering spatial geometry, preference is given to visualization and interactivity. One of the most widely used tools for this purpose is the Tinkercad program. It is a very easy-to-use

3D modeling platform developed by Autodesk, which runs on a cloud basis (online). The program allows even schoolchildren and students without technical education to easily master the skills of three-dimensional modeling.

Accessibility: Tinkercad is a web application that works directly through the browser. Does not require installation, can be used from any device connected to the internet.Intuitive interface: the user interface is very simple. Teachers and students can learn the basic functions in a few minutes and start building uncomplicated figures.Basic 3D figure base: the library of bodies such as Pyramid, cone, cylinder, ball is presented ready-made. You can drag them onto the screen, resize them, merge them, or crop them.Intersecting and joining: you can easily create complex bodies by intersecting, overlapping, or adding shapes.3D printing adaptation: Ready-made models can be saved in STL or OBJ format and reproduced on a 3D printer.Through Tinkercad, students can visually explore the geometric combination of a pyramid and a cone. The modeling process in this program is simple:

Creating a structure:

From the" Shapes " library, select the Cone (cone) and Pyramid (pyramid) figures and place them in the work area;

You can easily change the size (height, base radius, wall) of each figure with drag & resize.

Creating a combination:

The figures can be stacked or stacked together and combined using Union, Hole, Group tools;

It is especially suitable for visualizing the size of the crossed part.

Show cross section:

If necessary, you can view the internal structure by switching one part to "Hole" mode and cutting it out.

Visual research:

By changing color, turning, zooming in/out, the student sees figures from different angles and develops spatial imagination skills.

The role of digital resources in teaching geometry is growing every year. Especially in teaching combinations of pyramid and cone-shaped spatial bodies, online platforms such as Math3D and 3D Calculator occupy an important place. These tools provide the ability to clearly represent a spatial representation, build an interactive model based on a formula, and perform analytical calculations.

Math3D is an interactive 3D graphics editor used through a browser, especially for the mathematical representation of spatial functions, planes and bodies. If the user enters a description of the figure in the form of a function or coordinate, the program immediately displays it in space.

Building a model based on a function – for example, inserting a cone as z = sqrt(x^2 + y^2) and representing it in space;

Display of planes and intersection points;

Allows you to draw bodies such as coordinate segments, vectors, spheres, cones and pyramids;

Show the area of intersection of any bodies – you can visualize the points of overlap of the pyramid and cone;

With the Rotate, Zoom functions, it is possible to rotate the model and see from different directions;

Establishes communication with the formula and develops analytical understanding.The student can enter the Cone and Pyramid formula, draw them in a common coordinate system and find the intersection area.Can be used to see the beveled part of the cone cut through the plane or the section of the pyramid to the base.3D Calculator is an online tool that calculates the properties of geometric bodies and automatically visualizes them in space. It combines algebra and geometry, allowing you to describe figures in the form of formulas, measures and coordinates.

Automatic volume and face area calculation by entering the parameters of the body;

Setting individual parameters of pyramids and cones (base radius, height, apothem, angles);

3D visualization automatically displays the model in space;

There are tools to calculate the common part of two crossed bodies;

The model can be exported as PDF or PNG – this is useful for drawing illustrations for a dissertation. The student can enter the size of the cone and pyramid, calculate the intersection area or the remaining volume, and compare the result with a visual model;

Math3D and 3D Calculator are modern online tools for working with geometric shapes, including a combination of a pyramid and a cone. They not only develop students ' spatial imagination, but also teach them the skills of analytical thinking, working with formulas, visually solving real problems. Using such platforms, it is possible to create an opportunity to master geometric modeling in depth.

2. METHODOLOGY

2.1 Methodology for teaching complex problems related to the combination of a pyramid and a cone

The combination of a pyramid and a cone represents one of the most intricate and intellectually engaging problems in the field of stereometry. Solving such problems significantly contributes to the development of spatial reasoning skills, as well as to the improvement of students' ability to visualize and analyze complex three-dimensional figures. This area of geometry holds not only theoretical value but also has practical significance across a variety of disciplines, including engineering, architecture, construction, physics, and other technical sciences where the understanding of spatial structures is essential. Investigating the relationship between a pyramid and a cone enables a deeper comprehension of their geometric properties, mutual placement, and interactions.

The process of solving these problems often involves the integration of several mathematical methods such as analytical techniques, trigonometric functions, coordinate geometry, and sectional analysis. In particular, great importance is given to the axial sections of cones and the heights of pyramids, as these elements play a crucial role in deriving formulas for volume and surface area. Furthermore, the comparison of the volumes and surface areas of these two solids helps in identifying internal symmetries, proportional relationships, and geometric similarities. This type of comparative analysis is valuable not only in solving textbook problems but also in real-world applications where understanding the geometric configuration of composite structures is necessary.The main aim of this research is to develop a structured methodology for solving advanced problems related to the combination of cones and pyramids and to demonstrate its practical use in mathematics education. The study proposes a systematic approach to problem-solving through the use of visual aids, geometric modeling, and illustrative examples that enhance students’ comprehension and engagement with spatial geometry.

To address complex geometric problems involving a cone and a pyramid, several key mathematical approaches were utilized:

1. Analytical Method
   This method forms the foundation for solving volumetric problems. The volume of a pyramid is determined by the formula
2. Geometric Method
   This method involves solving problems based on geometric properties and relationships. It allows one to compare shapes by analyzing their dimensions, angles, and symmetries. For instance, by comparing the base areas and heights of a cone and a pyramid, we can identify proportional relationships in their volumes.
3. Trigonometric Method
   Trigonometry is applied to determine unknown angles and side lengths. This

method is especially useful when working with slanted or inclined elements, or when calculating the height of shapes that are not perpendicular to the base.

4. Sectional Method
   The sectional method involves slicing the three-dimensional shape with a plane to study its internal structure. For cones, the axial section forms a triangle that reveals the radius and height, while for pyramids, cross-sections help to analyze slant heights and face angles.
5. Coordinate Method
   By placing the figures within a three-dimensional coordinate system, this method enables accurate calculations of distances and angles. It is particularly effective for solving problems involving positioning, transformation, and alignment of geometric solids in space.

These methods complement each other and, when used in combination with interactive geometry software such as GeoGebra or Solid Geometry tools, they provide a powerful framework for exploring complex problems and enhancing the learning experience in stereometric topics.

Figure 2.1.1 Indicators of the pyramid and cone

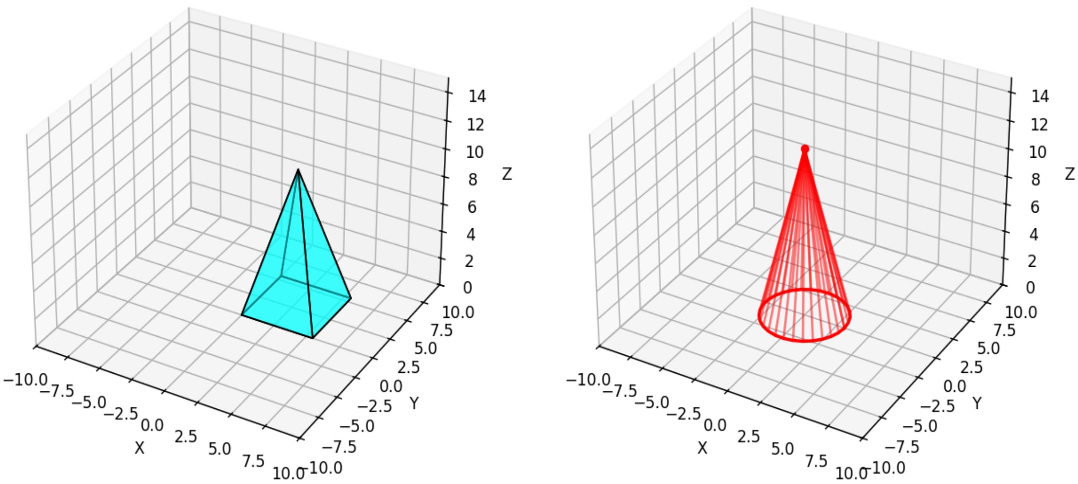


Table 2.1.1 Numerical indicators of the pyramid and cone.

Indicator	Pyramid	Cone
Height (h)	10 см	12 см
Area of the base (S)	30 см²	40 см²
Volume (V)	100 см³	160 см³

The volumetric characteristics of the pyramid and Cone were shown using graphs. The results showed that by analyzing the geometric relationship between the two figures, accurate calculations can be made regarding their volume and area..

Table 2.1.2 Advantages and scope of application of the methods.

Method	Advantages	Scope of application
Analytical method	High accuracy, based on formulas	Engineering, construction
Geometric method	Ability to explain visually	Education, theoretical research
Trigonometric method	Convenient for calculating angles and distances	Navigation, aerospace
Section method	Allows you to analyze the internal structure of figures	Mechanics, design
Coordinates method	Allows you to model spatial bodies	Computer geometry, physic

Graphs and diagrams are used to visually represent the results obtained. The features of each method and the effectiveness of their application are proved using specific examples. Although the analytical method shows high accuracy in solving complex problems, for its application it is necessary to clearly understand the formulas. Since the geometric method is based on visualization, it is effective in learning. The trigonometric method is convenient for calculating angular measurements and distances. The sectional method allows you to study the internal structure of spatial bodies. The coordinate method helps in three-dimensional modeling.

Analytical method

Example: the base of the pyramid is square, the side is 6 cm, the height is 10 cm. Let's calculate its volume:$S_{\text{the basis}} = 6^2 = 36\text{см}^2 V = \frac{1}{3} \times 36 \times 10 = 120\text{см}^3$

Geometric method

Example: if a cone and pyramid have equal heights and the same base area, what is the ratio of their volumes?$\frac{V_{\text{pyramid}}}{\text{cone}} = \frac{h/3}{\pi R^2 h/3}$

As a result, the ratio of volumes will depend on the shape of the base of the pyramid and cone.

Trigonometric method

Example: the base radius of the cone is 5 cm, the angle created by the jaw surface is 30°. Let's find the height.$h = R \times \tan(30^{\circ}) = 5 \times \tan(30^{\circ}) \approx 2.89$ см

Section method

Example: the axial section of a cone is in the form of an equilateral triangle. What is the sectional area if its height is 12 cm and the length of the base is 10 cm?$S = \frac{1}{2} \times 10 \times 12 = 60$ см^2

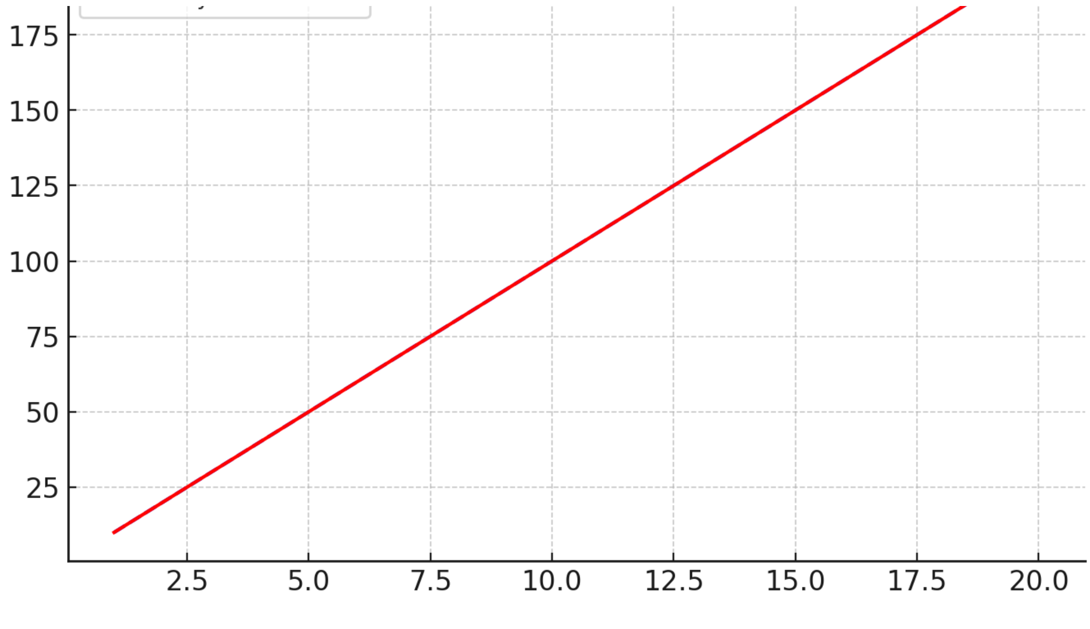
Coordinate method

Example: the vertex of the pyramid is (0,0,6), and the four corners of the base are located in coordinates (±3, ±3, 0). Finding the height by the vector method: h=6 cm (since the vertex coordinates are located vertically upwards).

The results of the study show that different methods can be used for different problems. You can get a full-fledged solution only when the advantages of one method are supplemented by another. For example, by combining the analytical method with the coordinate method, you can achieve high accuracy in solving spatial problems. The possibility of classifying complex bodies into simple parts using the cross-section method was studied. In addition, it was found that the trigonometric method plays an important role in calculating angular measurements.

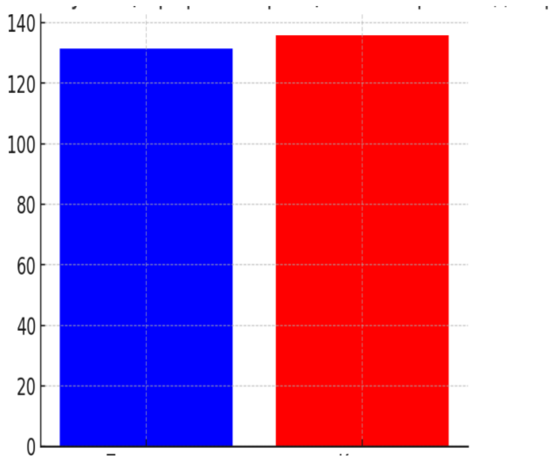
The use of graph methods increases the visibility of calculations and makes it easier to understand. The following shows the dependence of the volumes of the pyramid and cone on the height.

Figure 2.1.2 Comparative graph of the volumes of the pyramid and cone.



The graph shows how the volumes of the pyramid and cone change when the height changes. It was found that if the base area is the same, the volume of the cone will be 1.5 times the volume of the pyramid.

Figure 2.1.3 Comparison diagram of the side faces of the pyramid and cone.



This diagram shows how the surface areas of the two figures change. It has been shown that the lateral surface of a cone is larger than that of a pyramid because its lateral sides are curved.

Practical application

In engineering and construction

The geometric features of the pyramid and cone are widely used in engineering and construction. For example, in architecture, pyramidal and conical structures are important for strength and stability.

Examples of use:

In architecture-pyramidal structures give stability (for example, Egyptian pyramids);

In civil engineering – used in the calculation of minarets and domes (for example, mosque domes);

In mechanics-conical mechanisms are used in rotation mechanisms.

Aerospace and navigation

Since the conical shape is aerodynamically effective, rocket heads and aircraft parts are made in this form.

Examples of use:

Rocket technologies-the conical shape of the rocket head reduces air resistance;

In optics – for lenses and reflectors, cones calculated with precision are used.

The analysis of the combination of a pyramid and a cone allows you to study their geometric relationships more deeply. Ways to solve complex problems using various methods have been identified. The combined use of trigonometric, coordinate and analytical methods increases the accuracy of calculations. The use of such approaches in the system of mathematical education improves the spatial thinking of students. The results of the study can be applied in engineering, architecture and Applied Sciences. The proposed methods demonstrate the effectiveness of solving complex problems. In the upcoming research, it is planned to use three-dimensional modeling technologies. The development of new approaches to solving complex problems related to the combination of a pyramid and a cone remains relevant.

This work considered the methodology for solving complex problems related to the combination of a pyramid and a cone. Five main methods were described: analytical, geometric, trigonometric, section and coordinate methods. The features and effectiveness of each method were shown with specific examples. The results of the study show that different methods can be used for different problems. These methodologies are widely used in the field of Education, Engineering and scientific research. In the future, it is proposed to conduct additional research in order to improve research methods. the study of the combination of a pyramid and a cone is not only of theoretical importance, but also finds real application in various fields. The explanation of geometric dependencies through graphs and diagrams makes it easier to solve complex problems. The use of three-dimensional modeling methods in future research will be considered.

An important role in teaching geometry is played by mastering the properties of spatial figures, developing students ' logical thinking and spatial imagination. By teaching complex problems related to spatial bodies, such as pyramids and cones, students not only gain theoretical knowledge, but also learn to model real life situations. This methodological section discusses ways to effectively organize the process of teaching complex problems, built on the basis of a combination of a pyramid and a cone.

The combination of a pyramid and a cone is a geometric model obtained by combining the common properties, elements and conditions of mutual arrangement of two spatial bodies. In the problems associated with such combinations, it becomes necessary to use such properties of both bodies as volume, surface area, height, cross-section and symmetry.

During the training, the following knowledge, skills and abilities are formed:

Ability to distinguish the structural elements of a pyramid and a Cone (Hill, base, height, apothem, creator) ;

Using multiple formulas in combination to find the volume and area of combined bodies;

Creating a step-by-step solution to complex geometric problems;

Development of geometric thinking through drawing and graphic models;

Geometric approach to natural and technical problems.

Methodological approaches

For effective training of complex problems related to the pyramid and cone, the following methodological approaches are proposed:

1.modular learning – allows students to repeat the individual properties of bodies, and then move on to their combination. This method is based on the principle of gradual complication.

2.Creating a problem situation – presenting a geometric problem to the student in a specific situation related to life, the solution of which is unknown in advance. For example,"full size calculation if a cone-shaped tower is installed on a pyramid-shaped roof."

3. visualization and modeling – creating models of three-dimensional bodies using such programs as GeoGebra, Math3D. This facilitates spatial visualization and understanding of the content of the report.

4.Comparison and analysis – explanation of properties in a combination by analyzing the similarities and differences between a pyramid and a cone.

5.pair and group work – the formation of collective thinking skills through the discussion and joint solution of problems, organizing students to work in small groups.

The structure and stages of the lesson

The lesson of teaching complex problems can consist of the following stages:

1.arouse interest – give students a practical example from everyday life.

2.interpretation of new material – the concept of a combination, properties, formulas, solution methods.

3.working with a drawing – depicting a combination of a pyramid and a cone on a piece of paper or on a digital medium.

4.demonstration by example – solving a problem under the guidance of a teacher.

5.independent work – assignment of tasks to students and organization of their independent execution.

6.conclusion and reflection – analysis of the work performed by students, identification of difficulties.

Evaluation criteria

When assessing students ' knowledge, the following criteria are taken into account:

Understand the report and analyze the condition correctly;

Accurate and complete drawing execution;

Correct application of formulas;

Logical and argumentative representation of how to solve the problem;

Correct conclusion formulation.

Conclusion

Teaching complex problems related to the combination of a pyramid and a cone is one of the most informative and important areas in the geometry course. This topic expands students ' geometric knowledge and develops logical and spatial thinking. The proposed methods and lesson structure increase students ' interest in the subject and help them achieve high results. In addition, the skills acquired at this level of training will also benefit in future professional-oriented disciplines.

2.2 Solving complex problems related to the combination of a pyramid and a cone

This section discusses methods and ways to solve geometric problems related to the bodies of a pyramid and a cone. These problems are widely used in stereometry and improve students ' spatial thinking, logical reasoning, and mathematical language. In addition, in the process of solving problems, geometric concepts and formulas such as volume, surface area, sections, axial symmetry, coordinate method, etc.are used.

The problems presented in this section are selected according to the level of complexity, and in the process of solving them, methods such as the analytical method, solving by drawing, using formulas, the coordinate method and the volume comparison method were used. These problems are aimed at a deeper understanding of the properties of pyramid and cone bodies and the application of theoretical knowledge in practice.

Modern School Mathematics attaches special importance to the development of spatial thinking, logical and analytical abilities of students. Within the geometry course, working with spatial figures is an effective way to improve students ' spatial imagination, volumetric thinking. In particular, teaching to solve complex problems by analyzing the similarities and differences between cone – shaped and pyramid-shaped figures, their combinations-contributes to the application of students ' theoretical knowledge in practical problems. However, in the school curriculum, these topics are considered in a limited way, and methods for solving complex problems based on their combination are not systematically studied. In this regard, the development of a special methodological system aimed at increasing students ' creativity, awakening interest in mathematics is one of the most pressing problems. Solving complex problems related to the pyramid and cone

Pyramid and cone are types of geometric bodies in space that have a high degree of importance, are common and are often found in real life. The properties and elements of these bodies – their base, height, apothem, sections and side surfaces – form the basic concepts of the stereometry section. Solving complex problems related to these bodies – not only increases students ' spatial imagination, but also develops logical thinking, the ability to work with formulas and the ability to build a geometric model.

Reports on this topic are often found in the general education program, UNT format, university exams, as well as in engineering and technical specialties. For this reason, deep mastery of the methodology for solving complex problems related to the pyramid and cone is considered the basis for the professional training of a future specialist.

A pyramid is a polyhedron that is obtained from a single polygonal base and from a single point (Vertex) located outside this plane of the base through segments drawn to each vertex of the base. And a cone is a body of space that is formed when a region bounded by a closed line in a plane is connected by a single point (Vertex) located outside this plane. The main feature of both bodies is their ability to have axial and central symmetry and the location of the height perpendicular to the plane of the base.

In the process of solving complex problems, the following methods are most often used:

Each of these methods is effectively used depending on the type of report. For example, if a pyramid has all sides equal, then you can use the properties of equilateral triangles. And if the vertical section of the cone is given, then it can be considered as an equilateral triangle. At the same time, the formulas of the Pythagorean theorem and the area of the Triangle are often used.

The meeting of pyramid and cone bodies in many Applied Problems is an indication of their experimental importance. In architecture, civil engineering, design, and Aerospace Industries, shapes similar to these bodies are common. Towers, roofs, domes, radars and missile heads are all structures that are geometrically similar to pyramids and cones.

Also, in the process of solving problems related to the pyramid and cone, the use of Integral and differential methods, computer modeling, 3D drawings and graphs is becoming widespread. For example, with programs such as GeoGebra, Math3D and SolidWorks, you can visually draw the shape of these bodies and accurately calculate the volume and area.

The six problems presented in this section deal with various complex situations related to the pyramid and cone. They are aimed not only at developing the ability to calculate, but also at developing research and analytical skills. The content of each problem is aimed at improving the student's logical thinking, spatial imagination and skill in working with formulas.

Problem 1: given SABCD given a right square pyramid and so given a cone with the center of the base at the height of the pyramid.The E - point is located in the center of the SD edge , and the F-point is located in the AD edge so that AF=3/2fd. The 2 vertices of the Triangle in the axial section of the cone are on the CD line ,and the other one is on the EF . If AB=4, SO=3 ,then find the volume of the cone.

Solution: let's say that the vertices of the axial section of the K and M – cone lie on the line SD. So the segment km cannot be the diameter of the base of the cone, if the points K and M located on the so Line Q would be more symmetrical than the center of the base of the cone, and CD and so are intersecting lines. Then, point M or K will be the cone Vertex. Let's consider Point m as the cone Vertex, where ML and MK are the cone maker, and MQ is clearly the cone height steel.

Let's consider all possible segments, one end of which lies on the line SD and the middle on the line SO. It is the second end of such segments that lies in the plane γ, while AL is the orthagonal projection of this plane γ.

The ASD plane is perpendicular to the γ plane because it passes through the AD line, and the AD line is perpendicular to the γ plane. Therefore, the perpendicular ET from the vertex E descended to the line AL, where AL is the intersection line of these two planes and perpendicular to the plane γ. Hence, point T is the projection of Point E onto the plane γ.

The H-point lies in the middle of the line AD, in the right triangle SOH we find this:

$$SH = \sqrt{SH^2 + OH^2} = \sqrt{9+4} = \sqrt{13}$$

$AR = SH;\ a\,T -$ the point is the middle of the AR

$$AT = \frac{1}{2}SH = \frac{1}{2}\sqrt{13}$$

$\frac{LA}{LT} = \frac{FA}{ET}, LT = LA + AT = LA + \frac{1}{2}\sqrt{13}, ET = AH + \frac{1}{2}AH = 2 + 1 = 3, AF = \frac{3}{5}AD = \frac{3}{5}\cdot 4 = \frac{12}{5}$, онда $\frac{LA}{LA+\frac{1}{2}\sqrt{13}} = \frac{\frac{12}{5}}{3} = \frac{4}{5} \Rightarrow$ LA=$2\sqrt{13}$

$BK_1 = CK = AL_1 = 4, L_1K_1 = 12, L_1K = \sqrt{L_1K_1^2 + KK_1^2} = \sqrt{144+16} = 4\sqrt{10},$ ALL_1, KLL_1 *through the right triangles we find:*

$$LL_1 = \sqrt{LA^2 - AL_1^2} = \sqrt{52 - 16} = 6,$$

$$KL = \sqrt{LL_1^2 + L_1K^2} = \sqrt{36 + 160} = 14.$$

The OQ-line is the midline of the Triangle *KLL1* ,so OQ=$\frac{1}{2}\,LL_1 = 3, QL = QK = \frac{1}{2}KL = 7.$

SPQ in the Triangle PO is the median of the triangle ,so QP=SP=SH=$\sqrt{13}$.

QP$\perp MK$ We consider the Triangle MQK by the theorem regarding Perpendiculars:

$$QK = 7, QP = \sqrt{13}, PK = PC + CK = 2 + 4 = 6.$$

$$MP = \frac{QP^2}{PK} = \frac{13}{6}$$

$$MQ = \sqrt{QP^2 + MP^2} = \sqrt{13 + \frac{169}{36}} = \frac{7}{6}\sqrt{13}.$$

If h is the height of the cone R is the radius of the pan, then V is the volume r= $QK = 7\,, h = MQ = \frac{7}{6}\sqrt{13}$ from this

V=$\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\cdot 49\cdot\frac{7}{6}\sqrt{13} = \frac{343\sqrt{13}}{18}\pi$

Answer: $\frac{343\sqrt{13}}{18}\pi$

Problem 2: Given that the base of a pyramid, the base of which is an isosceles triangle inside the cone, is drawn internally at the base of the cone, and the top of the pyramid is located at the creator of the cone. The angle of an isosceles triangle at the base of the pyramidα $\left(\alpha \geq \frac{\pi}{3}\right)$ if, then find the ratio of the volume of the cone to the volume of the pyramid .

Solution: let MO be the height of the cone drawn externally to the pyramid DABC, point O is the center of the circle at the base of the cone, the circle is drawn externally to the triangle ABC.

AB=BC ,$\angle ABC = \alpha, \left(\alpha \geq \frac{\pi}{3}\right)$

DK –the height of the pyramid is because all the sides of the pyramid are not inclined at the same angle to the base, that is, the two-sided angles at the base are equal. And the point K $\Delta ABC-$ center of the inscribed circle.

Points o and k lie along the line BN, because ΔABC it is known to be an isosceles triangle. MO∥ DK, this is because the lines MO and DK, the pyramid and cone are perpendicular to the plane on which the bases lie.

The plane passed through, passes through the parallels MO and DK, where MV is the cone maker. Then, the point D corresponding to this MOB plane is located on the side surface of the cone, and therefore the MB corresponding to the cone maker.

MOB $\Delta MOB\sim\Delta DKB$ there $\frac{MO}{DK}=\frac{OB}{BK}$.

OB=R let be the radius at the base of the cone.

There $AB=BC=2R\sin\angle BAC=2R\sin\left(\frac{\pi}{2}-\frac{\alpha}{2}\right)=2R\cos\frac{\alpha}{2}$.

$S_{\Delta ABC}=\frac{1}{2}AB^2\sin\angle ABC=\frac{1}{2}\cdot 4R^2\cos^2\frac{\alpha}{2}\sin\alpha=2R^2\cos^2\frac{\alpha}{2}\sin\alpha.$

$\Delta BKC\ \ \angle CKB=\frac{\alpha}{2},\ \angle BCK=\frac{1}{2},\ \angle ACB=\frac{\pi-\alpha}{4},$

$\angle BKC=\pi-(\angle CKB+\angle BCK)=\pi-\frac{\pi+\alpha}{4}.$

$\frac{BC}{\sin\angle BKC}=\frac{BK}{\sin\angle BCK}.$

$BK=\frac{BC\sin\angle BCK}{\sin\angle BKC}=\frac{2R\cos\frac{\alpha}{2}\sin\frac{\pi-\alpha}{4}}{\sin\left(\pi-\frac{\pi+\alpha}{4}\right)}=\frac{2R\cos\frac{\alpha}{2}\cos\frac{\pi+\alpha}{4}}{\sin\frac{\pi+\alpha}{4}}=2R\cos\frac{\alpha}{2}ctg\frac{\pi+\alpha}{4}$

Cone volume: $V_1=\frac{1}{3}\pi R^2\cdot MO$

Volume of the pyramid: $V_2=\frac{1}{3}S_{\Delta ABC}\cdot DK$

There $\frac{V_1}{V_2}=\frac{\pi R^2\cdot MO}{S_{\Delta ABC}\cdot DK}=\frac{\pi R^2\cdot OB}{2R^2\cos^2\frac{\alpha}{2}\sin\alpha\cdot BK}=\frac{\pi R^3}{4R^3\cos^3\frac{\alpha}{2}ctg\frac{\pi+\alpha}{4}\sin\alpha}=\frac{\pi tg\frac{\pi+\alpha}{4}}{4\cos^3\frac{\alpha}{2}\sin\alpha}$

Answer : $\frac{\pi tg\frac{\pi+\alpha}{4}}{4\cos^3\frac{\alpha}{2}\sin\alpha}$

Problem 3: a vertical cone is drawn externally and internally on a right triangular pyramid. Pyramid height h=4, length of the base of the externally drawn cone l= $\sqrt{3}\pi$ if so, find the difference in the volumes of the cones drawn externally and internally.

Solution: L=2 $\pi R=\sqrt{3}\pi$ from here we find the radius of the circle drawn on the outside $R=\frac{\sqrt{3}}{2}$.

Formula for the radius of a circle drawn externally on a right triangle R=$\frac{a\sqrt{3}}{3}$ through we find the side of the Triangle $a=\frac{3}{2}$, Formula for the radius of a circle inscribed in a right triangle :

$r=\frac{a\sqrt{3}}{6}=\frac{1{,}5\sqrt{3}}{6}=\frac{\sqrt{3}}{4}$,

$V=\frac{1}{3}\pi R^2 H$

$H_{cone}=H_{pyramid}$

$V_{externally\ drawn\ cone}=\frac{1}{3}\pi\cdot\frac{3}{4}\cdot 4=\pi$

$V_{internally\ drawn\ cone}=\frac{1}{3}\pi\cdot\frac{3}{16}\cdot 4=\frac{1}{4}\pi$,

$$V_{externally\ drawn\ cone}-V_{internally\ drawn\ cone}=\pi-\frac{1}{4}\pi=\frac{3}{4}\pi$$

Answer: $\frac{3}{4}\pi$

Problem 4: a rectangle with one side A and the other three sides B of the base of the pyramid drawn internally to the cone. The pyramid roof is located right in the middle of the cone maker. If the angle between the height of the cone and the creator is equal to a, then find the volume of the pyramid.

Solution: Let R be the radius of the base of the cone drawn externally to the ABCD quadrilateral at the base of the pyramid ,AB=BC=CD=b,AD=a, Point F AC, BD intersection of dioganals

$$\angle ADF=\beta\ if\ ,there\ \angle FDC=\angle FBC=\ \beta\Rightarrow AD\parallel BC,\angle B=180°-2\beta\Rightarrow$$

$S_{ABCD}=S_{ACD}+S_{ABC}=\frac{1}{2}absim2\beta+\frac{1}{2}b^2sin2\beta.$

But

$S_{ABCD}=\frac{AC\cdot BD}{2}\cdot sin\angle AFD=\frac{(AC)^2}{2}\ sin2\beta(AC=BD,\angle AFD=180°-2\beta)\Rightarrow$ $AC=\sqrt{b^2+\text{ab}}.$

$\triangle\ ACD$ by the theorem of Sines for,R=$\frac{AC}{2sin2\beta}$.

It is known that according to the calculation condition, the height of the pyramid is equal to half the height of the cone, that is, it $\frac{1}{2}Rctg\alpha$ equal $\Rightarrow$

$V_{EABCD}=\frac{1}{6}Rctg\alpha\cdot\qquad S_{ABCD}=\frac{1}{6}\cdot\frac{AC}{2sin2\beta}\cdot ctg\alpha\cdot\frac{(AC)^2}{2}\ sin2\beta=\frac{ctg\alpha}{24}(b^2+ab)^{\frac{3}{2}}.$

Answer: $\frac{ctg\alpha}{24}(b^2+ab)^{\frac{3}{2}}.$

5-problem: The base of the pyramid is a right triangle drawn inward to the base of the cone. The vertices of the pyramid and cone face at one point. The angle that the two sides of the pyramid make with the legs of the right triangle at the base α and β. Find the ratio of the volumes of the pyramid and the cone.

Solution: Let the DO-segment be the common height of the DABC pyramid and the cone, O is the center of the circle , since ABC is drawn externally into a right triangle O point AC is half of the hypotenuse . From Point O, we draw the Perpendiculars OE and OF to the legs AB and BC in the plane of the foot.

There $\mathrm{DE} \perp \mathrm{AB}, \mathrm{DF} \perp \mathrm{BC}$ then $\angle\mathrm{DEO} = \alpha, \angle\mathrm{DFO} = \beta$.

$$\text{If } DO - \alpha, \text{ there } \Delta DOE\ (\angle DOE = 90°): EO = DOctg\angle DEO = arctg\beta.$$
$$\Delta DOF(\angle DOF = 90°): FO = DOctg\angle DFO = arctg\alpha.$$
$$AB = 2 \cdot FO = 2arctg\alpha, BC = 2 \cdot EO = 2arctg\beta.$$
$$\Delta ABC(\angle ABC = 90°): AC = \sqrt{AB^2 + BC^2} = 2\alpha\sqrt{ctg^2\alpha + ctg^2\beta}.$$
$$Area\ of\ the\ base\ of\ the\ pyramid: \ S = \frac{1}{2}AB \cdot BC = 2a^2ctg\alpha\ ctg\beta,$$
$$volume: \ V_1 = \frac{1}{3}S \cdot DO = \frac{2}{3}a^3ctg\alpha\ ctg\beta.$$
$$Radius\ of\ the\ base\ of\ the\ cone: \ R = \frac{1}{2}AC = \alpha\sqrt{(ctg^2\alpha + ctg^2\beta)}.$$

$$volume\ : V_2 = \frac{1}{3}\pi R^2 \cdot DO = \frac{1}{3}\pi a^3(ctg^2\alpha + ctg^2\beta)$$

now we find relationships: $\frac{V_1}{V_2} = \frac{2ctg\alpha\ ctg\beta}{\pi(ctg^2\alpha+ctg^2\beta)}$

Answer : $\frac{2ctg\alpha\ ctg\beta}{\pi(ctg^2\alpha+ctg^2\beta)}$

6-problem: The bilateral angle of the lateral side of a right triangular pyramid is equal to 2α. Find the volume of the cone drawn externally on the pyramid if the height of the pyramid is h .

Solution: Let ABCD be the lateral side of the Pyramid at an angle φ with the plane of the base ,and the side of the right triangle ABC with the base ABC is equal to A, and from point A we draw AF perpendicular to CD. If O is the center of the base ,then DO=h is the height of the pyramid. The straight line OC is an orthogonal projection of the CD inclined to the plane of the pyramid . CO$\perp AB$ since, then by the theorem of three Perpendiculars CD$\perp AB$.

Thus ,the line CD is perpendicular to the two lines AF, AB, intersecting the plane of the Triangle AFB. This means that the CD line is perpendicular to this plane. Therefore, the AFB CD on the side side of the pyramid is the linear angle of the two-sided angle ABCD . Under the terms of the report $\angle AFB = 2\alpha$. AFB the height of an isosceles triangle FM is its median and bisector.

Therefore

$$MF = AMctg\angle AFM = \frac{\alpha}{2} \cdot ctg\alpha$$
$$MF = CMctg\angle MCF = \frac{\alpha\sqrt{3}}{2} \cdot sin\varphi$$
$$\frac{\alpha}{2} \cdot ctg\alpha = \frac{\alpha\sqrt{3}}{2} \cdot sin\varphi$$
$$sin\varphi = \frac{ctg\alpha}{\sqrt{3}}$$

$\cos\varphi = \sqrt{1 - sin^2\varphi} = \sqrt{1 - \frac{1}{3}ctg^2\alpha}$,

$tg\varphi = \frac{sin\varphi}{cos\varphi} = \frac{\frac{ctg\alpha}{\sqrt{3}}}{\sqrt{1-\frac{1}{3}ctg^2\alpha}} = \frac{ctg\alpha}{\sqrt{3-ctg^2\alpha}}$.

From the right triangle COD we find:

OC$=\frac{OD}{tg\varphi}=\frac{h\sqrt{3-ctg^2\alpha}}{ctg\alpha}$.

r=OC$=\frac{h\sqrt{3-ctg^2\alpha}}{ctg\alpha}$.

V$=\frac{1}{3}\pi r^2 h = \frac{\frac{1}{3}\pi h^3(3-ctg^2\alpha)}{ctg^2\alpha} = \frac{\frac{1}{3}\pi h^3(3tg^2\alpha-1)}{3}$

Answer : $\frac{1}{9}\pi h^3(3tg^2\alpha - 1)$

2.3 Experiment on learning problems related to the combination of a pyramid and a cone.

Purpose and objectives of the study.

The main goal of this master's thesis is to develop an effective methodology for solving complex problems related to combinations of cones and pyramids within the framework of the school geometry course, as well as to determine the influence of this methodology on the quality of students ' knowledge, the ability to think logically and the level of spatial imagination.

To achieve this goal, the following specific tasks were set and systematically implemented in the course of the study:

Drawing up complex problems by combining the properties of the geometric elements of the cone and pyramid.

At this stage, the individual and common characteristics of spatial figures were studied, and on their basis a set of problems of different levels (initial, average, complex) was developed, depending on various circumstances. Since the problems were built in a clear geometric context, they were aimed at developing spatial logic and analytical thinking of students.

Development of methodological approaches to solving problems of a new type.

As part of this task, algorithms were developed to identify spatial relationships and connections that describe the combination of a pyramid and a cone, and visual tools, graphical models, and step-by-step solutions were proposed for students to easily understand them.

Testing the proposed methodology in a specific educational process.

In order to test the effectiveness of the methodology, it was practiced in a specific educational institution, and the initial and final indicators of students were compared. During the experiment, special attention was paid to how quickly, correctly and deeply students can complete tasks of a new type.

Analyze the results of the experiment and prove the effectiveness of the proposed methodology.

The scientific validity of the study is proved by comparing educational achievements in quantitative and qualitative terms. In addition to statistical indicators, the opinions of students and the results of observations were also taken into account.

Information about the place and participants of the study

The experimental stage of the research was held on the basis of the school-Lyceum Qalam High School, located in Almaty. A total of 32 students in Grade 11 took part in the study. The participants were divided into two groups:

The experimental group consisted of 17 students. In this group, training was conducted on the basis of a new methodology, using interactive and visual tools.

The control group consisted of 15 students. This group was trained on the basis of traditional teaching methods.

The peculiarity of the division into groups is that the selection was carried out purposefully, and not in a random way. In particular, depending on the specifics of the research work, classes with an in-depth curriculum of the natural and mathematical direction were selected. Since the experiment to be carried out was based on complex spatial geometric combinations, in particular, on the conditions of intersection of cones and pyramids and the calculation of their volumetric characteristics, a high level of logical and mathematical training was required to perform these tasks.

Such a targeted selection not only ensured the accuracy of the research results, but also made it possible to test the effectiveness of the methodology in a realistic and subject-oriented environment.

The level of knowledge in geometry of the students who participated in the experiment was approximately equal, the average performance indicators were close to each other. Thus, the difference between the experimental and control groups was only due to the teaching methods used. This increased the accuracy of the research results and ensured that scientific conclusions were justified.

In general, the structure of the study, goals and objectives, as well as specific data on the participants reflected in this section, fully reveal the theoretical basis and practical significance of the dissertation work.Organization of the experimental training program

The experiment consisted of three stages: initial diagnosis, training stage and final assessment. Each stage had specific goals and content structure:

Stage 1-diagnostic: in order to determine the level of basic knowledge of students on the topics of cones and pyramids, preliminary test tasks were performed. These initial data were important for comparison with subsequent results.

Stage 2-experimental training: a specially designed methodology was used in the experimental group. It provides in-depth study of theoretical material, visualization using three-dimensional models, solving problems related to real life, and the use of information and communication technologies. Students were also offered tasks that allow them to make decisions on their own. In the control group, traditional teaching methods were used, and the educational process was mainly limited to textbook reports.

Stage 3-final assessment: each group was given final test tasks and the results of their implementation were analyzed in comparison. The evaluation criteria and score scale were the same for all participants.

Educational materials and methodological approaches

The training methodology used in the experimental group was based on the following basic principles:

Visualization: Dynamic models of cone and pyramid combinations were demonstrated using interactive programs such as GeoGebra.

Modeling: students were given tasks aimed at developing the ability to spatial imagination by creating geometric shapes from paper and plastic materials.

Project method: each student or group was asked to make a simple layout of an architectural object, which includes a combination of cones and pyramids, and calculate its dimensions, volume, sections.

Differential learning: depending on the level of students, tasks were presented in a complex way.

Reflection and self-assessment: at the end of each lesson, students described what they understood and shared the points that caused the problem.

Structure and content of the lesson.

A short-term module consisting of six lessons was created. Each lesson was planned based on specific didactic goals:

Lesson 1: repetition of the basic properties of the cone and pyramid.

Lesson 2: building a combination: placing shapes inside or on top of each other.

Lesson 3: solving problems related to the volume and surface area.

Lesson 4: intersection and construction of sections.

Lesson 5: practical work — recognize geometric combinations from objects in real life.

Lesson 6: generalization and generalization: solving complex problems, performing test tasks.

3. RESULTS

Evaluation system and test structure
The final assessment was organized in the following structure:
Test of theoretical knowledge (10 points);
Main points (30 points);
Complex problems related to combinations (60 points).
General Rating Scale:
85-100 points - "very good";
70-84 points — "good";
50-69 points — "satisfactory";
0-49 points - "unsatisfactory".
The results of the experiment and *their analysis*

Table 3.1 Initial test conclusion:

Group	Avarage score (100)
Experimental	62
Control	64

Table 3.2 Final test result:

Group	Avarage score (100)
Experimental	88
Control	72

These data showed a significant increase in the level of knowledge of students in the experimental group. An increase of 26 points is an indicator that clearly proves the effectiveness of the new methodology. Although an increase was also observed in the control group (8 points), this figure is much lower compared to the experimental group. At the same time, the number of students who received more than 70 points in the experimental group was initially 13, and in the end it reached 17. And in the control group, this figure increased only from 9 to 10.

As an example of qualitative changes, the opinion of some students is as follows:

"It was very interesting to see through three-dimensional models and build by hand. Previously, it was difficult to imagine such reports in abstract form."(Class 11B, Aidana)

"I liked reports that require reflection and proof. The connection with Ordinary Life — Increased my interest in the report."(11a class, Nurzhan)

Conclusion and scientific and methodological recommendations.

The results of the conducted study clearly showed the effectiveness of the proposed methodology. In the process of solving complex problems associated with combinations of cones and pyramids, students ' abilities for spatial imagination, logical thinking, and acting in a problem situation have increased. In addition, the new method opened the way for students to increase their interest in the subject, to reveal their creative potential.

Recommendations:

Development of an adapted version of the proposed methodology for implementation in schools at the level of the Republic;

Adding methodological materials to advanced training courses for teachers;

Integration of virtual models and video recordings on distance learning platforms;

Compilation of a methodological manual or workbook for schoolchildren;

Organization of an in-depth elective course or elective on this topic.

The results of the study showed that mathematics is of great importance in the development of innovative areas of teaching, improving the functional literacy of students and improving the efficiency of the educational process. Therefore, this methodology may in the future become part of the modern model of teaching geometry.

Summing up the experimental study and future research directions

Conclusion:

The results of the conducted experimental research showed that the methodology for solving complex geometric problems associated with combinations of pyramids and cones has a positive impact on the quality of education of schoolchildren. The effectiveness of this methodology has paid off not only at the theoretical level, but also in the real educational process. In the experimental group, it was observed that students ' interest and activity in geometry increased significantly. In the process of solving tasks, they developed not only the use of ready-made formulas, but also the skills of spatial imagination, logical thinking, comparison and analysis. Through this, the cognitive abilities of students were strengthened, motivation and confidence in the subject increased.The methodology contributed to the students ' understanding of the spatial relationship, geometric interrelationships between them by considering the pyramid and the cone not individually, but in an integrated way. This way of learning turned out to be much more productive than traditional methods. As concrete evidence, it can be said that the average score of students in the experimental group was much higher than in the control group. Such data clearly demonstrate the practical value of this methodology and its potential in the educational process.It is important to note that the conclusions made on the basis of this work can have a broad impact on the future research and educational process. Therefore, based on the results of the study, the following relevant scientific directions can be developed in the future:

1. expanding the methodology based on complex combinations of geometric bodies

Further scientific research should consider ways to solve complex problems related to combinations between not only pyramids and cones, but also other spatial bodies-such figures as a cylinder, sphere (sphere), prism. This direction deepens the content of teaching geometry and provides new opportunities for the development of students ' spatial imagination. Problems – especially with regard to complex spatial structures consisting of three or more figures-can be an important preparatory stage for students of engineering and technical profiles.

2. increase the level of visualization: using AR and VR technologies

Global trends in modern education are aimed at the widespread use of technologies in teaching. For this reason, the use of AR (Augmented Reality – Augmented Reality) and VR (Virtual Reality – Virtual Reality) technologies will be very effective in teaching spatial geometry. Through these technologies, students can clearly see how geometric bodies are arranged in space and interact with them. The fact that students work with figures in a way that they can see and touch – dramatically increases their level of understanding, memory and interest. In addition, AR/VR methods allow us to provide a convenient and adaptive learning approach even for students with special educational needs.

3. development of special training tools that develop spatial thinking

The next stage of the study should be the development of author's manuals, sets of tasks, methodological manuals aimed at spatial thinking for use in school geometry. These tools need to be adapted to the age and level of training of students at different levels. It is better to include graphic tasks, 3D models, design problems, situational tasks related to life in the collections. Such tools, in addition to increasing students ' interest in the subject, improve their ability to think independently, build problem-solving strategies.

4. adaptation of the methodology to the curricula of schools in Kazakhstan

The methodology should be gradually brought into line with specific curricula and subject standards of schools in Kazakhstan and introduced into general education schools, gymnasiums, lyceums and specialized schools for gifted children. It should take into account different levels of training and educational content, and methodological recommendations should be made variatively. Approbation in various educational organizations, such as rural schools, city schools and Nazarbayev Intellectual Schools, allows us to clearly demonstrate the flexibility and adaptability of the methodology.

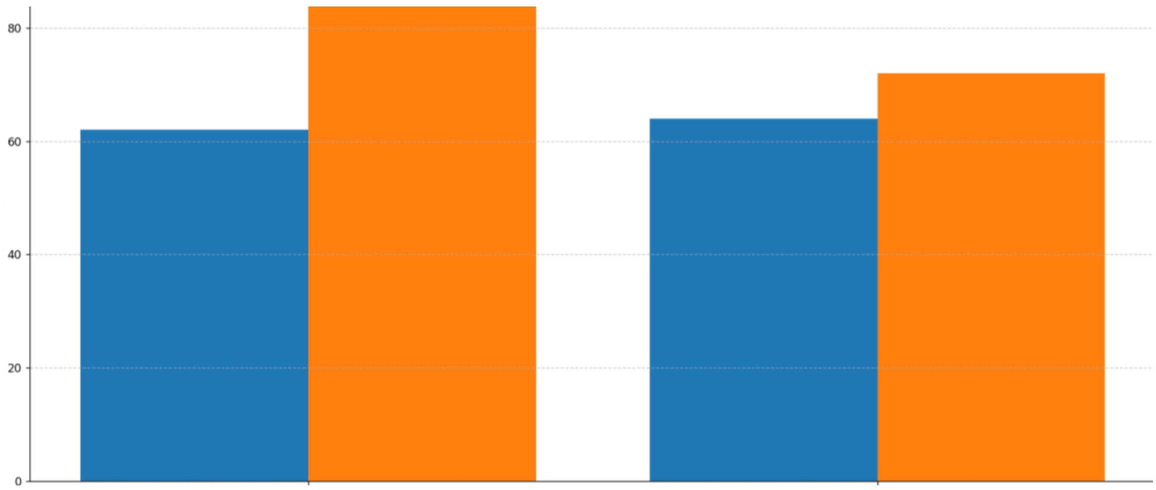
5. conducting long-term research

To assess how the new methodology will affect the academic success of students in the long term, it is necessary to conduct observations, comparisons, analyzes lasting several academic years. In this direction, it is possible to clarify the long-term effectiveness of the methodology by repeatedly observing the control group and the experimental group, conducting dynamic research on such indicators as academic performance, logical thinking, UNT results, motivation for the subject. In addition, by analyzing both the achievements of students in higher education, it will be possible to assess the impact of this methodology on the future professional direction.

The above-mentioned future directions will expand the range of teaching methods for complex problems solved by combining spatial figures such as a pyramid and a cone in the educational field and provide a great opportunity to develop them in accordance with modern requirements. This study is not limited to the consideration of a single topic, but is a concrete step towards the implementation of such important goals as the development of spatial thinking, the improvement of logical and analytical abilities of students and the introduction of modern teaching tools.

The proposed methodology and prospects for its development can make a significant contribution to improving the quality of education and modernizing the system of teaching school geometry not only in Kazakhstan, but also at the global level. Therefore, the continuation of work in this direction is a common task of the scientific and pedagogical community.

Figure 3.1 Results of the experiment: initial and final points.



Hypothesis: the use of interactive learning methods increases students ' academic performance.

Comment:

The experimental work carried out as part of this scientific study was aimed at testing an effective methodology for teaching complex geometric problems related to the combination of a pyramid and a cone. The main goal of the experiment was to determine the effect of the proposed innovative, that is, interactive teaching methods on improving the level of spatial thinking of students, the skills of logical analysis and the quality of knowledge, comparing them with traditional teaching methods.

The experiment was carried out in two groups: the experimental group and the control group. In the course of training in the experimental group, interactive methods were widely used, including visualization, graphic modeling, geogebraic simulations, group work and project tasks. Through these approaches, students were given the opportunity to visualize the connections between complex geometric shapes in space and clearly identify logical connections. And in the control group, traditional teaching methods were used-that is, the teacher's interpretation, working with the board, memorizing formulas.

According to the results of the study, it was noted that the educational indicators of the experimental group increased significantly. In particular, according to the difference in the results of the initial and final tests, the average score of students in the experimental group increased by 26 points, while in the control group this indicator was only 8 points. These data are clearly shown in the diagram and table compiled as a result of experimental work. Such a clear difference fully proves the effectiveness and scientific validity of the proposed methodology.

The substantive, cognitive and motivational effects of interactive learning methods used in the experimental group were particularly striking. In particular, students ' interest in the lesson increased, the ability to think creatively developed in the process of working with complex figures, and they tried to draw and prove their own conclusions. In the reflexive notes and comments given by students, as their difference from traditional teaching, it is stated that interactive lessons are understandable, interesting and connected with life. Three-dimensional modeling, practical problems and project tasks based on real life — all this increased the student's activity in the learning process.

In general, the results obtained fully confirmed the reality of the hypothesis initially put forward. The statement" interactive teaching methods improve the quality of students 'knowledge, develop spatial thinking and logical computing abilities" was supported by practical evidence. In addition, such methods increase the motivation of students for the subject of study and contribute to the constant maintenance of their interest in the lesson.

Based on the results of this study, the proposed training model can be successfully integrated into the school geometry training system. This model is able not only to master the content of education, but also to develop students ' functional literacy, logical-analytical skills, and creative abilities. That is, in accordance with modern requirements, it forms not only theoretical knowledge, but also the ability of students to apply it in life situations.

At the same time, the effectiveness of interactive methods was noted not only at the level of one lesson, but also in the process of long-term learning in general. Students in the experimental group were also active in the following chapters and showed high results in solving geometric problems. This means that their cognitive thinking style has changed, that is, from a passive student to an active seeker.

As a conclusion of the research work, the following conclusions can be expressed:

Interactive learning methods are powerful tools that increase students ' ability to think spatially, make logical decisions, and develop creatively.

Problems created by combining complex spatial figures such as a pyramid and a cone — increase the cognitive activity of students and allow them to relate to life.

In contrast to traditional teaching methods, visualization and practice – based approaches are more effective in improving the quality of Education.

The proposed methodology can be used not only in the school curriculum, but also in the engineering, architectural and pedagogical directions of higher educational institutions.

In conclusion, this study is an important step towards updating the content of teaching, improving the methodological base and increasing the level of functional training of students. The fact that the experimental results are accurate gives the basis for further development of the proposed training model and its implementation on a large scale. The effectiveness and science of the new methodology has been proven, and the possibility of its application in the educational process has been determined.

Test: properties of the cone and Pyramid

The radii of the bases of the truncated cone are 6 cm and 3 cm, and the height is 8 cm. Find the volume of the truncated cone.

452 cm3

603.2 cm3

723.4 cm3

301.6 cm3

2.the bases of the truncated pyramid are 36 cm2 and 16 cm2, and the height is 10 cm. Find the volume of the truncated pyramid.

200 cm3

260 cm3

320 cm3

400 cm3

3.determine the similarity of the cone and pyramid figures.

The sides of both are arched

The bases of both are polygons

The vertex of both meet at one point

The volume of both is found by the same formula

4. What is the main difference between a pyramid and a cone?

The base of the pyramid is a circle

Base of cone Polygon

The sides of the cone are curved, the sides of the pyramid are straight

Both are no different

5. which figure can consist of a combination of a truncated pyramid and a truncated cone?

Tower-like construction

Balloon

Parallelepiped

Cylinder

The height of the truncated cone is 12 cm, the base radii are 5 cm and 3 cm. Find the volume of the truncated cone.

402 cm3

502 cm3

602 cm3

702 cm3
What is the formula for the volume of a truncated pyramid?
$V = (1/3)\pi r^2 h$
$V = (1/3)h(S_1 + S_2 + \sqrt{(S_1 \cdot S_2)})$
$V = S * h$
$V = (1/2) * (a + b) * h$
What is the difference between a truncated pyramid and a full pyramid?
There is no Vertex point in a truncated pyramid
A truncated pyramid has only one base
A truncated pyramid has only a circular base
No difference
What figures does the truncated cone record consist of?
Side face in The Shape of two circles and a trapezoid**
Only one circle
Two circles and a rectangle
Three circles
If the base of a pyramid is a square, then what kind of pyramid is it called?
Triangular pyramid
Square pyramid
Pentagonal pyramid
Cone
How many times does the volume increase if the height and radius of the cone are doubled?
2 times
4 times
6 times
8 times
The areas of the bases of the truncated pyramid are 25 cm2 and 9 cm2, the height is 6 cm. Find the volume.
192 cm3
170 cm3
160 cm3
200 cm3
What do cone and beveled cone have in common?
Both have two paws
The bases of both are polygons
The lateral surface is formed as a result of rotation
No roof
Which figure can you get a jagged cone by cutting it out?
Cylinder
Pyramid
Cone
Prism
What is the height of a truncated pyramid?

Side side length
Perpendicular from the lower sole to the upper sole
Foot wall
Diagonal from ceiling to sole
Calculate the volume of the truncated cone: $R = 6$ cm, $r = 2$ cm, $h = 9$ cm.
452π cm3
368π cm3
332π cm3
248π cm3
How is the lateral surface area of a truncated pyramid calculated?
Sum of areas of all side faces
The sum of the areas of the upper and lower foot
Product of the heights of trapezoids
Sum of all facets
The creator of the truncated cone (l) when the base radii are 5 cm and 3 cm, the lateral surface area is 80π cm2. Find a creator.
4 cm
5 cm
8 cm
10 cm
What flat figures can be in the record of a truncated pyramid?
Two circles and sector
Polygons and trapezoids
One square and a circle
Triangles only
Show the difference between a truncated pyramid and a prism:
In a truncated pyramid, the soles are not the same
There is no ceiling in the prism
In a truncated pyramid, the sides are a rectangle
The bases of the prism are round
By what condition does a truncated cone appear on an intersecting plane?
Parallel to Axis
The ceiling
Perpendicular to the axis
Intersecting at right angles
Calculate the volume if the base areas of the truncated pyramid are $s_2 = 100$ cm2 and $S_2 = 36$ cm2, the height is 12 cm:
784 cm3
972 cm3
1152 cm3
1216 cm3
What is the surface area of a truncated cone?
Side face area only
Lower foot area + upper foot area

Side face area + two foot area
Difference in the area of two paws
What figure is all the sides of a truncated pyramid?
Triangle
Trapezoid
Rectangle
Parallelogram
What is the axial section of a truncated cone?
Circle
Trapezoid
Triangle

CONCLUSION

In this research work, the methodology for solving complex geometric problems related to the combination of a pyramid and a cone is considered in detail, their role and importance in the development of spatial thinking of students are differentiated. Based on the results of the research and experiments carried out, the following conclusions can be drawn. Foundation of geometric knowledge.

To begin with, it has been proven that in order to solve complex problems, students need to deeply master the personal qualities of the pyramid and cone. Each of these spatial figures has its own characteristic elements, properties and formulas. For example, such features of the pyramid as The Shape of the apothem and base, the consideration of the cone as the creator and the body of rotation play a key role in solving problems.

In the case of a combination of a pyramid and a cone, it is especially important to know exactly the features of the mutual arrangement of two bodies. Here you need not only to understand the condition of the problem, but also to have the skills to imagine it through imagination and depict it in a drawing. For example, in cases where a pyramid is located inside a cone or vice versa, the common radii of their bases, the points of descent of their heights and the intersections on the plane must be explained by geometric modeling.

Spatial thinking and analytical ability

Of particular importance in solving problems in this direction is the ability of the student to think spatially and use logical and analytical approaches. Through such tasks, students master high-level cognitive processes such as the placement of figures in space, comparison, modification and clarification of their properties.

In the process of solving problems, it is not enough to use only formulas. The student needs to draw logical conclusions and systematically analyze the structure of the problem. From this point of view, the problem – solving process develops in parallel such abilities as, on the one hand, mathematical accuracy, and on the other- pictorial representation, modeling and abstract thinking. Regular performance of such tasks – increases the level of creativity of the student and teaches the ability to connect mathematical knowledge with life.

The structure of the methodology and the results of the experiment

In the course of the study, it was found that the effective methodology for solving problems consists of several stages:

Understanding the text of the report and building a geometric model;

Determining the nature of the mutual arrangement of figures;

Selection and application of the necessary formulas;

Calculation of the area or volume of newly formed figures.

During the experiment, the results of students ' performance of these tasks were analyzed in comparison. The ability of the experimental group to spatial imagine, make logical conclusions and solve complex problems showed higher results than the control group. This proves the effectiveness of systematic training of complex problems based on a combination of a pyramid and a cone.

Practial importance

The tasks compiled as a result of the study were divided into initial, medium and complex levels in accordance with the level of training of students. This structure allows the teacher to carry out differential work with students. In addition, the presented methodological recommendations are suitable for use as an additional tool for the school curriculum.

Theoretical and practical blocks were also developed in the course of the work, which can be included as additional material in the courses of spatial geometry taught in higher educational institutions. Through this, research work can be effectively applied not only at the school level, but also in the higher education system.

The relationship between theory and practice

One of the most important results of the work is a clear demonstration of the relationship between theoretical knowledge and experience in solving practical problems. In the course of the study, drawings, models, and methods of graphic representation were widely used. Through such approaches, the student learns to solve the problem by imagining it not only on paper, but also in real space.

In addition, this work was recognized not only as a scientific interest of the author, but also as a methodological tool that can be clearly used in teaching practice. Test models for assessing students ' abilities, step-by-step problem solving plans, and visualization methods can be of effective help to both teachers and students. In general, this study systematically examined the problems related to the combination of pyramid and cone and proposed effective methodologies used in their teaching. The results of the work allow not only to deepen geometric knowledge, but also to develop students ' spatial and logical thinking.

In the future, continuing research in this direction, the study of combinations of other spatial bodies, their analytical models and the scope of practical application will be an important step towards the development of the field of spatial geometry.

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