Using Visualization Methods in Teaching 6th Grade Students to Solve Word Problems

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List of Abbreviations

2D – Two-dimensional

3D – Three-dimensional

df – Degrees of freedom

IBM SPSS – Statistical Package for the Social Sciences

IFI – Interpretation figural information

M – Mean

M.Ed. – Master of Education

MS Excel – Microsoft Excel

OGE – Unified State Examination

p – Probability value

PhD – Doctor of Philosophy

SD – Standard Deviation

SDU – Suleyman Demirel University / SDU University

t – t-statistic

VP – Visual processing

ABSTRACT

This study focuses at the efficiency of employing visualization tools to solve mathematical word problems for sixth graders. The study topic is focused on students' difficulty converting content of word problem into mathematical expressions (modeling) and their tendency to depend solely on "keywords" in the content. To overcome this issue, an one-group pretest and posttest quantitative research methodology was implemented. The study included 30 sixth-grade students recruited by convenience sampling. A pretest to identify the beginning level was used to gather data, followed by a one-month pedagogical intervention to teach visualization tools such as modeling and schematic diagrams, and finally a posttest. To assess the research findings, all 300 replies (10 questions·30 students) were examined using a specialized structural modeling descriptor ranging from 0 to 3. The t-test for dependent samples revealed a substantial increase in scores between the pretest (M=14.37) and posttest (M=15.87) ($p < 0{,}001$). The calculated Cohen's d effect size of 0.311 further confirms a small-to-medium practical impact of the visualization intervention on the students' modeling and word problem-solving abilities. Quantitative study revealed that most of students effectively advanced from a low level of competence to graph word problems (0-1 points) to a level of proper structural modeling (2 points). Finally, this study statistically verified that systematic training of visualization tools minimizes cognitive burden while considerably improving 6th grade students' ability to properly model and solve mathematical word problems.

Keywords: visualization tools, mathematical word problems, sixth-grade students, schematic diagrams, mathematics education, quantitative research.

АҢДАТПА

Бұл зерттеу жұмысы 6-сынып оқушыларына математикалық мәтіндік есептерді шығаруда визуализация құралдарын қолданудың тиімділігін қарастырады. Зерттеу мәселесі оқушылардың жазбаша ақпаратты математикалық өрнектерге айналдыру (модельдеу) кезіндегі қиындықтарына және мәтін ішіндегі тек "кілт сөздерге" ғана сүйену әдетіне негізделген. Бұл мәселені шешу үшін бір топтық претест пен посттест сандық зерттеу дизайны қолданылды. Зерттеуге ыңғайлы іріктеу әдісімен таңдалған 30 алтыншы сынып оқушысы қатысты. Деректер бастапқы деңгейді анықтауға арналған претест, бағандық модельдеу мен схемалық диаграммалар сияқты визуализация стратегияларын үйретуге бағытталған бір айлық педагогикалық тәжірибе және қорытынды посттест арқылы жиналды. Зерттеу деректерін бағалау үшін оқушылардың барлық 300 жауабы 0-ден 3-ке дейінгі арнайы құрылымдық модельдеу дескрипторы арқылы талданды. Тәуелді таңдамаларға арналған t-test нәтижелері претест (M=14.37) және посттест (M=15.87) ұпайлары арасында статистикалық айтарлықтай маңызды өсім бар екенін дәлелдеді ($p < 0{,}001$). Коэннің d эффект өлшемі (0.311) визуализация құралдарының оқушылардың модельдеу және есеп шығару дағдыларына шағын және орташа деңгей аралығындағы нақты тәжірибелік әсер еткенін растады. Сандық талдау оқушылардың басым бөлігінің мәтіндік есептердің сызбасын құру қабілеті төмен деңгейден (0-1 ұпай) дұрыс құрылымдық модельдеу деңгейіне (2 ұпай) сәтті ауысқанын көрсетті. Қорытындылай келе, бұл зерттеу визуализация стратегияларын жүйелі түрде оқыту 6-сынып оқушыларының танымдық жүктемесін азайтатынын және математикалық мәтіндік есептерді сауатты модельдеу әрі шешу дағдыларын айтарлықтай жақсартатынын сандық тұрғыдан растады.

Түйінді сөздер: визуализация әдістері, математикалық мәтіндік есептер, 6-сынып оқушылары, схемалық диаграммалар, математикалық білім беру, сандық зерттеу..

АННОТАЦИЯ

Данное исследование изучает, насколько эффективным является использование инструментов визуализации при решении математических текстовых задач учащимися шестого класса. Привычка полагаться только на «ключевые слова» и трудности преобразования письменной информации в математические модели составляют основу проблемы. Для решения проблемы был использован количественный дизайн в одной группе с тридцатью учениками шестого класса, которые были отобраны с помощью удобной выборки. Предварительное тестирование, обучение методам визуализации в течение месяца и итоговое тестирование составили три этапа сбора данных. Для оценки 300 ответов учеников использовался дескриптор структурного моделирования от 0 до 3. Результаты t-теста показали статистически значимое увеличение между предварительным (M=14,37) и итоговым (M=15,87) тестированием. Умеренное практическое влияние методов визуализации на решение задач было подтверждено размером эффекта d Коэна (0.311). Количественный анализ показал, что большинство учащихся успешно перешли от плохого графика (0-1 балл) к хорошему структурному моделированию (2 балла). В заключение, исследование количественно подтвердило, что систематическое обучение стратегиям визуализации снижает когнитивную нагрузку и значительно улучшает способность учащихся 6-го класса решать текстовые задачи.

Ключевые слова: Методы визуализации, математические текстовые задачи, шестиклассники, блок-схемы, математическое образование, количественные исследования.

1. INTRODUCTION

As a link between abstract mathematical ideas and practical applications, word problems are an important part of the mathematics curriculum. This level is especially important for 6th graders, as this class deals with complex relationships such as ratios, fractions, and proportions. Successful mastery of these topics requires not only computational skills, but also the cognitive ability to represent data quantitatively. The practice of depicting the structure of a problem through diagrams, charts, or graphs is called visualization and has been recognized as an effective method for helping to move to concrete solutions. This pedagogical direction was emphasized by the President of the Republic of Kazakhstan Kassym-Jomart Tokayev, who repeatedly stated that the quality of human capital is the main advantage of a modern state. In his addresses on the modernization of the education system, President Tokayev paid special attention to the need to strengthen the teaching of natural sciences and mathematics. This pedagogical direction was emphasized by the President of the Republic of Kazakhstan Kassym-Jomart Tokayev, who repeatedly stated that the quality of human capital is the main advantage of a modern state. In his addresses on the modernization of the education system, President Tokayev paid special attention to the need to strengthen the teaching of natural sciences and mathematics. He explicitly remarked: "We must prioritize the teaching of mathematics and technical subjects... The younger generation must be competitive, pragmatic, and open to new knowledge". Furthermore, the President has criticized the tendency towards formal education where students acquire grades without understanding. "Quality education is the core of a civilized society and the driving force of the state",- The President Tokayev stated. This national direction emphasizes the urgent need to shift away from dry memorization and mechanical calculation methods, which frequently impede the teaching of sixth grade mathematics, and toward new alternatives such as visualization. By helping students truly "see" and understand mathematical concepts, visualization methods directly contribute to the President's vision of cultivating a functionally literate and intellectually advanced generation.

Research Problem. There is a noticeable difference in students' problem-solving and calculation capabilities in real-world educational settings. Students in 6th grade sometimes struggle to solve word problems using the same numbers, despite their great accuracy in mathematical operations. The fundamental real-world issue is "keyword reliance." Students frequently scan the text for certain keywords (such as "total" and instantly adding) without comprehending the context, rather than examining the semantic structure of an issue. In non-standard or multi-step issues, this shallow approach results in chronic inaccuracies. Students do poorly academically and lack motivation because they lack a methodical way to convert the "story" of the problem into a mathematical equation.

Research Questions. This study was based on the following quantitative research questions:

- What is the effect of the visualization methods on the accuracy of the sixth-grade students in answering mathematical word problems statistically and practically?
- Does using visualization methods result in substantial gains in students' ability to model word problems relative to their baseline performance?

Research Aim: The aim of this study is to use quantitative analysis to thoroughly assess how well the visualization approach works as a pedagogical intervention for sixth-grade students' mathematical word problem solving abilities.

Research Objectives: To achieve this aim, the following tasks were assigned:

- To examine the evidence on the cognitive benefits of visualization in mathematics instruction and devise effective techniques.
- To create a pretest and posttest instrument for assessing students' accuracy in addressing non-standard word problems.
- To carry out a planned intervention employing visualization tactics with a convenience sample of students.

- To examine quantitative data from pretest and posttest scores to see if there was a statistically significant improvement in students' problem-solving ability.

Hypothesis: If visualization tools are implemented to teach word problem solving, then 6th grade students will show improved accuracy and modeling skills, resulting in significantly higher posttest scores compared to their pretests.

2. LITERATURE REVIEW

2.1 The role and place of word problems in teaching mathematics

Enhancing mathematical literacy in the current educational system is crucial while answering word problems helps students strengthen their logical thinking, language understanding, and capacity to simulate real-world scenarios in addition to expanding their mathematical knowledge and abilities. In this situation, improving students' word problem-solving skills is crucial. Word problems are regarded as a crucial, multipurpose teaching tool. Students' functional abilities are also developed by their capacity to convert text into mathematical models and apply theoretical knowledge to real-world scenarios (Batyrbayeva et al., 2024). Verschaffel et al. (2020) stated that word problems have a long history, extending back to Egyptian papyri, Chinese, and Indian texts from roughly 4000 years ago. It is still used in elementary and lower secondary school mathematics today (Swetz, 2009; Verschaffel et al., 2020). In addition to mathematical expertise, word problems call on logical reasoning, analysis, linguistic comprehension, and the capacity to simulate real-world scenarios (Kaskataeva & Khasanova, 2024). Moreover, recent studies bring to focus the "motivational and values-oriented" role of word problems. Word problems which have their roots in real situations, such as those portrayed in regional environmental statistics or local economy-related situations, make students realize the relevance of mathematical learning (Nurtayeva et al., 2023). One or more questions are requested to be answered in word problems, which are characterized as verbal explanations of problematic circumstances. Mathematical processes utilizing numerical data and quantities are used to determine the responses to them (Verschaffel et al., 2000, as cited in Verschaffel et al., 2020). However, extensive research documents significant challenges. Verschaffel et al. (2020) conducted a comprehensive survey of 50 years of research, identifying word problems as "among the most difficult kinds of problems that mathematics learners encounter." Students struggle with both comprehension and solution strategies. Word problems are among the most challenging topics in school

mathematics. According to Verschaffel et al. (2020), words problems are often defined as verbal descriptions of problem situations with one or more questions to respond. They provide a link between occurrences and numbers. Word problems can be defined as tasks originating from real-world situations (Verschaffel et al., 2010, as cited in Fatmanissa & Kusnandi, 2017). A.P. Tonkikh's description of a word problem was employed by Asanbayeva and Tagybergen (2025). This is a word problem is one which utilizes mathematical language to explain a particular scenario or process, shows how quantities relate to one another, and specifies the procedures needed to solve the problem. Isakova (2024) defines word problem activities as the main way to improve students' mathematical literacy and points out that by using new mathematical ideas and techniques, they aid in knowledge consolidation. The relationship between mathematics and natural language is a unique aspect of word problems. In other words, the student recognizes known and unknown amounts by comprehending the content. By doing this, students compare, organize, and think about the problems rather than just solving them. This enhances the student's cognitive abilities (Asanbaeva and Tagybergen, 2025). Sarkulova and Omarbaeva (2025) have emphasized the unique role that word problems play in this respect, even as they stress the need of professional preparation for aspiring educators. As Verschaffel et al. (2000) defined a successfully solved word problem as including a mathematical model and equation, a solution, and a presentation of the result. Also, Vessonen et al. (2026) discovered that primary school students have challenges in answering word problems and proposed that the presence of visual aids would facilitate their resolution. As Gusev and Bataeva (2020) point out, word problems are the most traditional issue in mathematics. Dyachkovskaya and Spiridonova (2023) agreed with this idea that word problems play an essential part in the current educational process. They also stated that their solution should receive special attention in both primary and secondary schools. Gusev and Bataeva (2020) stated that solving them is the most crucial talent student may develop for the future. They claim that by tackling real-world challenges, students will be able to solve difficulties more effectively in the future. Also, students' logical thinking, analytical

abilities, capacity to abstract from real-world challenges and practical application of mathematical knowledge are all enhanced by word problems. Word problems are said to have a unique place in mathematics education since they foster students' analytical thinking and problem-solving capabilities (Andrianova, 2024). In her article, Tuxtayeva (2025) asserts that word problems are crucial and that students must apply logical reasoning, analytical thinking, and prior knowledge to solve the word problems. In addition to these abilities, it has been observed that solving word problems calls on the ability to read, comprehend, build a mathematical model, and then assess it (Reys et al., 2008; Ryan & Williams, 2007; Verschaffel et al., 2010, as cited in Fatmanissa & Kusnandi, 2017). In other words, these abilities are seen as crucial components of the prerequisites for problem-solving. Additionally, they lengthen students' attention spans (Spiridonova & Dyachkovskaya, 2023). According to Rudenko's (2020) study, students begin learning to tackle word problems in the elementary grades. According to him, in grades 5 and 6, an algebraic method of solving word problems is formed. In this aspect, the teacher makes a vital role. The teacher should present students in these grades algebraic solutions to a variety of problems. Students will then be able to solve them with ease. Solving a text problem is difficult for students, hence Ayvazyan (2017) recommends teaching students a variety of strategies. That is, the act of addressing a word problem has its own definition. It entails looking for principles to identify the solution to the problem. One of the most challenging assignments for students is to solve mathematical word problems, according to Dyachkovskaya and Spiridonova (2023). Students are "scared" when they view a lengthy sentence, they claim. As a result, improving students semantic reading abilities is seen as crucial. Furthermore, students struggle to comprehend word problems when they come across novel symbols and terms. Students may become less motivated because of being given "unclear" problems. (Kaskatayeva&Khasanova, 2024). According to Sarkulova and Omarbaeva's (2025) research, students frequently make mistakes when answering word problems, namely misinterpreting the circumstances and failing to properly create a mathematical model. In her review of the well-known book "Making sense of

word problems", Novotła (2000) makes the case that word problems' typical shape, components, functions, and solution method all contribute to its theoretical framework. She also shows that word problems in math classes are not appropriate for practical applications. According to Kusuma and Retnawati (2019), mathematics is a topic that is taught from concrete to abstract, and since it plays a significant part in daily life, it is crucial to understand every facet of the subject to the point that it can be used in daily life. Drawing on D. Poya's work, Kaskatayeva and Khasanova (2024) identify word problems that, according to scholars, commonly occur in daily life and connect numbers to the operations carried out on them. They also point out that the most significant and useful mathematical tool is word problems. Translating the supplied conditions and problem formulation into mathematical language is the primary challenge in addressing a word problem (Zhuravleva and Smirnova, 2019). The studies stated above claim that word problems aid in the growth of analytical and logical abilities. Word problems can therefore be categorized as developmental activities. In this sense, Asanbaeva and Tagybergen (2025) noticed in their research the skills that may be enhanced by Yu. M. Kolyagin's developmental efforts. Among them are:

- Students learn to compare, abstract, improve, and recognize when to use these skills;
- They will be able to form conclusions using both inductive and deductive reasoning;
- They will learn how to develop predictions, arguments, and hypotheses;
- They will be able to develop mathematical models and analyze existing models, such as graphs, tables, and diagrams;
- They will learn to classify and categorize utilizing their previous knowledge;
- They will get the capacity to build methods for accomplishing goals;
- Students will grasp their relationship to the environment, its usage and application in real life, and its relevance;
- Critical thinking, written precision, speaking eloquence, attention, and logical consistency will be improved (Asanbaeva and Tagybergen, 2025).

In their research, Asanbayeva and Tagybergen (2025) drew on the opinions of several Russian and Kazakh academics who have studied word problems. A.E. Abylkasymova, one of them, observed that there are four primary purposes for mathematical problems. Specifically:

- educational
- upbringing
- development
- control

The goal of education is to shape students' knowledge, skills, and capacities. It was observed that this function varies according to the degree of educational material: it manifests as extended and in-depth content, as well as the fundamental knowledge outlined in the curriculum.

2.1.1 Types of word problems

There are no distinct types of word problems. Its kinds differ by school and age (Daroczy et al., 2015). Poprygina (2021) demonstrated that there are two categories of word problems. These are integrated word problems and word problems with multidisciplinary connections. The material that depicts the connections between items is where they diverge. Additionally, Sarkulova and Omarbaeva (2025) discuss and analyze the significance of employing this integrated teaching method. Authors like Shaunova and Kerimbayeva stated in their research that they have examined this method. Sarkulova and Omarbayeva (2025) claim that integrated education teaches students how to use theoretical knowledge in real-world situations and how disciplines are related to one another. Based on their experiments with solving word problems, Kaskatayeva and Khasanova (2024) concluded that the kinds of word problems should follow the concept of accessibility and be suitable for the students' age. Lozhkina (2009) clearly defines and provides three types of constructing word problems. His study indicates that a query or a condition should be developed for the first type of constructing word problems. A mathematical model should be used to organize the second types of problems. In other words, the learner should be given problems to

complete that have distinct questions but the same answer. This implies that students can repeatedly learn from problem-solving. A graphic or image should be used to formulate the third type of problem. (Lozhkina 2009). Word problems can be categorized according to their level of complexity. Semantic connections and difficulty level are the two categories used in a research by Pavlin-Bernardić (2008) for elementary school students. Three different kinds of semantic connections are found in the study. These are comparison, change, and combination. In other words, problems related to the first type should be those for which combining is the only way to get the solution. For instance, there ought to be issues like Alua has three apples, Kanat has two, and how many apples are there overall. Additionally, a second type of problems may be found in the changing format. In other words, there is a quantity that varies throughout time. Additionally, a third-type problem is one that compares one to another or the third with all (Bernardić and Pavlin, 2008). Additionally, four semantic types for arithmetic word problems were found by Surav Mandal et al. (2021). According to Pavlin-Bernardić (2008), they consist of combine, comparison, and change. Additionally, Surav Mandal (2021) found fourth type of semantic type is division-multiplication. There are subcategories inside each of them. According to Surav Mandal's research, these categories contributed to an automated word problem solution accuracy of 94.22%. But all these categories are for elementary school students. Classifying word problems is more challenging for sixth graders. There are, however, three primary categories of word problems for sixth graders, according to a study of the literature. Specifically, one-step, hidden information, and extra information problems. In their work, Schwanebeck et al. (2008) provide a precise description of these categories. Additionally, Kusuma et al. (2019) separated word problems containing integers, fractions, and decimals according to the topic. Also, Chilingaryan (2017) examines at a collection of word problems by the context and concludes that, according on their structure, they may be roughly categorized into six groups. These are:

- problems about percentages;

- problems with quantitative connections;
- sequence-related difficulties;
- motion-related problems;
- work-related problems;
- problems with alloys and solutions.

Meanwhile, Khayitova (2021) breaks a word problem into three major components. They are:

- the problem statement, which is the connection between known and unknown numerical values that characterize the scenario under discussion;
- the problem requirement, which may be discovered in the numerical connections inside the problem conditions;
- the problem formulation, which is a series of numerically connected activities necessary to solve the problem statement.

It is crucial to classify the tasks' intrinsic complexity in order to methodically examine the cognitive demands made on students while they solve mathematical problems. Students must negotiate a multi-step cognitive process when working with word problems, which include deciphering linguistic information, recognising quantitative links, and converting these textual parts into precise mathematical models. The famous methodologist V.A. Dalinger created a fundamental theoretical framework that successfully divides mathematical word problems into three different hierarchical levels after realising these important obstacles. Teachers can assess the different levels of abstraction and cognitive load that students encounter during the modelling phase according to this tripartite classification, which offers an essential teaching tool. Teachers can assess the different levels of abstraction and cognitive load that students encounter during the modelling phase according to this tripartite classification, which offers an essential teaching tool. Using this approach enables teachers to better grasp these transitional challenges and create focused instructional interventions, as shown and further explained in the recent empirical work by Batyrbayeva et al. (2024). Table 2.1.1.1 below provides a thorough summary of these three hierarchical levels.

Table 2.1.1.1. Three Levels of Word Problems According to V.A. Dalinger

The name of the level	Description
Acceptance	Problems at this level should include more mathematical content than plot material and be like real-world situations. This level's objective is to assess students' knowledge and proficiency in the areas of mathematics they have learned.
Formalizing a connection	This level is comparable to the integrated approach. The ability to differentiate between problems with mathematical and non-mathematical substance is the primary goal of this level. They should be conscious of their distinctions and commonalities at the same time.
Reasoning	The problems have to be created with real-world scenarios in view. They ought to be large as well. They learn how to look up information from different sources.

2.1.2 Solving methods of word problems

The "practice–theory–practice" paradigm is the foundation for word problem solving, according to Asanbayeva and Tagybergen (2025). To put it another way, word problems are analyzed in the context of real life, translated into mathematical language, and utilized to create a model since they are frequently based on real-world circumstances (Asanbayeva & Tagybergen, 2025). Teachers should educate students to employ a variety of approaches for solving word problems. Solving word problems may be divided into various steps:

- identify problem elements;
- determine problem circumstances and questions;
- the capacity to distinguish between known, unknown, and uncertain variables;
- the capacity to extract and alter words (phrases) that indicate connections between quantities from the problem's circumstances;

- proficiency in documenting functional connections and formulas;
- the capacity to interpret quantitative connections into mathematical expression;
- the capacity to convey the outcome of the solution in the language of the problem;
- the capacity to verify the answer (Urban, 2018, as cited in Kupreeva, 2023).

Zhuravleva and Smirnova (2019) observed that there isn't a single algorithm for word problem solution. The findings of reviews such as the Unified State Examination (OGE) also show that elementary school students have difficulty with basic word problems, according to their research. It all comes down to the student's inability to visualize the problem, their lack of comprehension of what is provided, and their lack of understanding of what needs to be discovered. This indicates that one of the most crucial phases is being overlooked. Put another way, the first step in solving a word problem is to comprehend and visualize the information provided. Karrison and Carroll (1991) developed a five-step procedure to solving word problems. According to the studies Karrison and Carroll (1991) conducted, the following five-step procedure works best for solving word problems. It consists of the following steps:

- reading the task;
- selecting the required computing operation;
- deciding which information should be processed;
- doing the calculation;
- determining the appropriate units for the outcome.

Several authors have supported that strategy. Furthermore, the students require all these skills at once. For instance, Gusev and Bataeva (2020) contend that teachers should be able to recognize the following roles in word problem solving: using illustrations to make the problem's logic easier to understand and that this is a unique approach. They claim that geometric pictures, as opposed to vacuous logic, make the problem easier to comprehend. According to their research, teaching word problems to students in grades 5-6 via diagrams, lines, segments, rays, and tables is more

successful. Tuxtayeva (2025) provides experiment to solving word problems for elementary classes using innovative technology. In the classroom, gamification, electronic games, logic games, and interactive crossword puzzles and etc. were utilized. "Learning by discovering" was the foundation of her teaching approach. Here, students have to make their own discoveries after grasping the word problem. In other words, they use their own approach to address it. In the end, students in the experimental group outperformed those in the control group because of the new innovative technology and technique of discovery. Four methods are the article of Ayvazyan's (2017) discussion on L.P. Stoylova's classifications. They are practical, pictorial, and arithmetic and algebraic. The arithmetic method is applied to elementary school. In other words, the arithmetic method uses arithmetic processes to determine the question's solution. And in doing so, the solution to the problem is discovered. For grades 5-6, the algebraic method is preferable. Rudenko (2022) also indicated in his research that the algebraic method needs to be utilized for solving word problems in the fifth and sixth grades, and that it was introduced in those grades. According to his research, a teacher should not begin using the algebraic method straight immediately. Since Kaskatayeva and Khasanova (2024) noted in their study that these difficulties - specifically, the difficulties students in grades 5 and 6 encounter when switching from arithmetic to algebraic methods - are all related to the inadequate attention given to the preparatory stages of algebraic methods in elementary school. Additionally, they observed that students struggle with word problems when they start secondary school because they become stuck and are unable to complete whatever they are doing. That is why Rudenko (2022) implies first, the teacher performs and discusses problems using algebraic and arithmetic methods. A student gradually realizes that algebraic methods are more efficient and simpler than the arithmetic method (Rudenko, 2022). In their work, Lokshin and Lavrova (2024) also emphasize the use of algebraic, arithmetic, and geometric methods to solve a variety of problems. To put it another way, they employed each of these methods to address the problems. They conclude that "none of them is considered a part of another". In other words, each method needs

to be considered separately. Additionally, it concludes that the student must be able to visualize a problem scenario and determine which method to use for a given problem. Due to the fact that this method is seen as difficult for elementary school. By developing an equation, the solution may be found using the algebraic method. Additionally, the pictorial method allows for the establishment of algebraic, arithmetic, and geometric relationships through problems that may be solved just by examining the illustrations or drawings. This approach may be used to help students in elementary school understand concepts that they have not yet mastered and that they find challenging by using the illustration. The practical method to word problems involves handling items to get the solution. In other words, this procedure is regarded more complicated. There, it is vital to establish hypotheses and theories (Ayvazyan, 2017). Arithmetic, algebraic, geometric, practical, logical, tabular, method of trial and error, and mixed methods to word problem resolution were discussed by Pilyukova and Voistinova (2018). Using arithmetic method on numbers is the definition of the arithmetic method that is the answer shown on the solving only arithmetic operations. The algebraic method entails creating an equation or inequality to find the solution. Additionally, the solution is regarded as having been acquired via various approaches if it is achieved by creating one or more equations. The geometric method employs geometric laws and geometric figure construction to arrive at the solution. The logical method utilizes logical thinking instead of solving any equations. The practical method involves building models to identify the solution. In the tabular method, a comprehensive table is created to determine the solution. One or more approaches are merged in the mixed method. The method of trial and error. Using this method, problems that have intuitive solutions are solved. To put it another way, you'll have to make a guess. In other words, you'll find the right answer by experimenting with a few alternative answers. The Polya procedure is employed by Novitasari et al. (2023) to solve word problems. Lee (2017, as cited in Novitasari et al., 2023) asserts that employing the Polya procedure enhances students' learning efficiency and makes the problem simpler to grasp. Powell (2011) specifically noted that since the Polya

procedure's four steps were introduced, teachers have started offering methodical advice on word problem resolution. According to Wiworo (2021), the Polya method works effectively. Novitasari et al. (2023) established four steps using this method. These include comprehending a problem, formulating a remedy, carrying it out as implementation step, and evaluating the outcomes as the review. The student is required to write down all the material provided during the comprehending step, along with any unknown. At this point, the problem is recognized. Planning the solution, the second step is developing a mathematical model and organizing the anticipated solution to the problem. They operate in accordance with their strategy during the third step, known as the implementation step. They focus on the correctness and reasoning of the response at the fourth step, known as the review stage. According to Powell's (2011) systematic analysis of 12 research, the schema-based method helps students who struggle with learning to solve word problems. Additionally, schemes may be used to arrange the primary structure of the word problem, which are represented by diagrams, illustrations, or equations (Powell, 2011). The following approach to word problem resolution has been suggested, according to Mardliyana and Amir (2023). There are four steps in this process. These include adding parts, splitting a whole into units, multiplying groups of items, and using logical reasoning. The first step involves reading the provided material, writing down the questions, and then identifying the components of the response. Using the information provided about the situation, a plan is created in the second step. The quotients for each component are combined together to get a single result in the third stage. Each set of x is divided repeatedly until the division is finished in the fourth step. Nevertheless, Mandal and El-Hmoudova (2021) note that students typically struggle with understanding problems, identifying formulae, and performing calculations – proposing an implementation remains difficult, regardless of the specific circles. In order to enhance teacher professional growth, Sarkulova and Omarbaeva (2025) gave students a variety of problems. For instance, they used problems involving motion, concentration, and percentages. Algebraic methods were used to solve each of them. Additionally, they thought that

explanations should make use of integrated knowledge; for example, concentration should be connected to chemistry and mathematics, whereas motion-related problems should be taught using contrasting teaching methods. They added that word motion-related problems benefit from animated visual aids. In 2024, Kaskatayeva and Khasanova worked with students in grades 5 and 6. They discovered that switching from arithmetic to algebraic approaches is the most significant obstacle for students in this age range. They determined that to solve this problem, it was necessary to determine students' lower, medium, and upper levels and to create problems that were suitable for each level. They believe that each class has students that fit into these three levels. They also found out how these three levels of students see the assignments. As a result, they described how students at every level should be taught by them. Kupreeva (2023) addressed word problems using a case study methodology. The main idea behind this method is that group work is crucial for students. In the framework of a case study, students must comprehend real-life problems and come up with answers. Here, the primary goal of case study learning is to work as a group with the teacher to solve the word problem. By doing this, students can create their knowledge rather than just receiving pre-made information. The definition of the term "case" is relevant in this situation. Both theoretical and practical interest may be found in it. It distinguishes between two elements of a case: work-related duties and supplementary data. To put it another way, while employing a case study method to solve word problems, the teacher first splits the class into groups, asks the students questions to help them comprehend the case better, and then evaluates their work. After that, the students got to work. They create potential solutions while considering the opinions of their group members, note the right response, and plan future assignments in this format. Andriyanova (2024) claims that teaching word problems using a component-based method is the algorithm for addressing problems in mathematics education. There are four steps in the generic problem-solving algorithm:

- examining the problem's theoretical component;
- figuring out how to solve the problem;

- confirming the selected method of solution;
- self-evaluation of the finished problem.

In the first phase, the teacher assists students in understanding the problem's theoretical component. After then, students try to determine the problem's known and unknown values as well as the precise requirements. Their comprehension of the functional link is strengthened by this procedure. At this point, students may easily understand the connection between the amounts and the event if the teacher makes a table, diagram, or drawings. They create methods for resolving the word problem in the second phase. In other words, they decide which part of the equation will be the primary one. In the third phase, they put their strategy and methods into practice and try to find a solution. An analytical evaluation occurs in the fourth phase. At every level, the students' activities are evaluated. As they arrive at the solution, they assess their own performance. This approach improves the student's word problem-solving skills even more. Asanbayeva and Tagybergen (2025) also discuss the steps needed in completing a problem like general. They, as well, are classified into four categories:

- the first step: analyzing the nature of the problem;
- the second step: deciding how to solve the problem and developing a strategy;
- the third step: executing the strategy and looking for a solution;
- the fourth step: it involves validating the obtained solution.

These four steps can be identified as after the ones reported by Andriyanova (2024) in her research. Furthermore, Asanbayeva and Tagybergen (2025) state that if all of these processes are completed sequentially, the result will be right and accurate. However, if any step is omitted, the answer may be inaccurate. To make sense of a problem's context, Asanbayeva and Tagybergen (2025) advise students to ask themselves the following questions. These questions consist of:

- What is the core of the issue?
- What are we aware of?
- What information is required?
- What information is not known?

This is an illustration from Asanbayeva and Tagybergen's (2025) research, which offered other instances in relation to similar queries. The problem is as follows: Two children are strolling in the same way along a road. They are initially separated by two kilometers. The first child moves at a pace of 4 km/h, while the second is moving at a speed of 5 km/h. A dog rushes from the first child to the second, then back again, and so on, until the second child catches up to the first. The dog can go at eight kilometers per hour. How far did the dog run before children lined up? Asanbaeva and Tagybergen (2025) were categorized by this problem based on the questions as follows:

- What exactly is the problem?

It involves two children and a dog who are in motion. Everyone knows their beginning speed and position.

- What do we need to find?

The total distance the dog has traveled during the excursion.

- What information is known?

(a) Both children are heading in the same direction.

(b) The beginning distance is two kilometers.

(c) The first child's speed is 4 km/h, whereas the second's speed is 5 km/h.

(d) The dog has a speed of 8 km/h. What is unknown?

How long will it take for the second child to catch up to the first, and how far will the dog go in that time?

- What is the nature of the problem?

It's a distance problem. That is, we must determine the length of the path traveled by the dog during that period.

According to the authors, the answers to these questions will help students recreate the problem statement. According to Chilingaryan (2017), there are many processes involved in solving a typical word problem. Among them are:

- using the letters x and y to represent the problem's unknown values;
- utilizing the known quantities given in the problem to create an equation or a system of equations;

- after which this equation or series of equations is solved;
- choosing solutions which satisfy the problem's requirements.

The author claims that applying these processes to solve the problem assists in the development of a mathematical model. This demonstrates the significance of building everything according to the conditions specified in the challenge and not ignoring any conditions. In this instance, solving word problems demonstrates to students the beauty of using mathematics in daily life and how they may appreciate it, despite the fact that word problems are quite challenging to solve (Khoshaim, 2020). In their study, Batyrbayeva et al. (2024) used the idea of the right construction of the word problem as the primary guideline for addressing word problems. They discovered the following principles because of their research:

- giving students problems based on topics they have already learned is crucial. Additionally, this allows for the evaluation of their prior knowledge;
- giving students problems that are similar to both nature and man-made events is crucial. They can then make use of their personal experiences;
- when presenting a problem to students, it should be structured so that they come across something novel. Their comprehension of the novel idea will thus develop more quickly;
- the last concept is the significance of obtaining the quantity of assignments that align with the duration of the course.

It is crucial to select word problems at a level that doesn't interfere with other educational activities. Furthermore, it is said that the problems must be suitable for the level of the students. In other words, the authors support the necessity of paying close attention to the word problem's structure by demonstrating these ideas.

2.1.3 Challenges in solving mathematical word problems

According to a research conducted by Fatmanissa et al. (2017), word problems remain one of the most challenging topics for students when compared to other mathematical topics. However, this trouble with word problems is not limited by language or culture, hence it affects students of all ages, grades, and cultures

(Khoshaim, 2020). That is, it is clear that the challenges they produce in students stem from language factors. In general, students solve mathematical challenges by memorizing rules and processes. Khoshaim (2020) also noted that challenges of addressing word problems in mathematics is not novel. Boonen et al. (2016, as cited in Khoshaim, 2020) discovered that this is insufficient to grasp mathematical ideas. Chilingaryan (2017) divided word problems into six groups according to their sentence structure, as was previously indicated. He also talked about the challenges that students in these six categories encounter while attempting to solve word problems. For instance, students frequently struggle with problems involving combinations and alloys because they do not identify them with chemistry, according to Chilingaryan (2017). The instructor has to clarify that "concentration" is a chemical concept (Chilingaryan, 2017). Students won't be able to grasp this crucial concept till then. Finding a portion of a whole, figuring out the whole from a given part, and computing a percentage are methods that are seen to be challenging for students to understand when it comes to topic about percentages. Additionally, before students try the task, professors must explain these algorithms to them. Meanwhile, Darocci et al. (2015) found three major components of difficulty in word problems after evaluating 227 articles:

- the linguistic complexity of the word problem;
- the numerical complexity;
- their interaction.

Similarly, Fatmanissa et al. (2017), who examined 35 publications, found that difficulties in word problems were linked to many semiotic systems, vocabulary, grammar, and syntax. Khoshaim (2020), utilizing brief surveys as well as interviews, indicates that students struggle to solve word problems due to their limited skills and negative attitudes toward mathematics. According to the results from surveys, these concerns cause students to become more stressed. Khoshaim (2020) claims that the mathematics education community has long recognized the complexity of word problems. Khoshaim (2020) contends that, while numerous studies have indicated the need to incorporate more word problems into classes to assist students grasp word

problems, it is unclear why teachers avoid them and students find them challenging. In his research, Khoshaim (2020) reported that Schwanebeck (2008, as cited in Khoshaim, 2020) discovered that persistent word problems might cause dissatisfaction and low self-esteem. Naturally, low self-esteem reduces students' ability to answer word problems and creates the perception that only intelligent students are capable of doing so (Lambdin, 2003, as cited in Khoshaim, 2020). Otherwise, to solve word problems, students' past knowledge and abilities are crucial (Craig, 2001, as referenced in Khoshaim, 2020). Gorbunova's research from 2025 demonstrates the challenges students have while resolving word problems. Her research showed that students had trouble with three-step word problems based on an examination of common mistakes and challenges. To put it another way, they are unable to translate between word problems and abbreviations and vice versa, nor can they comprehend the relationship between amounts. Specifically, making a table was the primary challenge for addressing word problems. The authors created a method for making tables to answer word problems for elementary school students in order to address this problem. As a result, the following standards were created to assess tabular models: the capacity to recognize the structural components of a word problems, the capacity to obtain and apply information from tables to carry out tasks and solve problems, the capacity to solve word problems in one to three steps, the capacity to apply relationships between speed, time, and distance traveled, performance, working time, and volume when solving word problems and the capacity to create a tabular model of word problems. Additionally, Ulmasbaeva and Soloshchenko (2017) point out the most frequent error made in word problem solution. They claim that reading the problem is the first step. Put another way, hurrying to find a quick solution without doing any research or deliberation. When tackling word problems, this causes numerous types of problems. Put differently, it impacts the student's capacity for autonomous and rational thought. Finding links between the known and the unknown is the primary goal of solving any word problem. Additionally, Ulmasbaeva and Soloshchenko (2017) observe that only the most accomplished students write problems on the board during math classes,

which is a challenge for them because they frequently leave off stages in the solution, writing only the major portion or the final equation. Their thinking is constrained by the problem at hand. Ulmasbaeva and Soloshchenko (2017) came to the conclusion that teacher general responsibility is to help "weaker" students solve challenges. Additionally, they ought to let students help solve the problem. Ulmasbaeva and Soloshchenko (2017) conclude once more that, especially in physics-related situations, one should always verify the accuracy of the solution before analyzing it. Khoshaim (2020) performed research with 18 teachers and invited them to conduct interviews with students they advised as part of an experiment, resulting in the following conclusions: Students are not used to thinking and memorize the template, so "new concepts" - new and never-before-seen words given in the word problem - make it difficult to understand what's written. Students' unpreparedness can also be attributed to the teacher, who requires seminars, also students have weak skills, and skips steps in solving the word problem. Kusuma et al. (2019) conducted interviews with sixth-grade students on fractions and integers in word problems and discovered the following challenges. The author categorizes it into four main causes. They are:

- students have trouble grasping word problems;
- students do not know the operations associated with fractions;
- students with limited counting abilities;
- students have struggled with accuracy.

Due to these factors, Kusuma et al. (2019) conclude that students' inability to answer the provided word problem is mostly caused by a lack of knowledge of the topic. That is, they are unable to create computation processes or mathematical models because they do not comprehend the meaning of the problem's conditions. Furthermore, the inability to discern the information in the problem, i.e. what is known and what is unknown, makes problem resolution harder. This is the most typical challenge. They are unable to solve word problems because of their inability to conduct fractional operations. The inability to count accurately, or to swiftly count and add numbers without discriminating between decimals and whole numbers, is also a

hindrance. The author states the following concerning the last factor: inaccuracy. That is, students are rushing to solve problems, however, they are unable to provide an appropriate solution. This is one of the challenges faced not just while solving word problems, but also when tackling additional challenges. Several foreign scholars have also drawn attention to the linguistic problem, most notably Fatmanissa et al. (2017). Put another way, it is asserted that students who do not speak English as their first language find it difficult to understand word problems. However, in Kazakhstan ordinary schools, math classes are not taught in English. For this reason, this problem is not seen in Kazakhstan. However, while analyzing problems in a foreign culture, one could encounter incorrect translations.

2.2 What is the visualization method

Nowadays, technology is always developing. Kvetko and Avdoska (2023) discuss the necessity of new instructional technology in the sphere of education for this same reason. These tools aid in the visualization of learning, making everything obvious and accessible to students and simplifying difficult concepts. When working with abstract mathematical topics, visualization is crucial. This implies that knowledge is obtained through visualization based on practical experience rather than formalism (Kvetko and Avdoska, 2023). Evteeva (2024) indicated that visualization should be given top importance while addressing problems as it serves as the foundation for students' further growth of geometric thinking. Additionally, Novikova and Vlasova (2022) pointed out that advances in this field have made it easier and faster to find everything one needs. They also highlighted the following characteristics of visualization: information analysis, source selection, reliability assessment, and future use of the chosen information. The act of transforming information from textual form into symbolic, or mathematical, form utilizing mathematical and symbolic tools, as well as organizing, generalizing, reconstructing, and vice versa, is known as visualization (Novikova and Vlasova, 2022). Particularly for teachers when introducing new content, visualization helps to increase brain activity, produce ideas, and lessen learning challenges (Subbotkina, 2020). One particular type of mathematical

method of teaching is visualization. Visualization in mathematics is the process of creating, interpreting, and reflecting upon pictures and images to represent mathematical concepts and relationships. According to Abu et al. (2001), mathematical visualization is “the art of creating a tangible experience with abstract mathematical objects and concepts”. Mathematical visualization also encompasses modeling mathematical problems using computer graphics and animation. This enables students to have hands-on experience with abstract mathematical notions. Based on Russian schoolchildren's involvement in worldwide research on reading and mathematics literacy, Novikova and Vlasova (2022) found that about 45% of students had trouble finding information that is clearly expressed in the text. For this reason, Novikova and Vlasova (2022) emphasize that information cannot be considered knowledge until it is conveyed graphically. De Guzman (2002) asserts that visualization enables mathematicians to examine structure by offering alternate ways of visualizing notions that might be overly complex for analytical visualization. Based on this rationale, visualization of mathematics, proven valuable for presentation, problem-solving, and investigation, involves using concrete representations for abstract understanding. This perspective is well-represented within the realm of mathematical cognition and communication. Aligning their perspective, researchers like Clements (2014) believe that visualization within mathematics involves using visual imagination for problem resolution. Nonetheless, students sometimes differ within their visualizations, even if they are algorithmic thinkers. This happens even if their performance level within visualization exercises is high. The method is particularly powerful because it leverages human visual perception, allowing learners to “see” mathematical relationships. Rahim et al. (2011) notes that visualization is not an innate skill but a learned process of “seeing to think”, which can be taught and developed to enhance mathematical understanding. Information comprehension is enhanced by visualization, which may be converted into graphic, spatial, and sensory representations. The following scientists' statements support this information. One of the primary learning activities in grades 5 and 6 is solving word problems, during which students acquire

fundamental abilities in logical reasoning, mathematical modeling, and abstraction (Nevostrueva & Ruzanova, 2021). Derkach (2025) examines at the views on visualization of O.A. Fedosova and E.N. Sokolina. Derkach (2025) defines visualization as the presentation of information via the use of graphs, tables, diagrams, maps, and other visual aids. At first, it was mostly demonstrative, but with time, it evolved into the creation of images meant to grab attention and promote educational activities. Since people initially began collecting data in antiquity, visualization has existed (Serbina and Kruk, 2020). Serbina and Kruk (2020) define visualization as a graphical depiction of data. This graphical representation's primary goals are to communicate information using pictures, graphs, tables, diagrams, and other visual aids and to make information easier to perceive. Serbina and Kryuk (2020) state that one specific subject must be the foundation of every visualization employed in mathematics. This presents an issue: visualization will not accomplish its goal if the student is not familiar with the graphical elements and the relationships between them—that is, lacks the knowledge to comprehend them. According to Serbina and Kryuk (2020), visualization is an external factor that promotes learning. This external factor can take many different forms, such as a computer program, an instructor, or a textbook. Facilitating the comprehension of ideas, concepts, activities, connections, algorithms, etc. is the primary goal of visualization. A link between the student's prior experience and the new information being studied is necessary to accomplish this goal. The student's level of knowledge must thus be considered in order for visualization to be successful. According to Rathour et al. (2022), the usage of visual aids in mathematics instruction relies on the subject's structure and the student's level of expertise. Additionally, it is necessary to employ visual aids to pique students' interest so that the lesson may be simply explained to them. These visual aids make the course easier to grasp. Furthermore, visual aids are crucial in bridging the gap between theory and practice (Rathour et al., 2022). In his thorough examination of the idea, Clements (2014) states that visualization is an action that a person does in his mind that aids in the creation of a mental image. He then reviews the theories of several scientists. For

instance, Zimmermann and Cunningham (1991, as cited in Clements, 2014) proposed that mathematical notions may be represented geometrically or graphically through visualization. Additionally, Norma Presmeg wrote on the definition of visualization and believes it is comparable to Clements' theory, which states that visualization describes both geographical and mental pictures. Furthermore, the notion of visualization in terms of problem solving is the foundation of Alan Bishop's concept. In other words, it is claimed that problem solving involves the use of visualization. Clements (2014) also discussed the viewpoint of Tall and Vinner. They described concept images. In other words, every mental image and property is described by a concept image. Additionally, Rathour et al. (2022) discussed the views of a number of academics. For instance, Arcavi defines visualization as the capacity, process, and result of producing, interpreting, and utilizing pictures, diagrams, and illustrations in our minds, on paper, or through technological means in order to represent and communicate information, think about, develop previously undiscovered concepts, and foster understanding. Arnheim's concept of visual thinking also caught the attention of Rathour et al. (2022). In other words, he described visual thinking as using visual processes to think. Additionally, he described visual thinking as the capacity to envision images in the mind as opposed to merely illustrating them. According to Presmeg (2006), all mental thoughts may be converted into visual pictures through visualization, which is crucial to solving mathematics. Presmeg (2006) makes the case once more that visualization is an effective mathematical tool because it may give abstract ideas a “concrete” portrayal. According to Rathour et al. (2022), visualization is the use of efficient technology in education to communicate the process under study and visually build linkages. According to Rathour et al. (2022), the visualization principle is essential to teaching mathematics. Furthermore, visualizing is regarded as an outside aid for mental work. There are two kinds of visualization, according to what the author stated. They are subject visualization and symbolic visualization. Drawings, tables, graphs, schemes, diagrams, and graphs of binary relations are all used in symbolic visualization. Models of geometric bodies, spatial scenarios, and flat figures

are examples of subject visualization. Based on Solomka (2025), to keep students interested, teachers should encourage them to utilize their imaginations more. Additionally, that points out that this kind of visualization might foster students' creativity and critical thinking abilities. Subsequently it goes on to say that there are several kinds of visual information. These might contain animations, films, and diagrams, for instance. Mathematical visualization can operate as a "cheat sheet" for problem-solving can be viewed as a link between the imagination and abstract numbers (Solomka, 2025). According to Solomka (2025), visualization makes it possible to accomplish three goals:

- to identify a straightforward and elegant solution; Rathour et al. (2022) separated the significance of visualization into three categories based on the concept of visualization provided by several authors, particularly Arcavi. They are:
- it is crucial for improving students' mathematical communication abilities and may convey mathematical concepts through both internal and exterior visual representations;
- mathematical ideas and their connections can be given significance through visualization;
- visualization is an effective method for investigating and resolving issues.

According to Ochkov et al. (2022), teachers can assist students solve problems by creating their own visual representations. According to Fokuo et al. (2024), all interactive and virtual gadgets are considered visualization tools. The students can utilize their innate capacity to comprehend visual information thanks to these visual aids. Additionally, they make abstract theories easier to comprehend. Tools for mathematical visualization aid in creating a connection between learning objectives and the content of the instructional materials. Additionally, by eliminating superfluous information and promoting additional knowledge retention, visualization helps streamline the learning process (Serbina and Kryuk, 2020). According to Rathour et al.

(2022), visualization has a long history and is in line with mathematical methods. Three distinct phases of knowledge may be identified based on their definition. They are:

- perception;
- creativity;
- abstract thought.

Additionally, they describe two phases of knowledge perception:

- produce rational;
- sensory writing.

A unique space for imagery characterizes the sensory stage. Additionally, they claim that to demonstrate the outward characteristics of mathematical objects as well as their relationships, distinctions, and similarities, visualization is required. Additionally, they claim that even at the third level of knowledge - abstract thinking - visualization aids in accurate thought. Depending on Tarabrina's work, Osmanova (2023) illustrated how visualization might support the following goals in elementary school:

- the growth of critical thinking
- a thorough method for resolving issues of any complexity;
- gathering, analyzing, and using of new information;
- assessment of one's own work;
- growth in emotional intelligence.

Students' critical thinking and logical skills can be developed by employing the visualization method in this manner. Their focus, creative inquiry, and goals may be impacted by this. Osmanova (2023) asserts that contemporary information technology advancements make it feasible to locate the information one wants quickly. She contends that information technology and multimedia may be used to illustrate visualization. The following results are made possible by using information technology to visualize lessons:

- providing information in an approachable and accessible manner;
- planning research projects;

- putting together a teaching strategy that is differentiated;
- enhancing emotional involvement;
- fostering an environment that encourages creative engagement;
- quick transitions between tasks;
- the use of technology based on games;
- how group-based learning is organized, etc. The following abilities can now be developed by the teacher in the elementary school using such visual aids:
- utilizing visible, contemporary, and recognizable technology to include students in the learning process;
- increasing the variety of learning activities students engage in and providing them with fresh material;
- boosting students' enthusiasm via individualized and visual instruction;
- enhancing the energy and interest of lessons (Osmanova, 2023).

For students in grades 5 through 11, visualization is seen to be a useful technique for improving mathematics comprehension and reasoning. In addition to improving their conscious and high-quality learning, the usage of such visual aids helps them acquire transferable learning abilities. Students can learn mathematics more easily and comprehensibly through visualization (Subbotkina, 2020). The beneficial effects of visualization on students have been divided into many important categories by Kovalenko and Ivanova (2022):

- it fosters critical and visual thinking in students and aids in their efficient organization and analysis of information;
- helps students make a harmonious connection between the information they have learned and a geometric object;
- aids in the consolidation of new knowledge;
- facilitates the rapid absorption of a large amount of information; - allows them to reconstruct and replicate different processes;
- allows them to arrange educational materials in an interesting manner.

2.2.1 Types of visualization method

There are two types of visualization in education: internal and external. Internal and external visualization in mathematics are complementary cognitive processes that play a crucial role in mathematical thinking and learning. Hegarty et al. (2004) identified three primary relationships between internal and external visualization:

- External visualizations might substitute for internal visualizations
- Comprehension of external visualizations depends on internal visualization skills
- External and internal visualizations can mutually augment each other

Schoenherr et al. (2023) specifically noted that external visualization is central to mathematics education, encompassing various types from pictorial to abstract representations. Mathematical visualization methods can be categorized into two primary types: physical resemblance visualizations and structural resemblance visualizations. Schoenherr et al. (2023) identified a comprehensive framework of visualization techniques, including:

Physical resemblance visualizations:

- pictorial representations
- abstract visual representations

In their systematic review, Hadi and Csíkos (2025) thoroughly examined the function of vision in word problem solving. They shared their thoughts, particularly with relation to schematic and pictorial visualization. They contend that while merely drawing a picture does not convey the problem's meaning, the ability to appropriately produce schematic visualization during the production of a word problem helps to effectively create a problem. Future descriptors will be developed based on this result.

Structural resemblance visualizations:

- Length-based visualizations
- Area-based visualizations
- Relational visualizations

The research suggests this visualization methods are crucial for enhancing mathematical understanding, particularly in helping students and researchers interpret complex mathematical concepts through visual representations. Presmeg (1985) identified five categories of visualization, which Clements (2014) cites. Here are:

- concrete visualization happens when relationships are envisioned in some manner;
- formula visualization happens when abstract information is pictured in concrete pictures;
- pattern visualization happens when relationships are imagined in some manner;
- when an image is shown in shape or pattern, kinesthetic visualization takes place; when pictures are pictured mentally, dynamic visualization takes place.

Also, Serbina and Kryuk (2020) examined a variety of visualization applications. Additionally, they showed that it falls into two categories:

- data analysis;
- sharing of information.

The application of visualization tools is known as data analysis. To put it another way, information communication is the visual representation of data, whereas it is utilized to find correlations between data points or collections of data points inside massive databases. Its goal is to successfully communicate with the addressee (Serbina and Kryuk, 2020). Hajieva (2020) observed information visualization models in her research. She was especially intrigued by the models created by P. Grech and L. Holtzapfel. The following categories comprise visualization models:

- verbal (written or spoken description);
- graphical (graph, picture, diagram);
- mathematical (using formulae for description);
- tabular (object-to-object, object-to-property).

The transformation between these models was explained by Khadzhieva (2020). Now proceed to examine each of them independently. Transformation from a verbal model:

- if the data is given verbally, a graph is created based on the text to transform it into a visual representation.
- specific values and data are recognized and documented in the text when switching to a tabular form.
- the phenomena being described must be articulated using mathematical equations in order to go on to the mathematical model.

Transformation from the graphical model:

- for the purpose of switching to the verbal model, a description and an explanation of the final graph's meaning are given.
- the values of the points along the graph are read and recorded into a table when switching to the tabular model.
- depending on the kind of graph created, an appropriate approximation function is chosen for conversion to a mathematical model.

Transition from a tabular model:

- when switching to a verbal model, a conclusion is reached after reading aloud the numerical values and data in the table.
- a graph is created by plotting the table's points on a coordinate plane in order to transform them into a graphical representation.
- the regression method is used to calculate the data in the table and determine the relevant function (formula) to move to a mathematical model.

Transition from a mathematical model:

- an explanation of the formula's logical or physical significance is given when switching to a verbal model.
- a related diagram or graph is created using the formula to switch to a graphical model.

- the formula is changed to a tabular model by substituting numbers, calculating the results, and entering the results into a table.

A particular method of information visualization is described by Klendershikova and Butko (2023). They separate it into two phases.

- the initial phase is regarded as theoretical. Using accessible graphs, diagrams, and other graphic elements, the teacher exposes students to a variety of methods for visualizing textual information.
- the second phase is seen as pragmatic. Students build their own visualization talents at this point. In other words, they create graphics and graphs on their own. They learn to think critically and do in-depth study in this way.

Five categories of visualization in math classes were recognized by Subbotkina (2020):

- diagrams and other comments that demonstrate the topic's structure;
- diagrams and graphs that allow for relationship analysis;
- tables that arrange data;
- geometric models and drawings;
- digital technology for dynamic visualization.

Each type has its own educational purpose and is designed to fit the structure of each lesson. Furthermore, visualization plays an important role in the development of mathematical concepts in grades 5–7. This is because visual models, drawings and diagrams are used to study fractions, percentages, coordinate planes and geometric shapes. This helps pupils move from concrete concepts to abstract thinking (Subbotkina, 2020). Four categories of visualization principles were studied by Novikova and Vlasova (2022). These are:

- the principle of radial
- the principle of trees
- the principle of sequential
- the principle of tabular

The radial principle was explained by Novikova and Vlasova (2022) using Harry Adler's approach. In accordance with this, the focus is positioned in the middle of the page, and several connected features are noted in the periphery, which are divided by rays. The radial principle and the tree-like principle are not the same. Like the branches of a tree, additional themes diverge from the primary and major themes in this instance as well. Cause-and-effect relationships are frequently demonstrated using this principle. Everything is confined within the chain when using the sequential principle. Novikova and Vlasova (2022) used quadrilaterals as examples to illustrate this idea. Simple tables serve as the foundation for the tabular principle. To put it another way, key information is arranged in columns and rows in memory. Diagrams, tables, pictures, mind maps, and roadmaps are categorized as visualization forms by Kovalenko and Ivanova (2022). They also emphasize how quickly knowledge can be recalled. According to Safonova (2000), visualization is a geometric approach to word problem solution that includes the following types:

- diagrams in one dimension
- diagrams in two dimensions
- graphs

Line segments are used in one-dimensional graphics to depict values. In two-dimensional diagrams, the area of a rectangle represents the product of values. For linear functions, graphs are utilized.

2.2.2 Theoretical analysis of the concept of "visualization" in pedagogy and psychology.

Subbotkina's (2020) analysis goes into further depth on a theoretical examination of the psychological and pedagogical underpinnings of the term of "visualization." Subbotkina (2020) claims that visualization is based on students' perceptions of the qualities of their own thinking as well as the fact that students absorb information faster, comprehend mathematical objects right away, and attempt to make connections between them because of visual representations. Regarding the accessibility of learning items, the use of visual aids is in line with the didactic teaching principles. The

evolution of functions, geometric objects, statistical data, and algebraic relationships all demonstrate the efficacy of visualization (Subbotkina, 2020). Osmanova (2023) offered some findings on the pedagogical definition of visualization in her article. In order to maximize the learning process, she considers visualization in education as a method of presenting instructional material in the form of pictures, graphs, tables, diagrams, and multimedia tools. Additionally, based on K.D. Ushinsky's theories, Osmanova (2023) explores from a psychological and educational standpoint how using students' different senses—visual, aural, and kinesthetic—allows them to comprehend and remember the content more effectively. The physiological elements of color perception are also examined. According to Kokoreva and Vendina (2020), graphic models convert an idea's substance into a visual representation that shows logical links. Presmeg (2006) divided the psychological components of visualization into two categories:

- the interpretation of images is known as IFI (Interpretation figural information). This is the capacity of the learner to extract mathematical meaning from a particular graph, figure, or picture.
- visual processing, or VP for short. This is the process of creating, modifying, and working with mental or written pictures.

According to Fokuo et al. (2024), visualization's psychological components are as follows: the students can utilize their natural capacity to interpret visual information by using visualization tools. This increases their motivation in studying and results in a stronger comprehension of mathematical ideas. Furthermore, Meyer and Moreno's (2010) cognitive theory of multimodal learning is cited by Fokuo et al. (2024). It has been observed that an overabundance of animations or visualizations might divert students' attention from the primary material and impede their ability to learn. Fokuo et al. (2024) compared visualization to a bridge between abstract mathematical concepts and concrete visuals to illustrate the pedagogical usefulness of visualization. They viewed visualization tools as an approach that enhances learning. Additionally, from an educational standpoint, visualization facilitates comprehension; as Fokuo et

al. (2024) pointed out, visual aids allow students to investigate complicated topics through visual representations, which helps them comprehend abstract ideas more deeply. Klendershikova and Butko (2020) wrote about the psychological characteristics of information perception in visualisation. Drawing on research by psychologists, they noted that people receive 80 per cent of information through visual channels. They also noted that visualisation, in accordance with didactic principles, is one of the main principles of teaching – the implementation of the principle of clarity. At the same time, it is based on the fact that visualisation provides a person-centred approach to the individual psychological characteristics of each student. Using studies by British psychologist Tony Buzan, Novikova and Vlasova (2022) emphasized the psychological components of visualization. According to his study, information needs to be conveyed through associations and visuals in order to maximize the capacity of the human brain. This indicates that the use of visualization is essential. Additionally, according to Novikova and Vlasova (2022), the psychologists who carried out the study found that employing both hemispheres of the brain throughout the learning process aids in the efficient development of cognitive skills. They also observed, using mind maps, that colors may enhance cognitive functions, activate the right hemisphere of the brain, and have a good impact on the human brain. According to Novikova and Vlasova (2022), visualization techniques are a useful tool for helping students improve their information processing abilities (structuring, generalization, analysis). According to his study of the findings, he suggested utilizing visualization starting in fifth grade and progressively progressing from basic diagrams to intricate mind maps while preserving the order and continuity of learning. He also emphasized the unique position of the teacher in this respect, pointing out that the student should act autonomously once the teacher has guided them first. He also discussed how well this visualization method works in group projects. He specifically showed how students acquire communication skills including engaging with others, defending their own beliefs, and comprehending the perspectives of others. From a psychological standpoint, Kovalenko and Ivanova (2022) explained how visualization affects human cognitive processes. Based on

psychologists' theories, they contended that organized, or visual, information presentation facilitates students' faster and more efficient understanding of new ideas. Additionally, they claimed that knowledge stored in visual-spatial memory is more likely to be maintained. Additionally, it was discovered that visual pictures reduce word sequences into a succinct, schematic representation. Kovalenko and Ivanova (2022) investigated the opinions of several academics from an educational standpoint. Visualization is thought of as the act of projecting internal pictures outside, similar to a projection, according to K. Volker and M. Minsk. A.A. Verbitsky's idea states that visualization may be used to summarize thoughts into a visual representation, which can then be reworked and utilized as a foundation for logical reasoning. The educational goals of visualization were also explained by Kovalenko and Ivanova (2022). They specifically mentioned how visual technologies promote learning activities, speed up and intensify the learning process, and encourage visual thinking and visual-perceptual processes. Moreover, enhancing visual literacy is another educational goal of visualization (Kovalenko and Ivanova, 2022). M.N. Grablev's model was introduced concurrently. It was demonstrated that he has created four models for visualization technologies. These are:

- a collection of educational expertise;
- pedagogical knowledge can be presented visually;
- technological and visual ways to communicate information;
- a collection of psychological techniques for using visual-spatial thinking.

Therefore, visualization is presented by Kovalenko and Ivanova (2022) as a sophisticated pedagogic system that improves students' cognitive engagement and develops their spatial thinking rather than just as a picture or example. Danilenko and Reshetnikova (2022), who presented visualisation as modelling, found in the psychological and educational literature that the definition of modelling refers to research. In other words, modelling is not studied directly within the process itself, but is primarily associated with a cognitive method. According to Danilenko and Reshetnikova (2022), there are three categories of mathematical modeling:

- schematics
- diagrams
- formulas

Additionally, they contend that spatial models, scales, and graphs may be employed. Safonova (2000) examined visualization in relation to employing geometric models to solve word problems. According to her, it has psychological and pedagogical value since it affects students' ways of thinking. Put differently, it allows students to consider their own ways of thinking. It affects their cognitive functions as well. In other words, students determine for themselves whether to use an algebraic or arithmetic approach. It affects their mental growth as well. In other words, it causes their memory and skills to grow. It makes the lesson easier to comprehend by using geometric models, illustrations, and diagrams. Nevostrueva and Ruzanova (2021) discussed the pedagogical and psychological benefits of visualization, which aid students in improving their memory, logical reasoning, and clarity of mind. While schematic representations set the stage for a shift to a symbolic image, students in grades 5 and 6 can observe a specific problem in a real-life scenario. Visualizing their word problems and visuals helps students enhance their logical thinking skills. In the meanwhile, using tables aids in improving students' focus and recall. Graphs and diagrams improve students' capacity to make autonomous judgments in novel circumstances, according to research. According to Kochurova (2018), one of the psychopedagogical components of visualization is mental representation. Put otherwise, a student has to be able to visualize a problem as soon as they come across it. They are able to differentiate between the problem's qualitative and quantitative aspects in this way. L.M. Friedman's claim that "every story problem is a verbal model of a certain real-life problem situation, and it is always possible to reconstruct this situation from the text of the problem (mentally or in the form of a dynamic mental model)" was cited by Kochurova (2018). Nevertheless, this is insufficient to address the narrative problem. The necessity for students to do semantic analysis was explained as follows: "It is also necessary to establish all the relationships specified in the

problem through semantic analysis of the problem, and after having identified them, find the solution to the problem". Additionally, students must be able to understand the purpose of a word problem by placing themselves in the shoes of a character.

2.3 Practice of using the visualization method in teaching word problem solving

Instead of only memorizing solution steps, visualization is an efficient way to solve word problems since it allows for better comprehension. According to Novikova et al. (2022), mental maps, radial diagrams, and tree-like information structures are examples of visualization strategies that successfully develop meta-subject abilities for dealing with information in mathematics education. The findings of their experiment demonstrated that students' capacity to organize word problems and convert it into symbolic representations was much enhanced by the methodical application of visualization methods. Andrianova (2024) outlined a number of steps involved in applying the visualization approach to solve an word problem. A teacher begins by outlining the theory underlying the problem. After that, the student has to sketch an image, table, or diagram and write a concise summary of the problem. The student must be able to create an equation in the second stage using their notes and drawings. The solution should be prepared by the third step. The most crucial stage is the fourth one. At this point, the student has to recognize their errors and try to fix them. To put it briefly, they review their own work. Additionally, three primary phases of mathematical modeling may be found in the problem-solving process, according to Kazakova's (2015) research that illustrates the various word problems modeling:

- first, the problem must be analyzed and translated into mathematical language to develop a mathematical model. The basic data and the requisite values must then be ascertained.
- second, the selected mathematical model must be followed when solving the problem. To get the value of the numerical expression, the equation or inequality must be solved.

- third, an explanation of the outcome is required. This entails converting the mathematical solution into a natural language.

Furthermore, Kazakova (2015) categorized the many forms of visualization, or modeling, in the "Mathematics" textbooks issued by the Russian Academy of Sciences' "Просвещение" publishing house (authors: G.V. Dorofeev, T.N. Mirakova, and T.B. Buka):

- a graphic model is a drawing.
- a graphic model is a sketch.
- graphical model: schematic illustration (diagram)
- diagram-based graphical model
- expression of the signal model
- signal model: a succinct explanation of the problem
- table of the signal model.

As students advance through the years, this type of modeling, visualization - becomes more complicated, but it is also advantageous, according to Kazakova (2015), who gave examples of such projects for grades 1-4. In other words, it teaches students how to analyze a word problem, gather the information they need, assess if a specific piece of writing poses a challenge, identify the key elements of this kind of assignment and its components, and make links between them and their importance. Additionally, modeling enables students to recognize qualities and connections that are not immediately obvious to them. Modeling aids students in understanding the reasoning behind problem-solving, especially those who are having difficulty. Instead of depending on pre-made templates, students may solve problems creatively by comprehending their significance via the use of visualization. Yumova and Garmayeva (2024) examined scholars' work on word problem solution in school mathematics across several nations. Specifically, based on the research of D. Costa, S. Chen, and M. Courtney, they found that visualization enhances students' performance in word problem solving. Additionally, Yumova and Garmayeva (2024) examined the textbooks by S. A. Zaytseva and I. I. Tselishchev and found that students find it simpler

to master problem-solving when they use drawings and diagrams to solve word problems. Additionally, these researchers discovered a model for word problems. Yumova and Garmayeva (2024) examined the work of other scholars, including A. Lewis, who employed diagramming methods for the college students. As practice shows, students' visualization skills improve once they learn to solve real-world problems. Yumova and Garmayeva (2024) split the word problem solving algorithm into three steps in the methodology section:

- introduction part
- main part
- conclusion part

The student must attentively study the problem statement and jot down the criteria and question during the introduction part. After that, they have to decide what is known and what is not. They must come up with a solution plan and mentally create the solution throughout the main part. The answer is polished and verified for accuracy during the last part. Additionally, Yumova and Garmayeva (2024) suggested employing gamification to solve word problems. The visualization itself incorporates gamification. For instance, entering information in a table (such as "weight of one box," "number of boxes," "total weight," or "speed," "time," and "distance") might assist solve problems with proportional quantities and motion and build a mathematical model. Yumova and Garmayeva (2024) explain how gamification can be used to solve word problems and explain how using digital technologies, like multimedia presentations, animation, visualizations, and illustrations, makes the problem-solving process more interesting and understandable. Özkan et al. (2018) employed visualization in math classes to enhance sixth-grade students' visual abilities. Fifty sixth-graders from Istanbul were chosen for the study using a convenience sample. Özkan et al. (2018) conducted the investigation using a quantitative method as well. Ten arithmetic problems were provided to the students in the research, and they were entirely permitted to utilize visualization to answer them. Regretfully, only one student out of fifty was able to correctly complete each of the ten problems. Furthermore, many

of students claimed that visualization has no foundation and that they don't employ it in their classes because it's simpler to compute utilizing a straightforward formula. The study's findings indicate that the majority of students demonstrated a reluctance to employ the visualization method going forward since they did not comprehend it. However, the researcher used visualization to assist individuals comprehend challenging ideas in the future. The lack of vision in the instruction was the cause of the problem at hand. Rif'at (2020) studied how to improve mathematical visual skills, particularly in geometry. Using an qualitative method, he experimented with applying visualization methods to geometry difficulties. Using Cramer's method, he assessed the experiment's results, yielding a value of 0.82. In the early tasks, students were shown to rely more on algebraic calculations, however in the latter tasks, visual calculations were more prevalent. According to the study's findings, many students still need to enhance their use of visual aids and learn how to put theories into practice. According to Şahinkaya et al. (2024), using visualization to solve word problems enhanced first-grade student academic performance. In particular, according to the methods section, 51 students were chosen using a homogeneous sample approach, however only 41 students took part in the experiment following pre-testing using a mixed method. They were gathered via interviews and provided straightforward computations devoid of images. The p-value, mean, and Wilcoxon signed-rank test between the pre-test and post-test were computed by the author using this data. Students' academic performance increased as a result of the visual method. In their study, Bakhmat et al. (2019) investigated how visualization affected themes related to probability and statistics. For their investigation, they employed the visual coding approach. The following categories of visual coding were suggested by them:

- tree of probabilities
- traditional table
- graphs

Development hypotheses will be included in every branch of the likelihood tree. This is comparable to Novikova and Vlasova's study's tree principle (2022). In other

words, more details permeate the branches from the core subject. Word problems are easier to categorize and more compact in the traditional table. Graphs can also be used to derive sophisticated algorithms. They stated that these approaches enhance students' comprehension of the problem. Students contended that by completing the table, they improve their cognitive activity and functional literacy. Yumova and Garmayeva (2024) employed gamification methods used their study, but they also employed tables and diagrams to address problems. They discovered that the problem was modeled through visualization. Students' use of tables therefore raised their level of cognitive effort. Research on elementary school students was carried out by Provalova (2022). She examined an experiment word problem by using visuals. A sheet of paper containing visual diagrams was handed to each student. Information was presented as an image or learning model using visualization. Students improved their preliminary diagrams and demonstrated the connections between the data with the teacher's assistance. Students were better able to comprehend and differentiate mathematical ideas because of visual modeling. Novikova and Vlasova (2022) investigated the aforementioned visualization types (radial, tree-like, tabular, and mind map) on students aged 13 to 14. Both experimental and control groups participated in the study. Initially, with a teacher's help children worked with pre-made radial or different types. Subsequently, the method of doing research got more intricate. In other words, they filled it out and put it together themselves. Students were then able to choose and recognize the various types on their own. The methodical application of methods of visualization has been shown to be a useful tool for producing meta-subject outcomes and has a good impact on the development of students' cognitive, regulatory, and communication abilities. Additionally, Ochkov et al. (2022) employed animation and visualization to address problems raised by high school students. Students used graphical ways to simplify the problem after first failing to solve it using symbolic methods. The study also made use of a variety of methods of visualization. They located the precise root using techniques like root portraiture. It has been demonstrated that students' critical thinking, quicker and more effective problem-solving,

motivation, and enthusiasm in learning all improved. An experiment on graphic modeling for students in elementary schools was carried out by Kokoreva and Vendina (2020). They used actual photos, sketches, schematic diagrams, and problem flowcharts to conduct their investigation. Throughout the experiment, the teacher used flowcharts to convey the material, asked questions to evaluate it, and chose mathematical operations. Students' visual representations made the problem's content comprehensible as a consequence of the study. Additionally, the procedures were chosen based on the problem's circumstances. By making flowcharts, they discovered how to address non-standard situations with ease. Teaching strategies in grades 5 and 6 were examined by Danilenko and Reshetnikova (2022). G.V. Dorofeev et al., S.M. Nikolsky et al., S.F. Gorbov et al. (the developmental teaching system—the Elkonin-Davydov method), and N.Y. Vilenkin et al. (the commonly used conventional technique). Lastly, G.V. Dorofeev observed that modeling is included in the textbook, albeit in tiny amounts, whereas Vilenkin and Nikolsky, who teach using conventional technique, discovered that modeling is frequently used just as an illustration with no explicit assignments for constructing models. It was discovered that the bulk of exercises (up to 60%) in Gorbov's developmental approach textbook are focused on working with models and their interpretation. A knowledge gap among teachers was also discovered throughout the investigation. In other words, teachers frequently create the model (diagram) themselves, which prevents the model from serving as a tool for students' autonomous thought. In her research, Safonova (2000) provided a table detailing the abilities that students in grades 1–8 need have to solve word problems. She turned at how geometric models are used. Actions including creating questions based on a diagram, filling up a table of functional connections between values, and choosing an equation or diagram that fits the problem's conditions were among the variables taken into consideration. It was discovered that middle school students do, in fact, struggle with word problem solving. Additionally, the use of geometric models into arithmetic and algebraic methods produced favorable outcomes. Students should be able to draw, add, subtract, and utilize line segments as quantities in calculations

when using one-dimensional diagrams (segments) as part of the geometric method. to use line segments to create equations. Students in grades 1–6 should possess abilities on above. The ability to depict the product of numbers (such as the product of speed and time-distance) as the area of a rectangle, which is regarded as a talent in grades 5–8, should be present in the form of two-dimensional diagrams. Additionally, the following graph-based abilities should be present: creating a coordinate system, producing graphs of linear functions, and locating their points of intersection, which is regarded as a talent in grades 6–8. Problems are provided orally in grades 5 and 6, along with a graphical representation. Additionally, they come with a thorough chart or diagram for examination (Golovanova, 2016, as cited in Nevostrueva & Ruzanova, 2021). Nevostrueva and Ruzanova (2021) contend that students should be proficient in both arithmetic and algebraic methods. Additionally, the research indicates that avoiding frequent errors made by students when calculating the "remainder" using diagrammatic drawings and offers examples of how to answer the same problem using two distinct approaches (arithmetic and algebraic). In this case, logical difficulties were solved using the method of removing impossible possibilities by appending "+" or "–" marks to a table. While recognizing the "mental-economic" benefit of the algebraic approach, Nevostrueva and Ruzanova (2021) emphasize the significance of the arithmetic approach in fostering students' linguistic abilities and methodical reasoning. Seven algorithms were put up by Kochurova (2018) to address word problems. Visualization methods were one of these. Below is a summary of the seven algorithms:

- step 1: Reading the content is the first step. Being able to comprehend the content efficiently is crucial at this point. The only way to see the following stages, she said, is to pay close attention to every word and phrase.
- step 2: Ascertain what is known. Understanding the relationship between the condition and the inquiry is crucial at this point.
- step 3: Visualizing the problem is the third step. In other words, the capacity to visualize the problem is crucial to comprehending it completely and

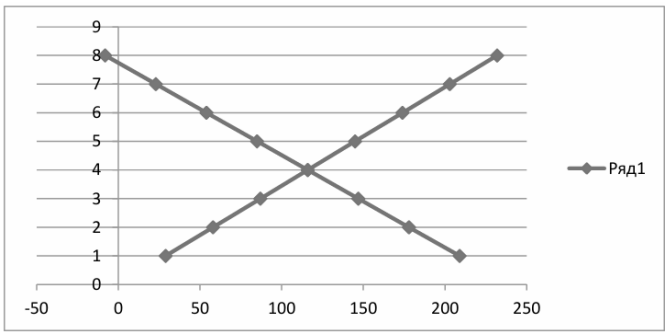
coming up with answers. Additionally, if students visualize themselves as a part of the problem, that is, within the problem, they will be more productive.

- step 4: Modeling is the fourth step. At this point, a diagram, sketch, table, or other succinct explanation of the problem must be created. According to Kochurova (2018), building a model in this manner is the first a bit analysis.
- step 5: Specifying the order of operations. At this point, reasoning occurs and a solution is found using the model built in the fourth phase.
- step 6: Explaining the answer. The solution is justified in this phase. Students' capacity to verify if something is correct is enhanced by this.
- step 7: Testing the answer is step seven. They just put the solution into practice and test it in this phase.

2.3.1 Examples of problems using visualization and analysis of their effectiveness

It is important to note that there are several approaches to solving word problems, including arithmetic, algebraic, graphical, and geometric (Zhuravleva and Smirnova, 2019). This is evident when examining certain research that used methods for visualization to answer word problems. Algebraic questions, or word problems requiring the construction of equations, are also common in sixth grade. Let's now examine the visualization of word problems by Zhuravleva and Smirnova (2019). The problem is as follows: At 25 km/h and 35 km/h, respectively, two visitors from two separate places set out to meet. The cities are 240 kilometers apart. How long will it take for the visitors to get together? Put another way, motion and speed determine the kind of problem. Arithmetic, algebraic, and graphic approaches are used to describe this topic. The graph is displayed here since the method of graphics is visible.

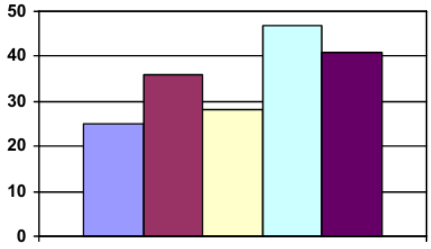
Figure 2.3.1.1. Distance-Time Graph of the Motion Problem



Note. The graph illustrates the intersection point representing the meeting time and distance of the two objects.

Travel time (in hours) is represented by the horizontal axis of a rectangular coordinate system, while distance (in kilometers) is represented by the vertical axis. Each visitor's movement is shown on the graph. The first visitor's mobility function is $y = 25 \cdot x$, whereas the second visitor's is $y = 240 - 35x$. The moment at which the hikers meet is shown by the graph's intersection, which is 4. For trainee teachers, Gusev and Bataeva (2020) provide examples of how to employ visual aids in a word problem for grades 5–6. The problem was as follows: Vitya weighed 47 kg, Nina weighed 41 kg, Tanya weighed 28 kg, Yura weighed 25 kg, and Sasha weighed 36 kg. This challenge was created specially to show how to utilize a diagram. Each student's weight was represented by a rectangle, and one kilogram was represented by a rectangle that was one millimeter long. Yura's weight was represented by a rectangle that measured 25 mm in height, Sasha's 36 mm, Tanya's 28 mm, Vitya's 47 mm, and Nina's 41 mm.

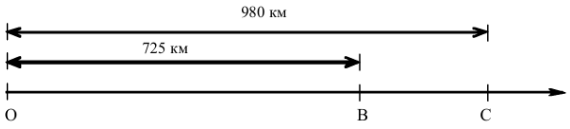
Figure 2.3.1.2. Bar Chart Representation of Students' Weights



Source: Adapted from Gusev and Bataeva (2020).

This diagram allows students to respond to a variety of queries, including who weighs the heaviest, according to the Gusev and Bataeva (2020). On the circumstances below, an automobile covered 980 kilometers in three days. It traveled 725 kilometers on Friday and Saturday. How many kilometers did the automobile go on each of these days if it went 123 kilometers more on Saturday than on Sunday? The coordinate method was employed by Gusev and Bataeva (2020) to solve the problem. In other words, they used point O, which represents the car's starting distance, as the ray's origin and the ray's direction as the car's moving direction. Following the drafting of line segment OC, which represents the car's three-day journey distance from point O, they drew line segment OB, which represents the car's Friday and Saturday travel distance of 725 kilometers. The distance traveled on Sunday, approximately 255 km, is represented by the resultant line segment BC. Upon closer inspection, it is evident that traveled 378 kilometers on Saturday and 347 kilometers on Friday.

Figure 2.3.1.3. Line Segment Diagram for Distance Calculation



Source: Adapted from Gusev and Bataeva (2020).

According to the Gusev and Bataeva (2020), the following actions should be considered while utilizing the coordinate method to solve word problems:

- students should be taught a variety of techniques for determining the coordinate axis's origin and direction;
- when using the coordinate axis, select a unit segment.

Rudenko (2020) offered many answer steps and demonstrated how to use an algebraic method in a table. The problem involved purchasing 30 kilograms of apples and pears for 5120 rubles. A kilogram of apples costs 180 rubles, whereas a kilogram of pears costs 160 rubles. What is the cost of one kilogram of apples and one kilogram

of pears? Firstly, to solve the problem, you had to comprehend the information provided and record it as follows:

- One kilogram of apples costs 180 roubles.
- 160 roubles for one kilogram of pears
- 30 kilos and 5,120 roubles in total.

Question: How many kg of apples and pears were purchased?

Secondly, finding the unknowns and the information provided. For instance, x kg represents the quantity of apples, whereas (30 – x) kg represents the quantity of pears. Thirdly, equating equal quantities. Fourthly, establishing an equation. Fifth, solving the equation. Sixth, verifying the equation's solution. Seventh, write down the response. It was also mentioned that a table might be used to display each of these steps.

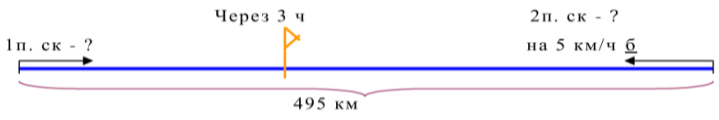
Figure 2.3.1.4. Tabular Representation for Constructing a Linear Equation

	Цена	Количество	Стоимость	Уравнение
Яблоки	180 руб	x кг	$180x$	$180x = 160(30 - x)$
Груши	160 руб	$(30 - x)$ кг	$160(30 - x)$	

Source: Adapted from Gusev and Bataeva (2020).

Although producing a representation might be thought of as an illustration of the problem, it is uncommon to not only write down the supplied content but also to illustrate it. As a result, a problem becomes more dynamic and visible (Nevostrueva & Ruzanova, 2021). This problem is demonstrated by Nevostrueva and Ruzanova, (2021): At the same time, two trains depart from two cities that are 495 kilometers apart and travel in the same direction. Three hours later, they get together. What is each train's speed if one is 5 km/h faster than the other?

Figure 2.3.1.5. Schematic Diagram of Trains Moving in the Same Direction



Source: Adapted from Gusev and Bataeva (2020).

Drawing the problem statement in this format, i.e., drawing an illustration, allows you to clearly understand the relationship between the data and the desired solution and aids in identifying one of the problem's critical conditions that caused the most difficulty in solving it (Nevostrueva & Ruzanova, 2021). Furthermore, the table aids in the solution of non-standard or logical problems, as demonstrated by the following problem explanation: "Korneev, Dokshin, Mareev, and Skobelev are citizens of our city; their vocations are baker, doctor, engineer, and policeman. The policeman is older than the doctor and the engineer, and the baker always walks to work. Dokshin and Korneev are neighbors, and Korneev always beats Skobelev at table tennis. Determine who does what".

Figure 2.3.1.6. Logic Grid for Solving a Profession Deduction Problem

	Пекарь	Врач	Инженер	Милиционер
Корнеев	–	+	–	–
Докшин	–	–	+	–
Мареев	+	–	–	–
Скобелев	–	–	–	+

Source: Adapted from A. A. Nevostrieva and B. B. Ruzanova (2021).

According to Nevostrueva and Ruzanova, (2021), putting minus marks on pairings of impossible items in the table indicates that can solve the problem.

3. METHODOLOGY

3.1 Research design

The use of visualization to make a support tool sixth-grade students solve word problems was the focus of a research. This study used the use of quantitative method. The quantitative method was also used in the investigations of Özkan (2018) and Abate et al. (2022). Using this quantitative method, Kusuma and Retnawati (2019) also studied sixth-grade students. The quantitative method was selected based on these researchers' findings. One-group pretest-posttest design is the method used in this research. Instead of grouping students, Ayten Özkan (2018) used a broad exam to gauge their proficiency. As a result, there are no experimental or control groups in this research, and every group underwent the same experiment using the same methodology.

Participants. Students in the sixth grade at secondary school No. 148 in the village of Bidaikol, Shieli district, Kyzylorda region, participated in the educational experiment for this research. Thirty students took part in the experiment. These students were chosen by convenient sampling and were selected from two sixth-grade classes. Although convenience sampling seeks to reduce time, money, and manpower losses, it was selected (Büyüköztürk, 2009, as cited in Özkan, 2018). Students between the ages of 11 and 12 who were learning linear equations and inequalities took part in the experiment. Students in this grade should employ visualization to solve word problems, according to Gusev and Bataeva (2020).

3.2 Informed consent and confidentiality

In order to adhere to the research's ethical guidelines, each participant's parents were informed about the research and had their signatures taken. Additionally, the school's administrative stamp was attached and approval was granted by the administration. This was done in order to protect the student's and parents' privacy. This is compliant with international research ethics' "Anonymous consent" standard. Throughout the study, no student's identity will be disclosed; instead, they will be

categorized as student 1. The goal of this study was to see how much the visualization approach improved the students' ability to solve word problems rather than to assess the students' uniqueness.

3.3 Data collection instruments

Students were required to provide honest answers to ten pretest and posttest questions in this research. In his research, Özkan (2018) also created a pretest with ten questions. Thirty students took the pretest throughout the first week, from February 9 to February 13, 2026. Beginning the next week, from 16.02.2026 to 16.03.2026, students were given practical applications and demonstrated how to use visualization to address word problems related to each topic. All 30 students who took part in the experiment responded to posttest questions on March 18, 2026. During the students' ten-day break, all pretest and posttest results were reviewed. To verify it, a unique descriptor was developed. The following four-level descriptors have been created in order to statistically examine and objectively evaluate student performance in Appendix D

3.4 Validity and Reliability

Validity of pretest and posttest questions. In terms of content validity, pretest questions were developed based on chapters 4-5 of the 6th grade textbook written by T. A. Aldamuratova, K. S. Baisholanova, and E. S. Baisholanov. Furthermore, the school practice and dissertation supervisors accepted all these queries. Every pretest question was chosen from sixth-grade problems found in academic journals that were included in the literature review. Problems 1, 3, 5, and 8 are derived from the Sarkulova and Omarbaeva (2025) article. The purpose of their piece, which was published in Kazakh, was to assess the word problem-solving skills of aspiring educators. The article by Nevostrueva and Ruzanova (2021) is the source of problems 2 and 4. They included word problems for grades 5-6 in the article. Task 6 is derived from Chilingaryan's (2017) article. Additionally, he supplied linear equation construction questions for these sixth-grade students in his article. The Batyrbayeva et al. (2024)

article is the source of tasks 7, 9, and 10. The posttest questions are a modified version of the pretest, which means that they are measured using comparable metrics while maintaining the pretest's content. The fact that the pretest and posttest questions were created with Bloom's taxonomy levels in mind further supports their validity. In other words, two primary criteria were used to choose the problems from the articles. In other words, the first is connected to the subject matter that sixth graders are studying, as was already indicated, and the second is that they match the levels of Bloom's taxonomy. Bloom's taxonomy's remembering level is the first problem. Since pretest and posttest questions are included in Appendices A and B. The first question there required the student to recollect formulas and facts that they previously knew. In other words, fundamental operations like converting kilograms to grams are carried out. Additionally, the second problem involves understanding Bloom's taxonomy at the second level. Because the student must comprehend the problem and identify a mathematical model to solve this problem. In other words, the student is aware that he must deduct the specified weight from the overall weight. The Applying level includes the third and fourth problems. Due to the student applies the method they have learned - calculating utilizing inverse operations or formulating an equation - in a particular problem. The analyzing level includes the fifth and sixth problems. This is since creating a single basic equation is insufficient for solving the problems. A complicated structure must be divided into its component elements by the student. In the butter problem, the student constructs and evaluates a logical link between weight, price, and cost; in the train problem, the relationship between speed, time, and total distance is examined. The evaluating level includes the seventh and eighth problems. This is why the student develops a novel solution by combining many rules. For instance, the student integrates number theory (the idea of an "even number") with the perimeter calculation for one aim in problem 7. In problem 8, the student builds a sophisticated motion model by combining the circumstances for late departure and overtaking. Additionally, the ninth and tenth problems are at the creating level. After all, the student does more than simply compute, find the solution, synthesize, and do

research. In other words, problem 9 determines which car can pass by comparing millimeters to meters. They optimize, select the most effective (least traveled) alternative, and provide a rationale for their decision in problem 10. Pretest questions are more than just computation-focused puzzles. Rudenko (2020) states that students start using the algebraic method to solve problems in grades 5-6. As a result, the problems were made up that need to be calculated visually and may be answered by developing equations.

Reliability of pretest and posttest questions. To evaluate the pretest's reliability, a pilot study was carried out. Ten sixth-graders who did not take part in the experiment were chosen for this investigation from a parallel class. The practice supervisor chose these students based on their quarterly grades. This indicates that the heterogeneous sampling approach was used in the selection process. In other words, students in the experiment's participating classes were leveled. These ten students were divided into three categories: low achievers (those who studied with three), ordinary achievers (those who studied with four), and high performers (those who studied with five). This was done to confirm that the examination is appropriate for students of all skill levels and to see whether it can be accessed by students of all skill levels. The visualization method may be utilized to help low achievers comprehend the problem's content and assess how effectively high achievers can come up with a strategic solution, according to the hypothesis of Hadi and Csíkos (2025). Students who took part in the pilot test had their findings assessed using 0 and 1 points. In other words, one point is awarded for a right response and zero points for a wrong one. Özkan et al. (2018) employed IBM SPSS 23 software as part of their evaluation procedure. The students' names were coded in the table below to assess the outcomes. This systematic coding process was essential to ensure the complete anonymity of the participants, thereby maintaining the ethical integrity of the research and preventing any potential bias during data analysis.

Table 3.4.1. Coded Student Responses and Term Grades for Evaluation

Student №	Term grades	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
1	5	1	1	1	1	1	1	1	0	1	1
2	5	1	1	1	1	0	1	1	1	0	1
3	5	1	1	1	0	1	1	0	1	1	1
4	4	1	1	0	1	1	0	1	1	0	1
5	4	1	0	1	1	0	1	0	1	1	0
6	4	1	1	0	0	1	1	0	1	0	0
7	4	0	1	0	0	1	0	1	0	1	0
8	3	0	0	1	0	0	1	0	0	1	0
9	3	0	0	0	1	0	0	1	0	0	0
10	3	0	0	0	0	1	0	0	0	0	0

Students 1, 2, and 3, who performed well, scored 8–9 points. Additionally, students who performed on average № 4,5,6,7 scored between 4 and 7 points. This indicates that there is a large variance in the students outcomes. This demonstrates that average-performing students are capable of doing well on this pretest. The remaining underachievers received scores ranging from one to three points. These low-performing students can respond to basic pretest questions based on their responses, or the outcomes. Cronbach's alpha was computed from these test data using Python code. The resulting Cronbach's alpha coefficient is 0.711, which exceeds the threshold of 0.70. This indicates that the test is reliable and suitable for measuring the mathematical problem-solving skills of 6th-grade students.

3.5 Lesson structure

Students in the sixth grade participated in a one month study on the application of methods of visualization to word problem solving. The novelty of this method is that, according to Kaskatayeva and Khasanova (2024), students' primary concern is being able to comprehend word problems with ease and use it as a bridge to the algebraic method. In addition to using the visualization method to display visuals, students were also able to envision and readily construct algebraic equations in their heads via the problems that were presented in the form of tables, graphs, diagrams, scales, etc. The algorithm for solving word problems was based on the findings of

Andriyanova (2024) and Kochurova (2018), and it included the following four basic stages:

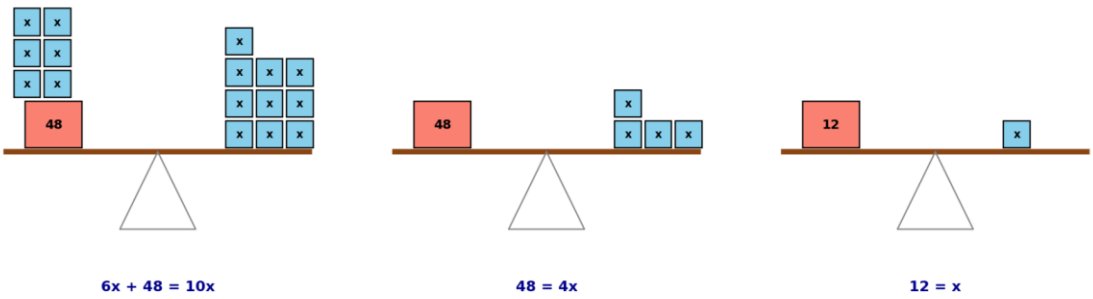
- first, read the supplied problem and identify it. That is, you must determine what is and is not offered. As Asanbayeva and Tagybergen (2025) demonstrate, the student must answer the following questions: What is the essence of the problem? What are we aware of? What information is needed? What information is unknown? The unique aspect of this step is that no numerical computations are performed; all that is required is an understanding of what is presented.
- in the second stage, the student applies the visualization method. At this point, students turn the problem into a structural similarity visualization. Depending on the sort of difficulty, students should visualize cuts, illustrations, schemes, diagrams, and so on in their notebooks with the teacher's supervision.
- in the third stage, these visual models should be mathematically represented. That is, students should apply the algebraic method to create an equation.
- finally, they assess their outcomes.

They compare the outcomes to the problem's conditions and determine their correctness. Students pose questions like, "How did our drawing (scheme) help them solve the problem?" and share their responses. This will help them improve their metacognitive skills and comprehend the value of visualizing. Thanks to the visualization method, students can overcome the problems of memorizing the solution to the problem and achieve the level of being able to imagine it in their thoughts. They will comprehend the significance of each mathematical operation they complete. The Appendix contains the short-term plan for the sixth-grade courses, which was developed by incorporating the visualization method into the word problems, written on above these four stages were implemented during the experiment. First, now examine a sample word problem from that short-term plan.

Condition of the problem: A rectangle's side AB is 8 cm longer than its side BC. The lengths of the two sides will be equal if side AB is extended six times and side BC

is extended ten times. Determine the side AB's length. Let's use the four steps mentioned above to address this problem: First, identify the unknown quantities. Since the long side AB is 8 cm longer than the short side BC, it is represented as $x + 8$ (cm). Next, we must develop a visual model. We shall now illustrate this problem using a scale. We should attempt to balance the two sides of the scale using the equality criterion. In other words, side AB is positioned on the left end of the scale after being extended six times. Student's picture this as six cubes with a total weight of forty-eight $(6 \cdot 8)$. The scale's right end represents a 10-fold extension of BC, or $10x$ cubes. The mathematical formula provided by the visual model is $6x + 48 = 10x$. In order to maintain this equilibrium, students visually remove six cubs. To right side of the scale in the third level. At that point, $4x$ is still on the right side of the scale while 48 is still on the left. Students now determine how many x substitutions will add up to 48. In other words, they comprehend the equation's primary goal through visualization. Fourth, students review the final response. They also talk about the benefits of mutual vision. The image below illustrates how the visualization method was used to measure this problem using a scale.

Figure 3.5.1. Visualization of the Equation Using a Balance Scale



Source: Compiled by the author.

Problem 840: A car departs town A, travels at 50 km/h, and arrives at town B on time. The car will arrive in town B one hour ahead of schedule if it accelerates by 10 km/h. What is the distance in kilometers between towns A and B? We first identify the unknowns and the knowns in order to tackle this challenge. In other words, we use t to

represent the car's time. Additionally, we use $S = 50t$ to represent the separation between A and B. In the second stage, we use visualization to build the equation. The visualized Python animation below is used to create the equation.

Figure 3.5.2. Python Animation Illustrating the Catching Motion Problem

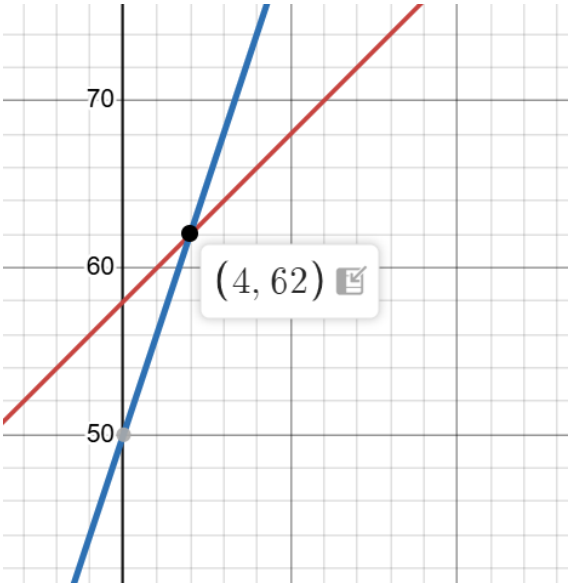


Source: Developed by the author using Python.

Students will develop an equation pertaining to the catching motion after watching this animation. The equation $50t = 60(t - 1)$ will be solved by them.

Problem 845: The grandfather is 58 years old; his son is 32, and his grandchildren are 7 and 11 years old. How many years will it take for the grandfather's age to match the total of his son's and grandchildren's ages? We shall employ the tabular-graphical model from Safonova's (2000) and Subbotkina's (2020) research to address this problem. The dynamics of the pupils' yearly age growth must be explained to them before this problem is reduced to an equation. In other words, his children gain three years if the grandfather gains just one. We illustrate this dynamic using the Desmos program. The graphical approach enables students to understand the dynamic relationship between distance, speed, and time by converting abstract numerical data into a visual representation.

Figure 3.5.3. Graphical Solution of the Age Problem Using Desmos

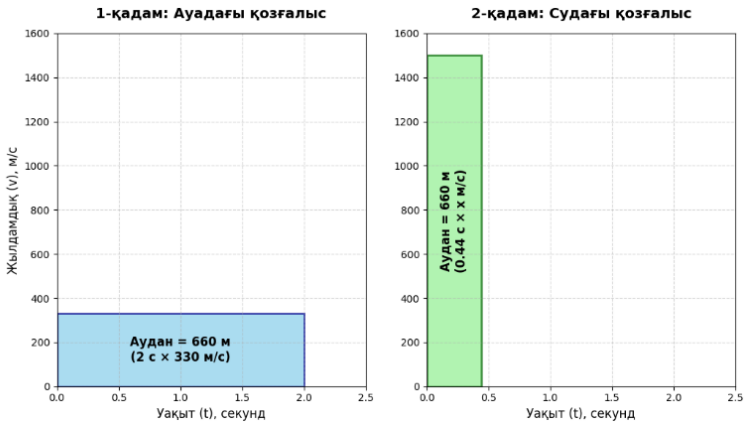


Source: Developed by the author using Desmos.

Students will discover that the two lines are at the position x=4 when examining the graph.

Problem 863: Sound travels at a speed of about 330 meters per second in air. In water, sound travels the same distance in 0.44 seconds as it does in 2 seconds in air. Determine the sound speed in water and contrast it with the sound speed in air. According to Safonova's (2000) two-dimensional diagram method to tackle this problem. In other words, students draw a rectangle whose width equals velocity ($v = 330$m/s) and whose length equals time ($t = 2$ s) to depict the first movement in the air. The distance traveled is represented by the area of the rectangle ($s = 330 \cdot 2$). They then draw a second rectangle with a shorter length ($t = 0{,}44$ s) and an unknown width (x m/s) to depict the movement in water. $s = 0{,}44x$ is used as its area. Students will equate $330 \cdot 2 = 0{,}44x$ as follows, knowing from the Python graphic below that the two rectangles are congruent.

Figure 3.5.4. Visual Comparison of Congruent Rectangles Representing Movement



Source: Developed by the author using Python.

Problem 879: On the first day, a turner produced x parts, and on the second day, y parts. Determine how many more pieces the turner produced on the first day compared to the second if $x > 120$ and $y < 100$. To fix this problem, a table will be utilized. The Figure below shows that the difference between x and y is always more than 20.

Figure 3.5.5. Tabular Representation of the Algebraic Model for Pattern Recognition

1-ші күн (x мәні) Шарт: x > 120	2-ші күн (y мәні) Шарт: y < 100	Айырма (x - y) Бағалау
121	99	**22**
130	90	**40**
150	50	**100**
200	0	**200**
350	-50	**400**

Заңдылық: Кестеден байқағанымыздай, айырма (x - y) әрқашан 20-дан үлкен.
Қорытынды алгебралық модель: x - y > 20

Source: Compiled by the author.

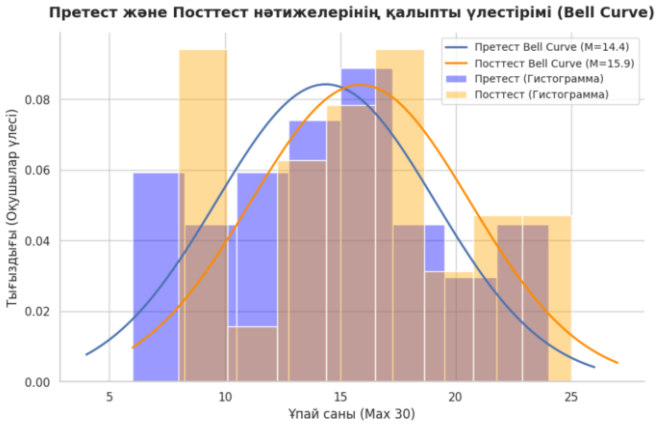
Every course employed this kind of presentation and visualization of different word problems. Students' knowledge was broadened and their ability to think independently may be enhanced by presenting them using various visualization techniques. Most significantly, visualization in the lesson plan was utilized directly as a cognitive tool for creating algebraic equations and demonstrating mathematical laws

rather than just serving as a visual aid. A final posttest was administered to all participants at the conclusion of the course cycle in order to objectively assess the degree to which students had mastered such visual-mathematical modeling skills and the overall efficacy of the suggested methodology. The results of the pretest and posttest, along with their conclusions, are analyzed mathematically and statistically in the next section.

4. ANALYSIS AND RESULTS

Throughout the experiment, the scientific hypothesis was tested and the quantitative data (pretest and posttest scores) was processed using statistical and mathematical analysis methods. The unique Microsoft Excel product "Data Analysis ToolPak" and the Python programming language (Matplotlib, Seaborn libraries) were used for data processing and computation. The whole data was processed using descriptive statistics, inferential statistics, and normality testing. The findings of the pretest and posttest results were compared to determine how the visualization method affected the performance of 30 students who took part in the study. The students' paired t-test was employed for statistical analysis of the findings. The pretest and posttest were compared with the constant sample since the sample remained unchanged. Additionally, the bell-shaped graph below demonstrates that the distribution is normal. The distribution regularity of the gathered data was examined to choose the appropriate statistical criterion. The data was found to follow a normal distribution after a histogram was made and the normal distribution curve (Bell curve) was visually evaluated. The Central Limit Theorem indicated that the sample size ($N \geq 30$) was enough, hence parametric statistical techniques were used for analysis. Additionally, the Python library was used to plot the data below.

Figure 4.1. Normal Distribution and Histogram of Pretest and Posttest Results



Source: Developed by the author using Python.

The MS Excel program's "Data Analysis" feature was used to carry out the computations. The mean (μ) and variance (s^2) were computed to evaluate the dynamics of changes in the overall educational level of research participants (N=30). The Table below shows them.

Table 4.1. Comparison of Mean Scores and Variances Between Pretest and Posttest

Type	Number of students	Mean	Variance
Pretest	30	14.37	23.21
Posttest	30	15.87	23.29

The table demonstrates that students' pretest average score was μ = 14.37, while their posttest average score improved to μ = 15.87 when the suggested visualization method was incorporated into the learning process. The proximity of variance markers 23.21 and 23.29 suggests that students' understanding in the groups follows a normal distribution. These findings are now shown separately in the Appendix. To compare the variation in students' mean scores between the pretest and posttest periods, a paired-samples t-test was selected. When comparing the outcomes of the same group at two distinct times (before and after experiment), this criterion is the best approach. The error threshold for educational research, α = 0.05 (i.e., 95% confidence interval), was used in all statistical computations. To ascertain the degree of correlation between the two measurements, Pearson's correlation coefficient (r) was also computed.

Table 4.2. Paired-Samples T-Test Results for Pretest and Posttest Averages

Variable	M	SD	t	df	p
Variable 1	15.87	4.83	5.74	29	<0.001
Variable 2	14.37	4.82			

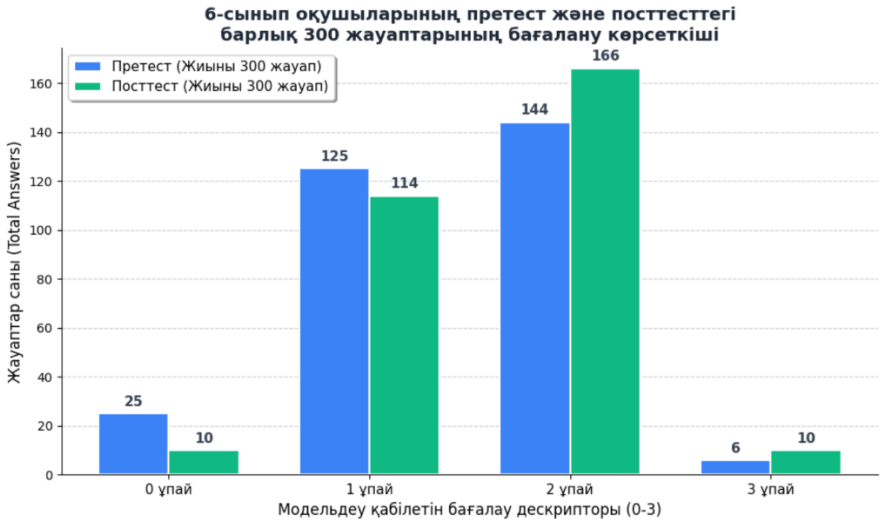
Note. M = Mean, SD = Standard Deviation. The t-statistic evaluates the difference between Variable 1 and Variable 2. p <0.001.

A paired-samples t-test was conducted to evaluate the effectiveness of the applied teaching methodology on students' problem-solving competencies. The analysis revealed a statistically significant increase in students' scores from the pretest (μ = 14.37, s^2 = 23.21) to the posttest (μ = 15.87, s^2 = 23.29), t (29) = 5.74, $p < 0.001$. Additionally, a strong positive correlation between the pretest and posttest scores (r = 0.96) indicates a highly consistent pattern of growth among the participants. Findings robustly confirm that pedagogical intervention had a significant and positive impact on improving students' solving the word problems using visualization methods.

To further evaluate the practical significance of the visualization method, the effect size was calculated using Cohen's d. The computation was performed using the Python programming language. Based on the descriptive statistics-specifically the pretest variance ($s^2 = 23.21$) and posttest variance ($s^2 = 23.29$) - the pooled standard deviation ($SDpooled$) was calculated to be 4.822. Subsequently, Cohen's d was determined by comparing the pretest and posttest means ($M_{pre} = 14.37, M_{post} = 15.87$). The resulting Cohen's d value is 0.311. According to standard interpretation guidelines, this value indicates a small-to-medium effect size. This confirms that the short-term pedagogical intervention utilizing visualization tools had a positive, observable practical impact on improving the 6th-grade students' word problem-solving skills, complementing the statistically significant results obtained from the paired-samples t-test.

The study's second question was to demonstrate that students' modeling abilities have improved. To answer this question, students' pretest and posttest 10 problems were evaluated using the assessment descriptor for each question. It was determined that if 30 students answered 10 problems, there would be 300 answers, so the following chart was generated using Python code in Google Colab.

Figure 4.2. Distribution of Assessment Scores for Pretest and Posttest Responses



Source: Developed by the author using Python in Google Colab.

The depicted Figure 4.2. shows that many students had low pretest results. That is, 0 points for 25 answers and 1 point for 125 answers were assigned. This suggests that prior to the experiment, students struggled to transform word problems into a structural representation. However, after including visualization methods into the learning framework, the posttest findings revealed a considerable and genuine increase in students' structural modeling abilities. The 2-point category showed the most positive dynamics, which means that the problem's model diagram is totally right but there are minor arithmetic problems. That instance, the indication for this category increased from 144 to 166 responses on the pretest. At the same time, the number of answers worth three points rose from six to ten. And the number of replies with no answer or all mistakes fell by 2.5 times, from 25 to 10. The dynamics of the mass transition from low 0 and 1 points to average 2 points clearly demonstrate quantitatively that the visualization method, i.e., tables, diagrams, the scale method, and visual interpretations, is a very effective pedagogical tool for correctly and competently teaching the mathematical model of word problems to 6th grade students. That is, these students make fewer arithmetic errors while improving their ability to analyze the problem circumstances and create a model.

5. DISCUSSION AND CONCLUSION

The primary goal of the study was to objectively analyze the influence of visualization tools on sixth-grade students' ability to answer mathematical word problems. The statistical analysis revealed a substantial rise ($p<0.001$) in the pretest ($M=14.37$) and posttest ($M=15.87$) indices. This positive dynamics demonstrates the direct usefulness of the visual modeling tools utilized in the experiment. Furthermore, the practical magnitude of the intervention is explicitly demonstrated by the calculated Cohen's d value of 0.311. In the context of educational experiments, this small-to-medium effect size provides critical depth to the discussion. It indicates that while the pedagogical intervention lasted for only one month, it generated a meaningful and observable cognitive shift in how sixth-grade students structurally represent mathematical relationships. This moderate effect size perfectly reflects the objective educational reality highlighted by Kusuma et al. (2019), which suggests that while visualization tools successfully mitigate text comprehension and conceptual modeling barriers, they cannot immediately remediate deep-seated mechanical or arithmetic calculation errors. Therefore, Cohen’s d result robustly confirms that visualization serves as an effective cognitive bridge for translating word problems into equations, even though baseline computational fluencies require more prolonged developmental time.

First, students demonstrated a reduction in cognitive burden on abstract information processing, which is a significant barrier to addressing word problems. This discovery is directly congruent with the findings of Novikova and Vlasova (2022). That example, their research showed that after applying the visualization approach, students dramatically increased their capacity to arrange word problems and convert them into symbolic expressions. Instead of being terrified of large texts, students learnt to break down the problem and convey it as a graphic or table.

The second study question glanced at how much students improved their structural modeling skills (on a scale of 0 to 3). After reviewing all 300 replies during the trial, it was discovered that many students improved from 0-1 to 2 points (creating

a valid structural model). This phenomenon is addressed by Hadi and Csikos (2025): they found that just sketching an illustrative image does not disclose the meaning of the problem, however developing a good "schematic visualization" immediately aids in the construction of a mathematical equation. The "scale model" and 2D graphics utilized in our experiment helped students understand the connections between variables.

According to Kaskatayeva and Khasanova (2024), the most challenging transition for students in grades 5–6 is from simple arithmetic to complicated algebraic procedures. Visualization served as an effective "bridge" in this transformation. Students were able to effectively utilize Andrianova's (2024) four-step problem-solving process by developing drawings and tables (understanding the theory, drawing a picture/diagram, creating an equation, checking the result). Gusev and Bataeva (2020) shown that geometric drawings and diagrams make it much simpler to comprehend the substance of a problem than dry logic.

However, the lack of a significant rise in the proportion of students who obtained all three points on the posttest reflects the age features of sixth graders. As Kusuma et al. (2019) demonstrated, even when students fully comprehend the problem's condition and structure, mechanical mistakes occur while working with components or owing to inattention. This establishes the objectivity of our investigation. That is, while visualization removes all barriers to understanding and modeling problems it cannot immediately replace basic arithmetic calculation abilities.

In conclusion, the hypothesis proposed at the start of the study was fully confirmed: the use of visualization tools successfully increases the correctness of word problem solving and modeling abilities in sixth grade students. Visualization techniques turned students from "passive listeners" who memorized ready-made formulae to "active researchers" who autonomously built mathematical models. This pedagogical technique is completely congruent with the national orientation of Kazakhstan's President, K. Tokayev, to eradicate dry memorization from the educational system and increase students' logical and functional literacy levels.

Based on the findings of this study, the following suggestions are offered to math teachers:

- Begin tackling word problems using schematic and structural visualization (drawings, sections, diagrams) rather of writing algebraic equations right away.
- Students must build a realistic mathematical model that demonstrates the link between quantities, not merely an illustrative visual.
- Prior to remembering mechanical rules, use a scale model and tabular approaches to expose the physical and geometric meaning of equations.

Building upon the findings and addressing the limitations of this study, several directions for future research are recommended:

- Experimental design with a control group: Future studies should employ a quasi-experimental or true experimental design that includes both control and experimental groups. This would allow researchers to isolate the effects of visualization tools and directly compare them with traditional arithmetic or algebraic methods.
- Larger and diverse sample sizes: Since this study was conducted with a convenience sample from a single secondary school in Bidaikol village, future research should include a larger, randomized sample encompassing multiple schools. Comparing urban and rural educational settings would provide a broader understanding of the method's generalizability.
- Longitudinal studies: The pedagogical intervention in this research lasted for one month. Conducting longitudinal studies over a full semester or academic year would help determine whether students retain structural modeling skills and if their cognitive reliance on visualization
- Expanding the mathematical scope: This study focused on 6th-grade students learning linear equations and inequalities. Future researchers are encouraged to explore the efficacy of visualization strategies across other grade levels

and more complex mathematical domains, such as probability, statistics, and advanced geometry.

- Integration of digital visualization tools: Given the successful demonstration of tools like Python animations and Desmos in explaining complex mathematical models, further research should specifically investigate the cognitive differences between using dynamic digital visual aids versus traditional paper-and-pencil schematic diagrams.

In conclusion, we can certainly state that the visual modeling approach is the most successful pedagogical tool for maximizing complicated concepts in current mathematics teaching while also raising student interest in the subject.

6. LIMITATIONS

While this study provides valuable insights into the effectiveness of visualization tools for solving mathematical word problems, several limitations must be acknowledged.

- First, the study utilized a relatively small sample size consisting of 30 sixth-grade students. Furthermore, the participants were selected using a convenience sampling method from a single educational institution, Secondary school No. 148 in the village of Bidaikol. This restricts the generalizability of the findings, as the results may not fully represent the broader, more diverse student population across different urban, regional, or cultural contexts.
- Second, the research employed a one-group pretest-posttest quantitative design, meaning there was no control group included in the experiment. Although the paired-samples t-test demonstrated a statistically significant improvement in students' posttest scores, the absence of a control group makes it challenging to definitively isolate the visualization intervention as the sole variable responsible for the improvement. External educational factors or regular maturation over the study period could have partially influenced the outcomes.
- Third, the pedagogical intervention was conducted over a relatively short duration of one month. While short-term gains in structural modeling skills were clearly demonstrated, longitudinal study would be required to determine whether students retain these visualization strategies over an extended period.

Finally, the scope of the mathematical content was primarily limited to word problems involving linear equations and inequalities tailored for sixth graders. The applicability and effectiveness of these specific visualization techniques in other abstract mathematical domains require further investigation.

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LIST OF APPENDICES

Appendix A – a pretest of the level tests

Претест (Бастапқы тест)

Мақсаты: Оқушылардың мәтіндік есептерді шығарудағы базалық деңгейін анықтау.

1) Білу: Орманда екі қыз таңқурай жинады. Бірінші қыз 1 кг 250 г таңқурай жинады, ал екіншісі одан 300 г артық жинады. Екі қыз бірігіп қанша грамм таңқурай жинағанын табыңыз.

(Дереккөз: Саркулова А.Қ., Омарбаева Б.Қ., 2025)

2) Түсіну: Қабырғаларды бояу үшін 4 бірдей құты ақ бояу және 3 кг жасыл бояу жұмсалды. Барлығы 19 кг бояу жұмсалған болса, әр құтыда неше килограмм ақ бояу болған?

(Дереккөз: Невоструева И.Л., Рузанова Е.А.)

3) Қолдану: Мен бір сан ойладым. Егер оны 4-ке бөліп, шыққан бөліндіден 2-ні азайтса, 7 шығады. Мен қандай сан ойладым?

(Дереккөз: Саркулова А.Қ., Омарбаева Б.Қ., 2025)

4) Қолдану: Ұзындығы 256 м арқан екі бөлікке бөлінді. Бір бөлігі екіншісінен 7 есе ұзын. Арқанның әр бөлігінің ұзындығын табыңыз.

(Дереккөз: Невоструева И.Л., Рузанова Е.А.)

5) Талдау: Екі қаладан бір уақытта бір-біріне қарама-қарсы екі пойыз шықты, олардың арақашықтығы 495 км. Олар 3 сағаттан кейін кездесті. Бір пойыздың жылдамдығы екіншісінен 5 км/сағ артық болса, әр пойыздың жылдамдығы қандай?

(Дереккөз: Саркулова А.Қ., Омарбаева Б.Қ., 2025)

6) Талдау: Сарымай мен ірімшік үшін 2,28 рубль төленді. Бағасы 3,6 рубльден 0,4 кг сарымай алынды. 1кг ірімшік 1кг сарымайдан 0,8рубльге арзан. Қанша килограмм ірімшік сатып алынды?

(Дереккөз: Чилингарян А.В.)

7) Жинақтау: Мектептегі жылыжайдың пішіні тіктөртбұрыш тәрізді, оның периметрі 12 метр, ал ұзындығы жұп санға тең болса, жылыжайдың ұзындығы мен енін табыңыздар.

(Дереккөз: Батырбаева Г.А., 2024)

8) Жинақтау: Аэродромнан жылдамдығы 210 км/сағ тікұшақ ұшып шықты. Арада 2 сағат өткен соң дәл сол аэродромнан тікұшақтың соңынан ұшақ ұшып шығып, 3 сағаттан кейін тікұшақты 840 км-ге басып озды. Ұшақтың жылдамдығын табыңыз.

(Дереккөз: Саркулова А.Қ., Омарбаева Б.Қ., 2025)

9) Бағалау: Көпір алдында биіктік шегі 3,5 м деген жол белгісі тұр. Биіктігі 3770 мм, 3400 мм, 2900 мм және 3350 мм болатын көліктердің қайсысына бұл белгі арқылы өтуге тыйым салынады?

(Дереккөз: Батырбаева Г.А., 2024)

10) Бағалау: Ақжар ауылында 50 оқушы, Қосбұлақ ауылында 100 оқушы тұрады. Елді мекендер арасындағы қашықтық 5 км құрайды. Оқушылар мектепке баратын жалпы қашықтық мейілінше аз болу үшін мектепті қай жерге салу керек?

(Дереккөз: Батырбаева Г.А., 2024)

Appendix B – a posttest of the level test

Посттест (Қорытынды тест)

Мақсаты: Визуализация әдісін қолданғаннан кейінгі оқушылардың білім деңгейін бағалау.

1) Білу: Орманда екі дос саңырауқұлақ жинады. Біріншісі 2 кг 450 г жинаса, екіншісі одан 850 г артық жинады. Екеуі барлығы қанша грамм саңырауқұлақ жинаған?

(Модификацияланған, негізі: Саркулова А.Қ.)

2) Түсіну: Мектепке 6 бірдей орындық және 15 000 теңге тұратын бір үстел сатып алынды. Барлық сатып алуға 51 000 теңге жұмсалса, бір орындықтың бағасы қанша?

(Модификацияланған, негізі: Невоструева И.Л.)

3) Қолдану: Ойлаған саннан 15-ті азайтып, нәтижені 3-ке бөлгенде 12 шықты. Ойлаған санды табыңыз.

(Модификацияланған, негізі: Саркулова А.Қ.)

4) Қолдану: Ұзындығы 360 м сым екі бөлікке кесілді. Бірінші бөлігі екіншісінен 8 есе ұзын. Әр бөліктің ұзындығын табыңыз.

(Модификацияланған, негізі: Невоструева И.Л.)

5) Талдау: Арақашықтығы 640 км екі қаладан бір-біріне қарама-қарсы екі жеңіл көлік шығып, 4 сағаттан соң кездесті. Бірінің жылдамдығы екіншісінен 10 км/сағ артық. Әр көліктің жылдамдығын анықтаңыз.

(Модификацияланған, негізі: Саркулова А.Қ.)

6) Талдау: Дүкеннен 4,5 кг алма мен 2 кг алмұрт сатып алынды. Алманың 1 кг бағасы 450 теңге. Барлығына 3225 теңге төленсе, 1 кг алмұрттың бағасы алмадан қанша теңгеге қымбат?

(Модификацияланған, негізі: Чилингарян А.В.)

7) Жинақтау: Тіктөртбұрышты бақшаның периметрі 28 метр. Егер оның ұзындығы енінен 2 метрге артық болса, бақшаның ауданын есептеңіз.

(Модификацияланған, негізі: Батырбаева Г.А.)

8) Жинақтау: Жүк көлігі 60 км/сағ жылдамдықпен жолға шықты. 2 сағаттан кейін оның соңынан 90 км/сағ жылдамдықпен жеңіл көлік шықты. Жеңіл көлік жүк көлігін неше сағаттан кейін қуып жетеді?

(Модификацияланған, негізі: Саркулова А.Қ.)

9) Бағалау: Қоймада биіктігі 2,8 м болатын жүк тиегіш жұмыс істейді. Биіктігі 285 см, 2750 мм, 300 см және 2,79 м болатын сөрелердің қайсысының астынан бұл жүк тиегіш еркін өте алады?

(Модификацияланған, негізі: Батырбаева Г.А.)

10) Бағалау: "А" ауылында 120 адам, "Б" ауылында 180 адам тұрады. Ауылдар арасы 10 км. Пошта бөлімшесін екі ауыл тұрғындарының жүретін жалпы жолы ең аз болатындай етіп қай нүктеге орналастырған тиімді?

(Модификацияланған, негізі: Батырбаева Г.А.)

Appendix C – lesson plan

Бөлім:	6.3А Бір айнымалысы бар сызықтық теңдеу
Педагогтің Т.А.Ә:	Қырғызбай Арайлым
Пән/Сынып:	Математика, 6-сынып.
Сабақтың тақырыбы:	Қайталау
Сабақтың мақсаты:	• Бір айнымалысы бар сызықтық теңдеулерді шешу; • Мәтіндік есептерді визуализация әдістері (блок-модельдер, кестелер) арқылы шығару.

Уақыты	**Кезеңдері**	**Педагогтің әрекеті**	**Оқушының әрекеті**	**Бағалау**	**Ресурстар**
5 минут	Ұйымдастыру	Сәлеметсіздерме! Бүгін, Қайталау тақырыбын қарастырамыз. **Бүгінгі сабақта меңгеретініңіз:** - сызықтық теңдеу; - мәтіндік есептерді визуализация (сурет, кесте) әдісімен шығару. **Үй тапсырмасын тексеру. №835.**	Амандасады, үй тапсырмасына жауап береді. Жаңа әдіс туралы тыңдайды.	Мадақтау	Оқулық
10 мин	Жаңа сабақты бекіту	**«Ой қозғау»** С-1: Қандай теңдеуді бір айнымалысы бар теңдеу деп атайды? С-2: Мәндес теңдеу деп қандай теңдеуді	Сұрақтарға жауап береді. Таразы моделі арқылы теңдеудің қасиетін еске түсіреді. Тест	Смайлик (Өзін-өзі бағалау)	Үлестірме материал

		айтады? С-3: Теңдеудің қандай қасиеттері бар? *(Мұғалім тақтадан «Таразы моделін» көрсетіп, теңдеудің екі жағына бірдей сан қосқанда тепе-теңдік сақталатынын көзбен көрсетеді)* С-4: Бір айнымалысы бар теңдеуді шешу алгоритмін атаңдар. С-5: ax = b түріндегі теңдеуді шешудің неше жағдайы бар? «Білім таусылмайтын кен» (тест)	тапсырмасын орындайды.		

Уақыты	**Кезеңдері**	**Педагогтің әрекеті**	**Оқушының әрекеті**	**Бағалау**	**Ресурстар**
25 минут	Бекіту тапсырмасы (Визуализация тәжірибесі)	Оқулықтан №1-№7 теңдеулерді дәстүрлі түрде шығартады. **№8 Есеп. (Блок-модель әдісі)** *Шарты:* 1-себетте 2-себетке қарағанда 5 есе артық алма бар.	Механикалық түрде х жаза бермей, алдымен дәптерлеріне блок-модельдермен кестелерді сызады. Суретке қарай отырып, теңдеудің оң	Дескриптор арқылы «Басбармақ» әдісімен бағалау.	Оқулық, тақта, дәптер

		Егер 1-себеттен 8 кг алманы 2-себетке салсақ, алмалар саны тең болады. Алғашында әр себетте қанша алмадан болған? **Визуализация қадамы:** *Бастапқы жағдай:* 2-себет: 1-себет: *Түсіндірме:* Екеуі тең болу үшін 1-себет өзінің 2 блогын береді. *Теңескен жағдай:* *(осы 2 блок = 8 кг)* 1-себет: Бұдан шығатыны: 2 блок = 8 кг, 1 блок = 4 кг. **Алгебралық модель:** 5x - 8 = x + 8. **№9 Есеп. (Кесте моделі)** *Шарты:* Жол бойымен 4 км/сағ, тас жолмен 6 км/сағ жылдамдықпен жүрді. Тас жолмен жүрген уақыт 3 сағатқа аз. Жолдар тең	және сол жағын сауатты теңестіреді. Шыққан нәтижені есеп шартымен салыстырады.		

		болса, тас жолмен қанша уақыт жүрген? **Визуализация қадамы:** Жол | v | t | S Құм жол | 4 | x | 4x Тас жол | 6 | x - 3 | 6(x - 3) **Алгебралық модель:** 4x = 6(x - 3).			
	Жеке оқушымен жұмыс	Хайролла Тілеубекке арнайы «Есепті аяқта!» тапсырмасын береді.	Тез есептеу шапшаңдығын дамыта отырып, тапсырманы орындайды.	Мониторинг	Парақшалар
5 минут	Қорытынды (Кері байланыс)	**Бүгінгі сабақта:** - сызықтық теңдеулер; - мәтіндік есептерді визуализация арқылы шығаруды үйрендік. **Кері байланыс «Нысана»** Сары түс: «Бұл әдіс маған қызық болды...»	Сабақты қорытындылайды, визуализация әдісі бойынша өз пікірлерін білдіреді, стикерлерді жапсырады.	Ауызша бағалау	Стикерлер

		Қызыл түс: «Маған бәрі түсінікті!» Көк түс: «Маған көмек қажет» **Үйге тапсырма: №845.**			

Lesson plan №2

Мұғалімнің аты-жөні:	
Пән/Сынып:	Математика / 6-сынып
Күні:	
Тарау немесе бөлім атауы:	6.3В Бір айнымалысы бар сызықтық теңсіздіктер
Сабақтың тақырыбы:	Қайталау
Оқу мақсаты:	Өтілген материалдарды толықтыру және визуализация арқылы бекіту
Бағалау критерийі:	Бір айнымалысы бар сызықтық теңсіздіктер мен теңдеулерді визуалды модельдер (таразы, координаталық түзу, сызбалар) арқылы шеше алады, математикалық өрнектерге айналдырады.

Уақыты	**Кезеңдері**	**Педагогтің әрекеті**	**Оқушының әрекеті**	**Бағалау**
5 минут	Ұйым-дастыру	Сәлеметсіздер ме! Бүгін «Қайталау» тақырыбын қарастырамыз. Бүгінгі сабақта меңгеретініңіз: Оқушылардың теңсіздіктерді құрғақ жаттау емес, визуалды суреттер, таразы немесе Desmos графиктері	Амандасады, үй тапсырмасына жауап береді. Интерактивті тақтадағы визуалды модельге қарап, өз жауаптарын тексереді.	Мадақтау

		арқылы көзбен көріп, шешімін табу дағдыларын қалыптастыру. Үй тапсырмасын интерактивті тақтадағы анимациялық графикті көрсету арқылы тексеру.		
10 мин	Жаңа сабақты бекіту	1. Қайталау сұрақтары (Визуалды түрде): Мұғалім тақтадан сандық сәулелердің, кесінділердің түрлі-түсті сызбаларын (2D диаграммаларын) көрсетеді. 1. Экрандағы қызыл және көк түспен боялған аралықтар қалай аталады? 2. (2; 7) интервалы динамикалық сызбада қалай көрінеді? 3. [-3; 5] және [1; 9] аралықтарының қиылысуын қағазға сызып көрсетіңіз. 2. Сәйкестендіру тесті (Таразы моделімен): Таразы (balance scale) әдісі арқылы теңсіздіктердің	Тақырып бойынша ресурстарды қарап, танысады. Берілген теңсіздіктерді құрғақ есептемей, әрқайсысын таразы моделіне салып елестетеді (мысалы, 2x + 1 жағы 8 + x жағынан ауыр) және сәйкестендіреді.	Дескриптор: • Визуалды сызбаны оқи алады; • Интервал, сәуле, кесіндіні ажыратады.

		қай жағы ауыр екенін талдау. 1. $5x - 10 \geq 2x - 1$ 2. $7x - 8 \leq 5x + 2$ 3. $4x - 12 \geq x - 6$ 4. $2x + 1 > 8 + x$		
25 минут	Бекіту тапсырмасы	Жалпы сыныппен жұмыс. 4 қадамдық алгоритмді қолдану. №1. «Визуалды шешім» а) $2x \leq 10$ б) $x + 3 \geq 7$ в) $-4x < 8$ г) $-x < 5$ Мұғалім әрекеті: Оқушылардан алдымен белгісіз мәндерді анықтауды, сосын сызба салуды талап етеді. №2. «Қатесін тап» (Топтық жұмыс) Мұғалім тақтаға Python немесе Geogebra арқылы салынған дайын сызбаларды және қате шығарылған алгебралық теңсіздіктерді шығарады. 1) $5y - 9 \leq 3 - 7y$ 2) $3x + 1 \geq 4x - 5$ Тапсырма: Сызбаға (графикке) сүйене отырып,	Дәптерлеріне координаталық түзу сызып, бояп көрсетеді. 4 қадамды орындайды: 1. Белгісізді анықтайды; 2. Визуалды модель сызады; 3. Математикалық теңсіздік құрады; 4. Жауабын графиктен тексереді. Оқулықтан №1003, №1005 есептердің визуалды модельдерін құрастырады.	Дескриптор: • Теңсіздіктің визуалды моделін құрады; • Алгебралық шешімін табады; • Координаталық түзуде кескіндейді; • Жауапты сан аралығында жазады. «Басбармақ» әдісі

		алгебралық қатені табыңыз. Өз бетімен жұмыс. Тест тапсырмасы: Әр сұраққа сызба құру міндеттеледі. 1. (-∞; 5] сан аралығының аты... 2. 1-ден 4-ке дейінгі кесінді белгіленуі... 3. 1-ден 3-ке дейінгі жартылай интервал...		
5 минут	Кері байланыс	Бүгінгі сабақта: Визуалды модельдеу әдісі арқылы сызықтық теңсіздіктерді шығаруды бекіттік. Мұғалім оқушылардан: "Біз сызған диаграммалар мен таразылар есепті шығаруға қалай көмектесті?" - деп сұрайды. Кері байланыс: «Білемін - Білдім - Білгім келеді» кестесін толтыру. Үйге тапсырма: №1006. Әр есепке міндетті түрде 2D сызба немесе таразы моделін сызып келу.	Оқушылар кестені толтырады. Визуализация әдісінің тиімділігі туралы өз ойларымен бөліседі (метакогнитивтік дағды).	Тақырыпты меңгергенін анықтау

Appendix D – four – level assesment descriptors

Score	Descriptor	Scientific foundation
0	The student either merely creates pictorial of the problem or generates an inaccurate solution without creating any drawings, graphs, or illustrations.	The detrimental effects of pictorial visuals on report writing were investigated by Hegarty and Kozhevnikov (1999, as cited in Hadi & Csíkos, 2025).
1	When translating abstract information into structural resembling, the student often blunders when attempting to describe a problem or create a diagram or schema. In other words, the student is unable to create a comprehensive graphic and his response contains errors in mathematical computations. The illustration or schema is barely half drawn, though.	In their research, Finesilver (2022, as cited in Hadi & Csíkos, 2025) and Schoenherr and Schukajlow (2024, as cited in Hadi & Csíkos, 2025) have highlighted the challenges of creating a graph or schema when structural resemlance is present. They stated that it's a common mistake.
2	The student properly sketches a graph, or a diagram or etc. (schematic representation) of the problem. The problem's conditions are completely reflected in the mathematical model.	Schematic representation was discussed by Presmeg (1986, as cited in Hadi & Csíkos, 2025). In other words, he stated that it is beneficial to comprehend the problem.
3	Using the schematic representation the student have developed, the student accurately completes mathematical computations and determines the precise solution without making any mistakes.	Hegarty and Kozhevnikov (1999, as cited in Hadi & Csíkos, 2025) argued that the correct use of schematic representation can help in the successful production of word problems.