SDU
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Methodology of developing students` logical thinking abilities in mathematics

Nurbolat Kalambay

A thesis submitted to the Faculty of Education and Humanities

in partial fulfillment of the requirements for the degree of

MASTER OF PEDAGOGICAL SCIENCES

in Mathematics

«SDU University»

Department of Pedagogy of Natural Sciences

Thesis Advisor: Doctor of Pedagogical Sciences,
Associate Professor
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Faculty of Education and Humanities
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Kaskelen, 2025

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List of Abbreviations

PISA – Program for International Student Assessment

EG – Experimental group

CG – Control group

ABSTRACT

This study examines the role of developing students' logical thinking skills in teaching mathematics in grades 5. The participants were divided into two groups. The first experimental group, the second control group.

This study allowed us to identify a number of shortcomings in the development of students' logical thinking skills in mathematics lessons, and to a certain extent, it became a means of helping to compensate for these shortcomings.

In this study, an elective course "Logical Problems and Games" was developed for 5th grade students to develop students' thinking skills in mathematics.

The developed elective course allows students to generalize and deepen their knowledge of mathematics within the program, to see the uniqueness and high abstraction of mathematical objects. On the other hand, logical problems show great potential for application in life, practical work, and everyday life, teach them to use logic in finding the right solution, solving various problems, including everyday problems.

The program is based on solving logical problems and games for 5th graders and is organized in 34 hours (1 hour per week).

The purpose of this study: to develop general logical thinking skills, increase student interest in the subject, and create conditions for the development of problem-solving skills.

Keywords: problem solving, learning strategies, modern educated person, thinking skills, creative abilities.

АҢДАТПА

Бұл зерттеу 5 сыныптардың математика пәнін оқытуда оқушылардың логикалық ойлау қабілетін арттырудың рөлін зерттейді. Қатысушылар екі топқа бөлінді. Бірінші сынақ тобы, екінші бақылау тобы.

Бұл зерттеу математика сабақтарында оқушылардың логикалық ойлау қабілеттерін дамыту саласындағы заманауи білім беруді талдау бірқатар кемшіліктерді анықтауға мүмкіндік берді, әрі белгілі бір деңгейде сол кемшіліктің орнын толтыруға көмек құралы болды.

Бұл зерттеуде математикадан оқушылардың ойлау қабілеттерін дамыту үшін 5 сынып оқушыларына арналған «Логикалық есептер және ойындар» элективті курсы әзірленді.

Әзірленген элективті курс оқушыларға бағдарлама бойынаш математикадан білімдерін жалпылауға, тереңдетуге, математикалық объектілердің бірегейлігін, жоғары абстрактілігін көруге мүмкіндік береді. Екінші жағынан логикалық есептер, өмірде, практикалық қызметте, тұрмыста қолданудың зор мүмкіндіктерін көрсетеді, дұрыс шешімді табуда, әртүрлі есептерді, соның ішінде тұрмыстағы есептерді шешудегі логиканы қолдануға үйретеді.

Бағдарлама 5 сыныпқа логикалық есептерді шешу мен ойындарға негізделіп, 34 сағатта ұйымдастырылған (аптасына 1 сағат).

Бұл зерттеудің мақсаты: жалпы логикалық ойлау қабілетін дамыта отырып, оқушының пәнге деген қызығушылығын арттыру, есептерді шығаруда оқушылардың біліктілігін дамытуға жағдай жасау.

Кілт сөздер: мәселелерді шешу, оқыту стратегиялары, замануи білімді тұлға, ойлау қабілеті, шығармашылық қабілет.

АННОТАЦИЯ

В данном исследовании рассматривается роль развития логического мышления учащихся в обучении математике в 5 классе. Участники были разделены на две группы. Первая экспериментальная группа, вторая контрольная группа.

Данное исследование позволило выявить ряд недостатков в развитии логического мышления учащихся на уроках математики, и в определенной степени стало средством, помогающим компенсировать эти недостатки.

В данном исследовании для учащихся 5 класса был разработан факультатив «Логические задачи и игры» с целью развития у учащихся математических навыков.

Разработанный факультатив позволяет учащимся обобщить и углубить свои знания по математике в рамках программы, увидеть уникальность и высокую абстракцию математических объектов. С другой стороны, логические задачи показывают большой потенциал для применения в жизни, практической работе и повседневной жизни, учат их использовать логику в поиске правильного решения, решении различных задач, в том числе бытовых.

Программа основана на решении логических задач и игр для учащихся 5 класса и организована в объеме 34 часов (1 час в неделю).

Цель данного исследования: развитие общих навыков логического мышления, повышение интереса учащихся к предмету, создание условий для развития навыков решения проблем.

Ключевые слова: решение проблем, стратегии обучения, современный образованный человек, навыки мышления, творческие способности.

INTRODUCTION

Relevance of the topic. The current stage of development of Kazakhstan clearly demonstrates new priorities in the field of school education, which correspond to global trends. One of the main tasks of the State Program of Education in our country until 2025 is to prepare a modern educated person who can make independent decisions, is capable of cooperation, is distinguished by mobility, dynamism, and constructiveness. In this regard, the formation of a comprehensive learner in basic school is considered one of the priority areas.

With the passage of time and changes in the times, new paradigms that dominate the field of education, combining the traditional system of education with new requirements, have become the basis for establishing the necessary conditions for the formation of a personality and establishing directions for the formation of schoolchildren in accordance with social and public demands. Therefore, the new tasks that have been assigned to us since the year of independence, primarily involve the implementation of modern methods of today's domestic education. And modern methods of education require the development of the student's personality, his thinking and creative abilities to be given priority in the educational process.

As the results of the international PISA study show, the level of our schoolchildren is far from leading positions. According to the PISA results, Kazakh students took 46th place in 2023, compared to 56th place in 2018. One of the main conclusions made by Kazakhstani experts based on the results of international testing is the conclusion that our students have insufficiently developed logical learning skills: the ability to analyze, generalize, and think logically.

Logic allows a person to not only think correctly, but also to speak confidently, act reasonably, and act rationally.

Ya. A. Komensky, L. S. Vygotsky, V. A. Sukhomlinsky, K. D. Ushinsky, P. Ya. Galperin, V. V. Davydov, D. B. Elkonin, A. A. Lublinskaya, A. M. Matyushkin and others were engaged in the development of logical thinking of students in the primary school.

Among domestic scientists, examples and analyses of problems designed to develop students' logical thinking skills can be found in the works of G.A. Khamitova, T. Aitkazhy, and others.

An analysis of modern education in the field of developing students' logical thinking skills in mathematics lessons has made it possible to identify a number of shortcomings:

- teachers do not fully understand the concept of "logical thinking skills" and the technologies for their formation;
- lack of a system for developing students' logical thinking skills in mathematics lessons;
- Limited informational and methodological support for teaching staff on the implementation of state educational standards.

Research into the development of logical thinking skills in mathematics lessons has revealed a number of contradictions:

- traditional education system and modern demands of society;
- The need to develop logical thinking skills and the lack of systematic didactic material available to the teacher.
- The need for logical thinking skills in the PISA international testing and the lack of information on the development of logical thinking skills in domestic scientific literature.

This justifies the relevance of developing logical thinking skills in mathematics among primary school students. Because logical thinking is not only a reflection of the quality of education and the reputation of the state, but most importantly, it is a condition for a student to successfully live in the modern world.

Object of research: The process of teaching mathematics

Research topic: Developing students' logical thinking skills in mathematics

Research goal: To develop and implement a methodology for developing students' logical thinking skills in mathematics.

Responsibilities:

- Analysis of logical problems encountered in school mathematics courses;
- Studying the features of the development of students' logical thinking abilities;
- Identify the structure of logically oriented problems;
- Developing a methodology for developing students' thinking skills through logical problems and games.
- Check the effectiveness of the method.

Research hypothesis: If a methodology based on logical problems and games in mathematics is developed and implemented, using methodological recommendations for the educational process, the development of logical thinking in students will be more effective.

Research methods : The following set of methods was used in the study:

Theoretical: analysis of psychological, pedagogical and methodological literature; systematization, generalization, analysis of real data; observation method.

Empirical: analysis of psychological and pedagogical documentation; pedagogical experiment, which includes the stages of identification, formation and control; methods of processing the results (quantitative and qualitative analysis of the data obtained).

The study was conducted in three stages.

In the first stage (2023) - theoretical-research - an analysis of scientific literature on the research problem was carried out; the theoretical and methodological apparatus and work program of the study were developed; the theoretical justification of the diagnostic research methodology was provided.

In the second stage (2023-2024) - experimental-exploratory - a defined experiment and an analysis of the results obtained were conducted; during the formative experiment, a methodological tool for the development of logical thinking

skills in 5th grade students was developed, theoretically substantiated and implemented.

In the third stage (2024) - a conclusion-generalization - control experiment was conducted, the data obtained were analyzed and interpreted, the effectiveness of introducing a methodological tool for developing students' logical thinking was determined, conclusions were formulated, and dissertation research materials were formalized.

Theoretical importance:

- Theoretical ideas about the features and conditions for developing students' logical thinking in mathematics have been expanded.
- The use of logical problems and games in mathematics lessons was justified to develop students' logical thinking skills;
- Criteria and diagnostic tools for developing students' logical thinking skills have been identified.

Practical significance: the conclusions and results of the dissertation work can be used in the educational process of general education institutions.

Research base: Astana city, school gymnasium No. 49. 5th grade students participated in the experimental work.

To be defended:

- Elective course "Logical Problems and Games" for 5th grade students;
- methodological recommendations for using the compiled program of the elective course;
- results of a pedagogical experiment.

Structure: the dissertation work consists of normative references, definitions, abbreviations and notations, an introduction, two chapters and a conclusion, a list of used literature and an appendix.

1. LITERATURE REVIEW

1.1 Analysis of logical problems encountered in school mathematics courses

Logical problems constitute a broad class of non-standard problems. Non-standard problems are problems that do not have general rules that determine a specific program for solving them in a mathematics course. Logical tasks include, first of all, text tasks that require the recognition of objects or their arrangement in a certain order according to the available properties. In addition, some of the statements about the condition of the problem may have different truth values (can be true or false).

Non-standard problems in mathematics are found not only in optional classes and clubs, but also in the school mathematics course.

We will analyze the most widely used mathematics textbooks for students in grades 5-6, their authors are T. Aldamuratova, K. Baisholanova, E. Baisholanov [2,3], A. Abylkasymova, T. Kucher, Z. Zhumagulova [4,5].

Based on the analysis of school textbooks, Table 1 was created, which contains information on the presence of non-standard (logical problems) tasks. The table shows the frequency of occurrence of non-standard, including logical problems, in the books of the above-mentioned authors.

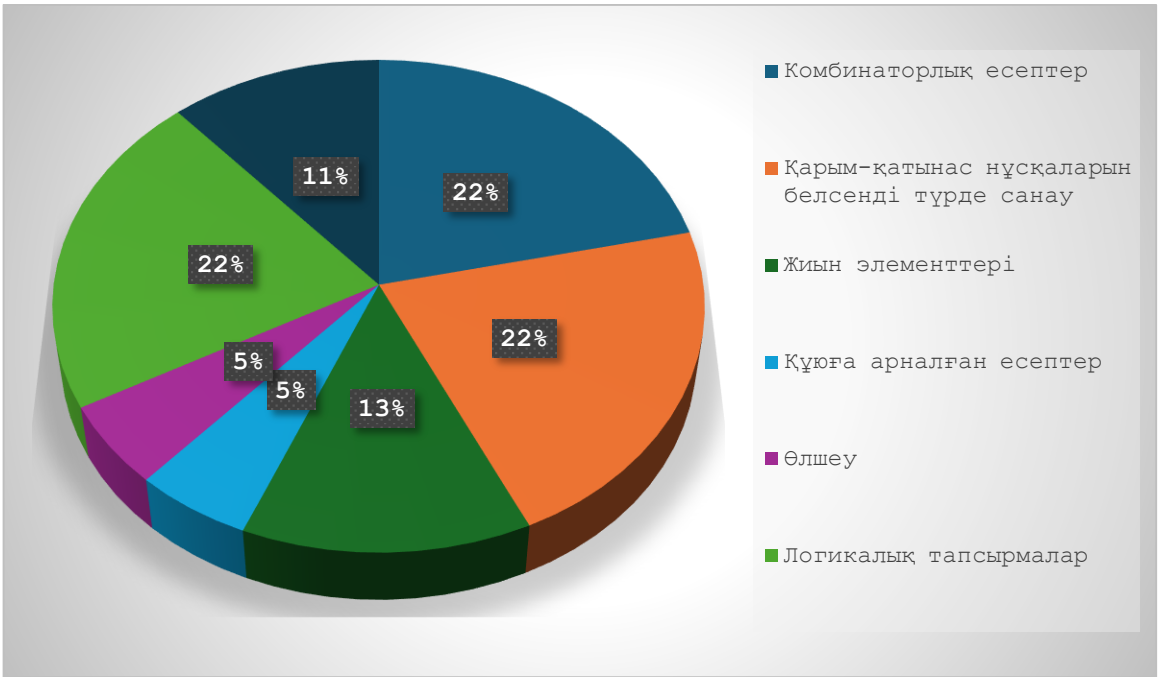
Table 1.1.1 Number of non-standard problems found in school textbooks

No.	Type of tasks	5th grade		6th grade	
		T. Aldamuratova, K. Baisholanova, E. Baisholanov	A. Abylkasymova, T. Kucher, Z. Zhumagulova	T. Aldamuratova, K. Baisholanova, E. Baisholanov	A. Abylkasymova, T. Kucher, Z. Zhumagulova
1	Combinatorial problems	7	6	5	6
2	Actively consider communication options	6	7	6	5
3	Set elements	3	4	4	4
4	Casting reports	1	1	2	2
5	Measurement	2	2	1	1

6	Logical tasks	5	5	7	7
7	Identifying functional, spatial, and temporal relationships	4	3	3	3

In mathematics textbooks for grades 5-6, we find average values for a certain type of non-standard problem (Figure 1.1.1).

Figure 1.1.1 Average number of non-standard problems in mathematics textbooks (grades 5-6)



Based on the data obtained, it can be concluded that when studying mathematics in grades 5-6, much attention is paid to logical-type problems. This is due to the fact that they are most suitable for developing the logical and cognitive abilities of children aged 11-13, since such tasks are a combination of both creative tasks and specific arithmetic tasks.

As we can see from Table 1, logical problems are common in the 6th grade mathematics course. In grade 5, the number of logical problems in both textbooks presented by the authors is 5.

The most important thing in solving logical problems is the stage of searching for a solution, which has unlimited possibilities for the student's all-round development, especially for the development of his abilities.

General method for solving the problem:

1) explore the content of the task through a picture, drawing, short note, or other visual illustration of the content;

2) if necessary, clarify the concept of the task, if possible, determine the type of task and remember a known method for solving it and other known information used to solve this type of problem;

3) collect additional information from the experience of solving other problems, transform the information taking into account the specifics of this task;

4) general analysis of the question from the condition; trial and error can be used;

5) if possible, divide the task condition or requirement into parts, develop a solution plan for each, and then combine them;

6) recall a problem similar to this solution, whose solution is known, compare them and develop a solution plan on this basis;

7) temporarily change the task condition or requirement so that the received task can be compared with the given task; then apply the analogy technique mentioned above;

8) change the condition of the problem to bring it closer to the question;

9) transform the problem in order to bring it closer to the condition;

10) replace the concepts contained in the task conditions or questions with their definitions;

11) choosing definitions of concepts that suggest (or shorten) a line of reasoning or replacing the definition of a concept with its symbol;

12) full application of the terms of reference;

13) if possible, highlight the special circumstances of the task and use the above-mentioned method of division into parts;

14) reformulate the task (several times), formulate a problem opposite to the given task and consider the possibility of solving it;

15) analyze all possible solutions and evaluate their effectiveness.

Each task, especially in the first lessons with 5th graders, should not contain many difficulties of a logical or semantic nature.

Logical problems presented in the mathematics course for grades 5-6 can be divided into the following types:

- tournament tasks;
- number puzzles;
- establish correspondences between elements of different sets;
- statement calculation tasks;
- Tasks of using graphs.

Tournament tasks. Here we consider a class of logical tasks related to determining the results of some tournaments. The tasks of this class, as a rule, provide incomplete information about the results of sports matches held, and it is necessary to obtain complete data through logical reasoning. Of course, in most cases, designing a tournament schedule based on the data obtained in a logical way helps to solve the problem. Of course, when solving a problem about a chess or football tournament, students should know the basic rules of such tournaments.

For example, 7 people participated in a chess tournament. Each person played one game with each other. What is the total number of games played? Solution: 6+5+4+3+2+1=21 games.

Numerical rebuses. Numerical rebuses include arithmetic expressions in which all or some numbers are replaced by symbols (letters, asterisks, etc.), most often numerical rebuses are numerical equations. Numerical rebuses represent a logical problem in which, through logical reasoning, it is necessary to reveal the meaning of the symbol and restore the numerical record.

For example, Rose, Tulip, and Jasmine presented baskets made of the flowers they grew, "rose," "tulip," and "jasmine," to the flower growers' competition. However, the girls' names do not match the names of the flowers they presented. What flowers did each of the girls grow? Answer: Rose - jasmine, Tulip - rose, and Jasmine - tulip.

	R	K	J
R	0	1	0
K	0	0	1
J	1	0	0

In the current methodological system of teaching, the emphasis has shifted from increasing the amount of information for students to developing general logical mathematical skills.

Let's dwell on the tasks in more detail.

Creating correspondence tables

When solving logical problems, we create a correspondence table that clearly shows the correspondence between concepts. Students in grades 5-6 mark correspondence as "+", and inconsistency as "-".

Task No. 1. Five classmates named Aisultan, Aigerim, Korkem, Enlik, and Miras became winners of the Olympiads in physics, mathematics, computer science, literature, and geography. Information about the students:

- The winner of the Informatics Olympiad teaches Aisultan and Aigerim how to work on a computer;
- Korkem and Enlik also showed interest in computer science;
- Aigerim doesn't really like physics;
- The winner of the Olympiad in art, Aigerim, and literature will participate in swimming together;
- Aisultan “spends little time on writing;

What Olympiad did each of these children win? The answer is given in the table.

	A	A	K	E	M
F	0	0	1	0	0
M	1	0	0	0	0
I	0	0	0	0	1
A	0	0	0	1	0
G	0	1	0	0	0

Task №2. When compiling the Friday lesson schedule, students want computer science to be assigned to the first or second lesson, physics to the first or third lesson, and history to the second or third lesson. Is it possible to satisfy all of these wishes at the same time? Answer: Computer Science History Physics or Physics Computer Science History.

Computer science	Physics	History			Computer science	Physics	History
1	0	0			0	1	0
0	0	1			1	0	0
0	1	0			0	0	1

When solving logical problems, it is necessary to place the correspondence on a line, and the first four conditions allow you to determine the order of placement on the line, and then the tabular representation of the correspondence.

Task #3. Before the tournament started, fans made the following predictions about their favorite athletes:

A) Max wins, Bill takes second place;

B) Bill takes third place, Nick takes first place;

C) Max comes last and John comes first.

After the tournament ended, only one of the fans' predictions was correct. What place did John, Nick, Bill, and Max occupy in the tournament? (List the vacant places of the participants in the order of their names in the answer.) Answer: 3124.

J N B M 3 1	A B C 1 1 0 0 0 0
J N B M 3 1 2 4	A B C 0 0 1 1 1 0

The next type of logical problems encountered in the 5th-6th grade mathematics course are calculation-based tasks.

Task #1. When my father was 31, I was 8 years old. Now my father is 2 years older than me. How old am I now?

Solution: 31-8=23 – the difference between the ages of the father and the son Let's say the son's age is 23: 23-8=15 – the number of years from the time the son was eight years old to the time he turned 23 31+15=46 – the father's age:

31-8=46-23

46/2=23

Answer: The son is 23 years old, the father is 46 years old.

Task #2. There is water in a 10-liter barrel and there are empty 7-liter and 2-liter containers. How can you divide 5 liters of water equally between the two containers?

Solution: 10-7=3

7-2=5

2+3=5

Answer: 16.

Task #3. Two containers of nine liters and four liters are given. How can six liters of water be removed from the tank using these containers? (The water can be poured back into the tank).

Solution: 9-4-4=1

9-4=5

5+1=6

Answer: 17.

Task #4. Given 8-liter, 5-liter, and 3-liter containers. The 8-liter container is full of water. Now, how can this water be divided into two containers of 4 liters each?

Solution: 8-5=3

5-3=2

3+3=6

6-2=4

Answer:18.

The following types of tasks are equation-based problems.

Task #1. I added one to the number I thought of, multiplied the sum by four. I divided the multiplication by four. I subtracted three from the quotient. The result was one. What number did I think of?

Solution: (((x+1)*2)/4)-3=1

x=7

Task #2. There are 35 students in the class. There are three more girls than boys. How many boys and how many girls are there in the class?

Solution: x+x+3=35

x=16(y)

x+3=16+3=19(k)

The given types of tasks represent different stages of application of mathematical properties, concepts, and laws studied in a single cognitive and various topics.

Thus, the analysis of logical problems encountered in the school mathematics course made it possible to determine their importance in the development of logical thinking, abstract thinking, problem solving and informed decision-making. Logical tasks not only develop the ability to analyze and solve complex problems, but also contribute to the development of students' creative thinking. In the course of studying the analysis of logical problems, we came to the conclusion that they are an integral part of mathematical education and play an important role in the formation of students' critical thinking.

This study allowed us to identify different types of logical problems, their features, solutions, and applications in the school mathematics course. We focused on the fact that logical problems can be used to develop students' skills, as well as to form their cognitive interests and increase their motivation to study mathematics.

1.2 Features of developing students' logical thinking skills

Before considering the development of students' logical thinking abilities, we define the concept of thinking as a mental cognitive process in general.

Thinking is one of the important mental processes, making the right decision in a problematic situation can be achieved through thinking. A person with developed thinking skills can abstract from the situation and draw theoretical or practical conclusions on the existing problem, based on his own experience. To understand the problem of forming logical thinking, it is necessary to define the concepts of "thinking" and "logical thinking".

The term "thinking" is presented in different sciences and understood in different ways. We will consider the definition of the concept of "thinking" in pedagogy and psychology.

Researcher A.V. Brushlinsky believes that thinking is a process similar to perception and memory. “Thinking is a socially conditioned, closely related to speech, mental process of search and discovery, that is, a process of correctly reflecting reality when applying analysis and synthesis” [6].

M.P. Palyanov gives the following definition: "thinking is the process by which a person knows a certain action" [7].

A.N. Leontiev believes that “thinking is a complex process of consciously representing reality in objective properties, connections, and relationships, including objects that are not subject to sensory perception” [8].

According to S.V. Lazarevsky, thinking in an individual arises only when there are certain tasks that make it the most relevant and in demand and require an urgent solution (i.e. acquired in the learning process or innate) [8].

Among the most famous foreign psychologists, the Swiss scientist J. Piaget proposed a theory of the development of thinking in childhood, which affects its development. The theory of the development of a child's thinking, proposed by J. Piaget, was called "operational". In the development of operational intelligence in children, J. Piaget identified the following four stages:

1. The sensorimotor intelligence stage, which covers the period of a child's life from birth to approximately two years of age. It is characterized by the development of the child's ability to perceive and recognize objects around them in terms of their permanent properties and characteristics.

2. The operational thinking stage, which covers the development from two to seven years of age. During this period, the child's speech develops, an active process of internalizing external actions with objects begins, and visual images are formed.

3. The stage of concrete operations with objects. This is typical for children from 7-8 to 11-12 years old. Here, mental operations are reversible.

4. Formal operations stage. It includes middle-aged children: from 11-12 to 14-15 years. This stage is characterized by the child's ability to perform mental operations using logical reasoning and concepts. Internal mental operations during this stage become a structurally organized whole [10].

The functions of thinking are different. V.D. Shadrikov identifies the following functions of thinking:

1. Understanding. This means discovering something important in the objects and phenomena of reality, understanding the essence and meaning of something, achieving understanding based on the connection of a person with past experience. "Understanding a phenomenon means determining the way of its occurrence, the rule, according to which this occurrence is carried out with a certain necessity in a certain set of conditions. This is the general formula for the formation of a concept, understanding."

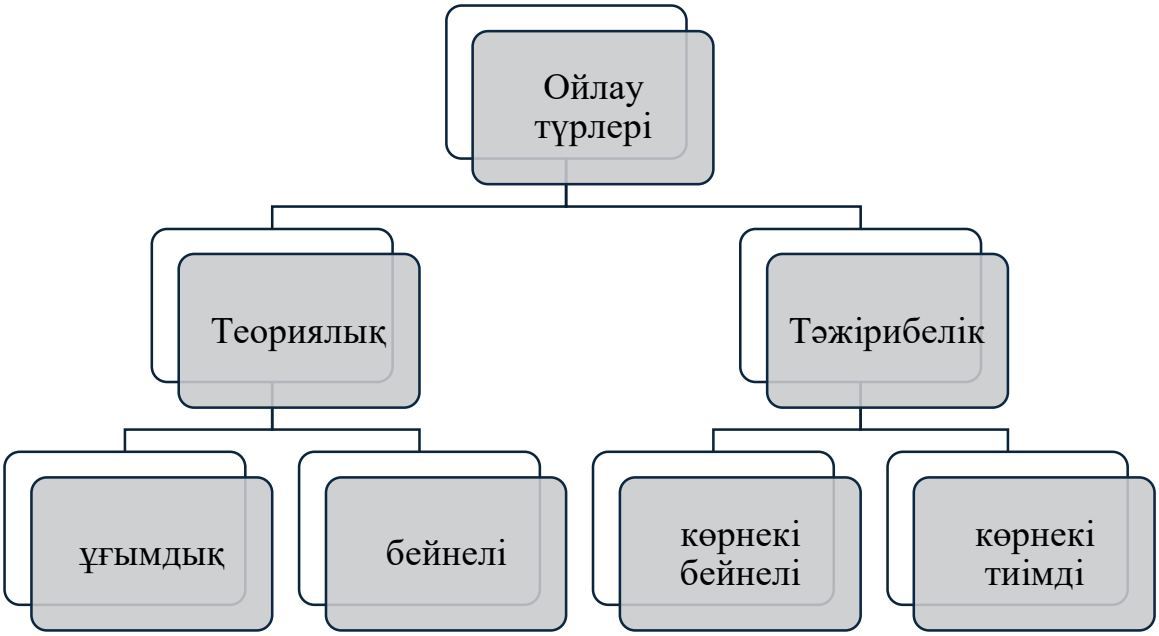
2. Solving problems and tasks. Thinking arises in situations where the means and methods of the subject's actions are insufficient to achieve the goals. The emergence of a problem means that as a result of the analysis, at least a preliminary solution to this and the unknown can be found.

3. Goal setting. This refers to the process of forming new goals in a person's thinking and actions. Forming an image of the future result of an action and taking this image as the basis for practical or mental actions, thus it can be involuntary and voluntary.

4. Reflection. Reflection is a person's thinking activity aimed at understanding knowledge, analyzing its content and methods of cognition, its actions, and self-knowledge [11].

Depending on the content of the problem being solved, the following types of thinking are distinguished in psychology: theoretical, experimental, theoretical-figurative, visual-figurative, visual-effective.

Figure 1.2.1 Types of thinking



Theoretical conceptual thinking, as defined by S.L. Rubinstein, is a type of thinking in which a person, in the process of solving a problem, resorts to concepts, performs mental operations, without directly dealing with the experience obtained with the help of the senses. He discusses and seeks to solve the problem from beginning to end, using the ready-made knowledge obtained by other people, expressed in forms, judgments, and conclusions [12].

Theoretical figurative thinking differs from conceptual thinking in that the material a person uses to solve a problem is not concepts, reasoning, or conclusions, but images taken from memory or creatively recreated by imagination.

A distinctive feature of visual-figurative thinking is that the thinking process in it is directly related to the thinking person's perception of the surrounding reality, and the images necessary for thinking are stored in his short-term and immediate memory [13].

A distinctive feature of visual-effective thinking is that the thinking process itself is a practical transformational action carried out by a person with concrete objects. In this case, the main condition for solving the problem is the correct actions with the corresponding objects [13].

In the research of AI. Karmanchikov, four types of thinking were identified: intuitive, logical, strategic, and emotional [14].

1. Owners of strategic thinking are characterized by a desire for freedom and independence. This, as a rule, is a generator of actions, which constantly requires the study of new spaces, new theories and "conquering commanding heights"; The main effective pedagogical method for such children is activity, originality, independence.

2. The logical type of thinking involves the pursuit of stability, standardized behavior, and the implementation of certain rules and instructions. The main effective pedagogical methods are logic, structure, clarity, stability, and consistency.

3. Children with an emotional type of thinking strive for a good psychological climate, positive emotional relationships with the teacher and other students. The main effective pedagogical methods are discussion, mutual understanding, and feelings.

4. For a child with intuitive thinking, the success of learning depends on motivation, and the positive perception of the teacher depends on his level of competence in certain issues. The main effective pedagogical methods are perspective, prediction, potential [14].

Of these, the most common type of thinking is logical thinking.

J. Piaget defines logical thinking as the grouping of various operations in the first stage, followed by the grouping of conclusions in the second stage, which leads to the correct decision [10].

Speaking about logical thinking, L.S. Vygotsky emphasizes that the basis of logical thinking is analytical activity.

Logical thinking is the highest stage of a child's mental development and goes through a long development process. It is characterized by the fact that it occurs in the form of abstract concepts and reasoning [15].

In the Brief Dictionary of Psychological Concepts, logical thinking is defined as "a type of thinking, the essence of which is to act with concepts, reasoning, and conclusions using the laws of logic."

Logical thinking, unlike practical thinking, is carried out only through verbal means. Teaching a child to reason requires that he or she be able to reason correctly.

According to O.K. Tikhomirov, logical thinking is one of the types of thinking that allows a student to synthesize and analyze a specific task. The author identifies the main elements of logical thinking:

1. Clarify the terms of the assignment;

2. The logic of the signs and facts to be verified.

The first element is searchable information. The second element is the informativeness of the signs and facts that serve as a search for solving tasks.

One way to develop students' logical thinking is to solve non-standard problems.

The concept of “non-standard problems” is used by many methodologists. The definition of a non-standard problem is given in L.M. Fridman’s book “How to Learn to Solve Problems”: “non-standard problems are tasks in a mathematics course that do not have general rules that determine a specific program for solving them” [16].

Non-standard problems, unlike traditional problems, cannot be solved directly using any algorithm. Such tasks do not limit the student to a strict framework of only one solution. It is necessary to find a solution that will contribute to the development of logical thinking and creative work. Such a task may even be simple, but it has a special content that requires the student's intelligence and logical thinking.

Solving non-standard problems involves the development of imagination, memory and curiosity, attention and flexibility of thinking. The student's thinking is sharpened, the skills of observing phenomena, analyzing, comparing and generalizing facts, drawing conclusions are formed. The students' thoughts become consistent, substantiated, logical, and their speech becomes clear, convincing, and convincing.

In the process of solving this type of problems, students' mathematical thinking expands, non-standard thinking is formed, the ability to use knowledge in non-standard situations, perseverance in achieving goals develops, and their interest in studying the subject increases. Independence, inquisitiveness, initiative, and activity are fostered. All this helps to develop creative and logical thinking in schoolchildren.

Based on the results of the analysis of the theory and practice of using non-standard tasks in the process of teaching mathematics, it is possible to distinguish their general and specific role. Non-standard tasks:

- teaching children not only to follow ready-made models of action, but also to independently search for new ways to solve problems, etc.
- encourage them to find their own ways of solving problems;
- influence the development of students' intelligence and resourcefulness;
- To create favorable conditions for students to increase the strength and depth of their knowledge, and to ensure their conscious mastery of mathematical concepts [17].

Solving non-standard problems activates the activity of schoolchildren. Students learn to compare, classify, generalize, and analyze, which contributes to a more solid and conscious acquisition of knowledge.

Experience has shown that non-standard problems are very useful not only for lessons, but also for extracurricular activities and Olympiad tasks, as they allow for a true differentiation of the results of each participant [18].

There is no generally accepted classification of non-standard problems, but B.A. Kordemsky identifies the following types of such tasks:

- Problems related to the school mathematics course, but of high difficulty - like problems in the mathematical Olympiad. Mainly intended for schoolchildren with a certain interest in mathematics; thematically, these tasks are usually related to one or another specific section of the school curriculum. The exercises here deepen the educational material, supplement and generalize the individual provisions of the school course, expand the mathematical thinking, and develop skills in solving difficult problems.
- Mathematical entertainment problems. They are not directly related to the school curriculum and usually do not require mathematical training. This does not mean that the tasks in the second category include only easy exercises. There are

problems with very difficult solutions and problems for which a solution has not yet been found [19].

To develop logical thinking, teaching is focused on creating learning activities that lead to the formation of theoretical thinking.

The main tool for developing students' mathematical abilities is tasks. In his work "What does it mean to master mathematics?", the famous modern mathematician D. Polya describes logical thinking as "the ability to solve problems that require not only standard, but also a certain independence of thinking, intelligence, originality, and ingenuity" [19].

One of the main reasons for students' difficulties in solving problems is that the mathematical problems included in the main chapters of school textbooks are usually limited to one topic. Solving such problems requires students to have knowledge, skills and abilities on any one issue of the educational material and does not consider broad connections between different sections of the school mathematics course. The function of such tasks is often to clarify the meaning of the material covered in the lesson and to illustrate it. Therefore, schoolchildren have some difficulties in solving logical problems.

Unfortunately, in the practice of teaching mathematics, solving problems is often considered a means of consciously mastering the program material by students. Even the increased difficulty in special collections for extracurricular work is mainly aimed at concentrating students' skills in solving standard problems, problems of a certain type.

And any non-standard task that is offered to students to solve can serve many specific learning goals. The main goal of non-standard problems is to develop students' creative thinking, increase interest in the subject being studied, and lead to the discovery of mathematical facts. This goal cannot be achieved using only standard tasks. Therefore, it is advisable to use non-standard problems more often in mathematics courses, along with standard problems [21].

Therefore, in the system of problems of the school mathematics course, tasks aimed at developing certain mathematical skills are needed. Special exercises are needed to teach students the methods of independent action, general methods of solving problems, to master the methods of scientific cognition of real reality and the methods of productive mental activity used by mathematicians when solving a particular problem. Students can be taught to observe, use analogy, induction, comparison and draw appropriate conclusions by using specially selected non-standard problems, carrying out training aimed at solving problems [22].

In lessons, it is necessary to systematically use non-standard tasks that contribute to the purposeful development of students' logical thinking, their mathematical development, and the formation of cognitive interest and independence. Such tasks require ingenuity, observation, creativity, and originality from schoolchildren.

Non-standard tasks are diverse, but they are united by the following:

- The method for solving non-standard problems is not known in advance. Their solution is characterized by a movement of thought, that is, trial and error leads

to a solution. In the process of searching for a solution, in some cases, guesses may arise that indicate the path to the desired solution.

- non-standard tasks contribute to increasing and maintaining interest in the subject being studied and serve as a motive for students' actions. The emotional reaction of children is determined by the specific nature of the plot, the way the task is presented, thereby encouraging students to find a solution;
- Non-standard tasks are created based on knowledge of the laws of thinking [23].

Thus, the study identified the main features of the development of students' logical thinking in mathematics. It was found that the successful and effective development of students' logical thinking depends on a number of factors, such as the quality of teaching, teaching methods, the level of students' motivation and interest in mathematics, etc.

To develop students' logical thinking, it is important to use a variety of teaching methods, including various tasks, games, puzzles, and other interactive learning methods. It is also important to encourage students to think independently and solve problems, which contributes to the development of their creativity and logical thinking.

Further research in this area could help improve mathematics teaching methods and help students develop logical thinking skills. Given the importance of logical thinking in today's world, this aspect of teaching is becoming increasingly relevant and important for the education and development of every student.

1.3 Structure of logically oriented problems

In mathematics teaching, a system of logical problems is often used. Their solution requires the construction of a clear chain of logical reasoning with correct intermediate and final conclusions. The selected logical problems help the student to consider the condition of the task from different angles, evaluate it and try to solve it in different ways. The success of solving such problems should not depend on the level of knowledge of students and their mastery of the program material [24].

When analyzing the literature, we found that there are various classifications of logical problems. In order to systematize these classifications, it is necessary to analyze the content of their classification headings. The results of the analysis showed that there are 31 classification headings in the existing classifications of logical problems. Let us present these classification headings on the following grounds:

A) plot logic problems:

1) problems with relations: 1.1) problems with transitive relations, 1.2) problems with incorrect conditions, 1.3) problems with non-transitive relations, 1.4) problems with multiple relations, 1.5) problems with equality relations, 1.6) problems for comparing elements in relations;

2) Passing exams;

3) calculations for casting;

4) reports on transitional relationships

5) traffic reports;

6) tasks about liars;

7) tasks for selecting weights to obtain the desired weight;

8) Tournament tasks;

9) tasks - jokes and funny stories;

10) tasks with matches;

11) entertainment tasks: rebuses, dominoes, checkers, combinatorial problems with magic squares, labyrinths, circular counters, genius games, geometric problems, sophisms and paradoxes [25].

B) logical problems on methodological methods of solution:

1) problems that require one specific answer;

2) problems whose conditions prompt the solver to take some action;

3) tasks that require performing some arithmetic operation with given operations;

4) problems whose conditions allow for the "negation" of a semantically correct solution;

5) Problems that need to be solved;

6) recovery reports;

7) problems for finding the number of remainders from division;

8) calculations to determine the truth of statements;

9) problems for predicting numbers;

10) problems solved using tables and diagrams;

11) problems solved using columns;

12) reports for selecting possible options;

14) problems for ordering sets [26].

C) logical problems using logical methods:

2) problems whose solution is based on reasoning;

3) problems for analysis and synthesis;

4) reports for removing excess items;

5) similarity problems;

6) problems for classification by a certain feature [27].

Analyzing the identified classifications of logical problems, it can be noted that the largest number of classifications of 16 headings was allocated by O.B. Bogomolova [28].

The average indicators of the content level of classification headings in the classifications of logical problems (5-10) were studied by the following authors: L.I. Teplova; E.Yu. Levina; F.M. Myazitova; L.V. Zankov; G.B. Polyak; L.M. Likhtarnikov and others.

The following types of problems are often found in the classification of logical problems:

a) By semantic content;

b) On methodological methods of solving;

c) By logical decision methods [29].

Not all authors mention entertainment problems in their classifications. In their classifications, authors very rarely mention the following classification rubrics: problems with relations; problems to determine the truth of statements; problems that impose one specific answer; problems whose conditions prompt the solver to perform any action.

Analyzing the proposed classifications, it was concluded that the classification of logical problems by O.B. Bogomolova is the most complete.

Other considered classifications of logical problems have the following drawback: the content of some classification rubrics overlaps with each other, and the combination of different classification rubrics does not form a complete set of logical problems.

O.B. Bogomolova divided logical problems into 6 sections, classifying them according to their semantic content and methodological methods of solution:

1. Relationship reports;
2. Problems using diagrams and tables.
3. Passing exams.
4. Problems that can be solved using graphs.
5. Tasks for sorting possible options.
6. Interesting tasks.

Analyzing the content of logical problems, it is found that in the textbooks of the mathematics course in the above sections, including the author's program on mathematics edited by G.A. Khamitova, 4836 different mathematical tasks are included, of which 1204 are logical problems [30]. The percentage of logical problems presented by G.A. Khamitova in the textbooks of the elementary mathematics course is 24.9%. With the help of the explanatory texts and the system of tasks of the textbook, students form the following operations of logical thinking: synthesis, analysis and highlighting of the main, comparison, generalization, classification, systematization, definition and explanation of concepts, clarification, proof and refutation, etc.

The author's program for mathematics, developed by T. Aitkazhi, includes 5180 different mathematical tasks. Of these, 802 are logical problems. The percentage of logical problems presented by T. Aitkazhi in the mathematics textbooks for grades 5-6 is 15.48%. T. Aitkazhi's textbook contains various logical problems that contribute to the development of logical problems [31].

Let's dwell on the typology of logical problems in the above-mentioned author's programs and school mathematics courses.

Tasks solved using the "correct reasoning" method

Many logical problems are solved by the method of "correct reasoning". The decision-making process is an analysis of various situations, selection of favorable situations and elimination of unnecessary ones. As a result of the decision, a way out of the difficult situation is found. The method of "correct reasoning" is used in solving problems for passing (the wolf, goat and cabbage problem), in measuring weight, etc. [32].

The wolf, goat, cabbage problem is one of the most popular and frequently asked logic problems in school mathematics courses.

Example: A goat, a wolf, and a cabbage need to be transported across a river. Only one of the three can fit in the carrier's boat. How can they be transported so that the goat does not eat the cabbage and the wolf does not eat the goat?

Solution. We need to consider different options for crossing. If the wolf is transported first, then the goat will eat the cabbage. And if there is cabbage, the wolf will eat the goat. So we need to transport the goat first. Then we transport the wolf, but if we leave it there, then it will eat the goat. So, we need to bring the goat back and transport the cabbage to the other side of the river. Finally, we can transport the goat.

Problems solved using tables.

Often, tables are used in solving logical problems, since tasks can include many conditions. Therefore, students need to create a table. It is created by carefully reading and analyzing the conditions of the task, and then all the information from the task is displayed in the table. This greatly simplifies the solution of the conditions of the problem, and sometimes it is the only way to solve it. With the help of tables, you can solve various problems, for example: correspondence problems between elements of different sets, problems of ordering sets, false statement problems, tournament problems, etc. [33].

1) Tasks to establish correspondences between elements of different sets.

This type of logical problems is associated with the consideration of several finite sets, usually with some dependencies between their elements. The simplest is the case when two sets with the same number of elements are given and it is necessary to establish a correspondence between them. In more complex cases, a larger number of sets are considered, the number of their elements is the same, and it is necessary to establish a correspondence between the elements of each pair of sets. Finally, several finite sets with dependencies between their elements, but not corresponding to each other, are considered. Various tables are used to solve these classes of problems. These tasks are entered into the appropriate cells of the table, for example: a positive result is marked with a “+” sign, and a negative result with a “-” sign. After applying all the conditions of the problem, the empty cells are filled in with a “+” or “-” sign using logical reasoning. If there are more than two sets, then it is necessary to consider several square tables or one rectangular table [34].

An example of two sets:

Task 1. Aizhan, Zhansaya, and Nazerke were asked what grades they received for their math test. The teacher replied: “The three of you have different scores. Aizhan doesn’t have a “3”. Nazerke doesn’t have a “3” or a “5”. Who got what grade?

Solution: The report can be divided into two sets: ratings and names. Each set consists of three elements. These are “3”, “4”, “5”. In the next column of the table, Aizhan, Zhansaya, Nazerke are written. Let's create a table of initial data. Aizhan does not match “3”, which means putting a “-” sign at the intersection of the “Aizhan” column and the “3” row. Nazerke does not match “3” or “5”. Therefore, a “-” sign is put at the intersection of the “Nazerke” column and the “3” and “5” rows. So, we can

see that the “4” sign is suitable for Nazerke, so we put a “+” sign in the corresponding cell. And if “+” is put at the intersection of the “Aizhan” column and the “5” column, then “+” should be put at the intersection of the “Zhansaya” and “3” columns.

2) Tasks with complex statements

Example: The assignment "The Case of Brown, Jones, and Smith." One of them committed a crime. During the investigation, each of them made two statements:

Brown: 1. I am not a criminal. 2. Jones is not a criminal either.

Jones: 1. Brown is not a criminal. 2. The criminal is Smith.

Smith: 1. The criminal is Brown. 2. I am not a criminal.

During the investigation, it was found that one of them lied twice, the second told the truth twice, and the third lied once and told the truth once. Who committed the crime?

Solution: Suppose that both of Brown's statements are true, then neither Jones nor Brown is a criminal, and we indicate this in the table, in the appropriate cells. Then Jones could have lied once and told the truth once, which means that Smith lied twice. From Jones's statement we get: Brown is a criminal and Smith is a criminal, and from Smith's testimony: Brown is not a criminal - he is a criminal. We indicate the obtained data in the appropriate cells of the table:

Criminal Brown - + -

Criminal Jones -

Criminals Smith + +

So, both of them committed the crime at the same time, which is impossible. Let's consider another option.

Brown is not the criminal, Smith is the criminal; Smith lied twice, meaning Brown is not the criminal, Smith is the criminal; then Brown lied and told the truth, meaning he is the criminal, and Jones is not the criminal. Let's show the result in a table:

Criminal Brown + - -

Criminal John -

Criminal Smith ++

The result is similar to the first option. Let's consider the next option.

This time, let's say Jones lies twice, Brown lies and tells the truth, and Smith tells the truth twice. According to Jones, we get: Brown is a criminal, Smith is a criminal. From Brown's testimony: Brown is a criminal, Jones is not a criminal. From Smith's words: Brown is a criminal, he is not a criminal. Let's mark the data in the table:

Criminal Brown + + +

Criminal John –

Criminal Smith - -

So, the criminal came to Brown. Answer: Brown committed the crime [35].

3) tournament reports.

Tournament problems are logical tasks related to determining the results of tournaments. Such tasks are given incomplete data on the results of sports matches. It

is necessary to obtain complete information about the tournaments held through logical reasoning. Solving the tournament problem is facilitated by drawing up the tournament schedule based on the data given in the task conditions, and then based on the data obtained in a logical way. In a football (hockey) tournament, the winning team of the match receives two points. A tie is evaluated as one point for each team, and a defeat is evaluated as zero points. In a football tournament, if the points of two teams are equal, the difference between the goals scored and the goals conceded is taken into account when dividing the places [36].

Based on the above typology of logical problems, the following structures can be formulated:

1. Working with logical expressions: declarative logic formulas, truth tables.
2. Proofs: methods and strategies for proving theorems, mathematical proof.
3. Mathematical operations and their properties: arithmetic operations, distributivity, commutativity, associativity, etc.
4. Mathematical Paradoxes: Fun and unconventional math problems and puzzles.
5. Logical problems: logic and reasoning, division and logical thinking tasks.
6. Set theory: basic concepts, operations on sets, properties of sets, and Euler-Venn diagrams.
7. Algebraic expressions: equations, inequalities, solving systems of equations, graphs of functions.
8. Geometric problems: problems for constructing and solving geometric problems, vertical and angular theorems.
9. Mathematical induction: the principle of induction, proof of sequences and recursive formulas.
10. Complex logical problems: logic, algebra and geometry, analysis and inference problems [37].

So, in conclusion, the main elements of logical problems - condition, question, data, solution - their interrelationships and importance in understanding and solving problems have been examined.

The section also examined different types of logical problems, their classification, and features.

Studying the structure of logical problems in mathematics allows you to improve your analytical, logical thinking, and problem-solving skills. Understanding the specifics of each type of problem helps you solve them more effectively and achieve more accurate and faster results.

To successfully solve logical problems, it is necessary to be able to clearly and precisely formulate the conditions of the problems, analyze the data, formulate the questions correctly, and logically derive the solution. Therefore, knowing the structure of logical problems in mathematics is the key to successfully and efficiently solving them.

2. METHODOLOGY

2.1 Developing students' thinking skills through logical problems

To develop students' thinking skills in mathematics, an elective course "Logical Problems and Games" has been developed for 5th grade students.

The developed elective course allows students to generalize and deepen their knowledge of mathematics within the program, to see the uniqueness and high abstraction of mathematical objects. On the other hand, logical problems show great potential for application in life, practical work, and everyday life, teaching them to use logic in finding the right solution and solving various problems, including everyday problems.

The program is based on solving logical problems and games for 5th graders and is organized into 34 hours (1 hour per week).

Goal: to develop general logical thinking skills, increase the student's interest in the subject, and create conditions for the development of problem-solving skills.

Responsibilities:

- To learn how to use the acquired knowledge in practice;
- Develop inquisitive and creative abilities;
- Mastering certain systems of thinking skills and skills;
- Organization of research and investigation work;
- Learn to solve logical problems using various methods;
- To demonstrate their practical importance in calculations;

Requirements for mastering the subject:

The student should know:

- Knowledge of how to solve logical questions and problems quickly and efficiently;
- Ability to construct equations to solve text problems;

The student should learn:

- Ability to think logically;
- A quick way to square some numbers;
- Solving logical problems using equations;
- Be able to use the knowledge gained in solving practical problems.

Elective course content:

"Fun" mathematics: "Magic" figures. Riddles, puzzles, crosswords. Mathematical tricks and sophisms. Entertainment account. Mathematical games.

Final lesson on the topic: counseling

Cutting problems: simple geometric shapes. Cutting problems.

Cutting tasks: completing the picture on the template. Tangram puzzle game. Laboratory work.

Logical problems: the concepts of "true and false", "negation". Contradictory words. Statements that contain both truth and falsehood at the same time. Solving logical problems by negating assumptions. Tasks for casting. Measurement tasks.

Logical problems solved using tables. Problems for dividing numbers. Problems solved using graphs. Game tasks. Combinatorial problems. Final lesson.

Interesting fraction problems: old fraction problems. Collaborative work tasks.

Olympic tasks: Solving Olympic tasks. Solving tasks of the mathematical game "Kangaroo".

The final lesson is a mini-Olympiad.

The program allows you to achieve the following results of mastering the educational program of basic general education:

Personal results:

The individual results of mastering the program of the elective course "Logical Problems and Games" are described:

Patriotic education: interest in the past and present of Kazakhstani mathematics, appreciation of the achievements of the Kazakh school of mathematics, and the application of these achievements in other sciences and applied fields.

Civic and spiritual and moral education: readiness to fulfill the duties of a citizen and exercise his rights, understanding of the mathematical foundations of the functioning of various structures, phenomena, and procedures of civil society (elections, surveys, etc.).

Labor education: active participation in solving practical problems in a mathematical direction, understanding the importance of lifelong mathematical education for successful professional activity and developing the necessary skills; conscious choice and creation of a personal educational trajectory and life plans, taking into account personal interests and social needs.

Aesthetic education: the ability to emotionally and aesthetically perceive mathematical objects, problems, solutions, and reasoning; the ability to see mathematical patterns in art.

The values of scientific knowledge: oriented towards the modern system of scientific ideas about the basic laws of the development of man, nature and society, understanding mathematical science as a sphere of human activity, its stages of development and its importance for the development of civilization; mastering the language of mathematics and mathematical culture as a means of knowing the world; mastering simple skills of research activity.

Physical education, health culture and emotional well-being: readiness to use mathematical knowledge in the interests of one's own health, maintaining a healthy lifestyle (proper nutrition, balanced mode of classes and rest, regular physical activity); formation of reflection skills, recognition of the right to make mistakes and the equal rights of another person.

Environmental education: orientation towards the use of mathematical knowledge to solve problems in the field of environmental protection, plan actions and assess their possible consequences for the environment; understanding the global nature of environmental problems and ways to solve them.

Individual outcomes that ensure the learner's adaptation to changing conditions of the social and natural environment:

- increase one's level of competence through practical activities, including the opportunity to learn from others and gain new knowledge, skills and competencies from the experience of others in collaborative activities;
- the need to form new knowledge, including the formulation of ideas, concepts, hypotheses about objects and phenomena, and to plan one's own development;
- The ability to understand a stressful situation, perceive a stressful situation as a challenge requiring countermeasures, correct decisions and actions taken, formulate and assess risks and consequences, and build experience.

Meta-disciplinary results:

The meta-disciplinary results of mastering the program of the elective course "Logical Problems and Games" are characterized by the mastery of universal cognitive actions, universal communicative actions, and universal regulatory actions.

1) Universal cognitive activities ensure the formation of basic cognitive processes of students (mastering methods of cognition of the surrounding world; using logical, research operations, skills in working with information). Basic logical activities:

- identify and describe the important features of mathematical objects, concepts, and relationships between concepts; formulate definitions of concepts; establish the important features of classification, the foundations of generalization and comparison, and the criteria for the analysis carried out;
- perception, formulation and transformation of judgments: positive and negative, unity, individual and general; conditional;
- identify mathematical patterns, relationships, and contradictions in facts, data, observations, and statements; propose criteria for identifying patterns and contradictions;
- draw conclusions using the laws of logic, deductive and inductive reasoning, and reasoning by analogy;
- analyze proofs of mathematical statements (direct and indirect), independently conduct simple proofs of mathematical facts, construct arguments, give examples and counterexamples; justify one's own ideas;
- choosing a method for solving a learning problem (comparing several solution options, choosing the most suitable one, taking into account independently allocated criteria).

Main research activities:

- using questions as a tool for researching knowledge; formulating questions that identify contradictions and problems, independently determining what is necessary and given, forming hypotheses, and proving one's position and opinion;
- Conduct a simple experiment according to an independently developed plan, a small study to determine the features of a mathematical object, and the interdependence of objects;

- independently formulate conclusions based on the results of the conducted observations and research, evaluate the correctness of the results, conclusions and conclusions obtained;
- predicting the possible development of the process, as well as making predictions about its development in new conditions.

Working with information:

- determine the insufficiency and redundancy of information and data necessary to solve the problem;
- choose a form of information presentation and illustrate the problems to be solved with schemes, diagrams, other graphics and their combinations;
- Assess the reliability of information according to criteria proposed by the teacher or formulated independently.

2) Universal communicative activities ensure the formation of social skills of learners.

Contact:

- accept and formulate judgments in accordance with the conditions and goals of communication; express one's point of view clearly, accurately, and competently in oral and written texts, provide explanations in the process of solving a problem, and explain the results obtained;
- During the discussion, ask questions about the essence of the topic, problem, or task being discussed, and express ideas aimed at finding a solution; compare their opinions with the opinions of other participants in the dialogue, identify differences and similarities in positions; formulate disagreements and their objections in the correct form;
- present the results of solving a problem, experiment, research, project; independently choose a speech format, taking into account the presentation objectives and the characteristics of the audience.

Collaboration:

- understand and use the benefits of teamwork and individual work in solving educational mathematical problems;
- Accepting the goal of joint activities, planning the organization of joint work, dividing the types of work, negotiating, discussing the work process and results; summarizing the opinions of several people;
- participation in group work (discussion, exchange of ideas, brainstorming, etc.);
- performing their part of the work and coordinating with other team members;
- assessing the quality of their contribution to the overall product according to criteria formulated by the interaction participants.

3) universal regulatory actions ensure the formation of a person's semantic views and life skills.

Self-organization: independently creating a plan, an algorithm for solving a problem (or part of it), choosing a decision-making method taking into account available resources and one's own capabilities, justifying and correcting solution options taking into account new information.

Self control:

- mastering methods of self-checking and self-monitoring the process and results of solving mathematical problems;
- Anticipate possible difficulties in solving the problem, make adjustments to the activity based on new circumstances, errors found, and identified difficulties;
- Assessing the compliance of the results of the activity with the set goals and conditions, explaining the reasons for achieving or not achieving the goal, finding errors, and evaluating the experience gained.

Subject results

After completing the elective course "Logical Problems and Games", the student will be able to:

- use non-standard methods for solving various mathematical problems;
- use of logical methods used in solving problems;
- logical reasoning in solving text-based arithmetic problems;
- using the methods learned to solve Olympiad problems;
- new methods of oral reporting;
- working with crosswords and rebuses;
- reasoning when solving logical problems, problems for ingenuity, problems for erudition and intuition;
- systematize data in a table format when solving problems, creating mathematical crosswords, and rebuses;
- Using non-standard methods to solve problems
- applying theoretical knowledge in solving problems;
- Identifying logical fallacies found in various statements in proof and refutation.
- Solving logical problems and entertainment problems based on the theoretical material of the science of logic.

The student will have the opportunity to:

- Under the guidance of a teacher, identify and state the simplest general rules of behavior (ethical norms) in cooperation for all people;
- Choosing how to act, based on ethical norms, in situations of communication and cooperation presented by the teacher, with the support of other participants in the group and the teacher.
- with the help of the teacher, determine and formulate the purpose of the activity; state the sequence of actions;
- learn to express one's own hypothesis (version); learn to work according to the plan proposed by the teacher;
- learning to distinguish a correctly performed task from a non-performed one;
- Emotionally evaluate the work of fellow students in collaboration with the teacher and other students.
- orientation in the education system: distinguishing between what is new and what is already known with the help of a teacher;
- learning to acquire new knowledge: finding answers to questions using life experience, information received from the teacher, and educational literature;

- learning to master measuring instruments.
- learning to express oneself;
- Mastering the skills of cooperation in a group in jointly solving educational tasks.

Table 2.1.1 Thematic plan

No.	Subject	Teacher's work, taking into account the work program	Number of hours
	"Fun" mathematics	to draw the attention of schoolchildren to the value aspect of the phenomena studied in the lesson, to organize work with socially significant information received in the lesson - to initiate its discussion, to express students' opinions on it, to develop their attitude towards it; to implement educational opportunities for various types of child activity (study, play, labor, sports, art, etc.); implementation of modern, including interactive, forms and methods of educational work both in the classroom and in extracurricular activities; establishing a trusting relationship between the teacher and his/her students, promoting a positive perception of the teacher's demands and requests by students, focusing their attention on the information being discussed in class, and increasing their cognitive activity; to encourage schoolchildren to observe generally accepted norms of behavior in the	6
1	"Magic" figures.		1
2	Riddles, rebuses, crosswords.		1
3	Mathematical tricks and sophisms.		1
4	Entertainment account.		1
5	Mathematical games.		1
6	Final lesson on the topic:		1
	Cutting tasks		5
7	Simple geometric shapes		1
8	Cutting tasks.		1
9	Cutting tasks		1
10	Finish the image in the template		1
11	Laboratory work		1
	Logical problems		13
13	The concept of "true and false"		1
14	Contradictory statements. Statements that contain both truth and lies at the same time.		1
15	Solving logical problems by refuting assumptions.		1
16	Problems to be solved later.		1
17	Casting tasks.		1
18	Measurement tasks.		1

19	Logical problems to solve	classroom, rules of communication with adults (teachers) and peers (students), principles of study discipline and self-organization.	1
20	Problems for dividing numbers.		1
21	Problems on Dirichlet's principle.		1
22	Problems solved using graphs.		1
23	Game tasks.		1
24	Combinatorial problems.		1
25	Final lesson (game)		1
	Fun tasks in parts		4
26	Old fraction problems		1
27	Old fraction problems		1
28	Joint work reports		1
29	Joint work reports		1
	Olympic reports		5
30	Solving Olympiad tasks.		1
31	Solving mathematical problems		1
32	Solving mathematical problems		1
33	Final lesson		1
34	Final lesson. Mini Olympiad.		1

Example tasks:

"Transitions".

A type of story task. Example of a task:

1. A wolf, a goat, and a cabbage. A farmer has a boat on the riverbank, and next to him are a wolf, a goat, and a cabbage. The farmer himself must cross the river and transport the wolf, goat, and cabbage to the other side. However, in addition to the farmer, either only a wolf, or only a goat, or only cabbage is loaded onto the boat. You cannot leave a wolf with a goat or a goat with cabbage unattended - the wolf may eat the goat, and the goat may eat the cabbage. How should the farmer transport the wolf, goat, and cabbage?

2. Two soldiers approached a river, where two children were sailing in a boat. If the boat could only hold one soldier or two children, and one soldier and one child could not, how would the sailors cross to the other side?

3. Tourists (a father, mother, and two twin children) need to cross a river. They have a small boat that can only hold one adult or two children. If both the adults and

the children know how to row, how can they organize the fastest crossing? If each trip across the river (one way) takes 10 minutes, how long will it take to cross?

4. Three travelers - Arman, Meiirkhan and Olzhas - had to cross a river from one bank to the opposite bank in a boat weighing no more than 100 kg. Arman weighed 54 kg, Olzhas 46 kg. but Meiirkhan weighed about 70 kg. What was the most rational action to take to cross the river?

5. Three merchants and three pirates came to the river at the same time. They all had to cross to the opposite bank. There was a boat on the bank that could only carry two people. The merchants were afraid of the pirates, because they might rob the merchants while crossing. If the number of merchants and pirates on the bank was the same when crossing the river, the pirates would not be able to touch the merchants. If the number of pirates was greater than the merchants, the pirates would attack the merchants. How could the merchants and pirates cross the river to the other bank? How many trips did the boat make?

Numerical puzzles.

Riddles about numbers. Numerical riddle conditions. Types of puzzles. Rules for restoring the numerical rebus record. Discussion of solving numerical riddles.

In most cases, the presented rebuses should have several correct decodings, which allows you to solve the problems by choosing. In this case, it is recommended to work on each task at two levels:

- find some solutions, find as many solutions as possible,
- find all solutions and prove that there are no other solutions.

In the second case, to prove correctly, it is usually necessary to analyze all the cases in the branched logical scheme.

Mathematical puzzles are a convenient object for teaching students to perform fairly complex (labor-intensive) logical thinking, which requires analyzing all possible situations.

The vast majority of problems that arise in practice can be solved in different ways. Mathematical puzzles can be used as a warm-up in class.

Tasks:

1. How many natural numbers are there less than 100:

a. Equally divisible by 2 and 3;

b. is divisible by 2, but not by 3;

c. is divisible by 3 but not by 2;

d. is divisible by 3 or divisible by 2. (It is divisible by at least one of these two numbers);

d. is not divisible by either 2 or 3;

2. I thought of a number, added 16 to it, and subtracted 7 from the resulting number. Then I subtracted the largest single-digit number from it, added 1 to it, and got 5. What number did I think of?

3. I thought of a number, added 17 to it, multiplied the result by 5 and got 95. What number did I think of?

4. I thought of two numbers, the sum of which is equal to the smallest four-digit number. If we divide the larger of these two numbers by the smaller, we get the largest single-digit number. What number did I think of?

5. Can you express the number 1000 using the same eight digits? In this problem, you can use multiplication and division symbols with digits.

6. It is easy to express the number 24 using the digit 8 three times: 8 + 8 + 8 = 24. Can you express the number 24 by taking another digit in place of the number 8 and using it three times? (There are several solutions to this problem.)

Geometry: cutting tasks.

Many scientists have argued that cutting problems have been important since ancient times. Solutions to many cutting problems were found with the ancient Greeks and Chinese. The first systematic treatise on this topic belongs to the Persian astrologer Abu al-Wafa Peru in the 10th century. Geometers began to seriously address the problems of cutting figures into the minimum number of parts and then constructing a certain new figure in the 20th century, primarily because there is no universal method for solving such problems, and each person solving them can fully demonstrate his ingenuity, intuition and ability. Given that deep knowledge of geometry is not required, amateurs can sometimes even surpass professional mathematicians.

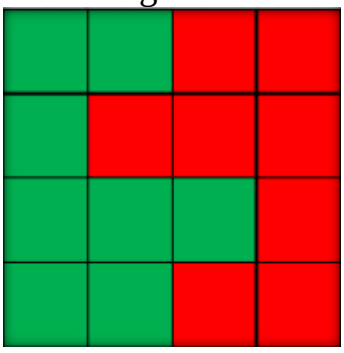
Cutting tasks help students form geometric representations of various materials as early as possible. When solving such problems, a sense of beauty, law, and order in nature emerges.

At the first stage, it is advisable to consider the tasks on checked paper. Tasks that involve cutting out shapes (mainly squares and rectangles) along the sides of the cells.

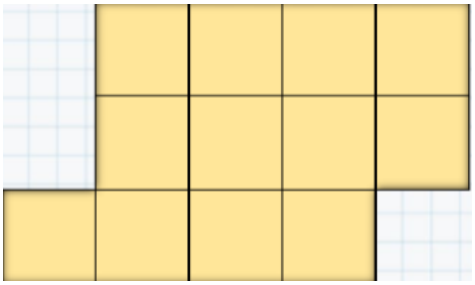
Next, problems related to pentomino figures can be considered. Pentominoes, originally (from another Greek. πέντα five, and domino) are five-cell polynomials, that is, flat figures, each of which consists of five identical squares connected to each other by edges,

Task examples:

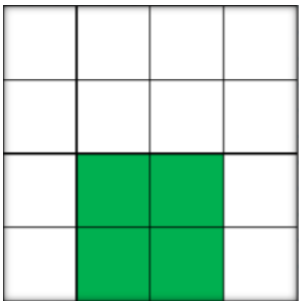
1. The figure shows a method of cutting 4 cells, the sides of the cells into 2 equal parts. Consider 5 other methods of cutting.



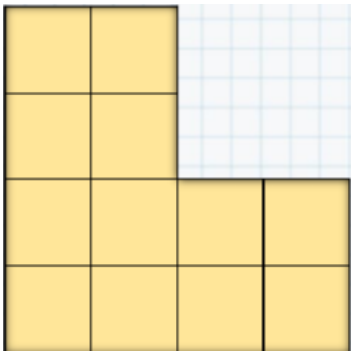
2. Cut into 2 equal pieces.



3. A square consists of 16 cells, four of which are colored. Cut it into 4 equal parts so that each cell has only one colored cell.



4. A local land dealer bought a plot of land of unusual shape for this occasion. He expected to sell it profitably in parts, but each of the eight buyers wanted a plot of land no less than his neighbor. Where should the dealer install a dividing hedge to obtain eight equal plots?



Intersection and union of sets. Euler circles.

The concept of a set, the intersection of sets, or their union. Euler circles are considered as a geometric scheme that can be used to depict relationships between subsets for the purpose of visual representation.

This topic is closely related to set algebra. The use of Euler circles gives clarity and simplicity to set algebra problems. Euler circles are successfully used in logical problems to represent the truth set of statements and in many other cases. Representing the condition of the problem using Euler circles usually simplifies the way to solve it.

This topic can be a good reason to talk to students about the life and work of Leonhard Euler and his works.

Examples of tasks:

1. Some children in our class like to go to the cinema. 15 children have seen the film "Inhabitable Island", 11 children have seen the film "Stylyagi", and 6 of them have

seen both the film "Inhabitable Island" and the film "Stylyagi". How many people have seen only the film "Stylyagi"?

2. There were 26 magical books on the shelf, all of which were read. 4 of which were read by both Harry Potter and Ron. Hermione read 7 books that neither Harry Potter nor Ron had read, and two books that Harry Potter had read. In total, Harry Potter read 11 books. How many books did Ron read alone?

Poisson's calculations (casting tasks).

One of the most famous transfusion problems is the problem of the famous French mathematician and physicist Simeon Denis Poisson. This topic deals with the solution of transfusion problems using various methods. The essence of these problems is as follows: having several vessels of different volumes, one of which is filled with a liquid, it is necessary to divide it in some way or pour part of it using other vessels for the minimum number of transfusions. In transfusion problems, it is necessary to indicate a sequence of actions in which the required transfusion is carried out and all the conditions of the task are met.

Simple and interesting examples of solving "casting" problems allow us to consider such important concepts as "command", "flow chart", "program". By solving problems, students learn to model simple algorithms. Solving problems of this cycle requires ingenuity, develops combinatorial thinking.

At the beginning of the lesson, it is necessary to summarize Poisson's calculations and tell his history. It is necessary to start solving the problem with simple, understandable tasks.

Task examples:

1. How can you use 3L and 5L jars to fill exactly 1L of water?

dishes	Casting			
5 liters	-	3	3	5
3 liters	3	-	3	**1**

2. How can you measure 4 liters of water using 3-liter and 5-liter containers?

dishes	Casting			
5 liters	-	3	3	5
3 liters	3	-	3	1

3. You need to bring 3 liters of water with a 5 liter and a 9 liter container.

4. A 10-liter canister is filled with kerosene. There are also 7-liter and 2-liter empty canisters. How can you pour kerosene into two canisters, each with 5 liters?

5. How can 12 liters of fermented drink in a 12-liter container be divided equally between two families with 8-liter and 3-liter containers?

6. One of the 8-liter, 5-liter, and 3-liter containers is filled with water. How can you fill the 2 containers so that each contains 4 liters of water?

7. How can you draw 13 liters of water from a water pipe using a 17-liter and a 5-liter container?

8. How can you get b liters of water from a water pipe using 4-liter and 9-liter containers?

9. How can you bring 6 liters of water using a 5-liter and a 7-liter bucket?

10. The first 8-liter container is filled with water. There are also two empty containers of 5 liters and 3 liters. How do we measure 1 liter of water using these containers?

11. The first 12-liter container is full of water. The 5-liter and 8-liter containers are empty. How can the water be divided into two equal parts?

10. How can you get 4 liters of water from a 3-liter and a 5-liter container?

Geometry: Mobius sheet.

The mysterious and famous Möbius strip (sometimes called the "Möbius strip") was invented in 1858 by the German geometer August Ferdinand Möbius, a student of the "king of mathematicians" Gauss. Historical essay on Möbius. A few words about topology. Möbius strip as a geometric object. Properties of the Möbius strip. One-sidedness. Continuity. Connection. Orientation. Riddles of the Möbius strip. Applications of the Möbius strip in life. Experiments with the Möbius strip.

It is very easy to make a Möbius strip, hold it in your hand, cut it, and do various experiments with it. Studying the Möbius strip is a good introduction to the elements of topology.

It is useful to prepare a sufficient number of strips of paper for the lesson, which students will work with (conduct experiments on). Strips that are 5 times longer than their width are best.

Examples of experiments:

1. If you start coloring a Mobius strip from one side without going all the way to the edge, what part of the strip will be colored?

2. What happens to a regular ring if you cut it in the middle?

3. What if the Möbius strip is cut in the middle (i.e. into 2 strips)? What is the result of cutting the Möbius strip into 3 strips?

Interesting tasks on percentages.

The concept of percentage. Finding the percentage of a number and its percentage. Examples of tasks:

1. The age of the elder brother is 40% of the age of the younger sister. What percentage is the age of the younger brother?

2. The moisture content of the watermelon was 99%. As a result of long-term storage, the moisture content decreased to 98%. How did the moisture content of the watermelon change?

Introduction to logic: "all", "some", negation

What does logic study? Historical sketch. Concept, reasoning, conclusion.

Statement. Negation as a logical operation. Quantifier.

Logical reasoning skills are important for everyone, not just for a select few. Although the entire school mathematics curriculum is saturated with logical ideas, the most important or unique methods of logical reasoning require special attention.

The topic is designed to form negative statements using the words “all” and “some.” In the language of mathematics, “all” corresponds to a general quantifier, and “some” to a specific quantifier.

Examples of tasks:

1. Say the same thing in a different way:

a) It is not true that all mammals live on land.

b) It is not true that the number 5 is divisible by 2.

c) It is not right for some fish to fly.

2. Construct the negation of the sentences using the word incorrectly and in a simple way.

a) It will be sunny today.

b) All dogs love cats.

c) Chicken is poultry.

d) Snow always melts in the spring.

e) 150 is less than 200.

d) Mathematics is an exact science.

3. Make your own propositions and construct their negation. Prove that the statement is false and construct its negation:

a) The number 0 is a natural number.

b) There are no natural numbers between 4 and 5.

c) The improper fraction is less than one.

Sum and arithmetic mean.

The concepts of "arithmetic mean", derivation of the corresponding formulas, study of the concepts of "average speed" and "average mass" and methods for finding them; ability to apply knowledge in practical tasks; consolidation of arithmetic operations with decimal fractions.

Examples of tasks:

1. A man walked at a speed of 4.6 km/h for 2 hours and at a speed of 5.1 km/h for 3 hours. At what constant speed would he have to walk to cover the same distance in the same time?

2. Ali Baizhanov's math grades in the journal: 4 5 3 4 5 4 3 3 4. What grade do you think Ali will get in the quarter? Why?

3. Mercy, Happiness and Beauty went up the mountain. They approached the forest and decided to take a break. Mercy had 2 cakes, Happiness had 4, and Beauty had 6. The children ate all the cakes equally. How many cakes did each person eat?

Paired problems (exchange, pairing).

The concept of a pair. Application of the idea of a pair: known statements. A pair of the sum and difference of several numbers. The idea of "pairing".

Problems using the concept of pairs are very common. Therefore, it is worth introducing schoolchildren to methods for solving these problems. The tasks are naturally divided into three cycles:

1. Pairing.

If the objects are divided into pairs, then their number will be even. Therefore, if several pairs are formed from an odd number of objects, then at least one object will remain unpaired. To solve such problems, it is necessary to see which pairs are actually divided in each case.

2. Replacement.

If a chain consists of adjacent objects of two different types, then all even positions will contain objects of one type, and all odd positions will contain objects of another type. The conclusion from this is that if the length of the chain is odd and the objects of both types are the same, then the length of the chain is even.

3. Even is odd.

The derivation of the equations is based on a simple observation: the sum of an even number of odd numbers is even. Generalizing this fact: the parity of the sum of several numbers depends only on the parity of the number of odd terms: if the number of odd terms is (odd) even, then the sum is (odd) even.

Examples of tasks:

1. Boys and girls are sitting at a round table. Prove that the number of adjacent pairs of opposite sexes is even.

2. The chess knight left square a1 and returned to it after several moves. Prove that it made an even number of moves.

3. There are three pucks on a hockey rink, A, B, and C. He does this 1999 times. Can all the pucks remain in their original positions after this?

4. A closed path is drawn on a checkerboard paper along the grid lines. Can its length be 1999? And its length be 2000?

5. All the dominoes were placed in a chain according to the rules. One end had 5 points. How many points were there on the other end?

6. There are 25 checkers on a 25 × 25 board and their arrangement is symmetrical about the diagonal. Prove that one of the checkers is located on the diagonal.

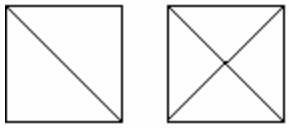
"Circular roads".

Examples of tasks.

1. Place 6 points on a plane and connect them with non-intersecting lines so that 4 lines emerge from each point.

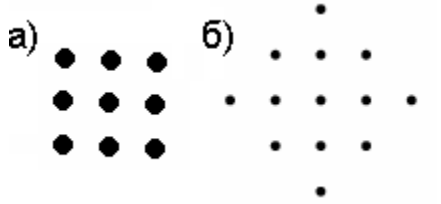
Draw 6 lines and mark 7 points on them, so that each line has three of the marked points.

An avant-garde artist painted “The Outline of a Square and Its Diagonals.” Could he paint his own picture without lifting the pencil from the paper or drawing any line twice? What if his painting was called “The Outline of a Square and Its Diagonals”?



2. a) Draw four lines through the 9 points shown in the image on the left, without lifting the pencil from the paper and without drawing a line twice.

b) Circle the 13 dots in the image on the right, in five sections, without lifting the pencil from the paper and without drawing any lines twice.



3. A pedestrian walks around six streets in a city, walking down each one twice. Can he walk around all the streets, walking down each street once?

a) 20 teams played a tournament according to the Olympic system (two teams meet, the winner plays further, the loser is eliminated). How many matches were played?

b) What if the tournament is played in a single round robin format? (each team plays each team once).

3. Dima from Vrunland said that there are several lakes here connected by rivers. Three rivers flow from each lake, and four rivers flow into each lake. Prove him wrong.

Matchstick puzzles

Task examples:

Construct six identical squares using 1.12 squares.

a) You need to create 3 identical squares by swapping the positions of three matchsticks.

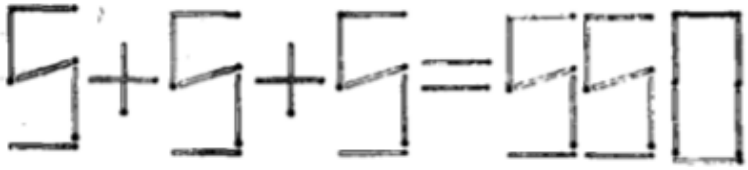
b) Remove 2 matches from the given figure to make 3 matches and 2 matches.

2. How do we construct a square and four triangles using 8 matchsticks?

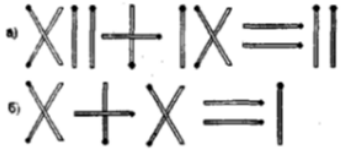
3. Using 12 matches, construct a square and eight triangles.

4. How can you construct six squares and 24 triangles using 36 matchsticks?

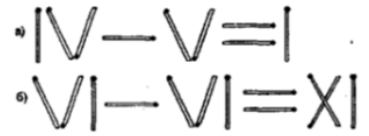
5. Achieve a direct tie by using 1 additional match.



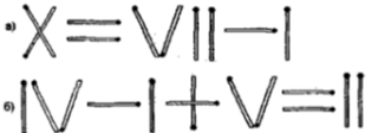
6. From the given incorrect equations, find the correct equation by replacing 1 matchstick:



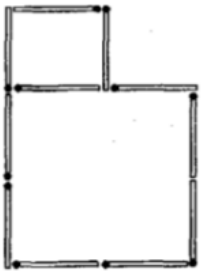
7. From the given incorrect equations, find the correct equation by replacing 1 matchstick:



8. From the given improper equations, find a correct one by changing 1 term:



9. If you rearrange 3 matchsticks correctly, you can make 5 instead of 2.



10. There are 9 matches on the table. Arrange them so that each row has a) 4; b) 2:

Kazakh black accounts

1. Cattle exchange

A shepherd decides to sell his livestock, so he goes to a villager who has a camel, a horse, and a cow and tells him his decision. The villager thinks for a moment and says: "For every leg of my donkey, I will give two sheep, for every leg of my mare, I will give three sheep, and for every leg of my cow, I will give one sheep." According to the agreement, the farmer drives the sheep, rides the horse he bought in exchange, and returns home, leading the donkey and the cow. How many sheep did the shepherd give in exchange in total?

2. How about three?

A father sent his 7-year-old son to the market, gave him 1 tenge and told him to buy 3 things. The boy went to the market and bought 1 melon.

"My son, what you brought is a melon," said his father.

"It's not just one thing, it's three things," says the son.

"How come there are three?" his father asked in surprise.

What did the son answer his father's question?

3. How many people eat meat?

Someone tells the neighbors that a butcher is slaughtering a horse. Buyers come from all over. He counts the people standing around the slaughtered animal with his eyes and says, "If I give each of them one bone, there will be three bones of meat left over. If I sell them two bones, there won't be enough meat for three people. Now how am I going to make ends meet?" He thinks for a moment. How many people have come to buy meat?

4. How much of each food?

One day, a father who wanted to teach his seven-year-old son arithmetic said: "In the next seven years, if our camels double, our horses triple, our cows quadruple, and our sheep five times, then the total number of them will be twice as many as you are now." At that time, how many horses, camels, cows, and sheep will there be in our house? The boy looked at his father and asked. How many camels, horses, cows, and sheep does this house have?

5. Serving a plate

Tattibek was a hospitable man. One day, a group of guests came to his house. Meat was being put in the cauldron. When the food was cooked and the plates were being served, the owner of the house came to his wife who was at the hearth and said, "If we serve one plate for two, one plate of meat will be missing, and if we serve one plate for three, one plate of meat will be left over. What should we do now?" He asked. How many guests are there in this house, and how many plates are there?

6. The meeting in Chapter Nine

In the lowlands of the Talas River, nine roads meet one after another. That is why the junction of the nine roads is called the Nine Branches. One year, nine hunters came to the well and shook hands with each other. How many times did we all shake hands? - one of them asked. Who knows, but I myself shook hands with eight people, - said the second traveler. How many times did the nine travelers shake hands with each other?

Repetition. Mathematical competition.

After the end of the lesson cycle, a general lesson is held, during which the studied material is reviewed, and also some kind of mathematical competition is held, which is most suitable for organizing work with students engaged in extracurricular activities. This can be a mathematical KVN, a mathematical auction, a mathematical regatta, a station game, mathematical hockey, mathematical lotto, brainstorming and other types of work.

The final Olympiad in mathematics is held as a form of a final lesson on mastering the program, which determines the objective level of knowledge and skills of students obtained as a result of participation in extracurricular activities. The event is held according to the principles of holding a classical Olympiad in mathematics. The work version is created by the teacher. The work includes tasks that are discussed in extracurricular classes.

2.2 Developing students' logical thinking skills through play

During the development and implementation of the elective course "Logical Problems and Games", games were held to develop students' logical thinking skills. The games were developed in accordance with the topics of the elective course. Based on the games, students' interest in the subject increased.

Let's look at examples of games held during the elective course.

1. Fun Math Club

Objectives:

- Develop logical thinking skills, reveal the child's talent
- Show that mathematics is an interesting subject
- Uniting the class team, getting to know children from other classes
- Develop ingenuity and creative thinking.

Preparatory work:

- Place 3 tables where teams will sit;
- Assign teams to prepare an emblem, theme, slogan, and greeting;
- Summoning a jury;
- Preparing maps for teams;
- Preparing sheets for competitions.

Plan:

1. Welcoming the teams
2. Jury recommendation
3. Warm-up exercises:
4. Oral report
5. Tangram
6. Captains' Competition
7. A task for fans
8. Conclusion, awards

I. Greetings

Name, motto, emblem. Jury recommendation.

II. Warm-up exercises

- Number order (2, 3, 4, 5, 6, 8, 9)
- Rebuses
- Solving expressions

1) 48+a–34	9) 276−(182+a)
2) b–92–19	10) 1903+1572+b−15
3) 95–(b–18)	11) 38−a−17
4) (86+m) −49	12) 5903−b−1572
5) 42+a+19+29	13) a+83−17
6) (166+x) −99+17	14) 101−b+38−5
7) y− 56−19−37	15) (139+m) −115
8) 286−a−172	

III. Oral report

Examples:

2*25*9*5*11*4

11*25*5*8*4*2

7*2*4*25*5*11

* – this is any arithmetic operation at the discretion of the teacher or presenter.

IV. Tangram

Answer the questions immediately without thinking.

1. In a running race, you overtook a strong runner. What position are you in?

2. You have passed the last runner. What position are you in?

3. Mary's father has 5 daughters: Chacha, Cheche, Chichi, Chocho. What is the name of the fifth daughter?

4. Think of 1000, add 40 to it. Add another 1000 to it. Add 30 to it. Add another 1000, add 20 to it. Add another 1000 to it. What is the answer?

V. Captains' Competition

Solve the equation while singing the song.

33−(18+x)=10

a−24−52=19

(51+b)−4=80

44+(a−59)=104

x+17−22=64

33−11−b=15

Conclusion, awards

2. A happy event

Objectives:

- Summarizing students' knowledge of the reading material they have read
- Cultivating interest in mathematics
- Develop attention, ingenuity, resourcefulness
- Fostering friendship, camaraderie, and teamwork

Preparation: Teams must prepare two questions for the opposing team and choose a captain.

Organizational moment: We gathered today to hold a mathematical "happy event" among 5th graders.

Now let's move on to the game.

I. Welcoming the teams

The teams take turns introducing themselves, displaying their emblems, and choosing a team captain.

II. Warm-up exercises

Questions for the first team	Questions for the second team
1. How many grades are there in a class? (3)	1. The smallest natural number. (1)
2. The formula for the area of a rectangle with sides a and b. (s=a×b)	2. What is the volume of the cube?
3. What is the unit of speed at sea? (Knot)	3. How to find an unknown divisor?

4. How to find an unknown dividend?	4. Is it possible to get the answer "0" when dividing? (yes)
5. Is it possible to get the answer "0" when multiplying? (yes)	5. The number (root) that makes an equation true
6. What is the volume of 1 kg of water? (1 liter)	6. How long is ¼ of an hour? (15 min)
7. What is the product of 13*25*0*0.1? (0)	7. A segment (radius) connecting a point on a circle to the center
8. Which is lighter: 1 kg of cotton or 1 kg of iron? (equal)	8. A girl was traveling to Astana. Three girls were traveling to Astana. How many people went to Astana? (one girl)
9. Three horses are galloping. Each horse has galloped 5 km. How many kilometers has the rider traveled? (5 km.)	9. How to find the unknown subtraction?
10. The area of a square is 36 cm^2. What is the length of the side of the square? (6 cm.)	10. The area of a square is 100 cm^2. What is its perimeter? (40 cm)
11. Which is larger: 34×54 or 43×55? (43×55)	11. Which is greater, 18*51 or 51-17? (18*51)
12. What is the one word for the sum of the lengths of all the sides of a polygon? (Perimeter)	12. There are two containers. The volume of the first container is 1 liter, the second is 10 dm^3. Which container has the smaller capacity? (the first)

III. The game "Tasks-confusions"

1. Lilies grew in the lake. Every day their number doubled, and on the 20th day the entire lake was covered with lilies. How many days did it take for the flowers to grow to half the lake? (2 points for the correct answer)

2. Two residents of two villages located 16 km apart set out to meet each other at 9 am and move at a speed of 8 km/h. If it is known that the residents returned to their villages at 12 pm, how many hours did they spend together? (3 points, answer: 2 hours)

3. A car driver set off from city A to village B. He drove for 3 hours at a speed of 80 km/h and got a puncture on a curved rail. A cyclist set off from village B to city A. He drove for 3 hours at a speed of 16 km/h and got a puncture on the same rail. Find the distance between city A and village B (3 points, answer: 288km)

4. Korkem and Tansholpan cut an apple in half and saw that a worm had eaten half of the apple. Tansholpan cut the apple in half from his part and removed the part eaten by the worm. Korkem did the same. What part of the apple did each worm get? (2 points, answer: ¼).

IV. The game "You to me, I to you"

Teams ask each other 5 questions. If a team finds it difficult to answer, the team that asked the question answers. For each correct answer, the team is awarded 1 point.

1. The name of a citizen's father is Daniyar Saginyshuvly, and the name of this citizen's son is Zhanserik Kuandykuly. What is the citizen's name? (Kuandyk Daniyaruvly)

2. The doctor prescribed 5 pills to Aikörkem, telling him to take each one 20 minutes apart. How long do these pills last? (1 hour 20 minutes)

3. The age of the grandfather is expressed by the smallest three-digit number written in different numbers. How old is the grandfather? (102 years)

4. There were 5 candles burning. Two of them were extinguished. How many candles are left? (5)

5. There were 10 birds sitting on a tree. The hunter shot one bird. How many birds were left on the tree? (There were none left)

6. A tailor has a piece of fabric measuring 16 m. He cuts 2 m every day. On what day will he cut the last piece? (7 days)

V. "Solve the crossword" game

Each team is invited to solve the crossword puzzle. 2 points for each guessed word. The competition ends when one of the teams solves the entire crossword puzzle.

Questions for the first team: 1. Unit of area. 2. Place occupied by a number in the notation of a number. 3. Distance between the ends of lines. 4. Line connecting a point of a circle with the center. 5. Two-digit number. 7. Rectangle. 8. Four-digit number. 9. The science of numbers.

Questions for the second team: 1. Polygon. 2. Rectangle. 3. Four-digit number. 4. Relationship between numbers. 5. Geometric figure. 6. Group of numbers in the notation of a number. 7. Mathematical operation. 8. Line of a coordinate ray.

Conclusion and awards

3. Geometric polygon

The game is played in two stages.

Stage 1. Geometric polygon

Stage 2. Problem solving tournament.

The goal of stage 1: to consolidate the knowledge students have acquired during the game. Students must correctly name geometric shapes.

The goal of stage 2: to determine how to perform logical calculations.

Stage 1. Polygon

I. Rules of the game

A table is set up in the middle of the room and the models are placed in the middle of the table. The cardboard squares are divided equally between the two teams. The teams play by sliding the cardboards against each other on both sides of the table. Both teams slide the cardboards at the same time. If the model falls to the floor, the student who knocked it over must name the geometric shapes drawn on the surface of the cardboard. The game ends when all the models have fallen off the table and the teams are dismantled.

II. Game conditions

1) For each knocked down model – 4 points.

2) 5 points are awarded for each correctly named figure, and 6 points are deducted for an incorrect answer.

3) It is allowed to pick up cardboard from the floor during the game.

4) A 3-point penalty will be imposed for any infraction during the game.

III. Organization of the game.

Square models are glued from thick paper with 5-8 cm on each side, 3x3 (square cm) squares are cut out of cardboard. The models are numbered. Various geometric shapes are drawn on the side faces of the cube: a line, a ray, a coordinate ray, a straight angle, an obtuse angle, a right angle, etc.

Then, a different set of problems is created for each model. The number of problems corresponds to the number of faces of the cube. The created problems are written individually on a card, which receives a number corresponding to the model number.

Stage 2. Tournament

Students sit at the table. The captain of each team is given cards corresponding to the numbers of the winning models. The goal is set: to solve as many tasks as possible. Mutual assistance is allowed. The tournament is given 20-30 minutes. Students who have solved the tasks approach the judge and announce the solution. For each correctly solved task, the team receives 5 points. The team that scores the most points in two stages wins. In order for the game to be organized and without violating the rules, the following is required:

a) Game results – counting points during the game so that each team can see their achievements. To do this, enter the number of points on each team's score sheet.

b) solving all problems

c) involve high school students as judges.

c) make the game colorful.

Examples of task cards.

1) A small military unit approached a river, they had to cross it. The bridge was broken, and the river was deep. How could it be? Suddenly, an officer saw two boys having fun in a boat on the shore. But the boat was so small that only one soldier or two boys could row it. However, all the soldiers crossed the river in this very boat. How?

2) In how many different ways can the sum of 19 tenge be converted into 3 and 5 coins?

3) Imagine a cube with sides of 3 dm, the entire surface of which is painted black. How many cuts are needed to divide the cube into cubes with sides of 1 dm? How many such cubes will there be?

4) I decided to find out the distance from my house to my friend's house. I walked at a steady pace and counted the steps along the way in pairs and half in threes, and the pairs were 250 more than the threes. How many steps are there to my friend's house?

4. Smart tournament

Purpose: to test ingenuity, knowledge of basic mathematical facts, to unite students, develop a sense of camaraderie, and responsibility.

Equipment: cards with tasks for each competition, a table on the board for entering the results.

This event was held among 5 classes. Each class is divided into two teams and a captain is chosen for each of them.

I. Warm-up exercises

1 point for each correct answer.

1 team

1. An instrument for measuring angles (protractor)

2. How long is 1/4 of an hour? (15 min)

3. What is a triangle with two equal sides called? (isosceles triangle)

4. Two fathers and two sons divided 300 tenge among themselves, each receiving 100 tenge. How is this possible? (Father, grandson and grandfather)

5. What does the fractional denominator show? (how many equal parts it was divided into)

6. The area of a rectangle is 36 cm^2. How many cm are the sides of a square with the same area as the rectangle? (6 cm)

2 teams

1. The formula for the area of a rectangle with sides A and B (s=a*b)

2. What part of an hour is 20 minutes (1/3).

3. What is a figure with three sides called? (Triangle)

4. There are four corners in a room. A cat is sitting in each corner. Opposite each cat are three cats. Next to each cat is 1 cat. How many cats are there in the room? (4)

5. What does the numerator of a fraction show? (how many parts were taken)

6. What is the result of addition called? (sum)

3 teams

1. A tool for drawing circles (compasses)

2. How many grams are in 1/4 kg? (250)

3. What is a triangle with an obtuse angle called? (obtuse)

4. A girl was traveling to Astana. Three girls were traveling to Astana. How many people went to Astana? (one girl)

5. How can you check if two geometric figures are equal? (put them on top of each other)

6. The area of a square is 100 cm^2. What is its perimeter? (40 cm)

4 teams

1. What is the formula for the area of a square with side A? (S=a2)

2. How many centimeters are there in 1/5 m (20)

3. Name a quadrilateral with all right angles (rectangle)

4. Three horses are galloping. Each horse has galloped 5 km. How many kilometers has the rider traveled? (5 km.)

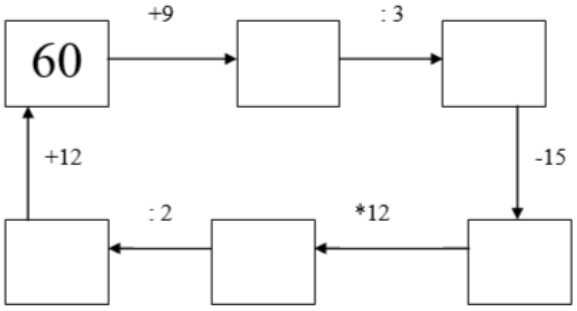
5. What is the name of the tool for measuring lines? (ruler)

6. The perimeter of a rectangle is 64 cm. What are the sides of a square with the same perimeter as the rectangle? (16 cm)

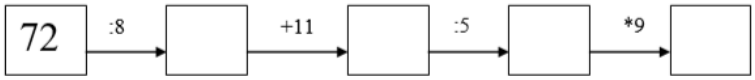
II. Captains' Competition

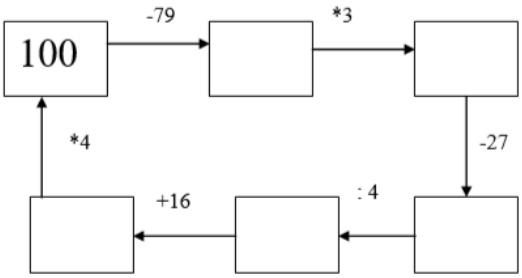
Captains are given cards with tasks. Each card has two tasks. You need to fill in the empty cells. 5 points for each correct sequence.

1 team captain:

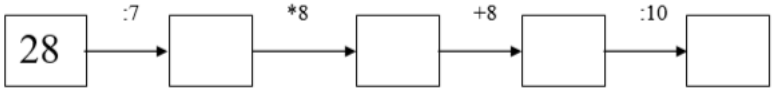


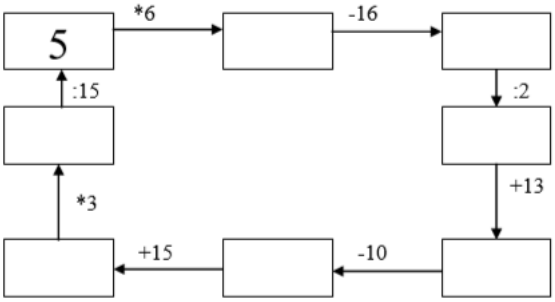
2 team captains:



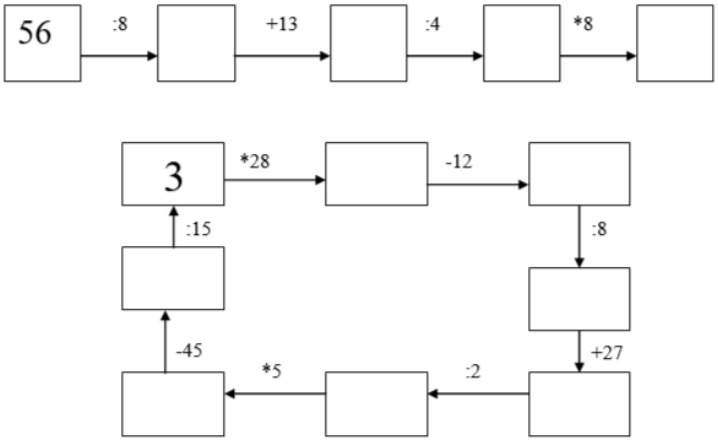


3 team captains:





4 team captains:



III. Tasks for teams during the captains' competition

You need to put the signs (+, -, *, :, (,)) to get the specified result.

1 team: 5555=3 [(5+5+5):5=3] 5555=5 [(5-5)*5+5=5] 5555=50 [(5*5)+(5*5)=50]	2 teams: 5555=7 [(5+5): 5+5=7] 5555=30 [(5:5+5)*5=30] 5555=130 [5*5*5+5=130]
3 teams: 5555=26 [(5*5)+(5:5)=26] 5555=75 [(5+5+5)*5=75] 5555=625 [5*5*5*5=625]	4 teams: 5555=4 [(5*5-5): 5=4] 5555=6 [(5*5+5): 5=6] 5555=120 [5*5*5-5=120]

5 points are awarded for each correct answer.

Conclusion and awards

5. Math Train

Preparatory work: a team of 6-7 people from each class, fans, gathers. The team comes up with a name, motto, emblem, and chooses a Captain. If fans participate in the quiz, they earn extra points for their team.

Game progress

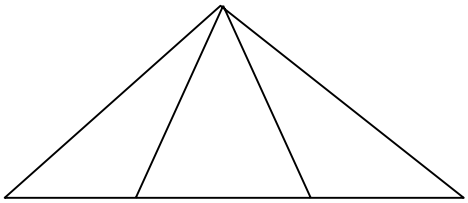
I. Geometrical stanitsa

1. Place the geometric shapes (cylinder, cone, cube, pyramid). For each correctly named figure – 1 point.

2. Name and draw flat geometric figures by hand. For each correct figure – 1 point.

3. Folding figures – 3 points.

4. Counting triangles – 2 points (correct answer 6).



II. Mathematical station

For each correct answer – 2 points.

1. How many lines can be drawn that intersect 2 points? (2)

2. How many straight lines can be drawn through two points? (Infinitely many)

3. There are 6 meters of canvas in one roll. 2 meters are sold every day. How many days will it take the seller to cut 2 meters? (2 days)

4. One and a half pike perch costs one and a half tenge. How much does one pike perch cost? (1 tenge)

5. The height of a tree is 5 m. If a snail climbs three meters during the day and slides 2 meters at night, find how many days it will take to reach the top of the tree (3 days)

6. What is the result of addition called? (sum)

7. 7×8=? (56)

8. How many minutes are there in an hour? (60)

9. Which is heavier, 1 kg of cotton or 1 kg of iron? (equal)

10. A rooster standing on one leg weighs 3 kg. How much does it weigh when standing on two legs? (3 kg)

11. What does half an apple look like? (Its other half)

12. Name the smallest two-digit number. (10)

13. What is the result of subtraction called? (subtraction)

14. How many ends does three and a half sticks have? (8)

15. What is the name of the device for measuring lines? (Ruler)

16. How do you find the area of a rectangle? (length x width)

17. What does the equality a+b=b+a mean? (commutative property of addition)

18. Plus sign (+)

19. What is the minus sign? (-)

20. The doctor prescribed 3 injections to the patient every half hour. The first injection was given at 8 o'clock. What time is the last injection given? (at 9 o'clock)

21. How to make 4 matches from 3? (IV)

22. How to find distance? (speed time)

23. What is the smallest natural number? (1)

24. How many musicians are there in a quartet? (4)

25. There are two sons and two fathers in a family. How many men are there in the family? (3)

During the game, students are divided into the following teams based on the type of problems:

1 wagon. Tanker (filling tasks).

2 wagons. Heavy truck (school coursework).

3 wagons. Goods (match reports).

4 cars. China (Chinese tangram).

5 cars. "Lux" (humorous issues).

1. How can you get exactly 1 liter of water using 3-liter and 5-liter jars?

Answer: 5+5-3-3-3=1

2. How can you measure 4 liters of water using 3L and 5L containers?

Answer: 5+5-3-3=4

3. How can you measure 6 liters of water if you have only two containers with capacities of 5 liters and 7 liters?

Answer.(7-5)+(7-5)+(7-5)

4. How can you measure 6 liters of water with only 4-liter and 9-liter containers?

Answer: 9+9-4-4-4=6

5. How can you measure 3 liters of water with only 5-liter and 9-liter containers?

Answer: 9+9-5-5-5=3

6. There are two containers with a capacity of 8 liters and 5 liters. How can you use them to pour 7 liters?

Answer: 5+5+5-8=7

7. If we have two containers of 5 liters and 7 liters, how do we pour 6 liters of water?

Answer: (7-5)+(7-5)+(7-5)=6

8. If we have two containers of 5L and 17L, how do we pour 13L of water?

Answer. 5+5+5+5-17+5+5=13

2 cars. Heavy truck

1. There were three freight trains at the railway station. The first train had 30 cars, and the second train had 5 more cars than the first. If the first train had 10 fewer cars than the third, how many cars were there in all?

Answer: 105.

2. A motorcyclist and a cyclist set off towards each other. The distance between them is now 272 km. If the cyclist's speed is 12 km/h faster and the motorcyclist's speed is 44 km/h faster, in how many hours will they meet?

Answer: 4 hours

3. At 3:00 PM, an electric train left the station at a speed of 80 km/h, and 1 hour later, a second electric train left the same station at a speed of 75 km/h. What is the distance between the trains at 6:00 PM that day?

Answer: 90 km.

4. An apartment consists of 3 rooms, a kitchen, a bathroom and a corridor. The area of the corridor is 4 m2. The area of the bathroom and kitchen is 4 times larger than the corridor. If the area of the corridor, bathroom and kitchen is 2 times less than the area of all the rooms, find the area of the entire apartment.

Answer: 60 m2.

5. I traveled from the city to the village where my friend lives for 4 hours by train and then another 2 hours by bus. The bus travels at a speed of 35 km/h, and the train travels at a speed twice as fast as the bus. How long did it take me to travel?

Answer: 350 km.

6. During the harvest, 612 tons of wheat were harvested from the first plot, 3 times less from the second plot, and 4 times less from the third. How many tons of wheat were harvested from the 3 plots?

Answer: 969 pages.

7. According to the plan, the factory was supposed to produce 2100 machines in 25 days. But the factory produced 21 machines per day ahead of schedule. How many days ahead of schedule was the plan fulfilled?

Answer: 20

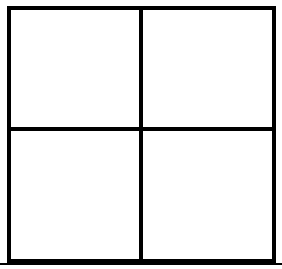
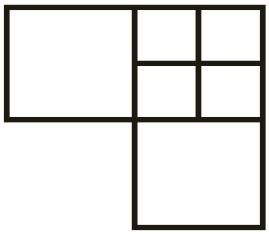
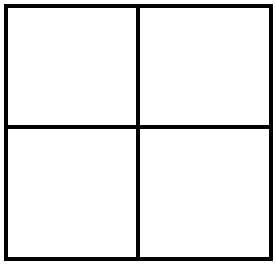
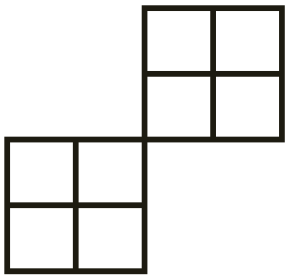
8. The distance between the village and the city is 144 km. If Olzhas went to the city by bus at a speed of 36 km/h and drove the truck back at a speed of 48 km/h, how long did it take to get to the city and back?

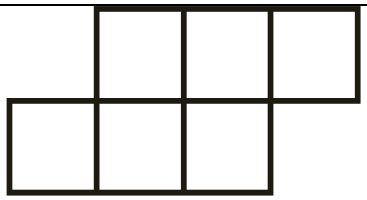
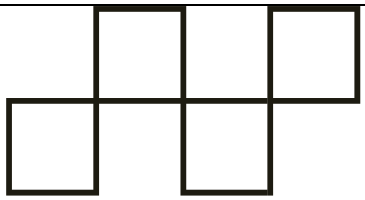
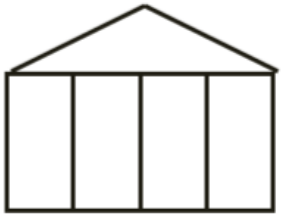
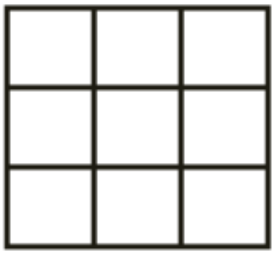
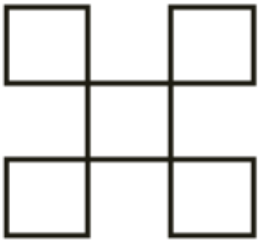
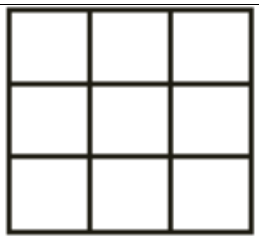
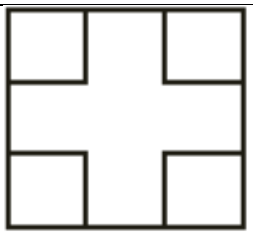
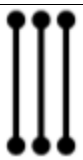
Answer: 7 hours

9. Two trains leave two stations 720 km apart at the same time. The speed of the first is 75 km/h, and the speed of the second is 10 km/h greater. How far apart will the trains be after 4 hours?

Answer: 80 km.

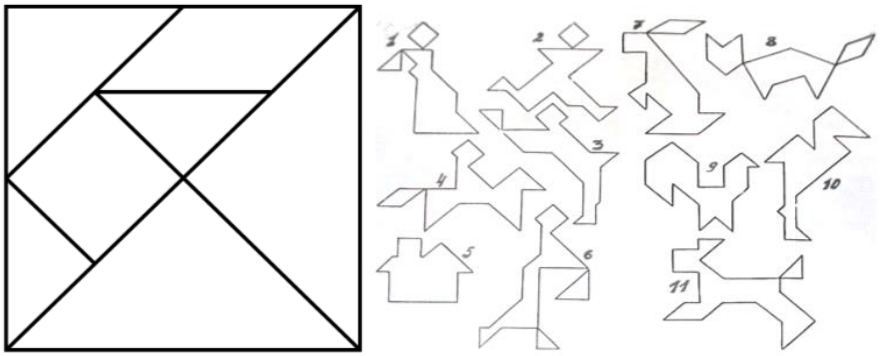
3 wagons. Goods

Place 2 matches so that you get 7 squares (it is allowed to put one match on another)		
Place 4 matchsticks to make 10 squares (you can place one matchstick on top of another)		
Remove two matches so that 4 identical squares remain.		

		
This temple is built from 11 matchsticks. Rearrange 2 matchsticks to make 11 squares.		
Remove 4 matches so that 5 identical squares remain.		
Move 2 matches so that the house turns in the other direction.		
Remove 4 matches, leaving 1 large and 4 small squares		
I have 3 tendons. If I add 2 more, I get 8.		
Arrange these matches so that there are 4 in each horizontal row.		

4 cars. Chinese

There is a fun Chinese game called tangram, known in China as chi-chao-chu ("clever pattern of 7 pieces"). They say that Napoleon spent hours making figures from tangram elements while in exile.



5 cars. "Lux"

1. Two fathers and two sons ate 3 eggs for breakfast, each of them ate one whole egg. How is this possible? (father, father, son)

2. How many ends does 5 and a half sticks have? (12)

3. What does a crow do after living for 3 years? (enters its fourth year)

4. What is warmer than skin tone? (2 skin tones)

5. What is the fastest time for a black cat to enter a house? (When the door is open)

6. How to choose a branch so as not to scare a bird? (wait for it to fly away)

7. What does half an orange look like? (to the other half)

8. What kind of tree does a crow sit on when it rains? (in a wet one)

9. What rocks are not found in the sea? (dry)

10. My brother, not my younger brother, but my father's son. Who is this? (myself)

11. What can you see if you close your eyes? (color)

12. How many identical letters are needed to write "Mom, Dad, Son, Daughter, Grandma, Grandpa"? (6 letters: Family)

13. One egg takes 4 minutes to boil. How many minutes will it take to boil 4 eggs? (4)

Thus, to develop the logical thinking abilities of 5th grade students, an elective course "Logical Problems and Games" was introduced and tested.

3. RESULTS

The experimental work was carried out on the basis of the school-gymnasium No. 49 of Astana. Students of grades 5 "B" and 5 "G" participated in the experimental work. Grade 5 "B" was the experimental group, grade 5 "G" was the control group. In accordance with the goals and objectives of the experiment, the study consisted of stages (identification, formation and control), each of which reflects the changes taking place in the program and includes an analysis of their content.

The purpose of the identification phase of the practical and experimental work was to determine the level of students' logical thinking abilities.

Theoretical analysis of scientific literature made it possible to identify and justify the criteria for assessing the formation of students' logical thinking abilities (low, medium, high), which are presented in Table 3.

Table 3.1 Criteria for assessing students' logical thinking abilities

Level of logical thinking skills	Evaluation criteria		
	down	average	high
Analyze objects to distinguish features (important and unimportant); synthesis as the assembly of wholes from parts; choosing the basis and criteria for comparing, generalizing, and classifying objects; building a logical chain of thought.	Analytical-synthetic activity, problems of generalization and isolation of patterns are insufficiently developed. Comparative activity is partially mastered. It is difficult to establish logical connections.	They are able to analyze, compare, generalize, and establish patterns, but they do so with errors. They establish logical connections, but it takes a long time to complete such tasks.	They are able to analyze, compare, establish patterns, generalize, and establish logical connections.

According to the criteria shown in Table 3, the preliminary level of logical thinking ability of 5th grade students was determined. Independent work tasks consisting of logical problems were offered to students of both grades. The text of the independent work is given in Appendix A. The levels of students were determined as follows based on the completion of the independent work:

- High – all tasks were completed correctly. There may be minor flaws that do not distort the overall picture of choosing the right solution;

- Average – 67-34% of tasks were completed, that is, 2 out of 3 tasks were solved correctly;
- low – less than 33% of tasks solved correctly (1 or all tasks solved incorrectly).

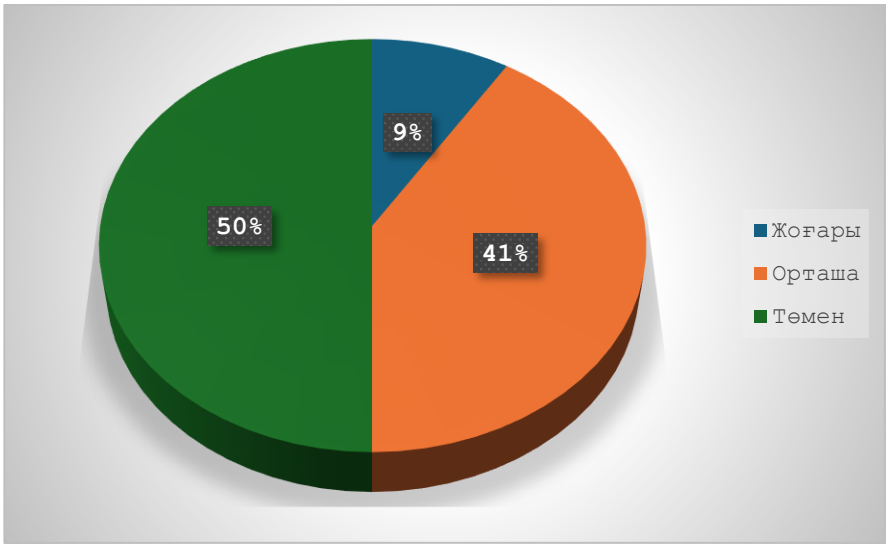
After completing the independent work, 2 students from the experimental group (grade 5 "B") showed high results. They completed the tasks of the independent work without errors. And 9 students showed average results. These students made 1 error while completing the tasks. And 11 students had low results. During the performance of these students, they completed only 1 task correctly or made all the errors.

Table 3.2 Level of logical thinking ability of students in the experimental group (grade 5 "B")

High level		Average level		Low level	
	number		number		number
	2		9		11

As we can see from Table 4, most of the students in the experimental group have a low level of logical thinking skills.

Figure 3.1 The level of logical thinking ability of students in the experimental group (grade 5 "B")



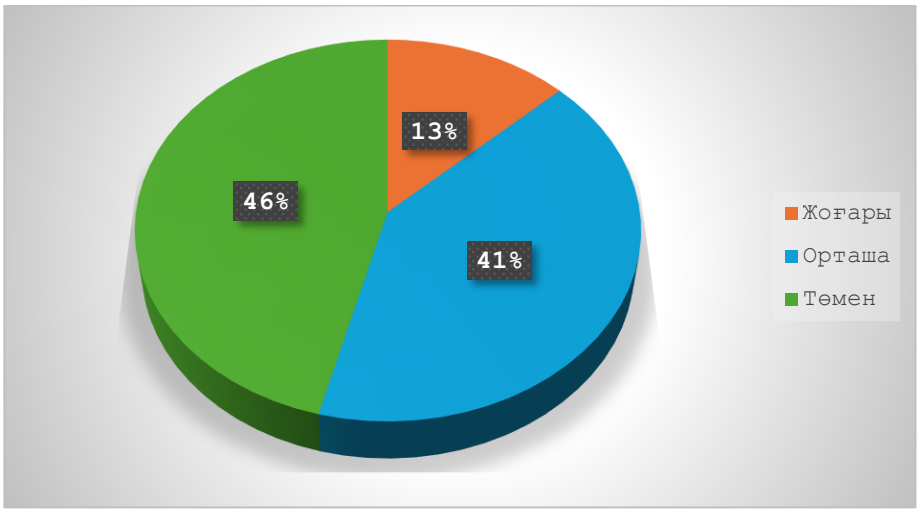
Among the students in the control group (grade 5 "G"), 3 students showed high results, 9 students showed average results, and 10 students showed low results.

Table 3.3 Level of logical thinking ability of students in the control group (grade 5 "G")

High level		Average level		Low level	
	number		number		number
	3		9		10

Based on the results of the control group students in Table 5, it can be concluded that the students have a low level of logical thinking ability.

Figure 3.2 Level of logical thinking ability of control group students (5th grade "G")



The low level of logical thinking skills of students allowed us to develop a methodology for developing logical thinking skills. At the formative stage, an elective course "Logical problems and reasoning" was developed and tested to develop students' logical thinking skills. The elective course was conducted for students of grade 5 "B".

The program is based on solving logical problems and games for 5th graders and is organized into 34 hours (1 hour per week).

The goal of the elective course is to develop general logical thinking skills, increase the student's interest in the subject, and create conditions for developing problem-solving skills.

After the course was implemented, the level of logical thinking of 5th grade students was re-tested. This was done during the observation period.

The purpose of the observation phase is to verify the effectiveness of the proposed methodology. To confirm the research hypothesis.

To re-diagnose the level of students' logical thinking skills, independent work based on logical problems was proposed. Independent work tasks are provided in Appendix B.

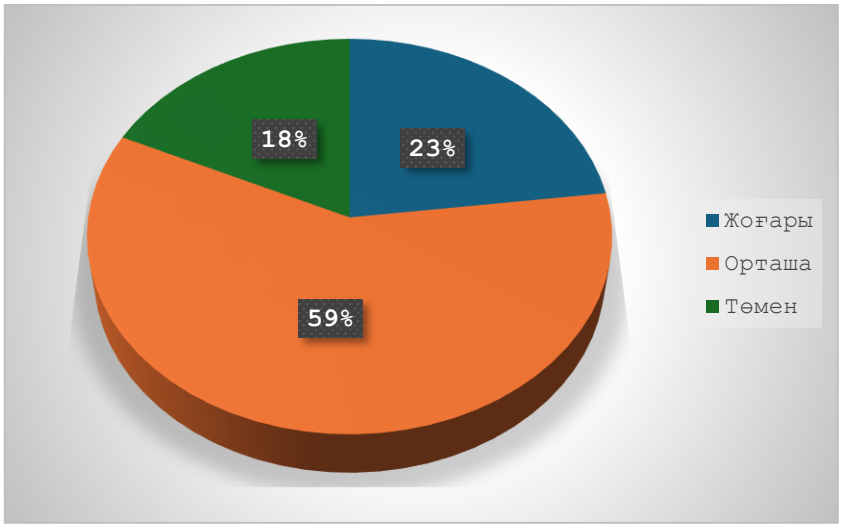
After completing the independent work, 5 students in the experimental group (5th grade "B") showed high results. They completed the tasks of the assigned independent work without errors. This indicates an increase in the number of high-level students by 3 compared to the detection period. And 13 students showed an average result. These students made 1 error while completing the tasks. This indicates an increase in the number of average-level students by 4 compared to the detection period. And 4 students had a low result. During the performance of these students, they completed only 1 task correctly or all of them were incorrect. This indicates a decrease in the number of low-level students by 7 students.

Table 3.4 Level of logical thinking ability of students in the experimental group (control period)

High level		Average level		Low level	
	number		number		number
	5		13		4

We present the diagram in Figure 4 to illustrate the results of the experimental group students during the observation period in Table 6.

Figure 3.3 The level of logical thinking ability of students in the experimental group (control period)

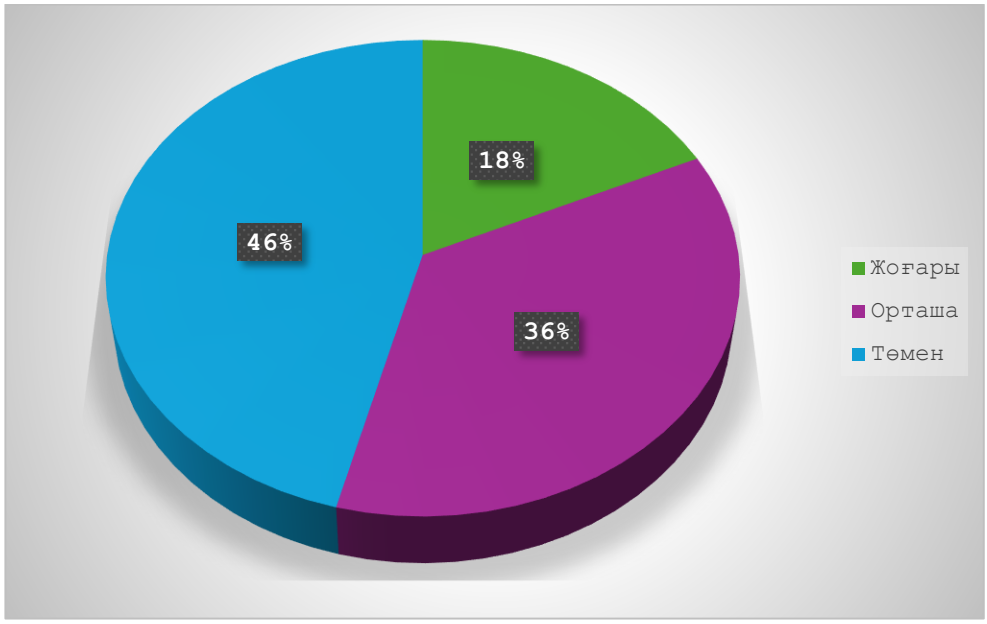


The results of the control group students changed only slightly from the period of determination. During the control period, only 18% of students in grade 5 "G" showed high results. The average result was 36%. And the low result was 46%.

Table 3.5 Level of logical thinking ability of students in the control group (control period)

High level		Average level		Low level	
	number		number		number
	4		8		10

Figure 3.4 Level of logical thinking ability of students in the control group (control period)



Thus, this study investigated the impact of the elective course "logical tasks and games" on the development of students' logical thinking. The results showed that the introduction of such a course can significantly increase the level of logical thinking of students, developing their analytical and creative abilities.

The results show a positive impact of the elective course on students, which confirms the importance of introducing such educational programs in schools. Further development of this direction is of interest for future research in the field of education and psychology.

In conclusion, we can conclude about the need to develop students' logical thinking, as well as the importance of introducing elective courses aimed at such goals. The introduction of such courses not only increases students' academic performance, but also contributes to the development of their cognitive skills and abilities.

The study also showed that the use of logical problems and game methods in mathematics lessons contributes to the development of students' logical thinking. Games can be an effective tool for stimulating mental activity, developing analytical skills, and increasing interest in the learning process.

Within the framework of this study, the main principles and methods of using games to develop students' logical thinking were identified. In addition, pedagogical experiments were conducted, which showed the positive effect of game methods on the level of students' logical thinking.

In general, the use of logical problems and games in mathematics lessons significantly improves the quality of learning, increases students' performance, and contributes to the development of their thinking skills. Thus, the introduction of game methods into the educational process can be an effective tool for developing students' logical thinking.

CONCLUSION

In the modern education system, emphasis is placed on the formation of students' general logical thinking abilities, rather than increasing the amount of information for their assimilation, since human intelligence is determined not by the amount of accumulated knowledge, but by a high level of logical thinking. In this regard, the teacher is faced with the task of teaching children to analyze, compare and summarize information obtained as a result of interaction with objects and phenomena.

Logical thinking skills: analysis, synthesis, comparison, abstraction, generalization, and others act as a necessary cognitive tool for the successful mastering of any academic subjects, contribute to the development of students' learning abilities, self-education, and self-development. Features of the development of students' logical thinking skills: they are associated with internal logic, therefore they can only be formed in a certain sequence.

Research objective of the dissertation work The aim was to develop and implement a methodology for developing students' logical thinking skills in mathematics.

To achieve this goal, the following tasks were solved:

- Analysis of logical problems encountered in school mathematics courses;
- Studying the features of the development of students' logical thinking abilities;
- Identify the structure of logically oriented problems;
- Developing a methodology for developing students' thinking skills through logical problems and games.
- Check the effectiveness of the method.

The work also presents the results of a study of the dynamics of the development of logical thinking in 5th grade students. Analysis of the dynamics during the control period of the experiment showed an increase in the level of development of logical thinking in the subjects. Such changes can be considered as a correct organization of the process of developing logical thinking in 5th grade students in the process of introducing the elective course "Logical Problems and Games". This indicates that the system of logical problems and games implemented in our experiment has a significant impact on the effectiveness of the process of developing logical thinking in 5th grade students.

The study found that using various methods effectively develops students' logical thinking in mathematics. In particular, using games, tasks, puzzles, and other interactive methods allows students to understand the material more deeply, develop the ability to analyze and solve complex problems.

As a result of the research, the main principles that help effectively develop students' logical thinking were identified: gradually increasing the complexity of tasks, using different types of material presentation, and encouraging independence and creativity in solving problems.

Thus, the methodology for developing students' logical thinking in mathematics is of great importance in the formation of critical thinking, logical literacy, and the

ability to analyze information. Further research in this area will allow us to improve the quality of education and increase students' progress in mathematics.

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APPENDIX

Appendix 1. Tasks for the discovery phase

1. When you cross out the first digit of a two-digit number, you get a number that is 31 times smaller than the first digit. Which digit of which number should be crossed out?

2. There are 5 apples. There are six children who come. How should these five apples be divided equally among 6 children? But no apple can be divided into 6 or more parts.

3. There are 28 houses in the village. 15 of them have camels, 8 have cows, and 6 have both cows and camels. How many houses do not have livestock?

4. Three fathers and six sons went for a walk. They bought 42 apples. Each of them got 6 apples. How did this happen?

5. There are chickens and rabbits in a barn. If there are 15 heads and 50 legs in total, how many chickens and how many rabbits are there in the barn?

Appendix 2. Tasks for the final stage

1. There were 76 people in two rooms. When 30 people left one room and 40 people left the other, the number of people in each room was equal. How many people were in each room at the beginning?

2. There are 6 children in a family. The age of each child is an even number. The ages of five children are 2, 6, 8, 12, and 14 years older than the age of the youngest. How old is the youngest child?

3. My mother filled a cup with tea. My younger brother accidentally dropped the sugar in his hand into the cup. However, the sugar remained completely dry. Why do you think?

4. Two crows are facing south, one facing north. They say to each other: "Friend, I see you." How do they see each other?

5. The first father gave 150 sheep to his son, and the second father gave 100 sheep to his son. So the total number of sheep that the two sons received from their fathers was 150. How did this happen?