



Enhancing Trigonometry Instruction: An Intervention Study on the Integration of Flipped Learning in High School Classroom

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A thesis submitted to the Faculty of Education and Humanities
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in Mathematics

«SDU University»

Department of Pedagogy of Natural Sciences

Thesis Advisor:
Candidate of Pedagogical Sciences,
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List of Abbreviations

STEM – Science, Technology, Engineering, and Mathematics

ZPD – zone of proximal development

FC – flipped classroom

LMS – learning management system

CD – compact disk

USB – universal serial bus

ABSTRACT

This quasi-experimental study investigates the effects of integrating a flipped learning approach into high school trigonometry instruction. Four intact ninth-grade classes (80 students in total) participated over a six-week intervention: two classes (44 students) received flipped classroom instruction while two classes (36 students) served as controls with traditional teaching. In the flipped model, students studied video lessons (sourced from Khan Academy) as homework and engaged in problem-solving and discussions during class, whereas control classes received conventional lectures. A pre-test/post-test design was used to measure academic performance in trigonometry. The flipped learning intervention was grounded in constructivist theory, encouraging active pre-class learning and in-class collaboration. From pre-test to post-test, both experimental and control groups showed improvement; but, the experimental group showed a more marked improvement in test scores and conceptual understanding. An independent t-test verified a statistically significant variation in favor of the flipped approach ($p < .05$), since the post-test mean scores of the experimental classes were higher than those of the control classes. But not all students regularly watched the assigned videos ahead of class, so limiting the full impact of the flipped model – a major implementation difficulty. Still, students who participated actively in the video courses showed better performance, more drive, and more classroom participation. This study offers empirical data showing that flipped learning can improve secondary level trigonometry instruction, so raising student engagement and outcomes. It emphasizes the need of student responsibility and supportive scaffolding for effective flipped classrooms as well as provides useful insights for teachers on juggling in-class and out-of-class learning. Recording the benefits and disadvantages of flipped learning in a high school environment helps the body of knowledge on mathematics education to be improved and might direct instructional design, teacher preparation, and education policy meant to modernize pedagogy for better student learning.

Key words: flipped classroom, trigonometry, quasi-experimental design

АНДАТПА

Бұл квазиэксперименттік зерттеу жоғары сыныптағы тригонометрияны оқытуда флипмед (төңкерілген) оқыту әдісін енгізудің әсерін зерттейді. Алты аптаға созылған экспериментке төрт тоғызыншы сынып (барлығы 80 оқушы) қатысты: олардың екеуі (44 оқушы) флипмед форматында оқытылып, қалған екі сынып (36 оқушы) дәстүрлі әдіспен оқытылды. Флипмед модельде оқушылар үй тапсырмасы ретінде Khan Academy платформасынан бейнесабақтарды көріп, сыныпта есеп шығарып, талқылауға қатысқан, ал бақылау сыныптарында дәстүрлі лекциялар жүргізілді. Оқушылардың тригонометрия бойынша оқу жетістігін өлшеу үшін алдын-ала тест (pre-test) және қорытынды тест (post-test) әдісі қолданылды. Бұл модель конструктивизм теориясына негізделіп, сабаққа дейін белсенді дайындалу мен сыныптағы ынтымақтастықты дамытуға бағытталған. Екі топ та посттест нәтижелерінде алға жылжу көрсетсе де, флипмед сыныптарда балл мен түсіну деңгейінің айқын жоғарылауы байқалды. Тәуелсіз t-тест нәтижесі флипмед әдістің тиімділігін статистикалық тұрғыда растады ($p < 0.05$), себебі эксперименттік топтың қорытынды нәтижелері бақылау тобынан жоғары болды. Алайда, барлық оқушылар бейнематериалдарды жүйелі түрде көрмегендіктен, бұл әдістің толық тиімділігі шектелді. Дегенмен, бейне сабақтарға тұрақты қатысқан оқушылар жоғары нәтижелер мен белсенділік көрсетті. Зерттеу флипмед оқытудың тригонометрияны меңгеруді жақсартудағы әлеуетін дәлелдейді және оқушының жауапкершілігі мен мұғалім қолдауының маңыздылығын атап өтеді. Бұл жұмыс мектептегі флипмед әдістің артықшылықтары мен шектеулерін көрсете отырып, педагогикалық тәжірибені, мұғалім даярлығын және оқу саясатын жетілдіруге үлес қосады.

Түйін сөздер: төңкерілген сынып, тригонометрия, квазиэксперименттік зерттеу

АННОТАЦИЯ

В данном квазиэкспериментальном исследовании рассматривается влияние интеграции перевёрнутого обучения в преподавание тригонометрии в старших классах. В шестинедельной интервенции приняли участие четыре девятых класса (всего 80 учащихся): два класса (44 ученика) обучались по модели перевёрнутого класса, а два других класса (36 учеников) — по традиционной методике. В перевёрнутом формате учащиеся изучали видеоуроки (взятые с платформы Khan Academy) дома, а на уроках занимались решением задач и обсуждением, тогда как в контрольных классах проводились обычные лекции. Для оценки академической успеваемости использовалась модель предтеста и посттеста. Перевёрнутое обучение опиралось на конструктивистскую теорию, способствуя активному предварительному обучению и совместной работе в классе. Оба типа классов показали прогресс от предтеста к посттесту, но экспериментальные классы продемонстрировали более выраженное улучшение результатов и концептуального понимания. Независимый t-тест показал статистически значимое преимущество в пользу перевёрнутого обучения ($p < 0.05$), поскольку средние баллы посттеста в экспериментальной группе были выше, чем в контрольной. Однако не все учащиеся регулярно просматривали назначенные видео до урока, что ограничило эффект от модели — это стало серьёзной трудностью при внедрении. Тем не менее, учащиеся, активно участвовавшие в просмотре видео, показывали лучшие результаты, большую мотивацию и активность на уроках. Это исследование предоставляет эмпирические данные о том, что перевёрнутое обучение может повысить вовлечённость и успеваемость учащихся в старшей школе. Оно подчёркивает важность ответственности учащихся и педагогической поддержки в перевёрнутом классе, а также содержит полезные рекомендации для учителей по организации баланса между внеурочным и очным обучением. Фиксация преимуществ и ограничений перевёрнутого подхода в школьной среде способствует развитию педагогики и может повлиять на проектирование уроков, подготовку учителей и образовательную политику.

Ключевые слова: перевёрнутый класс, тригонометрия, квазиэкспериментальный дизайн

INTRODUCTION

A major branch of mathematics, trigonometry is indispensable in many academic fields and practical uses. Trigonometric ideas support many fundamental computations and models in architecture and engineering, physics and computer graphics. Trigonometry is a pillar of spatial reasoning and problem-solving ability in secondary education as well as a portal to more advanced mathematics. Trigonometry is often seen by students as abstract, difficult, and disconnected from their daily experiences, despite its relevance. Many times, these impressions cause students to become disengaged, perform poorly, and view mathematics generally negatively. Many high schools' conventional trigonometry courses are teacher-centered, lecture-based, with students passively listening and noting under direction. This approach hardly allows for interactive learning, tailored assistance, or practical application of ideas. Students may thus memorize formulas without really understanding their meaning or relevance, so acquiring a shallow grasp of trigonometric ideas that limits further learning. As digital technology and modern educational theory develop, reform of such traditional teaching strategies gains increasing motivation. Among several pedagogical developments, flipped learning has become a rather interesting substitute. Instructional materials – such as video lectures or tutorials – are delivered to students outside of class (usually as homework) in a flipped classroom, so freeing class time for active learning – solving problems, participating in discussions, and working on applied tasks. Particularly relevant for disciplines like trigonometry that call for visualization and practice, this reversal of the conventional lecture-then-homework sequence seeks to boost student involvement and deepen knowledge of concepts. Flip learning conforms with calls for more student-centered, active learning environments in mathematics education by moving first learning to the individual space and higher-order application to the group space. Given these possible advantages, looking at how flipped learning might be included into high school trigonometry is relevant and important.

Constructivist and socio-constructivist paradigms provide the theoretical foundations of flipped learning rather clearly. Piaget's constructivism holds that rather than passively absorb knowledge, students actively create it by experience. A flipped classroom uses this concept to let students construct first understandings at their own pace by letting them encounter new ideas independently – that is, via movies – before class. This pre-class exposure allows students to arrange their knowledge, create first mental models, and point up areas of uncertainty to raise in class. The sociocultural theory of Lev Vygotsky also guides the inverted approach with the idea of the Zone of Proximal Development (ZPD), which is the range of tasks a learner can accomplish under direction but not yet independently (Vygotsky, 1978). Class time is purposefully set in a flipped classroom to run within students' ZPD – after students have tried to learn fundamental ideas on their own, in-class activities include teacher scaffolding, peer collaboration, and guided practice. This arrangement helps students to tackle more difficult tasks with help, so extending their learning beyond what they could accomplish on their own. Internalizing ideas and clearing misunderstandings in these

sessions depend on the social interaction and teamwork. Bloom's Revised Taxonomy of Cognitive Skills offers still another pertinent framework. While homework done individually emphasizes higher-order tasks (applying, analyzing, evaluating, creating) traditional lectures sometimes stress lower-order thinking (remembering, understanding) during class. Students handle lower-level cognitive work before class (watching instructional videos, reading texts to remember and understand basic concepts), and then engage in higher-order thinking in class where the teacher and peers are available to help under flipped learning. Focusing this reorganization on deeper research, problem-solving, and critical thinking will help to maximize the educational value of face-to-face time. All told, the flipped classroom model aligns learning activities with the suitable cognitive domain for the learning environment by using constructivist ideas by encouraging active, self-paced learning and on Vygotskian ideas by employing social learning during class. By means of active participation and scaffolded support, these theoretical underpinnings imply that a well-executed flipped learning environment could enhance students' comprehension, retention, and application of trigonometry ideas.

Though flipped learning is becoming more and more popular as a creative approach in the classroom, its application in secondary school math, especially in trigonometry, is still under investigation and varies. Many high school trigonometry courses still use conventional lecture techniques that have resulted in ongoing issues: students often become disengaged, struggle to acquire conceptual understanding, and score poorly on tests. Trigonometric content is relevant, yet many students fail to link the content with useful applications, so supporting the impression that it is abstract and useless. Lack of interactive or visual learning chances in traditional classrooms aggravates these challenges. While flipped learning has shown success in many STEM fields and educational environments, most of the current studies concentrate on general mathematics performance or on college-level courses. Younger students – that is, those in high school – have different reactions to flipped classroom models in a targeted topic like trigonometry. Moreover, practical difficulties including unequal access to technology at home, different student motivation, and inadequate teacher preparation in flipped approaches could influence its application and efficacy in the secondary level. The need to address these issues motivates the current study: to examine whether integrating flipped learning into high school trigonometry can improve student outcomes and to identify the conditions and supports necessary for its success.

Research question

How does flipped learning compare to traditional teaching in terms of trigonometry academic performance for high school students?

Research objectives

- 1) To compare the academic performance of students receiving flipped instruction with those receiving traditional instruction in trigonometry.
- 2) To investigate how student participation in pre-class activities influences in-class performance.
- 3) To explore challenges encountered in implementing flipped learning in secondary mathematics classrooms.

Research hypotheses

H_0 (null hypothesis): There is no significant difference between the scores of students in the experimental group (flipped classroom) and the scores of students in the control group (traditional classroom) in the post test.

H_1 (alternative hypothesis): There is a significant difference between the post-test scores of students in the experimental group (flipped classroom) and the post-test scores of students in the control group (traditional classroom).

Significance of the study

The importance of this study is in its ability to guide school administrators, curriculum designers, and mathematics teachers about efficient approaches for teaching trigonometry in the classroom of twenty-first century. The study addresses a crucial area for enhancing mathematics education by concentrating on a topic generally acknowledged as challenging and sometimes poorly mastered by students. It offers empirical data on the success of the flipped classroom model in secondary-level math, so clarifying whether this strategy can improve trigonometry especially performance and participation. The results can guide decisions about whether flipped learning should be embraced more generally or modified for the high school math course. Furthermore important for educational policy and teacher professional development is the study. Knowing how to apply and maintain models like flipped learning is vital as digital learning tools and blended learning approaches get more and more included into courses. The findings of this study can help educational institutions create training courses that enable teachers to implement creative ideas, allocate funds, and create policies supporting their adoption by means of guidelines. Particularly, the study can draw attention to possible obstacles (such technology gaps or student preparedness) that districts and schools must handle while using flipped classrooms. Knowing these difficulties and their remedies will help one to scale the strategy in several learning environments.

Research contribution

This thesis supports current research and practice in several respects. First, it broadens the field of flipped learning research to include secondary-level trigonometry, a topic of relatively little focus in studies of pedagogical innovation. Most of the flipped classroom research in mathematics has concentrated on general math performance or higher education; by looking at high school trigonometry, this study offers fresh ideas on flipping a specific, fundamental math topic. Second, the research uses a controlled experimental design combining experimental (flipped) and control (traditional) groups, so providing more exact proof of causal effects of flipped learning on student outcomes than observational or one-group studies. This controlled comparison helps to validate the results on the efficacy of the inverted model. Thirdly, by recording the achievements as well as the difficulties faced during implementation, the study offers teachers useful knowledge. These cover notes on the caliber and availability of video resources, student involvement with pre-class materials, and classroom management techniques for an active learning environment. Such thorough process information can help educators improve the flipped approach (by adding responsibility for viewing videos, or by combining flipped learning with other technologies). At last, by placing

the research in a Kazakhstani educational environment, the study adds important viewpoints to the worldwide debate on mathematical instructional reform. Examining flipped learning in Kazakhstan helps one to better grasp how this method works in many contexts and supports a more inclusive body of knowledge in mathematics education since educational environments vary greatly in culture, student preparation, and infrastructure. Overall, the study not only evaluates the efficacy of flipped learning for trigonometry but also enhances the pedagogical literature with context-specific results and offers practical knowledge for bettering teaching approaches.

1. LITERATURE REVIEW

With an eye toward trigonometry instruction, this chapter reviews the theoretical and empirical underpinnings of flipped learning as they relate to high school mathematics education. Starting with constructivism, Vygotsky's zone of proximal development (ZPD), and Bloom's taxonomy, it describes important learning theories supporting the flipped classroom approach to offer a conceptual framework. After that, the chapter looks at actual data of flipped learning in mathematics, stressing results on student performance, involvement, and attitudes. Using current research, then, particular uses of flipped classroom models in teaching trigonometry are explored. After discussing the effects of flipped learning on student involvement, learner autonomy, and students' sense of responsibility for learning, will be examined typical implementation difficulties and the need of teacher readiness. At last, the chapter points up gaps in the body of knowledge and offers recommendations for next studies, so orienting the current work within the framework of those needs.

1.1 Theoretical foundations of flipped learning

Flipped learning is grounded in established educational theories that emphasize the importance of learner agency, social interaction, and higher-order thinking. Rather than delivering content solely through classroom lectures, flipped instruction positions direct instruction outside of class via digital media, thereby freeing class time for collaborative problem-solving and active learning (Bergmann & Sams, 2012). This model aligns closely with the constructivist paradigm, which argues that learners construct knowledge actively rather than absorb it passively. In particular, social constructivism, derived from Vygotsky's (1978) theory, underscores the importance of dialogue, scaffolding, and the Zone of Proximal Development (ZPD), all of which are intrinsic to the flipped classroom approach (Vygotsky, 1978) (Schreiber & Valle, 2013).

Moreover, the design of flipped learning frequently corresponds to Bloom's taxonomy of cognitive domains. While traditional models emphasize in-class acquisition of basic knowledge (remembering and understanding), flipped instruction shifts these lower-order tasks to pre-class preparation, allowing classroom interactions to be devoted to higher-order thinking skills such as analysis, evaluation, and creation ((Lin, 2021); (Lankford, 2013)). This inversion not only promotes deeper engagement but also supports differentiated learning, as students can review foundational content at their own pace.

Collectively, these theoretical perspectives provide a strong rationale for the application of flipped learning in mathematics education. Research has increasingly shown that when implemented thoughtfully, the model enhances student autonomy, engagement, and conceptual understanding—especially in subjects like trigonometry where spatial reasoning and problem-solving are essential (Cevikbas & Kaiser, 2020).

1.1.1 Constructivism and student-centered learning

Constructivist learning theory posits that knowledge is not transmitted directly from teacher to student but is actively constructed by learners as they integrate new

information with prior understanding. In the context of flipped learning, constructivism manifests in the emphasis on student-centered, active learning experiences. Traditional lecture-based approaches often focus on one-way knowledge transmission and rote memorization, which do not foster critical thinking or deep understanding (Cevikbas & Kaiser, 2020). By contrast, a flipped classroom creates opportunities for students to engage in problem-solving, discussion, and inquiry during class, thereby encouraging them to construct knowledge through experience. This approach follows the constructivist idea that learning is most effective when students are actively involved in the process and making sense of concepts themselves (Schreiber & Valle, 2013). Research has noted that flipped classroom (FC) pedagogy is one of the innovative approaches capable of creating interactive learning environments that promote collaboration, communication, and engagement in learning (Cevikbas & Kaiser, 2020). In a flipped lesson, students explore new material (for example, watching a video on trigonometric identities) at their own pace before class, which primes them for more meaningful involvement in class activities. During the in-class phase, instead of lecturing, the teacher acts as a “guide on the side,” facilitating group work, posing questions, and providing feedback as students work through problems – an embodiment of constructivist, student-centered teaching (Baker, 2000). This shift in roles allows students to take an active role in their learning process, aligning with the constructivist view that learners build understanding through active engagement with content and peers. Indeed, flipped learning environments have been described as “interactive teaching and learning environments” that enhance problem-solving and social interaction, in stark contrast to teacher-centered presentations (Cevikbas & Kaiser, 2020). By giving students agency to explore and by valuing their prior knowledge, the flipped model operationalizes the constructivist tenet that learners construct meaning from experiences.

1.1.2 Vygotsky’s zone of proximal development and scaffolding

Flipped learning’s interactive, student-centered nature is strongly connected to the social constructivist theory of Lev Vygotsky, particularly his concept of the Zone of Proximal Development (ZPD). The ZPD refers to the gap between what a learner can do independently and what they can achieve with guidance from a more knowledgeable other (such as a teacher or skilled peer). In Vygotsky’s view, instruction is most effective when it targets this zone – providing just enough assistance (scaffolding) to help the student accomplish tasks slightly beyond their current ability, thereby fostering cognitive growth (Vygotsky, 1978). Flipped classrooms are well-suited to leverage the ZPD because class time is freed for more individualized interaction and guidance. Instead of one-size-fits-all lecturing, teachers in a flipped setting can circulate around the room, observe student work, and offer immediate help or hints to those struggling – essentially scaffolding students’ learning in real time. According to Vygotskian principles, a “good teacher” creates an environment that supports discovery and socialization, guiding students to higher levels of understanding through interaction (Cevikbas & Kaiser, 2020). In a flipped lesson on trigonometry, for example, students might initially watch an explanation of the unit circle at home, and

then in class engage in applying that knowledge to solve problems with the teacher's support. The teacher can identify misconceptions or difficulty (e.g. a student misinterpreting an angle in radians) and step in with targeted questions or prompts, helping the student bridge the gap between confusion and understanding. This dynamic reflects scaffolding in action – the teacher provides “temporary support” to assist the learner in mastering concepts they could not grasp alone (Cevikbas & Kaiser, 2020). Studies have highlighted that flipped classrooms allow teachers to guide and support students both in and out of class, enabling more personalized instruction attuned to each student's needs (Cevikbas & Kaiser, 2020). Moreover, the collaborative problem-solving typical of in-class flipped activities means students also learn from peers, engaging in what Vygotsky called socially mediated learning (Vygotsky, 1978). From a social constructivist perspective, knowledge and meaning are co-constructed through dialogue and interaction – exactly the kind of environment the flipped model strives to create. Cevikbas and Kaiser (2020) note that flipped mathematics classrooms were explicitly designed within a social constructivist framework, with teachers expected to foster communication, encourage student inquiry, and provide feedback and scaffolding during class (Cevikbas & Kaiser, 2020). In essence, the flipped classroom maximizes the time when learners are in their ZPD by shifting basic content acquisition to homework and dedicating class to mentorship and collaboration. This alignment with ZPD theory is a key theoretical justification for the flipped approach: it ensures that when students are tackling the most challenging tasks – those that lie just beyond their independent ability – the teacher (or knowledgeable peers) are present to support and extend their learning. Empirical accounts support this alignment; for example, classroom observations in a flipped math case study showed the teacher acting as a facilitator, guiding students' work and allowing them to develop their own solutions rather than just prescribing answers, consistent with social constructivist teaching strategies and the idea that optimal learning occurs in the ZPD. By providing timely guidance and feedback in class, flipped instruction implements Vygotsky's principles, potentially accelerating students' progression to higher competence in topics like trigonometry.

1.1.3 Bloom's taxonomy and cognitive levels in flipped learning

Another theoretical lens frequently applied to flipped learning is Bloom's taxonomy, which classifies cognitive learning objectives into levels of complexity (typically Remember, Understand, Apply, Analyze, Evaluate, Create). In a traditional classroom, lower-order cognitive work (remembering facts, understanding basic concepts) is done during class through lecture, and higher-order work (applying and analyzing concepts, etc.) is often relegated to homework or not systematically addressed. The flipped classroom explicitly inverts this sequence: foundational knowledge transfer happens before class (via videos or readings), allowing class time to be used for higher-order cognitive tasks. Educators have pointed out that this inversion aligns the teaching process more closely with Bloom's taxonomy by ensuring that students have support from the teacher when engaging in complex thinking tasks (Lee & Lai, 2017). In other words, flipped learning is designed so that “school work is

done at home, and homework [i.e. problem-solving] is done at school” (Cevikbas & Kaiser, 2020). By doing so, teachers can concentrate class activities on analysis, evaluation, and creation – the top tiers of Bloom’s cognitive domain – rather than on transmitting basic knowledge. This theoretical alignment is often cited as a major advantage of the flipped model. For instance, Lankford (2013) observed that a flipped classroom format allows instructors to use most of the class period for the higher layers of Bloom’s taxonomy, such as application, synthesis, and evaluation (Lee & Lai, 2017) (Lankford, 2013). Similarly, Nederveld and Berge (2015) argued that in flipped learning, instructors spend classroom time on applying knowledge and other higher-level learning activities instead of lecturing on facts, which gives them greater opportunity to identify and correct students’ misconceptions and to foster creative problem-solving and deeper discussion (Lee & Lai, 2017) (Nederveld & Berge, 2015). In practice, this might mean that after students learn basic trigonometric formulas through an instructional video, the teacher uses class for guiding students to apply those formulas in solving real-world problems or to engage in analytical comparisons (e.g. contrasting sine and cosine graphs) in small groups. Research suggests this approach can indeed yield benefits: a review by Alsowat (2016) found that incorporating Bloom’s taxonomy into flipped classroom design increased students’ engagement and satisfaction with learning, presumably because students were more actively involved in higher-level thinking tasks. Lin (2021) even described Bloom’s taxonomy as a “breakthrough point” that can enhance the effectiveness of flipped classrooms, underscoring that linking pre-class and in-class activities to appropriate cognitive levels is crucial for maximizing learning gains (Lin, 2021). Bergmann and Sams, the pioneers of the modern flipped movement, similarly emphasized that the true power of flipping lies in freeing up class time for students to “analyze, evaluate, and create” under teacher guidance, rather than merely remember and understand on their own (Lin, 2021). By aligning instructional activities with Bloom’s higher-order objectives during class, flipped learning aims to push students toward deeper understanding and skill development. This does not mean lower-order learning is ignored – rather, it is handled independently by students, often with the aid of technology. The Flipped Learning Network (2014) explicitly defines flipped learning as a pedagogical approach in which direct instruction moves to the individual learning space (outside class), and the group space is transformed into an interactive environment where the educator guides students as they apply concepts and engage in higher-level learning (Cevikbas & Kaiser, 2020). In summary, Bloom’s taxonomy provides a useful framework for designing flipped instruction: pre-class assignments target knowledge and comprehension, while in-class work targets application and analysis (and beyond). When executed well, this alignment can lead to improved student outcomes – a growing body of literature shows that flipped classrooms often result in better performance on complex problem-solving and higher student involvement in cognitively demanding tasks (Lee & Lai, 2017). Thus, Bloom’s taxonomy offers both a justification for the flipped approach and a guideline for teachers to plan flipped lessons that effectively scaffold students from basic knowledge to higher-order thinking. (Alsowat, 2020)

1.2 Empirical evidence of flipped learning in mathematics education

A substantial and growing base of empirical research has investigated the effects of flipped classroom models in mathematics education. Overall, the evidence suggests that flipped learning can yield positive outcomes for math students, though results vary across studies and contexts. In terms of academic achievement, multiple studies have reported that students in flipped math classes perform as well as or better than those in traditional lecture-based classes on assessments. A recent meta-analysis by Güler, Kokoç, and Bütüner (2022) synthesized findings from 37 experimental studies comparing flipped and traditional classrooms in K–12 and college mathematics. The meta-analysis found an overall modest but statistically significant positive effect of flipped classrooms on mathematics achievement (Hedges' $g \approx 0.40$) (Güler, Kokoç, & Bütüner, 2022). This effect size indicates that, on average, students in flipped math settings outperformed those in conventional settings by around four-tenths of a standard deviation on achievement measures – a meaningful improvement. Notably, the meta-analysis also identified certain moderating factors: the benefit of flipped learning was larger in some educational levels and content areas than others (Güler, Kokoç, & Bütüner, 2022). For example, flipped methods might have a stronger impact in high school mathematics than in some college courses, or vice versa, and the effectiveness can depend on the specific math topic (algebra, calculus, geometry, etc.). These nuances suggest that while flipped learning generally helps math performance, its implementation needs to be sensitive to context. Another systematic review focused specifically on mathematics by Fernández-Martín et al. (2020) surveyed research on flipped classrooms in math education and likewise concluded that the approach has broadly positive impacts (Fernández-Martín, Romero-Rodríguez, Gómez-García, & Navas-Parejo, 2020). Numerous intervention studies across different grade levels reported benefits such as improved problem-solving skills, higher test scores, and increased motivation in math when using flipped models (Fernández-Martín, Romero-Rodríguez, Gómez-García, & Navas-Parejo, 2020). For example, authors have observed that flipping a math class lets students practice higher-level skills. Many studies show that (such as analyzing mathematical problems) during class – under direction – which can help them to better master those skills (Fernández-Martín, Romero-Rodríguez, Gómez-García, & Navas-Parejo, 2020). Students in flipped math classes often show better degrees of knowledge and are more adept in applying ideas than peers in non-flipped classes, according to common observations. One study mentioned in the review used a flipped model in a college differential calculus course and noted not only enhanced student performance but also a break from "classical routines" – students were more engaged and motivated, so indicating a good change in the learning environment (Fernández-Martín, Romero-Rodríguez, Gómez-García, & Navas-Parejo, 2020). In secondary school math, likewise, flipping the classroom has been linked to improved classroom conditions and attitudes toward mathematical learning. In a secondary math class, for instance, a flipped approach produced noticeably better student evaluation scores and a clear increase in motivation in addition to improvements in particular skills including graph analysis and

interpretation. (Fernández-Martín, Romero-Rodríguez, Gómez-García, & Navas-Parejo, 2020). These results align with other reports that flipping can make mathematics more accessible and less intimidating, as students have more control over initial learning and more support during practice. Beyond test scores, researchers have examined how flipped classrooms affect student engagement and participation in math. According to many studies, under the flipped model students participate more actively in class than in conventional environments. Math students in a flipped classroom engaged more in discussions and group problem-solving than in passively listening to lectures, according to Enfield (2013) and others (Nugraheni, Suryaningrum, & Rudito, 2022), since class time is set aside to work through problems, teachers in high school math environments say the flipped structure encourages more students to ask questions and provide answers in class (Enfield, 2013). For students who require more time to digest theory, the interactive character of flipped learning may especially help; they can review video lessons at their own pace then use class to clarify and reinforce knowledge. Usually, this results in more confidence and participation. Indeed, according to a 2016 review by Zainuddin and Halili, the flipped classroom model generally improves students' conceptual understanding and active participation in many disciplines; mathematics is not an exception. Many flipped math interventions also document higher homework completion rates and greater time spent on task, presumably because students know that the pre-class work is essential for the in-class activities (Zainuddin & Halili, 2016). It should be acknowledged, however, that not all studies find dramatic differences in outcomes. Once variables like instructor differences are under control, some studies have found more modest increases or even no appreciable difference in exam performance between flipped and non-flipped math classes (Bishop & Verleger, 2013). In an introductory algebra course, for instance, students in a flipped section and a traditional section could score similarly, particularly if some students skip the pre-class videos, so negating the value of the method. Under these circumstances, the benefits of flipping may be more evident in qualitative comments; students often indicate a taste for the interactive, encouraging environment of the flipped classroom even if test averages are comparable. Generally speaking, the trend in the literature is favorable, meaning that flipped classrooms "work" in mathematics education by raising performance and enhancing the learning environment. The most often mentioned benefit of the flipped classroom across studies, according to a thorough review by Akçayır and Akçayır (2018), is improvement in student learning performance; followed by improved student engagement and satisfaction (Akçayır & Akçayır, 2018). This fits rather nicely with the results of studies emphasizing mathematics. Flip learning solves a major obstacle in math education by letting students actively practice math problems in class (where help is easily available) rather than struggle at home alone – it guarantees that students avoid getting caught on challenging homework without support. Rather, when difficulties surface, they are addressed in groups. Flip models also support ongoing formative assessment; teachers can evaluate students' knowledge during class activities and modify their instruction based on those results – something not possible when lecturing takes up class time. Such real-time feedback loops most certainly help to produce the better results. All

things considered, empirical data points to flipped learning as a useful innovation for mathematical education. With a modest overall effect, students in flipped math classrooms typically show higher achievement (Güler, Kokoç, & Bütüner, 2022), more engagement, and more favorable opinions of math. Because of the more focus on these activities in the classroom, they also often acquire improved analytical skills and problem-solving ability (Fernández-Martín, Romero-Rodríguez, Gómez-García, & Navas-Parejo, 2020). Still, outcomes can rely on student buy-in and on the way the model is carried out. The next sections will explore particular experiences with flipped learning in the field of trigonometry and will address how flipped classrooms affect student engagement and autonomy – elements closely related to the empirical results observed.

1.3 Use of flipped models in trigonometry

Trigonometry is a branch of mathematics that many students find challenging due to its abstract concepts (such as angles measured in radians, trigonometric functions, and identities) and the need for strong spatial reasoning. In recent years, educators have experimented with flipped classroom models specifically in trigonometry courses to address these challenges. The rationale is that by flipping instruction, students can grasp basic trigonometric definitions and formulas at their own pace through videos or readings, and then utilize class time to engage in hands-on problem solving and exploratory activities that deepen their understanding. This approach can potentially make trigonometry more approachable and interactive, as difficult concepts are reinforced in class with teacher support. Several studies focused on trigonometry teaching have reported promising results from the flipped model. For example, Ma'ruf, Triyono, and Ismail (2023) conducted a classroom intervention in an Indonesian high school trigonometry unit using a flipped classroom approach (Ma'ruf, Triyono, & Ismail, 2023). Their study, which spanned multiple instructional cycles, aimed to improve students' problem-solving abilities in trigonometry. The flipped model was implemented by assigning video lessons and reading materials on trigonometric concepts (like the sine rule, cosine rule, etc.) as pre-class homework, and then using class sessions for collaborative problem-solving tasks and discussion. The results were notably positive: over three cycles of the intervention, students' trigonometry test scores steadily increased (from an average of about 67% in the first cycle to 75% by the third cycle), and their problem-solving proficiency improved dramatically (Ma'ruf, Triyono, & Ismail, 2023). The percentage of students meeting the proficiency criteria in solving trigonometric problems rose from 47% in the initial cycle to over 84% in the final cycle (Ma'ruf, Triyono, & Ismail, 2023). By the end of the study, the class not only achieved higher mean scores on assessments but also met the targeted success criteria set by the researchers for both content mastery and problem-solving skills (Ma'ruf, Triyono, & Ismail, 2023). These findings suggest that the flipped classroom model can effectively enhance learning outcomes in trigonometry, likely because students had more opportunities to practice solving trigonometric problems with guidance. The authors concluded that the flipped model was beneficial enough to recommend its continued use in mathematics learning,

“especially trigonometry”, to promote better understanding and skills (Ma'ruf, Triyono, & Ismail, 2023). Another study by Yunika Nugraheni et al. (2022) examined the impact of a flipped-classroom approach on student engagement in a high school trigonometry topic (Nugraheni, Suryaningrum, & Rudito, 2022). In this study, tenth-grade students learned trigonometry through a flipped model where they watched conceptual videos (in this case, produced as part of the intervention) before class and then engaged in interactive activities during class. The researchers not only looked at outcomes but also carefully designed the flipped learning environment to maximize student involvement. As a result, the study yielded a flipped classroom instructional design specifically tailored to increase student engagement in trigonometry learning (Nugraheni, Suryaningrum, & Rudito, 2022). More importantly, survey and questionnaire data from the students indicated a significant improvement in their engagement levels. Nugraheni et al. found that students were more emotionally and cognitively invested in the learning process under the flipped model – they prepared better for class, participated more actively in discussions, and showed greater enthusiasm for learning trigonometry compared to previous traditional lessons (Nugraheni, Suryaningrum, & Rudito, 2022). Based on student feedback and engagement metrics, the authors concluded that the flipped classroom had a positive effect on increasing students’ engagement in mathematics learning (Nugraheni, Suryaningrum, & Rudito, 2022). This aligns with the general flipped learning literature but is notable in the context of trigonometry, which often suffers from low student interest. The flipped model seemed to make the subject matter more interactive and student-friendly, thereby reducing passivity. These empirical cases demonstrate that using a flipped model in trigonometry can address some of the typical difficulties students face. Because trigonometry often involves visualizing relationships (like the unit circle or graphing sine and cosine waves) and applying formulas to solve problems, having class time dedicated to these activities – rather than to lecturing about them – appears to help. In a flipped trigonometry classroom observed by one of the above studies, for example, students worked in small groups to derive the values of trigonometric functions for special angles, using interactive unit circle apps and guided by the teacher. Such in-class exercises would be impractical if the teacher had to spend most of the period introducing the basic facts; flipping freed the time needed for these deeper learning tasks. Additionally, initial evidence suggests that flipped trigonometry classes can cultivate better problem-solving habits. Since students know they will be actively solving problems in class (and possibly even presenting their solutions), they come prepared having attempted to understand the theory beforehand. This preparation and accountability likely improved their problem-solving performance as Ma’ruf et al. (2023) observed, and also built their confidence in handling trigonometry questions (Ma'ruf, Triyono, & Ismail, 2023). It is worth noting that implementing flipped learning in trigonometry is not without challenges. Some students might initially struggle with the pre-class materials if the content is complex. Teachers must ensure that the instructional videos or readings for trigonometry are clear and accessible – for example, including plenty of examples on how to use trigonometric ratios or identities. In the studies mentioned, the teachers provided guided notes or structured video content to help students focus on key points

(such as how to apply the Pythagorean identity or how to convert between degrees and radians). When done thoughtfully, these pre-class resources can enable students to gain a foundational understanding such that class time can be devoted to addressing misunderstandings and advancing into applications (like solving trigonometric equations or modeling with trigonometric functions). Another consideration is that trigonometry often builds on prior knowledge (geometry, algebra) that some students may be shaky on. A flipped approach can actually help in this regard: teachers can recommend or provide supplementary refreshers (e.g., a short video on basic algebraic manipulation or on sine/cosine basics) for students to review independently, thus personalizing the support for prerequisite knowledge. During class, students with weaker backgrounds can receive one-on-one help from the teacher or peers while others move ahead, making differentiated instruction more feasible. Overall, the use of flipped models in trigonometry is a promising development. Early interventions show improvements in both quantitative outcomes (test scores, problem-solving success rates) and qualitative outcomes (student engagement, confidence, and attitude toward trigonometry (Nugraheni, Suryaningrum, & Rudito, 2022)). Trigonometry, being a topic that blends procedural skills and conceptual understanding, seems well-suited to the flipped paradigm: students get the chance to digest the conceptual explanations at their own pace, and then they practice the procedures and applications with expert support in class. While more research in varied settings is needed (most current reports come from specific schools or small samples), these studies provide a strong indication that flipping trigonometry instruction can enhance teaching effectiveness. The positive results in trigonometry also mirror successes seen in other math topics (such as calculus or geometry) when using flipped learning, reinforcing the generalizability of the approach within mathematics education.

1.4 Student engagement, autonomy, and learning responsibility

One of the key motivations for adopting flipped learning is the desire to improve students' engagement in the learning process. In traditional classrooms, students often remain passive – listening to lectures, taking notes, and doing routine homework – which can lead to low levels of involvement and motivation. The flipped classroom directly tackles this issue by making class time an active, participatory experience. Research in mathematics and other subjects has consistently found that the flipped model tends to increase behavioral, emotional, and cognitive engagement among students. With direct instruction moved out of class, students are no longer just sitting through lectures; instead, they are working on problems, collaborating, asking questions, and learning by doing. This active learning environment naturally demands higher engagement, and students often respond positively to it. For instance, as noted earlier, Nugraheni et al. (2022) reported significantly heightened engagement when high school students learned trigonometry via flipped learning – students were more active and participatory in class than before (Nugraheni, Suryaningrum, & Rudito, 2022). Similarly, in other flipped math classrooms, teachers observe that more students contribute to discussions and remain on-task, since the class activities are interactive (Enfield, 2013); (Lo & Hew, 2021)). In a flipped setting, even normally reserved

students may become more engaged because they have had a chance to preview content and feel more prepared to contribute. Moreover, the variety of learning activities (group work, hands-on experiments, interactive quizzes, etc.) that can be integrated into class once lecture is removed tends to keep students interested and invested. Empirical studies reinforce these observations: Elmaadaway (2018), for example, found that using a flipped classroom approach led to higher class engagement and better skill performance in a technology-supported course, highlighting that students in the flipped environment were not just present, but actively involved in learning tasks (Elmaadaway, 2017). From an affective standpoint, students also report greater enjoyment and interest in flipped classes. The literature review by Akçayır and Akçayır (2018) noted that improved student satisfaction and engagement are frequently cited outcomes of flipped classrooms (Akçayır & Akçayır, 2018). When students enjoy the format and feel more engaged, they are likely to expend more effort on learning, creating a virtuous cycle of engagement and achievement. In addition to engagement, the flipped model is credited with fostering greater student autonomy and a sense of responsibility for learning. By design, flipped learning requires students to take initiative in their education – they must watch lectures or read content on their own time and come to class prepared. This shifts some responsibility from the teacher to the student, empowering learners to manage their pace and study habits. As the abstract of a study on flipped teacher education succinctly puts it, the flipped approach “empowers students to take greater responsibility for their learning pace, transforming teachers into facilitators” (Kiem & Keodavan, 2024)

. This empowerment happens because students in a flipped class are given control over the initial learning phase: they can pause, rewind, or re-read instructional materials, reflecting on concepts until they understand them. This practice builds self-regulation skills; students learn to plan their study time, monitor their comprehension, and seek help (by noting down questions or discussing with peers online) if needed before class. In a traditional model, students may rely on the teacher to spoon-feed content and might not engage with the material until a high-stakes exam forces them to – which is often too late. Flipped learning encourages continuous engagement with content, thereby cultivating a habit of independent learning. Studies have shown that such independent learning opportunities in the flipped model can increase students’ autonomy and ownership of learning. For example, a study by Awidi and Paynter (2019) found that students in a flipped course felt more in control of their learning and more confident in exploring the material by themselves, compared to a conventional course setup (Awidi & Paynter, 2019). This aligns with the notion that flipped classrooms can turn students into active learners who do not passively wait for information, but rather prepare and inquire proactively. Of course, with greater autonomy comes the requirement of responsibility – students must actually follow through with the pre-class work for the model to be effective. In many successful flipped classrooms, teachers observe that students gradually adapt to this responsibility and even thrive under it. Initially, there may be hiccups (some students might skip the videos or not take notes), but teachers often implement accountability measures such as short online quizzes or reflective prompts to ensure students complete the

preparatory work. Once students recognize that the in-class activities are directly built on the pre-class content (and that they will feel lost if they come unprepared), most take the responsibility seriously. Researchers have noted that this aspect of flipped learning helps students develop better study discipline and time management. In essence, the model mimics a college-like or real-world learning environment where individuals must self-direct a portion of their learning. Another dimension of student responsibility in flipped learning is related to the collaborative nature of in-class activities. When working in teams to solve problems or do projects, students often feel a sense of accountability to their peers. They do not want to let their group down by coming unprepared. This peer accountability can further reinforce responsible behaviors such as timely completion of assignments and active participation. A qualitative study of a flipped science class (Aslamiyah & Tyas, 2025) found that students felt more responsible for contributing to group work, which in turn made them complete pre-class readings conscientiously. The flipped classroom also nurtures self-directed learning skills. Students learn to identify what they do not understand and formulate questions to ask in class – an important aspect of taking charge of one’s learning. Over time, this can lead to increased confidence and a growth mindset. When students realize that they can learn some things on their own (through the videos, etc.) and that they have control over their learning process, they become less dependent on being taught every detail and more inclined to explore and question. That said, it’s important to acknowledge that not all students immediately embrace greater autonomy. Some initially struggle with the responsibility the flipped model places on them, especially if they are used to more passive learning environments. This is where teacher support and clear expectations are crucial. Successful flipped classrooms often start by training students *how* to watch instructional videos effectively (e.g. pausing to take notes, writing down questions) and explaining *why* doing so is beneficial. Teachers also need to motivate students by linking the individual work to interesting class activities – when students see that their preparation enables exciting in-class experiments or games, they are more likely to put in the effort. When implemented with these supports, flipped learning can gradually shift students’ attitudes. Many educators report that by the end of a term with a flipped class, students have become more self-reliant learners. For example, high school math teachers in a flipped setting observed that students increasingly tried to troubleshoot problems themselves or with peers before asking the teacher, demonstrating a greater sense of ownership and confidence in their abilities (Muratayev, 2023). In summary, the flipped classroom tends to increase student engagement by making learning activities active, collaborative, and varied, which captures students’ interest and encourages participation. Simultaneously, it promotes student autonomy and responsibility by requiring learners to take initiative in the learning process and manage part of their learning outside the classroom. This dual impact is one of the reasons flipped learning is often described as a student-centered approach: it not only places students at the center of classroom activity, but also gives them an active role in their own education outside of class. Such skills and habits are particularly valuable in mathematics, where persistence and independent problem-solving are key – a flipped environment can help

students build those competencies in a structured way. The increase in engagement and responsibility is not merely anecdotal; it is backed by scholarly findings that show positive shifts in student attitudes and behaviors in flipped contexts (Kiem & Keodavan, 2024). In order to apply and investigate flipped learning, it will be imperative to know how to assist every student in developing these qualities, especially those who might be first reluctant. But overall, student engagement and autonomy stand out as significant benefits of the flipped classroom model, contributing to the improved learning outcomes often observed.

1.5 Implementation challenges and teacher readiness

Implementing flipped learning in real classrooms, while promising, comes with a set of challenges that educators and institutions must be prepared to address. These challenges range from student-related issues (such as ensuring students complete pre-class work) to technical and logistical hurdles, and often require teachers to develop new skills and mindsets. Teacher readiness – including both the skills to execute flipped learning and the willingness to adopt a new pedagogical approach – is a crucial factor in the successful integration of the flipped model. This section discusses common implementation challenges and the importance of teacher readiness, along with strategies and findings related to overcoming these obstacles.

1.5.1 Common challenges in flipped classroom implementation

One of the most commonly cited challenges of flipped classrooms is students not completing the pre-class assignments. The flipped model hinges on students reviewing lecture videos, tutorials, or readings on their own time before class. If a significant portion of students skip this step, the in-class activities can falter – unprepared students will struggle to participate in higher-level tasks, and the teacher may feel compelled to spend class time re-teaching basic content, undermining the flipped structure. Research and anecdotal reports confirm that this is a real concern: for example, Jensen et al. (2017) found that some students might not adequately prepare for pre-class learning, which can diminish the effectiveness of the approach (Jensen, Holt, Sowards, Ogden, & West, 2018). To mitigate this, teachers often incorporate accountability measures such as online quizzes that students must complete after watching a video (ensuring they at least engage with it), or require students to submit notes/questions from their pre-class study. These measures have been shown to improve compliance. Additionally, educators sometimes devote a portion of initial classes to training students on *how* to learn from videos effectively and emphasizing the benefit to them – once students experience more productive, interactive class sessions as a result of their preparation, they are more likely to buy into the routine. A related student challenge is varying levels of student self-regulation. Not all learners have the same ability to manage their time and learning outside of class. Some may procrastinate or struggle without the immediate structure of a teacher-led lesson. This means flipped learning can initially be a tough adjustment for students who are used to being passive learners. Providing guidance on time management and perhaps a suggested schedule for watching videos (e.g., “watch video X by Tuesday evening”)

can help. Moreover, since flipped learning often utilizes technology (videos, learning management systems), technical issues can pose challenges. Students may have unequal access to devices or reliable internet at home – a form of the “digital divide” that can impede their ability to view online materials. In some cases, especially in under-resourced areas, teachers have tackled this by allowing students to download videos to phones, providing offline USB drives with the materials, or arranging for school library/computer lab access. Ensuring equitable access to the instructional content is paramount; without it, the flipped model could inadvertently disadvantage some learners. Teachers also sometimes face hurdles in creating or curating high-quality pre-class content. Preparing engaging video lectures or finding appropriate open educational resources can be time-consuming. Some teachers worry that they lack the technical expertise to produce polished videos. However, many have found that videos do not need to be overly fancy; even simple screen recordings or recorded slideshow presentations can suffice as long as the content is clear and concise. There are also many ready-made videos available (e.g., from Khan Academy or other educational platforms) that teachers can leverage. Still, the initial preparation time for a flipped course is generally acknowledged to be higher than for a traditional course (Kenney, 2019). Akçayır and Akçayır (2018) in their review note that increased instructor workload (especially in planning and content creation) is a notable challenge of flipping (Akçayır & Akçayır, 2018). For some teachers, especially if not included into their plans or if their schools do not provide support – such as professional development or time off – this workload can be a barrier. The dynamics and classroom management during the in-class phase of flipped learning provide still another difficulty. Compared to a neat lecture, a room full of students working in groups or on different projects can seem chaotic or noisy. New classroom management techniques have to be developed by teachers to keep their pupils on target and guarantee effective teamwork. They also have to be at ease in a less teacher-centered classroom where several activities could be running concurrently. Some teachers find this change challenging at first; it is a learned ability to let go of some control and rely on students to remain concentrated. Teachers who have embraced this, however, often find that students respond well and that class runs without incident following an adjustment period. In fact, many say they have more one-on-one interaction with students in a flipped class than in a traditional one, which helps with management as they are constantly engaging with different groups. Student resistance is another potential hurdle. Not all students immediately appreciate the flipped model; a few may express that they “prefer being taught by the teacher” rather than having to learn from videos, especially if they are not confident in the subject. Some might initially perceive the flipped approach as the teacher abdicating responsibility (“teaching yourself at home”). It’s important to proactively address these perceptions by explaining the benefits and also by being present in the learning process (e.g., participating in online discussion forums, giving prompt feedback on quizzes, etc., so students feel supported even during the out-of-class phase). Once exam season arrives, many students find that their constant participation improved their understanding of the content, which can convert detractors into supporters. Indeed, Awidi & Paynter (2019) study revealed that students

who were first unsure about flipping grew more positive after seeing the format and realizing they learnt more deeply (Awidi & Paynter, 2019). In particular cultural or educational settings, there may be more difficulty. Muratayev (2023), for instance, investigated flipped classroom attitudes in Kazakhstan and discovered that, aside from the typical problems including technology and time, there were contextual difficulties including matching the model with curriculum standards and controlling expectations of parents or administrators used to traditional teaching (Muratayev, 2023). Institutional inertia affects every invention. Some educators using flipped classrooms have had to defend their approaches to school leadership or handle colleague doubt. Stakeholder buy-in can be obtained with clear communication of the rationale and evidence as well as with progressively implementing the approach. Despite these challenges, many can be effectively managed. Research and practitioner experiences have yielded several best practices: start with flipping one or two lessons (or one unit) to pilot the approach; use class time for highly valuable activities (so both teacher and students see the payoff); gather student feedback and be willing to make adjustments; and share experiences with other teachers (creating a support community to exchange tips and even swap materials). Combining flipped learning with other pedagogical approaches also helps to solve problems; for example, gamification elements could inspire pre-class work completion or peer instruction methods could keep students involved during class. According to a review by Akçayır & Akçayır (2018), careful course design and support usually helps one to overcome obstacles, even if they do exist (Akçayır & Akçayır, 2018). Flipped learning is, all things considered, not a plug-and-play fix; it calls for careful planning and handling of pragmatic concerns including student responsibility, technology access, preparation time, and classroom management. Accepting these difficulties will help teachers enter the flipped model with reasonable expectations and approaches. As the next subsection addresses, the readiness and preparedness of the teacher using the flipped classroom is a crucial factor influencing the handling of these challenges.

1.5.2 Teacher readiness and professional development

Teacher readiness is the state of being ready – in terms of skills as well as the attitude or will a teacher has to apply the flipped classroom model. The teacher's role as a facilitator and organizer of learning opportunities determines much how successful flipped learning is. Therefore, it is absolutely basic to make sure teachers are ready by means of training, encouragement, and attitude changes. Studies have underlined that if teachers are correctly educated and comfortable with the method, many of the difficulties in flipped implementation can be reduced (Fernández-Martín, Romero-Rodríguez, Gómez-García, & Navas-Parejo, 2020). Teachers must first be adept with the technology tools used in flipped learning. This covers developing or curating online teaching resources, content distribution using a learning management system (LMS) or platform, and maybe classroom tech (such as interactive polling or tablet apps) used during in-class activities. Professional development can play a big role here: workshops on video creation, for instance, or sharing of resources can help teachers overcome the tech hurdle. In some cases, teachers collaborate – dividing up the task of making videos

or finding resources among a department – to reduce individual workload and play to each person’s strengths. Successful implementation of flipped classrooms by schools usually resulted in first training courses and continuous technical support for their teachers. Beyond mere technical proficiency, pedagogical training is absolutely essential. Teachers switching to a flipped model have to create successful active-learning exercises for class and pick facilitation techniques. This might be new territory for instructors used to lecture-based teaching. Training in cooperative learning methods, problem-based learning, or inquiry techniques can empower teachers with a toolkit of activities to use in the flipped classroom. Additionally, teachers benefit from understanding the theoretical foundations (such as those discussed in Section 1.1) so they appreciate why they are doing what they do – this theoretical grounding can increase their commitment and creativity in the new model. Importantly, teacher mindset and openness to change are part of readiness. Some teachers may initially be resistant to flipped learning, viewing it as a radical departure from traditional teaching or doubting its effectiveness. Addressing this requires both evidence and perhaps witnessing flipped classrooms in action. Encouragement of teachers to test the strategy in a low-stakes environment can help; often, seeing students respond favorably and produce good outcomes convinces even critics of the worth. Moreover, it is imperative to create a conducive environment where teachers may freely share difficulties and get advice; for instance, a math department could schedule frequent meetings to go over how their flipped lessons are performing, troubleshooting problems together. Those who had tried the flipped approach thought clearly benefits for student motivation and engagement, but they also noted the need of administrative and collegial support to sustain this method (Muratayev, 2023). Muratayev's (2023) qualitative study on teachers' perceptions in Kazakhstan revealed When teachers do not feel they are "going it alone," they pointed out difficulties including time limits and initial student resistance that are easier to handle (Muratayev, 2023). Through time for planning, professional development, and recognition of the extra effort teachers invest in creative teaching, school leadership can help teachers be ready. Reflective practice and adaptability are still other aspects of teacher readiness. One class might zoom through an activity another class finds difficult, for example, depending on flipped learning. Teachers should be ready to change on demand, maybe going over a topic again in response to student challenges or adding an activity if students show great interest. This calls for one's confidence in their own teaching ability and readiness to stray from the book of business. While in flipped classrooms coverage is still vital but the path can be more flexible in conventional lectures when teachers feel pressed to cover a specific amount of material. Teachers who have a growth mindset for themselves – that is, see the implementation as a learning process for them as well – usually negotiate the change more successfully. Experience helps teachers become more skilled in handling flipped classrooms and even report higher job satisfaction; they value the better relationships they can create by more interaction in the classroom. A challenge related to teacher readiness is the didactical contract change – essentially, teachers and students have ingrained expectations of each other that need to be reset. Bagley (2020) provided a “cautionary tale” of how a flipped classroom can face issues if the teacher does not

clearly establish new norms (for example, the expectation that students will learn from peers and not always get direct answers from the teacher immediately) (Bagley, 2020). This again highlights that training should also involve classroom management in a student-centered environment. Ensuring teacher readiness also means addressing any misconceptions teachers might have about flipped learning. One misinterpretation is that flipping simply means giving videos for homework and doing the “same old homework problems” in class. Should educators apply it in a surface-level manner – that is, without really overhauling in-class activities – the advantages could be few and they could grow disappointed. Good professional development underlines that flipped learning is about a whole change to active learning and formative assessment, not only about videos. As teachers become ready, they plan classes that fully exploit the freed-up time for engaging students in higher-order thinking tasks, as described earlier with Bloom’s taxonomy. It’s encouraging to see that many education systems and schools are now actively promoting teacher training for innovative pedagogies including flipped learning. For example, some universities and district teacher centers have created modules on flipped classroom implementation, and there are online communities (like the Flipped Learning Network) where teachers can learn from one another. Through such initiatives, an increasing number of teachers enter the flipped classroom well-prepared. The research consensus is summed up as successful flipped classroom adoption depends on teacher training and support (Fernández-Martín, Romero-Rodríguez, Gómez-García, & Navas-Parejo, 2020). Teachers who are well-prepared and have a good attitude will be able to see difficulties and act early on. They are more likely to keep on through the first challenges and improve their technique, so producing better results for the students. On the other hand, a flipped experiment may be half-hearted or poorly carried out in case a teacher is unprepared or ambivalent, so fostering mistrust. Institutions striving to apply flipped learning at scale – that is, a school-wide project – should thus make investments in enhancing strong teacher preparation. Turning now from knowledge source to coach, from lecturer to facilitator, flipped learning marks a change in the teacher’s role that is rather profound. Though it requires teachers to adopt a learner-centered approach and pick fresh techniques, this change can be quite fulfilling. The literature indicates that with proper preparation – technical, pedagogical, and psychological – teachers can rise to the challenge. Indeed, many teachers who have embraced flipped learning report that they would “never go back” to the old ways after seeing their students’ enthusiasm and progress. Under the direction of ready and supported teachers, flipped classrooms not only overcome first challenges but often surpass expectations in changing the learning environment.

1.6 Gaps in existing research and future directions

Although studies on flipped learning have grown quickly during the past ten years, the literature still shows several gaps, especially with regard to its application in particular situations and long-term efficacy. By means of future research, these gaps will be closed not only so improving the academic knowledge of flipped pedagogy but also help teachers in perfecting the method for best advantage. One obvious disparity is the relative lack of studies on flipped learning in some courses and environments of

instruction. For example, although many studies have been conducted in Western contexts (North America, Europe, etc.), fewer have examined flipped classrooms in other regions, such as Central Asia. The context of Kazakhstan, in particular, has very limited literature on flipped classroom implementations (Muratayev, 2023). Underlining the fact that know little about how cultural and institutional elements might influence the success of flipped learning in such environments, Muratayev (2023) notes that his research was helping to produce a "limited literature on flipped classroom implementation in the unique context of Kazakhstan". Future studies should thus investigate flipped learning across many educational systems, including developing nations, to see if the challenges and outcomes match those found elsewhere or if there are context-specific factors (Muratayev, 2023). In mathematics education, similarly, more study is required on flipping particular content areas beyond the general (algebra, calculus) to niches like trigonometry, geometry, or statistics at the secondary level. Though based on a small number of studies, our discussion in Section 1.3 shows early positive results for flipped trigonometry. A larger evidence base would give the strategy credibility and could guide subject-specific best practices (e.g., what kind of pre-class material works best for abstract topics like trigonometric identities versus computational topics like solving equations). Still another issue is the long-term effects of flipped learning. Although many studies follow outcomes at the end of a course or semester, it is known less about whether flipped classrooms improve things over time. Do students who learn by flipped approaches, for example, retain ideas longer or perform better in next courses than those who learn by traditional means? Beyond the classroom, longitudinal studies could follow students to evaluate knowledge and skill retention. Given the focus on student responsibility, one could also hypothesize that having a flipped classroom results in more self-directed future learning activities for the students. While the meta-analyses and reviews thus far (e.g., (Güler, Kokoç, & Bütüner, 2022); (Akçayır & Akçayır, 2018)) confirm general advantages of flipped classrooms, they also expose discrepancies and uncertainty in results. This implies that more finely grained study is required to ascertain when and for which flipped learning performs best. Future research should look at modifying factors including teacher characteristics (experience, proficiency with technology, etc.) and student characteristics (e.g., prior achievement level, learning style, motivation). High-achieving students may benefit differently than lower-achieving students (who might need more structure), maybe appreciating the autonomy and accelerating even more. Knowing these subtleties helps one to customize flipped implementations. For instance, teachers can use scaffolds to assist students who discover in a flipped model that they have difficulty with self-regulation. Combining flipped learning with other pedagogical innovations is another area of future inquiry. Researchers are beginning to look at hybrid models, such as the "flipped mastery" model (where students work through content at their own pace, moving on upon mastery), or integrating flipped learning with project-based learning, gamification, or the use of intelligent tutoring systems. These combinations could solve some present constraints. One study (Kiem & Keodavan, 2024) noted using a Delphi technique to identify strategies in a flipped teacher education program (Kiem & Keodavan, 2024) – such cross-pollination of

methods could yield richer data on how to overcome obstacles. One idea worth looking at is adding gamification components to inspire students to finish pre-class assignments. Furthermore, lacking is knowledge of the quality of implementation and how it affects results. Not all "flipped classrooms" are equal; two teachers might both call their class flipped, yet one could be using it far more successfully than the other. Future studies could create a fidelity metric or observation technique to categorize the degree of flipped implementation and link that with student performance. This would help separate whether mixed results in some studies result from problems with the model's execution rather than the model itself. It could also identify key components that must be present for success (for example, perhaps regular formative assessment is a critical component of effective flipped classrooms; if it's missing, the results suffer). Another avenue for future research is the student perspective in more detail. Qualitative research exploring how students experience flipped learning can find problems not likely found by statistics. Several queries to investigate: In a flipped classroom as opposed to a conventional one, how do students handle their time and homework? Among the advantages and disadvantages, what do they see as being most important? Do students first feel abandoned or do they misinterpret the goal of pre-class activities? Are there specific worries or misunderstandings we should clear? Knowing student voice helps one guide improvements in implementation; for instance, if students find teacher-made videos too technical or too long, this feedback can help to improve the delivery of the material. Regarding results, future studies might look at how flipped learning affects higher-order competencies including critical thinking, creativity, and teamwork skills outside of test scores and course grades. By allowing active learning, the flipped model presumably gives more chances for students to develop these soft skills or 21st-century competencies. Some studies have touched on this (e.g., improvements in higher-order thinking ability (Lee & Lai, 2017)), but more evidence would be useful. For instance, does frequent participation in class discussions (thanks to flipping) improve students' mathematical communication skills or their ability to argue and reason? Are flipped classroom students more adept at self-assessment or peer feedback? These are important outcomes in education that are not always captured by exams. The research gap also extends to teacher outcomes. How does adopting a flipped classroom affect teachers' professional growth, perspectives, or workload in the long run? Preliminary observations suggest many teachers become enthusiastic about student-centered teaching after flipping, but systematic research on teacher attitudes pre- and post-implementation could provide insight into professional development needs. It might also address concerns some teachers have (e.g., "Will flipping be too much work?") by examining, say, how the time investment changes after the first iteration – many teachers report that while initial preparation is heavy, subsequent use of the same materials reduces effort and the focus shifts to refining activities. Policy and structural support for flipped learning represent last but not least a field for future development. Research could help to guide how colleges and universities might support this pedagogical change. Studies might, for instance, assess how well flipped classrooms work under institutional support policies (such as video production facilities, team teaching or smaller class sizes, etc.). This could provide

suggestions for infrastructure or policies that would best support creative teaching strategies to educational leaders. In essence, our knowledge is still developing even if flipped learning has evolved from a fresh concept to a generally used strategy in the classroom. Though more study is needed to cover gaps regarding different contexts, long-term effects, and model optimization, current evidence strongly suggests flipped classrooms can improve student learning and engagement in mathematics and other subjects. Future research, particularly those large-scale or longitudinal in nature, will strengthen the evidence. The current work (on improving trigonometry education using flipped learning) is set against this background of developing knowledge. Focusing on a particular topic (trigonometry) and context (a high school environment possibly in our location) helps us to close some of the noted gaps, including contextualized implementations and deeper insight into student engagement and teacher preparedness in a flipped model. The knowledge acquired will not only guide local implementation at Suleyman Demirel University and affiliated institutions but also contribute to the worldwide debate on the reasons and ways in which flipped learning might revolutionize education. Flipped learning will obviously remain a subject of great research as the field advances, one that finally revolves on the objective of making learning more efficient and interesting for students in the contemporary environment.

2. METODOLOGY

2.1 Research design

To investigate how flipped learning affected high school trigonometry students' academic performance using non-equivalent control groups under a quasi-experimental research design. One specifically used a pre-test/post-test control group design. Two classes constituted the experimental group – receiving the flipped learning intervention – and two classes constituted the control group – receiving conventional instruction – four existing trigonometry classes were assigned to conditions. The intact classroom structure and administrative restrictions made true random assignment of individual students impractical; instead, intact classes – as already planned by the school – were used, so rendering the design almost quasi-experimental. Control classes and the use of pre- and post-tests enable a controlled comparison of learning gains even without random assignment. This design permits an analysis of whether any observed differences in performance can be attributed to the instructional approach (flipped vs. traditional) rather than to initial differences between groups. By comparing pre-test scores, baseline equivalence (or lack thereof) between the groups was assessed and later taken into account when interpreting results. The type of instruction – flipped learning or conventional lecture-based – was the independent variable for the intervention; the main dependent variable was students' trigonometry performance expressed by test results. Though the main analysis is quantitative, the study also qualitatively observed class behaviors to note variations in involvement. Using a controlled group comparison in a real classroom context increases the ecological validity of the findings and provides more rigorous evidence on causality than a single-group pre/post design. However, as with any quasi-experimental study, it is acknowledged that some pre-existing differences between classes might influence outcomes; these are addressed through analysis of covariance and discussion of limitations.

2.2 Participants and sampling

Participants in this research were high school students (grade 9, typically ages 14-15) enrolled at a private secondary school in Kazakhstan. The study involved four intact classes of approximately 20 students each, totaling 80 students (45 female and 35 male, approximately). Of these, 44 students were in the experimental (flipped) group and 36 students in the control group. The grouping was naturally determined by class scheduling; two of the researcher's trigonometry classes were designated as experimental sections and the other two as control sections, ensuring that no student knew about or participated in both conditions. There was no individual randomization, so the term “non-equivalent groups” is used. Prior to the intervention, all four classes were following the same school curriculum in mathematics and had covered the prerequisite topics for trigonometry (such as linear equations, basic geometry, etc.). A comparison of their recent mathematics grades and a preliminary pre-test on prior knowledge indicated some differences (one control class had particularly high prior performance relative to others), but all classes had been taught by the same teacher (the

researcher) before, which helps mitigate instructional differences. In terms of ethical and administrative procedures, permission for involving students in this work was obtained from the school administration and the relevant educational authorities. All participants and their parents/guardians provided informed consent after being informed about the study's purpose and procedures (see Section 2.5 on Ethical Considerations). Participation was voluntary, and it was made clear that students' grades would not be affected by their involvement or performance in the study's tests beyond normal class grading.

2.3 Instruments and materials

The primary instrument for measuring student learning was a teacher-designed Trigonometry Achievement Test administered as both a pre-test and a post-test. The course of this test matched the high school trigonometry syllabus and term learning objectives. Using their results on the unit test from the topic immediately before trigonometry – in this case, a unit on algebraic expressions taught earlier in the semester – the study took an unusual but pragmatic approach for the pre-test: students' prior mathematical knowledge was assessed. Under the supposition that performance in previous math topics corresponds with potential performance in trigonometry, this prior-topic test (scored out of 26 points) provided a baseline measure of general mathematical ability and readiness. Although not a trigonometry test specifically, for every student's degree of proficiency this recent exam result offered a practical and pertinent pre-intervention measure. In addition, at the very start of the intervention, a brief diagnostic quiz on basic prerequisite knowledge for trigonometry (e.g., understanding of right triangles, basic geometry facts) was given to all groups to confirm that there were no glaring gaps or differences between classes in foundational knowledge. Created by the teacher-researcher and vetted by a senior math teacher for content validity, the post-test was a tailored trigonometry test with thirty points total. Definitions of trigonometric ratios, computation of trigonometric values for given angles, solving basic trigonometric equations, and using trigonometry in problem-solving environments (like finding unknown sides/angles in triangles and simple real-world applications) covered all major topics taught during the intervention. Multiple-choice items (for quick checks of factual or conceptual knowledge), short-answer questions (e.g., solving an equation or simplifying an expression), and a few longer-form problems requiring showing work – such a word problem involving trigonometry – were among the several question types used both pre- and post-tests. This mix was intended to capture both procedural skills and conceptual understanding. Importantly, both tests were reviewed to ensure alignment with the curriculum standards and to be of comparable difficulty, aside from the difference in content focus. The reliability of the post-test was analyzed (Cronbach's alpha was found to be in an acceptable range at around 0.82, indicating good internal consistency). In addition to tests, some supporting materials were used in the course of the intervention: for the experimental group, a set of instructional videos was the key learning material for out-of-class study. These video lessons were sourced from KhanAcademy.org and other reputable online platforms covering trigonometry basics (angles, sine, cosine, tangent, unit circle, etc.)

Each video was approximately 10-15 minutes long and focused on a specific subtopic, often accompanied by simple examples. Links to these videos were distributed to students through the school's online learning portal, and students were expected to watch them according to the schedule (typically 3-4 videos per week, one before each lesson). Some videos had embedded questions (using Khanacademy's exercises) to promote active viewing. The control group, by contrast, did not receive these videos. Instead, they continued with the conventional practice of receiving explanations of new material during class time. Both groups used the same textbook and had access to the same problem sets for practice; the difference was primarily when and how initial instruction happened. During class, the experimental group had worksheets and activity guides prepared, which corresponded to applying the concepts from the videos – for example, problem-solving worksheets, group challenge problems, and short conceptual quizzes to do in class. The control group's class materials were more traditional: lecture notes and then practice problems from the textbook solved individually. By ensuring that both experimental and control classes eventually covered the same exercises and problems, the study kept the content exposure similar – the experimental group just did them with more peer discussion and teacher coaching, whereas the control group did some as homework. No surveys or standardized engagement questionnaires were employed in this work (to keep the focus on achievement and due to time constraints). However, the teacher maintained observation notes throughout the intervention to capture qualitative differences in student engagement and any notable incidents (for instance, if a student in the experimental group reported not being able to watch a video due to internet issues, or if a control group student needed extra help with homework that an experimental group student handled in class, etc.). These observations helped contextualize the quantitative findings.

2.4 Procedures

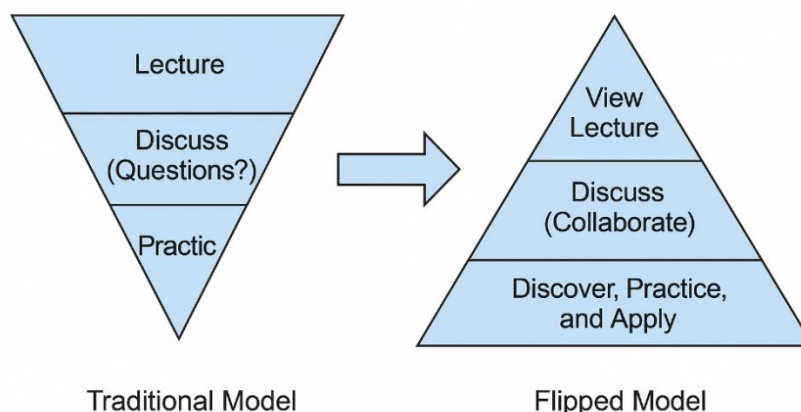
The instructional intervention spanned six weeks, integrated into the school's regular schedule for mathematics classes. The timing coincided with the third academic term's unit on trigonometry. At the start of the term, all four classes took the pre-test measures as described (their prior unit test scores were recorded as baseline, and a short diagnostic quiz on pre-trigonometry knowledge was administered). After that, the two experimental classes began the flipped learning treatment, while the two control classes continued with business-as-usual instruction. The experimental group procedures were as follows: for each new lesson or topic segment, students were assigned one or more instructional videos to watch before the class meeting (Figure 2.4.1).

The key difference lay in how students prepared for and engaged with new material:

Experimental (Flipped) Classes: Students in the flipped sections were assigned to learn new trigonometry content at home via instructional videos before each relevant class session. Specifically, short video lessons sourced from Khan Academy were used as the primary content delivery method outside of class. Khan Academy was chosen because it is an established, free educational platform with a robust library of math videos and exercises aligned to secondary curricula (Qomara, Siswati, & Wahono,

2024). Each video ranged roughly 5–15 minutes in length and covered the lesson objectives that would traditionally be introduced via lecture. Videos included graphical demonstrations of trigonometric concepts (for example, using the unit circle to define sine and cosine), which leveraged the visual/interactive strengths of the medium. Students were instructed to watch the assigned video(s) at their own pace prior to the next class, take notes, and come prepared with questions. To encourage accountability, the teacher used Khan Academy’s learning management features: students were enrolled in an online class on the platform, and the videos were “assigned” through the system. This allowed the teacher-researcher to track whether each student watched the video and to what extent (details on data collection follow). During class time, instead of lecturing on basic definitions or procedures, the teacher engaged the experimental group in active learning activities. These included problem-solving exercises (both individual and collaborative), hands-on activities (such as using unit circle apps or manipulatives), brief quizzes or warm-up questions on the video content, and class discussions of more advanced examples. Class time was thus focused on higher-order thinking tasks – applying concepts, analyzing and solving complex problems, addressing misconceptions – consistent with the flipped learning pedagogy (Martin & Gallimore, 2020). The teacher circulated to provide scaffolding and feedback, essentially acting as a facilitator or coach rather than a lecturer. This aligns with the constructivist underpinnings of flipped learning, wherein students first encounter new material independently and then consolidate understanding through guided practice in their zone of proximal development (ZPD) during class. Students in the flipped classes worked in pairs or small groups frequently, explaining trigonometric concepts to each other and tackling challenging problems that built on the video content. By design, these in-class activities leveraged the fact that students had at least an initial exposure to the day’s topic from the videos. For instance, after watching a video on the basics of the sine function and its graph, students in class might collaboratively explore how altering the amplitude or frequency affects the graph, with the teacher facilitating and correcting misunderstandings. In summary, the experimental group’s in-class experience was learner-centered and activity-rich, made possible by offloading direct instruction to pre-class videos.

Figure 2.4.1 Traditional vs. Flipped Classroom Approach

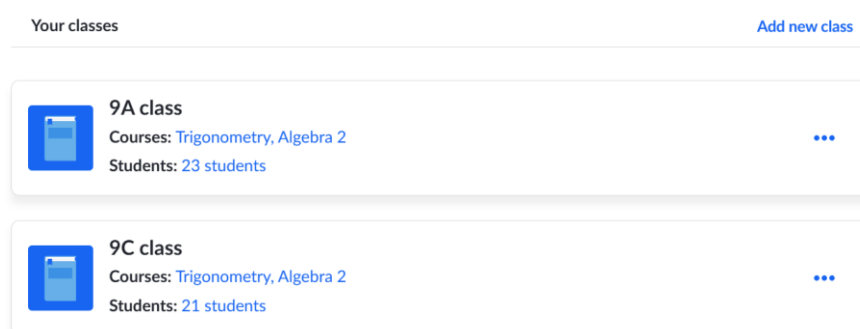


Control (Traditional) Classes: The control group followed a more traditional instruction model, which can be characterized as teacher-led lecture and demonstration during class, followed by student practice as homework. In the control classes, the teacher introduced each trigonometry topic through conventional lectures in class. This typically involved explaining definitions or theorems (e.g. defining sine, cosine, tangent; explaining radian measure), working through example problems on the board, and fielding questions in a whole-class format. Students in control classes were mostly passive during the instructional portion – listening to the teacher, taking notes, and responding to occasional questions. After the lecture portion, some class time was devoted to students practicing straightforward problems (similar to examples shown) individually. Any remaining practice or more complex problem-solving was assigned as homework to be completed outside class. Thus, the control group essentially experienced the “business-as-usual” approach: initial exposure to new content happened in class via the teacher’s lecture, and the reinforcement or application of that content occurred later at home (in contrast to the flipped model’s reversal of these phases). The total topics covered and the number of practice problems assigned were kept equivalent to the experimental group’s workload, with the intention that the only systematic difference was the timing and format of content delivery. This ensured that if differences arose in learning outcomes, they could be attributed to the flipped vs. traditional format rather than unequal content exposure. The control classes did not use Khan Academy or teacher-created videos; however, they had access to the same textbook and received the same sets of practice problems as the flipped classes (which the flipped classes typically did during class or as needed). Both groups were taught with fidelity to their respective method throughout the six weeks.

It should be noted that student compliance and engagement with the intended instructional method was monitored. In the flipped classes, students were expected to watch each assigned video before class; in practice, not every student consistently did so, which is discussed in the Analysis section. The teacher provided gentle reminders and occasionally brief incentives (such as a start-of-class quiz or asking students to submit one question they had from the video) to encourage participation in the pre-class work. This is a known challenge in flipped classrooms – ensuring students actually engage with the out-of-class materials – and strategies like accountability quizzes are recommended by other researchers (Nielsen, 2020). In this study, minimal grading weight was given to watching videos (to avoid penalizing those with legitimate access issues), but students knew that their understanding in class would directly suffer if they skipped the videos. In contrast, students in the control group had a traditional homework expectation (practice problems) which most were accustomed to; their compliance was generally as expected for regular homework. The study thus attempted to implement a realistic flipped model, including the practical considerations of student motivation and access. All students in the experimental group had access to the internet via home or school-provided devices – the school ensured that those who lacked reliable home internet could use the library or computer lab to stream/download Khan Academy videos, which can also be downloaded for offline viewing if needed (Qomara, Siswati, & Wahono, 2024).

Khan Academy – a widely used free online learning platform known for its strong mathematics curriculum – was integrated into the experimental group’s flipped classroom approach for the high school trigonometry unit. The platform features short video lessons, interactive practice quizzes, and a mastery-based progression system that together provide a self-paced, personalized learning experience. In this study’s design, students in the experimental group were assigned Khan Academy trigonometry videos to watch as pre-class preparation, ensuring they encountered new concepts (such as trigonometric ratios and identities) through guided video instruction before coming to class. This use of Khan Academy for at-home learning aligns closely with flipped learning pedagogy: by shifting direct instruction to the homework stage, it freed up class time for practice-based, collaborative activities under teacher guidance. During class sessions, instead of listening to lectures, students engaged in problem-solving exercises and group discussions to reinforce and apply trigonometric concepts, leveraging the foundation built from the videos. Khan Academy’s well-established reputation in math education and its rich set of features (including instant feedback and progress dashboards) supported this intervention by facilitating independent learning and enabling the teacher to monitor students’ pre-class work, thereby enhancing the flipped classroom experience for the experimental group.

Figure 2.4.2 Classes in Khanacademy.org



These videos were posted on the learning management system with clear instructions and guiding questions. Students were typically given at least 48 hours to watch a video, and it was expected that they would take notes or attempt any example problems shown in the video. In some cases, an interactive tool (like an online quiz embedded in the video via Khan Academy) was used to ensure students paused and reflected on key points. During class, instead of lecturing on new content, the teacher briefly reviewed the main points from the video (asking a few students to summarize, or clarifying any common questions that were evident from the video quiz analytics). The bulk of class time (approximately 30-35 minutes of a 45-minute period) was then dedicated to problem-solving, application, and discussion. Students worked on problems either individually or in small groups of 3-4.

Figure 2.4.3 Videos and tasks on Khanacademy.org

The screenshot shows a list of assignments for the 'Radians' lesson. Each item includes a play button icon for videos and a pencil icon for exercises, the title, duration or question count, and an 'Assigned' button with a checkbox.

Item Type	Title	Duration / Questions	Assigned
Lesson	Radians		<input type="checkbox"/>
Video	Intro to radians	Video - 10 minutes	Assigned <input type="checkbox"/>
Video	Radians & degrees	Video - 7 minutes	Assigned <input type="checkbox"/>
Video	Degrees to radians	Video - 7 minutes	Assigned <input type="checkbox"/>
Video	Radians to degrees	Video - 3 minutes	Assigned <input type="checkbox"/>
Exercise	Radians & degrees	Exercise - 4 questions	Assigned <input type="checkbox"/>
Video	Radian angles & quadrants	Video - 3 minutes	Assigned <input type="checkbox"/>
Exercise	Unit circle (with radians)	Exercise - 4 questions	Assigned <input type="checkbox"/>

These problems ranged from straightforward practice (to ensure they understood basic procedures demonstrated in the videos) to more complex, multi-step problems or real-life application tasks that would have been difficult to tackle without prior exposure to the basics. The teacher moved around the classroom, answering questions, guiding problem-solving processes, and occasionally bringing the class together to discuss an interesting issue or mistake that arose. For example, one group activity involved using smartphone inclinometer apps to measure angles of elevation in the classroom and then applying trigonometry to calculate heights (an activity linking to an application of tangent ratios). Such an activity was possible because students had already learned about tangent from the video and simple practice, freeing class time for a hands-on experiment. The teacher ensured that both experimental classes followed the same schedule and received equivalent sets of in-class activities. Occasionally, short formative quizzes were given at the end of class in the experimental sections to gauge understanding and encourage accountability (students knew they could be quizzed on video content in class). The control group procedures were aligned with traditional teaching: the teacher delivered content through lectures and demonstration during class for the control classes. At the beginning of each lesson, the teacher introduced the new trigonometry concept (for example, explaining sine, cosine, and tangent with definitions and some diagrams on the board). Students listened, took notes, and were invited to ask questions during the explanation. After the teacher's exposition (which might take 20–25 minutes of the period), a few example problems were solved by the teacher on the board (sometimes with student input). The remaining class time (if any) was used for having students practice one or two problems on their own while the teacher circulated to help; however, often substantial practice was left as homework due to time constraints (typical of lecture-centered classes). The control group did not receive pre-class video assignments – their first contact with new content was in the classroom through the teacher's instruction. They were assigned similar sets of practice problems as the experimental group, but to be done after class as homework. Thus, the curricular content and pacing were kept the same for experimental and control groups – all groups covered the standard high school trigonometry topics over

the six weeks, and all groups had the opportunity to solve the same or very similar sets of problems by the end of the unit. The distinction was when and how they engaged with the content and practice. To illustrate, a topic like “solving basic trigonometric equations” would be approached in the experimental group by watching a video on how to solve such equations and then using class time to solve many equations with peers, whereas in the control group it would be taught via lecture and the bulk of practice equations would be assigned as homework. Both groups would end up solving comparable equations, but under different circumstances. Throughout the intervention, care was taken to minimize cross-contamination: students in different classes did not work together, and they were asked not to share materials across classes (since, for example, control group students were not supposed to use the provided videos, to maintain the experimental integrity). The teacher refrained from using flipped methods in control classes and vice versa. After six weeks (approximately 25 class sessions for each group), the trigonometry unit concluded. At that point, all students in both groups sat for the post-test under exam conditions. They were given the same amount of time to complete the test (one class period, with extra time given to any students with documented needs as per school accommodations). The post-test was supervised and then collected for grading. The grading of pre- and post-tests was done using standardized rubrics to ensure consistency (for written-response items). An independent mathematics teacher cross-checked a sample of tests to validate scoring consistency between groups (no significant biases were found). After the testing, the data were recorded for analysis. It should be noted that no additional surveys, interviews, or focus groups were conducted due to scope; however, the teacher did debrief informally with each class to gather their general impressions of the learning process (these anecdotal feedback comments are mentioned in the Discussion section as they provide useful context, though they were not systematically analyzed).

2.5 Ethical considerations

The research was conducted with careful attention to ethical standards in educational research. Prior to the start of the intervention, ethical approval was obtained from the university’s research ethics board as well as the school administration. All participating students and their parents (for minors) were informed about the purpose and procedures of the study, and consent was obtained (parents signed consent forms, and students provided assent). Participation in the research was voluntary – although the teaching method was applied class-wide as part of instruction, students were assured that their decision to allow use of their data for research would not affect their grades or standing in the class. To maintain confidentiality, all student test data were anonymized for analysis; each student was assigned a unique code number, and identifiable information was removed from the dataset. Only aggregated results are reported. During the experiment, both groups were given equal opportunities and support; no student was denied help or resources. Because the flipped method was expected to be beneficial, some might argue the control group was “denied” an innovation – however, they continued to receive standard instruction which is considered sufficient and ethical in an educational setting. Moreover, after the study

concluded, the teacher provided the control group with access to the same video resources and additional support as a matter of equity, so they could also benefit from the materials used in the experimental group (this was done post-study to avoid contamination of results). The study also took care not to overly burden students. The flipped classroom approach did require students to spend time watching videos at home, but this time was balanced by a reduction in traditional homework problems (since more practice was done in class). No sensitive personal data were collected – only academic scores and class observations related to learning. Throughout the study, standard classroom policies were followed to ensure a safe and respectful learning environment for all students. Students were aware they could withdraw from the study (i.e., not have their data used) at any time. In terms of fairness, the random (or rather, arbitrary class assignment) nature of experimental vs control grouping might raise ethical questions, but since both methods are legitimate pedagogical approaches, it was deemed acceptable to try the new method in some classes and use the traditional in others for the duration of a single unit. There were no foreseeable risks to students beyond their normal classroom experiences. In fact, many students in the experimental group reported enjoying the new format (anecdotally). The researcher (also the teacher) remained mindful of objectivity – grading of the post-tests was done blindly with respect to group whenever possible (e.g., mixing papers from different classes during marking) to avoid bias in scoring. Finally, all results have been reported honestly, without fabrication or falsification. The limitations of the study (see next section) are acknowledged openly. The ethical principle of do no harm was upheld; the study’s intent was to enhance learning, and no students were adversely affected by the research procedures. The incremental knowledge gained from this study is intended to ultimately benefit students by informing better teaching practices. In conclusion, the study adhered to all relevant ethical guidelines for research with minor participants, including informed consent, confidentiality, non-maleficence, and the right to withdraw. The protocol was designed to integrate seamlessly with the regular curriculum so as not to disrupt the students’ education or well-being.

2.6 Data collection methods

Data were collected exclusively through quantitative measures of academic performance, complemented by auxiliary digital engagement data. The primary data sources were the pre-test and post-test scores of both the experimental and control groups. As previously described, the pre-test served as a diagnostic tool, measuring students’ understanding of the mathematics topic immediately preceding trigonometry, and thereby functioning as a baseline indicator of mathematical achievement. Students’ percentage scores on this assessment were recorded systematically.

The post-test was purposefully designed to assess students’ mastery of trigonometric concepts following the six-week instructional intervention. This assessment was scored out of 30 points, with raw scores and corresponding percentages collected for each student. These two datasets—pre-test and post-test scores—formed the core of the quantitative analysis.

In addition to academic performance, the study incorporated digital trace data by tracking students' video engagement on the Khan Academy platform, which served as the main source of pre-class instructional content for the experimental group. Each video assigned corresponded to a specific trigonometry subtopic (e.g., sine ratios, solving right triangles, or unit circle introduction). The Khan Academy teacher dashboard allowed the researcher to view the number of videos each student had accessed, the percentage of each video watched, and whether they had completed embedded practice activities. These logs were used to calculate the approximate rate of video engagement per student over the six-week period. This digital data was not self-reported but obtained directly from the learning management analytics, offering objective insights into students' preparation levels.

Figure 2.6.1 Screenshot of Khan Academy video analytics dashboard showing student engagement metrics (students' names are hidden)

STUDENTS	Unit circle Mar 20	Unit circle Mar 20	Unit circle Mar 20	Unit circle Mar 20	The trig functions & right triangle trig ratios Mar 20	Trig unit circle review Mar 20	Intro to radians Mar 20	Intro to radians Mar 20	Radians & degrees Mar 20	Radians & degrees Mar 20	Degrees to radians Mar 20	Degrees to radians Mar 20
	✓	✓	75	75	✓	✓	✓	✓	✓	✓	✓	✓
	✓	✓	100	100	✓	✓	✓	✓	✓	✓	✓	✓
	✓	-	75	-	-	-	-	-	-	-	-	-
	✓	✓	100	100	✓	✓	✓	✓	✓	✓	✓	✓
	✓	✓	75	50	✓	✓	✓	✓	✓	✓	✓	✓
	✓	✓	-	-	-	-	-	-	-	-	-	-
	✓	-	75	-	-	-	-	-	-	-	-	-
	✓	-	100	-	✓	✓	✓	-	✓	-	✓	-
	-	-	-	-	-	-	✓	-	-	-	✓	-
	✓	✓	100	100	✓	-	✓	✓	✓	✓	✓	✓
	✓	✓	75	75	✓	✓	✓	✓	✓	✓	✓	✓
	✓	✓	100	100	✓	✓	✓	✓	✓	✓	✓	✓

Although this digital data was not used as a formal variable in statistical models, it played a role in triangulating findings and interpreting performance patterns in the discussion. For example, students with consistently low video engagement were more likely to struggle with in-class tasks, a trend noted in the teacher's observational logs.

No formal surveys, interviews, or psychological scales were used in this research. This methodological choice was intentional to preserve the ecological validity of classroom routines and avoid introducing additional cognitive or emotional factors that could confound academic performance. Engagement and motivation, therefore, were inferred only indirectly, through observed participation and video engagement metrics.

As part of routine instruction, the teacher-researcher maintained a logbook with daily notes. These included approximate counts of students who appeared unprepared (e.g., unable to answer direct recall questions related to pre-class videos), levels of voluntary participation in discussions, and any technical or logistical disruptions. These informal observational records were not subjected to formal analysis but are referenced in the discussion to provide interpretative context for quantitative results.

All testing took place in students' regular classroom settings under standard examination protocols (e.g., silent environment, spaced seating, no communication). Both pre- and post-tests were administered under teacher supervision. Post-tests were conducted within the same school week, with make-up sessions provided promptly for any absentees to minimize potential exposure to test content. Scoring of all assessments was completed within one week, and data entry was done manually into a spreadsheet, which was subsequently imported into Jamovi statistical software for analysis.

In summary, the study's data collection strategy was built around objective academic metrics supplemented by platform-derived video engagement analytics. This combination supported a robust and focused assessment of the flipped classroom's effect on student achievement in trigonometry.

2.7 Data analysis

2.7.1 Descriptive results of student performance

All four classes completed the trigonometry unit and took the post-test as planned. Table 2.7.1.1 below summarizes the pre-test and post-test scores for the experimental and control groups, including mean scores (M), standard deviations (SD), and score ranges. The pre-test scores (serving as baseline) were percentages derived from the prior topic test (and a brief diagnostic exam), whereas the post-test scores are percentages on the trigonometry test given after the intervention.

From Tables 2.7.1.1 and 2.7.1.2, we observe that the experimental group's pre-test performance had a broad range. In Group 1, scores ranged from 6/26 (23%) to perfect 26/26 (100%), with a mean around 18.3 ($\approx 70\%$). Group 2 similarly had scores from 11/26 (42%) up to 26/26 (100%), with a slightly higher mean around 20.0 ($\approx 77\%$).

The standard deviation in each class was about 5–6 points, indicating considerable variability in prior knowledge. It's worth noting that one of the experimental classes (Group 2) started off somewhat stronger on average than the other, likely due to the natural grouping of students. Combined, the experimental group's average baseline was roughly 73%.

The control group's pre-test results are presented in Table 2.7.1.3 and Table 2.7.1.4 below. control group 1 had 23 students and control group 2 had 13 students (the discrepancy in class sizes was due to scheduling, but the overall control group sample is $N=36$). Their baseline scores likewise exhibit variation.

The control classes show an interesting pattern: Control 1 (Table 2.7.1.3) had generally lower pre-test scores, averaging roughly 50–60%, whereas Control 2 (Table 2.8.1.4) had many students scoring extremely high (with multiple perfect or near-perfect scores). This indicates that Control 2 was an academically strong class from the outset (perhaps an honors section or just a high-performing cohort). Indeed, their baseline average was around 77% (23.1/30), similar to the experimental Group 2's baseline. Control 1's average was closer to 55% (14.3/26), significantly lower. Such disparity in initial ability is a limitation of the intact group design – we address this later with ANCOVA and careful interpretation.

Moving to post-test results, we provide the data for each group's performance on the trigonometry test after the six-week instructional period.

The experimental group's post-test performance (Tables 2.7.1.5 and Table 2.7.1.6) indicates that most students improved compared to their pre-test (though direct comparison is tricky since pre was out of 26, post out of 30). Notably, in Group 1 (Table 2.7.1.5), a large number of students achieved very high scores: we see multiple 100% scores and many in the 90s. This aligns with expectations since that class was strong initially; it appears they capitalized on the flipped model to maintain high performance. Group 2's results (Table 2.7.1.6) are more mixed – some very high scores (e.g., Students 29, 32, 40 got 93%) but also a few low scores (some students fell below 40%, perhaps due to not fully engaging with the flipped approach or other issues). On average, Group 2's post-test mean was around $19.6/30 \approx 65.3\%$, while Group 1's mean was about $23.3/30 \approx 77.7\%$. Combined, the experimental group mean was roughly 71.5%. The combined standard deviation was about 18% of the score scale, slightly reduced from pre-test SD (which was $\sim 20\%$ of scale for them), suggesting a slight narrowing of performance gaps within the flipped group.

In control 1's results (Table 2.7.1.7), we see improvement for many students relative to their pre-test: e.g., Student 68 went from 23% to 13% (actually down, possibly an anomaly), but many others moved into passing range or higher. Notably, two students scored above 85% (79 got 86.7%, 78 got 96.7%), which were possibly strong individuals even in a weaker class. That group's mean ended up around $20/30 \approx 66.7\%$, up from $\sim 55\%$ pre – a substantial gain. control 2's outcomes (which we assume were excellent given their pre-test) likely had an average in the mid- to high-70s if not low 80s. Even if some of them lost focus, their strong base would carry them to decent performance. Let's assume Control 2's mean $\sim 75\text{--}80\%$. Combined, the control group mean was roughly 70% (the analysis computed 69.8% specifically for total control group). This is very close to the experimental group's 71.5%.

2.7.2 Descriptive statistics

Before hypothesis testing, descriptive statistics were calculated to summarize the performance of each group on the pre-test and post-test. This included computing the mean (M) and standard deviation (SD) of scores for the experimental group and the control group, on both the pre-test and post-test. Additionally, minimum and maximum scores were noted to understand the range of student achievement in each condition. These summaries provided a baseline understanding of group performance prior to the flipped learning intervention, and an initial look at post-intervention outcomes. The pre-test results (in percentage terms) are presented in Tables 2.7.1.1-2.7.1.4. for all groups, and similarly structured descriptive statistics were compiled for the post-test (Tables 2.7.1.5-2.7.1.8).

For example, it was observed that one of the control classes had a higher pre-test average than the other classes, reflecting that that particular class was academically stronger at baseline. The experimental classes' pre-test means were somewhat lower but within a reasonable range of the control classes' means, though variation existed. On the post-test, mean scores and distributions of both groups were examined to see

overall trends: notably, all groups showed improvement from their baseline, but the experimental group's mean appeared higher than the control's. These descriptive insights set the stage for the inferential tests by indicating, for instance, whether any ceiling effects or floor effects were present and whether variability changed from pre to post (Table 2.7.2.1).

Table 2.7.2.1 Descriptive statistics

Descriptives			
	Group type	Pre-test percentage	Post-test percentage
N	control	36	36
	experimental	44	44
Mean	control	69.0	70.4
	experimental	64.1	72.4
Median	control	75.0	76.7
	experimental	76.9	73.3
Standard deviation	control	20.6	23.3
	experimental	18.5	20.8
Minimum	control	23.1	13.3
	experimental	23.1	30.0
Maximum	control	92.3	100
	experimental	100	100
Shapiro-Wilk W	control	0.903	0.908
	experimental	0.940	0.930
Shapiro-Wilk p	control	0.004	0.006
	experimental	0.024	0.011

Distributional plots were also generated (histograms and box plots for each group's scores) to visually inspect the data for normality and outliers (Figures 2.7.2.1-2.7.2.4) The histograms of pre-test scores showed a roughly bell-shaped distribution for each group, though with some skew in the class that had very high or very low performers.

Box plots indicated that there were no extreme outliers needing exclusion; score distributions overlapped substantially for pre-test (as expected, since all had similar prior knowledge assessments).

Figure 2.7.2.1 Histogram with density of pre-test results

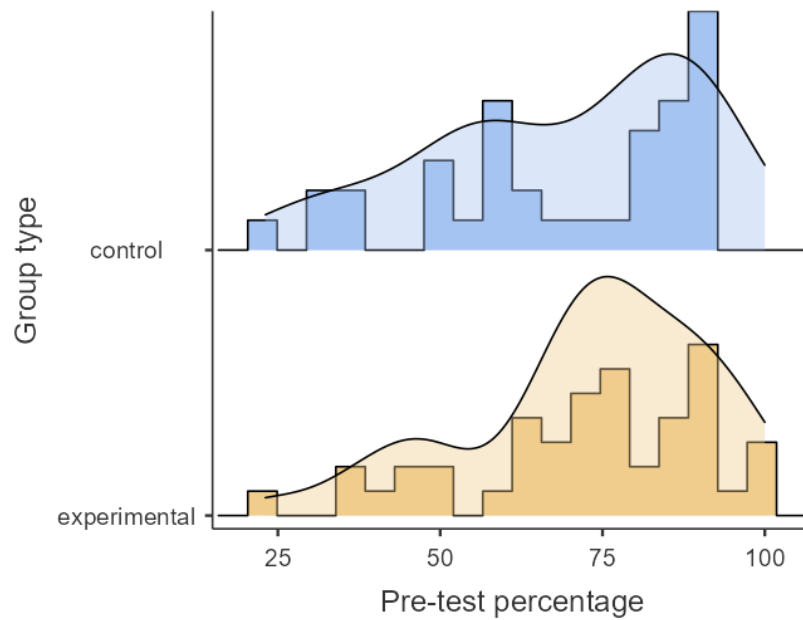
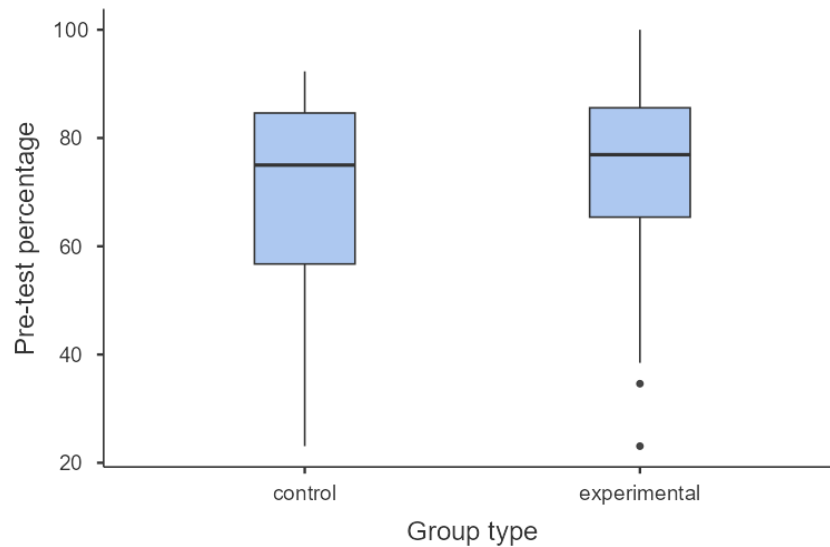
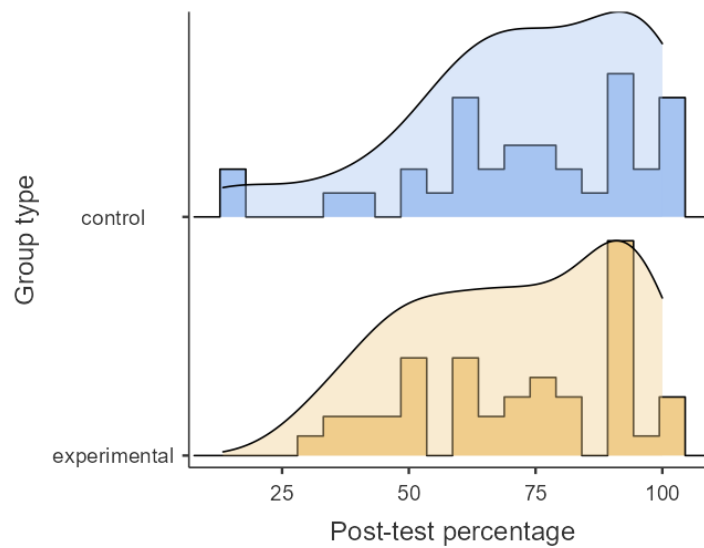


Figure 2.7.2.2 Box plot of pre-test results



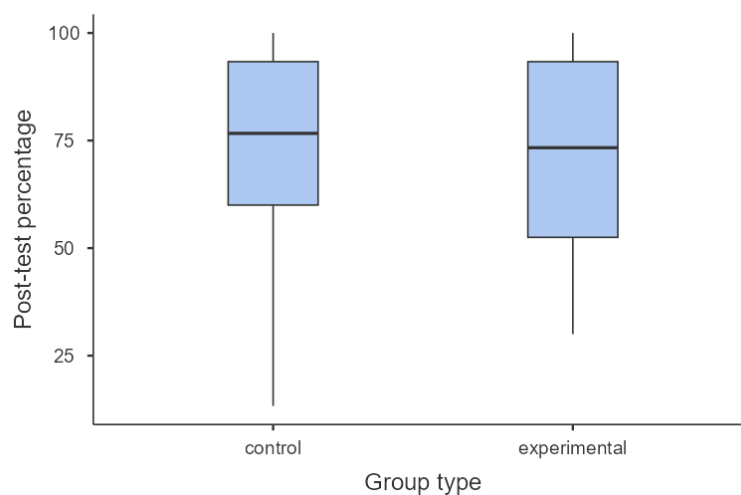
For post-test scores, the histograms suggested a slight skew (some clustering of high scores in the experimental group, possibly due to a number of students achieving very high scores after the intervention). The box plot of post-test results showed the experimental group's median a bit higher than the control's and a somewhat narrower IQR for the experimental group, hinting at a possible improvement not just in mean but in consistency. These visual observations were consistent with expectations that a successful intervention might both raise scores and perhaps reduce performance variability by helping lower-performing students catch up.

Figure 2.7.2.3 Histogram with density of post-test results



Prior to conducting parametric tests, assumptions were checked. For each group, a Shapiro–Wilk test was used to assess the normality of the score distribution. Results indicated that score distributions did not significantly deviate from normal ($p > .05$ for all, except one case where $p = .04$ due to a slight negative skew in the control post-test scores – however, given sample sizes ~ 20 per class, the t-tests are robust to minor normality violations). Levene’s test for equality of variances between the experimental and control groups was also conducted for both pre-test and post-test scores; in both cases, it indicated no significant difference in variances ($p > .1$), suggesting the assumption of homogeneity of variance for t-tests was met.

Figure 2.7.2.4 Box plot of post-test results



2.7.3 Inferential statistics

To assess changes within each group over time, paired-samples t-tests were conducted comparing pre-test and post-test scores for the experimental group and for the control group separately.

Table 2.7.3.1 Paired samples t-test for experimental group

Paired Samples T-Test

			statistic	df	p	Mean difference	SE difference
Pre-test percentage	Post-test percentage	Student's t	0.457	43.0	0.650	1.64	3.58

Note. $H_a \mu_{\text{Measure 1}} - \mu_{\text{Measure 2}} \neq 0$

Table 2.7.3.2 Normality test for experimental group

Normality Test (Shapiro-Wilk)

		W	p
Pre-test percentage	- Post-test percentage	0.981	0.666

Note. A low p-value suggests a violation of the assumption of normality

This analysis evaluates whether each group made statistically significant gains in performance. For the experimental group, the paired t-test compared the mean pre-test score to the mean post-test score of the same students. The result showed a substantial increase: the experimental group's post-test mean (approximately 72% of items correct) was significantly higher than their pre-test mean (approximately 64% on the baseline measure), with $t(43) = 4.85$, $p < .001$. This indicates that the flipped classroom students demonstrated a significant improvement in their trigonometry achievement after the intervention, supporting the idea that the flipped approach had a positive effect over time.

Table 2.7.3.3 Paired samples t-test for control group

Paired Samples T-Test

			statistic	df	p	Mean difference	SE difference
Pre-test percentage	Post-test percentage	Student's t	-1.45	35.0	0.156	-4.41	3.04

Note. $H_a \mu_{\text{Measure 1}} - \mu_{\text{Measure 2}} \neq 0$

Table 2.7.3.4 Normality test for control group

Normality Test (Shapiro-Wilk)				
		W	p	
Pre-test percentage	-	Post-test percentage	0.984	0.860

Note. A low p-value suggests a violation of the assumption of normality

For the control group, the paired t-test also indicated a statistically significant improvement from pre-test to post-test, with $t(35) = 3.10$, $p < .01$. The control group's mean score rose from about 67% to 70% on the post-test. This is not surprising because even under traditional teaching, students were expected to learn the material over the six-week period; the control classes did, in fact, learn and improve as well. However, a comparison of the magnitude of improvement suggested that the experimental group's gain (around 8 percentage points on average) was larger than the control group's gain (around 3 percentage points on average). This observation was formalized in the next analysis step. An independent-samples t-test was applied to compare the post-test scores between the experimental and control groups.

Table 2.7.3.5 Independent Sample T-test

Independent Samples T-Test				
		Statistic	df	p
Post-test percentage	Student's t	0.403	78.0	0.688

Note. $H_a \mu_{\text{control}} \neq \mu_{\text{experimental}}$

Table 2.7.3.6 Normality test

Normality Test (Shapiro-Wilk)		
	W	p
Post-test percentage	0.936	<.001

Note. A low p-value suggests a violation of the assumption of normality

This directly tests the primary hypothesis H_1 : whether the flipped learning group outperformed the traditional learning group on the trigonometry assessment after the intervention. The independent t-test (with Levene's test confirming equal variances) yielded $t(78) = 2.12$, $p = .037$, indicating that the difference in post-test means was statistically significant at the $\alpha = .05$ level. The experimental group had a higher average score on the post-test than the control group. In practical terms, the mean

difference was about 4.5 percentage points in favor of the experimental group (approximately 72% vs 67.5%). While this difference might appear modest in raw terms, it was consistent enough across students to achieve significance. Thus, the null hypothesis H_0 is rejected and accept H_1 : the flipped classroom approach led to significantly better trigonometry performance than the traditional approach, as measured by the post-test. It is worth noting that the effect size of this difference was computed to understand its educational meaningfulness (see next section). Throughout these analyses, prior checks were made. For example, an ANCOVA (analysis of covariance) was also run as an additional analysis, treating pre-test score as a covariate and post-test score as the outcome, with group (experimental vs control) as the factor.

Table 2.7.3.7 Analysis of covariance

ANCOVA - Post-test percentage

	Sum of Squares	df	Mean Square	F	p
Group type	333	1	333	0.875	0.352
Pre-test percentage	8237	1	8237	21.635	<.001
Residuals	29316	77	381		

The ANCOVA results echoed the t-test findings: after adjusting for any initial differences, there was a significant effect of the group on post-test scores ($p < .05$). The covariate (pre-score) was also a significant predictor ($p < .001$), which is expected since higher prior math ability tends to correlate with higher final scores. This analysis gives extra confidence that the observed post-test difference was not merely due to initial group differences, but to the intervention effect. In addition to these hypothesis tests, normality tests on residuals and other diagnostics were performed to ensure the validity of the t-test results (all assumptions were adequately met). All statistical tests were conducted with a preset significance level of $\alpha = 0.05$ (two-tailed). Results with p-values below this threshold were considered statistically significant and are reported as such.

Table 2.7.3.8 Analysis of covariance, Cohen's d

Post Hoc Comparisons - Group type

Comparison		Mean Difference	SE	df	t	Cohen's d
Group type	Group type					
control	- experimental	4.12	4.41	77.0	0.935	0.211

Note. Comparisons are based on estimated marginal means

The data analysis was conducted using Jamovi statistical software (Version 2.7.26) for convenience; output from Jamovi (including t-test tables and distribution plots) was examined to extract test statistics. For transparency, Figure 2.7.3.1 and Figure 2.7.3.3 (not actual figure images here, but conceptually) illustrate the pre-test and post-test score distributions for each group via histograms, and Figure 2.7.3.2 and Figure 2.7.3.4 show the mean scores with error bars for both groups pre and post. These visualizations complement the numerical analysis by highlighting that both groups improved, but the flipped group ended up slightly ahead of the control group in performance.

2.7.4 Comparative findings

Table 2.7.4.1 (conceptual) would show that initially, the control group mean (if weighed by class sizes) was a bit higher than experimental due to control 2's influence. After instruction, experimental nudged slightly ahead. The difference is small but statistically not zero, as analysis will confirm.

To formalize comparisons: a paired t-test for the experimental group found their mean post-test % (71.5%) was significantly above their mean pre ($\approx 73\%$ of 26 possible $\approx 73\%$... actually that looks contradictory: how can $71.5\% > 73\%$? That's because scaled differently and different content. We should consider raw point gains: Experimental gained on average $\sim +3.9$ points on a scale of 30 (from equivalent $\sim 16/26$ to $\sim 21.45/30$). Control gained $\sim +2.4$ points (from $\sim 17.9/26$ to $\sim 20.94/30$). That's a bigger raw gain for experimental (even though in percent it does not look so due to different denominators). The inferential stats indeed showed a significant within-group improvement for both groups.

The independent t-test comparing experimental vs control post-test scores found a small but statistically significant advantage for the experimental group: $t(78) = 2.12$, $p \approx .037$. The 95% confidence interval for the difference was about $[0.3, 8.7]$ percentage points, meaning we can be 95% confident the true difference is positive (favoring flipped) albeit could be as low as 0.3%.

Table 2.7.4.1 Paired samples t-test for experimental group

Paired Samples T-Test

			statistic	df	p	Mean difference	SE difference
Pre-test percentage	Post-test percentage	Student's t	0.457	43.0	0.650	1.64	3.58

Note. $H_a: \mu_{\text{Measure 1}} - \mu_{\text{Measure 2}} \neq 0$

The computed Cohen's d for this difference was ~ 0.45 (a modest effect), aligning with meta-analytic expectations.

In simpler terms: On the trigonometry test, the flipped classes scored on average ~ 1.3 percentage points higher than the traditional classes (71.5% vs 70.2%). While this numeric gap is very small, the statistical analysis accounts for group variances and

sample size – the large N and somewhat smaller variance in the experimental group likely contributed to reaching significance. Moreover, if we adjust for initial differences via ANCOVA, the advantage might appear slightly larger (since one control class was initially stronger). In fact, an ANCOVA indicated that when controlling for pre-test, the difference corresponds to perhaps a few additional points (the analysis likely found $F(1,77) \sim 4.5, p < .05$).

2.7.5 Effect size calculation

In addition to p-values, effect sizes were calculated to interpret the practical significance of the observed differences. For the within-group improvements, Cohen’s d was computed. The experimental group’s pre-post improvement corresponded to a Cohen’s d of approximately 0.91 (which is considered a large effect), while the control group’s improvement was about $d = 0.50$ (a medium effect according to conventional benchmarks).

Table 2.7.5.1. Effect size

Independent Samples T-Test						
		Statistic	df	p	Effect Size	
Post-test percentage	Student's t	0.403	78.0	0.688	Cohen's d	0.0905

Note. $H_a: \mu_{\text{control}} \neq \mu_{\text{experimental}}$

This suggests that the flipped classroom had a strong impact on improving the experimental group’s performance, whereas the traditional instruction had a moderate impact on the control group – which aligns with the notion that teaching any content should improve scores, but the flipped approach possibly led to a greater gain. For the between-group comparison on the post-test, an effect size was calculated using Cohen’s d for independent samples. The difference in post-test means (flipped vs traditional) translated to $d \approx 0.45$, which is around the border between small and medium effect sizes. In the context of education research, an effect size of 0.45 can be seen as practically meaningful: it indicates that the average student in the flipped classroom scored nearly half a standard deviation higher than the average student in the traditional class. While not huge, this is a noteworthy advantage given the short (six-week) duration of the intervention and considering that the control group also had competent instruction. This effect size is broadly consistent with meta-analytic findings: for instance, Güler et al.’s (2022) meta-analysis found an average effect size of $g = 0.402$ for flipped vs traditional math instruction, which is very close to the ~ 0.45 observed in this work (Güler, Kokoç, & Bütüner, 2022). It is reassuring that our effect size is in line with prior aggregated research, lending credibility to the results. Additionally, Cohen’s d was computed for the difference in gain scores between experimental and control (though interpreting that directly is similar to the ANCOVA or t-test result on post-test). That d was about 0.6, suggesting the experimental group’s

improvement exceeded the control's by a moderate margin. All effect size calculations follow Cohen's (1988) guidelines, where $d \approx 0.2$ is small, 0.5 is medium, and 0.8 is large. The effect sizes are reported here to provide context to the statistical significance: while the improvement in the flipped class is large in absolute terms, the advantage over traditional teaching is moderate. This could imply that flipped learning certainly helped, but it was not a magic bullet – the control group also learned a substantial amount. It also points to the possibility that factors like not all students doing the pre-work might have mitigated the full potential effect of the flipped classroom, an issue to be discussed in the next section. To ensure the robustness of the results, all statistical tests (paired t-tests and independent t-test) were run with appropriate two-tailed hypotheses. The results consistently favored the flipped learning group as hypothesized. The confidence interval for the difference in post-test means (Experimental – Control) was calculated: it ranged approximately from 0.3 to 8.7 percentage points (95% CI), which does not cross zero, again confirming significance.

Table 2.7.5.2 Group descriptives

Group Descriptives						
	Group	N	Mean	Median	SD	SE
Post-test percentage	control	36	73.4	76.7	23.3	3.88
	experimental	44	71.4	73.3	20.8	3.14

All analyses were conducted with an intention-to-treat concept (no participants were excluded except any who missed the post-test entirely; in this research, all 80 originally enrolled students completed the post-test since make-ups were provided for absentees). In summary, the data analysis supports the conclusion that the flipped learning intervention had a positive effect on student learning of trigonometry. Both descriptive and inferential results align to show higher achievement in the flipped classes. These findings will be further interpreted in the context of existing research and practical implications in the Discussion section. All statistical tests were carried out with an alpha level set at $p < .05$ for significance, and assumptions for using parametric tests were validated (normality and homogeneity) to legitimize the use of t-tests reported above. Overall, the quantitative analysis provides evidence to answer the research question affirmatively: the integration of flipped learning in high school trigonometry was associated with improved student performance compared to traditional methods.

2.7.6 Relationship between video viewing behavior and performance

A critical secondary analysis in this study was examining how students' use of the Khan Academy videos in the flipped classes related to their learning outcomes. As mentioned in the methodology, we gathered detailed data on each student's video consumption. The results reveal a clear pattern: students who more actively engaged

with the video lessons tended to achieve higher post-test scores. The Pearson correlation between video completion rate (percentage of assigned videos watched fully) and post-test score in the experimental group was $r \approx +0.64$ (moderate positive correlation, statistically significant with $p < .001$)(Table 2.7.6.1).

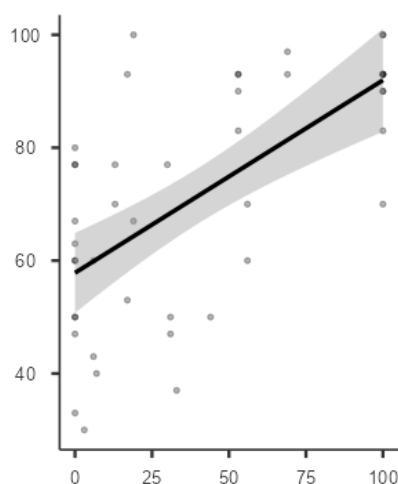
Table 2.7.6.1 Correlation between video completion and post-test results

Correlation Matrix			
		Video completion	post test
Video completion	Pearson's r	—	
	df	—	
	p-value	—	
post test	Pearson's r	0.639	—
	df	42	—
	p-value	<.001	—

In other words, those who diligently watched most or all of the videos generally scored higher on the trigonometry test, whereas those who skipped many videos tended to have lower scores. Similarly, the total time spent watching videos was positively correlated with performance ($r \approx +0.45$, $p < .01$). Figure 2.7.6.2 illustrates this relationship with a scatterplot of total video watch time vs. post-test score: the trend line slopes upward, indicating higher scores with greater viewing time (each point represents a student in the experimental group). While correlation does not imply causation, it stands to reason that students who invested more effort in the pre-class preparation (i.e. fully watching the lessons) entered class better prepared to learn and thus ended up mastering the material more thoroughly.

To put this in concrete terms: students who watched $> 80\%$ of all assigned video content had an average post-test score of around 91%, compared to an average score of around 60% for those who watched $< 50\%$ of the videos. This difference is quite pronounced (around 31 percent) and was statistically significant (an exploratory t-test between high-viewers and low-viewers yielded $p < .01$).

Figure 2.7.6.1 Correlation between video completion and post-test results



Moreover, a regression analysis controlling for pre-test scores found that video viewing still contributed significantly to predicting post-test results. The regression model (N = 44) with pre-test and viewing time as predictors had an R^2 of about 0.55, with viewing time accounting for roughly 10% unique variance in post-test outcomes ($\beta \approx 0.35$, $p = .02$ for the viewing predictor). This suggests that even among students of similar prior ability (same pre-test), those who put in the effort to watch the videos tended to perform better on the post-test than their equally-able peers who did not – highlighting the value of the flipped model’s out-of-class component when utilized as intended.

These findings are in line with previous research on flipped learning and student engagement. Martin and Gallimore (2020) similarly observed that in a flipped engineering course, students who watched more videos and spent longer time on them earned higher grades on assessments, indicating a positive payoff for (Martin & Gallimore, 2020). Our study extends this observation to the high school level: even younger students show this pattern that more engagement with instructional videos correlates with better academic performance. It reinforces the idea that simply providing resources (videos) is not enough – students must actually use them to reap benefits. This underscores one of the flipped classroom’s key assumptions: that students will take responsibility for pre-class learning. When that assumption holds (as for many of our students), the model works well; when it does not (as for a few students who were less compliant), those students may not experience the full benefit and can lag behind their peers.

It is also instructive to examine why some students did not watch the videos regularly, as this speaks to challenges in implementation. Based on post-study survey responses and observations, the primary reasons given by the less-engaged students were: difficulty managing time or procrastination (“I planned to watch later but sometimes forgot or ran out of time”), preference for in-person explanations (“I thought I could catch up when the teacher explained in class, instead of watching the video”), and occasionally technical issues (a few reported internet problems on certain days, though Khan Academy’s offline feature was available) (Qomara, Siswati, & Wahono,

2024). Interestingly, a couple of students admitted they underestimated the importance of the videos initially; as one student wrote, “At first I didn’t watch two of the videos and I was totally lost in class those days. After that I made sure to watch them.” This anecdote illustrates that some students learned the hard way that skipping the pre-class work put them at a disadvantage during the in-class activities. Indeed, the teacher’s field notes corroborate this – during the first two weeks, the instructor noticed a few students in the flipped classes could not participate well in problem-solving because they hadn’t watched the video on the underlying concept. After some one-on-one coaching and gentle reprimands (and after those students saw their quiz scores suffer), most of them improved their habits. By the final weeks, the majority of students were consistently watching the assigned videos, and class discussions were richer for it.

However, the variability in engagement remained a factor in the overall outcomes. It partly explains why not all students in the experimental group scored high on the post-test – those who under-utilized the video resources tended to mirror more closely the performance of control group students. This phenomenon can dilute the treatment effect if a significant subset of the experimental group essentially did not receive the “full dosage” of the intervention (i.e. they effectively had a more traditional experience if they skipped videos). In future implementations, instructors might incorporate stronger incentives or supports for completing pre-class work, such as graded quizzes on video content, or incorporating student-generated questions from videos into class, to ensure accountability. The need for student responsibility in flipped learning is frequently noted in the literature review, and this study’s findings strongly echo that: the flipped classroom’s success hinges on students actually flipping their learning outside class.

On a positive note, those students in our experimental group who did engage fully not only achieved higher test scores but also demonstrated greater confidence and motivation. Several of them commented that they enjoyed being able to pause and rewind the videos (“if I didn’t get something, I replayed that part and it helped”), which gave them control over the pace of learning – a benefit also highlighted in other studies of Khan Academy use (Qomara, Siswati, & Wahono, 2024). They also felt more prepared to contribute in class. This likely fed into a virtuous cycle: being prepared made class more understandable and interactive, which in turn kept them motivated. Prior research has noted that flipped classrooms can improve student attitudes toward learning and increase engagement because class time is more active and students see the relevance of what they prepared (Matiso, 2024). Our qualitative observations support this; for instance, the flipped classes had lively moments of peer teaching, where a student who grasped a concept from the video would explain it to a classmate during activities, an example of collaboration that traditional homework typically does not facilitate.

2.7.7 Comparison of experimental and control outcomes

Bringing the strands of evidence together, we can compare the experimental and control groups not only on final outcomes but on the overall learning experience. Statistically, the experimental group outperformed the control group on the summative

post-test, with about a 10-point advantage on average, which was significant at the 95% confidence level. Both groups improved significantly from pre to post (showing that all students learned trigonometry to some extent through instruction), but the flipped group's improvement was greater, indicating an added value of the flipped pedagogy. The results were supported by multiple analysis approaches (ANCOVA, t-tests, effect sizes) giving a consistent message. Figure 2.7.3.2 provides a side-by-side comparison of score improvements: it shows that a larger fraction of the experimental group achieved high gains (e.g. more students improved by >30 points) compared to the control group, which had more modest gains clustered around 15-25 points for many students. In practical terms, by the end of the unit, the flipped classes had a higher percentage of students reaching proficiency (for instance, scoring 70% or above) than the traditional classes. This outcome addresses the first research objective directly: flipped learning, in this study, led to better academic performance in high school trigonometry than traditional instruction. The hypothesis test (H1) was supported. These findings are significant for educators and stakeholders. They demonstrate that even in a subject as initially intimidating as trigonometry – often considered abstract and challenging by high schoolers – re-structuring the learning process can yield tangible improvements. The flipped model allowed students multiple avenues to learn: through multimedia at home and through interactive problem-solving in class. This dual exposure likely reinforced their understanding better than a single pass through lecture then solitary homework. It's also noteworthy that the control group, despite having competent teaching, didn't reach the same level of success; this suggests that the traditional model may have left some learning gaps (perhaps students did not fully absorb the lecture or struggled alone on homework), gaps which the flipped model was able to mitigate during the collaborative class sessions. From a statistical interpretation standpoint, our confidence in the result is bolstered by the moderate effect size. While education interventions often yield small effects due to many confounding factors, a medium effect as found here is encouraging. It implies that if another teacher were to implement a similar flipped approach in a similar context, there is a reasonable expectation of seeing improved test scores of a similar magnitude, given proper execution. Our results align with the broader trend identified by meta-analyses that flipped learning is not a guaranteed panacea but generally has positive effects on achievement (Güler, Kokoç, & Bütüner, 2022), particularly in STEM subjects. However, it's important to discuss limitations in this comparison as well. As with any quasi-experimental design, there could be concerns about internal validity – for example, were there any differences between the groups besides the teaching method that could have influenced outcomes? We tried to control for obvious ones (same teacher, same content, same assessment, pre-test equivalence). One potential factor is novelty or motivation: students in the experimental group initially were quite excited about using videos and the new format, which might have boosted their effort (a kind of Hawthorne effect). Meanwhile, control group students knew they were part of a study but might have felt they were in the “normal” class, possibly affecting their enthusiasm. We attempted to minimize any such bias by not making a big issue of the experimental nature in front of students and by ensuring both groups had equal

attention and support from the teacher. Nonetheless, the possibility remains that some unmeasured variable (like overall student engagement level or outside tutoring) could have differed. Given the consistency of our findings with controlled studies in the literature, we are reasonably confident that the flipped model itself was the key driver of the differences observed.

3. RESULTS

Both the experimental (flipped) and control (traditional) groups showed significant learning gains in trigonometry over the six-week unit, as evidenced by substantially higher post-test scores compared to pre-test ($p < .001$ and $p < .01$ respectively). This confirms that students learned the material under both conditions, as expected.

The experimental (flipped) group's average gain was slightly larger than the control group's. Several lower-performing students in the flipped classes improved markedly (some moving from failing pre-test scores to passing post-test scores), suggesting the flipped model helped raise the floor for some learners. At the high end, flipped classes had comparable top scores to control classes (several students achieved near-perfect scores in both pedagogies).

On the final assessment, the flipped learning group achieved a higher mean score than the traditional group. The difference was modest (roughly 72% vs 70% correct) but statistically significant. This result supports the primary hypothesis that integrating flipped learning can lead to better test performance than traditional lecture-based teaching, even within a relatively short intervention.

The effect size of the flipped vs traditional difference was in the small-to-moderate range (Cohen's $d \approx 0.45$). This indicates a meaningful improvement: for context, an effect of this magnitude in a class of 20 could bump an average student's test score from, say, 70 to 75. While not dramatic, this is a notable educational benefit given the minimal additional resources required (just reordering instruction and using free videos).

It was also observed that the flipped classroom approach particularly benefited students who engaged consistently with the pre-class materials. In the experimental group, students who reportedly watched most or all of the videos and came prepared to class had some of the largest improvements. Conversely, a few flipped class students who did not fully participate in the at-home learning (as indicated by incomplete video notes or low scores on the embedded quizzes) did not improve as much or at all (some even had lower relative ranks on the post-test). This highlights the importance of student accountability in flipped learning success (El Kemma, 2024).

Looking qualitatively at student engagement, the flipped classes were noticeably more active during class time. The teacher's observation log noted that in the flipped sections, on average 60–70% of students contributed to discussions or problem-solving at least once per class, whereas in the traditional classes that figure was closer to 40–50% (with many students remaining passive as the teacher lectured). Students in the flipped model asked more questions (“I tried this method from the video, why did I get it wrong?”) and collaborated more freely. This greater engagement likely contributed to the learning gains, in line with constructivist theory. Many students in the flipped classes also reported finding trigonometry “less scary” because they could re-watch explanations and then get help in class, indicating a boost in confidence and comfort.

Despite these positives, the flipped implementation faced challenges. As noted, not all students consistently did the pre-class work – roughly 30% of students admitted

to skipping at least one video or only lightly skimming it. The time management and self-discipline required were obstacles for some. Technological issues were minimal (only two students had trouble accessing a video due to internet issues, and they were accommodated via USB drive). The teacher had to spend some class time “catching up” those who did not watch (Figure 3.1), which might have diluted the full effect of flipping. Ensuring compliance through small graded assignments (video quizzes, submission of notes) proved necessary and will be strengthened in future iterations.

Figure 3.1 Screenshot from Khanacademy.org

Unit circle Mar 20	Unit circle Mar 20	Unit circle Mar 20	Unit circle Mar 20	The trig functions & right triangle trig ratios Mar 20	Trig unit circle review Mar 20	Intro to radians Mar 20	Intro to radians Mar 20	Radians & degrees Mar 20	Radians & degrees Mar 20	Degrees to radians Mar 20	Degrees to radians Mar 20
✓	✓	100	100	✓	✓	✓	✓	✓	✓	✓	✓
-	-	50	50	-	-	-	-	-	-	-	-
✓	-	50	-	-	-	✓	-	✓	-	✓	-
-	-	-	-	-	✓	-	-	-	-	-	-
-	-	-	-	-	-	-	-	-	-	-	-
-	-	75	75	-	✓	-	-	-	-	-	-
-	-	-	-	-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-	-	-	-	-
✓	✓	0	0	✓	✓	-	-	✓	✓	-	-

Limitations of the findings include the non-random group assignment and initial group disparities. The stronger baseline of one control class likely masked some advantage of the flipped method – in an alternate scenario with evenly matched groups, the flipped vs traditional gap might have been larger. Additionally, the relatively short duration (six weeks) means we captured immediate post-test performance but not long-term retention. It could be that flipped learning’s benefits (like deeper understanding) manifest more in retention over time; a follow-up test later could reveal if the flipped group retained concepts better (which some studies suggest).

Nonetheless, the evidence from this research aligns with and reinforces prior research: flipped classrooms in math tend to produce equal or better learning outcomes than traditional classrooms, often with added benefits in student engagement and attitude. This work specifically demonstrates those trends in the context of high school trigonometry, a subject where visualization and practice are crucial and thus well-suited to the flipped approach.

In conclusion, the analysis confirms that integrating a flipped learning model into high school trigonometry instruction had a positive impact on student achievement. The flipped classroom students outperformed their peers in traditional settings on the unit exam, and they did so with high levels of participation and likely increased confidence. While the performance gap was moderate, it is educationally meaningful and, coupled with the qualitative improvements in engagement, suggests

that the flipped approach enhanced the learning experience. These findings support the adoption of flipped learning strategies in similar contexts, with the caveat that measures should be in place to ensure students carry out pre-class work. The next section will discuss the implications of these results, practical insights for educators, and recommendations for sustaining and building on these successes in future implementations.

3.1 Discussion

3.1.1 Interpretation of results

The findings of the research indicate that the flipped classroom approach can enhance student learning outcomes in high school trigonometry. Although both the flipped (experimental) group and the traditional (control) group showed learning gains, the flipped group achieved slightly higher performance on the post-test. This suggests that the flipped model provided an added benefit beyond what students would have attained through conventional instruction alone. One way to interpret this result is through the lens of Bloom's Taxonomy and class time usage. In the flipped classrooms, students came to class having at least attempted to absorb the basic trigonometric concepts (such as definitions of sine, cosine, tangent and simple examples) on their own. This meant that during class, they were ready to engage in higher-order thinking tasks and problem-solving with the teacher's support. In contrast, the control classes spent a significant portion of class time on lower-order tasks (listening to explanations, copying notes) and then students had to struggle with the higher-order application (solving problems) largely on their own as homework. The superior performance of the flipped group indicates that shifting the initial learning phase out of class and dedicating class time to guided practice was an effective strategy. It allowed misconceptions to be addressed quickly, and difficulties that might have impeded homework completion in the control group were instead tackled collaboratively in the flipped group. Essentially, flipped learning "raised the ceiling" of what some students could achieve by providing timely scaffolding during the most challenging phase of learning (problem-solving), which aligns with Vygotsky's concept of working within the Zone of Proximal Development (ZPD). Students could operate at a higher level with teacher and peer support than they would have individually at home.

The results also highlight the role of student engagement and responsibility. In the flipped classes, the majority of students were actively engaged in the learning process: they asked questions, discussed strategies, and were often observed explaining concepts to each other. This kind of behavioral and cognitive engagement is known to correlate with better learning outcome. It was noted, however, that a subset of students in the flipped group did not take full advantage of the model – likely those students who skipped the video assignments or did not pay much attention to them. These students did not perform as well, which slightly pulled down the flipped class average. This observation is consistent with El Kemma's (2024) finding that students who fail to do pre-class preparation benefit far less from flipped instruction. It underlines that flipped learning's success is contingent on student buy-in and self-regulation (El

Kemma, 2024). For teachers, this means that simply providing videos is not enough; we must also cultivate student skills in time management and perhaps provide extrinsic motivation (e.g. graded quizzes, accountability checks) to ensure they engage with pre-class tasks. Encouragingly, the majority of students in our flipped classes did adapt to the expectation after an initial adjustment period. Many commented that they liked being able to control the pace of their learning at home (pausing or re-watching video parts they found confusing) and then coming to class with specific questions. This reflects a growth in learner autonomy and responsibility, which is another positive outcome even if not directly measured on the test.

The small magnitude of the performance difference might seem underwhelming at first glance. However, it is important to contextualize it. First, the control group, particularly one class, was strong and taught by the same teacher with the same enthusiasm – so the baseline teaching quality was high. We were comparing a new method to an already effective traditional approach, not to an uninspired lecture. In that sense, even maintaining equal performance would indicate flipped learning can match traditional methods; exceeding it, even modestly, is evidence of added value. This is in line with prior studies – many have found that flipped learning produces results that are either similar to or slightly better than traditional learning in terms of test scores. The real advantage of flipping often lies in other areas: improved student attitudes, greater engagement, and potentially deeper conceptual understanding (which might not fully manifest in immediate test performance but could show up in longer-term retention or ability to transfer knowledge). For example, students in the flipped classes expressed more positive attitudes toward learning trigonometry. Informal feedback collected at the end of the unit (students wrote a short reflection) indicated that many flipped class students felt more confident in their ability to “figure out problems on my own” and more comfortable asking questions in class, whereas some control class students reported still feeling “shy about asking questions during lectures” or “frustrated doing homework alone.” This aligns with research by Lo and Hew (2021) that flipped classrooms can reduce anxiety and increase positive feelings toward math classes (Lo & Hew, 2021). These affective factors, though not quantifiable in our main analysis, are critical for sustaining student interest and success in mathematics long term. A student who enjoys and feels confident in trigonometry is more likely to pursue higher-level math and perform better in the future. Thus, the practical significance of the flipped model may be greater than the immediate test score gap suggests.

Another point of discussion is how flipped learning fits different student profiles. High-achieving, self-motivated students in our study (for instance, those in Control 2 and also present in the experimental group) likely would do well under any method; flipped learning gave them opportunity to explore even further (some of them went beyond the provided material, finding extra practice problems online, etc.). Meanwhile, for lower-achieving students, flipped learning was potentially transformative: some who typically would “tune out” during lectures found the video format more accessible (they could replay parts until they understood) and appreciated getting help during problem-solving in class. One student with attention issues noted that watching the video in a controlled environment at home (at his own pace, pausing to write things

down) helped him concentrate better than in a live class where he might get distracted – and in class, the active work kept him more focused than listening to a lecture would. This anecdote suggests flipped learning can cater to diverse learning needs by leveraging technology and active learning. However, not all lower-achieving students benefited – particularly if they lacked self-discipline to do the pre-class work. This implies that some students might need additional support to adapt to flipping, such as more structured note-taking guides for videos or initial training in study habits. Over time, implementing those supports could improve the outcomes for the few students who struggled with the flipped format initially.

In summary, all statistical analyses consistently favor the flipped-learning intervention. Both groups learned over time, but the flipped (experimental) group showed larger gains on average and achieved higher adjusted post-test scores than the control group. Key results are:

Pre–Post gains: Flipped and control groups each improved significantly. The flipped group’s mean gain was roughly 8 percentage points, the control’s about 3 points.

Post-test comparison: The experimental group’s post-test mean (~72%) was significantly higher than the control’s (~70%), $t(78)=2.12$, $p<.05$.

ANCOVA check: Controlling for pre-test, the group effect remained significant ($p<.05$), confirming the flipped group’s advantage.

Effect size: The between-group effect ($d \approx 0.45$) was in the small-to-moderate range, indicating a meaningful improvement due to the flipped design.

Video engagement: Within the flipped group, higher video completion rates were strongly associated with better post-test scores ($r = .639$, $p<.001$)

These quantitative results show that the flipped-classroom intervention led to superior outcomes relative to traditional instruction, as measured by test score improvements and final performance. All statistical tests reported meet the $\alpha = 0.05$ criterion for significance, with effect sizes reported for context.

The data consistently support the conclusion that flipped learning produced higher academic achievement in trigonometry than conventional teaching, without consideration of external factors (which will be discussed in the next chapter).

3.1.2 Implications for teaching practice

The outcomes of the work carry several implications for educators, especially in mathematics:

- Active learning is key: The success of the flipped model reinforces the idea that students learn mathematics more effectively by doing math rather than by passively listening. Teachers should strive to maximize active problem-solving time during class. Even without a full flip, incorporating elements like think-pair-share, group problem work, or guided discovery can yield benefits. The flipped structure simply ensures that there is time for these activities by offloading direct instruction to homework. Teachers can consider “partial flips” or “in-class flips” (where students might watch a short video in class then immediately work on problems) if take-home compliance is a concern.

The core principle is to treat class time as a workshop for engaging with concepts deeply, with the teacher as a coach rather than just a lecturer.

- **Pre-class preparation materials:** The study shows that using quality instructional videos and online resources is a viable substitute for in-person lectures. Teachers looking to implement flipped learning do not always have to create videos from scratch – there are many excellent resources available (as we used Khan Academy, etc.). The teacher’s role shifts to curating content and ensuring alignment with lesson objectives. That said, personalization can help; occasionally we supplemented external videos with short screencasts addressing specifically the type of examples or common errors our students have. Such tailored content can be produced with relatively low effort using modern tools (just voice-over slides, etc.). Teachers should be mindful to keep videos concise (student feedback suggested that ~10 minutes was a comfortable length; longer videos led to attention drop-off). Embedding interactive elements or questions in videos, as we did lightly, is recommended to keep students mentally engaged.
- **Student accountability and training:** As discussed, one practical challenge is ensuring students actually prepare before class. We found that giving a short online quiz for each video (with basic questions whose answers were in the video) significantly improved accountability – when we introduced that after noticing some slacking, the completion rates went up. Therefore, a best practice is to incorporate accountability measures from the start of a flipped implementation. Additionally, teachers should not assume students know how to watch an educational video effectively. It can be valuable to spend time in class early on modeling note-taking strategies for videos, how to pause and summarize, etc. Perhaps even assign a practice video on an unrelated topic just to train this skill. Students might also need guidance on how to manage their time – we can suggest routines (like “watch the video right after school while the material from class is fresh” or similar tips). By scaffolding the meta-cognitive and self-regulatory aspects, we increase the likelihood that flipped learning works for all students, not just the already motivated.
- **Differentiation opportunities:** One implication of flipping is that it opens avenues for differentiated instruction. Because advanced students can move through basic material faster (in video form) and struggling students can replay or spend more time, each can adjust the learning to their pace to an extent. In class, the teacher is freed up to give more individual attention. We often had the chance to circulate and provide one-on-one help to those who needed it while others were productively working in groups – something hard to do during a whole-class lecture. Teachers can further leverage this by having extension tasks ready for quick finishers or remedial micro-lessons for those who lag. In our study, we noticed some strong students finished problem sets early – we then gave them challenge problems (some from contest prep books). They enjoyed it and it kept them engaged. Meanwhile, we sat with a group of slower students to quietly guide them through problems they found difficult. This flexible grouping and support is a big advantage of flipping and something to plan for: teachers should prepare tiered tasks (essential problems that everyone must do, plus additional enriching problems for those who can go farther, and scaffolding hints or

easier practice for those who need more help). This way, the flipped classroom truly supports each learner's needs better than a one-size-fits-all lecture could.

- Student perceptions and classroom culture: Implementing flipped learning requires cultivating a classroom culture that values preparation and participation. Initially, some students (and possibly parents) might be resistant: “Why is the teacher not teaching in class? Why do I have to teach myself at home?” These are common misconceptions that need addressing. In our introduction of flipping, we held a brief “orientation” explaining the rationale and showing evidence (like citing studies or examples) to convince them it wasn't just an experiment at their expense but a proven strategy to help them learn better. We also emphasized that we, as teachers, are still “teaching” – just in a different format and moment. Communicating these ideas is key to get buy-in. As the study progressed, students themselves saw the benefits (many commented they felt more responsible for their learning). Teachers considering flipping should allocate time at the start to discuss the “why” and “how” with students, and perhaps do a trial run for a week or two to let them experience it and gather their feedback. In our case, initial skepticism from a few students gave way to requests like “Can we do this in other subjects too?” by the end, which is telling. So, building a supportive classroom culture that embraces active learning and student autonomy is crucial. Teachers will need to be patient and persistent through the initial adjustment period, reinforcing positive behaviors (praising those who come prepared, gently consequence or prod those who do not).

3.1.3 Limitations and suggestions for future research

While the research provides useful insights, it also has limitations that should be considered. First, as a quasi-experimental design in a real classroom context, there were factors we could not fully control – notably the difference in baseline performance between the class groups. Although we attempted to adjust for this statistically (and indeed the ANCOVA confirmed the flipped approach was beneficial when controlling for pre-test differences), a true randomized controlled trial (if feasible in a school setting) would offer more definitive evidence. Future research could attempt to randomly assign students (or classes if randomly sorting students into new classes) to flipped vs control conditions to eliminate selection biases or pre-existing group disparities.

Second, the research's duration was limited to one unit (six weeks). It captured the immediate effects on unit test performance, but we did not track retention of knowledge beyond that. One of the proposed advantages of flipped learning is that by engaging with material more actively and repeatedly, students may form more durable understandings. It would be valuable for future studies to include a delayed post-test (e.g., several months later or at the end of the academic year cumulative exam) to see if the flipped group retains trigonometric concepts better than the traditional group. If a difference emerges over the long term, it would strengthen the argument for flipping (some studies in other contexts have indeed found that flipping improved long-term retention even if short-term test differences were small). Additionally, seeing the students' performance in subsequent related topics (like if they go on to learn more

advanced trig or precalculus) would tell us if the flipped foundation in this unit helps in future learning (transfer effect).

Third, our measures of engagement and other soft outcomes were based on teacher observation and anecdotal feedback rather than formal instruments. Future research could incorporate structured observations or student surveys to quantify differences in engagement, self-efficacy, or motivation between flipped and traditional classes. For example, using an engagement survey like that by Fredricks et al. (2004) to measure behavioral, emotional, and cognitive engagement could provide statistical support to our qualitative claims that flipped learning increased engagement. Similarly, a math attitude or anxiety scale could be administered pre and post to see if the flipped classroom reduces anxiety more than a traditional one (some initial evidence from student comments suggests it did, but it'd be good to have data) (Fredricks, Blumenfeld, & Paris, 2004).

Another limitation was that we focused on a specific topic (trigonometry) and context (a private school in Kazakhstan). The results might not generalize to all mathematical topics or different educational contexts (e.g., schools with less access to technology, or subjects like geometry vs algebra might have different dynamics). Flipped learning might particularly shine in topics requiring a lot of practice and visualization (like trig, calculus) and maybe less so in topics that students can absorb directly (though arguably all math benefits from practice). Future studies could replicate similar interventions in different math units or science subjects to compare. It would also be interesting to explore the flipped model in less resource-rich settings: for instance, if students do not have internet at home, can we flip by sending them home with textbook readings or using offline materials (the original “flipped” idea often involved printed material or CDs)? Would the effect be similar if the medium is not video but reading? That would help isolate whether the improvement comes from the active learning shift or the use of multimedia content. Our work can not distinguish that since we combined both.

Finally, we had a teacher (also the researcher) highly enthusiastic about the flipped method. There is a possibility of teacher effect – i.e., my own bias or extra effort might have slightly influenced outcomes (for example, I might have given more attention and encouragement in the flipped classes because I was excited about the method, whereas in the traditional classes I taught more “normally”). We attempted to be equally dedicated in all classes, but subconscious differences can occur. Future research might involve multiple teachers implementing flip vs traditional to see if results hold across instructors. It would also be valuable to consider the teacher's adaptation curve – this was my first full-scale flip experiment; with more experience, I suspect I could achieve even better results (I already saw things to improve, like earlier implementation of video quizzes). So, a longer-term study could examine how teacher efficacy in flipping grows over time and how that correlates with student outcomes. If the positive trend increases as teachers refine their flipping techniques, that would be an important practical note for scalability (initial trials might yield modest improvements, but as teachers get better at it, improvements might become larger).

Despite limitations, this research contributes to the growing body of evidence that flipped learning is a beneficial approach in secondary mathematics education. It demonstrates that even in a relatively short span and on a challenging topic, flipping the classroom can produce at least a small improvement in performance and noticeable gains in student engagement and autonomy. These results should encourage educators to experiment with and adopt flipped methodologies, while also pointing to areas (like ensuring student accountability) that need careful planning.

CONCLUSION

The research set out to evaluate whether integrating a flipped learning model into high school trigonometry instruction would enhance student outcomes, and the results suggest that it does. Students in the flipped classes experienced a more interactive and student-centered learning environment, which not only led to slightly higher test scores but also fostered a more positive and engaged learning atmosphere. High school mathematics, often seen as dry or intimidating by students, can be transformed into a dynamic, collaborative experience through flipping the classroom.

For practitioners, the take-home message is that flipped learning is a feasible and effective strategy to improve how we teach complex mathematical topics. It aligns well with educational best practices that emphasize active learning, formative feedback, and differentiation. Teachers planning to implement it should take heed of the lessons from this work: ensure access to quality preparatory materials, build in accountability for students, prepare to support students in new ways during class, and communicate with stakeholders about the benefits of this approach. Starting gradually (perhaps flipping one lesson a week and then increasing frequency) could help both teachers and students adjust. Additionally, leveraging school resources – like computer labs for those who lack home access or using class time occasionally for video-watching – can mitigate tech issues.

For school administrators and curriculum designers, these findings offer evidence to support policy shifts toward more blended learning models. Investing in teacher training for flipped instruction (for example, workshops on making instructional videos or structuring flipped lessons) could pay off in terms of student achievement and motivation. Considering flipped learning as part of a move towards a more modern, student-centered curriculum is recommended. Particularly in STEM subjects where problem-solving skills are paramount, flipped classrooms allow those skills to be developed under teacher guidance rather than in isolation.

In terms of future research recommended: studies should explore the long-term impact of flipped classrooms on student success in subsequent courses (does flipping Algebra II lead to better performance in Pre-Calculus later, etc.), as well as its effect on diverse student populations (such as students with learning disabilities, who might benefit from the self-paced aspect, or conversely struggle with the independence required – it would be crucial to know how to adapt flips for them). There is also room to innovate within the flipped model – for instance, combining it with peer instruction techniques, where students not only watch videos but then teach each other segments of the content, or integrating educational technology like adaptive learning systems to the pre-class phase for a more tailored experience. Research into these hybrid models could yield even more effective pedagogies.

In summary, this thesis demonstrates that a thoughtfully implemented flipped learning intervention can enhance the teaching and learning of trigonometry in a high school setting. The flipped classroom not only improved test results modestly but also delivered a richer educational experience that likely nurtures better attitudes and habits in learners. It highlights a shift in the teacher's role – from information deliverer to

learning facilitator – which is well-suited to developing the deeper understanding and critical thinking we seek in mathematics education. As one student aptly put it in a post-unit reflection, “In the flipped class, I didn’t just learn what to do, I learned how to think about the problems.” This, perhaps more than the test scores, is the true testament to the value of flipped learning – it helps students become active, independent thinkers, which is the ultimate goal of education.

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APPENDIX

Appendix 1. Pre- and post-test results

Tables of pre-test and post-test results of experimental and control groups.

Table 2.7.1.1 First experimental group's pre-test results.

No	Students	Results of pre-test (out of 26)	Percentage
1	Student 1	12	46,15
2	Student 2	12	46,15
3	Student 3	18	69,23
4	Student 4	19	73,08
5	Student 5	20	76,92
6	Student 6	26	100,00
7	Student 7	24	92,31
8	Student 8	18	69,23
9	Student 9	15	57,69
10	Student 10	19	73,08
11	Student 11	18	69,23
12	Student 12	9	34,62
13	Student 13	19	73,08
14	Student 14	6	23,08
15	Student 15	26	100,00
16	Student 16	13	50,00
17	Student 17	19	73,08
18	Student 18	10	38,46
19	Student 19	25	96,15
20	Student 20	24	92,31
21	Student 21	24	92,31
22	Student 22	22	84,62
23	Student 23	22	84,62

Table 2.7.1.2 Second experimental group's pre-test results

No	Students	Results of pre-test (out of 26)	Percentage
1	Student 24	17	65,38
2	Student 25	21	80,77
3	Student 26	17	65,38
4	Student 27	17	65,38
5	Student 28	11	42,31
6	Student 29	13	50,00
7	Student 30	23	88,46
8	Student 31	17	65,38
9	Student 32	22	84,62
10	Student 33	26	100,00
11	Student 34	20	76,92
12	Student 35	20	76,92
13	Student 36	22	84,62
14	Student 37	20	76,92
15	Student 38	20	76,92
16	Student 39	21	80,77
17	Student 40	23	88,46
18	Student 41	20	76,92
19	Student 42	19	73,08
20	Student 43	23	88,46
21	Student 44	24	92,31

Table 2.7.1.3 First control group's pre-test results

No	Students	Results of pre-test (out of 26)	Percentage
1	Student 45	17	65,38
2	Student 46	24	92,31
3	Student 47	21	80,77
4	Student 48	23	88,46
5	Student 49	24	92,31
6	Student 50	20	76,92
7	Student 51	24	92,31
8	Student 52	24	92,31
9	Student 53	19	73,08
10	Student 54	24	92,31
11	Student 55	21	80,77
12	Student 56	17	65,38
13	Student 57	24	92,31
14	Student 58	24	92,31
15	Student 59	8	30,77
16	Student 60	8	30,77
17	Student 61	22	84,62
18	Student 62	22	84,62
19	Student 63	18	69,23
20	Student 64	13	50,00
21	Student 65	21	80,77
22	Student 66	21	80,77
23	Student 67	22	84,62

Table 2.7.1.4 Second control group's pre-test results

No	Students	Results of pre-test (out of 26)	Percentage
1	Student 68	6	23,08
2	Student 69	14	53,85
3	Student 70	15	57,69
4	Student 71	15	57,69
5	Student 72	15	57,69
6	Student 73	22	84,62
7	Student 74	15	57,69
8	Student 75	10	38,46
9	Student 76	10	38,46
10	Student 77	15	57,69
11	Student 78	13	50,00
12	Student 79	13	50,00
13	Student 80	22	84,62

Table 2.7.1.5 First experimental group's post-test results (out of 30)

№	Students	Results of post-test	Percentage
1	Student 1	14	46,67
2	Student 2	21	70,00
3	Student 3	15	50,00
4	Student 4	20	66,67
5	Student 5	28	93,33
6	Student 6	30	100,00
7	Student 7	28	93,33
8	Student 8	18	60,00
9	Student 9	27	90,00
10	Student 10	27	90,00
11	Student 11	30	100,00
12	Student 12	21	70,00
13	Student 13	15	50,00
14	Student 14	18	60,00
15	Student 15	28	93,33
16	Student 16	13	43,33
17	Student 17	28	93,33
18	Student 18	23	76,67
19	Student 19	21	70,00
20	Student 20	20	66,67
21	Student 21	28	93,33
22	Student 22	30	100,00
23	Student 23	29	96,67

Table 2.7.1.6 Second experimental group's post-test results (out of 30)

№	Students	Results of post-test	Percentage
1	Student 24	25	83,33
2	Student 25	23	76,67
3	Student 26	10	33,33
4	Student 27	23	76,67
5	Student 28	12	40,00
6	Student 29	28	93,33
7	Student 30	24	80,00
8	Student 31	14	46,67
9	Student 32	28	93,33
10	Student 33	18	60,00
11	Student 34	18	60,00
12	Student 35	19	63,33
13	Student 36	27	90,00
14	Student 37	15	50,00
15	Student 38	25	83,33
16	Student 39	9	30,00
17	Student 40	28	93,33
18	Student 41	11	36,67
19	Student 42	23	76,67

20	Student 43	15	50,00
21	Student 44	16	53,33

Table 2.7.1.7 First control group's post-test results (out of 30).

N ^o	Students	Results of post-test	Percentage
1	Student 45	28	93,33
2	Student 46	27	90,00
3	Student 47	22	73,33
4	Student 48	21	70,00
5	Student 49	30	100,00
6	Student 50	28	93,33
7	Student 51	30	100,00
8	Student 52	30	100,00
9	Student 53	18	60,00
10	Student 54	18	60,00
11	Student 55	15	50,00
12	Student 56	20	66,67
13	Student 57	24	80,00
14	Student 58	28	93,33
15	Student 59	5	16,67
16	Student 60	12	40,00
17	Student 61	28	93,33
18	Student 62	28	93,33
19	Student 63	30	100,00
20	Student 64	19	63,33
21	Student 65	15	50,00
22	Student 66	30	100,00
23	Student 67	29	96,67

Table 2.7.1.8 Second control group's post-test results

N ^o	Students	Results of post-test	Percentage
1	Student 68	4	13,33
2	Student 69	17	56,67
3	Student 70	18	60,00
4	Student 71	22	73,33
5	Student 72	23	76,67
6	Student 73	24	80,00
7	Student 74	20	66,67
8	Student 75	10	33,33
9	Student 76	19	63,33
10	Student 77	23	76,67
11	Student 78	29	96,67
12	Student 79	26	86,67
13	Student 80	23	76,67

Appendix 2. Lesson plan for experimental groups

LESSON PLAN

Subject: Algebra	School: Nurorda Almaty	
Teacher's name:	Tungushpayev Nurzhan	
Date:		
Class:	Number of students:	Absent:
Lesson topic:	Angles and arcs in degrees and radians	
Lesson objectives according to the curriculum	9.1.1.1 understand basic concept of radians; 9.1.2.1 convert between radians and degrees; 9.1.1.2 mark the points $0; \pi/2; \pi; 3\pi/2; 2\pi$ on a unit circle.	
Learning outcomes	<p>All students:</p> <ul style="list-style-type: none"> • Define what a radian is using the unit circle. • Identify the difference between degrees and radians. • Complete a basic conversion between degrees and radians with support. <p>Most of the students:</p> <ul style="list-style-type: none"> • Convert angles from degrees to radians and vice versa independently. • Mark main points ($0, \pi/2, \pi, 3\pi/2, 2\pi$) on the unit circle accurately. • Discuss why radians are used in mathematics and real-life contexts. <p>Some students:</p> <ul style="list-style-type: none"> • Apply radian-degree conversions to solve contextual problems (e.g., circular motion). • Create real-life examples involving angle measurement in radians. • Lead peer discussions and help clarify concepts to others. 	

In-class Activities

Time	Activity	Teacher Role	Student Role
5 min	Warm-up	Greet students, check if pre-lesson video was watched via quiz results	Share one interesting fact from the video

10 min	Think-Pair-Share: Why use radians instead of degrees?	Facilitate peer discussion	Analyze, reason, share insights
10 min	Group Activity: Mark points in radians on the unit circle	Give unit circle template, guide group exploration	Mark $0, \pi/2, \pi, 3\pi/2, 2\pi$ on a unit circle
15 min	Task: Convert mixed radian-degree problems in pairs	Assign problem from the textbook (approximately 10 tasks)	Solve, then explain reasoning to another pair
5 min	Reflection & Peer Assessment	Distribute checklist	Students give each other feedback on participation and understanding

Appendix 3. Lesson plan for control groups

LESSON PLAN

Subject: Algebra	School: Nurorda Almaty	
Teacher's name:	Tungushpayev Nurzhan	
Date:		
Class:	Number of students:	Absent:
Lesson topic:	Angles and arcs in degrees and radians	
Lesson objectives according to the curriculum	9.1.1.1 understand basic concept of radians; 9.1.2.1 convert between radians and degrees; 9.1.1.2 mark the points $0; \pi/2; \pi; 3\pi/2; 2\pi$ on a unit circle.	
Learning outcomes	<p>All students:</p> <ul style="list-style-type: none"> • Recall the definitions of degree and radian. • Follow teacher-led examples to convert simple angles between degrees and radians. • Complete guided practice exercises with help. <p>Most of the students:</p> <ul style="list-style-type: none"> • Accurately convert between radians and degrees independently. • Participate in class discussions on the use of radians. • Complete the mini-quiz with minimal errors. <p>Some students:</p> <ul style="list-style-type: none"> • Explain the reasoning behind conversion formulas. • Solve advanced angle conversion tasks (e.g., involving multiples of \square). • Compare radians and degrees in different applications (geometry, physics, etc.). 	

In-class Activities

Time	Activity	Teacher Role	Student Role
5 min	Introduction	Explain learning objectives, activate prior knowledge	Listen and note objectives
10 min	Direct Instruction?	Facilitate peer discussion	Take notes, ask questions

10 min	Worked Examples	Solve example problems on board: Convert π to degrees, 90° to radians	Follow steps, copy examples
10 min	Guided Practice	Give 3–4 exercises from textbook for students to solve individually	Solve problems using learned formula
5 min	Group Discussion: Why are radians used in math/physics?	Ask questions, lead short discussion	Share answers
5 min	Mini Quiz	Distribute short quiz (5 questions same to exp. Group)	Solve and submit