Значит, данное множество формул можно расширить до полного φ-типа $t(x)$ над моделью M, который будет совместен с формулой $\Psi^i(x)$. Но тогда $t(x)$ несовместен с формулой $\Psi^{1-i}(x)$. Значит, тип $t(x)$ — это один из тех типов $s(x)$, которые несовместны с формулой $\Psi^{1-i}(x)$. Но если формула $\varphi^j(x;\bar{b}) \in s_i$, такая что в $t_1(x)$ мы выбрали ее отрицание, вспомним, что мы выбрали по одному дизъюнкту из $\neg S_i(x)$, то тип $s(x)$ никак не может равняться типу $s(x)$.

Получили противоречие, следовательно формул $q_i(x) \cup \{\Psi^i(x)\}$ не может быть совместно

Теорема доказана.

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A BOUNDARY VALUE PROBLEM FOR THE GENERALIZED HEAT EQUATION IN THE DOMAIN WITH MOVING BOUNDARY

Abstract. Method to solve the problem for heat equation for solid with variable cross-section with moving boundary is based on use degenerate hypergeometric function. Solution of problem is a linear combination degenerate hypergeometric function. The main idea of this method is to find coefficients and prove the convergence of series.Consider the surface generated by revolution of a curve $r = y(z,t)$ about z - axes. Let us assume that the radial component of the temperature gradient in the solid bounded by above surface is negligible in comparison with the axial component, i.e. the solid can be considered as a bar with a variable cross – section that has only axial component of heat flux.

Key words: degenerate hypergeometric function, generalized heat equation.

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Аңдатпа. Гипергеометрикалық функцияларды шекарасынашығa алатын айнымалы бөлімінде бар денеге арналған жылу проблемасын шешу әдісін ұқсанды пайдалану негізделген. Мәселенің шешімі нұқсанды гипергеометрикалық функциялар сызықтық комбинациясы болып табылады. Бұл әдістің негізгі идеясы коэффициенттері табу және қатарлардың жинақтылығы дәлелдеу болып табылады. Конверттеу қисық бұрау арқылы құрылған бетін қарастырайық. Қатты денедегі радиалды градиент температураның құрамдас болатын беті арқылы шектелген деп есептейік. Қатты дене айнымалы қимасы жылу ағынының тек осьтік компонент бар ретінде қарастырылалады.

Кілт сөздер: ағызатын гипергеометриалық функциясы, жалпылама жылу теңдеуі.

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Аннотация. Метод решения задачи теплопроводности для тела с переменным сечением с движущей границей основан на использовании вырождающейся гипергеометрической функции. Решение задачи представляет собой линейную комбинацию вырожденной гипергеометрической функции. Основная идея этого метода в отыскание коэффициентов и доказательстве сходимости рядов. Рассмотрим поверхность, образованную вращением огибающей кривой. Предположим, что радиальная составляющая градиента температуры в твердом теле, ограниченном указанной поверхностью, пренебрежимо мала по сравнению с осевой составляющей, т.е. твердое тело можно рассматривать как тело с переменным сечением, имеющим только осевую составляющую теплового потока.

Ключевые слова: вырожденная гипергеометрическая функция, обобщенное уравнение теплопроводности.

Introduction

Equation of heat conductivity for bodies with variable cross-section

$$\frac{\partial\theta}{\partial t} = a^2\left(\frac{\partial^2\theta}{\partial z^2} + \frac{2y'_z}{y}\frac{\partial\theta}{\partial z}\right) + \frac{1}{c\gamma}q(z,t) \qquad (1)$$

particular case (1) when $y(z,t) = z^{\nu/2}$ и $q(z,t) = 0$

$$\frac{\partial\theta}{\partial t} = a^2\left(\frac{\partial^2\theta}{\partial z^2} + \frac{\nu}{z}\frac{\partial\theta}{\partial z}\right) \qquad (1^*)$$

consequently, the function

$$S_{\beta,\nu}{}^{(1)}(z,t)=(2a\sqrt{t})^{\beta}\,\Phi(-\frac{\beta}{2},\frac{\nu+1}{2};-\frac{z^2}{4a^2t})$$

(2)

$$S_{\beta,\nu}{}^{(2)}(z,t)=(2a\sqrt{t})^{\beta}\,(\frac{z^2}{4a^2t})^{\frac{1-\nu}{2}}\,\Phi(\frac{1-\nu-\beta}{2},\frac{3-\nu}{2};-\frac{z^2}{4a^2t})$$

satisfies equation(1*). The linear combination of a degenerate hypergeometric function is a function of the second order $\Psi(a,b;x)$

$$\Gamma_{\beta,\nu}(z,t)=\frac{\Gamma(\frac{1-\nu}{2})}{\Gamma(\frac{1-\nu-\beta}{2})}S_{\beta,\nu}{}^{(1)}(z,t)+\frac{\Gamma(\frac{\nu-1}{2})}{\Gamma(-\frac{\beta}{2})}S_{\beta,\nu}{}^{(2)}(z,t)=(2a\sqrt{t})^{\beta}\Psi(-\frac{\beta}{2},\frac{\nu+1}{2};-\frac{z^2}{4a^2t})$$

Using the integral representation, the degenerate hypergeometric function can be expressed inthe form

$$\Phi(-\frac{\beta}{2},\mu;-z^2)=\frac{2\Gamma(\mu)}{\Gamma(\mu+\frac{\beta}{2})}\exp(-z^2)z^{-\mu+1}\int_0^{\infty}\exp(-x^2)x^{\mu+\beta}I_{\mu-1}(2zx)dx \tag{3}$$

Mathematical model

A boundary value problem for the generalized heat equation in a domain with a moving boundary can be formulated

$$\frac{\partial\theta}{\partial t}=a^2\left(\frac{\partial^2\theta}{\partial z^2}+\frac{\nu}{z}\frac{\partial\theta}{\partial z}\right)\qquad \nu\ge 0\ \ 0<z<\alpha\sqrt{t} \tag{4}$$

$\theta(0,t)=f(t)$ (5), $\theta(\alpha\sqrt{t},t)=g(t)$ (6)

given functions are analytic

$f(t)=\sum_{n=0}^{\infty}\frac{f^{(n)}(0)}{n!}t^n$, $f_n=f^{(n)}(0)$ и $g(t)=\sum_{n=0}^{\infty}\frac{g^{(n)}(0)}{n!}t^n$, $g_n=g^{(n)}(0)$

The solution of problem (4) is represented as a linear combination of a degenerate hypergeometric function

$$\theta(z,t)=\sum_{n=0}^{\infty}\left(2a\sqrt{t}\right)^n\left[A_n\Phi\left(\frac{-n}{2},\frac{\nu+1}{2};\frac{z^2}{4a^2t}\right)+B_n\Psi\left(\frac{-n}{2},\frac{\nu+1}{2};\frac{-z^2}{4a^2t}\right)\right] \tag{7}$$

Obviously, this function satisfies not only the equation (4), but also to the conditions(5) and (6). From the condition(5)we get

$$\sum_{n=0}^{\infty}\left(2a\sqrt{t}\right)^{n}\left[A_n+B_n\right]=\sum_{n=0}^{\infty}f_n t^n$$

Equating the coefficients of the same powers of t, we have

$$A_{2n}+B_{2n}=\frac{f_n}{(2a)^{2n}} \qquad (8)$$

$$A_{2n+1}+B_{2n+1}=0 \qquad (8^*)$$

from the condition(6)

$$\sum_{n=0}^{\infty}\left(2a\sqrt{t}\right)^{n}\left[A_n\Phi\left(\frac{-n}{2},\frac{\nu+1}{2};\frac{\alpha^2}{4a^2}\right)+B_n\Psi\left(\frac{-n}{2},\frac{\nu+1}{2};\frac{-\alpha^2}{4a^2}\right)\right]=g(t)$$

finally

$$(2a)^{2n}\left[A_{2n}\Phi\left(-n,\frac{\nu+1}{2};\frac{\alpha^2}{4a^2}\right)+B_{2n}\Psi\left(-n,\frac{\nu+1}{2};\frac{-\alpha^2}{4a^2}\right)\right]=g_n \qquad (9)$$

$$\left[A_{2n+1}\Phi\left(-\frac{2n+1}{2},\frac{\nu+1}{2};\frac{\alpha^2}{4a^2}\right)+B_{2n+1}\Psi\left(-\frac{2n+1}{2},\frac{\nu+1}{2};\frac{-\alpha^2}{4a^2}\right)\right]=0 \qquad (9^*)$$

The conditions (8) and (9) give the following system of equations A_{2n+1}, B_{2n+1}

$$\begin{cases}A_{2n+1}+B_{2n+1}=0\\ A_{2n+1}\Phi\left(-\frac{2n+1}{2},\frac{\nu+1}{2};\frac{\alpha^2}{4a^2}\right)+B_{2n+1}\Phi\left(-\frac{2n+1}{2},\frac{\nu+1}{2};\frac{-\alpha^2}{4a^2}\right)=0\end{cases}$$

The determinant of the matrix is nonzero, then the system of equations has only the trivial solution $A_{2n+1}=B_{2n+1}=0.$

Also, from conditions (8 *) and (9*) we derive a system of equations for A_{2n}, B_{2n}

$$\begin{cases} A_{2n} + B_{2n} = \dfrac{f_n}{(2a)^{2n}} \\ A_{2n}\Phi\left(-n, \dfrac{\nu+1}{2}; \dfrac{\alpha^2}{4a^2}\right) + B_{2n}\Psi\left(-n, \dfrac{\nu+1}{2}; \dfrac{-\alpha^2}{4a^2}\right) = \dfrac{g_n}{(2a)^{2n}} \end{cases}$$

the coefficients A_{2n}, B_{2n} are defined in the form

$$B_{2n} = \frac{1}{n!(2a)^{2n}} \frac{g^{(n)}(0) - f^{(n)}(0)\Phi\left(-n, \frac{\nu+1}{2}; \frac{\alpha^2}{4a^2}\right)}{\Psi\left(-n, \frac{\nu+1}{2}; \frac{-\alpha^2}{4a^2}\right) - \Phi\left(-n, \frac{\nu+1}{2}; \frac{\alpha^2}{4a^2}\right)}$$

$$A_{2n} = \frac{1}{n!(2a)^{2n}} \frac{f^{(n)}(0)\Psi\left(-n, \frac{\nu+1}{2}; \frac{-\alpha^2}{4a^2}\right) - g^{(n)}(0)}{\Psi\left(-n, \frac{\nu+1}{2}; \frac{-\alpha^2}{4a^2}\right) - \Phi\left(-n, \frac{\nu+1}{2}; \frac{\alpha^2}{4a^2}\right)}$$

Eliminating the singularities in $n = 0$ we represent the coefficients $A_0, B_0 = 0$. The convergence of the series (7) follows from the following estimates [1], [2] and is easily shown on the Ratio test for converges. For large values of t, the small value of the argument corresponds to the hypergeometric function tending to unity, while the last argument of the function tends to zero, then the series (7) converges.Verigin's inverse problem of water-oil contact under conditions of elastic regime is also solved using the degenerate hypergeometric function [3].

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