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Numerical method for solving the equation of navier-stokes

In given work we consider a new version of the fictitious domain method with the continuation of the leading coefficient. The essential point is that at the boundary of the auxiliary field given by the tangential component of velocity and pressure. This condition allows us to build for the numerical calculation of the efficient iterative method. It is proved that the rate of convergence of the iterative method does not depend from the range of variation of a small parameter.

Consider the boundary value problem of linear stationary Navier-Stokes equations in Ω

$$\mu\Delta \mathrm{v} - \nabla p = f, \quad (1)$$

$$div\mathrm{v} = 0, \quad \mathrm{v}|_S = 0. \quad (2)$$

In the work [1] given one method of fictitious domain method for the numerical solution of the equation (1)-(2). In the domain $D \supset \Omega$ we solve the helping problem

$$div(\mu^\varepsilon \nabla \mathrm{v}^\varepsilon) - \nabla p^\varepsilon = f, \quad (3)$$

$$div\mathrm{v}^\varepsilon = 0,$$

with given boundary conditions

$$[\mathrm{v}^\varepsilon]_S = 0, \quad [(\mu^\varepsilon \nabla \mathrm{v}^\varepsilon - \delta p^\varepsilon)\vec{n}]_S = 0,$$

$$\mathrm{v}^\varepsilon \cdot \tau|_{S_1} = 0, \quad p^\varepsilon|_{S_1} = 0, \quad (4)$$

where S_1 - border of domain D, τ, $\vec{n}$ - tangent vector and vector of interior line to border of S domain , and

$$\mu^{\varepsilon}=\begin{cases}\mu, x\in\Omega,\\ \dfrac{\mu}{\varepsilon}, x\in Q\setminus\Omega.\end{cases}$$

For simplicity, we assume a two-dimensional domain Ω, S - smoother. Existence theorems of generalized and strong solutions and their convergence with a solution of the $\varepsilon\to 0$ to problems (1), (2) that are proved in [1] - [3].

In this paper we propose an efficient numerical method for solving equations (3), (4).

In our work is given effective numerical method to solving equations (3),(4).

Then let's take Далее рассмотрим difference scheme approximating the equation (3)-(4). Domain D assume that the rectangular $D=(0,l_1)\times(0,l_2)$.

Construct three dimensional grid

$$\omega_h=\left\{x=(x_1,x_2)=(ih,jh),\ i=\overline{0,N_1},\ \ j=\overline{0,N_2},\ h=\frac{l_1}{N_1}=\frac{l_2}{N_2}\right\},\ \gamma_1=\overline{\omega}_h\setminus\omega_h,$$

$$\overline{Q}_h=\{x=(x_{1i-1/2},x_{2j})=((i-1/2)h,jh),\ i=\overline{0,N_1+1},\ \ j=\overline{0,N_2}\}$$

the border

$$\partial Q_h=\overline{Q}_h\setminus Q_h=\partial Q_h^1\cup\partial Q_h^2\cup\partial Q_h^3\cup\partial Q_h^4,$$

$$\partial Q_h^1=\{(x_{1-1/2},x_{2j})=(-1/2h,jh),\ j=\overline{0,N_2}\},$$

$$\partial Q_h^2=\{(x_{1i-1/2},x_{20})=((i-1/2)h,0),\ i=\overline{0,N_1+1}\},$$

$$\partial Q_h^3=\{(x_{1N_1+1/2},x_{2j})=((N_1+1/2)h,jh),\ j=\overline{0,N_2}\},$$

$$\partial Q_h^4=\{(x_{1i-1/2},x_{2N_2})=((i-1/2)h,N_2h),\ i=\overline{0,N_1+1}\},$$

$$\overline{l_h}=\{(x_1,x_{2j-1/2})=(ih,(j-1/2)h),\ i=\overline{0,N_1},\ \ j=\overline{0,N_2+1}\},$$

the border

$$\partial Q_h = \overline{Q}_h \setminus Q_h = \partial Q_h^1 \cup \partial Q_h^2 \cup \partial Q_h^3 \cup \partial Q_h^4$$

$$\partial G_h = \overline{G}_h \setminus G_h = \partial G_h^1 \cup \partial G_h^2 \cup \partial G_h^3 \cup \partial G_h^4,$$

$$\partial G_h^1 = \{(x_{1i}, x_{2-1/2}) = (ih, -1/2h),\ i = \overline{0, N_1}\}.$$

$$\partial G_h^2 = \{(x_{1N_1}, x_{2j-1/2}) = (N_1 h, (j-1/2)h),\ j = \overline{0, N_2+1}\},$$

$$\partial G_h^3 = \{(x_1, x_{2N_2+1/2}) = (ih, (N_2+1/2)h),\ i = \overline{0, N_1}\},$$

$$\partial G_h^4 = \{(x_{10}, x_{2j-1/2}) = (0, (j-1/2)h),\ j = \overline{0, N_2+1}\}.$$

Further notation is taken from [4]. Consider the difference scheme approximating the problem (3), (4)

$$\left(\mu^{\varepsilon} v_{1\bar{x}_1}\right)_{x_1} + \left(\mu^{\varepsilon} v_{1\bar{x}_2}\right)_{x_2} - p_{1\bar{x}_1} = f_{1n} \text{ в } Q_h,$$

$$\left(\mu^{\varepsilon} v_{2\bar{x}_1}\right)_{x_1} + \left(\mu^{\varepsilon} v_{2\bar{x}_2}\right)_{x_2} - p_{1\bar{x}_2} = f_{2n} \text{ в } G_h,$$

(5)

$$div_h v = v_{1x_1} + v_{2x_2} = 0 \text{ в } \omega_h,$$

with boundary conditions

$$v_{1\bar{x}_1}\big|_{\partial Q_h^1} = v_{1\bar{x}_1}\big|_{\partial Q_h^3} = 0,\quad v_1\big|_{\partial Q_h^2} = v_1\big|_{\partial Q_h^4} = 0,$$

$$v_{2\bar{x}_2}\big|_{\partial G_h^1} = v_{2\bar{x}_2}\big|_{\partial G_h^3} = 0,\quad v_2\big|_{\partial G_h^2} = v_2\big|_{\partial G_h^4} = 0,\quad p\big|_{\gamma_1} = 0.$$

(6)

Find a solution to (5) (6) iterations. We investigate a scheme such as the simple iteration

$$B^{\varepsilon} \frac{v_1^{n+1} - v_1^n}{\tau} = -p_{\bar{x}_1}^{n+1} + \left(\mu^{\varepsilon} v_{1\bar{x}_1}^n\right)_{x_1} + \left(\mu^{\varepsilon} v_{1\bar{x}_1}^n\right)_{x_1} + f_1, \quad (7)$$

$$B^{\varepsilon}\frac{v_2^{n+1}-v_2^{n}}{\tau}=-p_{\bar{x}_2}^{n+1}+\left(\mu^{\varepsilon}v_{2\bar{x}_1}^{n}\right)_{x_1}+\left(\mu^{\varepsilon}v_{2\bar{x}_1}^{n}\right)_{x_1}+f_2, \quad (8)$$

$$v_{1x_1}^{n+1}+v_{2x_2}^{n+1}=0,\quad n=0,1,2,... \quad (9)$$

$$v_1^0=v_{01},\quad v_2^0=v_{02} \quad (10)$$

and for v_1^{n+1}, v_2^{n+1}, p^{n+1} taken boundary conditions of (6). Here $B^{\varepsilon}=E+E_{\mu}^{\varepsilon}$, E - identity matrix, E_{μ}^{ε} - diagonal matrix that consisting of the diagonal elements of the operator

$$\left(\mu^{\varepsilon}v_{1\bar{x}_1}^{n}\right)_{x_1}+\left(\mu^{\varepsilon}v_{1\bar{x}_1}^{n}\right)_{x_2}.$$

To force (6)-(8) for pressure we take problem of Dirihle for equation of Puasson

$$div_h\left(K_h^{\varepsilon}\nabla_h p^{n+1}\right)=F_n\left(v^n,f\right), \quad (11)$$

$$p_n^{\varepsilon}\Big|_{\gamma_n}=0,\quad K_h^{\varepsilon}(x)=\frac{\tau}{1+\mu^{\varepsilon}}.$$

Pressure p^{n+1} is a unique way, with efficient iterative method proposed in [5]. For example, the alternating-triangular method. Problem (6) - (.8) is uniquely solvable.

and find p^{n+1} from (11) and substituting in (7), (8) we obtain the operator

$$-\nabla_h p^{n+1}+div_h\left(\mu^{\varepsilon}\nabla_h v^n\right)=A^{\varepsilon}v^n.$$

We denote V_h - the mesh vector space of functions, components v_1 and v_2 that belong $W_2^1(Q_n)$ and satisfy (6). In V_h introduce a scalar product

$$(u,\mathrm{v})_{V_h} = -\left(div_h \mu^e \nabla_h \mathrm{v}^\varepsilon, u^\varepsilon\right) = -\left(div_h \mu^\varepsilon \nabla_h \mathrm{v}_1, u_1\right) - \left(div_h \mu^\varepsilon \nabla_h \mathrm{v}_2, u_2\right) =$$

$$= \left(\mu^e \nabla_h \mathrm{v}_1, u_1\right)_{D_h} + \left(\mu^e \nabla_h \mathrm{v}_2, u_2\right)_{D_h}$$

and norm

$$\|\mathrm{v}\|_{V_h}^2 = \left\|\sqrt{\mu^\varepsilon}\nabla_h \mathrm{v}\right\|^2 + \left\|\sqrt{\mu^\varepsilon}\nabla_h \mathrm{v}\right\|^2.$$

Let's write (6)-(8) in operator form in space V_h

$$B^\varepsilon \frac{\mathrm{v}_1^{n+1} - \mathrm{v}_1^n}{\tau} = A^\varepsilon \mathrm{v}^\varepsilon + f.$$

$B^\varepsilon, A^\varepsilon$ - self-adjoint operators in the space V_h. Note

$$0 < \gamma_0\left(B^\varepsilon \mathrm{v}, \mathrm{v}\right) \le -\left(A^\varepsilon \mathrm{v}, \mathrm{v}\right) \le \gamma_1\left(B^\varepsilon \mathrm{v}, \mathrm{v}\right).$$

Where γ_0, γ_1 - does not depend on ε. Hence, the general theory of iterative methods [5], it follows that the solution to (6) - (8) converge to the solutions of (5) - (6).

$$\left\|\mathrm{v}^{n+1} - \mathrm{v}\right\| \le \frac{1-\xi}{1+\xi}\left\|\mathrm{v}_0 - \mathrm{v}\right\|,$$

$$\xi = \frac{\gamma_0}{\gamma_1},$$

So, we have shown that the rate of convergence of the iterative method does not depend on changes in the small parameter $\varepsilon > 0$.

References

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Түйін

Мақалада жалған анықталу облысын табу нәтижесіне негізделген Новье-Стокс шешімі қарастырылған. Әдістің ерекшелігі – қосымша облыстың шекарасы жылдамдық пен қысымның тангенциальды құрауыштары арқылы өрнектелгендігі. Интеративті әдістің жинақталу жылдамдығы кіші параметрдің өзгеруінен тәуелсіз болады.

Резюме

В статье рассматривается численный метод решения уравнение Новье-Стокса, основанный на методе фиктивной области определния. Существенной особенностью предложенного метода явлается, то что границы вспомогательной области определяется на основе тангенциальных составляющих скорости и плотности. Скорость сходимости итеративного метода не зависит от малого параметра.

Özet

Bu makalede kağıt a. tanımlamak için hayali etki yöntemine göre, denklem Nove-Stokes denklemleri çözmek için sayısal yöntem kabulYardımcı sahası hız ve yoğunluk teğet bileşenleri olarak tanımlanır ne Önerilen yöntem yavlaetsya, önemli bir özellik. Iteratif yöntemin yakınsama oranı küçük bir parametreye bağlı değildir.