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# DEVELOPING STUDENTS' LOGICAL THINKING BY SOLVING NON-STANDARD TASKS

**Abstract.** One of the priority tasks of the modern education system is the formation of a logically thinking, research-oriented personality. The ability to think logically is a necessary condition for the successful assimilation of educational material. The article deals with the use of various non-standard tasks for this purpose, which force us to abandon the stereotypes of thinking. The paper considers some pedagogical conditions that must be followed when learning to solve non-standard problems, and partially reveals the work at the stages of their solution. However, it should be noted that when learning to solve non-standard tasks, you can and should follow the same pedagogical conditions as when working with standard tasks. The article also deals with the development of logical thinking in the process of teaching mathematics. The key point of studying this problem is the study of the levels of logical thinking: the identification of cause-and-effect relationships, the formation of mental operations in the course of solving non-standard problems.

**Keywords**: mathematical tasks, non-standard problems, stages of solving non-standard problems, logical thinking.

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**Андатпа.** Жаңартылған білім беру жүйесінің басты міндеттерінің бірі – баланың логикалық ойлау қабілетін дамыту және ғылыми-бағдарлы тұлға етіп қалыптастыру болып табылады. Оқу бағдарламасын меңгерудегі маңызды шарты – логикалық ойлауды дамыту.Осы мақсатта мақалада түрлі стандартты емес тапсырмаларды қолдану қарастырылады және бұл біздің дәстүрлі ойлау стереотиптерімізден арылуға көмек береді. Сондай-ақ мақалада дәстүрлі емес тапсырмаларды шешу барысында қажетті педагогикалық шарттарды ұстану қарастырылған және олар тапсырма шешу барысында толық ашылып, игеріледі. Алайда, стандартты емес тапсырмаларды меңгеру барысында педагогикалық шарттардың стандартты тапсырмаларын қолдануға болатындығын және қолдану қажет екендігін атап өткен жөн. Сонымен қатар, мақалада математика пәнін оқыту барысында баланың логикалық ойлауын дамыту да жан-жақты қарастырылады.Осы педагогикалық проблемалық жағдаяттың басты да маңызды шешу жолдары: оқытудың себеп-салдарын анықтау, стандартты емес тапсырмаларды шешу барысында ақыл-ой операцияларын қалыптастыру, дамыту болып табылады.

**Түйін сөздер**: математикалық тапсырмалар, стандартты емес есептер, стандартты емес есептерді шешу кезеңдері, логикалық ойлау.

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**Аннотация.** Одной из приоритетных задач современной системы образования является формирование логически мыслящей, научно-ориентированной личности. Умение логически мыслить-необходимое условие успешного усвоения учебного материала. В статье рассматривается использование для этой цели различных нестандартных задач, которые заставляют нас отказаться от стереотипов мышления. В статье рассматриваются некоторые педагогические условия, которые необходимо соблюдать при обучении решению нестандартных задач, и частично раскрывается работа на этапах их решения. Однако следует отметить, что при обучении решению нестандартных задач можно и нужно соблюдать те же педагогические условия, что и при работе со стандартными задачами. В статье также рассматривается развитие логического мышления в процессе обучения математике. Ключевым моментом изучения данной проблемы является изучение уровней логического мышления: выявление причинно-следственных связей, формирование мыслительных операций в процессе решения нестандартных задач.

**Ключевые слова**: математические задачи, нестандартные задачи, этапы решения нестандартных задач, логическое мышление.

*Introduction*

Solving various tasks is one of the factors of mastering knowledge and skills, developing mental abilities and personal qualities. Since any activity, including educational, can be described as a system for solving problems, the effectiveness of achieving the goals of education and student development depends to a certain extent on the specific hierarchy of tasks used at each moment of training.

The development of logical thinking is one of the important tasks of training. The role of mathematics in the development of logical thinking is extremely great. When consciously assimilating mathematical knowledge, students use the basic mental operations: analysis and synthesis, comparison, abstraction and concretization, generalization; make inductive conclusions, conduct deductive reasoning. The change in the priority directions of the development of the modern education system poses the task of forming creatively thinking people who have a non-standard view of the problems, who possess the skills of research work. Unfortunately, in many educational institutions in the classroom, students almost all the time solve educational and training standard tasks, the purpose of which is that the search activity of students with each subsequent task of the same type gradually curtailed and

eventually completely disappeared. Getting used to performing standard standard tasks that have a single solution and, as a rule, a single answer, which is predetermined based on some algorithm, students get used to the same type of actions, begin to think according to the standard, practically do not have the opportunity to act independently, effectively develop their own intellectual potential, primarily logical thinking, creative activity. After all, creativity is the ability to abandon the stereotypes of thinking in order to create something new. The solution of non-standard tasks by schoolchildren opens up wide opportunities in this regard.

*Theoretical part*

The task, according to psychologists, is the core structural link of any activity, including cognitive. Various functions of tasks have repeatedly become the subject of research in psychology, pedagogy, and private methods. Pedagogical experience shows that effectively organized educational activity of students in the process of solving non-standard problems is the most important means of forming a mathematical culture, such qualities of mathematical thinking as flexibility, criticality, rationality, logic; their organic combination is manifested in the special abilities of a person, giving him the opportunity to successfully carry out creative activities. Various studies contain psychological characteristics of the process of solving a problem, including non-standard ones. The phases of the thought process in solving problems are considered, generalized methods of mental activity are highlighted, and the possibilities of pedagogical management of students' mental activity are considered (Аксёнов, 2010).

In the methodology of teaching mathematics, the questions of teaching students to solve problems are quite fully developed. In methodological studies (Далингер, 2012), the role and place of problems in the process of teaching mathematics are identified, the stages of solving the problem are characterized, the methods of finding a solution to the problem are systematized, the external and internal structure of the problem is analyzed. Seitova S.M. in her works defines the formation of logical thinking of students through solving problems (Сеитова, 2019). In recent years, a number of methodological studies have been carried out, which address the issues of activating the mental activity of students with the development of mathematical education in Kazakhstan and learning to solve problems that are the theoretical basis of the methodology of teaching mathematics, considered in the works of well-known methodologists Abylkasymova A.E., Baimukhanov B.B., Smagulov E.Zh., Iskakova L.T., etc. As shown by various psychological, pedagogical and methodological studies (Дрозина, 2007), students are lost when faced with non-standard tasks, which often leads to the rejection of attempts to solve the problem. Students do not have enough skills that determine the tactics and strategy of actions in solving various tasks, in particular, the ability to independently develop a certain action program, correlate it with the results obtained, monitor and evaluate the implementation

of the initial action program, and summarize the results obtained. The role of non-standard problems in determining mathematical abilities is unique. But first you need to define non-standard tasks. Non-standard tasks are tasks that are not solved by the usual methods and transformations. Such problems are usually found at the end of the math test and are tasks for the top five, since they have a higher level of difficulty. Extraordinary tasks do not go beyond the boundaries of the school level, but often you need to spend more time on their solution (Митенева, 2005). In math lessons, we often hear the words: non-standard problem, non-standard solution, non-standard situation, non-standard approach. We have specifically considered a number of interpretations of the concept of "non-standard task". In the literature on the theory and methodology of teaching mathematics, there is still no single interpretation that would be able to fully reveal the essence of non-standard problems. The "non-standard" nature of the problem is relative. A "non-standard" task for one student may not be the same for another, more prepared one. This means that when defining a "non-standard task", you must specify the subject that solves this task. In the research works of many domestic and foreign scientists, definitions of non-standard tasks are given. Some call such tasks problematic, others - research or search, and still others - non-standard. Examining the theoretical literature on this issue, we can see that different authors have their own interpretation of these concepts. For example, Y.M. Kolyagin gives this definition of a non-standard problem: "A non-standard problem is a problem whose solution for a given student is not considered a known chain of known actions" (Колягин, 1973). This definition allows the student to understand how to perceive and understand a non-standard task.

Students in the process of studying a particular topic should be able to solve specific types of tasks, and such tasks should be called "standard tasks". So, by "standard problem" we mean problems, the algorithm for solving which is considered in the school course of mathematics, to which the requirements of the educational standard and the curriculum are imposed. Then the tasks that do not belong to the standard tasks can be called "non-standard" (Жумалиева, 2017).

L.M. Fridman and E.N. Turetsky in the book "How to learn to solve problems" provide the following definition of a non-standard problem – "Non-standard problems are such problems for which there are no general rules and regulations in the course of mathematics that determine the exact and correct program for solving them" (Фридман, 1984).

G.V. Dorofeev, M.K. Potapov, and N.H. Rozov argue that non-standard tasks can be different. On the one hand, non-standard tasks may seem very unusual, so it is initially unclear how to "approach" them. Some problems may be hidden, on the surface it seems to be a simple quadratic equation, but it cannot be deduced by standard methods. And for some groups of tasks, you will need very skillful clear and clear logical thinking. Such unusual "non-standard tasks"

require not only ingenuity, fluency in various branches of mathematics, high logical culture and psychological training. They should also be part of the curriculum (Дорофеев, 1968).

Summarizing these and other definitions, we want to offer our own understanding of the topic. We believe that non-standard tasks are those that strongly ask for an unusual, even creative approach to their solution. Solving problems of a non-standard type is very difficult, requiring a special type of thinking, concentration, strength of spirit, will, inspired by the idea and meaning of something to comprehend. Sometimes solving such problems requires minimal information from the course of school mathematics, but logic, ingenuity and ingenuity will be simply necessary. It is quite difficult to make a complete classification of non-standard tasks. Some of them can be classified according to the way they are allocated. For example, logic problems, combinatorial problems, two-method problems produced by the computational method, graph problems, Dirichlet principle problems, invariant problems, coloring problems, rational strategy problems, etc. But many reports can belong to multiple groups. Among the non-standard problems, in addition to the above problems, we can include problems for proving inequalities, equations that are derived by non-standard methods. Equations and inequalities with a parameter can also be attributed to the number of non-standard problems, since methods for solving them are not considered in school. Among the geometric problems, we can distinguish the tasks for constructing and the tasks for finding the geometric position of a point. A non-standard problem is traditionally understood either as a problem whose solution is unknown to the student, or as a problem for which the mathematics course does not contain a rule defining the program for its solution.

The analysis of psychological, pedagogical and methodological literature shows quite convincingly that a lot of material has been accumulated on the issues of teaching students to solve problems. Thus, on the one hand, it is necessary to train students to solve non-standard tasks, since such tasks have a special role in the formation of logical thinking.

*Examples of non-standard mathematical tasks*

The complexity of the concept of a non-standard problem is related to the relativity of the term "non-standard". If we consider the task on the basis of which the cognitive activity of the student is organized as a chain: content-form of presentation — the process of solving (as a component of cognitive activity) - the result (as a result of the task and the result of the activity to solve it in the form of an increment of knowledge, skills, skills, the development of emotions, motives and interests, etc.), then we will consider the non-standard task in which one or more links of this chain are non-standard. At the same time, as a criterion of "standardness", we will take officially accepted programs and textbooks of school mathematics. Here are examples of tasks that are used by us in the

organization of teaching mathematics, focused on the formation of cognitive competence, and which can be considered non-standard.

***Problem №1***

*6 bags of gold coins were found on the sunken ship. In the first bag there were 60 gold coins, in the second 30, in the third 20, in the fourth 15. When counting all the bags, it was noted that there is a pattern in the number of coins. How many gold coins are there in the 5th, 6th bags?*

*Author : Kozlova E.G.*

*Book : Fairy tales and hints, Moscow, 2004*

***Solution:***

*60, 30, 20, 15 we must find some pattern between this numbers, but as we can see there is no pattern. Then let use number of bags*

*In the first bag 60 gold – 1 x 60 = 60*

*In the second bag 30gold – 2 x 30 =60, so we can see the pattern, that product of number of bag and number of gold in it is same*

*3 x 20 = 60*

*4 x 15 = 60*

*In fifth bag we get 60 : 5 = 12*

*In sixth bag we get 60 : 6 = 10*

***Answer: 12 and 10***

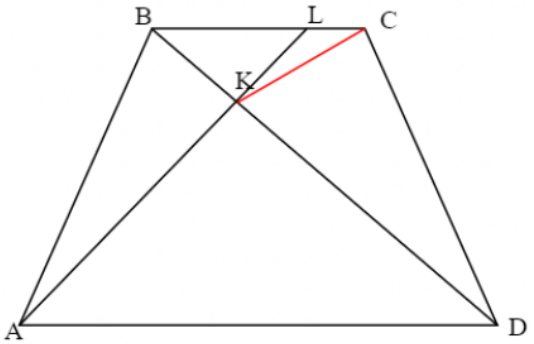
***Problem №2***

*In a trapezoid ABCD with an area of 36 $cm^2$, a straight line is drawn through the vertex A, which intersects the diagonal BD at point K, and the base BC at point L, with BK:KD = 1:3 and BL: LC = 2:1. Find the area of the quadrilateral DKLC.*

*Autho r: Kulanin E.D.*

*Book : 3000 competitive math problems, Moscow, 2003*

***Solution:***




*Let's draw a line from point K to point C, we get two triangles ΔBKL and ΔCKL. The area of ΔBKL is two times greater than area of ΔCKL, because of BL= 2LC. Let's assign area of ΔCKL as S, then area of ΔBKL = 2S.*

*Now let consider triangles$\Delta BCK$ and $\Delta CDK$. The area of $\Delta CDK$ is three times greater than area of $\Delta BCK$, because of $KD = 3BK$. So area of $\Delta BCK$ = area of $\Delta BKL$ + area of $\Delta CKL = 2S + S = 3S$, then area of $\Delta CDK = 9S$.*

*$\Delta BKL \approx \Delta DKA$, as $BC \parallel AD$, $\angle B = \angle D$, $\angle L = \angle A$*

$$\frac{BK}{KD} = \frac{KL}{KA}$$

$$\frac{1}{3} = \frac{KL}{KA}$$

$$KA = 3KL$$

*So in triangles$\Delta BKL$ and $\Delta ABK$ the area of $\Delta ABK$ is three times greater than area of $\Delta BKL$, because of $KA = 3KL$, so area of $\Delta ABK = 6S$. In triangles$\Delta ABK$ and $\Delta ADK$ the area of $\Delta ADK$ is three times greater than area of $\Delta ABK$, because of $KD = 3BK$, so area of $\Delta ADK = 18S$.*

*The area of trapezoid 36 $cm^2$*

$S_{ABCD} = S_{\Delta CKL} + S_{\Delta BKL} + S_{\Delta ABK} + S_{\Delta ADK} + S_{\Delta CDK}$

$36 = S + 2S + 6S + 18S + 9S$

$36 = 36S$

$S = 1\ cm^2$

$S_{DKLC} = S_{\Delta CKL} + S_{\Delta CDK} = S + 9S = 10S = 10x1 = 10\ cm^2$

***Answer: 10 $cm^2$***

***Problem №3***

*Some four-digit number is an exact square. If you remove the first digit on the left, it becomes an exact cube, and if you remove the first 2 digits, it becomes the fourth power of an integer. Find this number.*

*Author : Litvinov V.L*

*Book : 88 interesting and olympiad problems in mathematics, Samara, 2015*

***Solution:***

If you remove the first two digits, you get a two-digit number, which, according to the condition, is the fourth power of an integer. There are only two such two-digit numbers: $16=2^4$ and $81=3^4$;

If we remove the first digit on the left, we get a three-digit number-a cube by the condition. There are not many cubes among the three-digit numbers, here they are: $5^3$, $6^3$, $7^3$, $8^3$ and $9^3$. In this case, the last two digits must be 16 or 81. So only $6^3$ is suitable. It remains to find a number x such that x216 is a square. We can write taking this equality modulo 3, we get: (since the square of the number gives the remainder of either 1 or 0 when divided by 3). So, x is equal to 1, 3, 4, 6, 7 or 9. It could be explained more simply: x+2+1+6 - the sum of the digits of the desired number gives the same remainder of the division by 3 as the number itself [a well-known property]. You can iterate over the remaining numbers, or you can assume that the number is divisible by three. Then there are the numbers 3, 6, 9. Moreover, the desired number is divisible by 4. This means that the number is divisible by 12. It remains to check the squares of the numbers 36, 48, 60, 72, 84, 96. But the desired number ends in 6, so only 36 and 96 are

subject to consideration. A light check makes sure that 96 is suitable. The desired number is 9216

***Answer: 9216***

*Conclusion*

As one of the fundamental principles of the modern concept of teaching mathematics, the idea of prioritizing the developmental function of teaching mathematics is moving to the fore. In accordance with this, the main goal of mathematical education is not to study the basics of mathematical science as such, but to develop the ability to mathematically, and therefore logically and consciously investigate the phenomena of the real world. Therefore, the use of various kinds of non-standard tasks by the teacher in the educational process is a necessity.

Revealing the role of non-standard tasks as components of task systems, which in the competence-oriented teaching of mathematics are a form of representation of the content of education and a means of organizing cognitive activity, it can be noted that it is non-standard tasks that act as carriers of the above subject content, which largely determines the formation of those characteristics of a person capable of carrying out productive cognitive activity, which are reflected in the concept of "cognitive competence". It is the combination of standard and non-standard tasks in teaching mathematics that makes it possible to organize full-fledged cognitive activity and the development of program material, demonstrate the practical application of ideas and methods of mathematics in related fields of science, in the professional and social life of a person, involve subjects of the educational process in the management of their own cognitive activity, provide the motivational component of cognitive activity, maintain and develop cognitive activity and interest, update intellectual potential.

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