Improving the conceptual understanding of mathematics by solving real-life problems: a comparative analysis of methods and approaches.

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A thesis submitted to the Faculty of Education and Humanities

in partial fulfillment of the requirements for the degree of

MASTER OF PEDAGOGICAL SCIENCES

in Mathematics

«SDU University»

Department of Pedagogy of Natural Sciences

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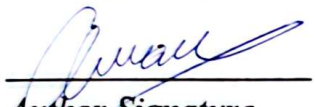

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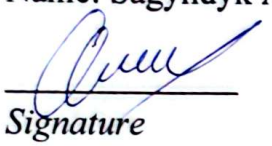

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List of Abbreviations

DOK – Depth of Knowledge
HTA – High Tech Academy
IB – International Baccalaureate
IBL – Inquiry-Based Learning
NIS – Nazarbayev Intellectual School
NCTM - National Council of Teachers of Mathematics
PBL – Problem-Based Learning
RLP – Real-life problem
STEAM – Science, Technology, Art, Engineering and Mathematics
STEM – Science, Technology, Engineering and Mathematics
TIMSS – Trends in International Mathematics and Science Study
UNT – The United National Testing
ZPD – Zone of Proximal Development

ABSTRACT

This study investigates how solving real-life mathematical problems can improve students' conceptual understanding. A quasi-experimental design was implemented in two Grade 9 classrooms at a Kazakhstani secondary school. While both groups worked with the same real-life performance tasks, the experimental group (9A) engaged with them through rich tasks and the "5 Practices" model (Smith & Stein, 2018), fostering inquiry-based learning and mathematical discourse. The control group (9B) used traditional instruction methods. Quantitative and qualitative data were collected through pre-tests, post-tests, performance tasks, observations, and reflection journals. Results showed significant improvement in both groups, but the experimental group demonstrated more consistent conceptual growth and deeper reasoning. The study concludes that combining real-life tasks with structured inquiry-based strategies enhances mathematical thinking, supports soft skill development, and contributes to more meaningful learning experiences.

Keywords: conceptual understanding, real-life problems, rich tasks, 5 Practices, mathematics education, inquiry-based learning

АҢДАТПА

Бұл зерттеу шынайы өмірге негізделген математикалық тапсырмаларды шешуде оқушылардың математиканы тұжырымдамалық түсінуіне қалай әсер ететінін қарастырады. Квазиэксперименттік дизайн аясында Қазақстандағы орта мектептің екі 9-сынып оқушыларымен жұмыс жүргізілді. Екі топ та бірдей тапсырмалармен жұмыс істеді, бірақ эксперименттік топ (9A) тапсырмаларды rich tasks әдісімен және “5 Practices” (Smith & Stein, 2018) моделі арқылы шешті. Бұл тәсіл зерттеушілік оқытуды және математикалық пікір алмасуды қолдады. Бақылау тобы (9B) дәстүрлі әдістермен оқытылды. Деректер алдын ала және кейінгі тесттер, орындау тапсырмалары, бақылау және рефлексиялық күнделіктер арқылы жиналды. Нәтижелер екі топта да жақсаруды көрсетті, алайда эксперименттік топта тереңірек және тұрақты түсінік байқалды. Бұл зерттеу шынайы өмірлік тапсырмаларды құрылымдалған зерттеушілік әдістермен біріктіру тиімді екенін көрсетеді.

Түйінді сөздер: тұжырымдамалық түсіну, шынайы өмір тапсырмалары, rich tasks, 5 Practices, математика оқыту, зерттеушілік оқыту

АННОТАЦИЯ

Данное исследование посвящено изучению того, как решение задач из реальной жизни способствует формированию концептуального понимания математики у школьников. В рамках квазиэксперимента два девятых класса казахстанской школы работали над одними и теми же задачами, приближёнными к реальности. В экспериментальной группе (9A) задания реализовывались через rich tasks и модель “5 Practices” (Smith & Stein, 2018), что позволило выстроить обучение на основе исследовательского подхода. В контрольной группе (9B) использовались традиционные методы. Сбор данных включал пред- и посттесты, performance tasks, наблюдения и рефлексивные журналы. Результаты показали улучшение в обеих группах, но у 9A прирост был более равномерным и глубоким. Исследование подтверждает эффективность сочетания задач из реальной жизни с осмысленной педагогической структурой, направленной на развитие критического мышления и метапредметных навыков.

Ключевые слова: концептуальное понимание, задачи из реальной жизни, rich tasks, 5 Practices, преподавание математики, исследовательское обучение

INTRODUCTION

Mathematics is widely recognized as one of the most important subjects in education. It provides students with the tools they need to analyze situations, interpret data, and solve complex problems. But more than that, it challenges them to think logically and apply what they learn in meaningful ways. One promising strategy for deepening mathematical understanding is the use of real-life problems—tasks that push students to think critically and creatively, while also showing them that math has value beyond exams and textbooks.

Still, for many learners, math feels distant and abstract—something disconnected from real life. Traditional instruction, which often emphasizes memorizing formulas and following step-by-step procedures, can lead to shallow learning. Students may be able to repeat algorithms, but they don't always understand why those steps work or how to apply them elsewhere. As the world changes—new technologies, evolving job markets, and a growing need for flexible thinking—there's a clear need to rethink how we teach mathematics. Research increasingly points to the importance of teaching in ways that make math meaningful, relevant, and useful. Real understanding, in this sense, becomes more than a goal—it's part of how students learn to reason, justify, and make sense of the world.

International studies like PISA and TIMSS have shown a consistent pattern: students tend to perform well on routine problems, but struggle when faced with unfamiliar or real-world contexts. This suggests that while procedural skills are being taught, conceptual understanding is often left behind. The challenge for teachers, then, is not just to teach students how to calculate, but to help them see mathematics as a way of thinking—a tool for making sense of the complex and changing world around them.

Recent research (Liljedahl et al., 2016; Kilpatrick et al., 2001) has emphasized that conceptual understanding is essential for long-term mathematical learning. It's not enough to simply know how to follow an algorithm; students need to grasp the underlying relationships between mathematical ideas. In fact, Liljedahl and colleagues argue that tasks centered around exploration and analysis help students uncover the structure and coherence of mathematical principles. Kilpatrick et al. take it further, suggesting that conceptual understanding empowers students to evaluate different solution strategies critically — and to apply them flexibly in unfamiliar situations. That kind of thinking isn't just helpful; it's foundational for building lasting mathematical skills.

One thing I've consistently seen — and which is backed up by the literature — is the power of real-life problems in deepening that understanding. Studies by Nilimaa (2023) highlight that tasks grounded in real-world contexts don't just improve critical and creative thinking. They also increase motivation. When students see that math connects to something outside the classroom — something relevant to their lives — their attitude shifts. They start to see math as meaningful. As something that's actually useful.

These kinds of tasks do more than just make math engaging. They help students build an intuitive sense of mathematical ideas. They give learners the chance to make personal connections, to relate what they're studying to their own experiences. And in my own teaching, that's when I've seen the biggest breakthroughs — when students stop asking "when will I ever use this?" and start saying, "this actually makes sense now."

What I've also noticed — and what research continues to support — is that real-world tasks do more than just teach math content. They strengthen critical thinking and problem-solving skills. Students begin to formulate hypotheses, test assumptions, justify their thinking, and make decisions even when information is incomplete or overwhelming. These aren't just math skills; they're life skills. In today's fast-changing world, the ability to think flexibly and make thoughtful decisions in uncertain conditions is more important than ever.

Realistic tasks also play a role in shaping how students *feel* about mathematics. When they see math applied in everyday life — whether it's budgeting, building something, planning a route, or analyzing data — they begin to understand its value. And when that happens, their motivation increases. They become more engaged. They start to believe that learning math is not just a school requirement, but something that genuinely matters. I've seen students go from indifferent to enthusiastic simply because they were given a chance to work on something real.

In short, integrating real-life tasks into math instruction is a powerful way to develop not only conceptual understanding, but also higher-order thinking and a more positive, purposeful approach to learning. Teachers who bring these kinds of problems into their classrooms are doing more than covering curriculum. They're preparing students to navigate a complex, rapidly changing, interconnected world — and giving them the tools to do it thoughtfully.

Research topic: Improving the conceptual understanding of mathematics by solving real-life problems: a comparative analysis of methods and approaches. The primary aim of this study is to explore the effectiveness of using real-life mathematical problems to support students' conceptual understanding. This includes examining the impact of instructional approaches that incorporate rich tasks and the 5 Practices model (Smith & Stein, 2018), as well as comparing their outcomes with those of traditional, procedure-focused teaching methods.

Participants: 9-grade students learning mathematics in settings where instruction incorporates real-life problem-solving. Scope of the Study: teaching methods and instructional strategies that are grounded in solving real-life mathematical problems.

Research questions:

1. To what extent does solving real-life problems improve students' conceptual understanding of mathematics?
2. Which teaching methods are most effective for integrating real-life problems into the classroom?
3. How do students' outcomes differ when learning through real-life problems compared to traditional approaches?

This study is built around two sets of hypotheses: one addressing the general effect of using real-life problems, and another comparing the effectiveness of two instructional approaches for implementing them.

Null Hypothesis 1 (H_0): The use of real-life problems has no significant impact on students' conceptual understanding of mathematics. Alternative Hypothesis 1 (H_1): The use of real-life problems leads to a significant improvement in students' conceptual understanding of mathematics.

Null Hypothesis 2 (H_0): A teaching approach based on rich tasks and the 5 Practices model shows no advantage over the traditional method of teaching real-life problems in terms of improving conceptual understanding. Alternative Hypothesis 2 (H_1): The teaching method grounded in rich tasks and the 5 Practices model is significantly more effective than the traditional approach to teaching real-life problems when it comes to developing students' conceptual understanding of mathematical concepts.

These two sets of hypotheses reflect different levels of analysis: the first focuses on the general effect of incorporating real-life problems into instruction, while the second compares specific instructional methods for implementing such problems. Testing the first pair of hypotheses helps determine whether using real-world contexts in the math classroom deepens students' mathematical thinking and enhances their ability to make meaningful connections between concepts.

The second set of hypotheses (H_0 and H_1) focuses on the qualitative differences between two instructional approaches: the traditional method, and a more structured approach that combines *rich tasks*—which are open-ended, layered, and closely connected to real-world contexts—with the 5 Practices model (Smith & Stein, 2018), which offers a pedagogical framework for guiding mathematical discussions. The underlying assumption is that this combination—open, meaningful tasks paired with intentional teacher moves—can have a more profound impact on students' conceptual understanding than simply adding real-life contexts to conventional explanation-and-practice lessons.

In this way, testing the proposed hypotheses allows not only an assessment of the general effectiveness of real-life problems in the classroom, but also an exploration of the specific instructional conditions that best support their use in school mathematics.

Keywords: conceptual understanding, real-life problems, rich tasks, mathematics teaching methods, active learning, constructivism, scaffolding, action research, quasi-experiment.

1. LITERATURE REVIEW

1.1 Definition of conceptual understanding

Developing conceptual understanding is considered one of the central goals of modern mathematics education. Within the International Baccalaureate (IB) framework, conceptual understanding is defined as a student's ability to engage meaningfully with mathematical ideas, make connections between concepts, operations, and real-life situations, and apply their knowledge in unfamiliar contexts (International Baccalaureate Organization, 2021). The IB curriculum places strong emphasis on transferable thinking skills: students are expected not only to master algorithms but to understand why they work and where they can be applied.

Key concepts such as form, function, connection, and modeling serve as the foundation for instruction. These ideas promote an interdisciplinary approach and encourage learners to see mathematics as a meaningful, interconnected structure rather than a collection of isolated procedures. This reflects a broader shift in educational philosophy — away from learning by repetition to learning by reasoning and connection, meaningful engagement with content.

In the Middle Years Programme (MYP) of the IB, mathematics instruction is built around clearly defined concepts that are explicitly aimed at developing deep conceptual understanding. According to the *MYP Mathematics Guide*, this is achieved through a focus on *key concepts* and *related concepts*, which help students interpret ideas, recognize patterns, and apply knowledge in real-world contexts (International Baccalaureate Organization, 2021).

The key concepts in the MYP mathematics framework represent broad, overarching ideas that run throughout the entire curriculum. In MYP Mathematics, these include:

- Form – the structure and organization of mathematical objects,
- Logic – reasoning processes and systems of proof,
- Relationships – the connections between quantities, properties, and concepts.

Engaging with these concepts helps students move beyond memorizing isolated facts. Instead, they are encouraged to grasp the underlying meaning of mathematical ideas and to interpret them across a variety of contexts. The goal is not just to *know*, but to *understand* — to see how mathematical thinking applies across domains and makes sense in the real world.

Related concepts complement the key ideas and provide subject-specific depth. These include concepts such as *Change*, *Equivalence*, *Model*, *Pattern*, *Quantity*, *Representation*, *Space*, and *System*. Rather than focusing solely on procedures, these ideas help structure the mathematics curriculum around meaning and understanding.

For instance, when studying functions, students might explore how quantities change (*Change*), create and analyze models (*Model*), interpret visual representations such as graphs (*Representation*), and identify underlying patterns (*Pattern*). This kind of work supports a more integrated and meaningful understanding of mathematical relationships.

In this way, the concept-based structure of the MYP Mathematics curriculum supports the development of flexible, transferable knowledge—knowledge that students can apply beyond the boundaries of the classroom. This directly aligns with the central aim of the present study: to identify instructional approaches that enhance students' conceptual understanding of mathematics through the use of problems rooted in real-life contexts.

Similar ideas emerge in research exploring the integration of digital technologies into mathematics education. In a chapter by Derek Pope (2022) focusing on platforms such as Desmos and GeoGebra, conceptual understanding is defined as "comprehension of mathematical concepts, operations, and relations," as well as "a holistic and functional grasp of mathematics that enables students to assimilate new ideas by connecting them with what they already know" (Pope, 2022, p. 108). According to Pope, it is through visualization, interactive experimentation, and inquiry-based tasks that students move from surface-level familiarity to deep mathematical understanding.

Digital tools like Desmos and GeoGebra allow learners to explore connections between algebraic and graphical representations, analyze the behavior of functions, and identify patterns on their own. These tools foster a learning environment where real-life problems can be introduced more effectively—problems that not only encourage cognitive engagement, but also make the learning experience more meaningful and relevant for students.

One of the most foundational contributions to this area comes from the U.S. National Research Council's report *Adding It Up: Helping Children Learn Mathematics*, where conceptual understanding is identified as one of five strands of mathematical proficiency—alongside procedural fluency, strategic competence, adaptive reasoning, and productive disposition (Kilpatrick, Swafford, & Findell, 2001). The authors define conceptual understanding as "an integrated and functional grasp of mathematical ideas," in which the learner not only knows how to apply a rule, but also understands why it works. This kind of knowledge is structured and connected, allowing it to be transferred to unfamiliar problems and situations.

This kind of understanding is particularly crucial in real-world contexts, where solutions are rarely straightforward or formulaic. Kilpatrick et al. (2001) emphasize that students with strong conceptual understanding are able to recognize, formulate, and solve problems based on generalized ideas, rather than by mechanically applying procedures.

Building on this perspective, Faulkenberry (2003) highlights that conceptual understanding involves making sense of the structure of mathematics—that is, the internal connections and logic underlying mathematical processes. He argues for the importance of moving from fragmented knowledge toward a more coherent and systemic view, which is especially relevant in areas such as school geometry and algebra.

More recent studies, such as those by Hussein and Csíkos (2022), have explored how concept-oriented instruction influences students' motivation and emotional well-being. Their findings suggest that approaches that foster conceptual understanding can

reduce math anxiety, as students experience a greater sense of control and meaning in their learning (Hussein & Csíkos, 2022). These insights are particularly valuable as education shifts away from a purely evaluative model and toward a more formative and developmental focus.

In their study, Birgin and Uzun (2022) define conceptual understanding as "a comprehensive and functional grasp of mathematical ideas" that allows students to build new knowledge upon prior experience. Their research shows that learners with higher levels of conceptual understanding are better equipped to adapt to unfamiliar problems and to work fluidly across multiple representations—graphical, numerical, and algebraic.

These findings are especially relevant to the present study, which centers on solving real-life problems. Such tasks require not just the recall of procedures, but the flexible application of knowledge in complex, often unfamiliar situations.

The study by Alqawas and Alhadad (2024)proposes a model that balances conceptual and procedural learning. The authors argue that instruction is most effective when conceptual understanding is supported—not replaced—by procedural practice. This idea aligns closely with the earlier conclusions of Kilpatrick et al. (2001) and reflects the emphasis in current educational standards, such as those of the NCTM and the International Baccalaureate.

According to their model, conceptual knowledge should come first and serve as a guide for procedural development: first comes meaning, then comes practice. This is particularly important in topics such as functions, equations, and other areas where understanding the relationships between representations is essential.

A valuable contribution to the evolving definition of *conceptual understanding* can be found in the recent study by Rupnow and Fukawa Connelly (2004), published in *Frontiers in Education*. In this work, the authors conducted a series of interviews with professional mathematicians who teach proof-based courses, aiming to explore how these educators define and foster deep understanding of mathematical concepts and definitions.

Their findings reveal that mathematicians do not associate conceptual understanding with memorizing definitions. Instead, they emphasize the ability to explain an idea in multiple ways, to interpret a definition in different contexts, and to connect it to broader mathematical structures. As one interviewee noted:

«If you really understand a theorem, you can explain why it's true in more than one way, and you can apply it in places it wasn't intended for» (Rupnow & Fukawa-Connelly, 2004)

The study concludes that educators often consider true understanding to be reflected in a student's ability to interpret, apply, and justify mathematical ideas within a framework of formal reasoning. This reinforces the position of other scholars (Alqawas & Alhadad, 2024), highlighting the ongoing shift from mechanical, rule-based instruction to a more meaning-centered, concept-driven approach.

In the context of school mathematics education, this body of research clearly demonstrates that even at the highest levels of mathematical training, it is conceptual knowledge—not mere procedural fluency—that remains the central focus. This is

especially relevant to the present study, which emphasizes tasks that demand flexible thinking and the ability to transfer knowledge into real-life contexts.
Bringing together the perspectives discussed above, several shared characteristics of conceptual understanding can be identified:

- It involves deep, connected knowledge.
- It requires the ability to transfer and apply ideas flexibly.
- It is grounded in reasoning and relational thinking rather than memorization.
- It develops most effectively through real-world tasks where context matters.
- It is strengthened by—but not replaced with—procedural knowledge.

All of the sources reviewed consistently underscore the same conclusion: conceptual understanding is the foundation of long-term, meaningful, and resilient mathematical thinking. For this reason, the present study aims to compare instructional methods that explicitly support the development of this kind of understanding—particularly through the use of real-life mathematical problems.

1.2 Definition of real-life problems

A recent study conducted by Portuguese educators Rocha and colleagues (2024) set out to explore how ninth-grade students approach inequality problems when they are embedded in realistic contexts. The authors observed that these tasks pushed students to engage in more meaningful mathematical thinking. As they noted:

«The real context of the problems seems to have the potential to develop in students a more integrated mathematics, focused on understanding and not so much on the repetition of mechanical and meaning independent procedures» (Rocha, Viseu, & Matos, 2024).

This insight reinforces the idea that real-life problems are not just about providing relevance — they encourage students to connect ideas, think relationally, and move away from rote procedures. In this way, authentic contexts become more than backdrops for computation; they serve as catalysts for conceptual integration and reflective reasoning. The inclusion of such problems in mathematics curricula, then, is not simply a matter of making learning more engaging — it's a deliberate pedagogical strategy for fostering understanding.

In a comprehensive review, Liljedahl et al. (2016) elaborate on contemporary approaches to problem-based mathematics education. They outline four interconnected dimensions that support deep engagement: heuristics, creativity, student-generated problems, and the use of technology. Together, these elements encourage students to work with problems that are grounded in practical reasoning rather than abstract manipulation. As the authors explain:

«Problem solving in mathematics education has… infused mathematics curricula around the world with calls for the teaching of problem solving as well as the teaching of mathematics through problem solving» (Liljedahl, Santos-Trigo, Malaspina Jurado, & Bruder, 2016).

This shift affirms a growing consensus: real-world problems are not simply contextualized exercises — they are complex, open-ended challenges that require

flexible thinking, creativity, and sustained inquiry. When used thoughtfully, they do more than make math "relevant"; they redefine what it means to understand.

According to the online resource *Ooodles Learning* (2024), a real-life problem is defined as:

«Teaching math through real-world problem-solving helps students view math as a concrete idea, encourages critical thinking, and boosts their confidence"

A key pedagogical insight here is the importance of translating abstract mathematical concepts into concrete, real-life situations. This process not only increases student motivation but also deepens understanding, as learners begin to see math as something meaningful rather than mechanical.

The *Accelerate Learning* blog (2022) emphasizes the value of connecting mathematics to everyday experiences. As the authors note:

«When real world connection is incorporated into lessons, students have the opportunity to see how math fits into their daily lives… math becomes a visible phenomenon in their routines instead of a set of abstract rules»

This perspective highlights a shift in how we frame mathematics for students — from something distant and theoretical to something lived, observed, and usable.

This definition confirms that real-life problems are not simply textbook exercises with a storyline — they are drawn from everyday practice and help students recognize mathematics as something applicable beyond the classroom. When students are able to see math as a tool for understanding their surroundings, its perceived value increases.

The Mathematical Association of America (Devlin, 2022) describes the process of solving real-world problems as a cyclical loop:

«We begin with some real-world situation… identify key features… formulate those features in mathematical terms… do the math… apply back to real world… sometimes round again»

This cycle captures the essence of mathematical modeling — translating life into formulas, graphs, and structures, and then interpreting those results back through the lens of lived experience. In this sense, modeling is not an add-on, but the bridge between abstract reasoning and contextual understanding. In her article, Larina (2016) offers a theoretical model for defining real-world math problems, drawing on both international research and empirical data gathered in Russian schools. Her aim is to identify which tasks, actually used by teachers in classrooms, meet the criteria of "realness" — and how those tasks are perceived by students. This dual perspective, combining pedagogical intent with student reception, provides a more grounded and practice-oriented understanding of how real-life problems function in education.

According to Larina (2016), three key criteria define what qualifies as a real-world mathematical task:

1. Realistic context – The task must be grounded in a life situation that is recognizable and relevant to the student.
2. Mathematical modeling – The problem should involve translating that situation into mathematical language, such as equations, formulas, or graphs.

3. Non-triviality – The task should not be solvable by a standard, memorized procedure. It should instead require strategic decision-making, interpretation of information, and analytical thinking.

Taken together, these criteria frame real-world problems not as decorative stories wrapped around standard exercises, but as genuinely meaningful tasks. They invite students to make sense of real situations through mathematics, to model, interpret, and reflect — rather than simply recall and reproduce. Such problems push students to think flexibly, evaluate methods, and engage in conceptual reasoning. They also foster motivation, as learners begin to see purpose in what they're doing. When mathematical content is embedded in context — and that context demands thought — learning becomes active, inquiry-driven, and intellectually honest. These conclusions echo the findings of Rocha et al. (2024), Liljedahl et al. (2016), Larina (2016), and the Mathematical Association of America (Devlin, 2022), all of whom argue that the role of real-world problems extends far beyond application: they are vehicles for deep, transferable understanding.

Additional research confirms that incorporating real-life problems into mathematics instruction fosters not only mathematical skills, but also broader cross-disciplinary competencies such as argumentation, decision evaluation, and collaboration (Boaler, 1998). Boaler emphasizes that students who consistently engage with contextualized tasks develop more robust and enduring mathematical knowledge — knowledge that is transferable to unfamiliar situations. This is particularly relevant in today's fast-changing world, where adaptability and reasoning are often more valuable than the ability to recall standard procedures. In this context, real-world problems do more than teach mathematics; they cultivate habits of thinking that extend well beyond the math classroom.

Research from the Freudenthal Institute (Netherlands) promotes the concept of *realistic mathematics education* — an approach grounded in the idea that students learn mathematics best through experiences rooted in real life. Rather than introducing concepts through formal definitions, this model emphasizes inductive learning: students encounter meaningful problems first, and through reflection and exploration, gradually build abstract understanding (Gravemeijer, 1994).

In this paradigm, students draw on strategies that are familiar from their everyday lives, making mathematics feel more accessible and purposeful. Learning becomes an active process of sense-making — not something delivered from the outside, but something constructed through engagement with situations that feel real and relevant. This vision of mathematics education supports not only conceptual understanding, but also learner agency and authenticity in the classroom.

Moreover, real-life problems contribute to the development of students' intuitive grasp of patterns, structures, and models. Tasks involving budgeting, travel planning, time estimation, percentages, or resource consumption offer opportunities for students to transfer abstract mathematical ideas into the realm of personal experience. Through such contexts, mathematics becomes not merely a subject to study, but a way of thinking — a cognitive tool for navigating everyday decisions.

A defining feature of these tasks is their openness. Rather than following predetermined steps, students are invited to investigate which information is relevant, identify reasonable assumptions, and explore how best to represent and interpret their findings. This element of uncertainty mirrors the nature of real-world problem-solving and aligns closely with principles of mathematical inquiry. In such environments, students are not passive recipients of knowledge but active constructors of meaning, making sense of mathematics through exploration, reflection, and dialogue.

It is also important to highlight that realistic tasks enable teachers to evaluate not only the final answer but also the student's reasoning process. This feature is particularly valuable within the framework of formative assessment, where the emphasis is placed on understanding rather than on the number of correct responses. Such insight into student thinking allows for more targeted and meaningful feedback, fostering further conceptual development.

In practice, designing effective real-life problems requires not only pedagogical expertise but also interdisciplinary thinking. Teachers must be able to connect mathematical ideas to current themes in fields such as environmental studies, economics, social sciences, or technology. This cross-curricular approach enriches both the relevance and depth of mathematical learning.

Within the context of this study, real-life tasks are positioned as a crucial component in the shift from traditional, procedure-focused instruction to a model grounded in conceptual understanding, modeling, and reflection. They serve as a bridge between classroom mathematics and the lived experiences of students — a claim supported by both empirical evidence and the theoretical foundations of constructivist learning.

1.3 Theoretical framework

This research is grounded in theoretical approaches that emphasize the development of students' conceptual understanding through the use of real-life problem solving. At its core, the study draws upon constructivist pedagogy, Vygotsky's theory of scaffolding, and the methodological frameworks of action research and quasi-experimental design. The instructional practices were structured around the "5 Practices" model (Smith & Stein, 2018), particularly in the 9A classroom, where rich tasks were employed as a tool to deepen mathematical thinking and promote meaningful engagement with mathematical ideas.

1.3.1 Constructivism and Vygotsky's theory

Constructivism, as a philosophy of education, posits that knowledge is actively constructed by learners through their engagement with the environment and social interactions. Although scholars such as Jerome Bruner, Jean Piaget, and Lev Vygotsky offer different perspectives on constructivist learning, they share a common emphasis on meaning-making and learner participation. Bruner highlights the importance of spiral curriculum and the representation of knowledge in three modes: enactive, iconic, and symbolic. Piaget concentrates on stages of cognitive development, while Vygotsky emphasizes the influence of social context and adult guidance in learning.

This study draws particular attention to Vygotsky's cultural-historical theory, which argues that learning can lead development and actively promote it when the learning environment is thoughtfully structured. A central concept is the Zone of Proximal Development (ZPD), referring to the gap between what a learner can accomplish independently and what can be achieved with guidance. Learning is most effective when it occurs within this zone, supported by interaction with a teacher or a more capable peer. This support is often referred to as *scaffolding*—a temporary framework that adjusts to the learner's needs and is gradually removed as independence grows (Vygotsky, 1978), (Bakker, Smit, & Wegerif, 2015).

Scaffolding can be implemented in a variety of ways, including guiding questions, structured group work, visual representations, or strategic prompts. It proves particularly effective in the context of real-life problem solving, where students are required to apply their knowledge in novel and often unpredictable situations. Research suggests that such scaffolded support fosters not only procedural mastery, but also the development of cross-disciplinary skills such as argumentation, hypothesis generation, and critical evaluation (Wibowo, Wangid, & Firdaus, 2025).

Constructivist pedagogy also holds that knowledge should be embedded within meaningful contexts. This principle makes real-world tasks especially valuable, as they allow students to recognize the relevance and applicability of mathematical ideas. Learners are encouraged to identify patterns, apply abstract concepts in tangible situations, and generate their own hypotheses. Such an approach fosters deep, durable, and transferable understanding.

1.3.2 Action research and quasi-experimental approach

The methodological framework of this study integrates action research with elements of a quasi-experimental design. Action research positions the teacher not merely as an observer but as an active participant in the educational process, implementing innovative strategies, monitoring their outcomes, and making informed adjustments based on feedback. This approach encourages reflective practice and enables change grounded in the specific teaching context.

A key feature of action research is its cyclical nature—planning, acting, observing, and reflecting. This cycle supports a flexible and adaptive process, allowing for adjustments tailored to the classroom environment and student needs. Moreover, it creates a bridge between theoretical frameworks and day-to-day teaching, facilitating immediate assessment of pedagogical effectiveness.

The quasi-experimental element involves a comparison between two classes, 9A and 9B. In the 9A group, instruction incorporated rich tasks and the 5 Practices model, whereas 9B followed a more traditional (Soviet-influenced) approach. This comparative structure supports a more rigorous evaluation of how different instructional methods influence students' conceptual understanding. The design includes baseline diagnostics, classroom observation, analysis of student work, and final assessment, providing a comprehensive view of learning outcomes.

This combined methodological strategy offers a deeper insight into instructional impact by accounting for individual learning trajectories and contextual factors. The

presence of both groups allows for exploration not only of academic achievement but also of learner motivation, engagement, and depth of reasoning throughout the learning process.

1.4 The 5 practices model and rich tasks

The core instructional tool used in the 9A classroom was *«The 5 Practices for Orchestrating Productive Mathematical Discussions»* framework (Smith & Stein, 2018), which offers a structured approach for teachers to guide meaningful and conceptually rich classroom discourse. The model outlines five interconnected steps:

1. Anticipating – considering the range of strategies students might use and predicting potential difficulties;
2. Monitoring – observing and collecting evidence of student thinking during the task;
3. Selecting – identifying specific student responses to highlight during discussion;
4. Sequencing – purposefully arranging the order in which selected responses will be shared;
5. Connecting – helping students make mathematical connections between different approaches and underlying concepts.

This approach enables the teacher to maintain a clear instructional goal while creating space for authentic student thinking to emerge and be used as a central resource for learning.

This structure enables the design of flexible yet purposeful lessons, where students are not merely reproducing knowledge but actively constructing and re-evaluating it. The 5 Practices model is particularly effective when combined with *rich tasks*—multi-faceted assignments that promote active thinking, integration of knowledge, and application to real-life contexts.

Rich tasks foster metacognition, creativity, and the ability to transfer knowledge across domains. They often transcend the boundaries of a single topic or unit, requiring students to synthesize different mathematical ideas and skills. For instance, a task involving budget planning for a school trip may incorporate arithmetic, proportions, graph construction, and argumentation in selecting the most efficient option.

Moreover, such tasks cultivate collaboration and reasoning skills, as they often involve group work, peer discussion, and justification of proposed solutions. They contribute to students' confidence and reinforce the role of mathematics as a tool for analysis and decision-making in real-world scenarios. This aligns with contemporary educational priorities, including the development of soft skills and functional mathematical literacy.

1.4.1 Theoretical foundation of the 5 practices model

«The 5 Practices for Orchestrating Productive Mathematics Discussion» (Smith & Stein, 2018) represents one of the most influential frameworks in modern mathematics education for supporting conceptual understanding through classroom dialogue. Developed over years of classroom-based research, the 5 Practices model

provides a structured yet flexible approach that empowers teachers to facilitate meaningful mathematical discussions based on students' own ideas. At its core, the model addresses the challenge of how to transition from teacher-centered instruction toward a more dialogic, student-centered environment, where mathematical thinking is not only performed but also verbalized, critiqued, and refined in a collaborative setting.
Breakdown of the Five Practices

1. Anticipating Anticipating involves the teacher's effort to envision how students might approach the task, including the different strategies—both correct and incorrect—that may emerge. It requires teachers to solve the task themselves in as many ways as possible and to think about how each strategy relates to the underlying mathematics. Teachers often benefit from anticipating together with colleagues, reviewing student work from previous years, or consulting research on common misconceptions. By anticipating potential solutions, teachers can prepare targeted questions to guide students and recognize deeper mathematical potential in students' strategies.

Figure 1.4.1 A chart for monitoring students' work on the Leaves and Caterpillars task. Reprinted from 5 Practices for Orchestrating Productive Mathematics Discussions by M. S. Smith & M. K. Stein, 2017, p. 12. Copyright 2017 by Corwin Press.

Strategy	Questions	Who and What	Order
Unit rate Find the number of leaves eaten by one caterpillar (2½) and multiply by 12, or add the amount for one 12 times to get 30 leaves.	• How did you get 2½? What does it represent? • Why did you multiply by 12? What does it represent? • *First you had 2C and 5L. Now you have 12C and 30L. How are these related?* • *Suppose you had to feed 100 caterpillars instead of 12. How many leaves would they need?*	Janine: multiplied 12 x 2½ (sticks representing caterpillars) Kyra: added 2½ 12 times (picture of leaves and caterpillars)	Janine – 3rd
Scale Factor Find that the number of caterpillars (12) is 6 times the original amount (2), so the number of leaves (30) must be 6 times the original amount (5).	• What does 6 represent? • Why do you multiply by 5? • *First you had 2C and 5L. Now you have 12C and 30L. How are these related?* • *Suppose you had 100 caterpillars instead of 12. How many leaves would they need?*	Jason: narrative description (language a little confusing—*count by twos until you come to half of 12*)	Jason – 4th
Scaling Up Increase the number of leaves and caterpillars by continuing to add 5 to the leaves and 2 to the caterpillars until you reach 30 leaves.	• How did you get 30 leaves? • How do you know that this is the right number of leaves? • *First you had 2C and 5L. Now you have 12C and 30L. How are these related?* • *Suppose you had 100 caterpillars instead of 12. Could you find the number of leaves needed without continuing your drawing or table?*	Jamal: table with leaves and caterpillars increasing in increments of 2 and 5 Martin and Melissa: 6 sets of leaves and caterpillars	Martin – 1st Jamal – 2nd
Additive Find that the number of caterpillars has increased by 10 (2 + 10 = 12), so the number of leaves must also increase by 10 (5 + 10 = 15).	• Why did you add 10 to 2 and 5? • How many leaves did each caterpillar get when there were only 2 caterpillars? • How many leaves does each caterpillar get now that there are 12 caterpillars? • *What if we want each caterpillar to get the same number of leaves no matter how many caterpillars we have? What could you do?*	Missy and Kate	
OTHER Multiply leaves and caterpillars together: 5 x 12 = 60.		Darnell and Marcus	

2. Monitoring Monitoring is the practice of observing students as they work on tasks—individually or in groups—and documenting the strategies used. Effective monitoring requires more than just watching students; it involves asking probing questions to clarify student thinking and promote deeper engagement. Teachers may use a "monitoring chart" to track which students are using which approaches and identify unexpected strategies that arise. This chart becomes a foundation for making informed decisions during whole-class discussions.

3. Selecting After observing student work, teachers deliberately choose specific students to share their work during class discussion. The goal is to highlight mathematically significant ideas that serve the lesson's learning objectives. Teachers select solutions not necessarily because they are correct, but because they represent important mathematical points or common misconceptions worth exploring.

4. Sequencing Once the students are selected, the teacher sequences the order of their presentations to build mathematical understanding progressively. For example, the teacher might start with a common or concrete approach and move toward more abstract or sophisticated reasoning. This helps scaffold the discussion, allowing all students to access the ideas and participate meaningfully. Purposeful sequencing can also surface key comparisons and contrasts between strategies.

5. Connecting The final practice involves helping students draw connections among the different solutions and to the central mathematical ideas of the lesson. Rather than treating each presentation as an isolated event, the teacher orchestrates discussion to synthesize patterns, generalizations, and key mathematical principles. Teachers ask students to reflect on similarities and differences, efficiency of methods, and broader applicability of strategies. Smith and Stein (2018) emphasize that while each of the five practices has value individually, they are most powerful when enacted in concert. Anticipating enhances monitoring; monitoring informs selection; and thoughtful sequencing and connecting turn a series of presentations into a coherent and purposeful learning experience.

Integration with rich tasks: the integration of the 5 Practices model with rich tasks further amplified its impact. Rich tasks, by design, are multi-faceted, open-ended problems that encourage students to reason, collaborate, and apply mathematics to real-world scenarios. They require students to synthesize knowledge across domains and construct meaning through exploration rather than memorization. When paired with the structure of the 5 Practices, these tasks become a powerful engine for deep learning. For instance, in one rich task involving traffic safety and quadratic modeling, students were asked to develop a model for braking distance, analyze driver reaction time, and make policy recommendations for city signage. The task demanded mathematical modeling, graph interpretation, and consideration of assumptions—skills aligned with high-level cognitive processes. Through the 5 Practices, the teacher was able to elevate student reasoning, orchestrate discussion sequences, and link strategies across the class to fundamental mathematical principles.

Additionally, rich tasks support the development of soft skills and metacognitive awareness. Students must justify their decisions, evaluate the reasonableness of models, and revise their thinking when new information arises. This cultivates not only mathematical proficiency but also perseverance, communication, and collaboration—competencies that are essential in the 21st-century world.

By combining rich tasks with the 5 Practices model, the instructional approach in 9A provided a coherent, inquiry-based experience that balanced structure with student ownership. This approach supported the goals of this study: to enhance conceptual understanding, promote reasoning, and align classroom practice with international standards of mathematical literacy.

1.4.2 The connection between rich tasks and active learning methods

Rich tasks are closely aligned with active learning methods, such as Problem-Based Learning (PBL)—an instructional strategy in which learning is organized around solving a complex, open-ended problem rooted in real-life contexts. Students work collaboratively, analyze the situation, formulate hypotheses, conduct research, and arrive at a solution. This process promotes critical thinking, interdisciplinary connections, and the ability to acquire knowledge independently (Hmelo-Silver, 2004). Inquiry-Based Learning (IBL) is another approach in which students explore a mathematical problem by generating questions, formulating hypotheses, conducting observations, and drawing conclusions. At the core of this method lies the active construction of knowledge and the development of an investigative stance among learners (Artigue & Blomhøj, 2013).

Collaborative learning involves structuring the learning process so that students engage in small groups or pairs, where they exchange ideas, articulate their reasoning, and work toward shared conclusions. This format enhances social interaction and promotes the development of communication and transversal competencies such as argumentation, coordination, and reflection (Gillies, 2016).

All these instructional strategies align with the principles of constructivist pedagogy and find efficacy when implemented using rich tasks. Unlike conventional exercises with a single correct solution, rich tasks encourage learners to engage in exploration, make strategic decisions, and construct well-reasoned arguments. This mode of learning resonates with constructivist ideas and supports progress within the learner's zone of proximal development.

Rich tasks often incorporate elements of hands-on learning, wherein students physically interact with materials or tools to explore mathematical ideas. Research by Blanco et al. (2019) indicates that this approach activates embodied cognition, a process of grounded conceptual understanding that is especially significant when teaching abstract concepts like geometry, proportions, or functions.

In this sense, rich tasks are not merely a type of mathematical activity; they serve as a pedagogical framework that facilitates active, inquiry-driven, and dialogic learning, where the learner plays a central, agentive role in knowledge construction.

1.5 Relevance of the problem

Despite numerous global and national reforms aimed at modernizing mathematics education, a substantial and persistent issue remains: students often excel in routine procedural tasks but struggle with applying mathematics in authentic, unfamiliar contexts. This trend has been documented across decades by international assessments such as PISA and TIMSS. According to the latest OECD report (2025), this “transfer gap” suggests that students may be trained to compute but are not learning to think mathematically in flexible, adaptive ways.

This distinction between procedural fluency and conceptual understanding is critical. Procedural knowledge—knowing how to perform algorithms or manipulate symbols—can be rehearsed and tested in structured formats. But conceptual understanding involves seeing the meaning behind the procedures, recognizing

patterns, justifying reasoning, and applying ideas in diverse contexts. As mathematics increasingly becomes a foundational skill for navigating data-rich, technology-driven environments, this deeper understanding becomes not optional, but essential.

The challenge is not only pedagogical but cultural. Traditional instruction in many school systems, including Kazakhstan, privileges certainty, speed, and correct answers. As such, students are rarely invited to struggle productively, make conjectures, or reflect on their thinking—yet these are the very behaviors associated with expert mathematical reasoning (Boaler, 2016). Without opportunities to explore mathematics as a human endeavor—full of questions, interpretations, and meaningful applications—students may disengage, viewing math as irrelevant or intimidating.

In this light, real-life mathematical problems serve a dual purpose: they are not only vehicles for content but also *contexts for thinking*. They offer a bridge between school mathematics and the world students live in. When designed well, they demand modeling, decision-making, interpretation, and critique—core components of mathematical literacy. However, research (Larina, 2016) cautions that many so-called "real-world tasks" in textbooks are pseudo-contextual, offering surface-level scenarios without cognitive demand or authentic decision-making.

This is where rich tasks come in. As described in international research (NRICH, 2021); (Liljedahl, Santos-Trigo, Malaspina Jurado, & Bruder, 2016), rich tasks are open-ended, often non-routine problems with multiple solution paths and entry points. They are rich not because they are more difficult, but because they invite students to reason deeply, justify choices, and communicate their thinking. These tasks are especially powerful when embedded in collaborative settings and supported by deliberate discussion.

Yet even strong tasks can fall flat without pedagogical support. That is why the 5 Practices model (Smith & Stein, 2018) is critical—it provides a roadmap for teachers to transform student responses into learning opportunities. Rather than relying on spontaneous facilitation or privileging dominant voices, the 5 Practices help teachers plan for productive struggle**.**

This model aligns with Vygotskian theory: learning happens not in isolation but through mediated, social interaction. It also reflects the core principles of constructivist pedagogy**:** knowledge is not transmitted but built. In classrooms where the 5 Practices are used with rich tasks, students take on the role of investigators, not just answerers. This cultivates not only understanding but also confidence and agency.

Across the world, these approaches are no longer fringe—they are central. Singapore's CPA (Concrete–Pictorial–Abstract) approach, Finland's emphasis on interdisciplinary learning, and the U.S. Common Core's Standards for Mathematical Practice all reflect a global movement toward making math education more meaningful and connected**.** This is further echoed in the IB Middle Years Programme, where key and related concepts frame learning as transfer-oriented and inquiry-driven.

In Kazakhstan, however, implementation has been uneven. While official curricula reference real-life problems and critical thinking, in practice many schools retain a recitation-based model with little space for inquiry. Teachers may be evaluated based on coverage of content rather than quality of understanding. Moreover, external

assessments like the UNT reward speed and accuracy more than reasoning. These structural pressures make it difficult for teachers to experiment with new methods—especially ones that demand flexibility, dialogue, and responsiveness.

Nevertheless, growing interest in international best practices and the emergence of innovative schools (e.g., High Tech Academy, Nazarbayev Intellectual Schools) demonstrate a readiness for change**.** This study contributes to that conversation by offering a model that is both theoretically grounded and practically feasible. It acknowledges the constraints of the system but also identifies a pathway forward—one where students not only do mathematics but *understand it, use it, and see its value.*

In sum, the relevance of this research lies in its response to multiple, intersecting needs:

- The need for deeper understanding, not just performance;
- The need for instructional coherence between task design and facilitation;
- The need for localized examples of innovation in teaching;
- The need for methods that align with global educational trends but work in Kazakhstani classrooms.

By framing the problem through both international evidence and national context, this research positions itself not just as an academic exercise but as a contribution to the ongoing evolution of mathematics education policy and practice.

1.6 Comparison of traditional and conceptual approaches

Modern approaches to mathematics education emphasize a shift away from algorithmic instruction toward tasks that support deep, conceptual understanding. Increasingly, educational systems around the world recognize that students need more than the ability to apply formulas—they need to understand the "why" behind the methods, and apply mathematics meaningfully in real-life situations.

Table 1.6.1 Comparison of traditional and conceptual approaches to teaching mathematics

Approach	Learning Objective	Sample Tasks	Expected Outcomes
Traditional	Mastery of procedures and algorithms	Solve a linear equation; Substitute values into a formula	Quick and accurate application of memorized rules; Procedural fluency
Conceptual	Understanding of mathematical ideas and connections	Model the budget for a school trip; Analyze survey data; Compare mobile phone plans	Informed decision-making; Mathematical modeling; Interpretation and reasoning

Traditional tasks primarily assess procedural fluency and recall of formulas. Students are typically asked to reproduce steps identical or similar to those demonstrated by the teacher. This approach is effective for training efficiency and accuracy but often fails to promote a meaningful understanding of why certain strategies work, how concepts are related, or in which contexts a particular method is most useful.

In contrast, conceptual approaches center on meaning-making. Tasks are framed to elicit student reasoning, connection of ideas, and reflection. For instance, a budgeting task might integrate skills in percentages, arithmetic operations, proportional reasoning, and data interpretation. More importantly, students are asked not only to compute but to justify their decisions, communicate their thinking, and analyze consequences. These tasks align with the development of higher-order thinking skills, such as critical thinking, argumentation, abstraction, and transfer of knowledge across domains.

Importantly, conceptual learning does not reject procedural knowledge—it repositions it. Rather than treating procedures as ends in themselves, they become tools students choose and apply based on understanding. This fosters mathematical autonomy, where students are not only able to solve problems but also to choose appropriate methods, explain their reasoning, and adapt flexibly to novel situations.
This distinction is at the heart of many international curriculum reforms. In countries such as Singapore, Finland, and Canada, education policy has shifted to prioritize inquiry-based learning, mathematical modeling, and performance tasks. These systems emphasize mathematical thinking over memorization, aligning assessment with real-world competencies.

In Kazakhstan, however, mathematics instruction in many mainstream schools remains predominantly traditional. Students are often trained for speed and accuracy on standardized assessments like the UNT, which emphasize correct answers over reasoning or modeling. Although textbooks increasingly include real-world contexts, these are frequently treated superficially and do not engage students in deeper inquiry or dialogue.

Within this study, the conceptual approach was operationalized through rich tasks and the 5 Practices framework. These methodologies created an instructional environment where students did not simply solve problems, but constructed meaning through collaboration, questioned assumptions, and communicated their strategies in structured discussions. By engaging with authentic, multi-step problems and by taking ownership of their learning, students in the experimental group (9A) were able to not only achieve correct solutions but also understand the structure and logic of the mathematics involved.

This shift from “how to solve” to “why and when to use a method” is more than a pedagogical nuance—it is a paradigm shift. It aligns mathematics education with the demands of a 21st-century knowledge economy and equips students with the cognitive tools needed for lifelong learning and informed citizenship.

1.7 International experience in mathematics education

Leading educational systems around the world demonstrate fundamentally similar strategies for transitioning from mechanical learning to a conceptual approach focused on deep understanding, critical thinking, and the application of mathematics in real-life situations. These principles are most vividly implemented in the practices of Finland, Singapore, and the United States.

The Finnish education system is traditionally regarded as one of the most successful globally, owing to its humanistic approach and emphasis on student well-being. Mathematics education in Finland is structured around real-life contexts. Curricula emphasize interdisciplinarity, practical relevance, and an inquiry-driven approach.

Textbooks contain open-ended problems that go beyond rote formula application. For example, instead of a standard task on calculating percentages, students may be asked to compute a family's annual electricity costs, compare energy tariffs, and propose cost-effective solutions. This approach demands analysis, model construction, data interpretation, justification of choices, and strategy development — all of which are key components of conceptual understanding.

The assessment formats also align with this philosophy: classrooms actively incorporate projects, oral explanations, collaborative tasks, and self-assessment. This shifts the focus from obtaining the correct answer to the process of reasoning and understanding.

A distinctive feature of the Finnish model is its high level of teacher autonomy: educators are free to choose methods and flexibly adapt materials to their class. This makes it possible to implement real-life tasks that students perceive as meaningful and motivating.

Singapore consistently ranks among the top performers in international assessments such as PISA and TIMSS. This achievement stems from a well-structured hierarchy of concepts, a logical progression of instruction, and a carefully designed model of visualization. A central element of teaching is the CPA approach — Concrete → Pictorial → Abstract — guiding students from hands-on experiences to visual representations and finally to abstract reasoning.

In practice, this is implemented through the use of bar models—visual diagrams that represent quantitative relationships. Students first manipulate physical objects, then use drawings, and only later transition to algebraic formulas. This sequence ensures a solid and meaningful understanding of the relationships between quantities. Bar models are widely used to teach complex problems involving percentages, fractions, proportions, and equations.

Singapore’s educational documents emphasize the importance of metacognitive strategies, modeling, reflection, logical reasoning, and explanation. Teachers are trained not only to deliver knowledge but also to engage students in dialogue and reasoning. This provides a strong foundation for solving problems grounded in real-life contexts. Moreover, the Singaporean model incorporates a well-structured research lesson format in which students are encouraged to independently arrive at a solution, explore alternative approaches, and draw conclusions—reflecting the principles of

inquiry-based learning. This makes the tasks more meaningful and the learning process more active.

In the American education system, conceptual understanding is promoted through initiatives and recommendations by organizations such as the National Council of Teachers of Mathematics (NCTM). Central to these frameworks are mathematical modeling, reasoning, justification, and classroom discussion.

The integration of *rich tasks* has emerged as a response to the need to develop 21st-century skills—critical thinking, argumentation, and collaboration. These tasks are specifically designed to avoid a single solution algorithm. Instead, they encourage students to formulate hypotheses, analyze conditions, choose strategies, and justify their answers. For example, students may be asked to select the most cost-effective mobile phone plan based on real data or assess the structural stability of a bridge given a schematic. Such tasks require not only the application of formulas but also the construction of models, interpretation of results, and explanation of choices—in essence, engaging in mathematical thinking. Moreover, American schools are increasingly adopting the *5 Practices for Orchestrating Productive Mathematics Discussions* model (Smith & Stein, 2018). This framework provides teachers with a structured approach to facilitating discussions —from monitoring student solutions to connecting different strategies. It transforms the classroom into a space of active, inquiry-based learning and deepens conceptual understanding through meaningful interaction.

The educational practices of Finland, Singapore, and the United States show that lasting mathematical thinking is not developed through rote memorization of formulas, but through meaningful engagement with context, dialogue, and reflection. Across these systems, several shared principles emerge:

- Real-life problems serve as a source of both motivation and relevance;
- Visual and modeling tools support understanding of mathematical structure;
- Group work and peer explanation are treated as essential learning processes;
- The role of the teacher is flexible—more of a facilitator than a transmitter of knowledge.

These ideas formed the foundation of the present study. In the 9A class, teaching was structured around real-world tasks designed as rich tasks and guided by the 5 Practices model. In contrast, class 9B followed a more traditional, procedural approach. This design not only allowed the study to align local teaching practices with global trends, but also enabled a critical evaluation of the effectiveness of both approaches within the real context of a classroom.

In Kazakhstan, the traditional approach to teaching mathematics has seen a partial incorporation of real-life contexts; however, these are often implemented in a limited, procedural manner rather than as tools for fostering conceptual understanding. While textbooks and classroom examples may include data or narratives that reference real-world scenarios—such as train timetables, market prices, or population figures—the instructional design tends to prioritize the execution of algorithms over critical thinking, modeling, or interpretation. In such lessons, teachers typically follow a lecture-based format: they present the concept, solve one or two typical problems, and

then assign similar exercises for individual practice. Real-world problems in classrooms are usually "pseudo-contextual," meaning they contain realistic content but do not require the student to make real decisions, analyze constraints, or justify reasoning. This is reinforced by the assessment system in Kazakhstan, which is heavily oriented toward national standardized tests like the UNT, where speed and accuracy take precedence over creativity and conceptual depth. Consequently, students are rarely encouraged to discuss multiple strategies, explore mathematical structures, or engage in argumentation. Whenever teachers attempt to introduce real-life scenarios, without intentional scaffolding and space for student inquiry, these tasks often degenerate into routine exercises. Compared to international best practices, this results in a missed opportunity: OECD (2025) findings suggest that students trained primarily through procedural approaches tend to underperform in tasks requiring mathematical reasoning in unfamiliar contexts. Therefore, while the surface features of real-world applications may be present in Kazakhstani math education, their implementation under the traditional paradigm limits their potential to develop students' conceptual understanding, problem-solving capacity, and critical thinking skills.

1.8 Challenges and limitations

Despite the well-documented benefits of incorporating real-life problems into mathematics instruction and their proven effectiveness in fostering conceptual understanding, this approach encounters a number of challenges—methodological, pedagogical, institutional, and psychological. This section aims to critically examine these obstacles within the context of the implemented study and the broader field of educational practice.

Rich tasks place significant demands on the teacher's professional competence. They require the ability to integrate mathematical content with authentic contexts while addressing the cognitive and social needs of students. Designing such tasks goes beyond simply wrapping traditional exercises in real-world narratives. It involves ensuring the scenario is realistic, allowing for open-ended exploration, offering students meaningful choices, and requiring modeling and justification. This process is time-consuming and calls for pedagogical expertise as well as deep subject knowledge. Unfortunately, many teachers face limited time for planning, restricted access to high-quality task banks, and insufficient methodological support—particularly in general secondary schools.

The shift from traditional instruction to a conceptually driven approach is often perceived as risky. Teachers accustomed to rigid routines—explanation, practice, and assessment—may lack confidence in facilitating class discussions, responding to open-ended student ideas, or navigating unpredictable lines of reasoning. Moreover, in high-pressure environments driven by standard assessments, instructional priorities may lean toward exam preparation rather than the development of deep mathematical understanding.

In many educational systems—including those in the post-Soviet region—a strong emphasis remains on producing a single correct answer, prioritizing accuracy, speed, and procedural fluency. This mindset stands in contrast to the ethos of rich tasks,

which value experimentation, trial and error, and the acceptance of mistakes as part of the learning process. Students raised in an assessment culture where making mistakes is stigmatized may experience anxiety, resistance, and a decline in motivation when engaging with open-ended problems. Transforming these deeply rooted attitudes requires long-term commitment and consistent support at the school level.

Assessing conceptual understanding demands different tools from traditional multiple-choice assessments. It involves evaluating students' reasoning, modeling, and the use of visual and verbal representations of thought processes—elements that are not easily standardized or quantified. This presents a dual challenge: teachers must develop the skills necessary for formative assessment, while educational systems must evolve beyond a sole focus on numeric scoring. The absence of appropriate assessment tools can discourage educators from implementing innovative approaches, as documenting student progress in a clear and acceptable manner becomes difficult.

Real-life tasks—especially those that are open-ended and multifaceted—often require a high degree of student autonomy. However, not all learners are prepared for such independent engagement. Some may struggle to comprehend the task, extract relevant information, or organize their problem-solving strategies. This underscores the need for teachers to differentiate instruction effectively, apply scaffolding techniques, and select tasks with an appropriate level of challenge—once again pointing to the importance of teacher training and access to sufficient resources.

The integration of rich tasks requires a significant investment of time—both in planning and in delivering lessons. A single lesson based on this approach often takes longer than a typical session focused on explanation and drill. In the context of a tight curriculum schedule and the pressure to cover all required topics, such lessons may be seen as a luxury. Furthermore, in the absence of collective support or collaborative planning among staff, the responsibility for designing these lessons often falls solely on the individual teacher.

The shift from traditional, tightly structured problems to open-ended, inquiry-based tasks can be stressful for students, especially those accustomed to routine and familiar formats. Working with real-life problems demands greater effort, autonomy, and self-management. Some students may perceive these tasks as "too difficult" or "unclear." This highlights the need to foster a learning culture in which mistakes are treated as a natural part of the learning process rather than as signs of failure.

Although educational literature provides substantial evidence supporting the use of real-life tasks, systematic data from regular school practice—particularly at the local level—remains limited. This lack of context-specific quantitative evidence can hinder the adoption of such practices at the school or policy level, where decisions often require measurable outcomes.

In summary, despite their strong theoretical foundation and practical promise, the implementation of real-life tasks calls for systemic change: in teacher preparation, in assessment practices, in policy frameworks, and in the broader learning culture. Acknowledging and critically examining these constraints is essential for designing realistic strategies to support pedagogical innovation in mathematics education.

The theoretical framework of this study integrates constructivist learning theory, Vygotsky's concept of scaffolded instruction, the methodological principles of action research, and elements of quasi-experimental design. In the mathematics classroom, particular emphasis is placed on the use of rich tasks and the 5 Practices model as tools for deepening conceptual understanding. These approaches not only support academic achievement but also help students develop key transferable skills—critical thinking, communication, collaboration, and creativity. Bridging theory and practice through the use of real-life contexts makes mathematics instruction more meaningful, engaging, and effective.

1.9 Summary of the literature review

The reviewed literature converges around the growing recognition that developing students' conceptual understanding in mathematics requires more than procedural knowledge or rote application of algorithms. Central to this shift is the influence of constructivist theories, especially those advanced by Vygotsky, Bruner, and Piaget. These frameworks emphasize the learner's active role in knowledge construction, the significance of social interaction, and the use of scaffolding as a means to guide students through zones of emerging competence.

Rich tasks appear consistently in the literature as a promising tool to bridge abstract mathematical concepts with real-life applications. Unlike traditional exercises, these tasks engage students in open-ended inquiry, promote multi-step reasoning, and require a synthesis of skills across mathematical domains. They also support the development of argumentation, collaboration, and metacognitive reflection—skills now widely recognized as essential in modern education systems.

The international examples from Finland, Singapore, and the United States further illustrate how systemic support enhances the practical integration of rich tasks. Common patterns include emphasis on meaningful contexts, flexible pedagogical frameworks, and teacher autonomy in lesson design. In Finnish classrooms, for instance, problem-solving is embedded within interdisciplinary scenarios that mirror authentic decision-making. Singapore's structured transition from concrete to pictorial and then abstract representations demonstrates how visual modeling supports deeper understanding. Meanwhile, American initiatives promote discourse-rich classrooms through strategic frameworks like the "5 Practices" model, placing mathematical dialogue at the heart of instruction.

At the same time, the literature acknowledges a range of practical challenges. Rich tasks demand thoughtful design, often requiring more time and cognitive effort from both teacher and learner. Teachers may face constraints related to curriculum pacing, assessment formats, and institutional expectations. In many contexts, particularly those influenced by rigid assessment cultures, the fear of deviating from standardized methods discourages experimentation. Additionally, students accustomed to closed problems and fixed answers may struggle with the ambiguity and independence that open-ended tasks require.

The issue of assessment emerges as particularly complex. Capturing conceptual growth demands tools that move beyond binary right-or-wrong formats and instead

focus on reasoning, representation, and interpretation. However, such assessments remain underdeveloped or inconsistently applied, making it difficult to document learning gains in ways that satisfy institutional demands.

In post-Soviet educational contexts, including Kazakhstan, these challenges are often intensified by systemic inertia, lack of access to rich task resources, and limited professional development opportunities. Still, the potential for rich tasks to transform classroom practices remains significant, especially when paired with well-scaffolded instruction and reflective pedagogical inquiry.

Several studies also highlight the importance of teacher collaboration and professional learning communities in facilitating the shift toward inquiry-based instruction. Without collective engagement, even the most innovative approaches may remain isolated within individual classrooms. Moreover, the successful implementation of rich tasks often depends on the creation of a classroom culture that values exploration, accepts mistakes as learning opportunities, and encourages persistence in the face of complexity.

Furthermore, sustained integration of such methods requires not only changes in instructional practice but also a rethinking of curricular goals and success criteria. When conceptual understanding is prioritized over rote mastery, the role of mathematics education expands—preparing learners not merely to solve equations, but to interpret, question, and act in mathematically informed ways. This reconceptualization aligns mathematics with broader educational aims such as civic reasoning, social responsibility, and adaptive expertise.

The insights gathered from the literature thus inform the rationale for this study. By embedding rich tasks within a real classroom setting and systematically comparing outcomes across different instructional approaches, the research seeks to explore how theoretical ideas translate into practical gains. In doing so, it contributes to an ongoing conversation about what constitutes meaningful mathematics education—and how schools can create the conditions for such learning to take root.

2. METHODOLOGY

2.1 Research design

This research followed a mixed-methods design, combining both quantitative and qualitative approaches. As a practitioner-researcher, I sought not only to measure the effectiveness of various teaching strategies but also to deeply reflect on my pedagogical practice. This dual focus naturally led to a design-based framework that integrated principles from action research (Kemmis & McTaggart, 1988) and quasi-experimental design (Cook & Campbell, 1979).

The mixed-methods approach allowed me to capture both measurable learning outcomes and the nuanced cognitive shifts that occurred in students' mathematical thinking. While the quantitative component tested my hypotheses using statistical procedures, the qualitative component provided insight into how and why students developed conceptual understanding through real-life problem solving. This methodology was informed by the constructivist view of learning (Vygotsky, 1978), emphasizing that knowledge is not passively absorbed but actively constructed through meaningful experience. Rich tasks, inquiry-based instruction, and structured classroom discussion—particularly as articulated in the 5 Practices Model (Smith & Stein, 2018) — are deeply rooted in this view.

In order to systematically monitor how students approached complex contextual tasks, I used task-specific monitoring charts, such as the one shown in Figure 1. These tools—originally presented in 5 Practices for Orchestrating Productive Mathematics Discussions—helped anticipate students' solution strategies, common errors, and conceptual breakthroughs. For instance, in the "Leaves and Caterpillars" task, students demonstrated reasoning patterns ranging from additive and multiplicative to scaling and unit rate strategies. By documenting these patterns, I was able to trace the development of their mathematical thinking over time.

Furthermore, this research design allowed me to embed formative assessment moments directly into the classroom discourse. I paid close attention not only to final answers but to the processes students used: the questions they asked, the representations they chose, and the justifications they offered. These elements became critical data points in understanding conceptual shifts and the role of real-world problem solving in facilitating them.

2.2 Context and participants

The study was conducted at High Tech Academy (HTA), a secondary school in Almaty, Kazakhstan, during the second half of the 2024–2025 academic year. The participants were two ninth-grade classes:

- Class 9A ($n = 21$) – experimental group, taught by the researcher using rich tasks and the 5 Practices model.
- Class 9B ($n = 22$) – control group, taught by another experienced mathematics teacher using a traditional lecture-based method. The researcher acted as a co-teacher and observer in this group.

In both classrooms, a two-teacher model was used, allowing for deeper insight and comparison of instructional approaches.

2.3 Research timeline and phases

The implementation of the study followed a structured timeline:

- 2023–2024: The researcher completed the *Teaching Mathematics through Rich Tasks* professional development course (American School Foundation of Monterrey).
- September 2024: Initial integration of rich tasks into classroom practice.
- January 2025: Full implementation of the 5 Practices model (Smith & Stein, 2018) in 9A. Group differentiation begins.
- December 2024: Pre-test administered to both groups to assess baseline conceptual understanding.
- February–March 2025: Performance tasks conducted in both classes, aligned with real-life contexts.
- May 2025: Post-test administered to both groups.
- Throughout January–May 2025: Ongoing classroom observations, reflection journals, and collection of student work samples.

A detailed lesson plan illustrating the implementation of rich tasks and the 5 Practices model is provided in appendix 1.

2.4 Instruments

2.4.1 Conceptual understanding test

The main assessment tool was a **custom-developed diagnostic test**, inspired by PISA-type tasks. The test measured **conceptual understanding** through scenarios requiring reasoning, modeling, estimation, argumentation, and use of multiple representations. Unlike procedural exercises, these tasks assessed how well students understood underlying principles.

Examples of task types included:

- Interpreting and modifying graphs representing real-life contexts.
- Budget planning with constraints and trade-offs.
- Scaling and proportional reasoning in design problems.

2.4.2 Performance tasks

The performance tasks were designed to simulate real-life decision-making. One task, for example, required students to evaluate different mobile data plans based on family needs and build a mathematical model to support their recommendation. These tasks were scored using rubrics adapted from NCTM (2014) guidelines on mathematical practices.

Unlike traditional word problems that often focus on isolated skills, performance tasks demand integration of multiple concepts, reasoning strategies, and communication. They provide students with a complex scenario, typically open-ended,

where there is no single correct method, and often no single correct answer. This structure allows learners to demonstrate not only what they know, but how they think.

According to Webb's Depth of Knowledge (DOK) framework, performance tasks often operate at Levels 3 and 4, where students are required to engage in strategic thinking, justify their reasoning, and apply mathematical ideas to unfamiliar contexts. This depth promotes transfer of learning and nurtures conceptual understanding.

In the context of this study, performance tasks were framed around authentic contexts such as budgeting for a school trip, interpreting survey data, and comparing ecological footprints. Each task encouraged students to construct mathematical models, interpret data, and make reasoned decisions, thereby linking classroom learning to their lived experience.

The use of performance tasks also served as a formative assessment tool. By observing how students approached the task—what assumptions they made, how they organized information, and how they justified their conclusions—I gained insight into their level of conceptual understanding. This information informed my instructional decisions and provided opportunities for timely feedback and scaffolding.

Moreover, because performance tasks require sustained cognitive engagement, they foster perseverance and productive struggle—two essential components of mathematical growth (Boaler, 2016). Students were encouraged to revise their models, refine their reasoning, and discuss alternative approaches with peers, promoting a classroom culture of sense-making rather than answer-getting.

2.4.3 Qualitative tools

- Observation checklists and field notes captured student behavior, participation, and language use.
- Student reflections (written exit tickets and journals) offered insight into their thought processes.
- Teacher reflection logs documented pedagogical decisions, student responses, and emerging themes.

2.5 Data analysis

Given the modest sample size, I applied a combination of descriptive and inferential statistical techniques, complemented by thematic qualitative analysis.

2.5.1 Quantitative analysis

- Descriptive statistics summarized mean scores, standard deviations, and distribution trends.
- Normality tests (Shapiro–Wilk) were applied to check the assumptions of parametric testing.
- Paired t-tests were conducted to assess changes between pre- and post-test scores within each group.
- Cronbach's alpha was used to measure internal consistency of the test instruments.

- To ensure reliability with a small sample, I used bootstrap oversampling, a non-parametric resampling technique, which increased the robustness of inference.

2.5.2 Qualitative analysis

Qualitative data were analyzed through inductive thematic coding. I identified recurring patterns related to:

- Conceptual talk and mathematical language.
- Student persistence and collaboration.
- Types of reasoning (e.g., deductive, analogical).
- Shifts in metacognitive awareness.

Findings were triangulated with quantitative results to ensure validity and depth.

2.6 Ethical considerations

All participants and their guardians provided informed consent prior to data collection. Student anonymity was preserved, and participation in the study had no impact on students' academic grading. Ethical approval was obtained from the university's academic board in alignment with institutional research policies. Additionally, all classroom observations were conducted with full transparency, and teachers were informed in advance about the purpose and scope of the research. Care was taken to minimize disruption to normal teaching and learning routines.

2.7 Sample lesson plan: shipping context and linear inequalities

To illustrate the pedagogical design used in the experimental group (9A), one sample lesson is included below. This lesson was aligned with the 5 Practices framework (Smith & Stein, 2018) and incorporated a rich task centered on systems of linear inequalities using a real-world context — shipping dimensions and packaging constraints.

Learning Objective: Students will be able to solve systems of linear inequalities in a real-life context.

Real-World Context (Rich Task): *"Shipping companies impose extra charges for oversized packages and those requiring special handling. Oversized packages exceed a combined length and girth of 84 inches. Special handling applies if the length exceeds 60 inches. Which package dimensions qualify for both conditions?"*
Students interpret this context and translate it into a system of inequalities:

- $l + g > 80$
- $l > 60$

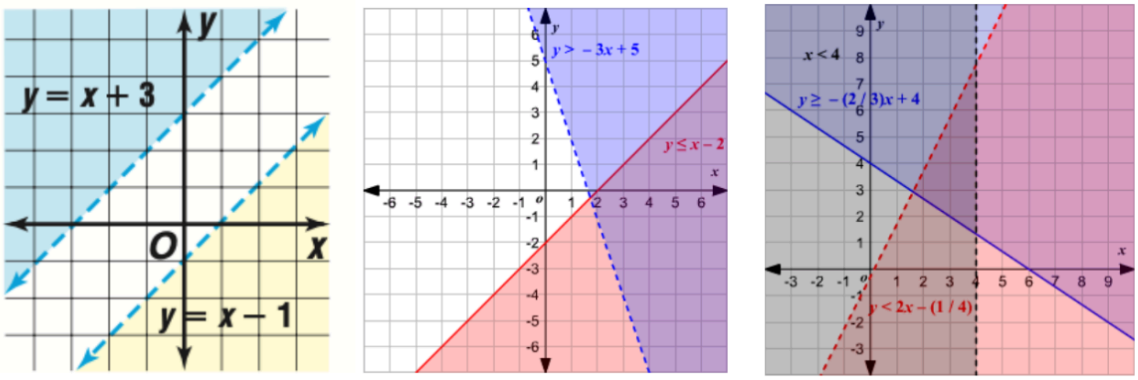
They then analyze the shaded region on a coordinate grid that represents the feasible solutions, and evaluate whether specific ordered pairs satisfy both conditions.

Table 2.7.1 Lesson activities for a rich task on graphing inequalities

Time	Activity	Description
15 min	Rich Task Introduction	Students are introduced to the shipping scenario, analyze the inequalities
10 min	See-Think-Wonder	Students reflect on the graph and context visually
10 min	Group Discussion & Modeling	Groups work on graphing and identifying correct solutions
5 min	Exit Ticket	Students choose correct coordinate pairs that satisfy both inequalities

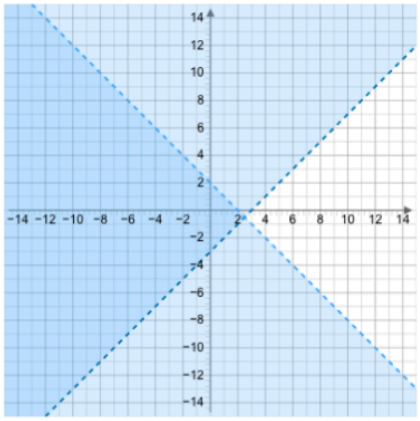
Figure 2.7.2 Visual tasks on inequalities used in See–Think–Wonder and Exit Ticket activities. Designed to support inquiry-based learning and formative assessment.

See-Think-Wonder activity

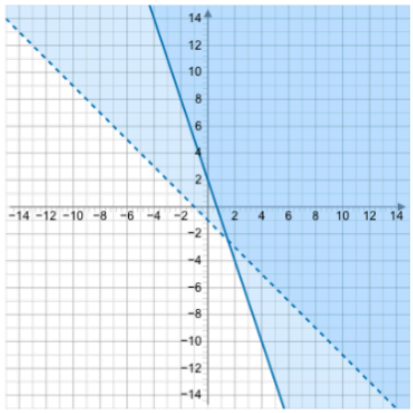


Exit ticket

Select the coordinate pairs that are solutions to the system. (There may be one or more correct answers.)



- ○ $(0, 0)$
- ○ $(-2, -5)$
- ○ $(4, 3)$
- ○ $(3, 4)$
- ○ $(-3, -4)$



- ○ $(0, 0)$
- ○ $(-4, 14)$
- ○ $(-2, -4)$
- ○ $(2, 3)$
- ○ $(1, 1)$

Connection to the 5 Practices Model:

- Anticipating: The teacher anticipates misconceptions (e.g., including the boundary lines, incorrect inequality direction).
- Monitoring: While students work on graphing solutions, the teacher collects sample approaches.
- Selecting: Representative student strategies are selected for class discussion.
- Sequencing: Responses are sequenced to progress from simple to more complex reasoning.
- Connecting: The class identifies the shared structure across different approaches and connects algebraic and graphical models.

Student Reflection: Through See-Think-Wonder and class discussion, students link inequality solutions to real-world constraints. This encourages deeper reasoning about feasible regions and contextual interpretations, which are essential in conceptual learning. Additional lesson plans are included in appendix 4.

Sample lesson plan 2: graphing linear systems in real-life contexts.

To further illustrate the pedagogical approach used in the experimental group (9A), this lesson demonstrates how students explored the concept of solving systems of linear equations through graphing, using real-world financial decision-making scenarios. The lesson follows the 5 Practices Framework (Smith & Stein, 2018) and includes structured inquiry and reflection components. Visuals and performance-based questions are used to promote applied conceptual understanding.

Learning Objective: students will be able to solve systems of linear equations using graphical methods and interpret their solutions in context.

Real-World Context (Rich Task). Consumer Choice Scenario:

Madison is planning to rent a car for two years. She has two rental options:

- Option A: Pay $326 per month after an initial $200 deposit
- Option B: Pay $1600 upfront, followed by monthly payments of $226

Students are asked:

"Which payment option is more cost-effective for a two-year rental if the initial deposit is not a concern?"

They then build mathematical models for each option and graph them to identify points of intersection or cost equivalence.

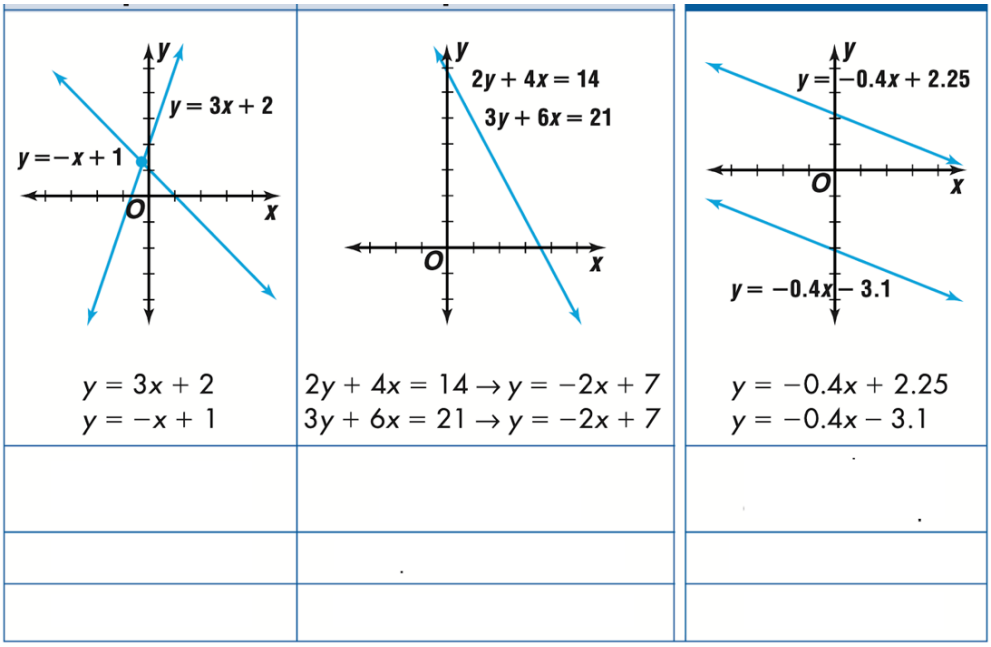
Table 2.7.3 Lesson activities for a rich task on graphing linear systems

Time	Activity	Description
10 min	Warm-up & Connection	Review the concept of a “solution” to a linear equation with graph examples
15 min	Rich Task: Consumer Choice	Students construct models, graph both options, and interpret total costs
10 min	Group Modeling & Discussion	Students explain reasoning, revise graphs, and compare solution strategies
5 min	Exit Ticket	Solve a new system and report the solution coordinates

See–Think–Wonder prompts:
Students are presented with sets of linear graphs and asked:

- What do you notice?
- What do you wonder?
- What can this tell you about the cost difference or break-even point?

Figure 2.7.4 Visual examples of linear systems used during See–Think–Wonder and Exit Ticket activities. Designed to provoke reasoning about types of solutions: unique, infinite, or none — in support of inquiry-based learning and formative assessment.



Exit Ticket Example:
Solve the system of equations graphically and show your answer in the form $x \cdot y$:

$$\begin{cases} 2x - y = 3 \\ x + 5y = 7 \end{cases}$$

Connection to the 5 Practices Model:

- Anticipating: Teacher predicts common errors (e.g., scale mismatch, intersection point misreading).
- Monitoring: Collects strategies during graphing.
- Selecting: Chooses contrasting examples for discussion.
- Sequencing: Moves from simpler (equal slopes) to complex (realistic cost models).
- Connecting: Compares algebraic and graphical interpretations of cost equivalence.

2.8 Examples of performance tasks

All the performance tasks listed below were provided to both groups (9A and 9B) as core content. However, the distinction lay in the pedagogical approach used. The experimental group (9A) engaged with these tasks through rich-task pedagogy supported by the "5 Practices" model (Smith & Stein, 2018), emphasizing inquiry, discussion, scaffolding, and conceptual understanding. The control group (9B), on the other hand, approached the same tasks via a traditional method, which focused on direct instruction, teacher-led explanation, and individual problem solving.

Example 1: Performance task — vehicle stopping distance (main task).

This task was embedded into the real-life rich task and used in both groups. It models the stopping distance of a truck using quadratic modeling based on data.

Prior to the full performance task, students engaged with a focused conceptual pre-task (see Figure 2.8.1) to build a foundational understanding of how quadratic models can represent braking distance. In this pre-task, they developed and compared two models (Model A and Model B) based on given data, helping them interpret vertex behavior, assess model fit, and discuss the relevance of mathematical models in realistic driving contexts. This conceptual warm-up served to scaffold the more open-ended performance task that followed, allowing students to approach it with greater mathematical insight and confidence.

Scenario: "You are an analyst working for the Almaty Transport Department. You have been tasked with improving traffic safety at a busy intersection where several minor traffic accidents have occurred. It is believed that drivers fail to stop in time before the traffic light. Your mission is to analyze the stopping distance of an average truck in Almaty and determine the optimal placement of speed limit signs that allow for safe stopping."

Task Features:

- Develop quadratic models based on tabulated data;
- Use Desmos or a graphing calculator to estimate model parameters (Model A and Model B);
- Plot and interpret graphs;
- Calculate percent error based on actual vs. modeled data;
- Analyze the role of driver reaction time in total stopping distance;
- Provide recommendations supported by mathematical reasoning.

This task, adapted from the IB Mathematics HL Specimen Paper, involves modeling braking distances using quadratic functions (International Baccalaureate Organization, 2018).

[Maximum mark: 17]

The braking distance of a vehicle is defined as the distance travelled from where the brakes are applied to the point where the vehicle comes to a complete stop.

The speed, $s\ m/s$, and braking distance, $d\ m$, of a truck were recorded. This information is summarized in the following table.

Speed, $s\ m/s$	0	6	10
Braking distance, $d\ m$	0	12	60

This information was used to create Model A, where d is a function of s, $s \geq 0$.

$$\text{Model A: } d(s) = ps^2 + qs, \text{ where } p, q \in Z$$

At a speed of 6 m/s, Model A can be represented by the equation $6p + q = 2$.

a) (i) Write down a second equation to represent Model A, when the speed is 10 m/s.
(ii) Find the values of p and q. [4]

b) Find the coordinates of the vertex of the graph of $y = d\ (s)$. [2]

c) Using the values in the table and your answer to part (b), sketch the graph of $y = d(s)$ for $0 \leq s \leq 10$ and $-10 \leq d \leq 60$, clearly showing the vertex. [3]

d) Hence, identify why Model A may not be appropriate at lower speeds. [1]

Additional data was used to create Model B, a revised model for the braking distance of a truck.

$$\text{Model B: } d(s) = 0.95s^2 - 3.92s$$

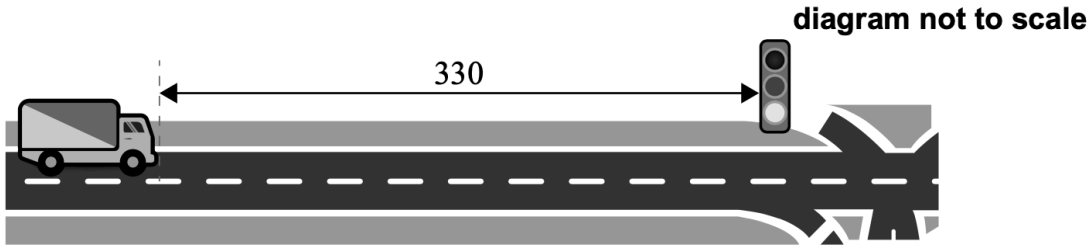
e) Use Model B to calculate an estimate for the braking distance at a speed of 20 m/s [2]

The actual braking distance at 20 m/s is 320 m.

f) Calculate the percentage error in the estimate in part (e). [2]

It is found that once a driver realizes the need to stop their vehicle, 1.6 seconds will elapse, on average, before the brakes are engaged. During this reaction time, the vehicle will continue to travel at its original speed.

A truck approaches an intersection with speed $s\ m/s$. The driver notices the intersection's traffic lights are red and they must stop the vehicle within a distance of 330 m.



g) Using model B and taking reaction time into account, calculate the maximum possible speed of the truck if it is to stop before the intersection. [3]

Rubric A — Performance task assessment rubric.

The following dimensions were evaluated:

1. Presentation — Clarity, structure, and logical organization of the mathematical work.
2. Mathematical Communication — Relevance and consistency of mathematical argumentation.
3. Personal Engagement — Creativity, curiosity, and initiative in conducting the investigation.
4. Reflection — Evidence of critical evaluation and awareness of the limitations and validity of the mathematical model.

5. Mathematical Accuracy — Use of appropriate mathematics and correct computations.

(Rubric table is available in Appendix 1 as Table 1.4)

Example 2: Probability and gambling.

- Context: Two gambling games involving dice were analyzed for expected value.
- Focus: Mathematical expectation, fairness of games, and long-term casino advantage.
- Additional Component: Reflection on the role of math in real-life financial decisions.

Example 3: Modeling bacterial growth.

- Scenario: Students conducted an experiment to model bacterial reproduction at different temperatures.
- Mathematical focus: Graphing exponential functions, calculating rates of change (derivatives), and interpreting scientific data.

Example 4: Combinatorics — license plate problem.

- Students investigated how many Kazakh license plates are "lucky," defined by numerical relationships between digits.
- Mathematical tools: Combinatorics, logical reasoning, probability estimation.

These performance tasks were integrated into the rich task sequence as contextual anchors for conceptual understanding. They required collaboration, investigation, and application of mathematics to realistic, often interdisciplinary problems. All tasks were supported by feedback, aligned with the "5 Practices for Orchestrating Productive Mathematical Discussions" (Smith & Stein, 2018), in the experimental group.

2.9 Tasks used to develop conceptual understanding

These PISA-style questions were selected to activate and assess a variety of conceptual dimensions outlined in the MYP Mathematics framework. Their design integrates real-life contexts with mathematical reasoning, encouraging students to engage with problems beyond routine procedures. For example, the water tank problem prompts learners to explore how the geometric form of an object (Form) influences the rate of change in height over time when filled at a constant rate (Change, Quantity). The moving walkway task emphasizes understanding of systems (System) and representations (Graphical Representation), as students must analyze and compare distances over time across multiple motion scenarios. The kite sail problems engage learners in mathematical modeling (Model), involving both reasoning (Space) and application of trigonometry, while also requiring logical explanation (Logic) for interpreting relationships. These tasks also involve Pattern and Equivalence, as students identify structures, compare quantities, and apply proportional reasoning. Altogether, such tasks provide rich opportunities to develop and observe conceptual understanding in ways that reflect the goals of inquiry-based and meaning-oriented mathematics instruction (Australian Council for Educational Research, CITO, 2006).

Figure 2.9.1 illustrates examples of the PISA-released items used in this study.

Question 1: WATER TANK

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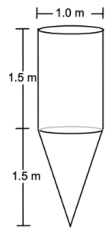
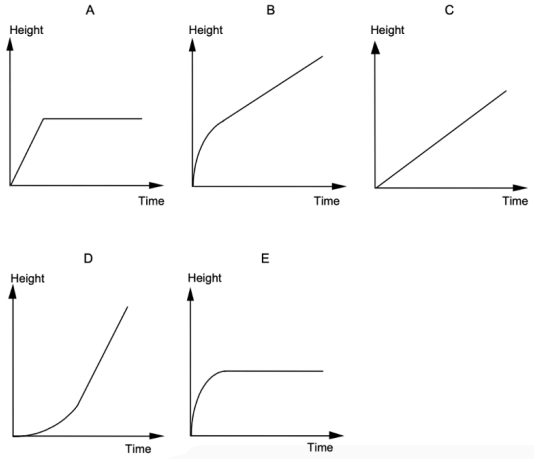
A water tank has shape and dimensions as shown in the diagram.

At the beginning the tank is empty. Then it is filled with water at the rate of one litre per second.



Water tank

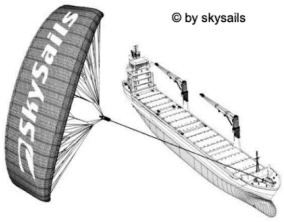
Which of the following graphs shows how the height of the water surface changes over time?



MATHEMATICS UNIT 2: Sailing ships

Ninety-five percent of world trade is moved by sea, by roughly 50 000 tankers, bulk carriers and container ships. Most of these ships use diesel fuel.

Engineers are planning to develop wind power support for ships. Their proposal is to attach kite sails to ships and use the wind's power to help reduce diesel consumption and the fuel's impact on the environment.



Sailing Ships – Question 1

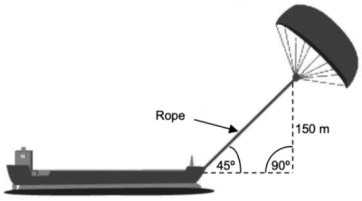
One advantage of using a kite sail is that it flies at a height of 150 m. There, the wind speed is approximately 25% higher than down on the deck of the ship.

At what approximate speed does the wind blow into a kite sail when a wind speed of 24 km/h is measured on the deck of the ship?

A 6 km/h
B 18 km/h
C 25 km/h
D 30 km/h
E 49 km/h

Approximately what is the length of the rope for the kite sail, in order to pull the ship at an angle of 45° and be at a vertical height of 150 m, as shown in the diagram opposite?

A 173 m
B 212 m
C 285 m
D 300 m



Note: Drawing not to scale.
© by skysails

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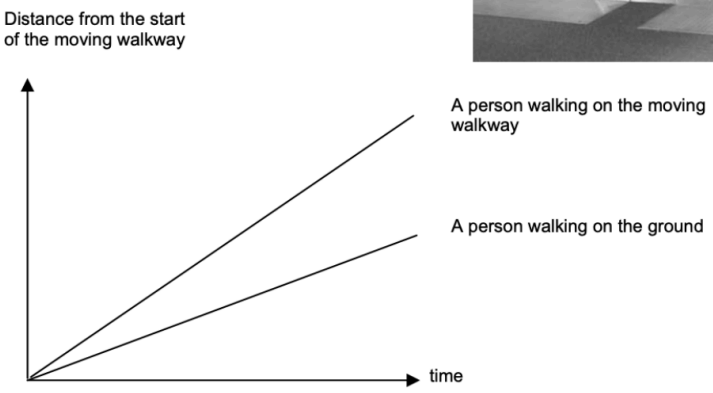
Question 1: MOVING WALKWAYS

On the right is a photograph of moving walkways.

The following Distance-Time graph shows a comparison between "walking on the moving walkway" and "walking on the ground next to the moving walkway."



Assuming that, in the above graph, the walking pace is about the same for both persons, add a line to the graph that would represent the distance versus time for a person who is standing still on the moving walkway.

Appendix 2 – PISA-style Tasks for Developing Conceptual Understanding in Mathematics.

3. RESULTS

This section presents the results of the study in relation to the two main hypotheses outlined in the introduction. Both quantitative and qualitative data were analyzed to explore whether solving real-life mathematical problems improves conceptual understanding, and whether using rich tasks + the 5 Practices model is more effective than traditional instruction.

3.1 Quantitative results: descriptive statistics

Table 3.1.1 Benchmark and post-test scores for students in classes 9A (experimental group) and 9B (control group) across the academic year. Data illustrate changes in performance over five assessment points from September 2024 to May 2025.

09 grades				Sept 18	Dec-24	Jan-25	Apr 23 2025	May-25
				Benchmark	Pre-test results	Benchmark	Benchmark 2.0	Post-test result
Total points		Age	Gender	15	50	25	16	50
Student 1	9A	15	female	7	18	9	4	24
Student 2	9A	16	female	10.5	17	9		30
Student 3	9A	14	female		21	10	7	35
Student 4	9A	15	male		11	7	2	23
Student 5	9A	15	male		7		3	17
Student 6	9A	14	male	5	21	12	12	32
Student 7	9A	14	male	4	11	4	1	9
Student 1	9B	15	female	8	5	2	2	18
Student 2	9B	14	male		29	9	7	26
Student 3	9B	15	female	9	26	9		31
Student 4	9B	15	male	8	8	7	5	18
Student 5	9B	15	female	12	25	9	5	44
Student 6	9B	15	female	11	31	9	14	44
Student 7	9B	14	female	6	12	4	14	38.5

Full table can be found in appendix 3**.**

To provide a clearer overview of students' performance trends, descriptive statistics were calculated for both groups (9A and 9B) on pre-test and post-test results. These metrics include mean scores and standard deviations, offering insight into not only central tendencies but also the variability of performance within each group.

Table 3.1.2 Descriptive statistics and normality test (Shapiro–Wilk) for pre-test and post-test results. Includes sample size, central tendency, variability, and distribution characteristics to assess suitability for parametric analysis.

Descriptives

	Pre-test results 9 grades	Post-test results 9 grades
N	40	43
Missing	3	0
Mean	16.9	29.3
Median	18.0	29.0
Standard deviation	7.85	9.72
Minimum	4.00	8.00
Maximum	32.0	45.5
Shapiro-Wilk W	0.962	0.971
Shapiro-Wilk p	0.190	0.337

Interpretation of Normality

- Shapiro–Wilk W (Pre-test): 0.962
- p-value: 0.190
  $p > 0.05 \rightarrow$ *no significant deviation from normality*
- Shapiro–Wilk W (Post-test): 0.971
- p-value: 0.337
  $p > 0.05 \rightarrow$ *no significant deviation from normality*

The Shapiro–Wilk test results show that both the pre-test and post-test scores are approximately normally distributed. Since the p-values for both tests are higher than 0.05 ($p = 0.190$ for the pre-test and $p = 0.337$ for the post-test), we can assume the data follows a normal distribution. This means it is appropriate to use standard statistical methods, such as t-tests, to analyze the results.

Table 3.1.3 Descriptive statistics and normality test (Shapiro–Wilk) for pre-test and post-test results of 9A and 9B.

Descriptives

	Pre-test results 9A	Post-test results 9A
N	19	21
Missing	24	22
Mean	16.8	27.7
Median	18.0	29.0
Standard deviation	6.35	7.96
Minimum	7.00	9.00
Maximum	27.0	41.0
Shapiro-Wilk W	0.938	0.967
Shapiro-Wilk p	0.246	0.664
	Pre-test results 9B	Post-test results 9B
N	21	22
Missing	22	21
Mean	17.1	30.9
Median	18.0	29.5
Standard deviation	9.16	11.1
Minimum	4.00	8.00
Maximum	32.0	45.5
Shapiro-Wilk W	0.939	0.924
Shapiro-Wilk p	0.212	0.094

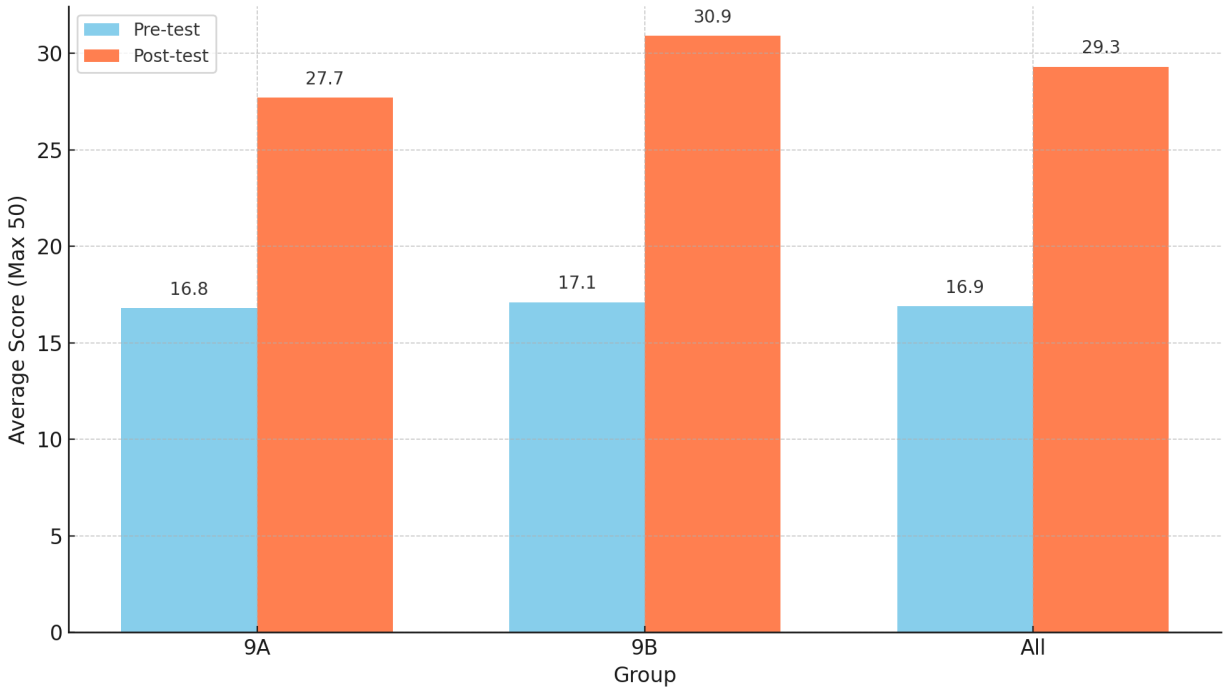
The Shapiro–Wilk test results for both 9A and 9B groups indicate that the data distributions for pre- and post-test scores do not significantly deviate from normality. All p-values are above the conventional threshold of 0.05 (e.g., $p = 0.246$ for 9A pre-test, $p = 0.094$ for 9B post-test), meaning the assumption of normality is reasonably met for subsequent parametric analysis. In addition to normality, other descriptive measures provide further insight into the structure and spread of student performance. For instance, the mean scores in both classes increased markedly from

pre- to post-test: from 16.8 to 27.7 in 9A and from 17.1 to 30.9 in 9B. This upward trend is also reflected in the median scores, which rose from 18.0 to 29.0 in 9A and from 18.0 to 29.5 in 9B, suggesting that performance gains were broadly distributed across both groups rather than being skewed by a few high achievers.

Standard deviation, however, paints a more nuanced picture. In 9A, the increase from 6.35 to 7.96 suggests a modest widening in score variability, possibly reflecting differences in how individual students responded to the intervention. In contrast, 9B's standard deviation grew from 9.16 to 11.1, indicating an even larger spread of post-test scores. This could imply that while 9B scored higher on average, the performance was more uneven, with a wider gap between the highest and lowest performers.

Taken together, the descriptive statistics support the inference that both groups improved overall, but with different patterns of variability and central tendency. These distinctions may warrant further exploration in relation to teaching methods, classroom dynamics, or student characteristics.

Figure 3.1.4 Mean comparison of pre- and post-test scores



This visual representation highlights the relative improvements from pre- to post-test across both groups. The general trend of growth is evident, with Group 9B showing a slightly larger increase in average performance. Specifically, students in 9A increased their mean score from 16.8 to 27.7 (a gain of +10.9 points), while students in 9B improved from 17.1 to 30.9 (+13.8 points). Although both groups demonstrated notable progress, Group 9B exhibited a slightly higher gain. These results suggest that both instructional approaches had a positive impact on students' learning outcomes, with the control group (9B) showing a more pronounced improvement in this metric. The observed improvements in both 9A and 9B confirm that real-life problem-solving tasks can enhance students' performance in mathematics. However, the slightly higher gains in 9B (+13.8 vs. +10.9 points) raise an important consideration: conceptual

understanding does not always translate into the highest short-term test scores. While 9B followed a more traditional, procedural approach, 9A was engaged in inquiry-based learning, emphasizing deeper reasoning, collaboration, and mathematical thinking.

This suggests that although traditional methods may lead to quicker performance gains in test-like settings, rich-task pedagogy may foster more sustainable and transferable understanding — benefits that might not be fully captured in standardized assessments. Therefore, future evaluation should consider not only test scores but also the quality of reasoning, confidence, and long-term retention — all of which were more visible in 9A during qualitative observation.

3.2 Within group pre- and post-test comparison

Hypothesis 1 (Effect of real-life problems):

- H_0: The use of real-life problems has no significant effect on students' conceptual understanding.
- H_1: The use of real-life problems significantly improves students' conceptual understanding.

Table 3.2.1 paired samples t-test results comparing pre-test and post-test scores for 9A and 9B groups.

Paired Samples T-Test

			statistic	df	p
Pre-test results 9A	Post-test results 9A	Student's t	-5.83	18.0	<.001
Pre-test results 9B	Post-test results 9B	Student's t	-7.70	20.0	<.001

Note. H_a $\mu_{\text{Measure 1 - Measure 2}} \neq 0$

Quantitative results:

- 9A Group (experimental):
  - Mean pre-test: 16.8
  - Mean post-test: 27.7
  - Paired t-test: $t = -5.83, p < 0.0001$
- 9B Group (control):
  - Mean pre-test: 17.1
  - Mean post-test: 30.9
  - Paired t-test: $t = -7.70, p < 0.00001$

Interpretation:

Both groups showed statistically significant gains between pre- and post-tests, indicating that solving real-life problems led to meaningful improvements in conceptual understanding. This supports H_1, providing strong evidence that RLPs enhance conceptual learning, even when delivered through different pedagogical methods. As the teacher, I observed a noticeable change in how students approached unfamiliar tasks. Initially, many relied on memorized procedures. However, by the end

of the study, they were more comfortable exploring multiple strategies and justifying their reasoning. It is also noteworthy that while both groups improved, the experimental group (9A) showed a more pronounced shift in reasoning style. Their progress was not just quantitative but qualitative, marked by deeper classroom dialogue, increased peer interaction, and greater willingness to tackle open-ended problems.

This suggests that the learning environment shaped by rich tasks fostered not only academic improvement, but also confidence, perseverance, and reflective habits—traits essential for long-term mathematical growth.

Conclusion: H_0 is rejected. The data support H_1: real-life mathematical problems improve students' conceptual understanding. While this improvement was observed in both groups, the instructional approach in 9A created additional benefits beyond test scores, such as enhanced reasoning skills and greater engagement. Future studies may investigate how these cognitive gains translate into long-term academic resilience or higher-order problem solving in unfamiliar contexts.

Between-group post-test Comparison

Hypothesis 2 (Comparing instructional methods):

- H_0: Teaching real-life problems via rich tasks and the 5 Practices is not more effective than traditional instruction.
- H_1: Teaching real-life problems via rich tasks and the 5 Practices is more effective than traditional instruction.

Results:

- Independent t-test (Pre-Test):
  - $t = 0.18, p = 0.856 \rightarrow$ groups were comparable at baseline.
- Independent t-test (Post-Test):
  - $t = -0.72, p = 0.476 \rightarrow$ no statistically significant difference.

Interpretation:

Although 9A and 9B both improved, the difference in final scores was not statistically significant. Thus, we cannot reject H_0. While I observed deeper reasoning and richer classroom dialogue in 9A, this did not translate into higher scores—possibly due to differences in how performance was measured or the added cognitive load in 9A's open-ended tasks. Another plausible factor could be the short duration of the intervention. Conceptual gains, especially those nurtured through inquiry-based methods, often require extended time to fully manifest in standardized assessments. Moreover, students in the control group may have had an advantage in navigating test formats, having been more accustomed to structured tasks and time-constrained problem-solving. It is also worth noting that students in 9A were frequently engaged in metacognitive reflection and peer discussion, which are crucial for long-term retention but may not immediately influence quantitative outcomes.

Conclusion: The data do not confirm H_1. Although the 5 Practices may foster deeper thinking, this was not reflected in test outcomes. H_0 is not rejected. However, this statistical result should not be interpreted as evidence against the pedagogical value of rich tasks or inquiry-based learning. The quantitative gains may have been modest, but qualitative classroom observations suggested important shifts in reasoning,

collaboration, and engagement—dimensions that are not always fully captured by summative assessments.

Moreover, given the relatively short intervention period and the cognitive complexity of open-ended tasks, it is possible that the benefits of the 5 Practices model require more time to translate into measurable performance. Future research might explore longitudinal effects or incorporate mixed methods to better understand the broader impact on students' mathematical thinking.

3.3 Performance tasks (ANOVA and task-level comparison)

Hypothesis 3 (Task-Level Impact):

- H_0: Rich tasks + 5 Practices do not lead to significantly better performance on applied tasks.
- H_1: Rich tasks + 5 Practices lead to significantly better performance.

The analysis shown in Table 3.3.1 is based on performance task scores collected from students in both 9A and 9B groups. Detailed datasets and scoring rubrics for each task are provided in (Figure 3.2) appendix 3**.**

Table 3.3.1 *Results of One-Way ANOVA (Welch's) comparing student performance on various real-life mathematical tasks across groups.*

Task Name	F (Welch's)	p-value	Significant?
Truck Tunnel	8.664	0.007	Yes
Braking Distance	0.444	0.510	No
General Task	1.895	0.184	No
Probability & Gambling	3.597	0.072	Marginal
Probability & Gambling	3.597	0.072	Marginal

Only the "Truck Tunnel" task showed a statistically significant advantage for the experimental group. Two others were close to significance, suggesting a trend in favor of rich task instruction.

Interpretation:

These results suggest that rich tasks may be more effective for certain types of conceptual thinking, particularly spatial modeling or estimation, but not uniformly across all domains. Some tasks may have benefited from procedural fluency, where the control group had the advantage.

Conclusion: Partial support for H_1. Rich tasks can outperform traditional methods in specific tasks, but the overall difference was not consistently significant.

Reliability and factor analysis. Internal Hypothesis (Tool Validity):

- H_0: The test instruments are not reliable measures of conceptual understanding.
- H_1: The instruments demonstrate good reliability and construct validity.

Results:

- Cronbach's Alpha (full pre-test): $\alpha = 0.780$ (acceptable)
- After removing misfitting items: $\alpha > 0.80$
- Key loading items (CFA):

- "Calculation" $= 0.4391, p < 0.001$
- "Order of Operations" $= 0.4745, p < 0.001$

Interpretation: these metrics indicate that the diagnostic instruments had acceptable internal consistency and construct clarity, especially after item refinement. Some items (e.g., in Geometry and Statistics) showed weak correlation and will require revision for future use. Conclusion: H_0 is rejected. The tools were statistically reliable for measuring conceptual understanding.

3.4 Discussion and interpretation

This study was designed to evaluate the extent to which solving real-life problems (RLPs) in mathematics—especially through rich tasks and the 5 Practices model—can support the development of conceptual understanding among Grade 9 students. The results showed promising outcomes but also presented certain tensions and paradoxes that warrant deeper reflection.

3.4.1 Conceptual gains in both groups

One of the clearest findings was that both 9A and 9B demonstrated statistically significant improvements from pre-test to post-test. This supports Hypothesis 1 (H_1) and aligns with global research that emphasizes the power of context-rich, meaningful tasks in mathematics (Kilpatrick, Swafford, & Findell, 2001).

From my own perspective as a teacher, I noticed how students became increasingly autonomous over time. In both groups, students began to articulate reasoning, question assumptions, and connect mathematical ideas to real-world experience. Even in the traditionally taught class (9B), the inclusion of realistic contexts likely contributed to engagement and meaning-making. This suggests that contextualization itself—regardless of method—can be a lever for deeper learning.

However, it is important to distinguish between performing well on a test and thinking mathematically. While both are desirable, they are not synonymous. The results raised precisely this distinction. Furthermore, the development of mathematical discourse was evident even in students who were previously passive participants. Engaging with meaningful contexts enabled them to visualize abstract content and contribute to peer dialogue. This indicates that rich contexts may serve as an equalizing factor in diverse classrooms.

3.4.2 Paradox: higher scores in the traditional group

At first glance, it may seem counterintuitive—and even disappointing—that the control group (9B) scored higher on most performance tasks and even had comparable post-test results. However, this outcome is revealing. It surfaces a deeper question about what we mean by "learning" and how we assess it.

There are several plausible explanations:

1. Test Strategy Familiarity: Students in 9B were used to exams and structured worksheets. They were likely more efficient in identifying what a question "wants," especially under pressure. Their thinking may not have been deeper, but it was more aligned with the format of assessment.

2. Measurement Misalignment: The performance tasks, while designed to be realistic, still relied on score-based rubrics. Students in 9B may have "played the game" better—not because they understood more, but because their responses fit the rubric.
3. Delayed Impact Hypothesis: Conceptual understanding is often slower to develop but has longer-term effects. As noted by Vygotsky (1978), internalization takes time. The full effect of rich tasks might only become evident in future problem-solving or abstract reasoning contexts.

Therefore, the lack of statistically significant superiority of the experimental method does not invalidate its pedagogical value. It simply highlights that measured outcomes and learning quality do not always coincide. It also calls for reassessment of current evaluation frameworks, which may inadvertently privilege procedural proficiency over depth. In doing so, educators and policymakers risk undervaluing the competencies most needed in real-world settings.

3.4.3 The role of the 5 practices

The implementation of the 5 Practices (Smith & Stein, 2018) played a central role in shaping the experimental group experience. These five stages—anticipating, monitoring, selecting, sequencing, and connecting—provided a roadmap for turning classroom discussions into meaningful mathematical dialogue.

As a teacher, I found this framework powerful yet demanding. It forced me to think deeply about student thinking rather than just content delivery. During lessons, I often changed my plan in real-time to amplify student voices or respond to emerging misconceptions. Students responded with more engagement, often suggesting multiple solutions or challenging one another's assumptions.

However, I also encountered challenges. The effectiveness of the 5 Practices depends heavily on the teacher's pedagogical content knowledge (PCK) and classroom orchestration skills. The quality of each phase—especially "connecting"—varied across lessons. I view this as an area for future professional growth. Moreover, the anticipatory phase proved crucial: lessons where I could accurately foresee student strategies led to more coherent discussions. This underlines the importance of pre-lesson planning, which is often underestimated in traditional approaches.

3.4.4 The value and limitations of performance tasks

Performance tasks were chosen as the primary tool for assessing applied conceptual understanding. According to McTighe & Wiggins (2004), such tasks are "authentic, complex, and open-ended," and require students to think like practitioners in real contexts.

In practice, these tasks revealed much about how students approach modeling, estimation, and strategic decision-making. I observed that students in 9A often used visual reasoning, verbal justification, and flexible representations. In contrast, 9B students were faster and more mechanical, but sometimes lacked depth. Still, the scoring rubrics may not have fully captured the depth of student thought—especially when responses were incomplete or unpolished. In the future, I would recommend

including oral defense, process portfolios, or peer evaluation to triangulate understanding. Furthermore, many performance tasks sparked spontaneous group collaboration, even when not explicitly required. This reinforces their value not only as assessment tools, but also as drivers of social learning. However, balancing creativity with clarity in student responses remains a scoring challenge.

3.5 Regression analysis

Table 3.5.1 Individual student-level comparison between average performance task scores and post-test results (9 grades).

Average results of performance tasks	Post-test results 9 grades
50.13209464	24
82.48634868	30
72.47377944	35
64.45714817	23
25.6215604	17
62.3387199	32
63.60820548	9
56.49733857	35
87.34764294	25
54.27557467	32
71.72216471	31
72.94966201	35
73.96256923	16
67.85626073	29
89.2952726	21
71.17923059	38
53.72584718	32
20.62658155	29
64.37549504	28
73.35913804	41

Full table can be found the appendix 3.

Table 3.5.2 Shapiro–Wilk normality test for regression residuals: Assumption of normality is not violated (p = 0.186).

Normality Test (Shapiro-Wilk)	
Statistic	p
0.963	0.186

Table 3.5.3 Linear regression coefficients and model fit: Predicting post-test scores based on performance task results (N = 43).

Model Fit Measures

Model	R	R^2
1	0.238	0.0568

Note. Models estimated using sample size of N=43

Model Coefficients - Post-test results 9 grades

Predictor	Estimate	SE	t	p
Intercept	18.701	6.9334	2.70	0.010
Average results of performance tasks	0.154	0.0979	1.57	0.124

To gain additional insight into whether students' engagement with real-life performance tasks had a measurable influence on their conceptual understanding, a simple linear regression was conducted. The model examined the relationship between students' average results on performance tasks (as a cumulative indicator of their formative, process-oriented mathematical reasoning) and their post-test scores (as a summative measure of individual achievement after the intervention). These two variables were chosen because they reflect two distinct but complementary aspects of mathematical learning: applied reasoning over time versus final independent performance.

The regression analysis yielded a weak positive correlation ($R = 0.238$), with the model explaining only about 5.7% of the variance in post-test scores ($R^2 = 0.0568$). While the intercept was statistically significant ($p = 0.010$), the predictor variable—average performance task results—was not statistically significant ($\beta = 0.154$, $p = 0.124$). The assumption of normality was met (Shapiro-Wilk $p = 0.186$), validating the model's applicability.

This weak correlation suggests that while students who consistently performed well on rich tasks tended to score higher on the post-test, the relationship was not strong enough to indicate predictive power. Several interpretations are possible. First, rich performance tasks required extended reasoning, collaboration, and contextual understanding, while the post-test was time-constrained and more structured. Thus, students with deeper thinking skills may not have had the opportunity to demonstrate them fully on the test. Second, this finding may point to the challenge of aligning formative, inquiry-based assessment with summative evaluations. Lastly, motivational and linguistic factors may have influenced task engagement and test performance differently.

In essence, the weak correlation signals that while there is some connection between performance in real-world tasks and conceptual understanding as measured

by testing, it is insufficiently robust to assume direct causality. This result reinforces the need for a broader range of assessment tools that can better capture the depth and nuance of conceptual growth fostered through inquiry-based pedagogy.

3.6 Challenges and limitations

Despite the structured design and thoughtful implementation, the study faced several real-world challenges, both methodological and contextual.

3.6.1 Limited sample size and scope

With only two classrooms (43 students in total), this research falls into the category of small-scale field study. Although statistical analysis was strengthened through bootstrap methods, the findings cannot be generalized to all schools or grade levels. For stronger claims, replication with more diverse and larger populations is needed. Moreover, both classes came from the same school (HTA), which already supports progressive pedagogies. This may not reflect the broader spectrum of schools in Kazakhstan, where traditional, test-focused instruction is more dominant. Additionally, the school's progressive context may have provided an advantage in implementing rich tasks, with resources and administrative support that are not always available elsewhere. The study's insights, while meaningful, must be situated within this relatively privileged setting.

3.6.2 Teacher-as-researcher tension

My dual role as teacher and researcher created both opportunity and risk. On one hand, it allowed for authentic, embedded insights. On the other, it posed a risk of confirmation bias and subjective interpretation. While I made every effort to remain objective—including maintaining field journals and consulting with colleagues—I recognize that my pedagogical preferences may have influenced my interpretation. In future iterations, I would recommend the use of external observers or video analysis to strengthen data credibility. Furthermore, emotional investment in student outcomes may have subtly shaped expectations and interactions. Balancing involvement with analytical detachment remains a persistent challenge for practitioner-researchers and should be openly acknowledged in future research protocols.

3.6.3 Assessment alignment and scoring

While real-life tasks were valuable, their evaluation posed challenges. I often felt that student reasoning was richer than their written responses suggested. For example, a student might have a strong verbal justification but write only a brief answer due to time pressure or uncertainty.

The rubrics used were based on NCTM process standards, but even so, they may have favored structure over exploration. This mismatch may partially explain why the control group scored higher despite shallower reasoning. Summative assessments often reduce learning to numerical scores. To effectively assess conceptual understanding, more formative, interactive, and reflective methods are required. Moreover, current assessment systems do not always accommodate multi-step reasoning, visual modeling, or exploratory dialogue. As such, the richness of mathematical thinking

demonstrated during class may remain invisible in official records, underscoring the need for broader evaluative frameworks

3.6.4 Time and curriculum pressures

Implementing rich tasks and facilitating structured discussion took more time than traditional instruction. As a result, we covered fewer topics in 9A compared to 9B. This raises a tension between depth and coverage, especially in systems where time is tight and exams are looming. Some colleagues questioned whether it was “worth it” to sacrifice test prep for deeper learning. I believe the answer is yes—but this belief must be supported by policy, leadership, and long-term planning. Additionally, the inflexible nature of curriculum pacing guides may prevent teachers from fully engaging with inquiry-based instruction, even when they recognize its benefits. Without systemic adjustments, the pressure to "keep up" with syllabus content may continue to limit pedagogical innovation.

3.6.5 Student readiness and transition

Not all students were ready to engage with open-ended tasks. Some lacked the confidence or language to express their ideas. Others were so used to being told what to do that they found inquiry-based learning “confusing.”
This highlights the need for explicit scaffolding, including:

- Modeling “how to think” about a problem;
- Providing sentence starters or thinking frames;
- Allowing time for metacognitive reflection.

Rich tasks are not just academic tools—they are cultural shifts. They demand time, trust, and training.

3.6.6 Systemic and institutional constraints

Beyond the classroom, systemic issues presented additional obstacles. Educational policies in Kazakhstan, like in many post-Soviet contexts, remain heavily exam-oriented. As a result, innovation in pedagogy is often constrained by rigid curriculum structures, centralized assessments, and lack of flexibility in pacing. Teachers may feel compelled to prioritize efficiency and test preparation over deeper learning experiences. In this environment, integrating rich tasks can appear as a deviation rather than a best practice. Administrative support and curricular autonomy are essential preconditions for sustainable implementation. Without these, even the most well-intentioned pedagogical innovations risk becoming isolated experiments rather than systemic change. Furthermore, differentiated support strategies must be embedded consistently. Bridging the gap between passive learning habits and active problem-solving requires a developmental approach, recognizing that cognitive autonomy is cultivated gradually through practice, guidance, and encouragement.

3.6.7 Resource and professional development gaps

Another major challenge lies in the availability of suitable resources. Designing and delivering rich tasks requires time, access to real-world contexts, and high-quality materials. Unfortunately, many teachers work in resource-constrained settings and lack

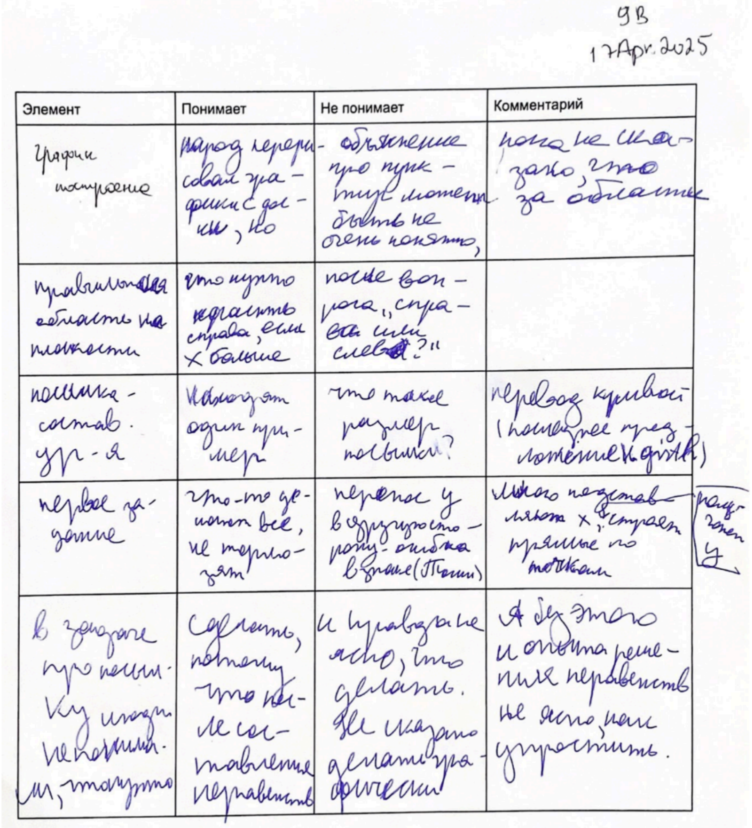
both the tools and the training to create such tasks independently. Professional development opportunities that focus specifically on inquiry-based learning and rich task design remain scarce. While interest among teachers is growing, institutional mechanisms for ongoing mentorship, peer collaboration, and reflection need to be strengthened. Teachers also need access to examples of best practice and opportunities to adapt tasks collaboratively, rather than reinventing them alone. Investment in professional learning communities could play a pivotal role in scaling up such practices across diverse school contexts.

3.7 Qualitative results: teacher observations in 9A and 9B

Teacher observation records from both 9A and 9B classes provide rich qualitative evidence for evaluating the impact of pedagogical approaches on students' conceptual understanding of mathematics. In the 9A class—where instruction was grounded in rich tasks and guided by the 5 Practices model—students demonstrated higher levels of engagement, autonomy, and mathematical reasoning. Observational notes describe students in 9A actively discussing multiple solution strategies, making connections between graphs and real-life contexts, and justifying their choices. Teachers also noted an increased willingness to take intellectual risks, such as proposing unconventional solutions or challenging assumptions made by peers. This openness contributed to a classroom atmosphere where mistakes were seen as opportunities for learning rather than failures. For example, several groups questioned the reasonableness of their models and adjusted them accordingly after collaborative reflection. These behaviors reflect an emerging sense of mathematical agency and deeper conceptual grasp.

In contrast, observations from the 9B class—where traditional instruction dominated—highlight a more passive learning environment. Students often required direct prompting and exhibited uncertainty when transitioning between representations (e.g., from verbal to algebraic forms). Comments such as *"I don't know what to do with this graph"* or *"Is this x or y?"* indicate that many students struggled to make sense of the context without explicit scaffolding. They demonstrated a reliance on procedural knowledge without understanding *why* a method worked, suggesting the absence of a strong conceptual foundation. This contrast reinforces the importance of instructional design in shaping students' depth of understanding and their ability to transfer knowledge across contexts. Moreover, students in 9B rarely engaged in mathematical dialogue with peers, often working individually and awaiting teacher validation. This limited interaction may have further hindered the development of reasoning and problem-solving skills.

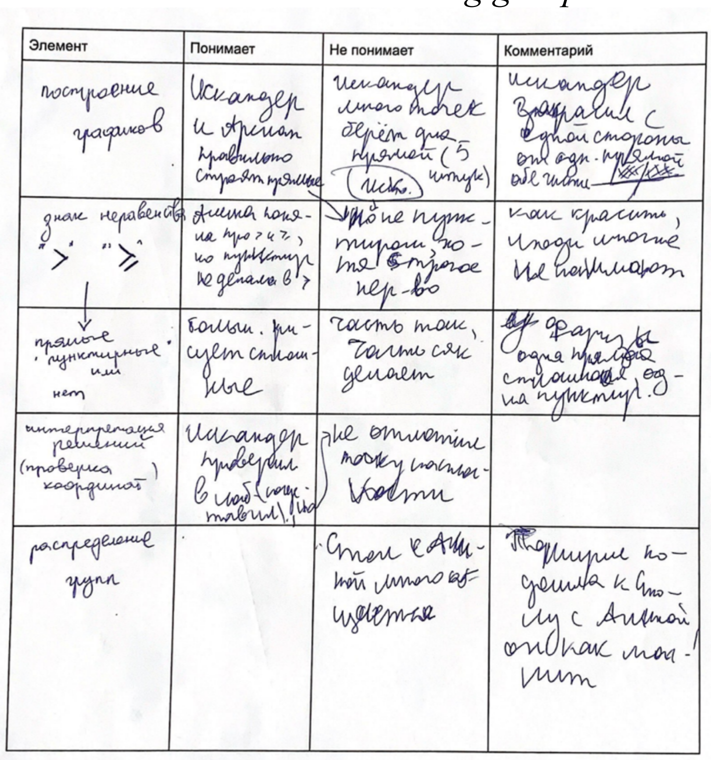
Figure 3.7.1 Teacher observation notes from 9B class on students' understanding during a mathematics lesson (April 17, 2025). Columns indicate what students understood, misunderstood, and teacher's comments for clarification.



9B
17Apr.2025

Элемент	Понимает	Не понимает	Комментарий
графики построение	народ перерисовал графики с доски, но	объяснение про пунктир может быть не очень понятно,	пока не сказано, что за область
правильная область на плоскости	что нужно красить справа, если x больше	после вопроса „справа или слева?"	
пошагово-составл. ур-я	находят один пример	что такое размер посылки?	перевод кривой (пошаговое представление [illegible])
первое задание	что-то делают все, не торопятся	перенос y в другую сторону - ошибка в знаке (Тони)	много подставляют x, строят прямые по точкам [illegible] y
в задаче про посылки [illegible] непонятна, т.к. нужно	сделать, потому что не ясно составление неравенств	и правда не ясно, что делать. Не сказано делать графически	А без этого и опыта решения неравенств не ясно, как упростить.

Figure 3.7.2 Teacher observation notes from 9A class during a lesson on graphing inequalities. The table records elements students understood or struggled with, along with teacher comments during group work.



Элемент	Понимает	Не понимает	Комментарий
построение графиков	Искандер и Арман правильно строят прямые	Искандер много точек берёт для прямой (5 штук) (Иск.)	Искандер закрашивает с одной стороны от одн. прямой обе части [illegible]
знак неравенства „>" „≥"	Алина поняла про ≥ и >, но пунктир не делала в >	что не пунктиром, хотя строгое нер-во	как красить, иногда многие не понимают
прямые „пунктирные" или нет	больш. рисует сплошные	часть так, часть сяк делает	[illegible] фигуры одна прямая сплошная, одна пунктир.
интерпретация решений (проверка координат)	Искандер проверил в лоб (подставил), но	не отмечают точку пересечения	
распределение групп		Стоп с Алиной много [illegible]	Тимирлан подошёл к [illegible] с Алиной он как мешает

Additional observation records can be found in appendix 3**.**

More specifically, five themes emerged from the 9B observation data:

1. Limited conceptual understanding**:** Students showed difficulty interpreting graphs, selecting appropriate axes, and translating word problems into mathematical models. While they could follow steps, it was not always clear that they understood the meaning behind the procedures.
2. Lack of representational flexibility: Learners struggled to shift between contextual scenarios and abstract representations. For example, they had trouble recognizing how a verbal description mapped onto a graph or an equation, reflecting compartmentalized rather than integrated thinking.
3. Need for scaffolding**:** Many students were unable to organize their solution paths independently. Comments like *"I can't do this without being shown"* highlight the importance of anticipatory support—aligning with the need for structured teacher facilitation as described in the 5 Practices model (Smith & Stein, 2018).
4. Importance of visual modeling and interim discussions: The observations indicate that visual aids and whole-class discussions were beneficial, but not sufficient on their own. Students needed opportunities to reflect and engage with the process, not just the outcome.
5. Clear contrast between procedural and conceptual approaches: Although both groups worked with the same real-life tasks, students in 9B focused on executing formulas, while those in 9A were guided to explore meanings and relationships. This confirms that real-life contexts alone do not guarantee deep understanding unless supported by inquiry and reflection.

Despite these challenges, gradual progress was noted in both groups. In 9B, several students began to engage in more reflective practices by the end of the lesson—posing clarifying questions, re-evaluating errors, and showing a willingness to revise their work. Meanwhile, in 9A, the inquiry-based structure appeared to accelerate this growth: students confidently moved through problem-solving cycles, demonstrated stronger metacognitive awareness, and showed signs of transfer beyond the immediate task.

In summary**,** the comparison reinforces the central claim of this study: real-life problem solving has the potential to enhance conceptual understanding**,** but only when embedded in a pedagogical approach that supports inquiry, scaffolding, visual reasoning, and student-led reflection. While both groups had access to the same tasks, the instructional context shaped not only how students approached them, but how deeply they understood the mathematics involved.

3.8 Summary and recommendations

3.8.1 Main conclusion

This study set out with a central question: *Can solving real-life mathematical problems improve students' conceptual understanding?* Over the course of several months, through lesson observations, performance tasks, and post-assessments, it became clear that the answer is yes — but with conditions.

Both classes in the study, 9A and 9B, worked with the same types of tasks. Both groups were presented with real-world contexts that required interpretation, modeling, and reasoning. Yet their experiences — and their outcomes — were strikingly different. In 9A, where the rich tasks were supported by the 5 Practices model (Smith & Stein, 2018), students did more than just solve problems. They reflected, asked critical questions, debated alternative approaches, and revised their thinking. Their classroom environment allowed space for error, exploration, and, perhaps most importantly, sense-making.

In contrast, 9B followed a more traditional, teacher-directed approach. While students were able to follow instructions and apply procedures, they often relied on memorization. Their responses, both written and verbal, lacked depth. When asked to explain their reasoning or apply their knowledge to a new situation, many hesitated. The tasks were the same, but the thinking behind them was not.

What this tells us is important. Real-life problems on their own are not a magic fix. Without structured opportunities for students to *grapple* with concepts — to talk, to visualize, to connect — the potential of these problems remains untapped. The real difference lies not in the task itself, but in how it is used. The 5 Practices framework created a predictable rhythm in 9A: anticipate, monitor, select, sequence, connect. Over time, students began to internalize this rhythm, developing habits of inquiry and reflection. It's also worth noting that students in 9A didn't just score higher; they thought differently. They showed stronger metacognitive awareness — knowing when their solution didn't make sense, and being willing to go back and revise. They used a wider range of representations and were more confident transitioning between graphs, equations, and verbal descriptions. They also demonstrated stronger reasoning when working collaboratively, showing that conceptual learning is both individual and social. Their engagement extended beyond individual tasks, creating a culture of mathematical dialogue. This suggests not only deeper understanding but also greater *transferability* of knowledge — a key indicator of conceptual learning. To put it plainly: conceptual understanding doesn't emerge from procedures. It grows from experiences that demand reasoning, flexibility, and ownership. And those experiences have to be designed and facilitated with care.

3.8.2 Recommendations for teachers and schools

1. Based on these findings, the following recommendations are offered to educators and school leaders: Embed real-life problems into the heart of instruction. Don't treat them as enrichment or "bonus" activities. When used intentionally, these tasks help students connect mathematical ideas to real experiences, which supports lasting understanding. Such tasks also help students see the relevance of mathematics in their own lives, increasing motivation and participation. They foster skills that are aligned with 21st-century competencies such as creativity and critical thinking.
2. Train teachers in structured frameworks like the 5 Practices. Knowing how to choose rich tasks is only half the challenge. Teachers also need tools for orchestrating classroom dialogue, anticipating student thinking, and using

mistakes as learning opportunities. Professional development should include real-time lesson planning, reflection sessions, and modeling of facilitation techniques. Teachers need ongoing support to confidently navigate open-ended and unpredictable student responses.

3. Design assessments that reflect conceptual goals. If we want students to reason, explain, and connect ideas, our tests and rubrics must measure more than right answers. Include prompts for modeling, written justifications, and reflection. Assessments should also capture process-based competencies such as strategic thinking and representation. This helps educators evaluate learning in a more holistic and meaningful way.
4. Allow for instructional depth. Resist the pressure to "cover" too much content. Give students the time to explore fewer ideas more deeply, and you'll see stronger, more transferable learning outcomes. Prioritizing depth over breadth creates space for productive struggle and reflection. This also helps close learning gaps by giving all students the opportunity to reach conceptual clarity.
5. Build a culture of inquiry and intellectual safety. Conceptual learning thrives in classrooms where students feel safe to make mistakes, ask hard questions, and challenge each other respectfully. Teachers can model curiosity and resilience, making it clear that confusion is part of the learning journey. Over time, students begin to value the process of understanding over simply getting the correct answer.
6. Encourage co-teaching or peer collaboration. Teachers implementing rich pedagogies like 5 Practices benefit greatly from having a thought partner to plan and reflect with. Collaboration allows teachers to exchange strategies and improve the quality of lesson design. It also fosters a sense of shared responsibility for student learning and innovation.

3.8.3 Recommendations for policymakers and researchers

The implications of this study go beyond individual classrooms. Policymakers, curriculum developers, and researchers also have a role to play in creating conditions that support deep learning:

- Incorporate real-life mathematics into national curricula and assessments. What we assess sends a message about what matters. Include modeling, reasoning, and multi-step problem solving in high-stakes exams. This change would align instructional practice with assessment and encourage schools to prioritize understanding over rote learning. Exam reform can be a powerful driver of instructional improvement.
- Fund long-term professional learning. Pedagogical change takes time. Teachers need ongoing access to workshops, coaching, and collaborative planning spaces. Sustained investment in professional learning communities can create a culture of continuous improvement. Training should also be context-specific, responding to local needs and resources.
- Support practitioner-led research. Encourage schools and teachers to document their use of rich tasks, collect student work, and share what they learn — both

successes and struggles. Teacher-driven inquiry can lead to contextually grounded innovations and build ownership of change. Policymakers should create platforms for disseminating such research.

- Develop shared repositories of rich tasks. Make high-quality, context-rich tasks widely available and adaptable. Include guidance on how to implement them in diverse classrooms. Tasks should be designed with multiple entry points, making them accessible to students with different backgrounds and learning profiles. Supplementary materials should include facilitation tips and scaffolds.
- Align educational structures with inquiry-based goals. For example, reduce pressure from pacing calendars that discourage deep exploration, and give teachers autonomy to respond to student thinking in real time. Revising institutional expectations will create space for innovation and meaningful learning. Educational systems must trust teachers as professionals capable of shaping curriculum with integrity.

3.8.4 Final thought

In a rapidly changing world, students don't just need to know how to compute. They need to know how to think — how to model, reason, communicate, and adapt. These are the competencies that real life demands. And these are the very skills that conceptual understanding supports.

This study does not claim to offer definitive answers. But it does suggest a direction: if we want students to truly understand mathematics — to see it as a meaningful, useful way of thinking — then we must teach in ways that reflect that belief. Real-life problems, when supported by thoughtful pedagogy, have the power to shift not only what students learn, but who they become as learners.

And perhaps that's the most important lesson of all. Mathematics is not just a subject. It's a language of structure, of logic, of possibility. When students experience it through problems that mirror the complexity of the world they live in, they begin to see themselves as problem-solvers, not just answer-givers. They gain the confidence to ask, "What if?", to challenge assumptions, and to pursue understanding — not because it's required, but because it's meaningful.

In this sense, conceptual understanding is not just an academic goal. It is a foundation for lifelong learning, for civic reasoning, and for navigating a world where clarity, adaptability, and critical thinking are more valuable than ever. Teaching for understanding is an investment — not only in better mathematics, but in better thinkers, better citizens, and better futures. By improving conceptual thinking through meaningful tasks, we prepare students not only for academic success but for real-life challenges where logic, ethics, and adaptability intersect. That is the true promise of education in the 21st century. As we look ahead, the challenge is not only to equip students with knowledge, but to inspire a mindset of curiosity, resilience, and responsible action. Mathematics education must rise to this moment—not by adding more content, but by deepening the experience of learning itself. Every task, discussion, and mistake becomes an opportunity to build the intellectual character needed in uncertain times.

Teachers are not merely instructors—they are designers of thinking environments. When we trust them to create rich, open-ended learning experiences, we acknowledge their role as agents of transformation. Likewise, students are not empty vessels to be filled, but thinkers in formation, capable of insight, empathy, and innovation. Our responsibility is to offer them not just tools, but a reason to use those tools thoughtfully.

In this evolving landscape, conceptual understanding becomes more than a curriculum goal—it becomes a moral imperative. It prepares learners to engage with complexity, question assumptions, and contribute meaningfully to their communities. By embedding real-life problems into the heart of instruction, we don't just teach mathematics. We teach students to navigate the world with integrity and imagination.

The future of education lies in this intersection: where relevance meets rigor, where content meets conscience, and where students grow not only as learners, but as citizens of a shared world. If we believe that students deserve an education that prepares them not just to pass tests, but to shape the world they live in, then we must act with intention. This means rethinking not only what we teach, but how we teach—and most importantly, why.

Schools must become laboratories of thinking, not just factories of answers. Teachers need time, trust, and tools to redesign instruction around real problems and student ideas. Curriculum designers must prioritize depth over coverage. Policy leaders must create systems that value the invisible work of conceptual growth.

It is also time to bring students into the conversation. Let them reflect on their thinking, contribute to lesson design, and evaluate what helps them learn most meaningfully. When students see themselves as co-creators of knowledge, learning becomes personal—and powerful.

Mathematics, in its richest form, teaches more than numbers. It teaches how to think carefully, to reason ethically, and to act wisely. The responsibility now lies with all of us—teachers, researchers, leaders, and communities—to make that kind of learning the norm, not the exception.

CONCLUSION

This research set out to investigate how the integration of real-life problem solving in mathematics classrooms can foster deeper conceptual understanding among secondary school students. The study emerged from a growing recognition—both globally and within Kazakhstan—that mathematical literacy today requires more than procedural fluency. Students need to be able to reason, interpret, and apply their knowledge to unfamiliar, authentic situations. Against this backdrop, the research aimed to compare two instructional approaches: traditional, teacher-centered instruction, and a more student-centered, inquiry-based method grounded in rich tasks and the 5 Practices model developed by Smith and Stein (2018). This focus aligns with international trends advocating for transferable, 21st-century skills within STEM education.

As economies become more data-driven and interconnected, conceptual understanding in mathematics is no longer optional—it is fundamental.

The research was grounded in a strong theoretical framework. Drawing from constructivist traditions, particularly the works of Vygotsky and Bruner, the study positioned learning as an active, socially mediated process. Real-life tasks were not simply used as motivational tools, but as vehicles for meaningful engagement with mathematical ideas. By encouraging students to model, reason, and communicate, these tasks aimed to shift learning away from memorization toward understanding. In this context, mathematics becomes a medium for inquiry rather than a collection of techniques to be memorized. Students were invited not just to solve problems, but to engage with uncertainty, ambiguity, and multiple representations. This mirrors the actual processes used by mathematicians, scientists, and professionals outside of school.

The empirical part of the study involved two Grade 9 classes in a Kazakhstani school: Class 9A (experimental) and Class 9B (control), with 21 and 22 students, respectively. While both groups explored the same content and completed identical performance tasks, the pedagogy differed significantly. In 9B, instruction followed a more conventional format focused on explanation and individual practice. In contrast, 9A lessons were built around rich, context-based problems, peer discussion, and structured teacher facilitation following the 5 Practices model. The contrast between these two settings enabled a meaningful exploration of how pedagogy affects mathematical thinking. Moreover, the use of co-teaching allowed for close observation and reflection throughout the intervention.

The results of the intervention were clear. Students in the experimental group demonstrated not only higher average gains in post-test scores, but also qualitatively richer thinking. They engaged more frequently in collaborative reasoning, questioned assumptions, and showed greater flexibility in applying mathematical concepts. Their written work and classroom discourse reflected an increased ability to justify solutions, consider alternative strategies, and connect different representations. These shifts were especially evident in performance tasks that required modeling and interpretation—skills central to conceptual understanding. While quantitative measures were

important, the qualitative insights provided a more nuanced picture of learning. In-class discussions, written reflections, and group interactions revealed students' emerging ability to think mathematically. In some cases, students who were previously disengaged became active contributors when real-world contexts were introduced.

Importantly, these gains were not purely academic. Many students reported greater interest and motivation when working on tasks that felt connected to real life. For example, lessons involving topics like traffic safety, pricing plans, or epidemic spread allowed students to see mathematics as something relevant and dynamic. This relevance fostered greater engagement and ownership, especially among students who might not normally identify as "strong" in math. The emotional engagement observed among students was an unexpected but powerful byproduct of this pedagogical shift. Students expressed that real-life task "made math make sense," a sentiment rarely heard in traditional classrooms This transformation suggests that conceptual understanding and student identity are closely intertwined.

However, the study also surfaced real-world complexities in implementing such approaches. Rich tasks require thoughtful design, flexible pacing, and a classroom culture that encourages risk-taking and exploration. Teachers must be comfortable not having all the answers immediately and be skilled in facilitating—not dominating—discussion. In this study, the co-teaching model helped manage these demands, but such structures are not always feasible in all schools. Moreover, some students initially struggled with the openness of inquiry-based lessons, especially if they were used to more structured and guided routines. Resistance from students and even colleagues was not uncommon during the early weeks of the intervention. Changing teaching practices also means shifting expectations—and that takes time and sustained support. Nonetheless, these challenges are part of the learning curve inherent in pedagogical transformation.

From a practical perspective, the findings point toward several implications. First, mathematics education in Kazakhstan—and elsewhere—can benefit from a shift in focus: from covering content to uncovering meaning. This does not mean abandoning procedures, but rather embedding them in contexts that promote sense-making. Second, teacher preparation and ongoing professional development are crucial. Educators need both time and training to design rich tasks, facilitate discussion, and assess understanding formatively. Third, school leaders play a vital role in supporting a culture where experimentation and reflective practice are encouraged. Policy reforms that emphasize flexibility, creativity, and inquiry in mathematics curricula could accelerate this transformation Equally important is building communities of practice among teachers, where collaborative lesson design and reflection become the norm.

The study also opens the door to further research. One limitation was the relatively short duration of the intervention. Longer-term studies could explore whether gains in conceptual understanding persist and how they influence performance in high-stakes assessments or students' attitudes toward STEM fields. There is also a need to adapt such approaches for younger learners or for students in rural and multilingual settings, which are common in Kazakhstan. Exploring how different student

populations respond to rich tasks can inform more inclusive instructional strategies. In addition, longitudinal research could uncover whether early exposure to inquiry-based learning affects later academic or career choices.

In conclusion, this research affirms that real-life mathematical problem solving—when integrated thoughtfully and supported by inquiry-based pedagogy—can significantly enrich students' conceptual understanding. It highlights the value of moving beyond routine exercises and toward more dynamic, student-driven learning experiences. Such approaches do not merely enhance academic achievement—they help cultivate habits of mind that are essential for lifelong learning. Mathematics, when grounded in real contexts and social interaction, becomes more than a subject—it becomes a way of thinking critically about the world. As Kazakhstan continues its efforts to modernize its education system and align with international best practices, approaches that promote thinking, reasoning, and real-world application will be essential. By equipping students not just with answers, but with the ability to ask meaningful questions, we empower them to become active participants in a complex and quantitative world.

While this study sheds light on the impact of real-life problem solving and inquiry-based methods on students' conceptual understanding, it also raises broader questions about the evolving nature of mathematics education. As global challenges become increasingly complex, there is a growing need for curricula that prepare students to navigate uncertainty, synthesize information, and construct arguments based on evidence. In this light, mathematics classrooms must evolve from sites of answer-getting to spaces of meaning-making.

One important area for further exploration is the relationship between conceptual understanding and assessment practices. Standardized tests often fail to capture the richness of student thinking, particularly in contexts that emphasize modeling, reflection, and creativity. Future research might examine alternative forms of assessment—such as portfolios, performance tasks, or oral defenses—that better align with inquiry-based instruction.

Another consideration involves scalability. While this study was conducted in a relatively well-resourced, progressive school, many institutions face significant constraints in terms of class size, instructional time, and teacher preparation. Investigating how rich tasks can be adapted for diverse learning environments, including rural or under-resourced schools, remains an important challenge. Similarly, ensuring that such pedagogies are inclusive and responsive to students of different linguistic, cultural, and academic backgrounds must be a priority.

Finally, the role of technology in facilitating conceptual learning through real-life problems deserves closer attention. Digital tools have the potential to provide dynamic representations, simulations, and collaborative platforms that enrich the problem-solving process. However, technology alone is not a solution—it must be integrated purposefully within pedagogical frameworks that value reasoning, dialogue, and student agency.

In sum, while this research confirms the value of rich, context-based mathematics instruction, it also highlights the complexity of transforming educational

systems to support such practices. Sustainable change will require coordinated efforts across teacher education, curriculum development, assessment reform, and school leadership. It is in this intersection—between research, practice, and policy—that the future of meaningful mathematics education lies.

Working on this research changed the way I think about teaching—more than I expected. I went into the study wanting to test an idea: that real-life problems could deepen students' understanding of math. But what I discovered was far more personal. I saw, firsthand, how messy and beautiful learning can be when students are given space to explore, question, and make sense of ideas on their own terms.

There were many moments when I had to stop myself from jumping in too soon. I used to think that helping meant explaining quickly. But I learned that real help sometimes means waiting—allowing students to wrestle with uncertainty and make their own connections. Some of the best learning happened not when I clarified, but when I stayed quiet and just listened.

Using the 5 Practices framework taught me how much value there is in anticipating students' thinking—not just the "correct" answers, but the in-between steps, the misconceptions, the unusual strategies. It wasn't always easy to adjust in the moment, but over time, I became more comfortable with the unpredictability. Teaching turned into something more collaborative, more responsive—and honestly, more alive.

Of course, it wasn't perfect. There were days when I questioned whether I was doing the right thing. As both teacher and researcher, I was deeply invested in my students' progress. I had to remind myself that this wasn't about proving something—it was about observing honestly, even when things didn't go as planned. I kept field notes, reflected after lessons, and had long discussions with my co-teacher. That process of reflection became just as valuable as the lessons themselves.

One moment that really stayed with me was when a student—someone who rarely spoke in class—offered an unexpected solution to a modeling task. It wasn't technically correct, but it was thoughtful and creative. And more importantly, it showed she was thinking independently. From that day on, she participated more often. That moment reminded me why I started this project in the first place.

I finish this research not with a list of perfect answers, but with a deeper belief in the power of meaningful, student-centered learning. I've seen how students can grow—not just in knowledge, but in confidence and identity—when we trust them as thinkers. And I've seen how we, as teachers, grow too, when we're willing to try something new, reflect honestly, and stay open to the unexpected.

While this study provided meaningful insights into the role of real-life problems and inquiry-based instruction in enhancing conceptual understanding, it also raised several questions that merit further exploration.

First, the short duration of the intervention—spanning just one semester—limited the opportunity to observe long-term effects. Future research could investigate whether conceptual gains persist over time and how they influence students' performance in high-stakes assessments or future mathematical learning. A longitudinal study following students beyond Grade 9 would offer valuable insights into the sustainability and transferability of conceptual growth.

Second, this study focused on two classes in one urban, progressive school. To increase generalizability, future studies could replicate the research in different educational settings, including rural schools, multilingual classrooms, or schools with limited resources. Understanding how contextual factors influence the success of rich task instruction would be especially relevant in diverse regions such as Kazakhstan.
Third, further investigation is needed into the impact of professional development on teachers' implementation of inquiry-based approaches. While this study benefited from co-teaching and collaborative planning, many schools may not have such structures in place. Research that explores how teachers can be supported—through coaching, lesson study, or digital communities—would help scale these pedagogies more effectively.

Fourth, future studies might explore the intersection of language and mathematics. In multilingual settings, language can either support or hinder conceptual understanding. Examining how real-life problems function in classrooms where students are learning math in a second or third language could yield valuable findings about scaffolding, representation, and accessibility.

Lastly, future research could examine assessment more closely. While this study used performance tasks and post-tests, alternative assessment models—such as process portfolios, student interviews, or peer assessments—may provide a richer picture of conceptual understanding. Comparing these formats could help develop more valid and reliable tools for evaluating deep learning.

In short, this study opens many paths forward. By continuing to explore how real-world context, thoughtful pedagogy, and equitable practices intersect, future research can deepen our understanding of how to teach mathematics not just for performance, but for meaning.

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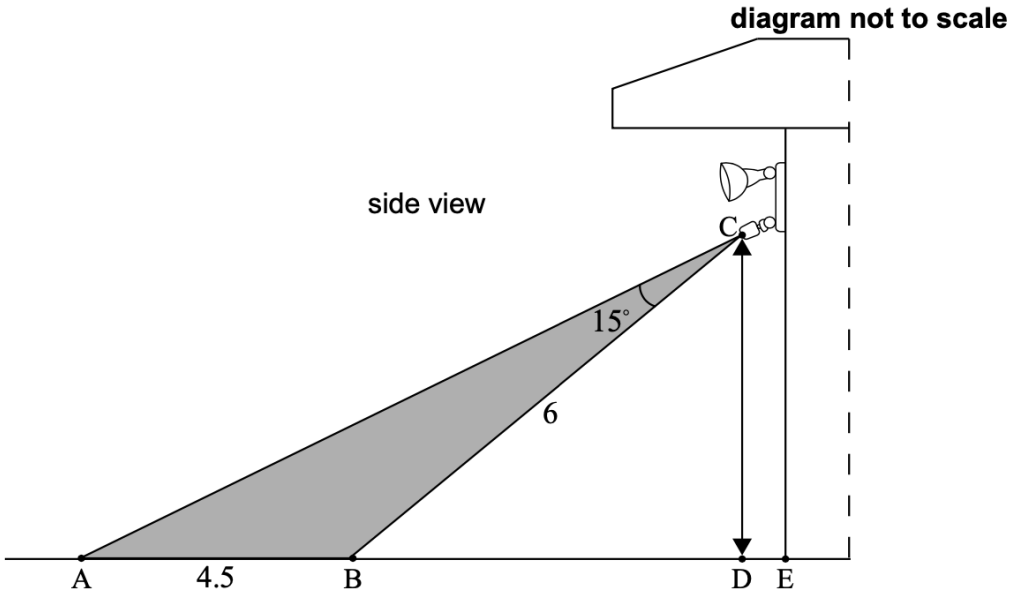
APPENDIX

Appendix 1. Examples of Performance Tasks and Assessment Rubrics

Figure 1.1 Real-life contextual problem involving geometry and trigonometry. Reprinted from Mathematics: Applications and Interpretation, HL, Specimen Paper 1 (International Baccalaureate Organization, 2018).

Ollie has installed security lights on the side of his house that are activated by a sensor. The sensor is located at point C directly above point D. The area covered by the sensor is shown by the shaded region enclosed by triangle ABC. The distance from A to B is 4.5 m and the distance from B to C is 6 m. Angle $A\hat{C}B$ is 15°.



(a) Find $C\hat{A}B$. [3]

Point B on the ground is 5 m from point E at the entrance to Ollie's house. He is 1.8 m tall and is standing at point D, below the sensor. He walks towards point B.

(b) Find the distance Ollie is **from the entrance to his house** when he first activates the sensor. [5]

Figure 1.2 Arithmetic sequence task involving ticket pricing in a real-world context. Reprinted from Mathematics: Applications and Interpretation, HL, Specimen Paper 1 (International Baccalaureate Organization, 2018).

The Osaka Tigers basketball team play in a multilevel stadium.

The most expensive tickets are in the first row. The ticket price, in Yen (¥), for each row forms an arithmetic sequence. Prices for the first three rows are shown in the following table.

Ticket pricing per game	
1st row	6800 Yen
2nd row	6550 Yen
3rd row	6300 Yen

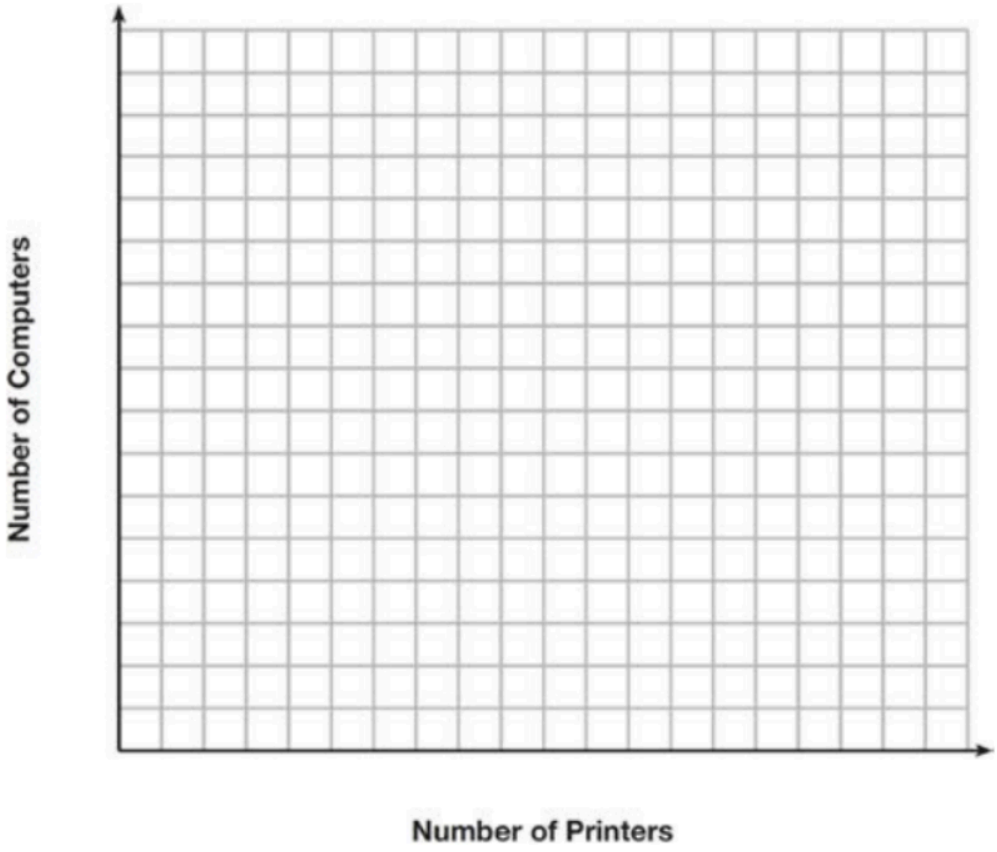
(a) Write down the value of the common difference, d [1]

(b) Calculate the price of a ticket in the 16th row. [2]

(c) Find the total cost of buying 2 tickets in each of the first 16 rows. [3]

Figure 1.3 Graphing a system of inequalities representing sales and shipping constraints for an electronics store.

Интернет-магазин электроники должен продавать как минимум на $2500 принтеров и компьютеров в день. Каждый принтер стоит $50, а каждый компьютер – $500. Магазин может отправить не более 15 товаров в день. На графике ниже изобразите систему неравенств, которые моделируют эти ограничения.



Определите комбинацию(количество) принтеров и компьютеров, которая позволит магазину удовлетворить все условия задачи. Объясните, как вы пришли к своему решению.

Sample 1 of performance task: Вы работаете аналитиком в транспортном департаменте города Алматы, и вам поручено улучшить безопасность дорожного движения на одном из оживленных перекрестков, где произош (International Baccalaureate Organization, 2018)ло несколько мелких аварий. Предполагается, что причиной этих аварий является то, что водители не успевают вовремя остановиться перед светофором. Ваша задача – проанализировать тормозной путь среднего автомобиля в Алматы и определить оптимальное размещение знаков ограничения скорости перед перекрестком, чтобы обеспечить водителям достаточное расстояние для безопасной остановки при необходимости.

(Resource) https://transavto7.ru/kalkulyator-rascheta-tormoznogo-puti-avtomobilya

Figure 1.4 Real-life modeling task on braking distance using quadratic functions. Adapted from Mathematics: Applications and Interpretation, HL, Specimen Paper 2 (International Baccalaureate Organization, 2018)

Задание (Максимальный балл: 15)

Тормозной путь автомобиля определяется как расстояние, которое транспортное средство проходит от момента применения тормозов до полной остановки.

Скорость x (м/с) и тормозной путь y (м)грузовика были записаны. Эти данные приведены в таблице ниже:

Скорость, x (м/с)	0	6	10
Тормозной путь, y (м)	0	12	60

Эти данные были использованы для создания Модели А, где y является функцией, зависимая от скорости x, $x \geq 0$.

Модель А: $y(x) = ax^2 + bx$, где $a, b \in \mathbb{Z}$.

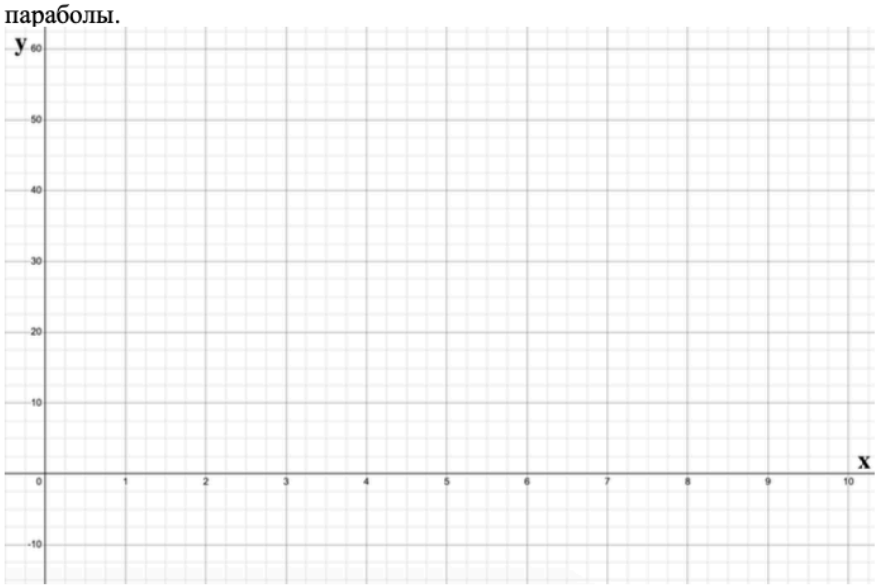
(a) Найдите значения a и b. Для этого ты можешь использовать **графический онлайн-калькулятор** Desmos (код для калькулятора: $y_1 \sim ax_1^2 + bx_1 + c$)

[2]

(b) Найдите координаты вершины графика функции $y(x)$.

[2]

(c) Используя значения из таблицы и ваш ответ к части **(b)**, постройте график $y(x)$ на промежутке $0 \leq x \leq 10$ и $-10 \leq y \leq 60$, четко обозначив вершину параболы.



[3]

(d) Объясните, почему модель А может быть неуместной при меньших скоростях.

[1]

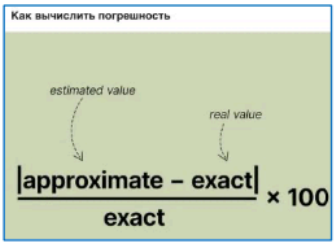
Дополнительные данные использовались для создания Модели B, улучшенной модели для тормозного пути грузовика.

$$\text{Модель B: } y(x) = 0.95x^2 - 3.92x$$

(e) Используя Модель B, рассчитайте примерное значение тормозного пути при скорости 20 м/с.

[2]

Фактический тормозной путь при скорости 20 м/с составляет 320 м.
(f) Рассчитайте процентную погрешность для оценки в части **(e)**.



[2]

Было установлено, что после того, как водитель понимает необходимость остановиться, проходит в среднем 1,6 секунды, прежде чем он начинает тормозить. За это время грузовик продолжает двигаться с той же скоростью.

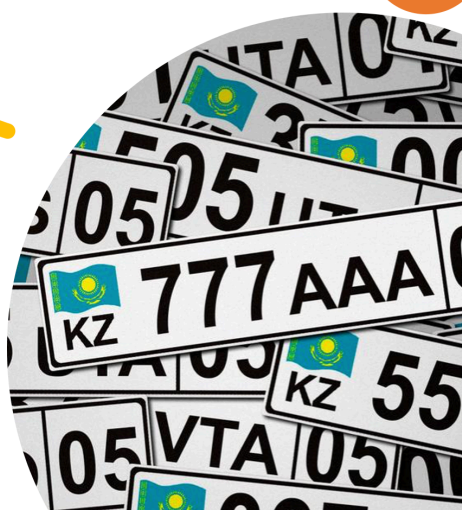
Грузовик подъезжает к перекрестку со скоростью s м/с. Водитель замечает, что светофор показывает красный свет, и ему нужно остановить грузовик до перекрестка, при этом расстояние до светофора составляет 330 метров.

(g) Используя Модель B и учитывая время реакции водителя, рассчитайте максимальную скорость грузовика, при которой он сможет остановиться перед перекрестком.

[3]

Sample 2 of performance task:

Что такое Комбинаторика?



Задание: Найдите, какие казахстанские номера считаются *"счастливыми"*, если под этим понимается, что из трех цифр можно составить верное равенство, используя сложение, вычитание, умножение или деление. Какова вероятность того, что случайный номерной знак будет *"счастливым"?*

Пример: Для номера 248 можно составить 2×4=8, поэтому он *"счастливый."*

Sample 3 of performance task:

Game problem:

a) В казино была представлена новая азартная игра:

Игрок ставит 8 долларов в обмен на бросок двух костей, где игрок выигрывает столько долларов, сколько составляет сумма двух показанных чисел на верхней стороне.

На какую сумму игрок может рассчитывать выиграть?

b) На втором казино была создана другая азартная игра:

Игрок ставит 8 долларов в обмен на бросок двух костей, если выпадают две шестерки, игрок выигрывает 252 доллара

Какая игра будет более прибыльной для казино в долгосрочной перспективе?

Вопросы на рефлексию:

1) Почему в долгосрочной перспективе казино всегда будет в выигрыше, независимо от того, как играет отдельный игрок?
2) Что такое «математическое ожидание» и как оно объясняет, почему игроки в целом теряют деньги, а казино выигрывает?
3) Как бы ты объяснил своему другу, который хочет начать играть в казино, что, хотя выигрыш возможен, на практике это всегда невыгодно для игрока?

Sample 3 of performance task:
Математическая модель размножения бактерий
Ситуация**:** В школе возникла необходимость изучить влияние условий окружающей среды на микрофлору для улучшения санитарной ситуации. Команде учеников поручено разработать и протестировать гипотезу о влиянии температуры на размножение бактерий.

Роль учеников**:** Вы биологи и математики, которые должны провести эксперимент, собрать данные и построить математическую модель роста бактерий. Ваша задача — представить исследовательский отчет и сделать выводы, которые помогут улучшить санитарные условия.
Действия учеников**:**

1. Проведение лабораторной работы с использованием чашек Петри, питательной среды и бактерий.
2. Наблюдение и запись изменений в количестве колоний при разных температурах.
3. Построение графиков роста с использованием Desmos и расчет производных для анализа скорости роста.
4. Написание итогового тезиса и представление выводов.

Продукт**:** Финальный отчет(тезис) с расчетами, графиками и выводами, который представляется учителю и другим учащимся.

Table 1.4 shows a rubric for assessing the structure and clarity of students' mathematical presentations across four levels of performance

РУБРИКИ ОЦЕНИВАНИЯ

	Превосходит ожидания	**Соответствует ожиданиям**	**Частично соответствует ожиданиям**	**Нуждается в помощи**
Презентация *Оценивается четкость, организация и лаконичность презентации математической работы.*	работа представлена последовательно, хорошо структурировано и организовано(не содержит несвязанных или избыточных повторяющихся вычислений, графиков или описаний.)	работа представлена последовательно и хорошо структурировано.	работа частично представлена последовательно и демонстрирует некоторую структуру.	в работе присутствует некоторая логика или некоторая структура.

Математическая коммуникация *Оценивается организация и структура математической презентации.*	математическая коммуникация актуальна, соответствует теме и последовательна на протяжении всей работы.	математическая коммуникация актуальна и соответствует теме	работа содержит некоторую соответствующую и актуальную математическую коммуникацию.	работа не соответствует уровню, описанному характеристиками выше.
Личное вовлечение *Оценивается уровень интереса, любознательности и креативности ученика. Оценивается, исследования.*	присутствует выдающиеся уровень личного вовлечения.	присутствует достаточный уровень личного вовлечения.	присутствуют признаки частичного личного вовлечения.	работа не соответствует уровню, описанному характеристиками выше.
Рефлексия *Оценивается умение ученика критически рефлексировать на свои исследовательские шаги и результаты, анализируя процесс мат исследования и обсуждая важность результатов, их ограничения и возможные направления развития.*	в работе присутствуют существенные доказательства критической рефлексии.	рефлексия есть, но может быть более глубокой и аналитической.	присутствуют признаки ограниченной (limited) рефлексии.	работа не соответствует уровню, описанному характеристиками выше.
Использование математики *Оценивается актуальность, корректность и обширность использования математики в работе.*	используется актуальная математика. Все вычисления/аспекты верны. Продемонстрировано глубокое знание и понимание.	используется актуальная математика. В большей части все вычисления/аспекты верны. Продемонстрировано хорошее знание и понимание.	используется актуальная математика. Вычисления и математические аспекты частично верны. Продемонстрировано некоторое знание и понимание.	используется актуальная математика с ограниченным пониманием.

Appendix 2.

Figure 2.1 illustrates examples of the PISA-released items used in this study.

СКОРОСТЬ ПАДЕНИЯ КАПЕЛЬ

Внутривенные капельные вливания используются для введения жидкости и лекарств пациентам.

Для осуществления вливания медицинским сестрам нужно вычислять скорость падения капель (D), в каплях в минуту.

Они используют формулу $D = \frac{k \cdot V}{60n}$, где

k – показатель «число капель в единице объема», который измеряется в каплях в миллилитре (мл),

V – объем вливания (в мл),

n – время (в часах), за которое требуется сделать вливание.

Вопрос 1: СКОРОСТЬ ПАДЕНИЯ КАПЕЛЬ

Медицинская сестра хочет увеличить вдвое время вливания.

Приведите точное описание того, как изменится значение D, если n **увеличить в два раза**, а k и V оставить без изменения.

СКОРОСТЬ ПАДЕНИЯ КАПЕЛЬ: ОЦЕНКА ОТВЕТА НА ВОПРОС 1

ЦЕЛЬ ВОПРОСА:

Описание: Объяснить, что произойдет, если одна величина в формуле увеличится в два раза, а другие останутся без изменения.
Область математического содержания: Изменение и зависимости
Контекст: Профессиональный
Познавательная деятельность: Применять

Вопрос 2: СКОРОСТЬ ПАДЕНИЯ КАПЕЛЬ

Медицинским сестрам также нужно вычислять объем вливания (V), используя скорость падения капель D.

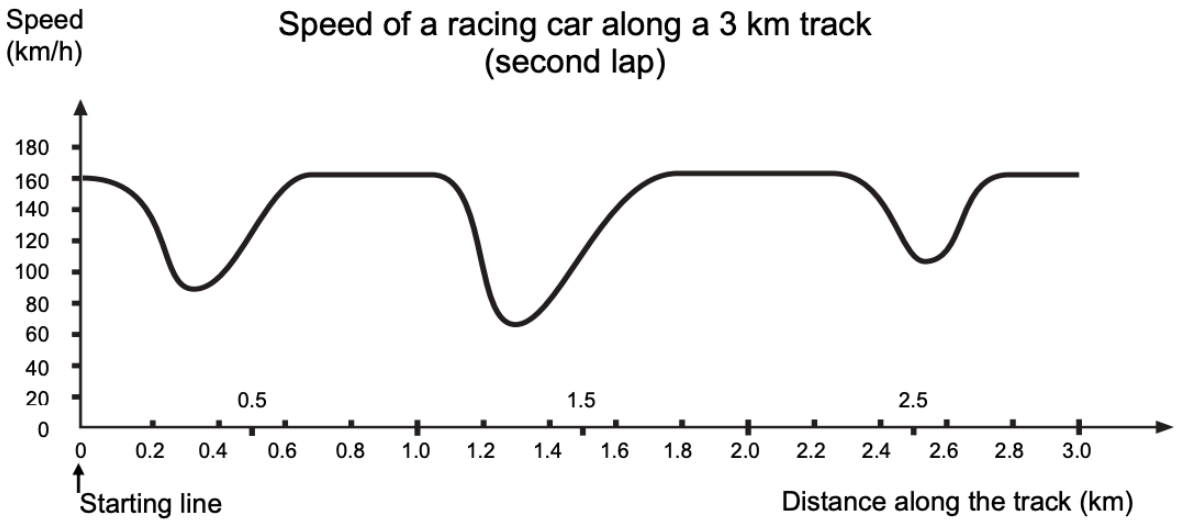
Вливание со скоростью 50 капель в минуту надо сделать пациенту за 3 часа. Показатель «число капель в единице объема» для данного вливания равен 25 каплям в миллилитре.

Чему равен объем вливания в миллилитрах?

Объем вливания: …….. мл.

M159: Speed of Racing Car

This graph shows how the speed of a racing car varies along a flat 3 kilometre track during its second lap.



Question 1: SPEED OF RACING CAR

M159Q01

What is the approximate distance from the starting line to the beginning of the longest straight section of the track?

A 0.5 km
B 1.5 km
C 2.3 km
D 2.6 km

Question 2: SPEED OF RACING CAR

M159Q02

Where was the lowest speed recorded during the second lap?

A. at the starting line.
B. at about 0.8 km.
C. at about 1.3 km.
D. halfway around the track.

Question 3: SPEED OF RACING CAR

M159Q03

What can you say about the speed of the car between the 2.6 km and 2.8 km marks?

A. The speed of the car remains constant.
B. The speed of the car is increasing.
C. The speed of the car is decreasing.
D. The speed of the car cannot be determined from the graph.

M432: Reaction Time

In a Sprinting event, the 'reaction time' is the time interval between the starter's gun firing and the athlete leaving the starting block. The 'final time' includes both this reaction time, and the running time.

The following table gives the reaction time and the final time of 8 runners in a 100 metre sprint race.

Lane	Reaction time (sec)	Final time (sec)
1	0.147	10.09
2	0.136	9.99
3	0.197	9.87
4	0.180	Did not finish the race
5	0.210	10.17
6	0.216	10.04
7	0.174	10.08
8	0.193	10.13

Question 1: REACTION TIME

M432Q01 - 0 1 9

Identify the Gold, Silver and Bronze medallists from this race. Fill in the table below with the medallists' lane number, reaction time and final time.

Medal	Lane	Reaction time (secs)	Final time (secs)
GOLD			
SILVER			
BRONZE			

Question 2: REACTION TIME

M432Q02 - 0 1 9

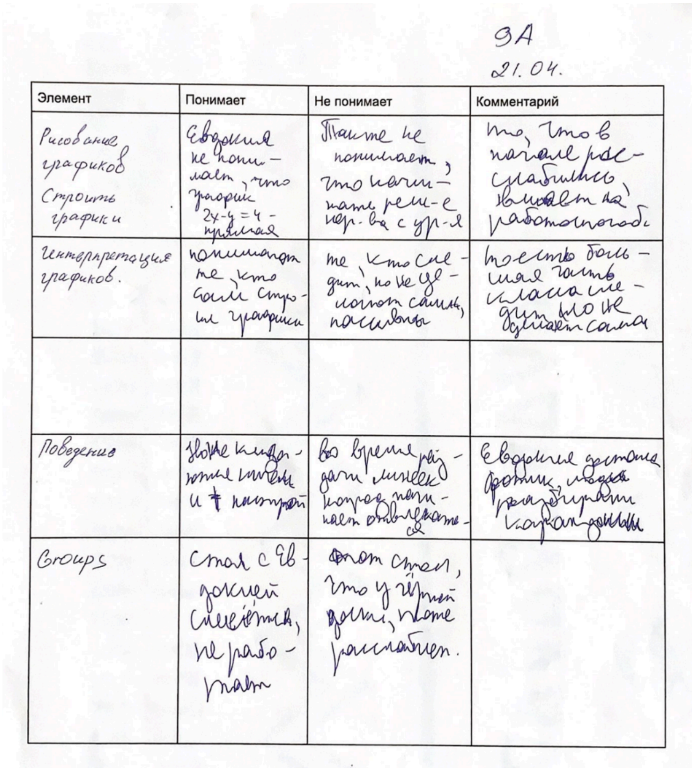
To date, no humans have been able to react to a starter's gun in less than 0.110 second.

If the recorded reaction time for a runner is less than 0.110 second, then a false start is considered to have occurred because the runner must have left before hearing the gun.

If the Bronze medallist had a faster reaction time, would he have had a chance to win the Silver medal? Give an explanation to support your answer.

Appendix 3. Test results and observations

Figure 3.1 Teacher observation notes from 9A and 9B classes on students' understanding during a mathematics lesson. Columns indicate what students understood, misunderstood, and teacher's comments for clarification.

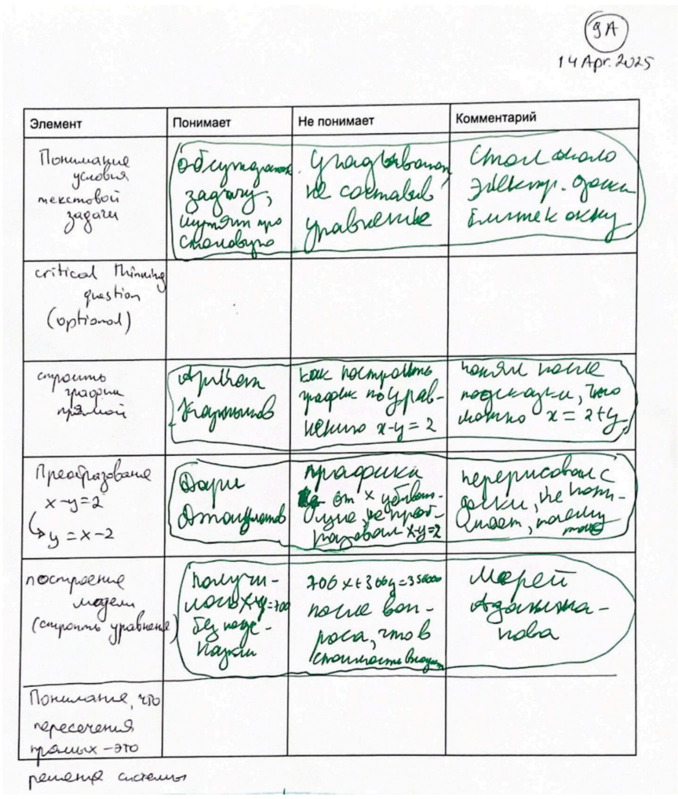


9A
21.04.

Элемент	Понимает	Не понимает	Комментарий
Рисование графиков. Строить графики	Евдокия не понимает, что график 2x-y=4 - прямая	[illegible] не понимает, что начинать реш-е нер-ва с ур-я	то, что в начале расслабились, выходят на работоспособ.
Интерпретация графиков.	понимают те, кто сами строили графики	те, кто следит, но не делают сами, пассивны	то есть большая часть класса следит, но не делает сама
Поведение	[illegible]	во время раздачи [illegible] начинает отвлекаться	Евдокия [illegible] разбирает [illegible]
Groups	Стол с Евдокией смеётся, не работают	[illegible] стол, что у [illegible] [illegible] расслаблен.	

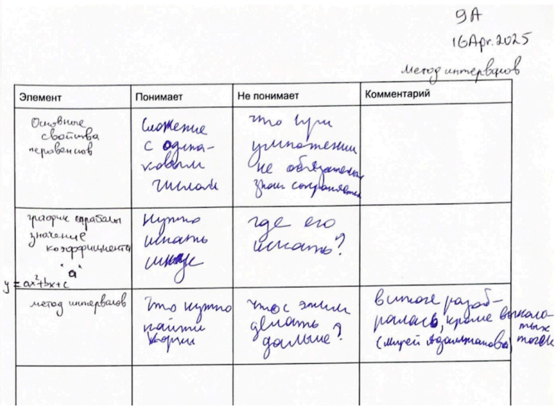
9B
15 Apr. 2025
Алина

Элемент	Понимает	Не понимает	Комментарий
Слушание задания от [illegible]	одна из учениц мнёт "тесто", что-то шутит	Это может отвлекать других, они тоже подключаются	Но это может быть важно для учениц как элемент успокоения и мелкой моторики
то же, потому что продолжается	люди вокруг слушают, не качаются на стульях	вопрос в том, насколько следят и усваивают	рядом с этой ученицей люди отвлекаются на неё, тоже "мнут" тесто; [illegible]
третья переменная, x, y, c	что нужно ввести c, много людей понимают	что делать дальше? не [illegible], не говорят про [illegible]	нужно не бояться развивать своё решение
решение задачи про [illegible]	но не продуктивно	тесто перешло на сосед. стол, но человек работает	важно понять, насколько отвлекает и чем [illegible]
задача про баскетбол	[illegible] понял, т.к. 3 переменных одна быстро узнаётся и дальше система из двух уравнений	пробивался с решением, быстро решилось, вопрос в уверенности (Искандер Каримов)	[illegible], которая тоже [illegible] тесто, ничего не поняла в задаче, пробовала даже уравнений начальных, переписала у соседки



9A
14 Apr. 2025

Элемент	Понимает	Не понимает	Комментарий
Понимание условия текстовой задачи	Обсуждают задачу, шутят про [illegible]	Угадывают, не составив уравнение	Стол около электр. доски ближе к окну
critical thinking question (optional)			
строить график прямой	Артем Каримов	как построить график по уравнению x-y=2	поняли после подсказки, что можно x = 2+y
Преобразование x-y=2 → y = x-2	[illegible]	графика от x убывающие, не преобразовал x-y=2	перерисовал с доски, не понимает, почему так
построение модели (строить уравнение)	получили x+y=700 без подсказки	700x+300y=350000 после вопроса, что в стоимость входит	Мерей Адамжанова
Понимание, что пересечения прямых - это решение системы			



9A
16 Apr. 2025
Метод интервалов

Элемент	Понимает	Не понимает	Комментарий
Основные свойства неравенств	сложение с одинаковыми числами	что при умножении не обязательно знак сохраняется	
график параболы значение коэффициента "a" $y=ax^2+bx+c$	нужно искать [illegible]	где его искать?	
метод интервалов	что нужно найти корни	что с этим делать дальше?	в итоге разобралась, кроме выколотых точек (Мирей Адамжанова)

9A

18 Apr. 2025

Элемент	Понимает	Не понимает	Комментарий
первое задание	Ученик сидит отдельно, ~~не в духе~~	Ученик не понимает, что происходит, пишет то, что делают, но без понимания, зачем	неправильно переносит слева направо выр-я (не меняя знак), ~~[illegible]~~ по X потом Y и т.д.
обсуждение планов у доски	два ближайших стола в диалоге с Сал.	дальний один стол следят, пока говорят с Сал., ученик отдельно, слушает	ученик не говорит с Сал. Ученик ушла с половины урока и вроде не вернулась
[второй урок начался с just dance	динамика — по крайней 4 ученика танц.	остальные, по ощущ., скучали	Ученик сидела в телефоне
разбор задачи про сосиску (объем и длина)	когда начали строить коорд. сист., отвлеклась планшет	ученик сидела в планшете, видимо играла, но	люди рисуют
обсуждаем, какую область красить, выше или ниже	студенты штрихуют.	не понимают, почему это нуждается в объяснении	подытожив — вот точки сказал решить Сал.

Figure 3.2 Summary of student performance data from 9A and 9B classes across multiple assessments, including homeworks, formative tasks, and summative evaluations. Highlighted cells indicate missing or exempted data.

				Benchmark 9 grade	Mock (ГОС) exam	Homework-1	Homework-2	Homework-3 (Теорема Виета)	Formative test-1	Homework (Построение параболы: графики)	Задание-Проход грузовика через туннель	Performance task(Тормозной путь)	Summative assessment	Performance Task	Основы комбинаторики Homework	Безопасный пароль	Формирующая работа по теме Комбинаторика	Summative assessment Q2	Теория вероятности и азартные игры
	Status (0-Old student; 1-New student)	Age	Gender (0-Girl; 1-Boy)	15	25	5	10	6	12	8	20	12	25	35	10	15	15	17	10
9A	0	15	0	7	9	5	10	6	9	5	0	0			9	12	11	7	
9A	**1**	**16**	0	10.5	9	5	10	6	8	8	17	12	22	34	8	13	13	6.5	9
9A	0	14	0		10	5	8	6	8	6	12	8	14	30	9	13	11	9	8
9A	0	15	1		7	5	8	6	7	6	12	10	13.5	28	8	13	5	7	7
9A	**1**	**15**	1			0	0	0	5	0	12	10			7	13	2	6.5	9
9A	0	14	1	5	12	5	8	4	9	5	12	10	14.7	27	0	15	11	8	
9A	0	15	1	7	14	5	7	0	9	0	11	7	16	30	9	12	11	9	6
9A	0	14	1		13	3	6	4	8.5	5	11	7	12	30	0	12	8.5	8	6
9A	**1**	**15**	0	12	12	5	10	6	10.5	8	18	12	20.5	33	9	13	12	13	9
9A	0	15	0	8	10	5	0	0	5	5	12	12	17	31	10	13	2.5	10	
9A	**1**	**14**	0	12.7		5	10	6		8	17	12	20.5	34	10	13	6.5	8.5	9
9A	0	14	0	12	18	5	10	6	10	0	0	8	19	30	10	13	10.5	11	8
9A	0	14	0	9		5	0	6	9	8	18	12	20	30	10	14	11	10	9
9A	0	14	0		12	5	7	6	6	7	18	12	5	30	10	14	8.5	6	9
9A	**1**	**14**	0	12	12	5	10	6	11	8	18	12	23.5	33	10	15	13	9	9
9A	0	15	1	9	10	5	8	6	5.5	6	15	12	12.5	35	10	13	7	10	7
9A	0	14	1	6	8	5	6.5	6	9	5	15		12		8	12	4	8	8
9A	0	15	0	9		0	0	0	5	0	12		3	31	0			8	
9A	0	14	0		13	5	9	4	5.5	6	13	8	12	33	8	13	6.5	4	9
9A	0	15	0		10	5	6	6	10.5	8	12	8	13	31	8	13	10	10	9
9B	0	15	0	8	2	5	9	6	10	8	18	9.6	18	30	9	11	11	10.5	9
9B	0	14	1		9	5	10	6	11.5	8	12	8.8	18	34	10	15	9	9	7
9B	0	15	0	9	9	5	8.8	6	10	8	15	10.4	22	33	10	14	10	14	8
9B	0	15	0			5	7	5	0	7	18	10.4	5	29	9	13	9	3	8
9B	0	15	1	8	7	5	8.8	6	9.5	6	17	10.4	16	32	7	13	11	12	8
9B	**1**	**15**	0	12	9	5	6	6	0	8	18	12	20	30	10	15	9	16.5	9
9B	**1**	**15**	0	11	9	5	10	6	0	8	18	12	24	30	9	14	10.5	16	9
9B	0	14	0	6	4	5	10	6	9.5	8	15	11.2	25	35	9	15	8	13	10
9B	0	14	0	5	5	5	9	6	10	5	18	12	21.5	32	10	14	11.5	12	7
9B	0	14	0	14	13	5	10	6	10	5	15	11.2	22	35	10	15	12	11	10
9B	0	14	1	7	5	5	10	5	6	5	12	8.8	15	34	8	13	9	10	7
9B	0	14	1	8	8	5	10	6	12	8	18	10.4	24	29	10	13	12	13	8
9B	0	15	0	6	5	5	0	0	7.5	0	18	9.6	10.5	32	6	14	9	11	8
9B	0	15	0	11	22	5	10	6	12	8	18	8.8	17	29	9	14	10	5	8
9B	0	15	0	8.5	5	5	9	6	9.5	5	17	10.4	14.5	32	10	15	6	10	9
9B	0	14	0			5	10	4.5	0	0	15	0	13	26	10	14	11	3	8
9B	0	14	1	7		5	6.5	6	7.5	5	12	8.8	6	34	8	15	7	7	8
9B	**1**	**15**	0	8		5	10	6	6	0	15	0	6.5	26	10	15	8.5	2	8
9B	0	15	1	3	10	5	8.5	6	6.5	7	15	10.4	17	33	10	15	10	12.5	8
9B	0	15	0			5	0	6	7	5.5	18	12	13	32	8	14	5.5	14	9
9B	0	14	0	10	14	5	10	0	11	6	18	8.8	15.5	29	10	13	11.5	14	7
9B	0	15	0	3	3	4	6.5	4	0	8	18	9.6	16	30	8	11	11	10	9
9B	0	15	0	13	6	5	9.7	6	12	8	18	9.6	18	32	10	14	12	12	10

Table 3.3 Benchmark and post-test scores for students in classes 9A (experimental group) and 9B (control group) across the academic year. Data illustrate changes in performance over five assessment points from September 2024 to May 2025.

09 grades				Sept 18	Dec-24	Jan-25	Apr 23 2025	May-25
				Benchmark	Pre-test results	Benchmark	Benchmark 2.0	Post-test result
Total points		Age	Gender	15	50	25	16	50
Student 1	9A	15	female	7	18	9	4	24
Student 2	9A	16	female	10.5	17	9		30
Student 3	9A	14	female		21	10	7	35
Student 4	9A	15	male		11	7	2	23
Student 5	9A	15	male		7		3	17
Student 6	9A	14	male	5	21	12	12	32
Student 7	9A	14	male	4	11	4	1	9
Student 8	9A	15	male	7	18	14	7	35
Student 9	9A	14	male		12	13		25
Student 10	9A	15	female	12	25	12	7	32
Student 11	9A	15	female	8	13	10	5	31
Student 12	9A	14	female	12.7	25		6	35
Student 13	9A	14	female	12	19	18		16
Student 14	9A	14	female	9			4	29
Student 15	9A	14	female		12	12	6	21
Student 16	9A	14	female	12	25	12	9	38

Student 17	9A	15	male	9	10	10	5	32
Student 18	9A	14	male	6	7	8	4	29
Student 19	9A	15	female	9			7	28
Student 20	9A	14	female		20	13		41
Student 21	9A	15	female		27	10	8	20
Student 1	9B	15	female	8	5	2	2	18
Student 2	9B	14	male		29	9	7	26
Student 3	9B	15	female	9	26	9		31
Student 4	9B	15	male	8	8	7	5	18
Student 5	9B	15	female	12	25	9	5	44
Student 6	9B	15	female	11	31	9	14	44
Student 7	9B	14	female	6	12	4	14	38.5
Student 8	9B	14	female	5	32	5	14	44
Student 9	9B	14	female	14	19	13	6	45.5
Student 10	9B	14	male	7	8	5	3	8
Student 11	9B	14	male	8		8	8	26.5
Student 12	9B	15	female	6	15	5	7	34
Student 13	9B	15	female	11	23	22	15	41
Student 14	9B	15	female	8.5	14	5	2	32.5
Student 15	9B	14	female		20		6	28
Student 16	9B	14	male	7	5			23

Student 17	9B	15	female	8	4		7	19
Student 18	9B	15	male	3	18	10	3	19
Student 19	9B	15	female		11		6	23
Student 20	9B	14	female	10	28	14	14	45
Student 21	9B	15	female	3	7	3	1	28
Student 22	9B	15	female	13	19	6	13	44

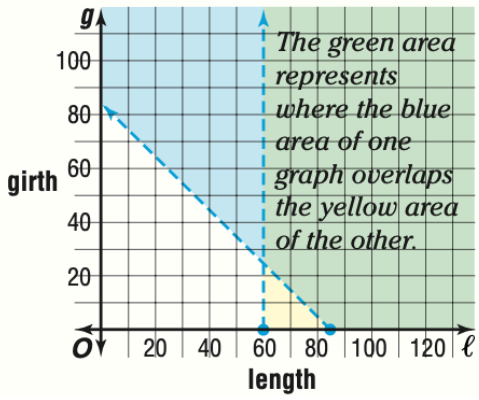
Table 3.4 Individual student-level comparison between average performance task scores and post-test results (9 grades).

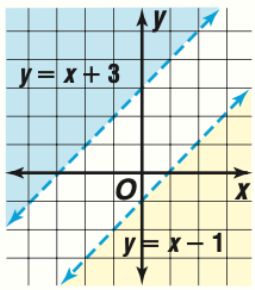
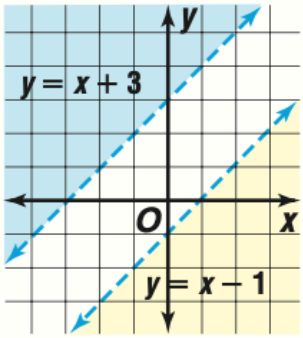
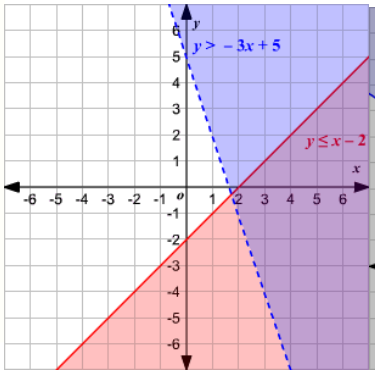
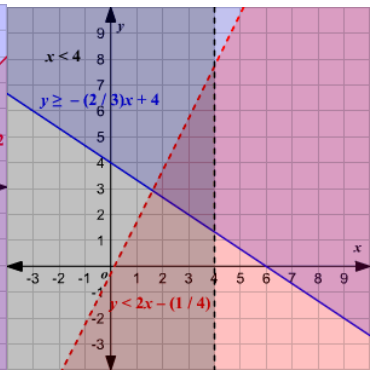
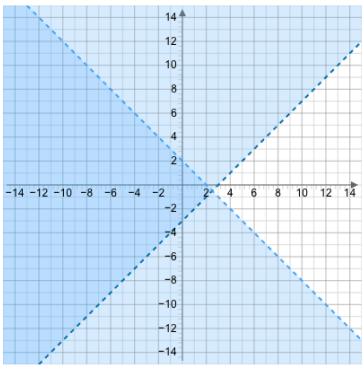
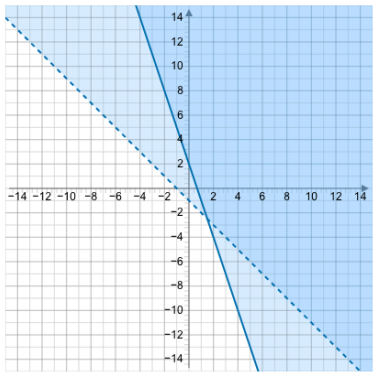
Average results of performance tasks	Post-test results 9 grades
50.13209464	24
82.48634868	30
72.47377944	35
64.45714817	23
25.6215604	17
62.3387199	32
63.60820548	9
56.49733857	35
87.34764294	25
54.27557467	32
71.72216471	31
72.94966201	35
73.96256923	16
67.85626073	29
89.2952726	21
71.17923059	38
53.72584718	32
20.62658155	29
64.37549504	28
73.35913804	41
79.07547911	20
80.71189326	18
84.68242112	26
57.16515369	31
76.99711384	18
75.59678293	44

80.41051388	44
84.51288333	38.5
82.35573163	44
85.72680275	45.5
69.72657536	8
88.30117322	26.5
54.09576897	34
81.54456035	41
73.73426855	32.5
55.57906868	28
61.53405974	23
56.02581798	19
78.46632956	19
64.06038205	23
77.95826565	45
65.67759551	28
85.75011414	44

Appendix 4.

Learning Objective: Решать систему линейных неравенств.

connection	
Rich task 15 min	**ДОСТАВКА** Службы доставки пакетов взимают дополнительные сборы за сверхразмерные посылки или посылки, требующие специальной обработки. Сверхразмерной считается посылка, в которой сумма длины и обхвата (girth) превышает 84 дюйма. Объем посылки — это расстояние вокруг нее. Для прямоугольной посылки объем — это сумма удвоенной ширины и удвоенной высоты. Посылка, требующая специальной обработки, — это посылка, длина которой превышает 60 дюймов. Какого размера должны быть посылки, чтобы они подходили под оба критерия — сверхразмерную и требующую специальной обработки?
Debrief 10min	**SHIPPING** **What size packages qualify for both oversize and special handling charges when shipping?** First write two inequalities that represent each type of charge. Let ℓ represent the length of a package and g represent its girth. Oversize: $\ell + g > 84$ Special handling: $\ell > 60$ Neither of these inequalities includes the boundary line, so the lines are dashed. The graph of $\ell + g > 84$ is composed of all points above the line $\ell + g = 84$. The graph of $\ell > 60$ includes all points to the right of the line $\ell = 60$. The green area is the solution to the system of inequalities. That is, the ordered pair for any point in the green area satisfies both inequalities. For example, (90, 20) is a length greater than 90 inches and a girth of 20 inches which represents an oversize package that requires special handling. 

Important moment!	Not every system of inequalities has a solution. For example, $y > x + 3$ and $y < x - 1$ are graphed at the right. Since the graphs have no points in common, there is no solution. 
See-Think-Wonder activity 10 min	**See-Think-Wonder**   
Exit ticket 5 min	Выберите пары координат, которые являются решениями системы. (Может быть один или несколько правильных вариантов).  ○ $(0, 0)$ ○ $(4, 3)$ ○ $(-3, -4)$ ○ $(-2, -5)$ ○ $(3, 4)$  ○ $(0, 0)$ ○ $(-2, -4)$ ○ $(1, 1)$ ○ $(-4, 14)$ ○ $(2, 3)$

Learning Objective:
Решать систему линейных уравнений. Графический способ решение.

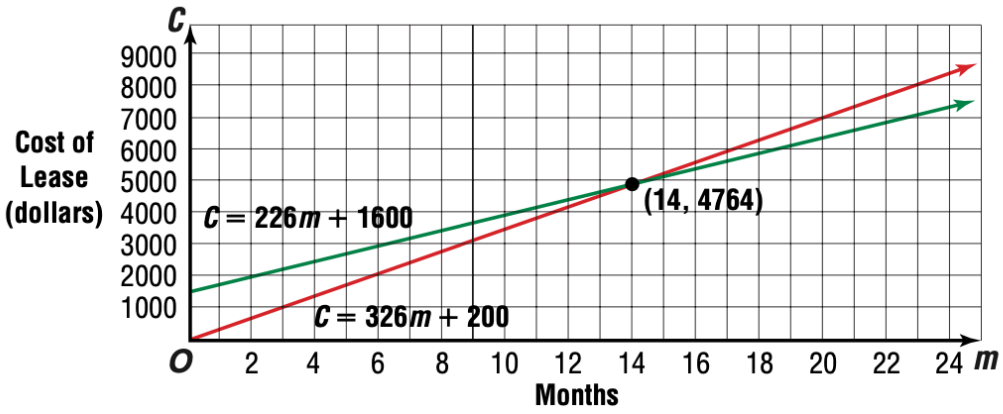
connection	
Warm-up	**Что означает "решение"?** Что означает, что точка является решением линейного уравнения? Например, если мы говорим, что $(2,5)$ - решение уравнения $y = 2x + 3$, как можно проверить это утверждение?
Rich task 10 min	ПОТРЕБИТЕЛЬСКИЙ ВЫБОР Мэдисон собирается взять машину в аренду на два года. В автосалоне ей предложили два варианта: Платить по 326 долларов в месяц при первоначальном платеже 200 долларов Или заплатить 1600 долларов сразу, и тогда ежемесячный платеж будет 226 долларов При каком сроке оба варианта обойдутся в одинаковую сумму? Какой вариант аренды на 2 года выгоднее, если сумма первого взноса для нее не имеет значения?
Debrief 10min	1. 💰 **Сколько Мэдисон заплатит всего по первому варианту за 2 года?** *(326 долларов × 24 месяцев + 200 долларов?)* 2. 💵 **Сколько она заплатит по второму варианту за 2 года?** *(226 долларов × 24 месяцев + 1600 долларов?)* **а.** Сначала составь уравнение, отражающее сумму, которую она заплатит по каждому варианту. Пусть C — это общая стоимость, а m — количество месяцев аренды. **Вариант 1** (первоначальный взнос 200 долларов, ежемесячный платёж 326 долларов): $$C = 326m + 200$$ **Вариант 2** (первоначальный взнос 1600 долларов, ежемесячный платёж 226 долларов) $$C = 226m + 1600$$ Теперь реши систему уравнений. Так как обе формулы содержат C, можно подставить значение C из одного уравнения в другое.

$$C = 326m + 200$$
$$226m + 1600 = 326m + 200 \quad C = 226m + 1600$$
$$1400 = 100m$$
$$14 = m$$

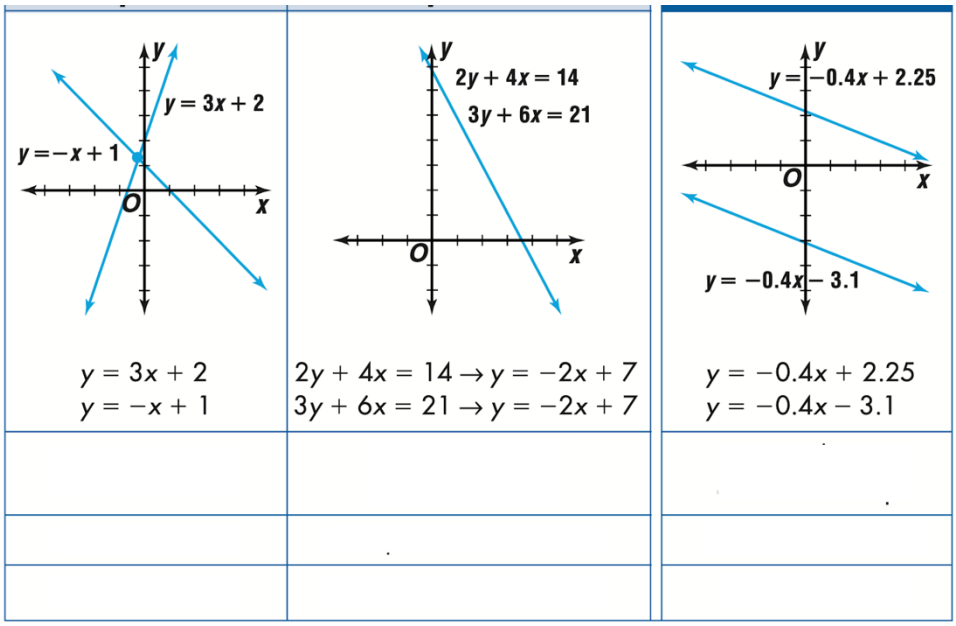
🔹 **После четырнадцатого ежемесячного платежа достигается точка безубыточности.**

b. График уравнений показывает, что после этой точки **вариант 1 становится дороже** при аренде на 2 года.

Значит, Мэдисон, скорее всего, стоит выбрать вариант 2.



Графический способ решения системы



Exit ticket

Решить систему уравнений и указать в ответе $x \cdot y$:

$$\begin{cases} 2x - y = 3 \\ x + 5y = 7 \end{cases}$$