



**Features of the use of trigonometric materials in solving complex problems related to
right triangles**

Assemgul Rakhymgalieva

A thesis submitted to the Faculty of Education and Humanities
in partial fulfillment of the requirements for the degree of

MASTER OF PEDAGOGICAL SCIENCES
in Mathematics

«SDU University»

Department of Pedagogy of Natural Sciences

Thesis Advisor: PhD., associate professor
Dzhanbulat Kayinbaev

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«SDU University»

Faculty of Education and Humanities
Department of Pedagogy of Natural Sciences

This is to certify that the Master's Thesis of

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Kaskelen, 2025

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25.06.2025

Date



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List of Figures

- Figure 1.1.1 Types of mathematical models
- Figure 2.4.1 Right triangle and its height
- Figure 2.4.2 Equilateral triangle constructed on right triangle
- Figure 2.5.1 Criterion of equality by two legs
- Figure 2.5.2 Criterion of equality by hypotenuse and acute angle
- Figure 2.5.3 Criterion of equality by hypotenuse and leg
- Figure 2.5.4 Application of triangle equality criterion
- Figure 2.5.5 Sine, cosine and tangent of acute angle
- Figure 2.5.6 Equality of sines and cosines of equal angles
- Figure 2.5.7 Basic trigonometric identity
- Figure 2.5.8 Pythagorean Theorem (1st proof)
- Figure 2.5.9 Right triangle with 30° angle
- Figure 2.5.10 Isosceles right triangle
- Figure 2.7.1 Using triangle equality criterion on a drawing
- Figure 2.7.2 Proving triangle equality
- Figure 2.10.1 Isosceles triangle and right angle
- Figure 2.10.2 Construction of a triangle based on given angles
- Figure 2.10.3 Problem solution from textbook (example)
- Figure 2.10.4 Triangle sides with exterior angle
- Figure 2.10.5 Triangle equality via shared side
- Figure 2.10.6 Equality by hypotenuse and angles
- Figure 2.10.7 Various cases of triangle equality
- Figure 2.10.8 Isosceles triangle with right angles at the base
- Figure 2.10.9 Construction and proof of equality
- Figure 2.10.10 Symmetry and equality in right triangles
- Figure 3.1 Comparative analysis
- Figure 3.2 Pre-test analysis
- Figure 3.3 Pro-test analysis

List of Table

- Table 1.1.1 Types of models of the textual task
Table 2.5 1 Values of $\sin\alpha$, $\cos\alpha$, and $\tan\alpha$
Tabel 2.6.1 Thematic plan on the topic “Right Triangles”
Table 2.7.2 Theorem and inverse on equality of right triangles
Table 2.10.1 Properties and inverse properties of a right triangle
Table 2.10.2 Measurements of the sides of a right triangle
Table 2.10.3 Squares of the lengths of a right triangle’s sides
Table 3.1 Descriptives
Table 3.2 Correlation matrix

List of Appendices

Appendix 1 Cards with Task

CONTENTS

INTRODUCTION	1
1. LITERATURE REVIEW	4
1.1 The concept of mathematical modeling	4
1.2 The role of mathematical modeling in solving word problems	7
1.3 Classification of word problems	9
1.4 The answer is the result of the actions received	11
2. METHODOLOGY	18
2.1 Difficulties in solving word problems	18
2.2 Organization of the experimental study	19
2.3 Common issues faced while working with verbal mathematical tasks	20
2.4 Right triangle and its properties	22
2.5 Signs of equality of right triangles	23
2.6 Methodological foundations of studying the topic "Right triangle"	31
2.7 Methodological recommendations for studying the topic "Right triangle"	35
2.8 Organization of repetition of the studied material	41
2.9 Unconventional form of traditional control	43
2.10 Sample lessons on the topic "Right triangle"	44
3. RESULTS	55
CONCLUSION	60
REFERENCES	62
APPENDIX	64

ABSTRACT

This thesis explores the specific features of using trigonometric materials in solving complex-level problems based on the topic of right-angled triangles. The primary goal of the research is to improve the quality of mathematics education in schools by introducing methodologies aimed at developing students' logical and spatial thinking skills. During the study, non-standard problems solvable through the properties of triangles and trigonometric ratios were selected and thoroughly analyzed. The research also identifies the difficulties students face when solving such problems and offers strategies to overcome them. The dissertation includes methodological recommendations for effectively integrating this material into the educational process, along with a set of practical tasks for classroom use. This study is intended for students in general secondary schools.

Key words: right-angled triangle, trigonometric ratios, complex problems, teaching methodology, practical tasks.

АҢДАТПА

Бұл диссертациялық жұмыста тік бұрышты үшбұрыштар тақырыбына негізделген күрделі деңгейдегі есептерді шешуде тригонометриялық материалдарды қолдану ерекшеліктері зерттеледі. Жұмыстың басты мақсаты – оқушылардың логикалық және кеңістіктік ойлау қабілеттерін дамытуға бағытталған әдістемелерді ұсыну арқылы мектеп математикасында білім сапасын арттыру. Зерттеу барысында үшбұрыштың қасиеттері мен тригонометриялық қатынастарды қолдану арқылы шешілетін стандартты емес есептер іріктеліп, олардың шешу жолдары толық талданды. Сонымен қатар, есептерді шешу барысында кездесетін қиындықтарды анықтау және оларды жою жолдары қарастырылды. Диссертациялық жұмыста оқу процесінде осы материалдарды тиімді қолданудың әдістемелік нұсқаулары мен сабақта пайдалануға арналған практикалық тапсырмалар топтастырылған. Бұл зерттеу жұмысы жалпы орта білім беретін мектеп оқушыларына арналған.

Кілт сөздер: тік бұрышты үшбұрыш, тригонометриялық қатынастар, күрделі есептер, оқыту әдістемесі, практикалық тапсырмалар.

АННОТАЦИЯ

В данной диссертационной работе исследуются особенности применения тригонометрического материала при решении задач повышенного уровня сложности, основанных на теме прямоугольных треугольников. Основная цель работы — повысить качество школьного математического образования путём внедрения методик, направленных на развитие логического и пространственного мышления учащихся. В ходе исследования были отобраны нестандартные задачи, решаемые с использованием свойств треугольников и тригонометрических соотношений, и подробно проанализированы методы их решения. Также рассмотрены трудности, возникающие при решении таких задач, и предложены пути их преодоления. В диссертации представлены методические рекомендации по эффективному использованию данного материала в учебном процессе, а также практические задания, предназначенные для использования на уроках. Данное исследование ориентировано на учащихся общеобразовательных школ.

Ключевые слова: прямоугольный треугольник, тригонометрические соотношения, сложные задачи, школьная математика, логическое мышление, методика преподавания, практические задания.

INTRODUCTION

The study of trigonometry occupies a fundamental place in the system of mathematical education and is particularly significant in the context of solving geometric problems involving right triangles. Among the various branches of mathematics, trigonometry serves as a bridge between algebra and geometry, providing tools that allow for precise analysis of angles, lengths, and relationships within triangles. One of the most important applications of trigonometric knowledge occurs in the process of solving complex, multi-step problems related to right triangles, where direct geometric methods may not be sufficient or efficient. The effective use of trigonometric materials—such as sine, cosine, tangent functions, and their inverses—enables students and professionals alike to approach problems with analytical precision and a structured strategy. In the context of secondary and post-secondary education, the development of skills in applying trigonometric identities, solving equations, and interpreting contextual word problems is crucial not only for academic success but also for practical applications in fields such as physics, engineering, architecture, and technology.

However, many students encounter significant difficulties when transitioning from basic triangle properties to solving problems that require layered reasoning and synthesis of multiple concepts. These difficulties often stem from a superficial understanding of trigonometric functions or a lack of connection between theoretical knowledge and practical use. Therefore, there is a growing need to examine the pedagogical strategies and curricular materials that influence students' ability to apply trigonometric principles effectively. By focusing specifically on right triangles—a geometric figure that serves as the foundation of much of trigonometry—this research aims to identify and analyze the key features of using trigonometric materials in educational settings. This includes exploring how students comprehend the relationships between angles and sides, how they use trigonometric ratios in problem-solving, and how instructional approaches can enhance conceptual understanding and procedural fluency. This study is grounded in both theoretical analysis and practical observation, seeking to bridge the gap between mathematical theory and classroom practice. It also examines the cognitive and didactic challenges faced by learners and suggests methodological approaches that may improve the teaching and learning of complex trigonometric problems. Ultimately, the research contributes to the broader goal of mathematics education: equipping students with the tools to think critically, solve real-world problems, and apply mathematical reasoning across disciplines.

The research problem addressed in this study is the development of an effective methodology for teaching students to solve textual (word) problems within the framework of the algebra course in basic school. Despite the widespread recognition of the importance of mathematical modeling in modern mathematics education, the integration of this approach into the teaching of textual problems at the basic school level remains a challenge. Many students experience difficulties in understanding the structure of a word problem, translating the narrative into mathematical language, and constructing appropriate equations to represent real-world scenarios. These challenges

often result from a lack of systematic instructional strategies that bridge theoretical algebraic knowledge with its practical applications. Therefore, this research seeks to provide a pedagogically sound and systematically structured methodology that supports students in developing both conceptual understanding and procedural fluency through mathematical modeling.

The object of the study is the process of teaching mathematics in basic school, specifically the didactic and cognitive mechanisms by which mathematical knowledge is transferred, assimilated, and applied by students. Within this broad educational context, the subject of the study is the use of mathematical modeling as a core tool in the process of solving textual problems in the algebra curriculum of basic school. This includes the analysis of how mathematical models are constructed from word problems, how they are used to represent quantitative relationships, and how students interpret, manipulate, and validate these models in the problem-solving process.

The purpose of the study is to identify and define the methodological features and pedagogical approaches that can effectively support students in solving textual problems by means of mathematical modeling within the course of algebra at the basic school level. This involves both theoretical investigation and practical experimentation aimed at improving instructional strategies and enhancing student learning outcomes.

To achieve this purpose, the following research objectives are formulated:

- To analyze the concepts of mathematical modeling and textual (word) problems in the context of the algebra course at the basic school level;
- To explore existing approaches to teaching students to solve word problems and identify the limitations and gaps in current practices;
- To design a methodology for integrating mathematical modeling into the teaching of algebraic word problems, emphasizing step-by-step model construction, interpretation, and validation;
- To develop instructional materials and teaching tools that facilitate the application of modeling techniques in classroom settings;
- To conduct an experimental study to evaluate the effectiveness of the proposed methodology in improving students' problem-solving abilities;
- To provide methodological recommendations for mathematics teachers regarding the incorporation of mathematical modeling into regular classroom instruction.

To achieve the stated research objectives and ensure the reliability of the results, a set of complementary research methods was employed. These methods were selected to align with the theoretical and practical aims of the study and to provide a multidimensional perspective on the problem of teaching students to solve textual problems through mathematical modeling. First, a comprehensive analysis of pedagogical and methodological literature was conducted. This included reviewing scientific publications, academic dissertations, and educational guidelines that address the role of mathematical modeling in school mathematics, as well as its integration into the teaching of algebra and word problems. In addition, school programs and official curricular documents were examined to determine the scope and sequence in which word problems are introduced and addressed in basic school mathematics education.

The study also involved a close investigation of the practical experience of domestic mathematics teachers who have implemented modeling-based strategies in the classroom. By studying case examples, reflective reports, and published teaching methods, the research was able to incorporate practical insights and pedagogical innovations from current school practice. A comparative analysis of mathematics textbooks and teaching aids was also carried out to identify differences in the presentation, structure, and complexity of textual problems, as well as the degree to which these materials promote or hinder the development of modeling skills among students. Finally, the research involved the generalization and systematization of the collected material in order to construct a coherent methodological framework that can inform both instructional design and pedagogical practice.

The theoretical significance of the study lies in its reexamination of the role of the textual problem within the structure of school mathematics and its reinterpretation as a powerful tool for applying mathematical modeling. Rather than viewing word problems as isolated exercises, the study presents them as vehicles for developing deeper understanding of mathematical relationships, fostering abstract thinking, and bridging the gap between formal mathematical language and real-life situations. By positioning the textual problem at the intersection of applied mathematics and pedagogy, the research contributes to the theoretical discourse on how modeling can enhance mathematical thinking and support curriculum development.

The practical significance of the study is reflected in the development of concrete methodological materials designed for use in the basic school setting. These materials include modeling-based instructional strategies, lesson plans, sample problems with guided modeling steps, and assessment tools that can be implemented by mathematics teachers in their day-to-day practice. Moreover, the study offers a valuable resource for pre-service teachers and students of pedagogical specialties, who can utilize the proposed materials during their teaching internships and professional development activities. The practical recommendations presented in this study aim to enrich the methodological repertoire of teachers and help create a learning environment where students are encouraged to explore, reason, and apply mathematics in meaningful ways. Ultimately, the study offers both a conceptual and instructional foundation for improving the quality of mathematical education in basic schools through the integration of modeling into the teaching of algebraic word problems.

1. LITERATURE REVIEW

1.1 The concept of mathematical modeling

In this paragraph, we present definitions of such concepts as "model", "mathematical model", as well as consider the concept of mathematical modeling. In methodological literature there is no unambiguous understanding of the model and modeling process.

Gives the definition of a model in a broad sense: "Two systems of objects A and B are called models of each other (or modeling one another) if it is possible to establish such a homomorphic mapping of system A to some system A and a homomorphic mapping of B to some system B that A and B are isomorphic" (Chang & Keisler, 1990).

Since this interpretation has a broad meaning, it is necessary to emphasize the following model criteria:

- Object M is designed (or selected) according to a specific purpose. It is possible to construct different models for different purposes of one object.
- The model should reflect the properties of the object that are essential for the purpose of its construction.
- A model is always material. In the case when a model is a product of thinking, it can be materialized.
- The model allows for other interpretations, particularly in other branches of science.

A model is a research model if the purpose of its construction is to obtain new information about the object. Mathematical model is a research model of some object expressed by means of mathematics.

The approaches available in the literature to the concept of mathematical model allow us to give the following definition: "Mathematical model is an image of the original, expressed with the help of mathematical symbols (mathematical language) and allowing the properties of the object - prototype, its parameters, internal and external relations to describe in quantitative form, with the help of logical and mathematical constructions" (Anguelov, 2020a; Law & Kelton, 2015)

Models are divided into two groups according to the types of means used to construct them: iconic and schematized (Figure 1.1.1).

Sign models of textual problems, executed in mathematical language, are called solving models, because they are used to solve the problem. All other models are auxiliary models, by means of which the transition from the text of the problem to its mathematical model (Meyer, 2021)

Figure 1.1.1 Types of mathematical models

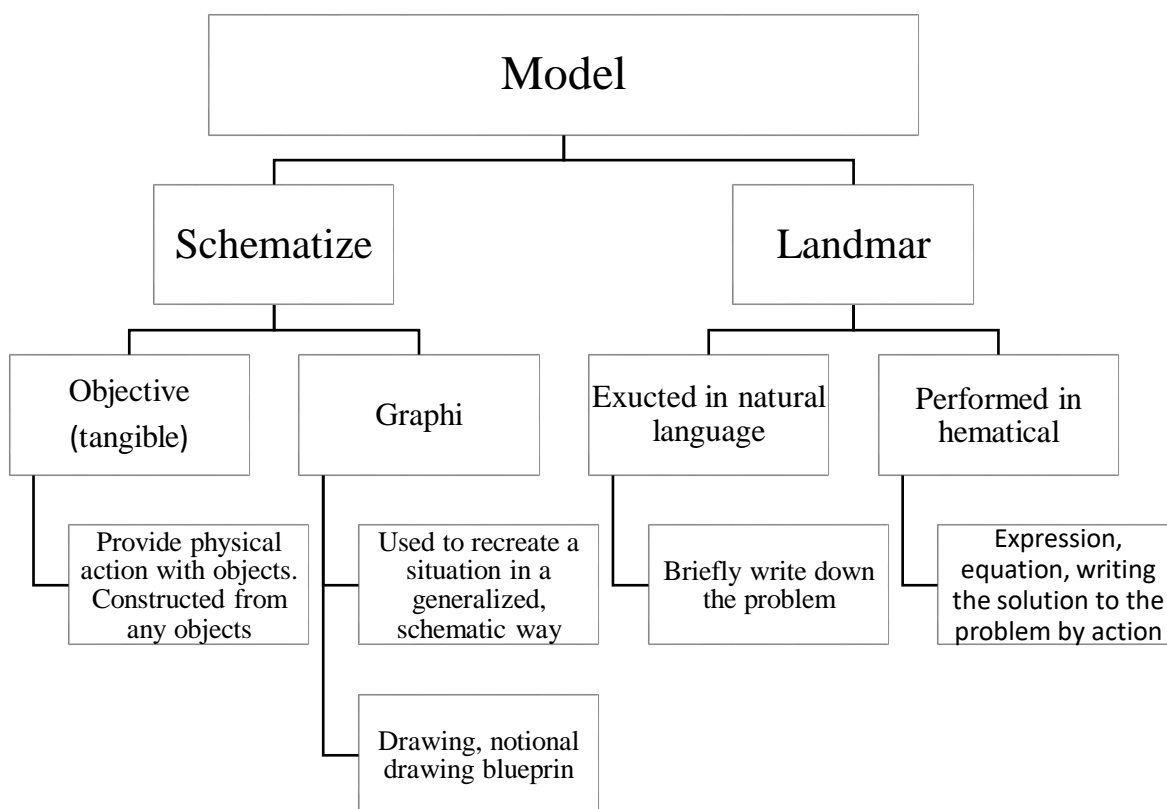


Table 1.1.1 Types of models of the textual task

Text task	Brief note	Decisive model
A model of a real situation, with the problem condition highlighting the properties of reality objects that are necessary to fulfill the requirement.	Model the textual task if: <ul style="list-style-type: none"> • it is necessary to solve the problem and helps the student; • it reflects all the properties of objects (essential for answering the question of the given task), relations between the objects referred to in the task, as well as the requirement of the task. 	An exploratory mathematical model that includes the following: - description of the components that make up the mathematical object (interpretive component); Mathematical object (formalization in mathematical language that includes connections and relations, according to the problem condition); mathematical object, which includes the formalization of the problem requirement (meaningful component).

Let us consider this classification from a different perspective. E.M. Lozhkina (*Geometriya 7th Grade: Lesson Plans Based on L. S. Atanasyan's Textbook*, 2004) cites the following main types of models that students encounter when solving a textual problem: the text of the problem; auxiliary models (drawing, table, diagram, etc.); solving models.

Based on the definition of the research model and on the above model criteria, some explanations are provided in the table (Table 1).

Thus, considering the process of learning to solve textual problems, we come to two directions in which models are used:

1. As a means of facilitating understanding and comprehension of the task (auxiliary models);
2. as a means of research activity (equations, inequalities, etc.).

When solving training tasks, distinguishes four main types of models (Anguelov, 2020b):

- logical and mathematical models (B generally in the general case the algebra of statements or predicates is the carrier of the model);
- analytical models (processes of real objects are written down in the form of explicit functional dependencies);
- geometric models (media data carriers models are geometric objects);
- stochastic models (statistical processing, data analysis, probabilistic characteristics of random events);
- mixed model type.

When choosing a particular method, the problem can be solved with the help of different models. Thus, when using the algebraic method, by making and solving absolutely different equations it is possible to get an answer to the question of the same problem (Aris, 1994).

Modeling is traditionally understood as the replacement of some object A (the original) with another available object M (model) in order to study the properties of the original. The modeling technique consists in the fact that studying a certain object, one builds such an object (model), the study and solution of which allows to reveal new information, and the result of the study is transferred to the original object (Meyer, 2021).

Also, mathematical modeling can be considered as a process in which consists of a sequence of tasks performed in order to obtain a mathematical representation of the real world (Anguelov, 2020a).

Selena Oswalt (2012) considers mathematical modeling as a method in which students apply mathematical knowledge learned earlier to new and unfamiliar situations.

Thus, based on the above, under mathematical modeling we will understand the description of any real process or phenomenon in mathematical language.

1.2 The role of mathematical modeling in solving word problems

Mathematical modeling plays a crucial role in solving word problems, particularly in the context of school algebra, where students are required not only to perform operations but also to understand and translate real-life or hypothetical situations into mathematical language. Word problems often present students with complex, multi-step scenarios that require careful analysis, abstraction, and application of appropriate mathematical tools. In this regard, mathematical modeling serves as a structured and logical bridge between the context of the problem and its mathematical solution. Through modeling, students learn to identify essential quantities, define relationships between them, and construct equations or systems that accurately represent the given situation. This process not only reinforces their understanding of algebraic concepts but also develops their ability to think analytically and make informed decisions based on available data.

The role of modeling in solving word problems is not limited to the mechanical application of formulas; rather, it involves a cognitive transformation of verbal information into symbolic representation. This requires the development of skills such as identifying known and unknown variables, interpreting conditions and constraints, and evaluating the reasonableness of solutions. When mathematical modeling is integrated into problem-solving instruction, it enhances students' ability to dissect problems into meaningful components and understand the underlying mathematical structure. Furthermore, it encourages metacognitive thinking, as students must justify their modeling choices, consider alternative representations, and reflect on the validity and applicability of their results.

In educational practice, modeling provides a context-rich approach that helps students see the relevance of mathematics beyond the classroom. It allows them to approach problems systematically, using real-world logic, and to connect abstract algebraic procedures with concrete phenomena. For example, when solving a problem involving the cost of products, travel distances, or rates of work, students can construct and manipulate linear equations or inequalities that reflect real conditions. This connection between mathematics and everyday life not only increases student motivation but also equips them with tools for critical thinking and problem-solving in practical situations.

Moreover, modeling promotes a deeper engagement with mathematics because it requires iterative thinking. Students often need to revise their models, reconsider assumptions, and refine equations to arrive at accurate solutions. This dynamic process mirrors the nature of mathematical inquiry and scientific reasoning, making it a valuable skill for further education and career readiness. It also aligns with the competencies emphasized in international educational standards, such as mathematical literacy, adaptability, and the ability to apply knowledge in novel situations.

In conclusion, mathematical modeling is a foundational component in the effective teaching and learning of word problem solving. It transforms passive problem-solving into an active, meaningful process that builds students' mathematical reasoning and prepares them for more advanced studies and real-life challenges. By fostering the ability to analyze, represent, and solve contextual problems, modeling

serves as both a didactic strategy and a cognitive tool that significantly enriches the learning experience in school mathematics. In teaching mathematics to schoolchildren, solving text problems takes a great place. When solving text problems, students not only apply their mathematical knowledge and skills that they have already acquired, but also improve their abilities (Muhammad & Fauzi, 2018). Let's determine the importance of text problems in the school course of mathematics. Solving text problems in a high school math course promotes:

- Development of students' logical thinking,
- The development of the idea of functional dependence,
- Improving computing culture.

A.V. Shevkin (2025) defines the role of text tasks in the course of mathematics as follows:

- Text problems are one of the most important means of teaching mathematics. In the process of learning to solve text problems, students realize interrelationships and relations between quantities, gain experience of working with them, and solve practical problems.

Using the arithmetic method of problem solving, students develop ingenuity, wit, that is, they develop natural language.

Arithmetic methods of solving text problems allow formulating and developing important general learning skills such as: analyzing task situations, drawing up a solution plan (taking into account the relationship between known and unknown values), explaining the results of actions within the problem condition, checking the correctness of the solution in various ways.

In the textbook edited by V.I. Mishin (*Geometriya 7th Grade: Lesson Plans Based on L. S. Atanasyan's Textbook*, 2004) it is written that problem solving forms the following general learning skills in pupils: the ability to carefully perceive educational information, motivate Each step of their activity and be able to plan it, rationally formalize the results of actions, self-control and verification.

Kapa (2001) and Muhammad & Fauzi (2018) consider mathematical modeling as a condition for the development of students' logical thinking. This article states that students' conscious problem solving and the development of their logical thinking are ensured through the formation of skills to build models of textual problems. It also indicates the necessity of improving the methodology of teaching schoolchildren to solve text problems, which contribute to the formation of generalized intellectual skills: to analyze and draw conclusions, to establish links between a particular object and other objects, to determine the essential features of the object. Mathematical modeling of text problems is of special importance, because with the help of this method pupils can see the essence of mathematical relations contained in various situations hidden in subject areas. L.A. Mamykina writes that the method of mathematical modeling used in the study of mathematics contributes to the systematization of knowledge, allows finding applications of the applied orientation of the course of mathematics for a better understanding of the processes of the modern world and the essence of scientific theories ("Mathematics at School," 2001). Note that in the process of solving textual

problems by the method of mathematical modeling students establish interdisciplinary links with other disciplines.

In the second chapter of the presented work, when considering examples of text problems, we will take into account the following general learning skills of schoolchildren, formed in the mathematical modeling of text problem solving: planning their own activities; perception and analysis of educational information; motivation at each stage of their own activities; competent presentation of the obtained results; exercising control over one's own activity.

1.3 Classification of word problems

The classification of word problems is an essential component in the methodology of teaching mathematics, as it allows educators to organize problem types according to their structural, logical, and contextual characteristics. This, in turn, facilitates the selection of appropriate teaching strategies and helps students develop effective problem-solving skills. Word problems can vary widely in form and complexity, but they all share a common feature: the requirement to translate a verbal or contextual narrative into mathematical representation in order to find a solution. A well-structured classification provides a pedagogical framework for analyzing the types of cognitive operations involved, the mathematical models required, and the types of reasoning necessary for successful completion.

One of the most basic distinctions is between simple and complex word problems. Simple problems typically involve one mathematical operation and a direct question, making them accessible for learners at initial stages of mathematical instruction. In contrast, complex or multi-step problems may involve several operations, require intermediate calculations, and necessitate logical sequencing and the formulation of systems of equations. Another common basis for classification is the type of mathematical relationship represented in the problem. This includes problems involving arithmetic relationships (addition, subtraction, multiplication, division), proportional reasoning (ratios, percentages, rates), algebraic patterns (equations, expressions, inequalities), geometric concepts (area, perimeter, angles), or statistical analysis (averages, distributions).

A more detailed classification considers logical structure and problem-solving strategy. For instance, problems can be classified as "change problems" where quantities increase or decrease over time; "comparison problems" where two or more quantities are evaluated relative to each other; "equalizing problems" where balance or equivalence is achieved; and "part-whole problems" that involve combining or partitioning sets. Each of these categories requires different interpretive and modeling strategies from students. Additionally, word problems may be contextualized in everyday scenarios, such as shopping, travel, or work tasks, or they may be abstract and symbolic, focusing solely on mathematical relationships without a real-life frame.

From a methodological perspective, classification also includes problems by their level of openness. Closed problems have one correct answer and a defined solution path, while open-ended problems may allow for multiple solutions or modeling approaches. This distinction is particularly important in developing

creativity, critical thinking, and flexible problem-solving strategies. Furthermore, classification may consider whether a problem is "well-structured," where all data is clearly provided and the goal is explicitly defined, or "ill-structured," where data may be incomplete, and students must make assumptions or decide which information is relevant.

In educational settings, using a classification system helps teachers scaffold instruction, beginning with simpler problems and gradually introducing more complex or abstract forms as students' skills develop. It also assists in diagnosing student difficulties and in designing differentiated instruction to meet diverse learning needs. For students, understanding the types of word problems they may encounter enhances their ability to recognize patterns, select appropriate strategies, and develop confidence in approaching unfamiliar tasks. In conclusion, the classification of word problems serves both theoretical and practical functions in mathematics education. It contributes to curriculum design, instructional planning, and assessment development, while also supporting learners in building systematic approaches to problem-solving. A well-conceived classification system empowers both teachers and students by providing clarity, structure, and insight into the varied and often complex world of mathematical word problems. Let us consider different approaches to the concept and definition of a text (story) task. The first approach is related to the concept of a textual task as some description of a real situation (Iakovleva, 2020). E. Iakovleva, believe that: "A textual task is a description of some situation in natural language with the requirement to give a quantitative characteristic of some component of this situation, to establish the presence and absence of some relation between its components or to determine the type of this description" (Selena Oswalt, 2012). To obtain this model, we can reformulate the problem, build its graphical model, and introduce appropriate notations.

From the point of view of L.M. Friedman (Friedman et al., 2010), a task is a requirement and a question to which it is necessary to find an answer, relying on and taking into account those conditions specified in the task.

(Friedman et al., 2010) writes that in order to teach students to independently solve non-standard problems, to develop a general approach to solving any problems, to form the ability to reasonably search for a way to solve problems, it is necessary to give students elementary knowledge of the theory of problems along with problem solving during the whole training (Friedman et al., 2010; ЯКОВЛЕВА, 2020).

The second approach is related to the structure of the task (Giordano et al., 2014).

According Oganesyanyan et al., (1980) understands a text (story) task as follows: "A story task is a task in which the data and the relationship between them are included in the fabula".

Constituents of the task:

- The condition is what is known in the problem.
- The question is what is required to know in the task.
- The solution is to perform the actions.

1.4 The answer is the result of the actions received

Oganesyan et al., (1980) writes that the content of a textual task in most cases represents a certain situation close to life. Such tasks are mainly important for the formation of students' ability to represent mathematical relations for mastering the modeling method, for the development of interest in mathematics.

R. Aris (1994) writes that a mathematical model is at a higher level of abstraction than the textual task to which it corresponds. Textual tasks of different specific content can have the same logical and mathematical model. Let us give an example of the following two tasks:

Two typists retyped a manuscript in a hours. In how much time could each typist, working alone, retype the manuscript if the first typist spent b hours more than the second typist?

Two pumps fill a swimming pool in a hours. In how much time could each pump, working alone, fill the pool if the first pump takes b hours longer to fill the pool?
- corresponds to the same logical and mathematical model:

$$\frac{1}{x} + \frac{1}{x+b} = \frac{1}{a}$$

The construction of a mathematical model is performed after translating the conditions of both problems into the language of this model.

- In the first case: x is the time taken by the first machine;
- in the second case: x is the time taken by one first pump.

Material problems have common mathematical characteristics. This approach is based on the formation of mathematical models based on text problems (where x is the working time of one typist/pump). This approach to modeling implies the construction of formal equations and solution paths from text data.

According to contemporary research (Krawitz, Chang, Yang et al., 2021), understanding text is the initial stage in mathematical modeling, because if students have a good understanding of the semantic structure of the text, it will be easier to build a model. In this regard, increasing students' comprehension of text problems in the context of learning should be supported by the introduction of modular tools and preliminary explanation questions. Such an approach contributes to the development of their modular competence.

In a study conducted by Jupri & Drijvers (2016), it was found that the difficulties in creating mathematical models in text problems are significant. In particular, errors occurred during the equation generation stage and analytical shortcomings were identified in the graphical or symbolic explanations. This indicates the need to systematize the modeling process.

In general, the following components are important in teaching mathematical modeling:

- Semantic analysis: Interpreting the content of the text, identifying key parameters. This stage is evidenced by the research of Krawitz et al. (2021).
- Mathematical description: Selecting notations such as X and creating related equations.

- Solution and interpretation: Solving the resulting equations and analyzing the results in a real-life context.
- Verification: Checking the protocols and assessing the validity of the results.

Also, researchers such as Blomhoj and Kjeldsen (2006) and Blum (2002, 2015) consider modeling as a key element of the learning process. They emphasize that modeling determines the ability to analyze the domain, abstract and apply it to a specific context.

This approach does not allow children to model until they fully master the content of the problem, until they delve into its givenness, rather than relying on traditional formulas. Therefore, it is a good idea to introduce modular explanations into didactics and lesson plans. This method allows for the adaptation of differentiated instruction among students of equal abilities.

The modeling approach proposed by the Minnesota STEM Teacher Center (Alexander et al., 2023) emphasizes that the transition from verbal situations to abstract concepts is an important stage of the transition. The typists and pump problems you gave as examples are calculated using this same model formalization.

Based on a different designation of the variables, a different mathematical model could be obtained,

Thus, under a textual task we will understand a description of some real situation or phenomenon in natural language with a requirement and a question to which it is necessary to find an answer, given the conditions of the task.

1.5 Improving students skills through mathematical modeling

Improving students' skills through mathematical modeling is a critical objective in contemporary mathematics education, as it fosters not only computational proficiency but also analytical thinking, problem-solving ability, and real-world application of knowledge. Mathematical modeling encourages students to engage with mathematics in a meaningful and purposeful way by requiring them to interpret a real-life or theoretical scenario, identify relevant variables, establish relationships between these variables, and construct a mathematical representation—typically an equation, inequality, graph, or function—that reflects the structure of the given situation. This process inherently promotes the development of higher-order thinking skills, such as abstraction, generalization, evaluation, and validation of results.

The modeling process goes far beyond rote application of formulas. It nurtures an exploratory mindset in students, where they must analyze the problem context, make justified assumptions, and choose appropriate mathematical tools. This transforms students from passive recipients of knowledge into active participants in the problem-solving process. As students engage with modeling tasks, they become more adept at interpreting data, forming logical connections, and recognizing underlying mathematical principles within various contexts, such as finance, geometry, motion, or environmental phenomena. These experiences help develop mathematical literacy and the ability to transfer skills across disciplines, which is essential in science, economics, and everyday life.

Moreover, mathematical modeling provides a context for collaborative learning

and communication, as students often work in groups to interpret problems and test models. In this setting, they learn to articulate their reasoning, critique the ideas of others, and refine their thinking based on feedback—skills that are critical not only in academic environments but also in professional and social contexts. As students iterate through the modeling cycle—formulating, solving, interpreting, and validating—they develop a more nuanced understanding of the limitations and scope of mathematical solutions. This reflective element strengthens their metacognitive skills and deepens their conceptual understanding.

Another key benefit of using modeling to improve student skills is that it bridges the gap between theoretical knowledge and practical application. Many students struggle to see the relevance of algebra or functions when taught in isolation. However, when these concepts are used to model real situations—such as predicting population growth, calculating the cost of goods, or analyzing speed-time relationships—students begin to perceive mathematics as a useful and powerful tool. This relevance increases motivation, engagement, and persistence when faced with challenging tasks.

Mathematical modeling also supports differentiated instruction. Because modeling problems can be designed with multiple entry points and levels of complexity, they allow teachers to address a wide range of abilities and learning styles. More advanced students may explore multiple solutions or test various assumptions, while others may focus on constructing a basic model and interpreting its meaning. This flexibility promotes inclusiveness and ensures that all students are developing skills at an appropriate pace.

In conclusion, mathematical modeling serves as a catalyst for skill development in mathematics by providing a dynamic, interactive, and contextualized approach to learning. It enhances students' abilities to reason quantitatively, communicate mathematically, and apply their knowledge to unfamiliar and meaningful problems. As such, incorporating modeling into the mathematics curriculum is essential for preparing students not only for academic success but also for the demands of the modern world, where critical thinking, adaptability, and applied reasoning are indispensably.

Enhancing students skills via mathematical modeling remains a crucial component of modern mathematics education. This approach supports not only students mastery of computation but also nurtures their analytical thinking, problem-solving strategies and real-world reasoning abilities. Through modeling, students are encouraged to interpret real-life or hypothetical scenarios, identify key variables and establish mathematical relationships between them. These relationships are then represented using mathematical structures such as equations, graphs or functions, which reflect the original problem's framework. This practice contributes significantly to the cultivation of higher-order thinking skills, including abstraction, generalization, critical evaluation and the validation of results.

Unlike repetitive application of formulas, mathematical modeling involves a deeper engagement with the problem. Students are required to investigate context, justify assumptions and select suitable mathematical methods. This transforms learners from passive knowledge receivers into active problem solvers. As they engage with modeling

based tasks, students improve their ability to interpret data, establish logical connections and recognize fundamental mathematical principles across various domains like geometry, economics, physics or environmental studies. Their learning experiences foster mathematical literacy and support knowledge transfer across subjects which is vital in real world professions.

Text problems related to right-angled triangles are an important part of school mathematics. They teach students to apply theoretical knowledge in practice and develop mathematical thinking. Especially in the trigonometry section, to solve such problems, it is necessary to construct a geometric figure and connect the given quantities using trigonometric functions. This topic is also relevant in the modern education system. As the results of Kazakhstani research and foreign literature show, mastering trigonometric materials and using it through modeling in solving complex problems in right-angled triangles is one of the most effective ways to learn. This literature review examines the theoretical foundations of teaching trigonometry in the school mathematics course, pedagogical aspects of text problem solving methods, the method of mathematical modeling, the importance of using trigonometric functions in practice, and compares Kazakhstani and foreign studies.

Theoretical foundations: In the school curriculum, trigonometry is taught in grades VIII–X. In Kazakh schools, trigonometry begins in the VIII grade in the geometry course with the introduction of functions of an acute angle in a right triangle, and then in grades IX–X, the topics of solving functions and triangles are taught. Trigonometric functions are considered as a key concept connecting algebra and geometry during the lesson. For example, as Adilkhanova (2020) noted, trigonometric functions describe the relationship between the sides and angles of triangles, allowing students to understand the concept of a function more deeply.

The history of trigonometry teaching methodology has considered various approaches: in some textbooks, sine, cosine, tangent are first introduced for the acute angles of a right triangle, and then these concepts continue from 0° to 180° . The structure of the teaching material is also changing: if earlier trigonometry was taught as a separate subject, now it is integrated into the courses of geometry and algebra, and in elementary courses (for example, through the study of the heritage of Al-Farabi) attention is paid to mastering historical and basic concepts. For example, Kazakh researchers consider the introduction of Al-Farabi's algorithm for calculating sine and creating tables in the school curriculum to increase the scope and historical and cultural significance of trigonometry.

In addition, as suggested by A.G. Mordkovich, it is necessary to use the "digital circle" model in teaching trigonometry at the beginning, and work with trigonometric transformations should be the main thing. This opinion is widespread in teaching methodologies - that is, it calls for paying more attention to visual modeling and explaining their meaning than to memorizing traditional formulas.

Pedagogical aspects of solving word problems: Word problems are a key component of the learning process and an indicator of students' mathematical readiness. They connect theory with practice, creating a bridge between learning and life. Mathematics teachers can develop students' thinking skills such as analysis, synthesis,

abstraction, and comparison through word problems. However, as practice shows, students often have difficulty understanding the condition of the problem, extracting the necessary information, and creating a solution plan. Foreign studies also show the same thing: for example, as Lewis (2025) notes, the complexity of trigonometric word problems requires students to have spatial-associative thinking, and many initially lack these skills. For this reason, pedagogical methods focus on teaching students the logical steps in solving problems and visualizing problems.

For effective solving of word problems, it is recommended to use the modeling method. This approach emphasizes structuring problems by capturing them in diagrams, schemes, or symbolic expressions. The student depicts a given life situation in the form of a picture and inserts it into a mathematical model - a right triangle figure. For example, in problems of height and distance, he finds the ratio of the legs of a triangle to the hypotenuse, and uses the sine or tangent formula.

Thus, graphically representing the situation of the problem makes it easier to determine the necessary quantities and helps to systematically plan its solution. Predictive modeling - as the first "step", determines the physical state of the problem, and in subsequent stages transforms the model into a mathematical expression.

In the pedagogical literature, methods of formulating and transforming problems are widely discussed. Many authors indicate the need to teach students not only to solve given problems, but also to formulate new problems themselves and transform given problems. For example, mathematics teachers require students to write a simple "word" according to the content of the problem, create problems from it that involve two actions, and then compare the models of these problems. This approach is consistent with the practice of solving problems in the natural sciences and increases students' creativity.

Mathematical modeling method: Current research considers modeling as an effective method for solving problems. Mathematical modeling is the process of translating a real-life phenomenon into mathematical language. By teaching this method, students identify mathematical dependencies in a given problem and express them in the form of equations and functions. The level of mastery of modeling is the main condition for successful problem solving. It is carried out in stages: it is necessary to create a graphic or semantic model of the problem, and then convert it into a mathematical formula.

The modeling method is used at different levels of education. In higher grades, it is customary to build a real triangle model to solve trigonometric problems. The principles of mathematical literacy included in the modern school curriculum are also based on the concept of modeling problems. In this regard, Leskaleva and Yerkisheva, (2025) consider modeling as one of the effective ways of teaching, showing that it increases interest in solving applied problems and teaches to apply knowledge in practice. As they noted, the stages of modeling learning are explained in a simple diagram, which makes it easier for students to systematically solve practical problems.

As an example in practice, if a text problem is given regarding the calculation of height or distance, the student first represents the problem in the form of a right triangle. The corresponding ratio of the leg to the angle observed along the hypotenuse is

determined by the sine or cosine. Thus, through modeling, equations are created from the given data and the unknown quantity is found. Such steps teach the student to apply the formula to a real problem situation instead of memorizing it mechanically.

The importance of using trigonometric functions in practice: The practical application of trigonometric functions is very important in terms of connecting the educational material with life. In modern science and technology, functions such as sine and cosine play a huge role. For example, in astronomy they are used to calculate the distance to stars, in geodesy to measure distances on the Earth's surface, and to control the navigation of satellites. In addition, the principles of trigonometry are widely used in medicine (modeling of frequency processes, heartbeat biorhythm), physics (wave motion, repetition processes), architecture and engineering (planimetric design, construction and calculation of the strength of structures). As Baltabaeva (2020) notes, trigonometry is found in many areas: from music theory to biology and architecture.

Foreign studies also confirm this. Asamoah (2025) notes, trigonometry is a crucial tool in engineering, physics, architecture, and geodesy. Mastering trigonometric concepts helps students develop spatial awareness and analytical thinking. For example, a civil engineer often uses trigonometric formulas when calculating the diagonals of buildings, and a navigator often uses trigonometric formulas when tracking the movement of planets. Teaching such problems also increases practical interest in mathematics among Kazakhstani teachers and students: it is shown that knowledge is not only theoretical, but also applied in real life.

Comparison of Kazakh and foreign studies: Kazakh and foreign studies show a number of similarities in the issues of teaching trigonometry. Both sides use the modeling method as the dominant method and attach importance to connecting knowledge with practice. For example, Leskaleva and Yerkisheva (2025) in their study highlighted the importance of mathematical modeling when solving school problems and considered it as a method that increases students' interest. Foreign scholars also support this method, considering it effective in translating mathematical concepts into real-life situations.

On the other hand, there are also differences that should be noted. Kazakhstani studies are often tied to national curricula and historical and cultural contexts (e.g., the legacy of Al-Farabi, teaching methods at school). In foreign works (Asamoah, 2025) trigonometry is widely associated with STEM subjects, engineering and physical applications. In addition, foreign scholars pay attention to the cognitive aspects of students: for example, Seidimbek (2023) points out the need to develop students' spatial thinking skills. This is also relevant in Kazakhstan - Kazakh researchers suggest that in teaching, it is necessary to delve deeper into the meaning, not just memorize formulas.

Thus, studies by Kazakh and foreign authors jointly emphasize the importance of modeling and applied nature in teaching trigonometry. However, while Kazakhstanis often focus on the teaching methodology of the subject, foreign studies more broadly study the cognitive difficulties and practical applications of students.

Conclusion: Based on the studies discussed above, the following conclusions can be drawn. Teaching trigonometry in the school curriculum is a fundamental direction that combines algebra and geometry and allows students to apply their knowledge of mathematics to solving real-life tasks. The use of the modeling method when solving text problems teaches students to engage in creative work and understand mathematical concepts in a practical way. In addition, trigonometric functions are widely used in various fields of science and technology, so their teaching is of great practical importance. As shown by Kazakhstani studies and foreign experience, it is effective to focus not only on theoretical preparation in the teaching process, but also on methods of explaining problems through modeling and practical application. In the future, it will be relevant to develop new methods of teaching trigonometric text problems by combining the modeling method and information and communication technologies to improve teaching methodology.

2. METHODOLOGY

2.1 Difficulties in solving word problems

The methodology of this study is based on a comprehensive combination of theoretical analysis and practical exploration aimed at developing an effective and pedagogically sound approach to teaching students to solve textual problems through mathematical modeling within the framework of basic school algebra. The methodological foundation of the research incorporates both qualitative and comparative methods that allow for the analysis of existing pedagogical practices, evaluation of current educational resources, and development of new instructional strategies. The study began with an extensive review of pedagogical, psychological, and methodological literature related to mathematical modeling, word problem solving, and algebra instruction. This theoretical analysis served to clarify core concepts, such as the definition and structure of mathematical models, the cognitive demands of textual problems, and the role of modeling in bridging abstract mathematical reasoning with real-world contexts.

In addition to theoretical inquiry, the study involved the examination of national and regional curricula to determine how textual problems are introduced, sequenced, and assessed within the school mathematics program. Special attention was given to the standards for mathematical literacy and the expected learning outcomes at the basic school level. To gain insights into classroom realities, the experience of practicing mathematics teachers was analyzed through case studies, teaching journals, and published resources. This helped identify common instructional challenges, student misconceptions, and successful strategies already employed in practice.

A comparative content analysis of various mathematics textbooks and teaching aids was conducted to evaluate how modeling elements are integrated into the structure of word problems. This included identifying the types of problems presented, the presence of step-by-step modeling guidance, and the alignment between textbook tasks and real-life situations. The findings from this analysis provided a basis for designing the experimental component of the study, which involved the development of a new methodology focused on improving students' modeling competencies through structured teaching interventions.

The practical component of the research involved the creation of a set of instructional materials, including problem templates, modeling frameworks, and scaffolding tools, which were tested in selected basic school classrooms. These materials were designed to support students in identifying key information, selecting relevant variables, and formulating equations that accurately represent problem conditions. The methodology emphasized an iterative learning process, encouraging students to reflect on their reasoning, adjust their models as needed, and interpret results in context. The effectiveness of the proposed methodology was evaluated through observation, student performance analysis, and feedback from both learners and teachers. Throughout the research, data were collected through classroom observation, written student work, and semi-structured interviews with educators. The

data were analyzed qualitatively to identify patterns in student understanding, typical errors, and growth in problem-solving ability. The synthesis of these results informed the refinement of the methodology and led to a set of practical recommendations for teachers and curriculum developers.

In summary, the methodology of this study combines theoretical research, curriculum analysis, textbook comparison, and classroom experimentation to construct a comprehensive approach to teaching word problems through mathematical modeling. It is grounded in real classroom needs and aligns with contemporary educational goals, ensuring its relevance and applicability in the context of basic school mathematics instruction.

2.2 Organization of the experimental study

The organization of the experimental study was carried out in accordance with the objectives of the research and aimed at validating the effectiveness of the proposed methodology for teaching students to solve textual problems using mathematical modeling in the context of basic school algebra. The experimental work was designed as a pedagogical intervention implemented under natural learning conditions, ensuring that the findings would be applicable and relevant to actual classroom practice. The study was conducted in several stages, each with its own specific goals and procedures, beginning with the preparatory phase, which involved the selection of participants, educational institutions, and teaching materials.

Participants in the experimental study included students from the basic school level (grades 7–9), mathematics teachers, and student-teachers engaged in pedagogical practice. The sample was divided into two groups: an experimental group, which was taught using the newly developed modeling-based methodology, and a control group, which continued learning according to the standard curriculum without additional methodological adjustments. Both groups were taught the same algebraic content over a comparable time period to ensure consistency in subject matter exposure. Prior to the intervention, diagnostic testing was administered to assess the initial level of students' ability to solve textual problems and apply mathematical modeling techniques. This allowed for a comparative analysis of the students' baseline skills and ensured the groups were adequately balanced.

The core of the experimental phase involved the implementation of a series of lessons in the experimental group that were specifically designed to emphasize modeling in the solution of textual problems. These lessons incorporated tasks that required students to analyze real-life scenarios, identify relevant quantities and relationships, and translate verbal descriptions into mathematical models such as equations or expressions. A structured, step-by-step approach was applied to help students move through the stages of the modeling process: understanding the context, simplifying assumptions, choosing variables, building a model, solving it, and interpreting the solution. Teachers in the experimental group received methodological guidelines and instructional support to ensure fidelity to the research design and consistency across classrooms.

Throughout the implementation phase, qualitative and quantitative data were collected through observation, analysis of student work, teacher reflections, and interim assessments. Special attention was paid to students' problem-solving strategies, the types of errors they made, and the degree of independence and reasoning exhibited in their modeling efforts. Interim results were used to adjust the instructional strategies when necessary, ensuring that the methodology was responsive to the learners' needs.

At the end of the experimental cycle, post-testing was conducted using a set of textual problems that required the application of modeling skills. The results from the post-tests were compared with those of the initial diagnostic assessments and also with the control group's outcomes. This comparative analysis allowed the researchers to evaluate the progress made by students in the experimental group, both in terms of accuracy and the quality of reasoning, model construction, and interpretation. The data were statistically processed to determine the significance of the observed changes, while qualitative data provided insight into the cognitive and metacognitive development of the learners.

In conclusion, the experimental study was carefully organized to ensure methodological rigor, educational relevance, and practical applicability. It combined traditional assessment tools with classroom-based observation and instructional analysis to validate the effectiveness of integrating mathematical modeling into the teaching of textual problems. The findings from this experiment served as a critical foundation for the formulation of recommendations aimed at improving mathematics instruction in basic school settings.

2.3 Common issues faced while working with verbal mathematical tasks

Difficulties in solving word problems remain one of the most persistent and widespread challenges faced by students in mathematics education, particularly in the context of algebra at the basic school level. These difficulties stem from a combination of cognitive, linguistic, and methodological factors that interfere with students' ability to comprehend, analyze, and mathematically represent the problem situation. One of the most fundamental challenges is the process of translating verbal information into mathematical expressions or equations. Many students struggle to distinguish between relevant and irrelevant data in a word problem, leading to confusion or misrepresentation of the relationships between quantities. This is often a result of weak reading comprehension skills or the inability to interpret mathematical language embedded within narrative contexts.

Another significant source of difficulty lies in identifying the mathematical structure underlying the problem. Students may be able to perform arithmetic or algebraic operations in isolation but find it difficult to recognize when and how to apply them in a real-world scenario. This lack of strategic competence — the ability to select appropriate methods and apply them in context — leads to superficial problem-solving attempts or reliance on guesswork rather than systematic reasoning. Furthermore, students often perceive word problems as fundamentally different from abstract mathematical exercises and may feel less confident when facing tasks that require

contextual interpretation and modeling. This psychological barrier results in anxiety, avoidance, and reduced engagement with the task.

In many cases, difficulties are also associated with the inability to identify unknowns, define variables, or construct equations that accurately represent the situation. When problems involve more than one step or when intermediate results must be found before arriving at the final answer, students frequently become overwhelmed and lose track of their reasoning process. This suggests an underdeveloped sense of logical sequencing and problem decomposition. Moreover, the habit of memorizing solution patterns rather than understanding underlying principles can hinder students from adapting to new or unfamiliar problem types. When a problem differs even slightly from those previously encountered, students may lack the flexibility to adjust their approach. Didactic factors also contribute to these challenges. If instructional practices overemphasize procedural fluency without building conceptual understanding, students may lack the foundational knowledge needed to interpret and model problems effectively. Inadequate use of visual representations, insufficient practice with real-life contexts, and limited exposure to open-ended or non-routine problems can further limit students' problem-solving development. Teachers may also face constraints such as limited time, large class sizes, or lack of access to differentiated instructional materials, all of which affect the quality of word problem instruction.

Language barriers present an additional difficulty, particularly for students learning mathematics in a second language or those with limited academic vocabulary. The specific phrasing of a problem can obscure its meaning, and linguistic complexity may mask simple mathematical relationships. In such cases, students may fail not because they lack mathematical ability but because they misinterpret the wording or do not fully grasp the context. To address these issues, it is essential to integrate strategies that strengthen both mathematical and linguistic competencies. This includes explicitly teaching students how to analyze problem structures, identify key terms and phrases, define variables, and follow logical steps in constructing and solving models. The consistent use of graphic organizers, modeling frameworks, and guided practice with reflection can support deeper comprehension. Encouraging students to verbalize their thought process, explain their reasoning, and engage in collaborative problem-solving can also reduce cognitive load and increase confidence.

In summary, the difficulties students face in solving word problems are multifaceted and require a comprehensive, student-centered pedagogical response. By recognizing the cognitive, linguistic, and instructional dimensions of these challenges, educators can develop more effective strategies that promote not only the correct execution of mathematical procedures but also the development of reasoning, modeling, and communication skills essential for lifelong mathematical competence.

A triangle is a shape that consists of three points that do not lie on the same straight line, and three segments that connect these points in pairs. Points are called vertices of a triangle, and segments are called sides.

In Figure , we see a triangle with vertices A, B, C and sides AB, AC, CB.

A triangle is indicated by specifying its vertices. Instead of the word "triangle", the sign Δ is sometimes used. For example, the triangle in the figure is denoted as ΔABC . The three angles- $\angle BAC$, $\angle PCBA$, and $\angle PACB$ -are called the angles of triangle ABC. Often they are denoted by a single letter $\angle A$, $\angle B$, $\angle C$.

A triangle is called rectangular if it has a right angle. The sides of a right triangle have special names:

Figure 2.3.1 Right triangle



Hypotenuse - the side lying opposite the right angle; legs - the sides adjacent to the hypotenuse (Figure 2.3.1).

Since the sum of the angles in a triangle is 180° , a right triangle has only one right angle. The other two corners of a right triangle are sharp. The sum of acute angles in a right triangle is $180^\circ - 90^\circ = 90^\circ$.

Statement: In a right triangle, the hypotenuse is larger than the leg.

In fact, the hypotenuse lies opposite the right angle, and the catheter, against the acute one. Since the right angle is larger than the acute one, the hypotenuse is larger than the leg.

2.4 Right triangle and its properties

Let us consider the properties of right-angled triangles, which are established by the sum of angles theorem.

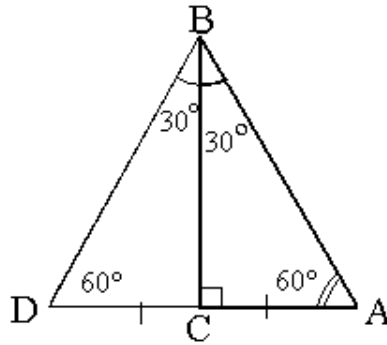
In a right-triangle, the hypotenuse is larger than the leg (a corollary of the theorem on the relation between sides and angles in a triangle).

1. The sum of two acute angles of a right triangle is 90° .

In fact, the sum of the angles of a triangle is 180° , and the right angle is 90° , so the sum of the two acute angles of a right triangle is 90° .

2. The leg of a right triangle lying opposite an angle of 30° is equal to half the hypotenuse.

Figure 2.4.1 Right triangle and height



Let ABC be a right-angled triangle with right angle C and angle B equal to 30° , which means that angle A is equal to 60° (Fig. 3). Let's construct triangle DBC equal to triangle ABC, as shown in the figure. Triangle ABD has all angles equal to 60° , so it is equilateral.

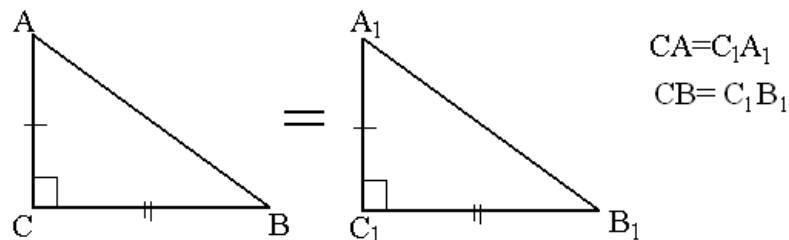
Since $AC = \frac{1}{2} AD$ and $AD = AB$, then $AC = \frac{1}{2} AB$. That's what I needed to prove.

3. If the leg of a right triangle is equal to half the hypotenuse, then the angle lying opposite this leg is 30° (inverse theorem).

Consider a right triangle ABC with an AC catheter equal to half of the hypotenuse AC (Figure 4a). Let's prove that $\angle ABC = 30^\circ$.

Let's attach the triangle DBC equal to it to triangle ABC as shown in Figure 4 (b). We get an equilateral triangle DBA. The angles of an equilateral triangle are equal to each other, so each of them is equal to 60° . In particular, $\angle DBA = 60^\circ$. But $\angle DBA = 2\angle ABC$. Therefore, $\angle ABC = 30^\circ$. That's what I needed to prove.

Figure 2.4.2 Equilateral triangle constructed on right triangle



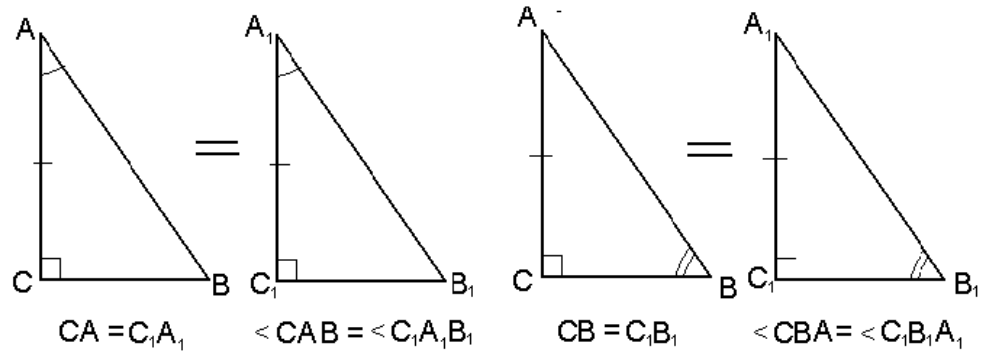
2.5 Signs of equality of right triangles

To establish the equality of right-angled triangles, it is enough to know that two elements of one triangle are equal to two elements of another triangle, respectively (excluding the right angle). This, of course, does not apply to the equality of two angles of one triangle to two angles of another triangle.

Since in a right triangle the angle between two legs is straight, and any two right angles are equal, the first sign of the equality of triangles follows:

If the legs of one right triangle are correspondingly equal to the legs of the other, then such triangles are equal (Figure 2.5.1)

Figure 2.5.1 Criterion of equality by two legs

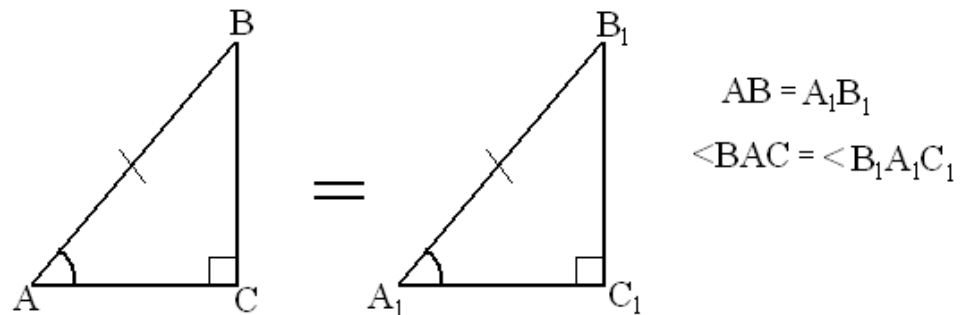


Then, from the second sign of the equality of triangles, it follows: If the leg and the adjacent acute angle of one right triangle are respectively equal to the leg and the adjacent angle of another triangle, then such triangles are equal (Figure 2.5.1)

Let's consider two more signs of equality of right triangles.

The Theorem: If the hypotenuse and acute angle of one right triangle are equal to the hypotenuse and acute angle of another triangle, then such triangles are equal (Figure 2.5.2)

Figure 2.5.2 Criterion of equality by hypotenuse and acute angle



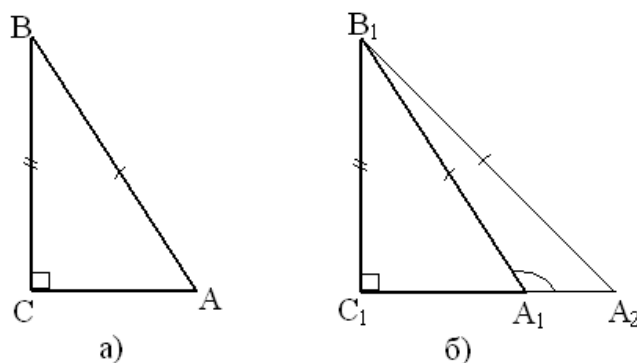
Proof. It follows from the property 1° §2 that in such triangles the other two acute angles are also equal, so the triangles are equal by the second sign of equality of the triangles, that is, by the side (hypotenuse) and the two adjacent angles.

That's what I needed to prove.

The Theorem: If the hypotenuse and leg of one right triangle are respectively equal to the hypotenuse and leg of the other triangle, then such triangles are equal.

Proof of Performance: Consider the triangles ABC and $A_1B_1C_1$, where the angles C and C_1 are straight, $AB = A_1B_1$, $BC = b_1c_1A$ (Figure 2.5.3)

Figure 2.5.3 Criterion of equality by hypotenuse and leg



Since $\angle C = \angle C_1$, triangle ABC can be superimposed on triangle $A_1B_1C_1$ so that vertex C is aligned with vertex C_1 , and sides CA and CB are superimposed on rays C_1A_1 and C_1B_1 respectively, since $CB = C_1B_1$, then vertex B is aligned with vertex B_1 . But then the vertices A and A_1 will also be aligned. In fact, if we assume that the point A coincides with some other point A_2 of the ray C_1A_1 , then we get an isosceles triangle $A_1B_1A_2$, in which the angles at the base of A_1A_2 are not equal (in the figure $\angle C = \angle C_1$ is acute, and $\angle A_1$ is obtuse as adjacent to an acute angle $\angle B_1A_1C_1$). But this is not possible, so the vertices A and A_1 will coincide. Therefore, the triangles ABC and $A_1B_1C_1$ are completely aligned, that is, they are equal.

That's what I needed to prove.

Its significance lies in the fact that most of the theorems of geometry can be derived from it or with its help. One of the theorems makes it possible to verify that if a perpendicular and inclined lines are drawn to it from a point outside the straight line, then: a) the inclined ones are equal if their projections are equal; b) the inclined one is larger, which has a larger projection.

The Pythagorean theorem was the first proof to relate the lengths of the sides of triangles. Then we learned how to find the side lengths and angles of acute-angled and obtuse-angled triangles. There was a whole science of trigonometry ("trigon" - in Greek means "triangle"). This science has found application in the field of hypocrisis. But even earlier, with its help, they learned to measure imaginary triangles in the sky, the vertices of which were stars. Now trigonometry is even used to measure the distances between spaceships.

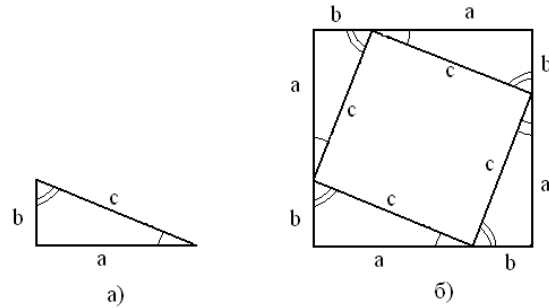
Using the properties of polygon areas, we now establish a remarkable relationship between the hypotenuse and the legs of a right triangle. The theorem that we will prove is called the Pythagorean theorem, which is the most important theorem of geometry. If we are given a triangle, and at the same time with a straight angle, then the square of the hypotenuse. We can always easily find:

1. We square the legs,
2. We find the sum of degrees
3. And in such a simple way
4. We will come to the result.

The Theorem. In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the legs.

Proof: Consider a right triangle with legs a , b , and c (Figure 2.5.4 (a)).

Figure 2.5.4 Application of triangle equality criterion



Let's prove that $c^2 = a^2 + b^2$. Let's complete the triangle to a square with side $a+b$, as shown in the figure (Figure 2.5.4 (b)).

The area of such a square with side $a + b$ is $(a + b)^2$. On the otherhand, this square is made up of four equal right triangles, whose area $\frac{1}{2}$ is ab , and a square with side c , so $S = 4 \cdot \frac{1}{2}ab + c^2 = 2ab + c^2$. Thus, $(a + b)^2 = a^2 + 2ab + b^2 = 2ab + c^2$, where $c^2 = a^2 + b^2$.

That's what I needed to prove:

- Corollary 1. In a right triangle, any of the legs is smaller than the hypotenuse.

Proof. According to the Pythagorean theorem, $AB^2 = AC^2 + BC^2$. Since $BC^2 > 0$, $AC^2 < AB^2$, i.e. $AC < AB$

- Corollary 2. For any acute angle α $\cos\alpha < 1$.

Proof. By the definition of cosine, $\cos\alpha = \frac{AC}{AB}$. But in corollary 1, it was proved that $AC < AB$, which means that the fraction is less than 1.

Right triangles whose sides are expressed as integers are called Pythagorean triangles.

We can prove that the legs a , b and hypotenuse c of such triangles are expressed by the formulas $a=2kmn$; $b=k(m^2-n^2)$; $c=k(m^2+n^2)$, where k , m and n are natural numbers such that $m > n$. Triangles with sides whose lengths are equal to 3, 4, 5 are called Egyptian triangles, because they were known to the ancient Egyptians.

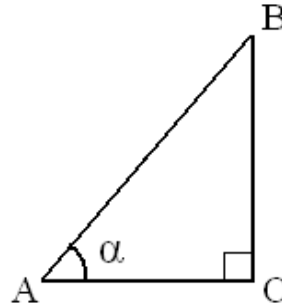
Inverse of the Pythagorean theorem: If the square of one side of a triangle is equal to the sum of the squares of the other two sides, then the triangle is rectangular (indicates a right triangle).

Proof. Let ABC $AB^2 + BC^2 = AC^2$ in triangle ABC . Let us prove that the angle C is a straight line. Consider a right triangle A_1B_1C with a right angle C_1 , which A_1 has $A_1C = AC$ and $B_1C = BC$. By the Pythagorean theorem, $A_1B_1^2 = A_1C_1^2 + B_1C_1^2$, which means that $A_1B_1^2 = AC^2 + BC^2$. But $AB^2 + BC^2 = AC^2$ by the condition of the theorem. Hence, $A_1B_1^2 = AB^2$, where $A_1B_1 = AB$. Triangles ABC and $A_1B_1C_1$ are equal on three sides, so $\angle C = \angle C_1$, that is, triangle ABC is rectangular with a right angle C .

That's what I needed to prove: Sine, cosine, and tangent of an acute angle in a right triangle. Consider a right triangle ABC with a right angle C (Figure 2.5.6). The leg BC of this triangle is opposite to the angle A , and the AC catheter is adjacent to this corner.

The cosine of the acute angle of a right-triangle is the ratio of the adjacent leg to the hypotenuse.

Figure 2.5.5 Acute angle of a right -triangle



- The sine of an acute angle of a right triangle is the ratio of the opposite leg to the hypotenuse.
- The tangent of an acute angle of a right triangle is the ratio of the opposite leg to the adjacent leg.

Sine, cosine, and tangent of an angle equal to α are denoted by the symbols $\sin \alpha$, $\cos \alpha$, and $\tan \alpha$ (read: "sine alpha", "cosine alpha", and "tangent alpha"). In the drawing

$$\sin A = \frac{BC}{AB}, (1)$$

$$\cos A = \frac{AC}{AB}, (2)$$

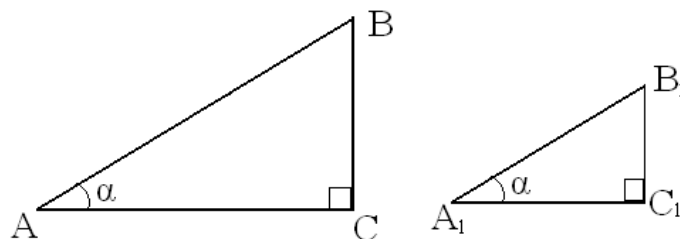
$$\tan A = \frac{BC}{AC}, (3)$$

From formulas (1) and (2) we obtain: $\frac{\sin A}{\cos A} = \frac{BC}{AB} \cdot \frac{AB}{AC} = \frac{BC}{AC}$. Comparing with formula (3), we find $\tan A = \frac{\sin A}{\cos A}, (4)$. That is, the tangent of an angle is equal to the ratio of the sine to the cosine of this angle.

Theorem. If the acute angle of one right triangle is equal to the acute angle of another right triangle, then the sines of these angles are equal, the cosines of these angles are equal, and the tangents of these angles are equal.

Proof: Let ABC and $A_1B_1C_1$ be two right triangles with right angles C and C_1 and with the same angle at the vertex A and A_1 equal to α (Figure 2.5.7a).

Figure 2.5.6 Equality of sines of equal angles



The triangles ABC and $A_1B_1C_1$ are similar in the first feature of triangle similarity, so $\frac{AB}{A_1B_1} = \frac{BC}{B_1C_1} = \frac{AC}{A_1C_1}$. From these equalities it follows that $\frac{AB}{BC} = \frac{B_1C_1}{A_1B_1}$, that is $\sin A = \sin A_1$.

Similarly $\frac{AC}{AB} = \frac{A_1C_1}{A_1B_1}$, that is $\cos A = \cos A_1$, and $\frac{BC}{AC} = \frac{B_1C_1}{A_1C_1}$, that is $\tan A = \tan A_1$.

That's what I needed to prove: Let us now prove the validity of the equality

$$\sin^2 A + \cos^2 A = 1, (5)$$

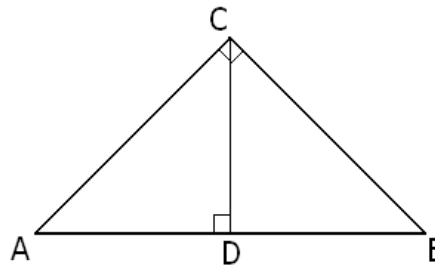
From formulas (1) and (2) we obtain $\sin^2 A + \cos^2 A = \frac{BC^2}{AB^2} + \frac{AC^2}{AB^2} = \frac{BC^2 + AC^2}{AB^2}$.

By the Pythagorean theorem $BC^2 + AC^2 = AB^2$, therefore $\sin^2 A + \cos^2 A = 1$.

Equality (5) is called the basic trigonometric identity. Let us present another proof of the Pythagorean theorem, based on the definition of the cosine of an angle in a right triangle.

Proof. Let ABC be a given right triangle with right angle C . Let's draw the height CD from the vertex of the right angle C . (Figure 2.5.8).

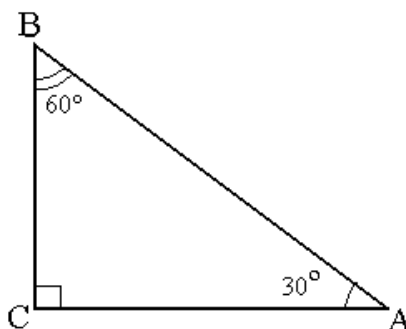
Figure 2.5.7 Basic trigonometric identity



By defining the cosine of an angle $\cos A = \frac{AD}{AC} = \frac{AC}{AB}$. From here $AB \cdot AD = AC^2$. Similarly $\cos B = \frac{BD}{BC} = \frac{BC}{AB}$. From here $AB \cdot BD = BC^2$. Adding the resulting equalities together in a straight line, and noting that $AD + DB = AB$, we get $AC^2 + BC^2 = AB \cdot (AD + DB) = AB^2$.

That's what I needed to prove: Sine, cosine, and tangent values for angles 45° and 60° . We first find the value of the sine, cosine, and tangent for angles 30° and 60° . To do this, consider a right triangle ABC with a right angle C , which has $\angle A = 30^\circ$, $\angle B = 60^\circ$ (Figure 2.5.9).

Figure 2.5.8 Pythagorean theorem



Since the leg lying opposite the angle of 30° is equal to half the hypotenuse, then $\frac{BC}{AB} = \frac{1}{2}$. But $\frac{BC}{AB} = \sin A = \sin 30^\circ$. On the other hand $\frac{BC}{AB} = \cos B = \cos 60^\circ$. So $\sin 30^\circ = \frac{1}{2} \cos 60^\circ = \frac{1}{2}$.

From the basic trigonometric identity, we obtain:

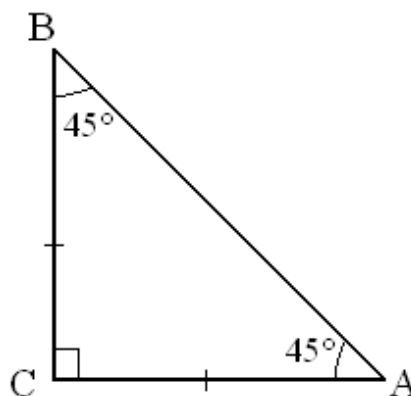
$$\cos 30^\circ = \sqrt{1 - \sin^2 30^\circ} = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2},$$

$$\cos 60^\circ = \sqrt{1 - \cos^2 30^\circ} = \sqrt{1 - \frac{3}{4}} = \frac{1}{2}.$$

By the formula (4) of clause 5.1. we find $\tan 60^\circ = \frac{\sin 60^\circ}{\cos 60^\circ} = \sqrt{3}$.

We now find $\sin 45^\circ$, $\cos 45^\circ$, and $\tan 45^\circ$. To do this, consider an isosceles right triangle ABC with a right angle C (Figure 2.5.10).

Figure 2.5.9 Right triangle with 30°



In this triangle, $AC = BC$, $\angle A = \angle B = 45^\circ$. By the Pythagorean theorem $BC^2 + AC^2 = AB^2$ $AB^2 = 2AC^2$, where $AC = BC$

$$\sin 45^\circ = \sin A = \frac{BC}{AB} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos 45^\circ = \cos A = \frac{AC}{AB} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan 45^\circ = \tan A = \frac{BC}{AC} = 1$$

Let's make a table of the values of $\sin\alpha$, $\cos\alpha$, and $\tan\alpha$ for angles α equal to 30° , 45° , and 60° .

Table 2.5.1 Values of $\sin\alpha$, $\cos\alpha$, and $\tan\alpha$

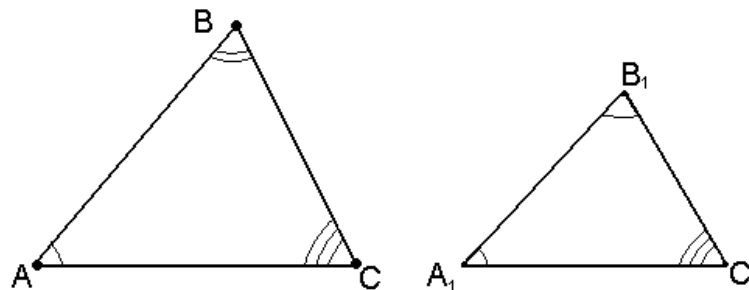
α	30°	60°	70°
$\sin\alpha$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\cos\alpha$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
$\tan\alpha$	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

In everyday life, there are objects of the same shape, but different sizes, for example, soccer and tennis balls, a small corner plate and a large round dish. In geometry, figures of the same shape are usually called similar. We introduce the concept of similar triangles.

Let two triangles ABC and $A_1B_1C_1$ have the same angles, respectively: $\angle A = \angle A_1$, $\angle B = \angle B_1$, $\angle C = \angle C_1$. In this case, the sides AB and A_1B_1 , BC and B_1C_1 , CA and C_1A_1 are called similar.

Two triangles are called similar if their angles are equal and the sides of one triangle are proportional to the similar sides of the other (Figure 2.5.11).

Figure 2.5.10 Isosceles right triangle



In other words, two triangles are similar if the notation ABC and $A_1B_1C_1$ can be entered for them, so that:

$$\angle A = \angle A_1, \angle B = \angle B_1, \angle C = \angle C_1, \quad (1)$$

$$\frac{AB}{A_1B_1} = \frac{BC}{B_1C_1} = \frac{CA}{C_1A_1} = k, \quad (2)$$

The number k equal to the ratio of similar sides of similar triangles is called the similarity coefficient. Denoted by $\Delta ABC \sim \Delta A_1B_1C_1$.

It turns out that the similarity of triangles can be established by checking only some of the equalities (1) and (2).

A right triangle has one right angle. Therefore, for the similarity of right triangles, it is sufficient that they have an equal acute angle.

Using this feature of similarity of right-angled triangles, we prove some relations in triangles.

2.6 Methodological foundations of studying the topic "Right triangle"

The method of teaching mathematics not only logically organizes the selected material, but also orients it to the characteristics of students of a particular class, using the patterns of memory, thinking, attention, etc., individual abilities of the age group.

The main role of a mathematics teacher in modern conditions is to educate students' personality, form their needs and motivational sphere, educate their abilities, moral ideals and beliefs. Teaching knowledge, skills and abilities in mathematics is an integral part of this education and the process in which this education is carried out.

Teaching mathematics contributes to the formation and development of moral personality traits: perseverance and purposefulness, cognitive activity and independence, discipline and critical thinking, the ability to argumentatively defend their views and beliefs. The study of mathematics makes a certain contribution to the aesthetic education of a person, forming an understanding of the beauty and grace of mathematical beliefs, contributing to the perception of geometric shapes and symmetry. Studying mathematics develops imagination and spatial representations.

Currently, there is an increased interest of mathematics teachers in psychological and pedagogical problems, in psychological knowledge. This interest is due to the fact that mathematics teachers in their daily practical activities encounter problems that can be solved only on the basis of psychological and pedagogical knowledge, as well as on the condition of a deep psychological understanding of the essence of these problems.

The need to take into account the age characteristics of students is mentioned everywhere, but it is not always indicated what this means, what features should be taken into account and how they should be taken into account. Meanwhile, it should be borne in mind that age - related abilities are something unchangeable and eternal, which is inherent in students of a certain age. These features themselves change quite dramatically over time.

Age psychology studies the peculiarities of a person's psychological development. The subject of her research is age dynamics, patterns and leading factors in the development of psychological processes and personality traits of a person at different stages of his life.

The correctness of the teacher's attitude to the student is associated with an understanding of the cardinal problems of age, specifics and leading trends that determine the features of the educational process, the strategy of teaching and upbringing. This means deciding for yourself what principles to put in the basis of planning the content and methods of teaching, what requirements to make to students, how to build communication with them, how to evaluate their knowledge, capabilities, abilities, i.e. how to determine the main directions and ways to implement the nature

of learning. An analysis of the age characteristics of students will help you build a lesson correctly and find the best approach to any of the teachers.

Let's look at some of the psychological features of a modern student, keeping in mind only those features that are important to take into account in the process of teaching mathematics. Having outlined the general characteristics of the age characteristics of students, we will try to outline the meaning of improving mental processes in adolescents.

A student is a growing, developing person. Having come to school at the age of seven, he finishes it at the age of 17 as a fully developed person of youthful age. During these ten years of training, the student goes through a huge path of physical, mental, and socio-moral development.

Students in grades 7-8 are mostly teenagers aged 12-13. It is at this age that important processes related to memory restructuring occur. Logical memory begins to develop actively and soon reaches such a level that the child moves mainly to the use of this type of memory, as well as arbitrary and indirect memory. Among the school subjects for the development of logical memory, geometry could not be better suited. A characteristic feature of adolescence is the readiness and ability for many different types of training, both in practical terms and in theory.

Another trait that is fully revealed for the first time in adolescents is the tendency to experiment, which manifests itself, in particular, in the unwillingness to take everything on faith. This age-related feature of students can help make geometry lessons on the topic "Right Triangle" very interesting for students themselves, if they are conducted in the form of some practical work, which can contribute to a good assimilation of many topics. After all, during this period, teenagers discover broad cognitive interests associated with the desire to double-check everything on their own, to personally verify the truth.

Adolescence is characterized by increased intellectual activity, which is stimulated not only by the natural age-related curiosity of adolescents, but also by the desire to develop, demonstrate their abilities to others, and receive high appreciation from them. In this regard, teenagers strive to solve the most complex problems, often showing not only highly developed intelligence, but also outstanding abilities, in the development of which geometry can play an integral role, with its laws, theorems and interesting solutions to some problems. Teenagers are characterized by an emotionally negative effective reaction to tasks that are too simple. In these cases, it is possible to give students the opportunity to deduce and prove the properties of right-angled triangles, the Pythagorean theorem. Give strong students the opportunity to do their own research or conduct a simple lesson.

At the same time, the thinking of this age is characterized by a desire for broad generalizations, where it is more necessary than ever to apply a broad knowledge of geometry to the development of logical and abstract thinking, and this topic touches on this aspect of studying this issue.

In middle school classes, instead of one teacher, several new teachers appear, who usually have different behavioral and communication styles, as well as methods of conducting classes. Different teachers make different demands on teenagers, which

forces them to adapt individually to each new teacher. In adolescence, differentiated attitudes towards different teachers appear: some are loved, others are not, and others are treated indifferently. Teenagers are more likely to appreciate knowledgeable teachers, strict but fair, who are kind to children, are able to explain the material in an interesting and understandable way, give fair grades, and do not divide the class into favorites and unloved ones. Especially highly appreciated by a teenager is the teacher's erudition, as well as his ability to correctly build relationships with students. The teacher should find an approach to students, try to attract them, and interest the lesson in it, built in the form of a teacher - student dialogue. As an example, the topic of properties of a right triangle, which is fully reviewed in the presented work, can serve as an example. Students will be interested and interested in the lesson if the teacher, when drawing out properties and proofs of these properties, will push students to draw up certain conclusions on the topic being studied.

Children at this age are already quite noticeably different from each other in their interests in learning, in the level of intellectual development and outlook, in the volume and strength of knowledge, and in the level of personal development. These differences determine their differential attitude to learning. The teacher must take into account the level of knowledge of each student, adapt to each, give knowledge to each. This circumstance determines the selective nature of attitudes to school subjects. Some of them become necessary and therefore more beloved by teenagers, while interest in others decreases. Often, the attitude of a teenager to a particular subject is determined by the attitude to the teacher who teaches this subject. Teenagers usually like the subjects taught by their favorite teachers. Many children's academic performance in secondary school temporarily falls because they develop strong, competitive interests outside of school.

Teenagers can formulate hypotheses, reason presumably. This skill will help to make lessons on learning right triangles more interesting. In turn, these classes will contribute to the development of skills. Students will be very interested in putting forward and trying to prove their hypotheses about the properties of a right triangle, about the signs of equality of right triangles. Seventh - and eighth-grade students are interested in studying alternative solutions to the same problem, as well as various ways to prove a theorem.

The sphere of cognitive, including educational, interests of adolescents goes beyond school and takes the form of cognitive independence - the desire to search for and acquire knowledge, to form useful skills. Taking into account this age feature, it is useful to conduct some lessons in the form of practical lessons.

Geometry contributes to the full development of the child, which is so necessary in adolescence. As psychologists' research shows, emotional development is the basis of general intellectual development. Its integral part is aesthetic education. It is geometry that provides huge opportunities for aesthetic development and aesthetic education.

For normal development, a teenager needs a full-fledged diet, just as for normal intellectual development, a variety of intellectual food is necessary. Today, mathematics, especially geometry, is one of the few environmentally friendly and full-

fledged products consumed in the education system. Geometry can and should become a subject that teenagers can use to balance the functioning of the brain and improve functional interaction between the hemispheres. Geometry is a vitamin for the brain.

Tabel 2.6.1 Thematic plan on the topic “Right triangles”

	Topic title	Number of lessons
7 CLASS		
1	Right triangle	20
2	Introduction to the concept of a right triangle	1
3	Properties of right triangles	1
4	Solving problems on the properties of a right triangle	1
5	Signs of equality of right triangles	1
6	Solving problems on the signs of equality of right triangles	1
7	Independent work on fixing the material	1
8	Preparation for the test work.	1
9	Test paper	1
10	Analysis of the test paper	1
8 CLASS		
1	Pythagorean theorem	1
2	Solving problems according to the Pythagorean theorem	1
3	Sine, cosine and tangent of the acute angle of a right triangle	1
4	PSolving problems on the sine, cosine and tangent of the acute angle of a right triangle	3
5	Independent work on fixing the material	1
6	Preparation for the test	1
7	Test paper	1
8	Analysis of test paper	1

Success in solving geometry problems largely depends on the professional level of the teacher and the degree of interest and readiness of students. Education in grades 7-8 is largely oriented. At this stage, the student should be helped to understand the degree of their interest in the subject and assess the possibilities of mastering it. The teacher can and should change their tastes depending on the class: simplify or supplement the material, rearrange topics, vary the number of hours allocated to a particular topic, conduct several more or less test papers.

Planning was carried out in accordance with the curriculum, according to which 5 hours per week are allocated for studying mathematics in grades 7-8, including 2 hours for geometry.

The intended planning of educational material for grades 7-8 is based on the textbook of L. S. Atanasyan et al. and is published in the collection of the Ministry of Defense of the Russian Federation "Program for general education institutions" (Moscow: Prosveshchenie, 2000)

2.7 Methodological recommendations for studying the topic "Right triangle"

Introduction of a right triangle, properties and signs of equality of right triangles

The purpose of this section is to provide methodological recommendations for the study of the "Right Triangle", such as the introduction of a right triangle, some properties of right triangles, signs of their equality. A possible sequence of problem solving is proposed. You can spend five hours studying these topics.

As a result of the study, students should: understand the concept of a "right triangle"; know the names of its sides; know that it has one right angle and two acute angles and that the sum of acute angles is 90° ; be able to draw a conclusion from a drawing or verbal data about which sides of a right triangle are legs and hypotenuse (for example, if it is given that in triangle ABC the angle B is a straight line, then the sides BA and BC are legs, and AC is the hypotenuse); know the formulations and proofs of some properties of a right triangle and special signs of the equality of right triangles; be able to apply them in solving problems.

Observations of the work of mathematics teachers lead to the conclusion that the formation of mathematical concepts in school does not fit in pure form into any of the logical systems of concept education.

Let's describe the methodological requirements for the concept formation. The initial stage is motivation. The essence of this stage is to emphasize the importance of studying the concept, to encourage students to engage in purposeful and active activities, and to arouse interest in studying the concept. Motivation can be carried out both by attracting funds of non-mathematical content, and in the course of performing special exercises that explain the need for the development of mathematical theory.

Definition of a concept is a list of characteristic, basic, necessary, and sufficient features of a concept.

Introducing the concept of a right triangle, we emphasize that this topic runs through many topics of the geometry course. Very often, a right triangle is used when solving problems. Properties and signs of equality and similarity of right triangles are

used. The concept of a right triangle is not defined; it is accepted as a fact. The distinctive properties of a triangle are distinguished by indicating its differences in origin. A right triangle is a triangle that has a right angle.

The definition should be abbreviated and specific. It should not be negative. The process of constructing a concept proceeds as a search for necessary conditions that are sufficient.

The next stage is to identify the essential properties of the concept that correspond to its definition. It is implemented mainly through exercises, the main purpose of which at this stage is to highlight the essential properties of the concept being studied and focus students attention on them. Emphasizing these properties helps students build a clearer the concepts.

In the first lesson, you need to introduce the concept of a right triangle, give the concept of the sides of a triangle. When solving numerous problems related to right triangles, students will need special knowledge of the concepts of "leg" and "hypotenuse" of a right triangle. The introduction of the names of the sides of a right triangle should be accompanied by oral exercises aimed at memorizing them and recognizing them in the drawing. For this purpose, you can offer the following tasks:

1. The segments AB and CD intersect at right angles at point O. Name the hypotenuses and legs of right triangles AOC and BOD.

2. In the MNK triangle, the height KD is drawn. Name the resulting right angle triangles, their hypotenuses, and their legs.

Statements about the angles of a right triangle, being direct consequences of the theorem on the sum of the angles of a triangle, are extremely simple to prove. Their proofs can be offered to students to conduct independently.

3. One of the angles of a right triangle is: a) 20° ; b) 30° ; c) 45° . Find the second acute angle of the triangle.

4. Determine the acute angles of a right triangle if one of them is 2 times larger than the other.

Next, consider the properties of a right-angled triangle. The study of point 34 on the properties of a right triangle can be started by solving problems 254 and 255. After that, consider property 1, which should be given special attention (the leg of a right triangle lying against an angle of 30° is half the size of the hypotenuse). Since students will use it when solving problems, and in the future - when getting the values of trigonometric functions of angles of 30° and 60° . The use of this property can be shown in the example of problem 265. Proving properties 2 and 3 should be done by the teacher himself, with the condition and conclusion of the forward and reverse statements written on the blackboard in the form of a table. Students should reproduce this table in their notebooks.

Table 2.7.2 Theorem and inverse on equality of right triangles

	Theorem	Inverse Theorem
Given	$\Delta ABC, \angle A = 90^\circ, \angle B = 30^\circ$	$\Delta ABC, \angle A = 90, AC = \frac{1}{2}BC$
Prove	$\frac{1}{2}BC < B = 30^\circ$	$\angle A = 90^\circ$

Then we recommend solving problems 257, 259, and 260.

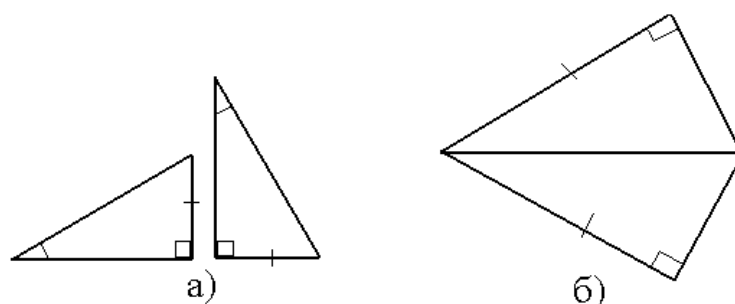
Before proving special signs of equality of triangles, it is useful to recall general signs, but not abstractly, in relation to right-angled triangles. This can be done by offering, for example, verbally using the finished drawing to conduct proofs:

- Prove that if two legs of one right triangle are respectively equal to two legs of the other, then such triangles are equal.
- Prove that two right triangles ABC and $A_1B_1C_1$ with right angles C and C_1 are equal if they have equal legs $BC = B_1C_1$ and adjacent acute angles: $\angle B = \angle B_1$.

After completing problem 2, we can make the observation that if in right triangles ABC and $A_1B_1C_1$ $\angle A = \angle A_1$, then $\angle B = \angle B_1$, since the angles B and B_1 complement up to 90° equal angles A and A_1 . So, it is possible to give the equality of these triangles along the leg and the opposite acute angle. It should also be noted that this feature and two other features that can be considered later are special features of right-angled triangles.

Proof of this trait can be offered to students to hang out on their own. To formulate a sign that right triangles are equal in terms of the hypotenuse and acute angle, the teacher can also ask students to prove it themselves. You can fix the proven attributes in the course of completing tasks. Justify the equality of the triangles in Figure 2.7.1 (a).

Figure 2.7.1 Using triangle equality criterion on a drawing



In Figure 2.7.1 (b) $\angle B = \angle D = 90^\circ$, $BC \parallel AD$. Prove that $\Delta ABC = \Delta CDA$. Or solve problems 261, 263 from the textbook.

Special attention should be paid to the proof of the sign of equality of triangles in the hypotenuse and leg. If the previous signs are proved very simply, then the proof of this sign requires additional constructions and complex logical reasoning. After the

teacher himself performs the proof of the sign of equality of right triangles by hypotenuse and leg, you can solve problem 267 by applying the considered sign.

To fix this feature, you can offer students a task: From the point D, which lies inside the angle A, the perpendiculars DB and DC are omitted to the sides of the angle. Prove that $\triangle ADB = \triangle ADC$ if $DB = DC$

When solving problems, students can take an additional step that is present in the proof of the first two signs, if they establish the equality of the second pair of acute angles and reduce the proof to the general signs of triangles. In this section, we study one of the most important theorems of geometry - the Pythagorean theorem and its inverse. The Pythagorean theorem will significantly expand the range of problems solved in the course of geometry. The further presentation of the theoretical course is largely based on it.

As a result of studying this section, students should:

- Know the statements of the Pythagorean theorem and its consequences; be able to reproduce the proof of the Pythagorean theorem, apply it to solving problems.

In order for Theorema to interest students and be assimilated by them, a thorough, comprehensive preparation is needed. Those who are not interested will not listen (listen "passively"), and the lesson will lose its meaning, it will not be a lesson. Before proving the Pythagorean theorem, it is advisable to carry out preparatory work on the finished drawings and repeat the basic concepts, definitions, terms; properties of areas, since the proof uses the area of a rectangle. When conducting the proof of the Pythagorean theorem, it is useful to bring students to the fact that they took a passive part in the formulation of the theorem; they mastered the formulation, highlighted the condition and conclusion. The teacher should prepare a drawing in advance, which is necessary for proving the theorem, and clearly show in the drawing the stages of completing the task.

It is necessary that students have experience in solving problems; they have mastered the first steps (they can make a drawing as close as possible to assimilation, enter everything that is given in the condition, enter the necessary notation), write down the condition and conclusion using the entered notation; They have elementary skills in finding solutions to problems.

To consolidate the theorem, we can offer students the following oral calculation tasks:

- a) Right triangle legs of 6 cm and 8 cm. Calculate the hypotenuse of the triangle.
- b) The hypotenuse of a right triangle is 5 cm, and one of the legs is 3 cm.

Determine the second leg.

Questions for repetition provide proof of the consequences of the Pythagorean theorem. These proofs are simple and are not explicitly mentioned in the textbook. When reviewing these proofs in class, you can ask students to write them down in their notebooks.

Another approach to studying the Pythagorean theorem is the problem situation method in geometry lessons.

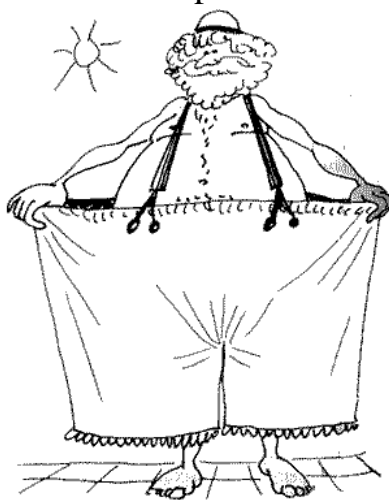
The learning process is carried out more actively in cases when the one is associated with solving problems of real situations, and the problems have a motivational basis, including a keen interest in the subject of study. Motivations stimulate, organize, and direct learning activities. Of considerable interest is the motivation for organizing the extra training program and the direction of students' thinking activity. Problems that the teacher may pose to students are usually resolved during one or more lessons. Most often, teachers create problem situations through experimentation, that is, by studying a particular case.

It is easy to organize a problem situation by offering students tasks that require new knowledge to solve. It is useful to maintain the intensity of activity with a chain of problematic issues that follow one another.

Before studying the Pythagorean theorem, a practical problem is considered, for the solution of which it is necessary to be able to calculate the length of the hypotenuse from the lengths of the legs.

The construction proves that there is a certain relationship between the legs and the hypotenuse, that two legs define a triangle in which the hypotenuse cannot be arbitrary. You can find an approximate solution graphically. Now the question arises: "Is it possible to express the relationship between the legs and the hypotenuse by the formula?". In this search for an answer, we will consider a convenient special case: a right triangle with acute angles of 45° degrees. We get the formula for it $c^2 = a^2 + b^2$ and we ask ourselves: "Is this formula correct for an arbitrary right triangle?".

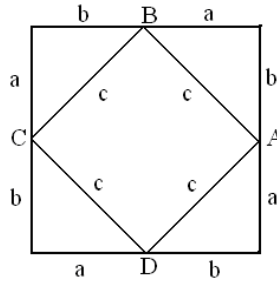
Further research can be based on the following scheme. Since the proposed formula includes the values a^2 , b^2 , c^2 that is, the areas of squares with sides a, b, c, c. Let's construct these squares. The first construction ("Pythagorean trousers") does not explain the idea of proof.



Then the teacher suggests linking the values a, b, and c in a combination of right triangles and squares in the way shown in the figure.

Let's look at this drawing. It is clear that on one side, the area of a large square is equal to the product of two sides, which are expressed as $(a+b)$. It follows that the area is $(a+b)^2$.

Figure 2.7.2 Proving triangle equality



On the other hand, the area of a large square is equal to the sum of the areas of the figures into which the given square is divided. In this case, it is the sum of a small square with side c and four equal triangles with sides a , b , and c .

It follows from Yuda that the area of a small square is equal to the difference between the area of a large square with side $(a+b)$ and the quad area of a triangle with sides a , b , and c , i.e.

$$c^2 = (a + b)^2 - 4 \cdot \frac{ab}{2}$$

$$2c^2 = 2a^2 + 4ab + 2b^2 - 4ab$$

$$2c^2 = 2a^2 + 2b^2$$

$$c^2 = a^2 + b^2$$

Can the formula be considered proven? If we proceed from the formula given in the drawing, then yes. Let us consider whether it is always possible to draw such a construction for any right triangle. We construct a square with side $(a + b)$ and construct a right triangle with legs a and b . Let's find out why all such triangles are equal. It remains to show that the shape formed by the hypotenuse and the resulting right triangles is a square. Note that all sides of this figure are equal as the hypotenuses of equal triangles. But is it enough for the figure $ABCD$ to be a square? - no. We prove that all the angles of this figure are straight, since they are equal to the difference between the expanded angle and the acute angles of this right triangle. Therefore, the Pythagorean theorem can be considered proven.

As a homework assignment, the teacher can instruct you to read the proof given in the textbook.

But the chain of questions related to the dependence of the sides of a right triangle can be continued.

Let us first ask: "Is the Pythagorean theorem valid for non-rectangular triangles?" - Obviously not, since the two sides of a triangle a and b do not uniquely determine its shape, and the third side changes its length depending on the value of the angle between sides a and b so that $a - b < c < a + b$ (if $b < a$).

Next problem: "Is the inverse of the Pythagorean theorem true?"

If the square of the larger side is equal to the sum of the squares of the other two sides, then the triangle is rectangular, namely: a right angle is the angle that lies opposite this larger side. Indeed, if it weren't a triangle whose sides a , b , and c are connected by a dependency

If $c^2 = a^2 + b^2$ were not rectangular, then its sides would not be able to satisfy this equality.

It is very useful to ask students to indicate a number of applications of the Pythagorean theorem.

The following issues may appear in the search for the answer to this question.

The land plot has the shape of a right triangle. The largest side of the plot faces the river and is swampy, so you can't go through it. How do I find the length of the largest side if the other two sides can be measured directly?

The length of the hour hand is 6 mm, and the minute hand is 8 mm. How much time does the clock show if the distance between the ends of the hands is 20 mm, and the minute hand is at "12"?

You can give students an excursion to East Asia, but a small one, so that students don't get tired of listening.

The history of the Pythagorean theorem is interesting. Although this theorem is associated with the name of Pythagoras, it was known long before him. In Babylonian texts, this theorem occurs 1200 years before Pythagoras. It is possible that at that time its proofs were not yet known, and the relationship between the hypotenuse and the leg itself was established experimentally on the basis of measurements. Pythagoras seems to have found a proof of this relation. An ancient legend has been preserved that Pythagoras sacrificed a bull to the gods in honor of his discovery, and according to other sources, even 100 bulls. Various other proofs of this theorem have been found over the last centuries. Currently, there are more than a hundred of them.

2.8 Organization of repetition of the studied material

In the process of teaching mathematics, an important place is given to organizing the repetition of the studied material. The need for repetition is caused by learning tasks that require a strong and conscious mastery of them.

Pointing out the importance of the process of repetition of the studied material, modern researchers have shown a significant role in this process of such dialectical techniques as comparison, classification, analysis, synthesis, generalization, which contributes to the intensive course of the memorization process. At the same time, flexibility, mobility of mind, generalization of knowledge are developed.

In the process of repetition, students' memory develops. Emotional memory relies on visual-figurative processes, gradually gives way to memory with logical thinking processes, which is based on the ability to establish connections between known and unknown components, compare abstract material, classify it, and justify one's statements.

Repetition of the teaching material in mathematics is carried out throughout the educational process system: when updating knowledge - at the stage of preparing and studying new material, when forming new concepts by the teacher, when consolidating

what was studied earlier, when organizing independent work of various types, when checking students' knowledge.

Through repeated learning, students' cognitive structures are systematically formed. In mathematics, in particular, repetition helps to improve logical analysis, abstraction, and reasoning skills. One study that supports this idea is Maglione et al. (2021), who found that students were more likely to understand concepts in depth and with greater understanding when repetition was implemented in a distributed rehearsal format. This approach emphasizes the need for repetition to be a planned process over time, rather than a mere concept. In addition, according to King and Morton (2022), reviewing repeated material in various educational formats develops students' critical thinking skills. For example, reviewing a certain trigonometric topic in a combination of graphs, equations, inequalities, and text descriptions increases students' conceptual aptitude.

The combination of emotional and cognitive memory plays a large role in revision. Byrne et al. (2023) showed that conducting revision in emotionally motivated lessons improves students' memory by up to 25%. For example, if emotional-psychological support is included in the revision lessons on the topic of triangles, in addition to visual drawings, this contributes to the acquisition of material at a deeper level.

The motivational aspect of repetition is also important. Wang et al. (2024) found that when a repetition component was methodologically integrated, students' confidence and interest in solving problems increased by 17%. This suggests that the repetition process affects not only memory, but also emotional and social development.

Some studies emphasize the methodological structure of repetition. For example, Pappas & Lakiotis (2021) suggest dividing repetition into four stages: reflection on previously acquired knowledge, re-presentation and practical application, independent application, and consolidation of knowledge through group reflection. This approach is effective in developing students' metareflective abilities, especially in solving trigonometric equations and models. Some researchers, such as Rodriguez & Solano (2022), recommend using visual models to enhance the ability to visualize. They found that the systematic repetition of the material presented in the visual component of repetition with symbols and pictures allows for a deep and stable mastery of logical models.

Based on these scientific approaches, it is recommended to use the following systematic model system when organizing repetition:

- Initial presentation - introduction of a new concept.
- Discussion with the teacher - comparison, classification, analysis.
- Distributed repetition - planning an interval and repeating the topic in different formats.
- Application phase - mathematical modeling, logical arguments, test tasks.
- Group reflection - students analyze their memory and application strategies, correct errors

The need to repeat the material studied earlier is caused by the very structure of the mathematics curriculum. The school curriculum is designed in such a way that,

without repeating previously studied material, it is difficult to understand the new one. Therefore, the repetition of the completed material is necessary for students. In practice, the importance and usefulness of generalizing repetition is felt. Generalizing lessons are the result of students' hard work on repetition and provide them with practical help in preparing for exams. The feedback of eighth-graders about these lessons, their awareness, logically correct answers, correct use of symbolic notation, and the ability to apply theoretical knowledge to solving problems indicate the great effectiveness of such repetition.

2.9 Unconventional form of traditional control

Traditional forms of monitoring, such as oral interviews and written independent work, require considerable time. In addition, during the oral control (survey), some students do not follow the answer. Written independent works require a lot of time and effort from the teacher, both for their compilation, and for checking and systematizing the mistakes made by the student. Serious disadvantages of such work include the fact that at best the results of verification are reported only in the next lesson, when the student has already managed to forget the course of solving the problem, and the problems of finding it are no longer relevant for him - he only needs an assessment. As a result, all comments on the implementation and design of decisions are ignored, and there is no need to talk about any correction of knowledge.

Therefore, while paying due attention to traditional forms of control, you should find time to use individual cards as well.

As you know, the learning process consists of several stages:

- reporting new facts (most often theoretical information);
- students' assimilation of this material (knowledge);
- use this information to prove other theoretical statements and solve problems (skills);
- correction of acquired knowledge - further work on the formation of basic methods of proof and problem solving (skills).

At each stage of training, the teacher needs to know how the learning process is going, what difficulties or shortcomings a particular student has in mastering knowledge and skills. Diagnostics of the level of assimilation of knowledge and skills at each stage of training allows you to optimally choose the forms and methods of training, as well as the forms of correction of errors and gaps in the assimilation and application of knowledge and skills. Traditional independent works and thematic control works cannot perform the function of operational control, and even more so they do not have the function of individual control, since they record the achievement or non-achievement of a certain mandatory level of mastering knowledge and skills.

To implement differential geometry training in grades 7-8, individual flashcards are used. They usually consist of one or more tasks and are separate materials. Most often, such cards are offered in class to a strong student, so that he does not "get bored". While the teacher is working with the class, either for the weak one, so that there is

something to give a grade for, or for a group of students. They record the achievement of a certain level.

In geometry lessons, work on individual cards, designed for 15-20 minutes, can serve, on the one hand, as a flexible control-diagnosis, and on the other - to perform a developing (training) function. The main purpose of including cards in the learning process is to quickly establish feedback. While solving tasks on the card, the student can ask the teacher a question related to the condition and the course of the solution. The information obtained as a result of working on the cards allows the teacher to draw a conclusion about the achievement of the basic level of knowledge at this stage of studying the geometry course by each student in the class. In addition, as part of this work, the teacher has the opportunity to help a weak student in solving problems and mastering theoretical material, and a strong student - to see the simplicity of geometry and demonstrate their knowledge. With this approach, the teacher will definitely notice any progress in mastering knowledge and skills.

Taking into account the heterogeneity of students in the class, it is necessary to prepare several cards for one topic: two cards A - for students with a low level of assimilation; four cards B - for students with an average level of assimilation of knowledge; two cards C - for advanced students.

The card should include two questions and tasks: the first is a theoretical question or a theoretical task; the second is a task (Appendix 1).

CARD A (for "weak" students)

- | |
|---|
| <ol style="list-style-type: none">1. Formulate the studied theorem, or reproduce or read the drawing.2. One-step task for "recognition" (saw - decided). |
|---|

CARD B (for students who reach the mandatory entry level geometric training)

2.10 Sample lessons on the topic "Right triangle"

This section contains four sample lessons on the "Right Triangle" theme, which can be used in preparing for lessons by both students - interns and teachers. These lessons are given because they are the most basic in this topic. These properties, signs of right triangles, and the Pythagorean theorem run through many topics of the geometry course.

When developing these lessons, the textbook Geometry 7 - 9 was used. Textbook for general education institutions. L. S. Atanasyan, Moscow. Prosveshchenie Publ., 2001.

Lesson 1: Some properties of right triangles

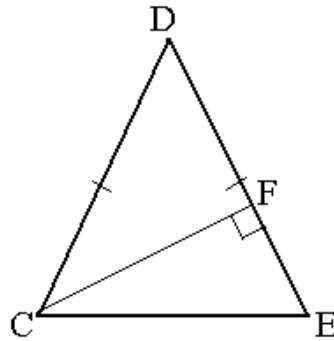
Objectives: To review some properties of right triangles and show how they are applied in solving problems.

Lesson progress:

I. Learning new material: Verbally solve problem # 254 of the textbook. Find the angles of an isosceles right triangle (use the demo isosceles right triangle). Solve

problem # 255 on the blackboard and in notebooks. a task. In an isosceles triangle CDE with base CE, the height CF is drawn. Find $\angle ECF$ if $\angle D = 54^\circ$.

Figure 2.10.1 Isosceles triangle and right angle



Decision: By the condition that the triangle CDE is isosceles, then $\angle E = \angle DCE = (180^\circ - 54^\circ):2 = 63^\circ$ (angles at the base of an isosceles triangle).

Since by the condition $CF \perp DE$, the triangle CFE is a right triangle with $\angle CFE = 90^\circ$, $\angle E = 63^\circ$; then $\angle ECF = 180^\circ - (90^\circ + 63^\circ) = 27^\circ$. Answer: 27° .

Consider property 1° and advise students to remember it, since it is often used in problem solving.

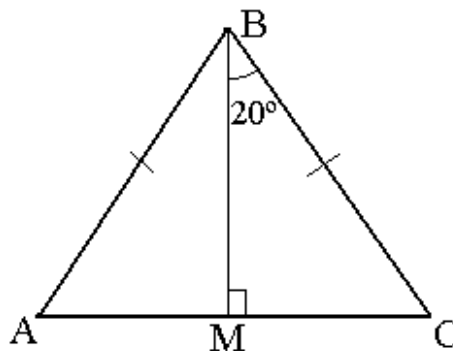
The proof of properties 2° and 3° should be performed by the teacher himself, with the condition and conclusion of the forward and reverse statements written on the blackboard in the form of a table. Students should reproduce this table in their notebooks.

Table 2.10.1 Properties and inverse properties of a right triangle

	Theorem	Inverse Theorem
Given	$\Delta ABC, \angle A = 90^\circ, \angle B = 30^\circ$	$\Delta ABC, \angle A = 90^\circ, AC = \frac{1}{2} BC$
Prove	$AC = \frac{1}{2} BC, \angle B = 30^\circ$	$\angle A = 90^\circ$

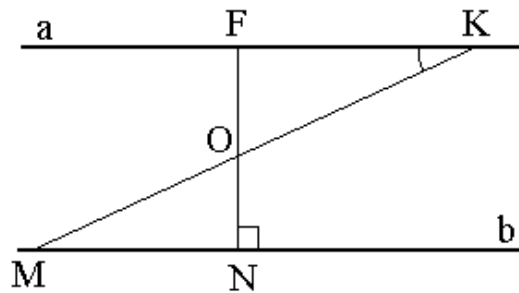
II. Pinning new content: Verbally solve problems based on ready-made drawings on the blackboard: Given a triangle ABC. Find the angles of triangle ABC.

Figure 2.10.2 Construction of a triangle based on given angles



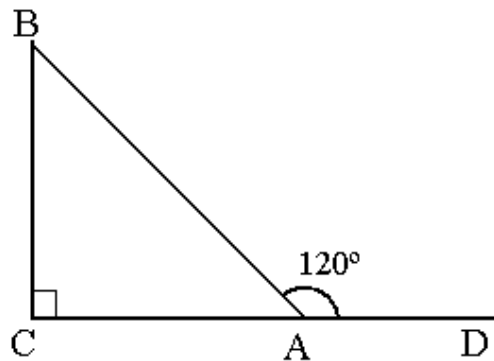
Two parallel lines a and b are given. Find the angles of the triangle MON.

Figure 2.10.3 Problem solution from textbook



Solve problem # 257 of the textbook on the blackboard and in notebooks.
 Task. In a right triangle ABC with right angle C, the outer angle at vertex A is 120° , $AC + AB = 18$ cm. Find AC and AB.

Figure 2.10.4 Triangle sides with exterior angle



Decision. $\angle CAB = 180^\circ - 120^\circ = 60^\circ$ (adjacent angles), then $\angle B = 90^\circ - 60^\circ = 30^\circ$ (according to the property 1°); $AC = \frac{1}{2} AB$ (property 2°: leg lying against the angle of 30°). By the condition:

$$AC + AB = 18 \text{ cm}; \frac{1}{2} AB + AB = 18 \text{ cm.}$$

$$1\frac{1}{2} AB = 18, AB = 12 \text{ cm, } AC = 6 \text{ cm,}$$

Answer: $AB = 12$ cm, $AC = 6$ cm.

Solve problem # 260.

The height drawn to the base of an isosceles triangle is 7.6 cm, and the side of the triangle is 15.2 cm. Find the angles of this triangle.

Decision. Given a triangle DMC; $DM = MC$; $MO \perp DC$; $DM = 15.2$ cm; $MO = 7.6$ cm. Find the angles of the DMC triangle.

Since $MO = \frac{1}{2} DM$, then by the property 3° $\angle D = 30^\circ$, then $\angle C = 30^\circ$,

$$\angle M = 180^\circ - (30^\circ + 30^\circ) = 180^\circ - 60^\circ = 120^\circ.$$

Answer: $\angle D = \angle C = 30^\circ$; $\angle M = 120^\circ$.

III. Lesson results: Your assignment: study point 34 of the textbook on some properties of a right triangle; repeat points 15-33 related to the signs of equality of triangles. Answer questions 10 and 11 on page 84; solve #256, 259.

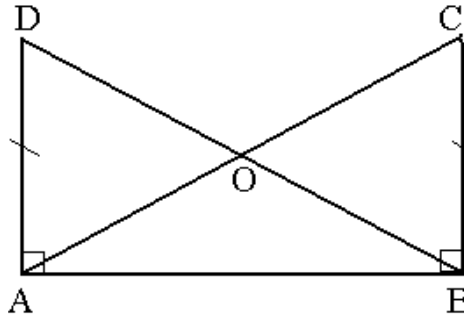
Lesson 2. Signs of equality of right-angled triangles

Objectives: to prove the equality signs of right triangles and show how they are applied in solving problems.

Lesson progress:

I. Repetition of the studied material; Formulate properties of right triangles; Recall the principle of equality of triangles.; Solve the problem: the hypotenuses BD and AC of right triangles ABD and ABC with a common leg AB and with equal legs AD and BC intersect at point O. Prove that triangle AOB is isosceles.

Figure 2.10.5 Triangle equality via shared side



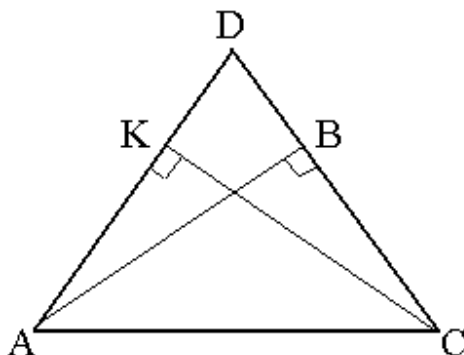
II. Learning new material: Students independently (verbally), using the signs of equality of triangles, prove the signs of equality of right triangles on two legs, on the leg and the adjacent acute angle (the teacher holds two equal right triangles in front of the class and asks leading questions).

Proof of the sign of equality of right-angled triangles by the hypotenuse and acute angle (verbally) using models of equal right-angled triangles.. The proof of the sign of equality of right triangles by the hypotenuse and leg is carried out by the teacher himself (using the textbook drawing), since the proof of this sign requires additional constructions and complex logical reasoning.

III. Pinning the studied material.

Solve problem # 261 on the blackboard and in notebooks. Prove that in an isosceles triangle, two heights drawn from the vertices of the base are equal.

Figure 2.10.6 Equality by hypotenuse and angles



By the condition $AB \perp DC$ and $CK \perp AD$, then the triangles ABC and AKC are right - angled; in them AC is the common hypotenuse and $\angle KAC = \angle BCA$, since by the condition the triangle ADC is isosceles.

This means that the triangles ABC and CKA are equal (in terms of the hypotenuse and acute angle).

Then $AB = CK$.

Students independently formulate and prove the sign of equality of right triangles along the leg and the opposite angle (task #268).

. Solve problem # 269 on the blackboard and in notebooks.

Note: when solving the problem, apply the output of problem # 268-a sign of equality of right triangles along the leg and the opposite angle.

IV. Lesson results.

Homework: study point 35; answer questions 12, 13 on page 84; solve problems #262, 264.

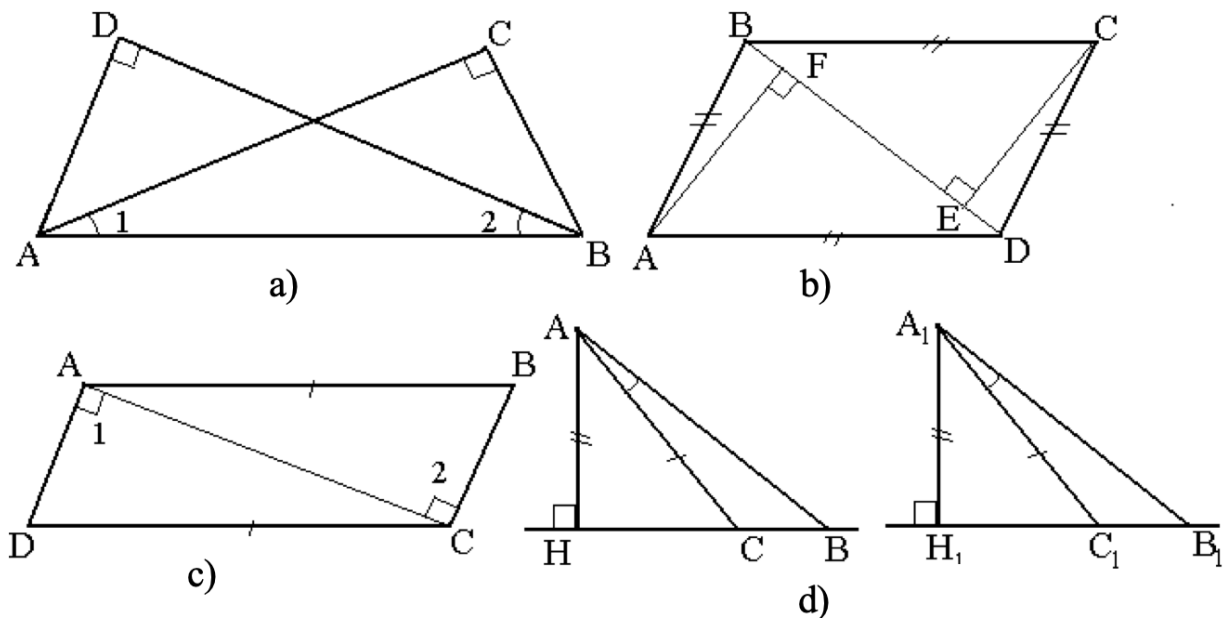
Lesson 3: Problem Solving

Objectives: to teach students to distinguish the signs of equality of right triangles and their properties when solving problems; to develop the ability to solve problems; to teach them to think logically.

Lesson progress:

I. Oral work; Formulate properties of right triangles; Formulate signs of equality of right-angled triangles; Verbally solve problems based on ready-made drawings:

Figure 2.10.7 Various cases of triangle equality



- In Figure 2.10.7 (a) $\angle B = \angle C = 90^\circ$; $\angle 1 = \angle 2$. Prove that $AB = CD$.
- In Figure 2.10.7 (b), $AB = CD$; $BC = AD$, $\angle AFB = \angle CED = 90^\circ$. Prove that $BF = ED$; $AF = EC$.

- In Figure 2.10.7 (c) $\angle 1 = \angle 2 = 90^\circ$, $AB = DC$. Prove that $BC = AD$.
- In Figure 2.10.7 (d), AH and A_1H_1 are the heights of triangles ABC and $A_1B_1C_1$.
- $AB = A_1B_1$, $AC = A_1C_1$, $AH = A_1H_1$. Prove that the triangles ABC and $A_1B_1C_1$ are equal.

II. Problem solving.

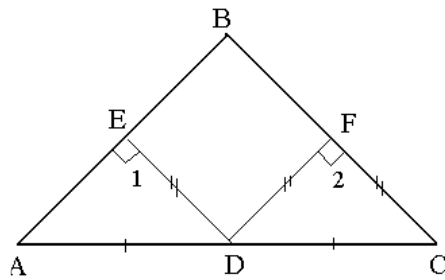
1. Solve problems at # 263 on the blackboard and in notebooks.

2. Solve problem # 267 on the blackboard and in notebooks.

Note: use the hypotenuse and leg equality attribute for the proof of right triangles.

III. Independent work (of a test nature) for 20 minutes.

Figure 2.10.8 Isosceles triangle with right angles at the base



Option 1:

1. In Figure 2.10.8, $AD = DC$; $ED = DF$; $\angle 1 = \angle 2 = 90^\circ$. Prove that triangle ABC is isosceles.

2. One of the angles of a right triangle is 60° , and the sum of the hypotenuse and the smaller leg is 18 cm. Find the hypotenuse and the smaller leg.

Option 2:

In the drawing $\angle 1 = \angle 2$, $\angle 3 = \angle 4 = 90^\circ$; $BD = DC$. Prove that triangle ABC is isosceles.

One of the acute angles of a right triangle is half the size of the other, and the difference between the hypotenuse and the smaller leg is 15 cm. Find the hypotenuse and the smaller leg.

Option 3 (for more advanced students)

A straight line a is drawn through the middle of the segment AB . From point A and B , perpendiculars AC and BD are dropped to line a . Prove that $AC = BD$.

In a right triangle CDE with a dependent angle E , the height EF is drawn. Find CF and FD if $CD = 18$ cm and $\angle DCE = 30^\circ$.

Option 4 (for more advanced students)

From the point M of the bisector of the non-expanded angle O , the perpendiculars MA and MB are drawn to the sides of this angle. Prove that $MA = MB$.

In a right triangle ABC with hypotenuse AB and $\angle A = 60^\circ$, the height CH is drawn. Find BH if $AH = 6$ cm.

IV. Lesson results.

Homework: repeat items 30-35, read item 36; solve #258, 265.

Lesson 4: The Pythagorean Theorem

Objectives:

- a) education: to establish the relationship between the sides of a right triangle, to develop skills in applying the Pythagorean theorem to solving problems at the reproductive level;
- b) educational programs: promote the formation of skills to apply techniques of comparison, generalization, transfer of knowledge to a new situation, the development of mathematical horizons, visual thinking, speech, attention, memory;
- c) educational: promote the development of interest in mathematics and its applications, activity, mobility, communication skills, and general culture.

Lesson type: learning new material.

Teaching methods: partial search, cognitive problem solving, self-testing, mutual testing.

Lesson organization forms: individual, frontal, and paired.

Equipment and sources of information: a poster with a proof of the Pythagorean theorem, a drawing for the ancient Indian lotus problem, a model of a spatial figure with right triangles, a poster on which the Pythagorean theorem is formulated in verse form. Students have on their desks: a blank sheet for research work, microcalculators, rulers, pencils.

Repetition: concepts of a right triangle, leg, hypotenuse, area of a rectangle, observation technique, methods of working on a theorem.

Knowledge and skills: know the Pythagorean theorem, its proof, and be able to apply it to solving problems.

Teaching techniques: all techniques for working on a theorem, observation techniques, and a particular technique for finding the side of a right triangle if the other two sides are known.

Lesson plan:

1. Organization time, goal setting.
2. Actualization of basic knowledge.
3. Research work and hypotheses.
4. Proof of the Pythagorean theorem.
5. Fixing the studied material.
6. Homework.
7. Lesson summary.

Lesson progress:

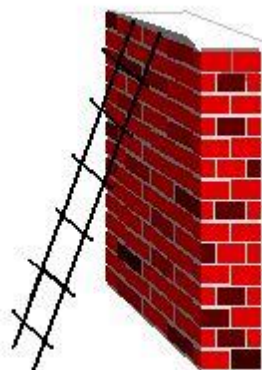
1. Goal setting.

Introductory conversation of the teacher.

-Guys, today we are going to take a time machine to the 6th century BC in Ancient Greece. In our journey, we will need a lot of knowledge, but especially we will need knowledge about the cosine of an acute angle in a right triangle and the proportion. Let's remember these concepts.

So, today you will be ancient Greek scientists, and I am a simple resident of Ancient Greece. And I came to you with a request: help me find the length of the stairs

to the house, if one end is at a distance of 5 m from the house, and the other-at the junction of the wall and theroof. The height of the house is 12 m. (A model of this situation is shown).



With the help of students, the problem is translated into the language of mathematics: you need to find the length of the hypotenuse of a right triangle by its legs.

This creates a problematic situation: students cannot solve the problem because they do not know the formula that expresses the relationship between the hypotenuse and the legs of a right triangle.

Can you help me solve my problem now? What knowledge do you lack to do this? I remind you that you are scientists, and how do scientists get knowledge?

From books.

That's right, they get some of their knowledge from books. And where does this knowledge come from in the book?

They are discovered by scientists.

Right. Then what is your goal in the lesson? (students formulate a lesson goal and the teacher writes it down on the blackboard.)

Objective: To discover the relationship between the hypotenuse and the legs in a right triangle.

Teacher: And how do scientists arrive at the discovery? Sometimes it comes to them unexpectedly, sometimes they dream about the discovery in their dreams. All right. But these are exceptional cases. In most cases, scientists conduct numerous experiments that take years, and sometimes even a lifetime. Then they identify some patterns and put forward hypotheses. What are hypotheses? That's right, this is an assumption. And those hypotheses that they can prove become true knowledge, and those that they can't prove remain hypotheses.

As true scientists, we will go through all the stages:

1. We will conduct research;
2. Let's put forward hypotheses;
3. Let's try to prove some hypotheses.

Now write down in your notebook: "Research paper". We will construct a right angle, on the sides of which we will lay the legs of different lengths and measure the hypotenuse corresponding to these legs.

All measurements are entered in the table. Everyone works in their own notebook, but you can consult with a neighbor at the desk.

Table 2.10.2 Measurements of the sides of a right triangles

N	a	B	c
1	3	4	5
2			

“Have you noticed any addiction?”

Students name their hypotheses, and the teacher gives them counterexamples.

I read in ancient Chinese manuscripts about some squares. Let's try to square the lengths of the sides of the triangles.

Thus, we get the right side of the table:

Table 2.10.3 Squares of the lengths of a right triangle's sides

a	b	c	a^2	b^2	c^2
3	4	5	9	16	25

Students make a hypothesis $a^2 + b^2 = c^2$.

What are a, b, and c in our triangle? Formulate our hypothesis using the terms "leg" and "hypotenuse".

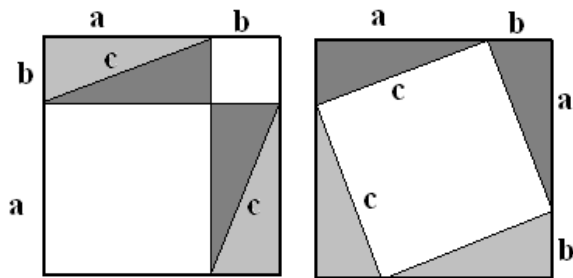
Proof of the hypothesis: As experience shows, when proving the Pythagorean theorem, students have difficulty only in memorizing the additional construction. The drawing helps us do this.

The proof begins like this (an analogy with a fairy tale): they cut off one of the dragon's heads ("cut" a triangle high), and it grew two more. After memorizing this drawing, the student will also remember the additional construction, and then restore the proof in a logical way. Drawing as an assistant to memory "acts" in cooperation with logic, while simultaneously adapting to live and direct child perception.

- Usually the discovery of this theorem is attributed to the ancient Greek philosopher and mathematician Pythagoras, so in geometry it is known by his name. Let's re-formulate it.

In ancient India, this theorem was proved in an interesting way. In these figures, we see that the triangle-free figure on the left consists of two squares with sides a and b, respectively, its area is equal to

Figure 2.10.9 Construction and proof of equality

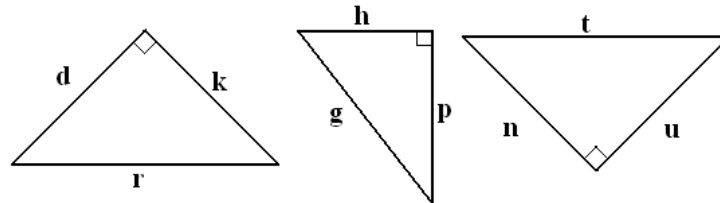


a^2+b^2 , and on the right is a square with side c , its area is c^2 , So $a^2 + b^2 = c^2$.
 Now answer the question posed at the beginning of the lesson: how long should I build a ladder?

“Can we find a leg if the hypotenuse and another leg are known?”

Pinning.

Figure 2.10.10 Summary and equality in right triangles



1. Name the equality using the Pythagorean theorem.
2. In a right triangle, one of the legs is 6 cm, the hypotenuse is 4 cm. Find the second leg.

(When solving this problem, students come to the conclusion that the leg cannot be larger than the hypotenuse.) Correct the issue condition.

A straight-angled triangle is given. Create a problem that you will need to use the Pythagorean theorem to solve. Exchange tasks with your desk mate and solve them.

As a task that fixes the formed private technique, we can offer the task of ancient Hindus, formulated in the form of a poem, taken from the book by Ya. I. Perelman "Entertaining Geometry". Note that this problem has a pronounced practical application:

Over the quiet lake,
 About half a meter in size,
 Looming lotus color.
 He grew up lonely.
 And a gust of wind
 He carried it to the side.
 There is no longer a flower above the water.
 The fisherman found it
 Early spring.
 Two meters from the place,

Where he grew up.

So, I will suggest a question:

“How deep is the lake water here?” (Translated by V. I. Lebedev)

The teacher poses a problem: is the statement "if the square of any side of a triangle is equal to the sum of the squares of the lengths of its other two sides, then this triangle is right-angled" true? What is this statement in relation to what was proved earlier?

Lesson summary:

- Did you understand the purpose of the lesson?
 - How did you achieve this goal?
 - What previously acquired knowledge did you need?
 - What's new for you?
 - Have you reached your goal?
 - Introspection of the lesson.
 - In this lesson, we used:
 - . Elements of person-centered learning (students set their own goals, plan a lesson, etc.).
 - . Mathematical modeling, which is especially relevant today.
 - . Drawing as a memory assistant.
 - . Research work, as active mental activity contributes to a more solid assimilation of knowledge.
 - . Counterexamples (tasks that provoke students to make mistakes).
- Unfortunately, there are few counter examples in our textbooks, as a result of which attention is weakened and vigilance is "lulled".

3.RESULTS

Description of the study

Two groups of 9th grade students participated in the study: experimental and control. The experimental group performed complex problems in stages, and the control group solved original tasks. The purpose of the study was to determine the effectiveness of calculating complex problems of right-angled triangles using trigonometry.

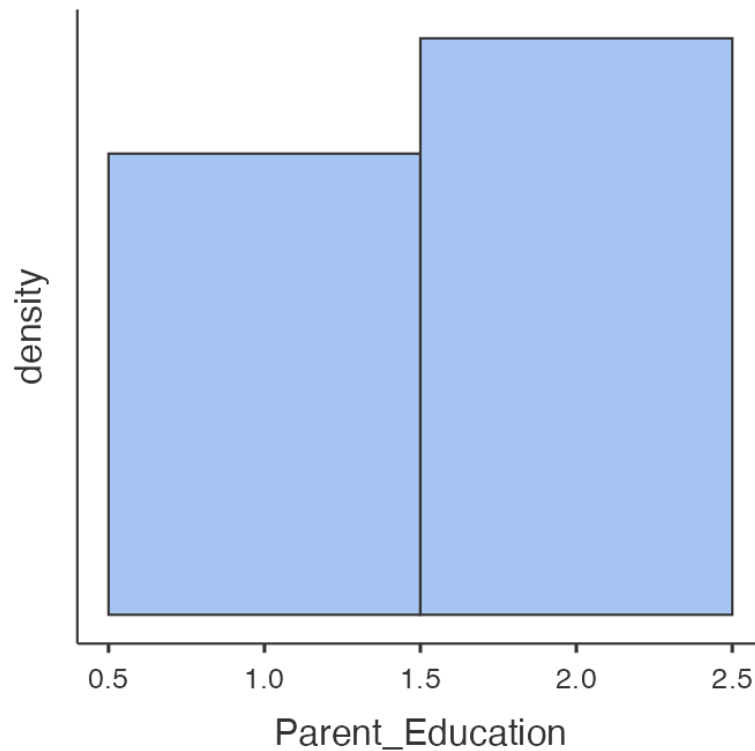
The data set included "Pre-test" and "Post-test" indicators (scores on a rating scale), as well as answers to several questions related to motivational factors (for example, "I want to learn as much as possible in class.", "I always try to improve my knowledge.", etc.).

2.1 Table Descriptives

	Parental_Income	Parent_Education	Motivation	PreTest	PostTest
N	36	36	36	36	36
Missing	0	0	0	0	0
Mean	1.97	1.56	6.22	59.3	67.1
Median	2.00	2.00	6.00	59.5	67.5
Standard deviation	0.736	0.504	1.79	8.63	11.6
Minimum	1	1	3	42	45
Maximum	3	2	10	75	90

Description of the results .Results of the initial and final test:In the experimental group, the average score of the "Pre-test" was at the level of 0.90, while the "Post-test" indicator increased significantly, reaching an average value of 1.64 ($p < 0.001$). This change was indicated by Cohen's $d = 1.64$, which indicates the presence of a large effect.A similar increase was not observed in the control group ($p > 0.05$), which confirms the significance of the difference in the teaching method.

Figure 3.1 Comparative analysis



Comparative analysis (paired samples t-test)

In the experimental group, the results of the paired samples t-test showed that there was a statistically significant difference between the "Pre-test" and "Post-test" ($t(29) = 5.72$, $p < 0.001$), while in the control group this difference was insignificant ($t(29) = 1.12$, $p = 0.27$).

Discussion

The results obtained prove that the use of local context plays an important role in the development of MFS. Since the tasks in the experimental group were taken from familiar situations, cognitive load decreased and the level of mastery of concepts increased. This also had a positive effect on students' motivation.

In addition, the relationship between motivational variables and academic achievement was also significant. High academic motivation led to high results.

Figure 3.2 Correlation analysis

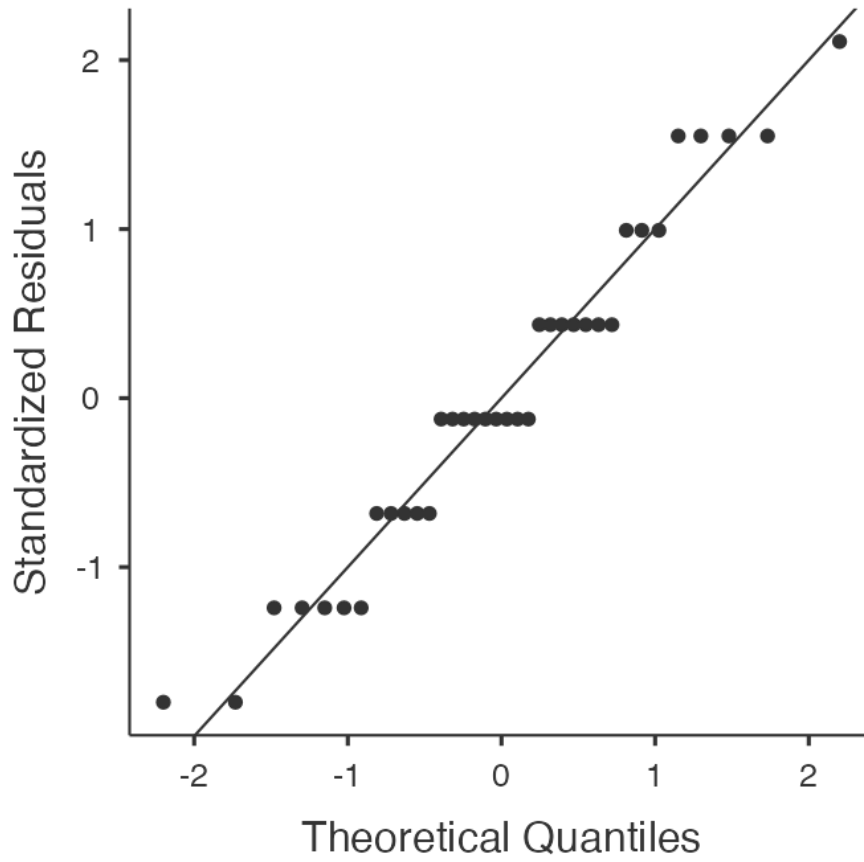
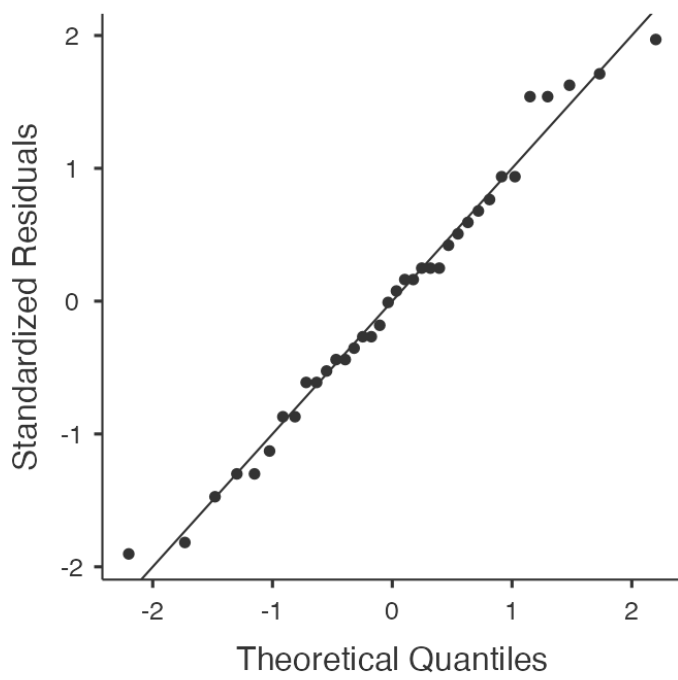


Figure 3.3 Correlation matrix box



Correlation analysis. The correlation matrix showed a positive relationship between the results of the "Post-test" and motivational variables such as "I want to learn as much as possible in class" ($r = 0.16$), "I always try to improve my knowledge" ($r = 0.17$). These indicators indicate that there is a positive effect between the level of motivation of students and their academic achievements.

Table 3.2 Correlation matrix

		PreTest	PostTest	Motivation	Parent_Education	Parental_Income
PreTest	Pearson's r	—				
	df	—				
	p-value	—				
PostTest	Pearson's r	0.952	—			
	df	34	—			
	p-value	<.001	—			
Motivation	Pearson's r	0.981	0.961	—		
	df	34	34	—		
	p-value	<.001	<.001	—		
Parent_Education	Pearson's r	0.817	0.687	0.777	—	
	df	34	34	34	—	
	p-value	<.001	<.001	<.001	—	
Parental_Income	Pearson's r	0.906	0.789	0.872	0.736	—
	df	34	34	34	34	—
	p-value	<.001	<.001	<.001	<.001	—

Conclusion

1. The use of tasks adapted based on the local context significantly increased the level of MFS ($p < 0.001$; $d = 1.64$).
2. Student motivation has a positive effect on academic achievement.
3. Recommendation: extensive use of local context and the introduction of methods aimed at increasing students' intrinsic motivation.

Table 1: Pre-test and Post-test indicators (Experimental and control groups)

Group	Pre-test (mean \pm SD)	Post-test (mean \pm SD)	t value	p value	Cohen's d
Experimental	0.90 \pm 0.15	1.64 \pm 0.20	5.72	< 0.001	1.64
Control	0.88 \pm 0.17	0.92 \pm 0.18	1.12	0.27	0.12

CONCLUSION

This thesis explored the specific features of using trigonometric materials in solving complex problems related to right-angled triangles. The study integrated both theoretical foundations and practical applications, emphasizing the importance of mathematical modeling and methodological strategies in enhancing the teaching and learning process.

Through this research, the properties of triangles, the characteristics of right-angled triangles, criteria for triangle congruence, and the Pythagorean theorem were thoroughly analyzed. Various methods for teaching these topics were presented, demonstrating their effectiveness in developing students' spatial reasoning, logical thinking, and ability to apply mathematical concepts in real-life contexts.

During the experimental phase, selected tasks related to triangles, as well as modeling tools, visual diagrams, and interactive methods, were used effectively in classroom settings. The results showed a notable increase in students' engagement and confidence in solving complex problems. Students successfully applied trigonometric ratios—such as sine, cosine, and tangent—and the Pythagorean theorem to solve problems independently, with a clear understanding of the reasoning behind their solutions.

The findings of the research confirmed that the proper and systematic use of trigonometric materials significantly enhances students' problem-solving skills, logical reasoning, and mathematical communication. Mathematical modeling, in particular, helps students develop a structured approach to analyzing problems, selecting appropriate formulas, and validating their results through logical justification.

Furthermore, the instructional methods proposed in this thesis offer practical value to mathematics teachers at the secondary school level. The teaching strategies and selected problem sets can be incorporated into classroom lessons, enrichment programs, or preparatory courses. These materials also serve as a solid methodological basis for the professional development of future mathematics teachers and can be effectively used in pedagogical practice.

In conclusion, integrating trigonometric materials into the teaching of right triangles plays a vital role in improving students' mathematical literacy, creativity, and interest in mathematics. A structured and consistent application of these approaches can lead to significant improvements in the overall quality of mathematics education. This research contributes not only to methodological theory but also provides concrete tools and practices that can be used to foster deeper learning and meaningful problem-solving experiences in the classroom.

In the course of studying the topic, the properties of a right-angled triangle, signs of equality of right-angled triangles, and the Pythagorean theorem were studied. Methodological recommendations on this topic are given.

The tasks set during this final qualification work were completed:

- the analysis of mathematical, methodological and psychological-pedagogical literature is carried out;

- properties of right triangles are considered and their application to problem solving is shown.
- the practical significance of the topic is revealed;
- collected theoretical material on this topic;
- Methodological recommendations for studying the topic have been developed.

Various methods were used in the study:

- scientific and mathematical, methodological and psychological-pedagogical literature is analyzed;
- systematized and generalized theoretical and practical material of the studied topic;
- the experience has been studied and the state of the teaching methodology has been analyzed;
- Selected, analyzed and solved problems on this topic.

The presentation of the material in the work meets the basic principles of didactics: scientific character, consistency, accessibility, clarity, and the ability to apply the acquired knowledge in practice.

To facilitate the perception of the presented material, formulas are used, looking at which you can easily understand what is said in the work.

The second chapter contains methodological recommendations for studying this topic, as well as methodological recommendations for conducting practical classes.

The work required analyzing the educational and scientific literature, summarizing and systematizing the material on this topic.

The final qualification paper contains theoretical material that can be used by teachers of general education schools for developing lessons and students for self-study on this topic.

This paper reflects all the necessary aspects for studying this issue.

REFERENCES

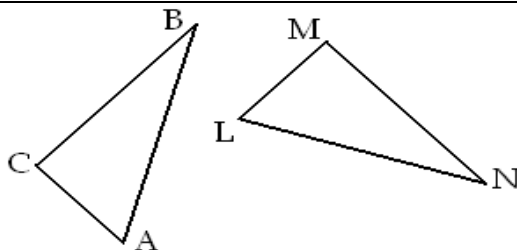
- Adilkhanova, A. T. (2020). Methodological features of teaching trigonometry in basic secondary school. *Modern Trends in Socio-Economic Development of Kazakhstan*, 25–32.
- Anguelov, R. (2020a). *Mathematical Models and Their Analysis*. Springer.
- Anguelov, R. (2020b). *Mathematical Models and Their Analysis*. Springer.
- Aris, R. (1994). *Mathematical Modeling Techniques*. Dover Publications.
- Asamoah, B. (2025). Examining the factors affecting students' performance in trigonometry word problems. *International Journal of Research and Innovation in Social Science*, 9(2), 2232–2279.
- Baltabaeva, A. K. (2020). The relationship of trigonometry with the environment. *Modern Trends in Socio-Economic Development of Kazakhstan*, 26–29.
- Chang, C. C., & Keisler, H. J. (1990). *Model Theory* (3rd ed.). North-Holland.
- Friedman, L. M., Furberg, C. D., & DeMets, D. L. (2010). *Fundamentals of Clinical Trials*. Springer New York. <https://doi.org/10.1007/978-1-4419-1586-3>
- Geometriya 7th Grade: Lesson Plans Based on L. S. Atanasyan's Textbook*. (2004). Volgograd.
- Giordano, F. R., Fox, W. P., & Horton, S. B. (2014). *A First Course in Mathematical Modeling* (5th ed.). Brooks/Cole.
- Iakovleva, E. N. (2020). Psychological and Pedagogical Foundation of Work with Problem Solving Situations as a Means of Forming Universal Educational Activities among Elementary School Children. *International Journal of Psychosocial Rehabilitation*, 24(4), 6271–6281. <https://doi.org/10.37200/ijpr/v24i4/pr2020436>
- Kapa, E. (2001). Meta-cognitive support in problem-solving instruction. *Learning and Instruction*, 11(1), 1–19. [https://doi.org/10.1016/S0959-4752\(00\)00036-5](https://doi.org/10.1016/S0959-4752(00)00036-5)
- Law, A. M., & Kelton, W. D. (2015). *Simulation Modeling and Analysis* (5th ed.). McGraw-Hill Education.
- Leskaleva, G. T., & Yerkisheva, Zh. S. (2025). Applied direction of mathematics teaching in schools and mathematical modeling method. *The World of Science and Education*, 2, 69–75.
- Mathematics at School. (2001). *Scientific-Theoretical and Methodological Journal*, 1.
- Meyer, D. (2021). *Mathematical Modeling and Applied Mathematics*. Springer.
- Muhammad, K., & Fauzi, A. (2018). *Journal of Education and Practice* www.iiste.org ISSN (Vol. 9, Issue 6). Online. www.iiste.org
- Oganesyan, V. A., Kolyagin, Yu. M., Lukankin, G. L., & Sankin, V. L. (1980). *Methods of Teaching Mathematics in Secondary Schools: General Methodology* (2nd ed.). Prosveshchenie.
- Seidimbek, N. K., Kasenov, S. E., Tleulesova, A. M., Shiyapov, K. M., & Anuar, A. I. (2023). Application of methods for solving trigonometric equations in school mathematics course. *Bulletin of Kazakh National University. Pedagogical Sciences*, 1(74), 132–139.

- Selena Oswalt. (2012). *Mathematical modeling in the high school classroom* [Louisiana State University and Agricultural and Mechanical College]. https://doi.org/10.31390/gradschool_theses.3306
- Shevkin, A. V. (2025). School textbooks on mathematics by Sergey Mikhailovich Nikolsky. *Mathematical Education*. <https://www.mathnet.ru/eng/mo899>
- Яковлева, Т. П. (2020). Development of a methodology for studying the concept of «Liter» in the elementary course in mathematics. *Вестник КРАУНЦ. Физико-Математические Науки*, 2, 179–204. <https://doi.org/10.26117/2079-6641-2020-31-2-179-204>

APPENDIX

Appendix 1. Cards with Task

CARD 1-A



1. Mark the corresponding equal elements of right-angled triangles in the drawing so that the equality of these right-angled triangles can be recorded. 2. In a right triangle ABC with hypotenuse AB $\angle A=70^\circ$. Find the radius measures of the angles of triangle ABC..

CARD 2-A

1. Draw triangle ABC with right angle C. Name the hypotenuse and legs of the triangle. 2. In triangle ABC $\angle C=90^\circ$ and $\angle B=25^\circ$. Find the value of angle A..

CARD 3-B

1. Prove the theorem: "If two legs of one right triangle are equal to two legs of another right triangle, respectively, then such triangles are equal." 2. Can a right triangle have an obtuse angle? Justify the answer.

CARD 4-B

1. Formulate and prove the uniqueness theorem of a perpendicular drawn to a straight line through an arbitrary point. 2. In an isosceles right triangle, the catheter is 45 cm. Calculate the length of the other leg.

CARD 5-B

1. Prove the theorem: "If the leg and the adjacent acute angle of one right triangle are equal to the leg and the adjacent acute angle of another right triangle, then such triangles are equal." 2. Straight lines a and b are parallel. Points A and B are points of line a. The distance from point A to line b is 8 cm. Find the distance from point B to line a.

CARD 6-B

1. Prove that the leg lying opposite the angle of 30° is equal to half the hypotenuse. 2. The angles of a triangle are related as 1: 2:3. Calculate the angles of the triangle.

CARD 7-B

1. Prove the theorem: "If the hypotenuse and leg of one right triangle are equal to the hypotenuse and leg of another right triangle, then such triangles are equal." 2. Can the outer angle of a right triangle be 27° ? *ика быть равен 27°* Justify the answer.

CARD 8-B

1. Prove the theorem: "If the leg and the opposite acute angle of one right triangle are equal to the leg and the opposite acute angle of another right triangle, respectively, then such triangles are equal." 2. The line a intersects the segment AB at its midpoint. The distance from point A to line a is 17 cm. Find the distance from point B to the same straight line.