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Conservative extension in various classes of complete theories models

THESIS

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ABSTRACT

This thesis is devoted to the in-depth study of the Bektur Baizhanov's "conservative extension of models of weakly o-minimal theories" and the solution of the problem whether it is possible to determine through a conservative extension, non-locally isolated type. I studied the concept of rational section quasi-rational section and irrational section, extension and elementary extension, orthogonality of types, basic properties of types. Notion of quasi-model (or Tarskii-Vaught type) and any types will be isolated or non-isolated and non-isolated type in turn divides by two kind. It's locally isolated (another word strictly definable) and non-locally isolated. Moreover any types will be definable and non-definable and any isolated type is definable. But is an non-isolated type definable? I was looking for the answer to that question and answered in this thesis. Using the concept of a control formula, I proved that a non-locally isolated type can be definable.

АННОТАЦИЯ

Этот тезис посвящен углубленному изучению «консервативного расширения моделей слабо о-минимальных теорий» Бектура Байжанова и решению проблемы, возможно ли определить с помощью консервативного расширения нелокально изолированного типа. Я изучал понятия рационального сечения, квазирационального сечения и иррационального сечения, расширения и элементарного расширения, ортогональности типов, основных свойств типов. Понятие квазимодели (или типа Тарского-Вота) и любых типов будет изолированным или неизолированным, а неизолированный тип, в свою очередь, делится на два вида. Он локально изолирован (другое слово строго определимо) и не локально изолирован. Более того, любые типы будут определимыми и не определимыми, а любой изолированный тип - определяемым. Но можно ли определить неизолированный тип? Я искал ответ на этот вопрос и отвечал в этом тезисе. Используя концепцию управляющей формулы, я доказал, что не-локально изолированный тип может быть определим.

АҢДАТПА

Бұл жұмыс Бектур Байжановтың “әлсіз о-минималды моделдің консервативті кеңейтулері” атты макаласын тереңірек зерттеуге және локалды изоляцияланбаған типтің анықталуын карастырады. Мен иррационалды қиылысу, квази-рационалды қиылысу жане де иррационалды қиылысуды, кеңейтуді жане элементарлы кеңейтуді, типтің ортогональдығын, типтің түрлерин қарастырдым. Квазимодель түсінігін (немесе Тарскии-Вот типін) және де кезкелген тип не изолационланған немесе изоляционланбаған болып бөлінетінін, және де изоляционланбаған тип өз ішінде локалды изоляционланған (немесе қатты анықталғын) не локалды изояционланбаған болып бөлінетінін карастырыған. Типтер тағы да анықталған және анықталмығын болып бөлінеді, ал кезкелген изоляционланған тип ол анықталған болады. Бірақ изоляционланбаған оның ішінді локалды да изоляционланбаған типті қалай анықтауға болады деген сураққа жауап іздеп осы тезисте дәлелдедім. Басқарушы формула қасиетін пайдалана отырып кезкелген изоляционланбаған оның ішінде локалды да изоляционланбаған типті анықталған және анықталмаған болып бөлуге болатынын корсеттім.

Contents

1. Complete type

1. Language, model structure, satisfaction

(1.1) **Language**

Collection of finite or infinite symbols L is called a language. There are three kind of symbols which define language:

- relation symbol
- function symbol
- constant symbol

For the function symbols each symbol associated with the number $n(f) \in \mathbb{N}^{>0}$, which called a arity of function f, and with the R-number $n(R) \in \mathbb{N}$, which is called a arity of relation R.

unction and relation symbols denoted respectively as f_i and R_j. Constant symbols denoted as c_k.

An $\mathcal{L}$- structure $\mathcal{M}$ is defined as follows:

- A set M is called the universe of $\mathcal{M}$.
- A function $f^M : m^{n(f)} \to M$ for every function symbol $f \in \mathcal{L}$ is called as an interpretation of f in $\mathcal{M}$.
- A subset R^M of $M^{n()R} \to M$ for every relational symbol $R \in M$ is called as an interpretation of R in $\mathcal{M}$.
- An element $c^M \in M$ for every constant symbol $c \in \mathcal{L}$ is called as an interpretation of c in $\mathcal{M}$.

The structure M has the following notation:

$$\mathcal{M} = (M, f_i^M, R_j^M, c_k^M | i \in I, j \in J, k \in K)$$

We have some cases when superscript M disappears, and we denote structure and its universe by only one letter. It makes confusion if we have more than one type of structure on M.

(1.3) Substructures. Consider M as an $\mathcal{L}$-structure. Then $\mathcal{L}$-substructure of M, or just substructure of M (if only one substructure), is a $\mathcal{L}$-structure N, with universe which is contained in universe of M and where interpretations of the symbols of $\mathcal{L}$ in N restricts interpretations of these symbols in M, as follows:

- The interpretation of f in N restricts function symbol of $\mathcal{L}$, f^M to $n^{n(f)}$,

- The interpretation of R in N restricts relation symbol of $\mathcal{L}$, $R^N = R^M \cap N^{n(R)}$.
- The interpretation of c in N restricts constant symbol of $\mathcal{L}$, $c^M = c^N$.

If a subset of M has all constants of $\mathcal{L}$ and also closed under functions of $\mathcal{L}$ (interpretations of M), then subset of M can be called the universe of substructure of M. It works for another way also. Also we should note that if we have language without constant symbols, then then empty set is the universe of substructure of M.[34]

(1.4) Instances of dialects, structures, and substructures. The solid structures considered in model hypothesis all originate from standard arithmetical precedents, thus the precedents given underneath will be extremely well-known to you.

Model 1 - The language of gatherings. The language of groups,$\mathcal{L}_G$, is the language $\{\cdot, ^{-1}, 1\}$ where $\cdot$is a 2-ary work symbol,$^{-1}$is an unary capacity image, and 1 is aconstant image.

Any gathering G has a characteristic $\mathcal{L}_G$-structure, gotten by translating $\cdot$ as the gathering augmentation, $^{-1}$ as the gathering reverse, and 1 as the unit component of the gathering.

A substructure of the gathering G is then a subset containing 1, shut under increase and reverse: it is essentially a subgroup of G.

This is a decent spot to comment that the thought of substructure is touchy to the language. While the converse capacity and the character component of the gathering G are retrievable (quantifiable) from the gathering augmentation of G, the thought of "substructure" intensely relies upon them. For example, a$\{\cdot, e\}$-substructure of G is just a submonoid of G containing e, while a$\{\cdot\}$-substructure of G can be unfilled.

Model 2 - The language of charts . The language comprises of a paired connection image. E. Diagrams which have all things considered one edge between two vertices are the {E}-structures: essentially decipher E (x,y) if and just there is an edge going from x to y. Charts in which there can be a few edges between two vertices need an increasingly refined language.

Precedent 3 - The language of rings. The language of rings,$\mathcal{L}_R$, is the language$\{+, -, \cdot, 0, 1\}$, where +,− and $\cdot$ are double capacities. 0 and 1 are constants.

A (unitary) ring S has a characteristic $\mathcal{L}_R$-structure, gotten by deciphering +,−,$\cdot$ as the typical ring tasks of expansion, subtraction and duplication, 0 as the personality component of +, and 1 as the unit component of S.

A substructure of the $\mathcal{L}_R$-structure S is then basically a subring of S. Note that it will specifically contain the subring of S created by 1, i.e., a duplicate of $\mathbb{Z}$ or of $\mathbb{Z}/p\mathbb{Z}$.

When one arrangements with fields, it is in some cases helpful to include a image for the multiplicative reverse (denoted^{-1}). By show $0^{-1} = 0$.

Precedent 4 - The language of requested gatherings, of requested rings.

One basically adds to $\mathcal{L}_G$, resp. $\mathcal{L}_R$, a paired connection symbol,$\leq$.

(1.5)Morphisms, embeddings, isomorphisms, automorphisms. Give M and N a chance to be two $\mathcal{L}$-structures. A guide $s:M \to N$ is an($\mathcal{L}$)- morphism if for all connection image $R\in\mathcal{L}$,work image $f\in\mathcal{L}$, and tuples $\bar{a}, \bar{b}$ in M, we have:

$$\text{on the off chance that } \bar{a}\in R, \text{ at that point } s(\bar{a})\in R;\ s(f(\bar{b}))$$

An implanting is an injective morphism s:M→N, which fulfills what's more for all connection $R\in\mathcal{L}$and tuple $\bar{a}$ in M, that

$$\bar{a}\in R \Leftrightarrow s(\bar{a})\in R$$

An isomorphism among M and N is a bijective morphism, whose reverse is additionally a morphism. At long last, an automorphism of M is an isomorphism M→M.

(1.6)Terms. We can begin utilizing the images of $\mathcal{L}$ to express properties of a given $\mathcal{L}$-structure. Notwithstanding the images of $\mathcal{L}$, we will think about a lot of images (which we guess disjoint from $\mathcal{L}$), called the arrangement of coherent images. It comprises of

– coherent connectives $\wedge,\vee,\neg$, and once in a while additionally (for comfort) $\rightarrow$ and $\leftrightarrow$.

– brackets (and),

– a (double connection) image = for equity.

– unendingly numerous variable images, more often than not indicated x,y,x_i, and so forth...

– the quantifiers $\forall$ (for all) and $\exists$ (there exists).

Fix a language $\mathcal{L}$. A $\mathcal{L}$-recipe will at that point be a series of images from $\mathcal{L}$ and legitimate images, complying with specific standards. We begin by characterizing $\mathcal{L}$-terms (or just, terms). Generally, terms are articulations acquired from constants and factors by applying capacities. In any $\mathcal{L}$-structure M, a term t will at that point characterize exceptionally a capacity from a specific cartesian intensity of M to M. Terms are characterized by acceptance, as pursues:

– a variable x, or a steady c, are terms.

– on the off chance that $t_1,\dots,t_n$ are terms, and f is a n-ary work, at that point $f(t_1,\dots,t_n)$ is a term.

Given a term $t(x_1,\dots,x_m)$, the documentation demonstrating that the factors happening in t are among $x_1,\dots,x_m$, and a $\mathcal{L}$-structure M, we get a capacity $F_t:M^m \to M$. Again this capacity is characterized by acceptance on the multifaceted nature of the term:

– in the event that c is an a consistent image, at that point $F_c:M^0 \to M$ is the capacity $\varnothing \to c^M$.

– in the event that x is a variable, at that point $F_x:M \to M$ is the personality.

– if $t_1,\ldots,t_n$ are terms in the factors $x_1,\ldots,x_n$ and f is a n-ary work image, at that point $F_{f(t_1,\ldots,t_n)} : (x_1,\ldots,x_n) \mapsto f(F_{t_1}(\bar{x}),\ldots,F_{t_n}(\bar{x}))$ $(\bar{x} = (x_1,\ldots,x_n))$.

(1.7)Recipes. We are presently prepared to characterize recipes. Again they are characterized by enlistment. A nuclear recipe is an equation of the structure $t_1(\bar{x}) = t_2(\bar{x})$ or $R(t_1(\bar{x}),\ldots,t_n(\bar{x}))$, where $\bar{x} = (x_1,\ldots,x_m)$ is a tuple of factors, $t_1,\ldots,t_n$ are terms (of the language $\mathcal{L}$, in the factors $\bar{x}$), and R is a n-ary connection image of $\mathcal{L}$.

The arrangement of sans quantifier equations is the arrangement of Boolean mixes of nuclear recipes, i.e., is the conclusion of the arrangement of nuclear equations under the activities of $\wedge$ (and), $\vee$ (or) and $\neg$ (refutation, or not). In this way, if $\varphi_1(\bar{x}), \varphi_2(\bar{x})$ are sans quantifier recipes, so are $\varphi_1(\bar{x}) \bigwedge \varphi_2(\bar{x})$, $\varphi_1(\bar{x}) \bigvee \varphi_2(\bar{x})$ and $\neg\varphi_1(\bar{x})$.

One regularly utilizes $\varphi_1(\bar{x}) \rightarrow \varphi_2(\bar{x})$ as a shortened form for $(\varphi_1(\bar{x}) \bigvee \varphi_2(\bar{x})$,and $\varphi_1(\bar{x}) \leftrightarrow \varphi_2(\bar{x})$ as a condensing for $(\varphi_1(\bar{x}) \rightarrow \varphi_2(\bar{x})) \bigwedge (\varphi_2(\bar{x}) \rightarrow \varphi_1(\bar{x}))$.

An equation ψ is then a series of images of the structure

$$Q_1 x_1 Q_2 x_1 \ldots Q_m x_m \varphi(x_1,\ldots,x_n)$$

where $\varphi(\bar{x})$ is a without quantifier recipe, with factors among $\bar{x} = (x_1,\ldots,x_n)$ and $Q_1,\ldots,Q_m$ are quantifiers, i.e., have a place with $\{\forall,\exists\}$. We may expect $m \leq n$.

Significant: the factors $x_1,\ldots,x_n$ are assumed distinct:$\forall x_1\ \exists x_1$...is not permitted. In the event that $m \leq n$, the factors $x_{m+1},\ldots,x_n$ are known as the free factors of the equation ψ. One for the most part composes $\psi(x_{m+1},\ldots,x_n)$ to show that the free factors of ψ are among$(x_{m+1},\ldots,x_n)$. The factors $x_1,\ldots,x_m$ are known as the bound factors of ψ. In the event that n=m,then ψ has no free factors and is known as a sentence.

In the event that all quantifiers $Q_1,\ldots,Q_m$ are $\exists$, at that point ψ is called an existential recipe; on the off chance that they are all $\forall$, at that point ψ is known as an all inclusive equation. One can characterize a progressive system of unpredictability of equations, by checking the quantity of alternances of quantifiers in the string $Q_1,\ldots,Q_n$.Let us basically state that a Π_2-recipe, additionally called a $\forall\exists$-equation, is one in which $Q_1 \ldots Q_n$ is a square of $\forall$ pursued by a square of $\exists$, that a Σ_2-recipe, likewise called a $\exists\forall$-equation, is one in which $Q_1 \ldots Q_n$ is a square of $\exists$ pursued by a square of $\forall$. In these definitions, either square is permitted to be unfilled, so an existential equation is both a π_2 and a Σ_2-recipe. Let us likewise notice that a positive equation is one of the structure $Q_1 x_1 \ldots Q_m x_m \varphi(x_1,\ldots,x_n)$, where $\varphi(\bar{x})$ is a limited disjunction of limited conjunctions of nuclear formulas.[34]

(1.8) Warning. I lied, this isn't the standard meaning of a recipe. An equation as in (1) is said to be in prenex structure. The arrangement of equations in prenex structure isn't shut under Boolean activities. One has anyway an idea of "coherent proportionality", under which for example the equations $Q_1 x_1 Q_2 x_2 \ldots Q_m x_m \varphi(x_1,\ldots x_m, x_{m+1},\ldots x_n)$ and $Q_1 y_1 Q_2 y_2 \ldots Q_m y_m \varphi(y_1,\ldots y_m, x_{m+1},\ldots x_n)$ are sensibly equal. At that point it is very simple to see that a Boolean mix of recipes in prenex structure is consistently proportionate to an equation in prenex structure. E.g.

$$(Q_1x_1Q_2x_2...Q_mx_m\varphi_1(x_1,....x_n))\bigwedge(Q'_1x_1Q'_2x_2...Q_mx_m\varphi_2(x_1,....x_n))$$

is sensibly equal to

$$Q'_1x_1Q'_1y_2...Q_mx_mQ'_my_m\varphi_1(x_1,....x_n)\bigwedge\varphi_2(y_1,....y_m,x_{m+1},....x_n)$$

In the event that one needs to be conservative about the quantity of quantifiers.

one notes that when all is said in done $\forall x\varphi_1(x,...)\bigvee\forall x\varphi_2(x,...)$ is consistently proportional to $\forall x\varphi_1(x,...)\bigwedge\forall x\varphi_2(x,...)$, and $\exists x\varphi_1(x,...)\bigwedge\exists x\varphi_2(x,...)$ is sensibly identical to$\exists x\varphi_1(x,...)\bigvee\exists x\varphi_2(x,...)$. For nullifications, one uses the intelligent equality of $\neg Q_1x_1Q_2x_2...Q_mx_m\varphi(x_1,....x_n)$ with $Q'_1x_1Q'_2x_2...Q'_mx_m\neg\varphi(x_1,....x_n)$, where $Q'_i=\exists$ if $Q_i=\forall$, $Q'_i=\forall$ if $Q_i=\exists$. Accordingly the invalidation of a Π_2-recipe is a Σ_2-equation, and so on.

Consistent proportionality can likewise be utilized to revamp Boolean blends, and one can demonstrate that any without quantifier equation $\varphi(\bar{x})$ is intelligently equal to one of the structure $\bigvee_i\bigwedge_j\varphi_{i,j}(\bar{x})$, where the $\varphi_{i,j}$ are nuclear recipes or invalidations of nuclear recipes.

(1.9) Comments and models. The definitions given above are totally formal. When thinking about solid models, they get particularly disentangled, to concur with current utilization. The primary thing to note is that the recipe $(\neg(x=y))$ is condensed by x≠y.

Model 1.$\mathcal{L}_G$={·, −1,,1}. A term is developed from 1,·,$^{-1}$. Also, some variables.E.g.,·(1, ^{1})(·(x_1,^{1} (x_1)))) is a term, in the variable x_1. On the off chance that we work in a subjective $\mathcal{L}_G$-structure, i.e., not really a gathering, this articulation can't be rearranged. On the off chance that we work in a gathering, at that point we will as a matter of first importance change to the standard documentation of xy rather than ·(x,y) and x^{-1} rather than $_{-1}$ (x); at that point enable ourselves to utilize the associativity of the gathering law to get free of unessential brackets. The term above then ends up $1(x_1x_1^{-1})^{-1}$, which can be additionally rearranged to 1 (presently utilizing the characterizing properties of ^{1} and of 1). Starting now and into the foreseeable future, we will accept that our $\mathcal{L}_G$-structures are gatherings.

A term in the factors x_1,...,x_n is then essentially a word in the images $x_1,...,x_n,x_1^{-1},.....x_{n-1}$. One can complete a further decrease: supplant events of the equation $\omega_1(\bar{x})=\omega_2(\bar{x})$ by $(\omega_1(\bar{x})\omega_2(\bar{x}))^{-1}=1$. A nuclear recipe will at that point be a limited disjunction of limited conjunctions of conditions $\omega(\bar{x})=1$ and inequations $\omega(\bar{x})\neq 1$. The accompanying equation is a Π_2-recipe:

$$\forall \bar{x} \exists \bar{y}(\omega_1(\bar{x}, \bar{y}, \bar{z})) = 1 \bigwedge \omega_2(\bar{x}, \bar{z}) \neq 1$$

where $\omega_1(\bar{x}, \bar{y}, \bar{z})$ is a word in the components of the tuple $(\bar{x}, \bar{y}, \bar{z})$ and their inverses, and $\omega_2(\bar{x}, \bar{z})$ is a word in the components of $(\bar{x}, \bar{z})$ and their inverses. The free factors of this

equation are the components of the tuple $\bar{z}$, while the components of $(\bar{x}, \bar{y})$ are the bound factors of the recipe.

On the off chance that all structures considered are free gatherings, containing two non-driving components a,b, at that point a sans quantifier equation can be composed as limited disjunction of recipes of the structure

$$\omega(\bar{x}, a, b) = 1 \wedge \omega'(\bar{x}, a, b) \neq 1$$

Precedent 2. The language of charts {E}. The main terms are factors (since there are no capacity or steady images). Consequently a nuclear recipe is of the structure E (x,y) or (x=y). A case of recipe in this language is e.g.,

$$\exists y_1, \dots, y_m(\bigwedge_{i=1}^{m-1} E(y_i, y_{i-1}) \wedge E(x_1, y_1) \wedge E(y_n, x_2) \wedge \bigwedge_{1 \leq i < j \leq m} y_i \neq y_j).$$

Note the utilization of x≠y rather than ¬(x=y). The free factors of this equation are x_1, x_2.

Precedent 3. $\mathcal{L}_R = \{+, -, \cdot, 0, 1\}$. Once more, terms as characterized officially, are incredibly revolting.

In any case, on the off chance that all LR-structures considered are rings, they can be revised in an increasingly characteristic

style. Starting now and into the foreseeable future, all $\mathcal{L}_R$-structures are commutative rings.

On the off chance that $n \in N^{>1}$ the term 1 + 1 + · + 1 (n times) will just be signified by n. Essentially x + x + ... + x (n times) is meant by nx, and x · . . . · x (n times) by x n. A self-assertive term will at that point be of the structure $f(x_1, \dots, x_n)$, where $f(X_1, \dots, X_n) \in Z[X_1, \dots, X_n]$.

Sans quantifier recipes are limited disjunctions of limited conjunctions of conditions and

inequations. Along these lines, in the ring $\mathbb{C}$, they will characterize the standard constructible sets. On the off chance that one adds ≤ to the language, and expect that our structures are requested rings, at that point without quantifier recipes can be changed as limited conjunctions of limited disjunctions of recipes of the structure

$$f(\bar{x}) = 0, \quad g(\bar{x}) > 0$$

where f, g are polynomials over $\mathbb{Z}$. Here, $x < y$ represents $x \leq y \wedge \neg(x \neq y)$, and one employments the equivalences $x \neq 0 \Leftrightarrow x < 0 \vee x > 0$, $x > 0 \Leftrightarrow (-x) < 0$. In the event that M is an arranged ring, at that point M without quantifier equations will be as above, then again, actually f and g are polynomials over M. On the off chance that M is the arranged field R, one gets the standard semi-mathematical sets.

(1.10) Satisfaction. Let M be an $\mathcal{L}$-structure, $\phi(\bar{x})$ an $\mathcal{L}$-formula, where $\bar{x} = (x_1, \ldots, x_n)$ is a tuple of variables occurring freely in ϕ, and $\bar{a} = (a_1, \ldots, a_n)$ an n-tuple of elements of M. We wish to define the notion M satisfies $\phi(\bar{a})$, (or $\bar{a}$ satisfies ϕ in M, or $\phi(\bar{a})$ holds in M, is true in M), denoted by

$$M \models \phi(\bar{a})$$

(The negation of $M \models \phi(\bar{a})$ is denoted by $M \not\models \phi(\bar{a})$.) This is done by induction on the complexity of the formulas:

– If $\phi(\bar{x})$ is the formula $t_1(\bar{x}) = t_2(\bar{x})$, where t_1, t_2 are $\mathcal{L}$-terms in the variable $\bar{x}$, then

$$M \models t_1(\bar{a})\ if\ and\ only\ if\ F_{t_1}(\bar{a}) = F_{t_2}(\bar{a}).$$

– If $\phi(\bar{x})$ is the formula $R(t_1(\bar{x}), \ldots, t_m(\bar{x}))$, where $t_1, \ldots, t_m$ are terms and R is an m-ary relation, then

$$M \models R(t_1(\bar{a}), \ldots, t_m(\bar{a}))\ if\ and\ only\ if\ (F_{t_1}(\bar{a}), \ldots, F_{t_m}(\bar{a})) \epsilon R^M$$

– If $\phi(\bar{x}) = \phi_1(\bar{x}) \bigwedge \varphi_2(\bar{x})$, then

$$M \models \phi(\bar{a})\ if\ and\ only\ if\ M \models \phi_1(\bar{a})\ and\ M \models \phi_2(\bar{a})$$

– If $\phi(\bar{x}) = \neg\phi_1(bar x)$, then

$$M \models \phi(\bar{a})\ if\ and\ only\ if\ M \not\models \phi_1(\bar{a})$$

– If $\phi(\bar{x}) = \exists y \psi(\bar{x}, y)$ where the free variables of ψ anc among $\bar{x}$,y, then

$$M \models \phi(\bar{a})\ if\ and\ only\ if\ there\ is\ b \epsilon M\ such\ that\ M \models \psi(\bar{a}, b).$$

– If $\phi(\bar{x}) = \forall y \psi(\bar{x}, y)$, then
$M \models \phi(\bar{a})\ if\ and\ only\ if\ M \models \neg(\exists y \neg\psi(\bar{a}, y))$ if and only if for all b in M, $M \models \phi(\bar{a}, b)$.

(1.11) Parameters and definable set. Let M be an $\mathcal{L}$-structure i.e structure in our language, $\phi(\bar{x}, y)$ a formula ($\bar{x}$ an n-tuple of variables, $\bar{y}$ an m-tuple of variables), and $\bar{a} \in M^m$. Then the set $\{\bar{b} \in M^m — M —= \phi(\bar{a}, \bar{b})\}$ is called a definable set. We also say that it is defined over $\bar{a}$ by the formula $\phi(\bar{a}, \bar{y})$, or that it is $\bar{a}$-definable. The cortege or tuple $\bar{a}$ is a parameter of the formula $\phi(\bar{a}, \bar{y})$ in our language $\mathcal{L}$.

Let M be an $\mathcal{L}$-structure. The set of M-definable subsets of M^n is clearly closed under unions, intersections and complements (corresponding to the use of the logical connectives $\bigvee, \bigwedge$ *and* $\neg$). If $S \subseteq M^{n+1}$ is defined by the formula $\varphi(\bar{x}, \bar{a})$, $\bar{x} = (x_1, \ldots, x_{n+1})$, and $\pi : M^{n+1} \to M$ is the projection on the first n coordinates, then $\pi(S)$ is defined by the formula $\exists x_{n+1}\varphi(\bar{x}, \bar{a})$, and the complement of $\pi(S)$ by the formula $\forall x_{n+1}\neg\varphi(\bar{x})$.[34]

(1.12) The examples, revisited. (1) $\mathcal{L}_G = \{\cdot, ^{-1}, 1\}\}$. Consider the formula $\varphi(x, y)$: $xy = yx$. Let G be a group (endowed with its natural $\mathcal{L}_1$-structure), and $g \in G$. Then the formula $\varphi(x, g)$ defines in G the centraliser of g in G, while the formula $\psi(y) = \forall x\varphi(x, y)$ will define the centre of G. The sentence $\forall x, y(xy = yx)$ will only be satisfied if G is abelian.

Other examples of definable subsets of a group G are: The conjugacy class of an element g (by $\exists y\ y^{-1}gy = x$); the set of commutators ($\exists y, z\ (y^{-1}z^{-1}yz = x)$); the set of squares ($\exists y\ (y^2=x)$), or more generaly of n-th powers ($\exists y\ y^n = x$); the set of elements of order $\leq$ n ($x^n = 1$).

Are usually not definable: the commutator subgroup; the set of torsion elements; the subgroup generated by the squares.

(2) $\mathcal{L}$ = E. The formula $\varphi(x_1, x_2) = \exists y_1, \ldots, y_m(\bigwedge_{i=1}^{m-1} E(y_i, y_{i+1}) \wedge E(x_1, y_1) \wedge E(y_m, x_2) \wedge \bigwedge_{1\leq i<j\leq m} y_i \neq y_j)$ will define in a graph Γ the set of pairs (x_1, x_2) for which there is a path of length exactly m + 1 going from x_1 to x_2.

Other examples of definable sets: the set of elements connected by an edge to at least two distinct elements ($\exists y_1, y_2(y_1 \neq y_2 \wedge E(x, y_1) \wedge E(x, y_2))$); the set of elements at distance $\leq$ n of a given element a ($\exists y_1, \ldots, y_{n-1}(\bigwedge_{i=1}^{n-2} E(y_i, y_{i-1}) \wedge E(a, y_1) \wedge E(y_{n-1}, x)$).

Are usually not definable: the connected component of an element a (unless all of its elements are at bounded distance of a); the set of pairs contained in a loop [4]

In this segment we will present numerous definitions and significant ideas, we will also mention the every important Compactness theorem, one of the crucial tools of model scholars.

(2.1) Speculations, models of hypotheses, and so forth.. Let $\mathcal{L}$ be a language. A $\mathcal{L}$-hypothesis (or just, a hypothesis), is a lot of sentences of the language $\mathcal{L}$. A model of a hypothesis T is a $\mathcal{L}$-structure M which fulfills all sentences of T, indicated by M$\models$T. The class of all models of T is indicated Mod(T). On the off chance that K is a class of $\mathcal{L}$-structures, at that point Th(K) indicates the arrangement of all sentences valid in all components of K, and Th(M) is mean by Th(M).

A hypothesis T is steady iff it has a model. On the off chance that φ is a sentence which holds in all models of T, this is indicated by $T \models \varphi$. Two $\mathcal{L}$-structures M and N are basically identical, signified M $\equiv$ N, iff they fulfill similar sentences, iff Th(M)=Th(N). A hypothesis is finished iff given a sentence φ, either T $\models \varphi$ or T $\models \neg\phi$.

Identically, if any two models of T are basically comparable. (See that in the event that M is a L-structure, at that point fundamentally Th(M) is finished).

Rudimentary proportionality is an equality connection between L-structures. Two isomorphic

L-structures are unmistakably simply comparable, anyway the opposite holds for

limited L-structures. A popular hypothesis (of Shelah) states that two structures are simply

proportionate if and just in the event that they have isomorphic ultrapowers, see definition in Section

(2.2) Rudimentary substructures, expansions, embeddings, and so forth. Let M $\subseteq$ N be $\mathcal{L}$ structures. We state that M is a basic substructure of N, or that N is a rudimentary expansion of M, indicated by M $\prec$ N, iff for any recipe $\phi(\bar{x})$ and tuple $\bar{a}$ from M

$$M \models \phi(\bar{a}) \Leftrightarrow N \models \phi(\bar{a})$$

A guide f : M $\rightarrow$ N is a basic inserting iff it is an installing, and if f(M)$\prec$N. At the end of the day, if for any recipe $\phi(\bar{x})$ and tuple $\bar{a}$ from M, $M \models \phi(\bar{a})$ if and just $N \models \phi(f(\bar{a}))$.

Utilizing acceptance on the intricacy of recipes, one can appear

Tarski's test. Give M a chance to be a substructure of N. At that point M $\prec$ N if and if, for each

recipe $\phi(\bar{x}, y)$ and tuple $\bar{a}$ in M, on the off chance that $N \models \exists y\phi(\bar{a}, y)$, at that point there exists b $\in$ M with the end goal that $N \models \phi(\bar{a}, b)$.

Note that while the component b is in M, the fulfillment is taken in N

Note that while the element b is in M, the satisfaction is taken in N

(2.3) Some useful facts.

(1) If $M_1 \prec M_2$ and $M_2 \prec M_3$, then $M_1 \prec M_3$.

(2) If $M_1 \subseteq M_2 \subseteq M_3, M_1 \prec M_3 and M_2 \prec M_3, then M_1 \prec M_2$

(3) Let $M_n, n \in N$, be a chain of $\mathcal{L}$-structures (i.e., $M_n \subseteq M_{n+1}$ for all n). At that point $M_\omega = \bigcup_{n \in N} M_n$ has a one of a kind $\mathcal{L}$-structure which makes the M_n's substructures of M_ω: the translation of the constants is the undeniable one, $R^{M_\omega} = \bigcup_{n \in N} R^{M_n}$, and $f^{M_\omega} = \bigcup_{n \in N} f^{M_n}$, for R a connection image, and f a capacity image.

Expect that $M_n \prec M_{n+1}$ for every n $\in$ N. At that point $M_n \prec M_\omega$ for every n $\in$ N.

(2.4) Disposal of quantifiers. Equations with in excess of two alternances of quantifiers are genuinely unbalanced, and normally hard to choose reality of. One in this manner attempts to "dispense with quantifiers".

Definition. A hypothesis T takes out quantifiers if for any recipe $\phi(\bar{x})$ there is a quantifier free equation $\psi(\bar{x})$ which is proportional to $\phi(\bar{x})$ modulo T, i.e., is such that

$$T \models \forall \bar{x}(\phi(\bar{x}) \leftrightarrow \psi(\bar{x})).$$

Note that the arrangement of free factors in ϕ and ψ are the equivalent. Accordingly if ϕ is a sentence, so is ψ. (On the off chance that the language has no consistent image, at that point one permits ψ to be either T (genuine) or F (false); on the off chance that the language contains a steady image c, at that point one can use rather the equations c = c or c $\neq$ c).

Definition. A hypothesis T is model finished iff it is steady and at whatever point M⊆N are models of T, at that point $M \prec N$.

Comments. Obviously, a hypothesis which disposes of quantifiers is additionally model total. An significant outcome concerning model total hypotheses is the accompanying:

A steady hypothesis T is model finished if and just, for any equation $\phi(\bar{x})$, there is an existential recipe $\psi(\bar{x})$ such that

$$T \models \forall \bar{x}(\phi(bar.x) \leftrightarrow \psi(\bar{x})).$$

Note that this is equivalent to every formula being equivalent modulo T to a universal formula: if $\neg\phi(\bar{x})$ is equivalent modulo T to the existential formula $\psi(\bar{x})$, then $\phi(\bar{x})$ is equivalent modulo T to the universal formula $\neg\psi(\bar{x})$

One can show that a theory T is model complete, if whenever $M \subseteq N$ are models of T, then M is existentially closed in N, i.e., if $\phi(\bar{x})$ is an existential formula, and $\bar{a}$ a tuple in M such that $N \models \phi(\bar{a})$, then $N \models \phi(\bar{a})$.

(2.5) Note this is proportionate to each recipe being identical modulo T to a widespread

recipe: if $\neg\phi(\bar{x})$ is equal modulo T to the existential equation $\psi(\bar{x})$, at that point $\phi(\bar{x})$ is equal modulo T to the general recipe $\neg\psi(\bar{x})$

One can demonstrate that a hypothesis T is model finished, in the event that at whatever point $M \subseteq N$ are models of T, at that point M is existentially shut in N, i.e., if $\phi(\bar{x})$ is an existential recipe, and $\bar{a}$ a tuple in M to such an extent that $N \models \phi(\bar{a})$, at that point $N \models \phi(\bar{a})$.

(2.5) Examples. (1) The hypothesis of mathematically shut fields, indicated ACF, is axiomatised

by the aphorisms for fields, in addition, for every n ¿ 1, the maxim $\forall x_1, \ldots, x_n \exists y(y^n + x_1 y^{n-1} + \Delta\Delta\Delta + x_n = 0)$. ACF takes out quantifiers.

(2) Consider the field of genuine numbers, first with its common $\mathcal{L}_R$-structure, at that point as a $\mathcal{L}_R \cup \{\leq\}$-structure. Let T_0 be its hypothesis in $\mathcal{L}_R$, T_1 its hypothesis in $\mathcal{L}_R \cup \{\leq\}$. At that point one can demonstrate that T_0 is model finished, yet does not dispense with quantifiers (in $\mathcal{L}_R$), while T_1 wipes out quantifiers. The quantifier one can't dispose of in $\mathcal{L}_R$ is the existential quantifier of $\exists y(y^2=x)$. This can be viewed as pursues: consider the substructure $Q(\sqrt{2})$ of R. As a field (i.e., as a $\mathcal{L}_R$-structure), it

has an automorphism sending $\sqrt{2}$ to $-\sqrt{2}$. This automorphism anyway does not regard the requesting. See section 4 for a proof of quantifier-end.

(2.6) Soundness and culmination hypothesis. Given a lot of sentences, there is a idea of evidence, i.e., which proclamations are deducible from the given articulations utilizing a few formal principles of conclusion, for example, modus ponens (from An and A $\rightarrow$ B conclude B), and some substitution rules (from a sentence of the structure $\phi(c)$ where c is a steady, find $\exists x\ \phi(x)$). A proof can be thought of hence as a limited grouping of sentences, each being gotten from the past ones by applying some conclusion rules. The principal result, the soundness hypothesis, reveals to us that our thought of fulfillment is well-characterized: If a hypothesis T has a model, at that point one can't get a logical inconsistency from T, i.e., one can't demonstrate from T the sentence $\forall x(x \neq x)$.

Godel's fulfillment hypothesis at that point expresses the opposite: If from a given hypothesis T, one can't determine the sentence $\forall x(x \neq x)$, at that point the hypothesis T has a model.

Another method for expressing this outcome is by saying that the arrangement of sentences deducible from a given hypothesis T is actually the arrangement of sentences valid in all models of T, i.e., in the documentation presented over, that it concurs with Th(Mod(T)).

(2.7)Decidability. A hypothesis T is decidable, if there is a calculation permitting to choose regardless of whether a sentence ϕ holds in all models of T or not. (Here, the trouble lies in having the option to tell when ϕ does not hold in all models of T). In the event that one can list a hypothesis T and one knows (by one way or another) that T is finished, at that point T is decidable: given a sentence ϕ, begin counting the evidences from T; inevitably you achieve a proof of either ϕ $or \neg\phi$. How can one demonstrate that a hypothesis T is decidable? One attempts to locate a decent arrangement of sentences $\sum$ contained in T, and demonstrate that some other sentence is proportionate, modulo $\sum$, to a sentence which one can choose. For example, by the aftereffects of quantifier end for arithmetically shut fields given over, one demonstrates that the hypothesis ACF of mathematically shut fields is decidable: the arrangement of sayings of ACF is obviously enumerable, any sentence is proportionate to a sans quantifier sentence, and there are not very many of those. Along these lines; given a sentence ϕ, one begins identifying every one of the evidences from ACF. In the long run one will achieve a proof of $(\phi \leftrightarrow \psi)$ where ψ is some sans quantifier sentence. At that point, one again counts verifications to choose whether ψ is proportionate to T, F, or whether it suggests a limited disjunction of sentences of the structure $p = 0$ or $p \neq 0$. The system will in the end and state whether ϕ holds in all arithmetically shut fields or not.

Also for the hypothesis ORCF of genuine shut fields (sayings for requested fields, any odd degree polynomial has a root, and each positive component has a square root). From which it pursues that the hypothesis of genuine shut fields in the language of rings is likewise decidable, since the requesting can be characterized by "the non-negative components are the squares".

How can one demonstrate that a hypothesis is undecidable? Generally one attempts to code the ring (Z, +, −,·, 0, 1) in some model of the hypothesis. Or on the other hand to demonstrate that any limited chart is codable in some model of the hypothesis.

(2.8) Compactness hypothesis. Give T a chance to be a lot of sentences in a language L. In the event that each limited subset of T has a model, at that point T has a model.

We will exhibit later a proof of this hypothesis utilizing ultraproducts. Note that it is moreover a result of the culmination hypothesis, since any confirmation includes just limitedly numerous components of T. It likewise has for outcome the primary portion of the following hypothesis.

(2.9) Lowenheim-Skolem Theorems. Give L a chance to be a language, T a hypothesis, and let M be a limitless model of T.

(1) Let κ be an unending cardinal, $\kappa \geq |M| + |\mathcal{L}|$. At that point M has a basic augmentation N with $|N| = \kappa$.

(2) Let X be a subset of M. At that point M has a basic substructure N containing X, with —N— $\leq$ —X— + —$\mathcal{L}$— + $\aleph_0$.

(2.10) Remarks. These outcomes enable us to utilize huge models with great properties. For example, expect that we have a set $\Sigma(x_1, \ldots, x_n)$ of equations in the factors $(x_1, \ldots, x_n)$, and that we realize that each limited piece of $\Sigma(x_1, \ldots, x_n)$ is satisfiable in some model M of T, i.e., there is a tuple ¯a of M which fulfills all recipes of this limited piece. At that point there is a model N of T containing a tuple $\bar{b}$ which fulfills all the while all equations of $\Sigma(\bar{x})$.

Utilizing different procedures, one can demonstrate that if $\bar{a}$ and $\bar{b}$ are tuples of a $\mathcal{L}$-structure M, which fulfill similar equations in M, at that point M has a rudimentary expansion M*, in which there is an automorphism which sends $\bar{a}$ *to* $\bar{b}$.

(2.11) Application of the minimization hypothesis. Give T a chance to be a hypothesis in a language $\mathcal{L}$,

what's more, Δ a lot of recipes in the (free) factors $(x_1, \ldots, x_n)$, shut under limited disjunctions. Let $\Sigma(x_1, \ldots, x_n)$ be a lot of equations in the free factors $(x_1, \ldots, x_n)$, to such an extent that each limited section of $\Sigma(x_1, \ldots, x_n)$ is satisfiable in a model of T. The accompanying conditions are proportional:

(1) There is a subset $\Gamma(\bar{x})$ of Δ with the end goal that, if $\bar{c} = (c_1, \ldots, c_n)$ are new steady images, then

$$T \bigcup \Gamma(\bar{c}) \models \Sigma(\bar{c}), \quad T \bigcup \Sigma.(\bar{c}) \models \Gamma(\bar{c}).$$

(2) For all models M and N of T, and n-tuples $\bar{a}$ in M and $\bar{b}$ in N, if $N \models \Sigma(\bar{b}) and \bar{a}$ fulfills (in M) all recipes of Δ that are fulfilled by $\bar{b}$ (in N), at that point $M \models, \Sigma(\bar{a})$.

Comment. On the off chance that the set $\Sigma(\bar{x})$ is limited. at that point so is $\Gamma(\bar{x})$. Consequently, taking $\phi(\bar{x})$ to be the combination of the equations of $\Sigma(\bar{x})$. one acquires that $\phi(\bar{x})$ is equal, modulo T. to a limited combination of recipes of Δ.

(2.12) Preservation hypotheses. This has for results a few conservation hypotheses. One bearing is trifling. the other one not.

Give T a chance to be a hypothesis in a language $\mathcal{L}$.

(1) The accompanying conditions are comparable:

(a) Whenever $M \subseteq N$ are $\mathcal{L}$-structures, and $M \models T$. at that point $N \models T$.

(b) T has an axiomatisation given by existential sentences.

(2) The accompanying conditions are comparable:

(a) Whenever $M \subseteq N$ are $\mathcal{L}$-structures, and $N \models T$. at that point $M \models T$

(b) T has an axiomatisation given by all inclusive sentences.

(3) The accompanying conditions are comparable:

(a) Whenever M_n, $n \in N$. is a chain of models of T. at that point S $n \in N$ M_n is a model of T.

(b) T has an axiomatisation given by $\forall\exists$ sentences.

(4) The accompanying conditions are comparable:

(a) Whenever M is a model of T, and $f : M \to N$ is a morphism. at that point $f(M) \models T$.

(b) T has an axiomatisation given by positive sentences.

Note that (3) infers that a model total hypothesis has an axiomatisation given by $\forall\exists$ sentences.

(2.13) Craig's insertion hypothesis. Let $\mathcal{L}_1 and \mathcal{L}_2$ be two dialects. Let ϕ be a sentence of $\mathcal{L}_1$ and ψ a sentence of $\mathcal{L}_2$. On the off chance that $\phi \models \psi$. at that point there is a sentence Θ of $\mathcal{L}_1 \cap \mathcal{L}_2$ with the end goal that $\phi \models \Theta and \Theta \models \psi$.

A to some degree distinctive insertion hypothesis is given by Robinson: Let $\mathcal{L}_1 and \mathcal{L}_2$ be two dialects. and $\mathcal{L}_0 = \mathcal{L}_1 \cap \mathcal{L}_2$. Accept that $T_1 and T_2$ are speculations in $\mathcal{L}_1 and \mathcal{L}_2$ individually. with the end goal that $T_0 = T_1 \cap T_2$ is finished. At that point $T_1 \cup T_2$ is steady.

(2.14) Utilization of the L¨owenheim-Skolem hypothesis to the hypothesis of mathematically shut fields. This will enable us to demonstrate that the hypothesis of arithmetically shut fields (signified ACF) dispenses with quantifiers. From that we will reason that the culminations of ACF (i.e., the total hypotheses broadening ACF) are acquired by determining the trademark. This will really be clear from the proof.[34]

Documentation. Give M a chance to be a $\mathcal{L}$-structure, and $\bar{a}$ a n-tuple in M. We mean by $tp_M(\bar{a})$ the arrangement of recipes fulfilled by $\bar{a}$ in M.

Hypothesis. Let T = ACF be the hypothesis of logarithmically shut fields. At that point T takes out quantifiers. Also, any two models of T of a similar trademark are basically proportionate.

Verification. By (2.11), it gets the job done to demonstrate that if M and N are mathematically shut, and if $\bar{a}$ and $\bar{b}$ are n-tuples from M and N separately, which fulfill a similar without quantifier equations, at that point they fulfill similar recipes, i.e., $tp_M(\bar{a}) = tp_N(\bar{b})$.

First note that on the off chance that $\bar{a}$ *and* $\bar{b}$ fulfill similar recipes, at that point the fields M and N have a similar trademark. Without a doubt, on the off chance that char(M) = p ¿ 0, at that point $\bar{a}$ fulfills (the sentence) p=0 (where p is a shortened form for the term 1+ 1 + · ·+1 p times). Henceforth the prime subfield of M is isomorphic to the prime subfield of N, and we will signify this field by k. By suspicion $\bar{a}$ *and* $\bar{b}$ fulfill a similar without quantifier recipes, and this suggests there is a field-isomorphism $f : k(\bar{a}) \rightarrow k(\bar{b})$ sending $\bar{a}$ *to* $\bar{b}$.

Let κ be a cardinal, κ ¿ —M—, —N—. By Lowenheim-Skolem, M has a basic augmentation M* of cardinality κ, and N has a basic expansion N* of cardinality κ. Select amazing quality bases X of M* over k($\bar{a}$) and Y of N* over $\kappa(\bar{b})$. At that point —X— = —Y — = κ, so that there is a bijection g : X → Y . At that point f∪g stretches out to an isomorphism h : $M* \rightarrow N*$. As isomorphisms save equations, this suggests $tp_{M*}(\bar{a}) = tp_{N*}(\bar{b})$. As $M \prec M*$ and $N \prec N*$, this demonstrates $tp_M(\bar{a}) = tp_N(\bar{b})$.

Additionally the verification demonstrates that any two mathematically shut fields of a similar trademark are simply comparable, as they have isomorphic basic expansions.

We will see later a significant outcome of this, the Lefshetz guideline.

(2.15) Many-arranged structures. Many-arranged structures resemble standard structures, then again, actually there are currently a few universes, normally disjoint (however not generally), with related sorts. The language will have sorts, connection images will have joined to them, an arity, yet in addition a tuple of sorts, and comparably for capacities. Recipes are assembled in the standard way, the main limitation being that factors currently have a sort joined to them. On the off chance that one has limitedly numerous sorts, state n, one can diminish to the standard case, by adding for example to the language n new unary connection images $R_1, ..., R_n$, with expected translation the universes of the sorts.

A formal definition is somewhat ungainly, and we will rather give four normal precedents. Most old style results hold in many-arranged rationale, with now and again the fitting adjustment. Specifically, the minimization hypothesis holds.

Model 1. Give G a chance to be a limitedly created gathering, state by $a_1, ..., a_n$. We consider the

language with two sorts: one sort is the gathering sort, the other one is the "length" sort. The language is:

– $\{\cdot, ^{-1}, 1, a_1, \ldots, a_n\}$, connected to the gathering sort. Here we have added to the language of gatherings n new steady images for the components $a_1, \ldots, a_n$(we indicate, harshly perhaps, the consistent and the component by a similar image).

– Any structure you need on the length sort. E.g., a steady image 0, and a twofold connection $\leq$. Perhaps likewise an image for expansion and subtraction

– A paired capacity image d, with area the gathering sort squared, and go the length sort.

The two-arranged structure we have as a primary concern is the structure

$$\mathcal{G} = ((G, \cdot, 1, a_1, \ldots, a_n), (\mathbb{N}, +, 0, \leq), d),$$

where $(G, \cdot, 1, a_1, \ldots, a_n)$ is our gathering with the recognized components $a_1, \ldots, a_n$, $\mathbb{N}, +, 0, \leq$ is the non-negative numbers with their characteristic expansion, subtraction and requesting, and d : $G^2 \to$ N is the separation work (on the Cayley diagram of G regarding the set $\{a_1, \ldots, a_n\}$ of generators). Note that for every n, to be at separation $\leq$ n is expressible by a first request recipe. In any case, there is no recipe communicating that each component is at limited separation from 1, except if G is limited.

Note additionally that a basic expansion of (G, N, d) will be a two-arranged structure (G^*, N^*, d^*), in which the separation capacity will in any case be onto, and fulfill certain ultrametric imbalances: yet its range will (when all is said in done) be a non-standard model of the whole numbers, so remove between two components of G^* might be boundless. One can likewise supplant N by any added substance requested subgroup of the reals, yet the guide d will then not be onto. We will return to this precedent in asymptotic cones.

Precedent 2. Consider the field $\mathbb{Q}_p$, and its valuation $v : \mathbb{Q}_p \to \mathbb{Z}$, buildup map $\mathbb{Z}_p \to \mathbb{F}_p$, it is standard to see this esteemed field as a 3-arranged structure $\mathbb{Q}_p$: the sorts are the field sort, the worth gathering sort, and the buildup field sort. We have two arrangements of field tasks, one for the field sort, and one for the buildup field sort. We additionally have the language of requested gatherings for the buildup sort, together with a recognized steady ∞. At long last, we have the valuation map $\mathbb{Q}_p \to \mathbb{Z} \cup \infty$, and the buildup map $\pi : \mathbb{Z}_p \to \mathbb{F}_p$(if one needs this guide to be characterized all in all field sort, one relegates the components of $\mathbb{Q}_p \setminus \mathbb{Z}_p$ to 0, for example).

Precedent 3. Give K a chance to be a field, and fix a whole number m ¿ 0. Think about the polynomial ring

R = $K[X_1, \ldots, X_m]$. We will characterize a ω-arranged structure (listed by the whole numbers) on R. For d $\in$ N, the components of sort d are the polynomials of all out degree $\leq$ d, and we mean the comparing set by R_d. We include an expansion, subtraction and augmentation, in the characteristic way. At that point + for example will send

$R_d \times R_e$ to $R_{sup\{d,e\}}$, while · will send $R_d \times R_e$ to R_{d-e}.

Basic proclamations about the ω-arranged structure R are expressible by conventional $\mathcal{L}_R$-equations in the field K: as all factors have a sort, evaluation over polynomials of degree $\leq$ d is accomplished by measuring over their coefficients, i.e., over N-tuples for a specific N calculable from m, d.

A comparable development should be possible for K[V], for V an arithmetical set characterized over K

Precedent 4. The 2-arranged language utilized for charts in which there can be a few edges between two vertices, is essentially the language with two sorts, the vertice sort, and the edge sort. It has three unary capacity images, σ, τ a . The spaces of σ *and* τ are the edge sort, and their range the vertice sort. The area and scope of $^-$ is the edge sort.

A diagram will be given by (V, E, $\sigma, \tau, ^-$), where V is the arrangement of vertices, E is the arrangement of edges, σ : E $\rightarrow$ V doles out to an edge e its beginning stage σ(e), and τ : V $\rightarrow$ E doles out to an edge e its endpoint τ (e). The guide $^-$ turns around the course of edges. Henceforth we have $\bar{\bar{e}}$ = e for each edge e, and $\sigma(\bar{e}) = \tau(e)$, $\tau(\bar{e}) = \sigma(e)$.

One can put extra structure on the chart, for example by shading the edges.

This is finished by including unary predicates the edge sort. Bipartitite charts can be treated along these lines.

(2.16) Additional remarks on quantifier-elimination. So, what does one do if a theory T in a language $\mathcal{L}$ does not eliminate quantifiers? One possibility is to form the Skolemization of T: for each formula $\phi(x_1, ..., x_n)$ one adds a new n-ary relation symbol R_ϕ, and adds to T an axiom saying that R_ϕ defines precisely the set of n-tuples satisfying ϕ. The resulting theory eliminates quantifiers. However, this does not bring any information about our theory T.

The hope is that one does not need to add much to the language to obtain quantifierelimination. For instance we saw that to get elimination of quantifiers of the theory ORCF of real closed fields, it is enough to add the ordering to $\mathcal{L}_R$.

Another interesting example is the theory of the field of p-adic numbers $\mathbb{Q}_p$, now viewed as an ordinary $\mathcal{L}_R$-structure. A beautiful result of Macintyre states that it is enough to add to the language a unary predicate Pn for each n ¿ 1, as well as the inverse function $^{-1}$. The interpretation of the predicates P_n *in* $\mathbb{Q}_p$ is the set of non-zero n-th powers, i.e.,

$$\mathbb{Q}_p \models \forall x(P_N(x) \leftrightarrow (x \neq 0 \wedge \exists y\ y^n = x))$$

Observe that the elements of positive valuation are then definable: v(x) ¿ 0$\Leftrightarrow$ $P_2(1 + px^2)$. The first-order theory of the expansion of $\mathbb{Q}_p$ to this enlarged language then eliminates quantifiers. This allows one to give a good description of the definable sets.

(2.17) Outlines. Graphs are intended to give an intelligent detailing of the accompanying properties: the structure N contains a duplicate of the structure M: there is a morphism f : M $\rightarrow$ N: there is a rudimentary installing f : M $\rightarrow$ N.

Definitions. Give M a chance to be a $\mathcal{L}$-structure. A $\subseteq$ M. We let L(A) be the language gotten by adding to the language new images of constants ca for each a $\in$ A. On the off chance that $\bar{a}$ is a tuple of components of A. we let $\bar{c}_{\bar{a}}$ indicate the tuple of constants comparing to the components of the tuple $\bar{a}$. $\mathcal{L}$(A) is then a real language, yet might be a lot bigger than $\mathcal{L}$. Note that M turns out to be normally a $\mathcal{L}$(A)- structure. when one translates the consistent ca by a. for a $\in$ A. This structure is generally indicated by (M. ca)a$\in$A. or (M. a)a$\in$A (i.c., we signify the consistent and the component by a similar image).

(1) The without quantifier chart of An in M. Δ(A). is the arrangement of all sans quantifier sentences $\phi(\bar{c}_{\bar{a}}) \in \mathcal{L}$(A) which hold in the $\mathcal{L}$(A)- structure M (i.e.. to such an extent that M$\models \phi(\bar{a})$).

(2) The positive graph of An in M. Δ+(A). is the arrangement of all positive sans quantifier

sentences $\phi(\bar{c}_{\bar{a}}) \in \mathcal{L}$(A) which hold in the $\mathcal{L}$(A)- structure M.

(3) The rudimentary chart of An in M. Diag(A). is the arrangement of all sentences $\phi(\bar{c}_{\bar{a}}) \in \mathcal{L}$(A) which hold in the $\mathcal{L}$(A)- structure M.

(4) Let $\mathcal{L}'$ be a language containing $\mathcal{L}$. A development of the $\mathcal{L}$-structure M to $\mathcal{L}'$is a $\mathcal{L}'$-structure M'. with same universe as M, and to such an extent that the translation of the images of L in M and in M' correspond. M is then called a reduct of M' to $\mathcal{L}$. For example. $(M. c_a)_{a\in A}$ is an extension of M to $\mathcal{L}$(A).

Reality. Give M and N a chance to be two L-structures. Coming up next are quick results of the definition:

(1) N can be extended to a $\mathcal{L}$(M)- structure which is a model of Δ(M) if and just if there is an inserting f : M $\rightarrow$ N.

(2) N can be extended to a $\mathcal{L}$(M)- structure which is a model of Δ+(M) if and just there is a morphism f : M $\rightarrow$ N.

(3) N can be extended to a $\mathcal{L}$(M)- structure which is a model of Diag(M) if and just if there is a rudimentary installing f : M $\rightarrow$ N.

(5) A L-hypothesis T is model finished if and if. for each model M of T. T $\cup$ Δ(M) is finished (in $\mathcal{L}$(M)).

(6) A $\mathcal{L}$-hypothesis T disposes of quantifiers if and if. for each M $\models$ T and subset An of

M. T $\cup$ Δ(A) is finished (in $\mathcal{L}$(M)).

(2.18) Questions and expectations on free gatherings. Sela has demonstrated that the hypothesis T of all non-abelian free gatherings (in the language $\mathcal{L}_G$) is finished, and in addition, that each equation is identical, modulo T, to a Boolean mix of ∀∃-recipes. From what I comprehend, he doesn't yet have an axiomatisation of the hypothesis T, nor does he knows whether it is decidable One of the exceptional open inquiries for model scholars, is whether the hypothesis T is steady. Solidness is a sure combinatorial property of a hypothesis, which enables one to utilize an immense apparatus to concentrate models and potential connections between determinable subsets. One meaning of strength is the accompanying: we state that a recipe $\phi(\bar{x}, \bar{y})$ is steady (for T) if there is exists a whole number n with the end goal that in a model M of T, any two successions of tuples $(\bar{a}_i)$ and $(\bar{b}_j)$ ordered by some underlying fragment of N, and fulfilling

$$M \models \phi(\bar{a}_i, \bar{b}_j) \Leftrightarrow i < j$$

have length $\leq$ n. The (total) hypothesis T is steady iff all equations are steady. Dependability precludes specifically the presence of an endless (determinable) direct requesting on models of T It is accepted that the hypothesis T of non-abelian free gatherings is steady. One inquiry is to see better the sets determinable in free gatherings. A discretionary Boolean blend of ∀∃-determinable sets is a significant unmanageable item. The expectation is that the complexity begins from certain quantifiable sets. I.e., that there are a couple of groups of ∀∃-perceptible sets, to such an extent that on the off chance that one includes reasonable connection images for these determinable sets, at that point one can wipe out quantifiers to a lower level. Preferably to existential recipes, yet one shouldn't be excessively idealistic.

There are additionally questions identified with instigated structure. For example, let g $\neq$ 1, and consider the stabilizer C(g) of g in the free gathering F. We realize that C(g) is isomorphic to the abelian bunch $\mathbb{Z}$. One inquiry is: what is the structure prompted by F on C(g)? On the off chance that one realizes that T is steady, this diminishes to the accompanying: let D $\subset$ F^m be perceptible (without parameters). Portray D $\cap$ $C(g)^m$. Is it quantifiable in the structure (C(g), ·, −1 , 1)? (Perhaps the response to this inquiry is inconsequentially no, however one can pose comparable to inquiries for any determinable subset of F^n). On the off chance that it were, at that point the prompted structure would be extremely basic: in fact the hypothesis of $\mathbb{Z}$ in the language of gatherings amplified by including unary predicates S_n for every one of the subgroups n$\mathbb{Z}$ of $\mathbb{Z}$, n ¿ 1, wipes out quantifiers. .

Here we are utilizing the accompanying property of a steady hypothesis: let M be a model of a

stable hypothesis T, let S be a 0-quantifiable subset of M^n, and let D be a M-determinable subset of M^{nm}. At that point D $\cap$ S^m is determinable with parameters from S. (Cautioning: the recipe may change), the equation with parameters in S characterizing D $\cap$ S m isn't really equivalent to the

equation characterizing D.)[34]

In this segment we will present a significant device: ultraproducts. They are at the focus of numerous applications, inside and outside model hypothesis.

(3.1) Filters and ultrafilters. Give I a chance to be a set. A channel on I. is a subset $\mathcal{F}$ of $\mathcal{P}(I)$ (the arrangement of subsets of I), fulfilling the accompanying properties:

(1) $I\epsilon\mathcal{F}, \varnothing \notin \mathcal{F}$

(2) $If\ U\epsilon\mathcal{F}\ and\ V \supseteq U, then\ V\epsilon\mathcal{F}$

(3) $If\ U, V\epsilon\mathcal{F}, then\ U \cap V\epsilon\mathcal{F}$

A ultrafilter on I is a channel on I which is maximal for consideration. Identically, it is channel $\mathcal{F}$ with the end goal that for any $U\epsilon\mathcal{P}(I)$, either $U\epsilon\mathcal{F}$ or I
$U\epsilon F$.

(3.2) Remarks. (1) Note that condition (1) denies that both U and I
U have a place with the same channel on I.

(2) Using Zorn's lemma (and in this manner the aphorism of decision), each channel on I is contained in a ultrafilter

(3) If $\mathcal{G} \subset \mathcal{P}(I)$ has the limited crossing point property (i.e., the convergence of limitedly numerous components of $\mathcal{G}$ is never vacant), at that point $\mathcal{G}$ is contained in a channel. The channel produced by $\mathcal{G}$ is then the arrangement of components of $\mathcal{P}(I)$ containing some limited crossing point of components of $\mathcal{G}$.

(3.3) Important and non-vital ultrafilters, Fr'echet channel. Give I a chance to be a set. A

ultrafilter $\mathcal{F}$ on I is primary in the event that there is $i\epsilon I$ to such an extent that $\{i\}\epsilon F$ (and after that we will have:$U\epsilon F \Leftrightarrow I\epsilon U$). A ultrafilter is non-main in the event that it isn't chief. Note that in the event that I is limited, at that point each ultrafilter on I is essential.

Give I a chance to be endless. The Fr'echet channel on I is the channel $\mathcal{F}_0$ comprising of all cofinite subsets of I. A ultrafilter $\mathcal{F}$ on I is then non-essential if and just if contains the Fr'echet channel on I. Note that on the off chance that $S \subseteq I$ is unbounded, at that point $\mathcal{F}0 \cup \{S\}$ has the limited crossing point property, with the goal that it is contained in a ultrafilter.

(3.4) Cartesian results of $\mathcal{L}$-structures. Fix a language $\mathcal{L}$. Give I a chance to be a record set, and $(M_i), I\epsilon I$, a group of L-structures. We characterize the $\mathcal{L}$-structure M = Q $i\epsilon I$ M_i as pursues:

— The universe of M is just the cartesian result of the M_i's, i.e., the arrangement of successions $(a_i)_{i\epsilon I}$ with the end goal that $a_i\epsilon M_i$ for each $i\epsilon I$.

— If c is a consistent image of $\mathcal{L}$, at that point $c^M = (c^{M_i})_{i\epsilon I}$.

— If R is a n-ary connection image, at that point $R^M = \prod_{i\epsilon I} R^{M_i}$

— If f is a n-ary work image and $((a_{1,i})_i, ..., (a_{n,i})_i)\epsilon M^n$, at that point

$$f^{M}((a_{1,i})_i, \ldots, (a_{n,i})_i) = (f^{M_i}(a_{1,i}, \ldots, a_{n,i}))i\epsilon I$$

(3.5) Diminished results of $\mathcal{L}$-structures. Give I a chance to be a set, $\mathcal{F}$ a channel on I, and (M_i), $i\epsilon I$, a group of L-structures. The diminished result of the M_i's over F, indicated by $\prod_{i\epsilon I} M_i\|\mathcal{F}$, is the $\mathcal{L}$-structure characterized as pursues:

— The universe of $\prod_{i\epsilon I} M_i\|\mathcal{F}$ is the remainder of $\prod_{i\epsilon I} M_i$ by the identicalness relation$\equiv_{\mathcal{F}}$ characterized by

$$(a_i)_i equiv_{\mathcal{F}}(b_i)_i \Leftrightarrow \{i\epsilon\|a_i b_i\}\epsilon\mathcal{F}$$

We signify by $(a_i)_{\mathcal{F}}$ the comparability class of the component $(a_i)_i$ for this identicalness connection. The structure on $\prod_{i\epsilon I} M_i\|\mathcal{F}$ is then just the "remainder structure", i.e.,

— The elucidation of c is $(c^{M_i})_{\mathcal{F}}$, for c a consistent image of $\mathcal{L}$.

— If R is a n-ary connection image, and if $a_1, \ldots, a_n\epsilon \prod_{i\epsilon I} M_i\|\mathcal{F}$ are spoken to by $(a_{1,i})_i, \ldots, (a_{n,i})_i\epsilon \prod_{i\epsilon I} M_i$, at that point we set

$$\prod_{i\epsilon I} M_i\|\mathcal{F} \models R(a_1, \ldots, a_n) \Leftrightarrow \{i\epsilon I\|(a_{1,i}, \ldots, a_{n,1})\epsilon R^{M_i}\}\epsilon\mathcal{F}$$

— If f is a n-ary work image and if $a_1, \ldots, a_n\epsilon \prod_{i\epsilon I} M_i, \mathcal{F}$ are spoken to by $(a_{1,i})_i, \ldots, (a_{n,i})_i\epsilon \prod_{i\epsilon I} M_i$, at that point we set

$$f^{M}(a_1, \ldots, a_n) = (f^{M_i}(a_{1,i}, \ldots, a_{n,i}))\mathcal{F}$$

The properties of channels ensure that the remainder structure is well-characterized. Note that the remainder map $:\prod_{i\epsilon I} M_i \rightarrow \prod_{i\epsilon I} M_i\|\mathcal{F}, (a_i)_i \mapsto (a_i)_{\mathcal{F}}$, is a morphism of $\mathcal{L}$-structures.

Definitions. On the off chance that all structures M_i are equivalent to a structure M, at that point we compose $M^I\|\mathcal{F}$ rather than $\prod_i M_i\|\mathcal{F}$, and the structure is known as a diminished intensity of M. On the off chance that the channel F is a ultrafilter, at that point $\prod_i M_i\|\mathcal{F}$ is known as the ultraproduct of the M_i's (regarding $\mathcal{F}$), and $M^I\|\mathcal{F}$ the ultrapower of M (as for $\mathcal{F}$).

(3.6) Los Theorem. Give I a chance to be a set, $\mathcal{F}$ a ultrafilter on I, and (M_i), $i\epsilon I$, a group of $\mathcal{L}$-structures. Let $\phi(x_1, \ldots, x_n)$ be a $\mathcal{L}$-equation, and let $a_1, \ldots, a_n\epsilon \prod_{i\epsilon I} M_i\|\mathcal{F}$ be spoken to by $(a_{1,i})_i, \ldots, (a_{n,i})_i\epsilon \prod_{i\epsilon I} M_i$. At that point

$$\prod_{i\epsilon I} M_i|\mathcal{F} \models \phi(a_1, \ldots, a_n) \Leftrightarrow \{i\epsilon I|M_i \models \phi(a_{1,i}, \ldots, a_{n,i})\}\epsilon\mathcal{F}$$

(3.7) Result. Give I a chance to be a set. $\mathcal{F}$ a ultrafilter on I. and M a $\mathcal{L}$-structure. At that point the common guide $M \to M^I||\mathcal{F}. a \to (a)_F$. is a rudimentary implanting. (Here $(a)_F$ is the identicalness class of the grouping with all terms equivalent to a).

(3.8) Remarks and remarks. Give I a chance to be an endless list set. and $\mathcal{F}$ a ultrafilter on I.

(1) If $\mathcal{F}$ is vital. state $j \epsilon \mathcal{F}$. at that point $\prod_{i \epsilon I} M_i||\mathcal{F} \simeq M_j$ for any group of $\mathcal{L}$-structures $M_i. I \epsilon I$

(2) Suppose that the M_i 's are fields. with perhaps extra structure (e.g.. a requesting. new capacities. and so on.). Consider the perfect $\mathcal{M}$ of $\prod_i M_i$ produced by all components $(a_i)_i$ with the end goal that $i \epsilon I|a_i = 0 \epsilon F$. At that point $\mathcal{M}$ is a maximal perfect of $\prod_i M_i$. and quotienting by the identicalness connection $\equiv_{\mathcal{F}}$ is equal to quotienting by the maximal perfect $\mathcal{M}$.

The quality of Los hypothesis is to reveal to you that the rudimentary properties of the M_i 's. counting the ones relying upon the extra structure. are protected. E.g.. that $\mathbb{R}^I||\mathcal{F}$ is a genuine shut field.

(3.9) Shelah's isomorphism hypothesis. Give M and N a chance to be two $\mathcal{L}$-structures. At that point M $\equiv$ N on the off chance that and just if there is a ultrafilter F on a set I with the end goal that $M^I|\mathcal{F} \simeq N^I|\mathcal{F}$.

Note the accompanying quick result: in the event that M $\equiv$ N. at that point there is M* in which both M and N install basically.

(3.10) Application 1: proof of the conservativeness hypothesis. Give T a chance to be a hypothesis in a

language $\mathcal{L}$. and expect that each limited subset s of T has a model M_s. At that point T has a model.

Confirmation. In the event that T is limited, there is nothing to demonstrate, so we will accept that T is boundless. Let

I be the arrangement of every single limited subset of T. For each $\phi \epsilon T$. let $S(\phi) = \{s \epsilon I|\phi \epsilon s\}$. At that point the family $G = S(\phi)|\phi \epsilon T$ has the limited crossing point property, and in this way is contained in a ultrafilter F. We guarantee that $\prod_{s \epsilon I} M_s|\mathcal{F}$ is a model of T: let $\phi \epsilon$ T. At that point, by presumption. $\{s \epsilon I|M_s| = \phi\}$ contains $S(\phi)$, and thusly has a place with $\mathcal{F}$. By Los' hypothesis. $\prod_{s \epsilon I} M_s|\mathcal{F}| = \phi$.

(3.11) Application 2: Lefshetz guideline. Let ϕ be a sentence of the language $\mathcal{L}_R$. The accompanying conditions are proportionate:

(1) $\mathbb{C} \models \phi$

(2) If K is a logarithmically shut field of trademark 0. at that point $K \models \phi$.

(3) There is a whole number n, to such an extent that in the event that p is a prime number ¿ n. at that point $\mathbb{F}_p \models \phi$

(4) There is a whole number n, with the end goal that if p is a prime number ¿ n and

K is a logarithmically shut field of trademark p. at that point K $\models \phi$.

Verification. The equivalences of (1) and (2), and of (3) and (4) are clear, by (2.14). Expect

(2). At that point ACF$\cup\{$p $\neq 0$ — p a prime $\}\models \phi$. By smallness, there are limitedly many prime numbers $p_1, \ldots, p_m$ with the end goal that ACF$\cup p_1 \neq 0, \ldots, p_m \neq 0 \models \phi$. Take n ¿ $\sup\{p_1, \ldots, p_m\}$. This demonstrates (2) infers (3).

Expect (3), and let $\mathcal{F}$ be a non-essential ultrafilter on the set P of primes. For each prime p, pick a logarithmically shut field K_p of trademark p. By supposition, $\{p \in P | K_p \models \phi\} \in$ F, and consequently $\prod_{p \in P} K_p|\mathcal{F} \models \phi$. For every p, the set {q $\in$ P — q $\neq$ p} is likewise in the ultrafilter. By Los' Theorem, $\prod_{p \in P} K_p|\mathcal{F}$ is a mathematically shut field of trademark 0, and consequently appears (2).

(3.12) Application 3: Orderable gatherings. Give G a chance to be a gathering, and expect that each

limitedly created subgroup of G is orderable. At that point G is orderable.

Evidence. Give I a chance to be the arrangement of limited subsets of G. For every s $\in$ I, let G_s be the subgroup of G produced by s, and fix a requesting $\leq$ on G_s. Let $\mathcal{F}$ be a ultrafilter on I containing all sets S(g) = {s $\in$ I — g $\in$ s} for g $\in$ G, and consider the $\mathcal{L}_G \cup \leq$-structure

$$(G^*, \cdot, ^{-1}, 1, \leq) = \prod_{s \in I}(G_s, \cdot, ^{-1}, 1, \leq)|\mathcal{F}$$

At that point G* is an arranged gathering, by Los hypothesis. We insert G in G* in the accompanying design:

for g $\in$ G, we set

$$f(g) = (g_s)_{\mathcal{F}}, where\ g_s = \begin{cases} g & if\ g \in G_s \\ 1 & otherwise \end{cases}$$

One watches that f is a gathering implanting, and f(G) is hence an arranged gathering.

(3.13) Types. Give M a chance to be a L-structure, A $\subseteq$ M, and $\bar{b} \in M^m$. The sort of $\bar{b}$ over An, indicated by tp($\bar{b}$—A) (or tp_M ($\bar{b}$—A)), is the arrangement of all recipes $\phi(\bar{a}, \bar{y})$ fulfilled by $\bar{b}$ in M, where $\bar{a}$ is a limited tuple of components of A.

One has the accompanying outcome: if $\bar{b}, \bar{c}$ understand a similar sort over An in M, i.e., in the event that tp($\bar{b}$—A) = tp($\bar{c}$—A), at that point there is a rudimentary augmentation M* of M, and an automorphism of M*, which fixes the components of An and sends $\bar{b}$ *to* $\bar{c}$.

All the more by and large, a n-type over a subset An of M is a set $\Sigma(\bar{y})$ of recipes $\phi(\bar{a}, \bar{y})$ in the n-tuple $\bar{y}$ of factors and with $\bar{a} \in$ A, which is limitedly feasible in M, i.e., is to such an extent that in the event that s $\subset \Sigma$ is limited, at that point there is $\bar{b} \in M_n$ fulfilling every one of the equations of s.

(3.14) Application 4: ω_1-immersion. Give I a chance to be a countable set. $\mathcal{F}$ a non-chief ultrafilter on I. and (M_i). I $\in$ I. a group of $\mathcal{L}$-structures, where $|\mathcal{L}| \leq \mathfrak{N}$. Let $A \subset M^* = i{\in}I\ M_i|\mathcal{F}$ be countable, and let $\Sigma(\bar{y})$ be a sort over A. At that point $\Sigma(\bar{y})$ is acknowledged in M*, i.e..there is $\bar{b} \in$ M* which fulfills every one of the recipes of $\Sigma(\bar{y})$.

Another method for expressing this property is to state that if (S_n), n $\in$ N. is a countable group of quantifiable subsets of M^{*m} with the limited convergence property. *then* $\bigcap_n S_n \neq \emptyset$

(3.15) A developments of $\mathbb{R}$. We will exhibit a development enabling one to build $\mathbb{R}$.

This development originate from non-standard investigation. Let $\mathcal{L} = \{+, -, \leq, 0\}$ be the language of abelian requested gatherings. and supply $\mathbb{R}$ with its characteristic $\mathcal{L}$-structure.

We fix a non-main ultrafilter $\mathcal{F}$ on I = $\mathbb{N}$. Accept that for every I we are given a $\mathcal{L}$-substructure Γ_i of $\mathbb{R}$ (i.e.. an added substance subgroup of $\mathbb{R}$ with the actuated requesting).

Consider the arranged gatherings $\Gamma^* = \prod_{i\in I} \Gamma_i|\mathcal{F}$ *and* $\mathbb{R}^* = \mathbb{R}^I|\mathcal{F}$. We have a characteristic implanting $\Gamma^* \to \mathbb{R}^*$ prompted by every one of the incorporations $\Gamma_i \subseteq \mathbb{R}^*$. Note that $\mathbb{R}^*$ likewise has a

ring structure, and in this way contains a duplicate of (the ring) $\mathbb{R}$. Characterize

$$R^{fin} = \{g c \mathbb{R}^* | \exists\, c \in \mathbb{N}\ \ -c \leq g \leq c\}$$

$$\mu = \{g \in \mathbb{R}^* | \forall\, c \in \mathbb{N}\ \ -1 \leq cg \leq 1\}$$

$$\Gamma^{fin} = \Gamma^* \cap R^{fin} \quad \mu_\Gamma = \Gamma^* \cup \mu$$

At that point R^{fin} is the curved body of $\mathbb{Z}$ *in* $\mathbb{R}^*$. Both Γ^{fin} *and* μ_Γ are arched subgroups of Γ^*. Henceforth, $\Gamma_{\mathcal{F}} = \Gamma^{fin}|\mu_\Gamma$ is an arranged gathering, which is plainly archimedean. There are three

potential outcomes for this gathering: one plausibility is that it is insignificant: this is the situation for example if each $\Gamma_i = i\mathbb{Z}$. The second plausibility is F that it is discrete, i.e., has a littlest positive component. It is then isomorphic to $\mathbb{Z}$. The third case is when $\Gamma_{\mathcal{F}}$ has no littlest positive component. It is then isomorphic to a thick added substance subgroup of $\mathbb{R}$, and we guarantee that it rises to $\mathbb{R}$. We realize that it is archimedean, and it in this manner gets the job done to demonstrate that it is finished. Let $(a_n)_{n\in N}$ *and* $(b_n)_{n\in N}$ be arrangements of components of $\Gamma_{\mathcal{F}}$ with the end goal that $a_n < b_m$ for all n, m. Lift these groupings to successions $(a'_n)_{n\in N}$ *and* $(b'_n)_{n\in N}$ *in* Γ^{fin}. At that point $a'_n < b'_m$ for all m, n. By ω_1-immersion of Γ^* (see (3.14)), there is a component $c' \in \Gamma^{fin}$ with the end goal that $a'_n < c' < b'_m$ for all m, n. The picture c of c' in $\Gamma_{\mathcal{F}}$ then fulfills $a_n < c' < b_m$ for all n, m, and this demonstrates the culmination of $\Gamma_{\mathcal{F}}$.

The third case happens specifically when the gatherings Γ_i are of the structure $r_i\mathbb{Z}$, where $r_i \in$ R, and $(r_i)_{\mathcal{F}} \in \mu$. Or then again on the off chance that they are thick in $\mathbb{R}$.

This development is utilized in non-standard examination, with all Γ_i equivalent to $\mathbb{R}$. The structure R^{fin} is then a valuation ring, with maximal perfect μ (on the off chance that a $\in R^{fin}$, $a \notin \mu$, at that point there is c $\in$ N to such an extent that 1/c ¡ —a— ¡ c, so 1/a likewise has a place with R^{fin}). The components of μ are called little, and the ring homorphism $R^{fin} \to \mathbb{R} = R^{fin}|\mu$ is known as the standard part.

(3.16) The hypothesis of Ax and Kochen. Consider the field $\mathbb{F}_p((t))$ of intensity arrangement over $\mathbb{F}_p$. This is an esteemed field (where v(t) = 1), with worth gathering $\mathbb{Z}$ and buildup field $\mathbb{F}_p$. The hypothesis of Ax and Kochen states that if $\mathcal{F}$ is a non-chief ultraproduct on the set p of primes, at that point there is an isomorphism of esteemed fields between the fields $\prod_{p-P} \mathbb{Q}_p|\mathcal{F}$ *and* $\prod_{p-P} \mathbb{F}_p((t))|\mathcal{F}$.

This suggests specifically that, given a sentence ϕ which holds in everything except limitedly numerous

of the $\mathbb{Q}_p$'s [resp., in everything except limitedly a considerable lot of the $\mathbb{F}_p$((t))'s], the sentence σ will hold in everything except limitedly a large number of the $\mathbb{F}_p$((t's)) [resp., everything except limitedly a considerable lot of the $\mathbb{Q}_p$'s].

(3.17) Ultraproducts of many-arranged structures. The ultraproduct development conveys over to many-arranged structures with no inconvenience. In the event that s is a sort, at that point the universe of sort s of the ultraproduct will just be the ultraproduct of the universes of sort s.

Capacities between tuples of sorts and relations will be characterized correspondingly. Possibly this is better observed with models.

Model 1. See Example 1 of (2.15) for the documentation. Give I a chance to be a record set, $\mathcal{F}$ a ultrafilter on I, and $\mathcal{G}_i, I \in I$, a group of 2-arranged structures, where $\mathcal{G}i = ((G_i, \cdot, -1, 1), (\Gamma_i, +, 0, \leq), di)$.

At that point $\mathcal{G}^* = \mathbb{Q}_{i\in I}\mathcal{G}_i|\mathcal{F}$ will be the 2-arranged structure

$$(\prod_{i-I}(G_i, \cdot, ^{-1}, 1)|\mathcal{F}, \prod_{i-I}(\Gamma_i, i, +, 0, \leq)|\mathcal{F}, d^*)$$

where

$$d^*((g_i)_{\mathcal{F}}, (h_i)_{\mathcal{F}}) = (d(g_i, h_i))_{\mathcal{F}}$$

Example 2. Give P a chance to be the arrangement of primes, $\mathcal{F}$ a ultrafilter on P, and consider the 3-arranged structures $\mathcal{Q}_p$ presented in the model 2 of (2.15).

At that point $\prod_{i} \mathbb{Q}_i|\mathcal{F} = (\prod_{p} \mathbb{Q}_p|\mathcal{F}, \prod_{p} \mathbb{F}_p|\mathcal{F}, (\mathbb{Z} \cup \{\infty\})^{p}|\mathcal{F}, r^*, \pi^*)$. where $\prod_{i} \mathbb{Q}_i|\mathcal{F}$ *and* $\prod_{p} \mathbb{F}_p|\mathcal{F}$ have their normal field structure, $\mathbb{Z}^{I}|\mathcal{F}$ the structure of an arranged gathering, with a recognized steady ∞, and the maps r^* and π^* are characterized by: $r^*((a_p)\mathcal{F}) = (r(a_p))\mathcal{F}, \pi^*((a_p)\mathcal{F}) = (r()a_p)\mathcal{F}$.

Precedent 3. Give I a chance to be a file set, $\mathcal{F}$ a ultrafilter on I, K_i, I $\in$ I, a group of fields, and for every I, $R_i = (R_{n,i})$ the ω-arranged structure characterized in (2.15).

At that point $R^* = Q \prod_{i \in I} R_i|\mathcal{F}$ is the ω-arranged structure, with universe of sort n $R^*_n = Q \prod_{i \in I} R_i|\mathcal{F}$. The expansion a duplication are characterized normally. See that R^* is basically the ω-arranged structure related to the polynomial ring in $X_1, \ldots, X_n$ over $K^* = \prod_{i \in I} K_i|\mathcal{F}$.

Precedent 4 - Asymptotic cones. We begin with a group of 2-arranged structures $\mathcal{X}_i = ((X_i, x_i, \Delta\Delta), (\Gamma_i, +, -, \leq, 0), d_i)$, where (X_i, d_i) is a measurement space with a recognized point $x(I \in I)$. The estimations of d_i are in the subgroup Γ_i of $\mathbb{R}$. We permit X_i to have additional structure, for example a gathering structure. Fix a non-key ultrafilter F on I. Consider

$$\prod_{i \in I} \mathcal{X}|\mathcal{F} = (X^*, \Gamma^*, d^*)$$

where $(X^*, x^*, \ldots) = \prod_{i \in I}(X_i, x_i, \ldots)|\mathcal{F}, \Gamma^* = \prod_{i \in I} \Gamma_i|\mathcal{F}$ *and* $d^*((y_i)\mathcal{F}, (z_i)\mathcal{F}) = (d_i(y_i, z_i))\mathcal{F}$ .We consider the arranged subgroup Γ^{fin} *and* μ_Γ characterized in (3.15). set

$$X^{fin} = \{y \in X^* | d^*(x^*, y) \in \Gamma^{fin}\},$$

also, remainder X^{fin} by the comparability connection

$$E(y, z) \Leftrightarrow d^*(y, z) \in \mu_\Gamma$$

to acquire a set $X_{\mathcal{F}}$. We call the guide $X^{fin} \rightarrow X_F$ the standard part, and signify it st. At that point d^* actuates a guide $d_{\mathcal{F}} : X_{\mathcal{F}} \rightarrow \Gamma_{\mathcal{F}} = \Gamma^{fin}|\mu\Gamma \subseteq R^{fin}|\mu = \mathbb{R}$. See that the structure $(X_{\mathcal{F}}, \mathbb{R}, d_{\mathcal{F}})$, being a remainder of the substructure $(X^{fin}, \Gamma^{fin}, d^*)$ *of* (X^*, Γ^*, d^*),will fulfill all quantifier free recipes (possibly with parameters) fulfilled by (X^*, Γ^*, d^*).In specific, the guide dF will fulfill the aphorisms of a separation, so that (X, $d_{\mathcal{F}}$) is a measurement space. On the off chance that each (X_i, d_i) is δ_i-hyperbolic, at that point $(X_{\mathcal{F}}, d_{\mathcal{F}})$ will be $(\delta_i)\mathcal{F}$ - hyperbolic. On the off chance that for each I, and y, z $\in$ X, , there is a separation safeguarding inserting [0, d(y, z)] $\cap \Gamma_i \rightarrow X_i$ which sends 0 to x and d(y, z) to y, a similar will be valid for $(X_{\mathcal{F}}, d_{\mathcal{F}})$. We will give the verification, as this shows how adaptable ultraproducts seem to be: one can include structure on the off chance that one needs it. Let a, b $\in X_{\mathcal{F}}$, be spoken to by $(a_i)\mathcal{F}, (b_i)\mathcal{F} \in X^*$ individually. For every I, fix

$f_i : [0, d_i(a_i, b_i)] \cup \Gamma \to X$ which is separation protecting and sends 0 to a_i, $d_i(a_i, b_i)$ to b_i. At that point consider $f^* : [0, d^*((a_i)\mathcal{F}, (b_i)\mathcal{F}) \cup \Gamma^* \to X^*$ characterized by $f^*((\gamma_i)\mathcal{F}) = (f(\gamma_i))\mathcal{F}$ for $(\gamma_i)\mathcal{F}[0, d^*((a_i)\mathcal{F}, (b_i)\mathcal{F}) \cup \Gamma^*$. This guide is separation safeguarding. Henceforth, as $(a_i)\mathcal{F}, (b_i)\mathcal{F} \in X^{fin}$, we may go to the remainder, and get a guide f_F, separation protecting, and sending some section $[0, \gamma]$ of $\Gamma_{\mathcal{F}}$ *to* $X_{\mathcal{F}}$, and with $f_F(0) = a, f(\gamma) = b$.

In the event that for every I, there is a gathering G_i following up on X_i, then the ultraproduct $G^* = \prod_{i \in I} G_i \mathcal{F}$ will likewise follow up on X^*. In any case, except if there is a whole number N with the end goal that for all $i \in I$, $g \in G_i$ and $y \in X_i$, one has $d_i(y, gy) \le N d_i(x_i, y)$, the gathering G^* won't follow up on X^{fin}.

On the off chance that anyway all gatherings G_i are equivalent, and for each $g \in G$, there is N = N(g) with the end goal that for all $i \in I$ and $y \in X_i, d_i(y, gy) \le N(g) d_i(x_i, y)$, at that point G will follow up on X^{fin} and furthermore on $X_{\mathcal{F}}$.

Accept since each X_i is in certainty a gathering $(G, \cdot, ^{-1}, 1, a_1, \dots, a_n)$, with $x_i = 1$, and that $d_i : G^2 \to \mathbb{R}$ approaches $(1/m_i)d$, where d is the separation on the Cayley chart of G as for the set $a_1, \dots, a_n$ of generators of G (we set d(x, y) = d(1, yx^{-1}), so d is invariant under right increase). On the off chance that the component $N = (m_i)\mathcal{F}$ is boundless, at that point the picture of $d_{\mathcal{F}}$ will be all of $\mathbb{R}$, gave obviously our gathering G is unending

The comparability connection E will relate to being in a similar left-coset of the subgroup $G(0 = g \in X^{fin} | d^*(1, g) \in \mu$ *of* X^{fin}. Likewise, both G(0) and X^{fin} are raised subgroups of X^*, however G(0) isn't really typical in X^{fin}, so that $X_{\mathcal{F}}$ does not really have a gathering structure. Be that as it may, the gathering X^{fin} acts (transitively) on $X_{\mathcal{F}}$.

For more subtle study algebraic system we need to what is the type. Let T-complete theory. for any n¿0 denote by $F_n(T)$ set of all formulas in language of theory T, not containing free variables differing from $x_1, \dots, x_n$. Two formulas F ang G is $F_n(T)$ is called equvalent, if $T \vdash F \leftrightarrow G$. Let [F]-class of equvalent formula F. We denote $B_n(T)$ Boolean algebra, composed of all classes [F], when F part of $F_n(T)$. Boolean operation in $B_n(T)$ determine by next equality:

$$[F] \cup [G] = [F \vee G], [F] \cap [G] = [F \wedge G], [\bar{F}] = [\neg F]$$

The formula $F(x_1, \dots, x_n)$ is called consistent with T, if

$$T \vdash (\exists X_1) \dots (\exists x_n) F(x_1, \dots, x_n)$$

The set $S \subseteq F_n(T)$ is called consistent with T, if any conjunction of finite elements in S consistent with T.

Maximal constistent subset p of set $F_n(T)$ is called n-type

Eg.1. Let T- theory of one relation equivalence in language $L = \{E^2\}$ such that for any $n < \omega$ every model for T has exactly one class of equivalence which has one n elements. Lets consider next formulas:

$$\varphi_1 := \forall y[E(x,y) \to y = x]$$

$$\varphi_n := \exists x_2 \dots \exists x_n[\wedge_{i=2}^{n}(x \neq x \wedge E(x,x_i)) \bigwedge_{2 \leq i \neq j \leq n} x_i \neq x_j \wedge$$

$$\wedge \forall y(E(x,y) \to y = x \bigvee_{i=2}^{n} y = x_i)]. \quad n \geq 2$$

Easy to understand that formula $\varphi_n(x)$ determine elements,which lying in an equivalence class containing exactly n elements. For any $i < \omega$ we will consider next set of formulas:

$$S(x) := \{\neg\varphi_i(x)\} \cup \{\neg\varphi_j | j < \omega, j = i\}$$

We can understand thet $S_i(x)$ consistent fo any $i < \omega$. Also consider the next set of formulas:

$$S(x) := \{\neq \varphi_j(x) | j < \omega\}$$

Obviously that $S(x)$ also consistent. If for every formulas $F \in p$ we see on |F|, then received subset of set $B_n(T)$, which also denoted through p, it forms ultrafilter. In this way.

(i) if $F \in p$ *and* $G \in p$, *then* $F \wedge G \in p$

(ii)$F \in p \Leftrightarrow \neg F \notin p$

Every consistent subset of $F_n(T)$ can be complete to n-type. Denote through $Sn(T)$ (or $S_n(\emptyset)$) st of all n-type theory T.

Let $\mathfrak{A} \models T$ *and* $a_1, \dots, a_n in A$. We say that sequence $\langle a_1, \dots, a_n\rangle$ realize n-type p$\in$ $S_n(T)$ in $\mathfrak{A}$. *if* $\mathfrak{A} \models F(a_1, \dots, a_n)$ for every formula $F \in p$ such that if $\langle a_1, \dots, a_n\rangle$ satisfy in $\mathfrak{A}$ for every $F(x_1, \dots, x_n) \in p$. In such case we write that $p = tp(\langle a_1, \dots, a_n\rangle/\emptyset)$ such that type p is type of cortege $\langle a_1, \dots, a_n\rangle$ over empty set. If $\mathfrak{B} \models T$ *and* $b_1, \dots, b_n \in B$, then

$$\{F(x_1, \dots, x_n) | \mathfrak{B} \models F(b_1, \dots, b_n)\}$$

is n-type, exactly that n-type which $\langle b_1, \dots, b_n\rangle$ realize in $\mathfrak{B}$.

Eg.2. Let $\mathfrak{A} := \langle \mathbb{Q}, <, c_i\rangle_{i \in \mathbb{Q}}$, when $\mathbb{Q}$-is set of rational numbers. Obviously that $|\mathbb{Q}| = \omega$. Understand that $|S1(emptyset)| = 2^{omega}$. We know, the set of rational numbers is subset of real numbers, and for any rational numbers $q1$ *and* $q2$ with condition $q1 < q2$ we can find irrational numbers s such that $q1 < s < q2$. lets consider $tp(\sqrt{2}/\emptyset)$. Obviously that type has the formulas $x > 1, 1; x < 1, 12; x > 1, 11; x < 1, 419$ etc. We can understand that type any irrational numbers determine by formula of this kind with constant in $\mathbb{Q}$. Since irrational numbers $|\mathbb{I}| = 2^{\omega}$, then 1-type over irrational numbers too 2^{ω}

Proposition 4.1

Let $\mathfrak{A}$ - infinite set of algebraic system. Y$\subset$A.

(1) If p$\in$ $S_n(Th(\langle\mathfrak{A}, y\rangle y \in Y))$ or $(p \in S_n(Y))$, then there exist a system $\mathfrak{B} \succ \mathfrak{A}$ such that n-type p realized in $\langle\mathfrak{B}, y\rangle_{y\in Y}$ and $card\mathfrak{B} = card\mathfrak{A}$.

(2) There exists a system $\mathfrak{B} \succ \mathfrak{A}$ such that every n-type $p \in S_n(Th(\langle\mathfrak{B}, y\rangle_{y\in Y}))$ realized in $\mathfrak{B}$ and $card\mathfrak{B} \leq card\mathfrak{A} * 2^{max(\omega, cardY)}$.

Proof.

(1)Let $c_1, \ldots, c_n$ - subject constants that are not included in formulas from $Th(\langle\mathfrak{A}, y\rangle_{y\in Y})$. Let S be $Th(\langle\mathfrak{A}, y\rangle_{y\in Y}) \cup \{F(c_1, \ldots, c_n) | F(x_1, \ldots, x_n) \in p\}$.

We can consider $Th(\langle\mathfrak{A}, y\rangle_{a\in A})$ as an extension of $Th(\langle\mathfrak{A}, y\rangle_{y\in Y})$, since $Y \subset A$. Then $(\exists x_1)\ldots(\exists x_n)F(x_1, \ldots, x_n) \in Th(\langle\mathfrak{A}, y\rangle_{a\in A})$ for each formula $F(x_1, \ldots, X_n) \in p$. It follows that S together. By the basic existence theorem, S has the following model $\mathfrak{B}$, that card B = card A,$\mathfrak{B} \prec \mathfrak{A}$ and p is implemented in $\mathfrak{B}$.

(2) follows from (1) and the principle of elementary chains.

Let $k = cardS_nTh(\langle\mathfrak{A}, y\rangle_{y\in Y})$. By virtue of the agreement on countability $k \leq 2^{max(\omega, cardY)}$. Let $\{p_\delta | \delta < k\}$ is the complete ordering of $S_nTh(\langle\mathfrak{A}, y\rangle_{y\in Y})$. Define by transfinite induction elementary chain $\{\mathfrak{A}_\delta | \delta \leq k\}$:

(i) $\mathfrak{A}_0 = \mathfrak{A}$

(ii)Suppose that $\mathfrak{A}_\delta$ is already defined and $\mathfrak{A}_0 \prec \mathfrak{A}_\delta$. Then $\langle\mathfrak{A}_0, y\rangle_{y\in Y}$, therefore p_δ, which is an element of $S_n(Th(\langle\mathfrak{A}, y\rangle_{y\in Y}))$, can be considered as $S_n(Th(\langle\mathfrak{A}, y\rangle_{y\in Y}))$. According to 4.1 (1), there is a system $\mathfrak{A}_{\delta-1} \succ \mathfrak{A}_\delta$ such that p_δ is realized in $\mathfrak{A}_{\delta+1}$ and $card\mathfrak{A}_{\delta+1} = card\mathfrak{A}_\delta$.

(iii) Suppose that $\mathfrak{A}_\delta$ is already defined for all δ smaller than some limit ordinal λ, and that $\mathfrak{A}_\delta | \delta < \lambda$ - Elementary circuit. We assume that $\mathfrak{A}_\lambda = \cup\{\mathfrak{A}_\delta | \delta < \lambda\}$. Then $\mathfrak{A}_\lambda \succ \mathfrak{A}_\delta$ for all $\delta < \lambda$

Let $\mathfrak{B} = \mathfrak{A}_k$. Then p_δ is realized in $\mathfrak{B}$, since $\mathfrak{B} \succ \mathfrak{A}_{\delta+1}$ and p_δ is realized in $\mathfrak{A}_{\delta+1}$. With using transfinite induction, it is easy to show that $card\mathfrak{A}_\delta \leq card\mathfrak{A} \times card\delta$ for all $\delta \leq k$.□

Saturated systems

Let $\mathfrak{A}$ be an infinite algebraic system, $Y \subset A$.

System $\mathfrak{A}$ is called saturated over Y, if every type $p \in S_1(Th(\langle\mathfrak{A}, y\rangle_{y\in Y})$ realized in $\mathfrak{A}$ (more accurately, in $\langle\mathfrak{A}, y\rangle_{y\in Y}$)

System $\mathfrak{A}$ is called saturated if $\mathfrak{A}$ is saturated over each $Y \subset A$ such that $cardY < cardA$.

Let k be an infinite cardinal. Then $\mathfrak{A}$ is called k-saturated if $\mathfrak{A}$ is saturated for each $Y \subset A$ such that $card Y < k$.

Examples.

(1) Every countable model of the pure theory of equality is saturated (ω-saturated).

(2) The set of rational numbers, considered as a dense linear order without the largest and smallest elements, there is a saturated (ω-saturated) system

(3) Let T be a theory of signature $\Sigma := \{=, c_i\}_{i \in \omega}$, and let it be given by the axioms $c_i \neq c_j, i < j < \omega$ (i.e., this is a constant extension of the pure equality theory with the addition of countable number of constants in signature).

Lets consider $p(x) := \{x \neq c_i | i < \omega\} \in S_1(\emptyset)$. Let $\mathfrak{A} := \langle A, =, c_i \rangle_{i \in \omega}$, when $A = \{c_i | i \in \omega\}$.Then $\mathfrak{A}$ - model of theory T. Obviously that system $\mathfrak{A}$ isn't saturated over $\emptyset$, because this type p not realized in $\mathfrak{A}$.

Let $\mathfrak{A}$ be a model of the theory T that implements the type p by exactly one element (we denote this element is through a_1). Then $\mathfrak{A}$ is saturated over $\emptyset$.

Consider the following set of formulas $p_1(x) := p(x) \cup \{x \neq a_1\}$. Obviously, p_1 is not implemented in $\mathfrak{A}_1$, i.e. $\mathfrak{A}_1$ is not saturated over $\{a_1\}$.

Let $\mathfrak{A}_\omega$ be a model of a theory T containing ω realizations of type p. Then $\mathfrak{A}_\omega$ is saturated.

Theorem 4.2 If $\mathfrak{A}$ and $\mathfrak{B}$ - is saturated system of same powered and $\mathfrak{A} \equiv \mathfrak{A}$, then $\mathfrak{A} \simeq \mathfrak{A}$.

Proof.

Let card $\mathfrak{A} = k, A = \{a_\delta | \delta < k\}$ and $\mathfrak{B} = \{b_\delta | \delta < k\}$. Define with transfinite induction set $\{\langle c_\delta, d_\delta \rangle | \delta < k\}$. Fix $\delta < k$ and assume that the set $\{\langle c_\gamma, d_\gamma \rangle | \gamma < \delta\}$ is already defined and $\langle \mathfrak{A}, c_\gamma \rangle_{\gamma < \delta} \equiv \langle \mathfrak{B}, d_\gamma \rangle_{\gamma < \delta}$. (If $\delta = 0$, then it just means that $\mathfrak{A} \equiv \mathfrak{B}$.)

Case 1.δ is even. Let c_δ - is set of elements $A | \{c_\gamma | \gamma < \delta\}$ with least index. We soppose

$$p = \{F(x_1) | \langle \mathfrak{A}, c_\gamma \rangle_{\gamma < \delta} \models F(c_\delta)\}$$

Then $p \in S_1(Th(\langle \mathfrak{A}, c_\gamma \rangle_{\gamma < \delta}))$. Let q be obtained from p by replacing each occurrences of c_γ in p on d_γ for all $\gamma < \delta$. Then $q \in S_1(Th(\langle \mathfrak{B}, d_\gamma \rangle_{\gamma < \delta}))$. Since $\mathfrak{B}$ is saturated, then q is realized in $\mathfrak{B}$ by some element b; we assume $d_\delta = b$. Then

$$\langle \mathfrak{A}, c_\gamma, c_\delta \rangle_{\gamma < \delta} \equiv \langle \mathfrak{B}, c_\gamma, c_\delta \rangle_{\gamma < \delta}$$

Case 2. δ is odd. We proceed in the same way as in case 1, reversing the roles of $\mathfrak{A}$ and $\mathbb{B}$. We set $h(c_\delta) = d_\delta$ for all $\delta < k$. The given construction guarantees us that h establishes a one-to-one correspondence between $\mathfrak{A}$ and $\mathfrak{B}$. From the relation $\langle \mathfrak{A}, a \rangle_{a \in A} \equiv \langle \mathfrak{B}, h(a) \rangle_{a \in A}$ implies that h is an isomorphism.

Type omiting

It is said that system $\mathfrak{A}$ omits the n-type p. if p is not implemented in $\mathfrak{A}$ by any element from A^n . It is clear that it is much more difficult to omit types than to implement them, since for lowering need to worry more about each element of the extension.

Let T be a complete theory and $p \in S_n(T)$. A type p is called principal if there is such a formula $F(x_1, \dots, x_n) \in p$ such that $T \models F(x_1, \dots, x_n) \to G(x_1, \dots, x_n)$ for all $G(x_1, \dots, x_n) \in p$: say that $F(x_1, \dots, x_n)$ generates p (The main types of p from $S_n(T)$ are correspondingly Branch to isolated points $S_n(T)$, considered as Stone space Boolean algebra $B_n(T)$: generators of principal types p correspond to atoms $B_n(T)$). So as T is complete, every model for T implements every major type p.

Example 1. Let T be the theory of one equivalence relation in the language $L = \{E^2\}$ such that for any $n < \omega$ each model for T has exactly one equivalence class containing exactly n elements (Example 1 of lecture 4).

Consider the following formulas, as well as the following sets of formulas:

$$\phi_1(x) := \forall y[E(x, y) \to y = x]$$

$$\phi_n(x) := \exists x_2 \dots \exists x_n [\bigwedge_{i=2}^{n} (x \neq x_i \wedge E(x, x_i))$$

$$\bigwedge_{1 \le i \ne j \le n} x_i \neq x_j \wedge \forall y(E(x, y) \to y = x \vee \bigvee_{i=2}^{n} y = x_i)], n > 1$$

$$S_i(x) := \{\phi_i(x)\} \cup \{\neg\phi_j(x) | j < \omega, j \neq i\} i < \omega$$

$$S(x) := \{\neg\phi_j(x) | j < \omega\}$$

It is obvious that $S_i(x)$ for each $i < \omega$, and also S (x) are consistent. Each $S_i(x)$ expands to the principal type $p_i(x) \in S_1(T)$, and the formula $\varphi_i(x)$ generates the type p_i. Lots of S(x) expands to a non-principal type $p(x) \in S_1(T)$.

Theorem 5.1. If the theory T is countable and $p \in S_n(T)$ is a non-principal type, then T has a model that omits p.

Proof.

It is carried out according to the Genkin method applied in the main existence theorem. For simplicity, we assume that n = 1.

Let $\{c_i | i < \omega\}$ is a sequence of subject constants that are not in the language of the theory T. All formulas with one free variable x (in the language of theory T with added constants c_i) are arranged in the sequence $G_j(x) | j < \omega$.

Let $h : \omega \to \omega$ satisfies the condition:

(i) $j < i$ attracts $h(j) < h(i)$;

(ii) If $j \leqslant i$, then c_i excluded in $G_j(x)$.

Throught by H_i denotes i-s Henkin's axiom, such that $(\exists)G_i(x) \to G_i(c_{i_{l(i)}})$.

By induction we can determine expanding sequence of theories $\{T_i; i < \omega\}$, $T_0 = T$

(1) We consider, that T_{2i} is consistent and $c_{l(i)}$ exculded in T_{2i}. We suppose that $T_{2i-1} = T_{2i} \cup \{H_i\}$. Then T_{2i-1} consistent, lika a proof on basic theorem of existing.

(2) Lets consider that T_{2i} consistent and $T_{2i-1} = t \cup \{K(c_{i_1}, ..., c_{i_n}, c_i)\}$ when $i \neq j$ for $1 \leqslant j \leqslant n$ and c_i excluded in T, then

$$T \vdash (\exists x_1)...(\exists x_n)K(x_1, ..., x_n, x) \to F(x)$$

for every $F(x) \in p$

If $(\exists x_1)...(\exists x_n)K(x_1, ..., x_n, x) \notin p$ then $\neg(\exists x_1)...(\exists x_n)K(x_1, ..., x_n, x) \in p$ and

$$T \vdash \neg(\exists x_1)...(\exists x_n)K(x_1, ..., x_n, x)$$

And T_{2i-1} not consistent contarary to sentenses. If $(\exists x_1)...(\exists x_n)K(x_1, ..., x_n, x) \in p$, thrn p is principal which contrudiction our contition.

Suppose $T_\infty = \cup\{T_i | i < \omega\}$. Henkin's constuction ends with a choice of some maximal consistent extention $S \supset T_\infty$. Like a basic theorem of existing, S determine a model for T, and elements which are class of equivalence $[c_i]$. Neither $[c_i]$ not realize p, because $\neg F(c_i) \in T_{2i-1} \subset S$ for some $F(x) \in p$. □

The theorem of 5.1 is essential.

Let k-infinite cardinal. The theory T is called k-categorical, if all its model with power k isomorphic.

Examples.

(1) The teory of first example is not k-categorical for any infinite kardinal k.

(2) The pure theory of equivalence is k-categorical for any infinite kardinal k.

(3) The theory of dense linear order without end points also is k-categorical for any infinite cardinal k.

(4) The theory one relation equivalence with two infinite equivalence classes is ω-categorical, but not ω_1- categorical.

Consequences 5.2. Let T - countable complete theory, not having end models.

Then the following conditions are equivalent:

(1) T is ω-categorical,

(2) $S_n(T)$ is finite for any $n < \omega$,

(3) Each counting model for T is saturated.

Proof.

(1) ⇒ (2) Suppose that $S_n(T)$ is infinite. Then $B_n(T)$ is infinite. Every infinite Boolean algebra $\mathfrak{B}$ has a non-principal ultrafilter. Therefore, $S_n(T)$ contains a non-principal type p.

Indeed, if $B_n(T)$ is infinite, then there exist an infinite number of pairwise nonequivalent formulas with n free variables. Then you can choose an infinite number of pairwise disjoint formulas with n free variables $F_1(\bar{x}), F_2(x), \ldots$.

Let $p(\bar{s}) := \{\neg F_n(\bar{x}) | n < \omega\}$, we understand $p(\bar{x})$ consistent, such that for any $k < \omega$ and $1 \leqslant i_1 < i_2 < \ldots < i_k < \omega$ performed $T \vdash \exists \bar{x}[\wedge_{j=1}^{k} \neg F_{i_j}(\bar{x})]$. If not, then exists finite number of formulas $F_{i_1}(\bar{x}), F_{i_2}(\bar{x}), \ldots, F_{i_k}(\bar{x})$ such that $T \vdash \forall \bar{x}[\wedge_{j=1}^{k} F_{i_j}(\bar{x})]$, with contrudiction to choise formula $F_i(\bar{x})$. In this way, $p(\bar{x})$ is consistent and its extention to not principal type $p' \in S_n(T)$.

According to 5.1, there exists a countable model for T, which omits p. According to 4.1, there exists a countable model for T, which implements p. Therefore, T is not ω-categorical.

(3) $\Rightarrow$ (1) If every countable model for T is saturated, then T is ω-categorical in 4.2.

(2) $\Rightarrow$ (3) Suppose that $S_n(T)$ is finite for any n $< \omega$.

Then each type $p \in\in S_n(T)$ is principal for any $n < \omega$. Let A be countable model for T. Then every n-type implemented in $\mathfrak{A}$ is paramount.

Fix $p \in S_1(Th\langle\mathfrak{A}, a_i\rangle_{1<i<n}))$ and show that p is realized in $\mathfrak{A}$. We set

$$p^* = \{G(x_1, \ldots, x_n, x) | G(a_1, \ldots, a_n, x) \in p\}$$

Since $p^* \in S_{n+1}(T)$, p^* is principal; let $H(x_1, \ldots, x_n, x) \in p^*$ generate p^*. Then $H(a_1, \ldots, a_n, x) \in p$ and it generates p. Since p is the principal type, it is implemented in each model for $Th(\langle\mathfrak{A}, a_i\rangle_{1<i<n})$.

Sentences 5.3. Let $\mathfrak{A}$-saturated system, $Y \subset A, card(Y) < card(A)$ and type $p \in S_n(Th(\langle\mathfrak{A}, y\rangle_{y \in Y}))$. Then p realized in $\mathfrak{A}$.

The proof held through induction by n. Let $n > 1$ and $p \in S_n(Th(\langle\mathfrak{A}, y\rangle_{y \in Y}))$.

We consider

$$p_{n-1} = \{(\exists x_n)F(x_1, \ldots, x_n) | F(x_1, \ldots, x_n) \in p\}$$

Then $p_{n-1} \in S_1(Th(\langle\mathfrak{A}, y, a_1, \ldots, a_{n-1}\rangle_{y \in Y}))$, so p_{n-1} realize in $\mathfrak{A}$ some sequences $\langle a_1, \ldots, a_{n-1}\rangle$. Will consider

$$p_1 = \{F(a_1, \ldots, a_{n-1}, x) | F(x_1, \ldots, x_{n-1}, x_n) \in p\}$$

Then $p_1 \in S_1(Th(\langle\mathfrak{A}, y, a_1, \ldots, a_{n-1}\rangle_{y \in Y}))$, so p_1 realize in $\mathfrak{A}$ some a_n. We understand that $\langle a_1, \ldots, a_n\rangle$ realize p_n. □

Lemma 5.4.

Let T be a countable complete theory that has no finite models. Then T has a countable saturated model if and only if $S_n(T)$ is countable for each $n < \omega$.

Proof: Suppose first that T has a countable saturated model A. By 5.3, each n-type $p \in S_n(T)$ is realized in $\mathfrak{A}$. Then $S_n(T)$ is (at most) countable, so like A^n countable.

Now suppose that $S_n(T)$ is countable for any $n < \omega$. Then $S_n(Th(\langle\mathfrak{B}, y\rangle_{y\in Y}))$ is countable for each $n < \omega$, any model B for T, and any finite subset $Y \subset B$. In fact, let $Y = \{y_1, \dots, y_m\}$. If $S_n(Th(\langle\mathfrak{B}, y\rangle_{y\in Y}))$ is uncountable, then $S_{m+n}(T)$ is uncountable, since there is a unique mapping of the first set to the second, induced by the transition from $F(y_1, \dots, y_m, x_1, \dots, x_n)$ to $F(x_1, \dots, x_m, \dots, x_{m+n})$.

Let $\mathfrak{B}$ be a countable model for T, and let $\{Y_i|i < \omega\}$ - enumeration of all finite subsets of B. Define an elementary chain $\{\mathfrak{B}_i|i < \omega\}$

(i)$\mathfrak{B}_0 = \mathfrak{B}$

(ii)$\mathfrak{B}_{i+1} \succ \mathfrak{B}_i$. In model $\mathfrak{B}_{i+1}$ realize every type $p \in S_1(Th(\langle\mathfrak{B}_i, y\rangle_{y\in Y_i}))$. Its a finite. The existing of $\mathfrak{B}_{i+1}$ follows from 4.1 and countability $S_1(Th(\langle\mathfrak{B}_i, y\rangle_{y\in Y_i}))$

We consider $\mathfrak{C} = \bigcup\{\mathfrak{B}_i|i < \omega\}$. Then $\mathfrak{C} \succ \mathfrak{B}$ by 3.2.

each n-type $p \in S_1(Th(\langle\mathfrak{B}_i, y\rangle_{y\in Y}))$ is required in $\mathfrak{C}$ for any finite subset of the properties $Y \subset B$and $\mathfrak{C}$ is countable.

The required saturated saturated model $\mathfrak{A}$ is the limit of the elementary chain $\mathfrak{A}_i|i < \omega$, satisfying conditions:

(iii) $\mathfrak{A}_0$ is a counting model for T.

(iv) $\mathfrak{A}_{i+1} \succ \mathfrak{A}_i$. Each $p \in S_1(Th(\langle\mathfrak{B}_i, y\rangle_{y\in Y}))$ realize in $\mathfrak{A}_{i+1}$ for every finite $Y \subset A_i$.$\mathfrak{A}_{i+1}$ countable.

2.Definability of types

$p = S_1(A)$

p-definable if for any $\varphi(x, y)$ $\exists\Theta_\varphi(\bar{y}, \bar{c})$ $\bar{c} \in A$

$$\forall\bar{a} \in A[\varphi(x, \bar{a}) \in p \Leftrightarrow \models \Theta_\varphi(\bar{a}, \bar{c})]$$

Example:

$\rightarrow 0 \leftarrow$

$\{0 < x < \frac{n}{m} / \frac{n}{m} \in Q^-\}$

$x < y = \varphi_1(x, y)$ $\forall a \in Q[\varphi_1(x, a) \in p_1 \Leftrightarrow Q \models 0 < a]$ $\Theta_{\varphi_1}(y, 0)$

$y < x = \varphi_2(x, y)$ $\forall a \in Q[\varphi_2(x, a) \in p_0 \Leftrightarrow Q \models a = 0 \vee a < 0]$ $\Theta_{\varphi_2}(y, 0)$

$\Theta_{\varphi_1}(y, 0) = 0 < y$

$\Theta_{\varphi_1}(y, 0) = y = 0 \vee y < 0$

If $tp(\bar{a}/A)$ is definable $\Rightarrow \exists\bar{c} \in A\Theta(\bar{x}, \bar{c})$ $\forall\Theta(M, \bar{c}) \cap A = H(M, \bar{a}) \cap A$

Proof:$H(x, \bar{z})$ $tp(\bar{a}/A) = q(z, A) \in S(A)$ definable

$\forall b \in A$ $H(b, \bar{z}) \Rightarrow \exists\psi(x, \bar{c})c \in A$

$H(b, \bar{z}) \in q \Leftrightarrow \mathfrak{M} \models \psi(b, \bar{c})$

$\forall b \in A[\mathfrak{M} \models H(b, \bar{a})]$

$\forall b \in A$ $\mathfrak{M} \models H(b, \bar{a}) \Leftrightarrow \mathfrak{M} \models \psi(b, \bar{c})$

$a \in [A \cap H(M, \bar{a})] \Rightarrow \mathfrak{M} \models H(a, \bar{a}) \Leftrightarrow \psi(a, \bar{c}) \Rightarrow a \in [A \cap \psi(M, \bar{c})]$ 1) □

$\mathfrak{M} \prec \mathfrak{N}$ pair of models is called conservative, if $\forall\bar{a} \in N\mathcal{M}$ $tp(\bar{a}, M)$ is definable

$\mathfrak{M} \prec \mathfrak{N}$ $H(N, \bar{a})$ $\psi_H(\mathfrak{N}, \bar{a}) \cap M$

$\psi(\mathfrak{N}, \bar{c}) \cap M = \psi(\mathfrak{M}, \bar{c})$

$\mathfrak{M} \prec \mathfrak{N} \Leftrightarrow \psi(\bar{a}, \bar{c})$ $a \in M$

$\mathfrak{M} \models \psi(\bar{a}, \bar{c}) \Leftrightarrow \mathfrak{N} \models \psi(\bar{a}, \bar{c})$

Example: $Q \prec R$

$\rightarrow \sqrt{2} \leftarrow$

Difference between model and quasi-model(TV-type)

p$\in S_1(A)$ A-model p=$\phi_1(x, \bar{a})|\bar{a} \in A$-finitely relizable in A

1. A-is a model of theory $T_A \vdash \exists x \bigwedge_{i \in I_0} \varphi_i(x, \bar{a}_i)$

$A \prec M \Rightarrow M \models \exists x\varphi_i(x, \bar{a}_i) \rightarrow \exists b_{I_0} \in A$ $M \models \bigwedge_{i=I_0} \phi_i(b_{I_0}, \bar{a}_i)$

2. A-is a set of model M

$p(x) \in S(A)$-quasi-model (named by Baizhanov Bektur Sembeivich | Tarskii-Vaught-type named by Saharon Shelah)

if $\forall I_0 \in_{finitely\ realizable} I$ $M \models \bigwedge_{i \in I_0} \phi_i(x; b_i)|b_i \in A$

Example of non-quasi-model:

$M = \langle \mathbb{Q}, =, < \rangle \qquad A = \mathbb{Z}$

$\{1 < x < 2\}$-isolated on A, but $M \models \exists x(1 < x < 2)$ and for example $0.5 \notin A$

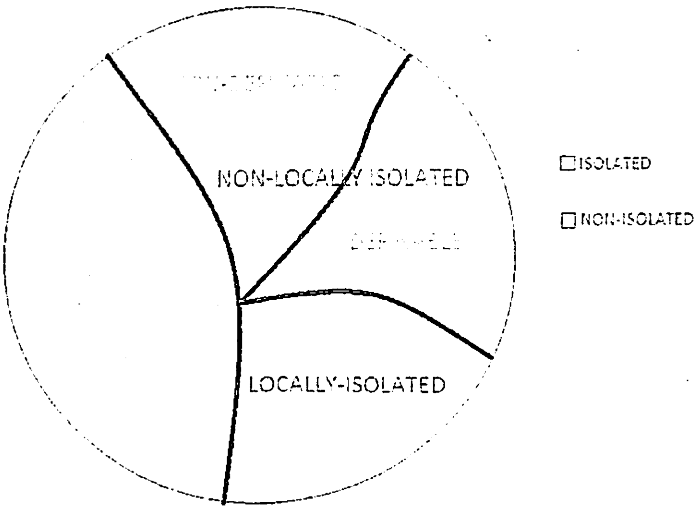
Claim 1. p-isolated$\Rightarrow$p-definable

$\Theta(x, \bar{c}) \quad \bar{c} \in A \qquad \forall \phi(x, \bar{a})[\phi(x, \bar{a}) \in p \Rightarrow A \subseteq M \quad M \models \forall x(\Theta(x, \bar{c}) \to \phi(x, \bar{a}))]$

p-non definable$\Rightarrow$ p-non-isolated($A \to B \Leftrightarrow \neg B \to \neg A$)

p-strictly definable(named by Baizhanov Bektur Sembeivich— locally isolated named by Saharon Shelah)

$\forall \phi(x, \bar{y}) \quad \exists \Theta_o(x, \bar{c}_o) \in p | c_o \in A$ such that $\forall \phi(x, \bar{a}) \in p \Leftrightarrow \forall x(\Theta_o(x, c_o) \to \phi(x, \bar{a}))$



Non-locally isolated

$\exists \phi(x, \bar{y})$ such that $\forall \Theta(x, \bar{c}) \in p$

$\exists \bar{a}_{\theta,c} \in A[[\varphi(x, \bar{a}_{\theta,c}) \in p \wedge \exists x(\Theta(x, c_o) \wedge \neg \phi(x, \bar{a}_{\theta,c}))] \vee [\varphi(x, \bar{a}_{\theta,c}) \notin p \wedge \forall x(\Theta(x, c_o) \to \phi(x, \bar{a}_{\theta,c}))]]$

Type $p \in S_1(A)$- non-locally-isolated, then formula $\phi(x, \bar{y})$ divide to two cases.

1. $\Theta(x, \bar{a}) \in p(x) \Rightarrow \models \exists x(\Theta(x, \bar{a}) \wedge \phi(x, \bar{a})) \wedge \exists x'(\Theta(x', \bar{a}) \wedge \neg \phi(x', \bar{a})) | a \in A$

2. $\forall x(\Theta(x, \bar{a}) \to \phi(x, \bar{a})) \wedge \exists x(\Theta(x, \bar{a}) \wedge \neg \varphi(x, \bar{a}))$

2. $H_\theta(\bar{y}, \bar{c}) | \theta(x, \bar{c}) \in p(x)$-we know that is a realized in A, and therefore it is a quasi model. Then every $H_\theta(\bar{y}, \bar{c}) \in q(y)$

$\Gamma(\bar{y}) = [H_{\theta_1}(\bar{y}, \bar{c}) \wedge H_{\theta_2}(\bar{y}, \bar{c}) \wedge H_{\theta_3}(\bar{y}, \bar{c}) \wedge \ldots H_{\theta_k}(\bar{y}, \bar{c})] \in q(y)$

$\Theta^{||}(x, \bar{c}') = [\Theta_1(x, \bar{c}_1) \wedge \Theta_2(x, \bar{c}_2) \wedge \Theta_3(x, \bar{c}_3) \wedge \ldots \Theta_k(x, \bar{c}_k) \wedge] \in p(x)$

$\Gamma(\bar{y})$-consistent. $\Gamma(\bar{y}) \subset q(y) \Rightarrow \exists F(y.\bar{d})$ such that $\Gamma(\bar{y}) \cup F(y.\bar{d})$ or $\Gamma(\bar{y}) \cup \neg F(y.\bar{d})$ is consistent by theorem of any locally consistent set of formulas we can complete to maximum consistent. But we have a question it will be a quasi model?

$A \models \exists \bar{y}[\bigwedge_{i=1}^{n} H_{\theta_i}(y_i.\bar{c}_i) \wedge \neg F(\bar{y}.\bar{d})] \Rightarrow \exists \bar{y}\neg[\bigwedge_{i=1}^{n} H_{\theta}(y_i.\bar{c}_i) \rightarrow F(\bar{y}.\bar{d})]$

$\exists \bar{y}(H'_{\theta_i}(\bar{y}.\bar{c}_j) \wedge F(\bar{y}.\bar{d}))$ we know every $d.c \in A$ then this things we can write $H_\vartheta(\bar{y}.\bar{c}) \Rightarrow$ quasi model.

Let $\Gamma(\bar{y}) \cup F(y.\bar{d})$ and $\Gamma(\bar{y}) \cup \neg F(y.\bar{d})$-is not a quasi model, then

$\exists y(\bigwedge_{i=1}^{n} H_{\theta_i}(\bar{y}.\bar{c}_i) \wedge F(\bar{y}.\bar{d})) = H_1$

$\exists y(\bigwedge_{i=1}^{n} H_{\theta_i}(\bar{y}.\bar{c}_j) \wedge \neg F(\bar{y}.\bar{d})) = H_2$

$A \models H_1 \wedge H_2 = H_3 \Rightarrow \exists \bar{y} H_3 \wedge (F(\bar{y}, \bar{d}) \vee \neg F(\bar{y}.\bar{d})) \Rightarrow \exists \bar{y}(H_3 \wedge F(\bar{y}.\bar{d})) \vee (H_3 \wedge \neg F(\bar{y}.\bar{d}))$ then in set A first part or second part always must be right. But it is not right because $F(\bar{y}.\bar{d})$ or $F(\bar{y}.\bar{d})$-non quasi model(contradiction).

p(x)-non locally isolated, then exist q(y). And now we know $\phi(M.a) \cap p(M) \neq \varnothing$ and $\neg\phi(M.a) \cap p(M) \neq \varnothing$ too. $a \in A$- which realized p(x) and q(y). $H(x) = \theta(x.\bar{a}) \in p(x) \Rightarrow A \models \exists x H(x) \wedge \phi(x.a)$ and $A \models \exists x' H(x') \wedge \neg\phi(x'.a) \Rightarrow p(x) \cup \{\phi(x.a)\}$ and $p(x) \cup \{\neg\phi(x.a)\}$- is consistent.

$K_\theta(\bar{y}.a) = \forall x(\phi(x.\bar{a}) \rightarrow H(x)) \wedge \exists x(H(x) \wedge \neg\phi(x.\bar{y})) \Rightarrow q(y) \not\vdash^{a} p(x)|\phi(M.a) \subset p(M) = \bigcup H(M)$ then Lemma1. $K_\theta(\bar{y}.\bar{a}) \in q(y) \Leftrightarrow H(x) \in p(x)$ it work for definability.

Proof:$\Rightarrow$ If q(y) is definable then $\forall v(\bar{y}.\bar{c})\exists F_{K_\theta}(\bar{a}.\bar{c}) \Rightarrow F_{K_\theta}(\bar{a}.\bar{c})$ be a control formula to $K_\vartheta(\bar{y}.\bar{a})$ $\Rightarrow$it must be control formula for every $H(x)$, we know that $\bigcap H(x) = p(x)$, then p(x) will be definable too.

2.1 Stable theories. Rank formulas Shelah

In Theory of model complete theory is called stable . if it hasn't many types. The basic purpose theory of classify is to share all of complete type on which model we can classify and which model very hard to classify. and classify all of model when we can do it. Another word if theory is not stable then it is vary hardly and so many to classify. and if theory is stable it can be some variant to classify it's model. special if theory is superstable or transcendental complete.

Theory of stability studied by Morley in 1965. he introduced some fundamental idea such that transcendental complete and rank of Morley. Concepts stability and superstable introduced by Shelah in 1969. that account of big part theory of stability.

Definition: Let T is complete theory

· T is n-stable (for infinity cardinal n). if for every set of A with cardinality equal to cardinality of all complete type over A has and it's cardinality equal to k.

· ω - stable. if stable for $\aleph_0$.

· Theory T is called stable if it is k-stable for some infinite cardinal.

· Theory T is called superstable if it is k-stable for all big cardinal k.

· Theory T is called non-stable if it is not k-stable for any infinite cardinal.

· Transcendental complete theory is these theory which any its formula has rank of Morley less then infinite.

Another word theory T is non stable if its can be used fo coding ordering set of natural numbers. If it has a model M and formula $\phi(A, B)$ in 2n variables A = $a_1, \ldots, a_n$ and B = $b_1, \ldots, b_n$ define of relation on M^N with infinity stable subset.

Examples:

1) Set of theory and arithmetic Peano is non-stable .

2) Theory of rational numbers if we consider as ordering set is non-stable.

3) Theory of sum natural numbers is non-stable.

4) Any infinite boolean algebra is non-stable.

5) Any monoid with canceling.

6) real closed field nonstable. because it has order.

Theory T is stable if it is k-stable for some cardinal k.

Example:

1) Theory any module over ring is stable

2) Theory of finite equivalence E_r for r natural numbers such that relation of equivalence has number of classes equivalence and every class equivalence E_i is combination of infinite numbers classes E_{r+1} stable, not not superstable.

3) Differential closed field is stable. if it has characteristic not equal to zero, it is not superstable, but if it has zero characteristic, then it is transcendental complete.

Theory T is superstable. if it is stable for all big cardinal, so all of superstable theory is stable. For countable theory T superstable equivalent stability for all $k \geq 2^{\omega}$.

Example:

1) Additive gruoup of integer numbers superstable. but not transcendental complete. It has 2^{ω} finite models.

2) Theory of finite unary relation P_j with models positive integer numbers. when $P_j(r)$ interpreted as pridicate. then r divide to r's prime number is superstable. but not transcendental complete.

3) Abelian group is superstable iff there exists only finite pair (q.r) with q prime, r natural number with $q^r A | q^{r+1} A$ is infinite.

$p(x)$-set of formulas $\quad p = \{\phi_i(x) / i \in I\}$

$$\forall i_1 ... i_n \models \exists x \bigwedge_{j=1}^{n} \phi_{i_j}(x)$$

Example: $\langle Q; =; < \rangle$

$(x) = (c < x < d)$

$R \models \{p_{\sqrt{2}} / c < \sqrt{2} < d\}$- set of such formulas

Stable - if it has not infinite 2-branching tree.

Rank it is number of floor of complete tree.

$r_\phi(x = x) = n \Leftrightarrow \exists \; complete \; \phi - n \; tree$

$\Leftrightarrow \neg\exists \; complete \; \phi - (n+1) \; tree$

$$T \vdash \exists y_1 \exists y_2 \exists y_{2^n - 1} \exists x_1 \exists x_{2^n} \bigwedge_{i,j-1}^{n} \varphi(x_i, y_j)$$

$(r_\phi(\phi(x, a_1)) = n - 1) \rightarrow (r_\phi(\neg\phi(x, a_1)) = n - 1 \vee n)$

Example:

$r_\phi(x = x) = 2$

$T \vdash \exists y_1 \exists y_2 \exists y_3 \exists x_{11} \exists x_{10} \exists x_{01} \exists x_{00} [(\phi(x_{11}, y) \wedge \phi(x_{11}, y_2) \wedge (\phi(x_{10}, y_1) \wedge \phi_{x_{10}, y_2}) \wedge (-\phi(x_{01}, y_1) \wedge \varphi(x_{01}, y_3)) \wedge (\neg\phi(x_{00}, y_1) \wedge \neg\varphi(x_{00}, y_3))\}$

Control formula $p \in S_1(A) \quad \phi(x, \bar{y})$

$r_\phi(x = x) = n$

$\forall \psi(x, \bar{b}) \in p \qquad r_\phi(\psi(x, \bar{b})) \leq n \qquad \{r_\phi(\psi(x, \bar{b})) = m / m \leq n - min\}$

$\psi(x, \bar{b}) \in p$

If $(\psi \in p \wedge H \in p) \Rightarrow (\psi \wedge H \in p)$

$\bar{a} \in A[\phi(x, \bar{a}) \in p \Leftrightarrow r_{\phi(x,y)}(\psi(x, \bar{b}) \wedge \phi(x, \bar{a})) = r_\phi(\psi(x, \bar{b}))] \models \Theta_{\phi(\bar{a}, \bar{b})}$

If theory is a stable then for every formula exists control formula

3. Definability of 1-types of weakly o-minimal theory [1]

Defenition 1. A partition (A, B) of a model M is called a section if $A < B$. Here $A < B \Leftrightarrow \forall a \in A \forall b \in B(a < b)$. A section is called rational if A has a maximal element or B is minimal, or one of them is empty. We say that a section is quasi-rational if A and, therefore, B are definable (with parameters). A non-quasi-rational section is called irrational. A model M is called Dedekind complete if its every section is rational. We say that M is a quasidedekind complete model if any of its sections is quasi-rational. Let be an elementary submodel of N. We say that a section (A, B) in M is realized in N if there exists $a \in N/M$ such that $A < a < B$.

Definition 2.[3, 4] A linearly ordered structure M is called o-minimal if any definable (with parameters) subset M is a finite union of points in M and intervals $(a, b), a \in M\bigcup\{-\infty\}, b \in M\bigcup\{\infty\}$

Definition 3.[5] Let M be an elementary submodel of N, where $N \models T$ and T is an o-minimal theory. A model M is called Dedekind complete in N if there is no irrational section in M, realized in N.

Definition 4. A subset A of a linearly ordered structure M is called convex if any element of M lying between two elements of A lies in A. In particular, any empty or single element set is convex. We say that the formula $\phi(x)$ is convex if the set $\phi(M)\{a \in M | M \models \phi(a)\}$ is convex.

Definition 5.[6, 7] A linearly ordered structure M is said to be weakly o-minimal if any definable (with parameters) subset M is a finite union of convex subsets

A theory T is weakly o-minimal if any of its models is weakly o-minimal.

Note that any o-minimal model is weakly o-minimal.

Definition 6. Let M be an elementary submodel of N, where $N \models T$ and T is a weakly o-minimal theory. We say that M is quasidedekindo complete in N if there is no irrational passage in M, realized in N.

Definition 7. Let A - set of a model M and model $M \models, n < \omega, p \in S_n(A)$. Type p is $\phi(\bar{x}_n, \bar{v})$-definable for $\phi(\bar{x}_n, \bar{v}) \in L(\bar{x}_n)$, if there exists a formula $R_\phi(\bar{v}) \in L(A)$ (A-definable formula) such that for any $\bar{a} \in A$ has $\phi(\bar{x}_n, \bar{a}) \in p$, iff $M \models R_\phi(\bar{a})$.

Then a formula $R_\phi(\bar{v})$ is called $\phi(\bar{x}_n, \bar{v})$- definability of type. We say that type p is definable, if p is $\phi(\bar{x}_n, \bar{v})$- definable for any formula $\phi(\bar{x}_n, \bar{v}) \in L(\bar{x}_n, n < \omega)$.

A tuple $\bar{\gamma} \in M$ is called ht-definable over A if its type over A is definable.

Recall [8]Lemma 2.3 that in the o-minimal theory for any section in the model M there is only one 1-type over M extending it. This type is defined if and only if the section is rational. Therefore, any o-minimal model of M is a D-1 model if and only if M is Dedekind complete. Any pair (M, N) of an arbitrary o-minimal theory is a D-1 pair if and only if M is Dedekind complete in N.

Theorema.8.[9] Any type over (R,+,·,0,1) definable (R-is a set of real numbers).

D.marker and C.Stainhorn determine this results on o-minimal theory.

Theorema.9.[5] Let T-o-minimal theory. $M \models T$. Then following is done:

1. Model M in S(M):=$\bigcup_{n<\omega} S_n(M)$ definale.

2. Let M - elementary submodel N. Then for any $\bar{a} \in N$, M type $tp(\bar{a}/M)$ definable iff M is dediking complete in N.

Some facts of definability of type.

The central concept in this paper is the notion of convergence of a formula to a type.

Let T be an arbitrary complete theory of the language L. N is sufficient a saturated model of the theory T. A $\subset$ N,$\bar{a} \in N$, and let q be a nonisolated type from S(A). and $\phi(\bar{x}, \bar{y})$ be an A-definable formula. we say thet $\phi(\bar{x}, \bar{b}), \bar{b} \in N$.devide $C \subset N^l$ (l-length of tuple($\bar{x}$). C not necessarily definable). if $\phi(N^l, \bar{b}) \cap C \neq \emptyset, \neg\phi(N^l, \bar{b}) \cap C \neq \emptyset$. Often we will write $\phi(N, \bar{b})$ instead of $\phi(N^l, \bar{b})$.

We say that the A-definable formula $\phi(x, y)$ weakly converges to type $q(\bar{x}) \in S(A)$ and we denote this by WEC($\phi(\bar{x}, \bar{y}), q(\bar{x})$), if for any of $\Theta \in q$ there is a $\bar{a} \in A$ such that $\phi(\bar{x}, \bar{a})$ divides $\Theta(N)$.

We say that the A-definable formula $\phi(\bar{x}, \bar{y})$ strongly converges to the type $q(\bar{x})$ and denote this by STC($\phi(\bar{x}, \bar{y})q(\bar{x})$). if for any in $\Theta \in q$ there is a $\bar{a} \in A$ such that $\phi(N, \bar{a}) \subset \Theta(N)$. Almost always we will omit $\bar{x}$ when writing $q(\bar{x})$.

Suppose that WEC($\phi(\bar{x}, \bar{y}), q(\bar{x})$) holds for $q \in S(A)$, and let $\phi(\bar{x}, \bar{y})$ be the graph of some A-definable function $f(\bar{y})(i.e. \phi(\bar{x}, \bar{y}) \equiv \bar{x} = f(y))$. Then STC($\phi(\bar{x}, \bar{y})q(\bar{x})$) holds. In this case, we say that the values of the function $f(\bar{y})$ converge to type q. and write STC($f(\bar{y}), q$).

We say that $\bar{a}$ tuple is weakly orthogonal to type q, if for any A-definable formula $\phi(\bar{x}, \bar{y})$. the formula $\phi(\bar{x}, \bar{y})$ does not divide $q(N) = \bigcap_{\Theta \in q} \Theta(N)$ (we denote it by through $\bar{a} \perp^{\omega} q$). and say that a is not weakly orthogonal to type q if there is an A-definable formula $\phi(\bar{x}, \bar{y})$ such that $\phi(\bar{x}, \bar{a})$ divides $q(N)(\bar{a} \not\perp^{\omega} q)$.

Note that $\phi(\bar{x}, \bar{a})$ divides q(N) if and only if for any in $\Theta \in q$ the formula $\phi(\bar{x}, \bar{a})$ divides $\Theta(N)$. Then for all $\bar{a} and \bar{\beta}$ such that $tp(\bar{a}/A) = tp(\bar{\beta}/A)$, we have $a \not\perp^{\omega} \leftrightarrow \bar{\beta} \not\perp^{\omega} q$. Thus, $p \in S(A)$ is weakly orthogonal to the type $q \in S(A)(p \perp^{\omega} q)$. if there exists $\bar{a} \in p(N)$ (in this case. any $\bar{a} \in p(N)$) is suitable such that $\bar{a} \perp^{\omega} q$ or, equivalently. $p(\bar{x}) \cup q(\bar{y})$ defines a complete $(l(\bar{x}) + l(\bar{y}))$ - type. Note that if $p \perp^{\omega} q$, then $q \not\perp^{\omega} p$

4. Definability of 1-types in o-stable theories [2]

Definition 1. Let s - partial 1-type over set B: A - is set. Lets set

$$S^1_s(A) =^{def} \{p \in S^1(A) : p \cup s - consistent\}$$

Everywhere in our work over $\mathcal{M} = (M, <, ...)$ denoted a structure with definable (without parameters) linear order which we will say linearly ordered structure, and its elementary theory - linearly ordered structure. For subset C and D of set M we will use a record $C < D$, if $c < d$ at any $c \in C$ and $d \in D$. Splitting (C,D) set M is called section, if $C < D$. section (C,D) definable partial type $s = s_{C,D} = \{x < d : d \in D\} \cup \{c < x : c \in C\}$ over a model. Its partial type will call a section too.

Let q-(partial) over set A. Type q is called B - definable, if for any formula $\phi(x, \bar{y})$ there exist B-formula (formula with parameters in B) $\Theta_\phi(\bar{y})$ such that for any tuple $\bar{a} \in A$ s true for $\phi(x, \bar{a}) \in q \Leftrightarrow \mathcal{M} \models \Theta_\phi(\bar{a})$. Partial type over set A definable. We will notice, that section $s_{C,D}$ definable iff set C define by formula (equivalent, set D is formulaically). In fact, consider the M-formula H (x), such that $H(\mathcal{M}) = C$. Then $\neg H(\mathcal{M}) = D$, and for formula $\phi(x, y_1, y_2) = y_1 < x < y_2$ its define $\Theta_\phi(y_1, y_2)$ will formula $H(y_1) \wedge \neg H(y_2)$

A definable section is called quasi-rational. A section (C, D) is called rational if either the set C has a maximal element or the set D is minimal. Obviously, a rational section is quasi-rational. An irrational indefinable section is called irrational.

Definition 2. (1) A linearly ordered structure $\mathcal{M}$ is called orderly stable in λ if, for any subset $A \subseteq M$, such that $|A| \leq \lambda$, and an arbitrary section s in $\mathcal{M}$ there are at most λ full types over A, which are compatible with type s, $i.e. |S_s(A)| \leq \lambda$.

(2) A theory T is called orderly stable in λ if each of its models is orderly stable.

(3) The theory T is orderly stable if there exists λ in which T is orderly stable.

(4) A theory T is orderly superstable if there exists a λ such that T is orderly stable in all $\mu \geq \lambda$.

The first description of a class of linearly ordered structures in terms of the number of extensions of an arbitrary section to complete 1-types over a model was made in [11]: a structure is o-minimal if and only if any section has a unique extension to a complete 1-type over structure. For weakly o-minimal structures, a similar result was established in [12]. In addition, the number of completions of a certain type image with respect to some continuous type mapping from one language to another is based on the definition E^*-stability [13]. In turn, the concept of E^*-stability is based on the concept of T^*-stability [14].

Theorem 1. If a theory is orderly stable, then it is not has the property of independence, i.e. is dependent.[2]

Theorem 2. A dependent theory is orderly stable if and only if it does not possess the property of local strict order. [2]

Theorem 3. There is a dependent linearly ordered theory that is not orderly stable

Proof: Let T be the elementary theory of the model.

$$\mathcal{M} = (M, =, <, \prec).$$

when Set $M = \mathbb{Q}\times\mathbb{Q}$, relation $<$ interpreted in the form of a lexicographic order, and the relation $\prec$ - in the form of a reverse lexicographic order.

Obviously, the formula $x \prec a$ is compatible with any section of the linearly ordered order $<$ of the set M. It is clear that for any two elements b and c of the set M, such that $b < c$, and for any arbitrarily large element of d,

$$R(\mathcal{M}b) \cap (d, +\infty) \subset R(\mathcal{M}, c)(d, +\infty)$$

when $R(x, y) = x \prec y$.Therefore, the formula $R(x, y)$ has the strict order property inside the section $+\infty$. By Theorem 2, the theory T is not orderly dependent.[2]

We now prove that the theory T does not have the independence property. A Boolean combination of formulas without the independence property does not itself have the independence property: therefore, it suffices to prove that the following

Approval 4. Elementary theory T of structure M permits elimination of quantifiers.

Proof.To prove the assertion, we use the Tarski criterion. Let the formula $F(x, \bar{y})$ be a conjunction of formulas of the following form and their negations: $(1) x = y$; $(2) x < y$; $(3) x > y$; $(4) x \prec y$; $(5) x \succ y$. We prove that the formula $\exists x F(x, \bar{y})$ is equivalent in the theory T to some quantifier formula.

We first show that without loss of generality, we can make the following assumption: all conjuncts of the 1–5 type are included in the formula $F(x, \bar{y})$ in a positive form. Indeed, the negation of equality is equivalent to the formula $x < y \wedge y < x$; the negation of the inequality $x < y$ is equivalent to the formula $x = y \wedge y < x$; the same can be said about the second linear order given by the relation $\prec$. The existential quantifier sweeps through a disjunction, so you can consider each disjunction separately.

If a conjunct of the first kind enters the formula $F(x, \bar{y})$, then we replace the occurrence of the variable x with the variable y everywhere in the formula $F(x, \bar{y})$ and obtain the quantifier-free formula. Thus, we can assume that the formula $F(x, \bar{y})$ does not contain subformulas of the first kind.

If $F(x, \bar{y})$ contains two conjuncts of the second type x ¡y $\wedge$ x ¡z, then we do the following operation: we replace this formula with the equivalent formula $(x < yy < z) \vee (x < z \wedge z < y) \vee (x < y \wedge y = z)$. It is known that a conjunct that does not depend on x can be put out of the brackets. Thus, it can be assumed that the formula $F(x, \bar{y})$ contains at most one conjunct of the second kind. The same procedure can be repeated for conjuncts of the third. fourth and fifth types. It remains to consider a formula of the form $\exists x(y < x < z \wedge u \prec x wedge x \prec v)$. It is equivalent to the formula $y < z \wedge u \prec v$. The assertion and the theorem are proved.□[2]

Theorem 4. Any linear order theory is orderly superstable. Proof.[2]

Theorem 5. Two increasing n $a_1 < ... < a_n$ from the chain C and $b_1 < ... < b_n$ from the chain D have the same type if and only if they satisfy the following conditions:

for each $i \leq n$. the elements a_i and b_i satisfy the same formulas;

for each $i < n$. the same sequences of formulas are realized between a_i and a_{i+1}. as well as between b_i and b_{i+1}, where for a given sequence $f_1(x), ..., f_n(x)$ formulas with a free variable x the language of chains and two points of the chain C say that this sequence of formulas is realized between given points. if you can find $c_1, ..., c_n$ in C, such that $c_1 < c_2 < ... < c_n$ and c_i implements $f_i(x)$ for all $i = 1, ..., n$.

Based on this theorem. in [15] the following conclusion was made: this theorem gives a simple description of non-principal 1-types over a model — this type is determined by its absolute type (that is, by its restriction on the empty set of parameters), by the section it defines, and by its the feasibility side (performed on the left side, on the right side. or on both, when the type is indefinable).

5. Conservative extensions in o-stable theories.

Definition 1. Let $A \subseteq M = T, p \in S(A)$. we say that type p is $\phi(\bar{x}, \bar{v})$-definable for $\phi(\bar{x}, \bar{v}) \in L(\bar{x})$, if there exists L(A)formula $\Psi_\phi(\bar{v})$ such that for all $\bar{a} \in A^{l(\bar{v})}$ is true: $\phi(\bar{x}, \bar{a}) \in p \Leftrightarrow M \models \Psi_\phi(\bar{a})$. Formula $\Psi_\phi(\bar{v})$ is called $\phi(\bar{x}, \bar{v})$- definable type p.

We say that type p is definable if it $\phi(\bar{x}_0, \bar{v})$ is definable for any its formula $\phi(\bar{x}_0, \bar{v}) \in L(\bar{x}_0)$. A tuple $\bar{\gamma} \in M$ ht-definable over A, if $tp(\bar{\gamma}|A)$ is definable.

Definition 2. Let $q(\bar{x}) \in S(A)$. $A \subset N$. We will say that type q strictly definable, if for any L(A)- formulas $\phi(\bar{x}, \bar{y})$ there exists L(A) formulas $\Theta_\phi(\bar{x}) \in q$ such that for any $\bar{a} \in A^{l(\bar{y})}$ is true

$$N \models \exists \bar{x}(\Theta_\phi(\bar{x}) \wedge \phi(\bar{x}, \bar{a})) \rightarrow \forall \bar{x}(\Theta_\phi(\bar{x}) \rightarrow \phi(\bar{x}, \bar{a})).$$

From these definitions, it follows that each isolated type is strictly definable, each strictly definable type $q \in S(A)$ is definable, and the $\phi(\bar{x}, \bar{y})$ definition of $q(\bar{x})$ is an L(A) formula $\Psi_\phi(\bar{y}) := \forall \bar{x}(\Theta_\phi(\bar{x}) \rightarrow \phi(\bar{x}, \bar{y}))$ or its equivalent $\Psi_\phi(\bar{y}) := \exists \bar{x}(\Theta_\phi(\bar{x}) \wedge \phi(\bar{x}, \bar{y}))$. We consider that classify of type, held by Shelah, for strictly definable type $p \in S(A)$, and for $B \subseteq A (B := \{b | \phi \in L, \Psi_\phi \in L(b)\})$ has $(p, B) \in F^1_{\aleph_0}$[16]

Fact 1. Let T - o-minimal theory, $A \subset N \models T$, $N - |A|^+$- saturated model, $p, q \in S_1(A)$, $q \not\perp^w p$, $a \in q(N)$. Then there exists $b \in p(N) \cap acl(A, a)$.

Proof. Recall that in an o-minimal theory, the set of realizations of a non-algebraic 1-type in a saturated model is a convex set without end elements, and the set of realizations of any formula is a finite union of intervals and one-element sets. Moreover, the ends of these intervals and these elements belong to the algebraic closure of the parameters of this formula [11]. Then, since $a \not\perp^w q$ and, therefore, some $L(A \cup \{a\})$ - formula divides the convex set $q(N)$, there exists $b \in p(N) \cap acl(A, a) \neq \emptyset$□

When studying the nature of elementary classes of models, an important role is played by properties and concepts associated with "good" elementary extensions of models (sets), such as simple, special, (strongly) constructivizable, saturated, conservative [10,11,16,17,18,19,20,22,23] and their modifications and generalizations - F-simple models, beautiful pairs, pairs of models, pseudo-small theories, axiomatized pairs of models [16,24-30]. Often, the existence theorems of such extensions are based on different versions of the theorems of realizations and / or type omissions (ϕ-types).

To construct a conservative expansion of a set with given properties, it is necessary to answer the following question:

Problem A(S). Let S - property of definable types and for $A \subset N$, $q(x), p(y) \in S_1(A)$ - definable types with property S. Does there exist a complete definable 2-type $r(x, y) \in S_2(A)$ such that $q(x) \cup p(y) \subsetneq r(x, y)$? We have two different cases:

A1 $q(x) \cup p(y)$ is not complete type, i.e $p \not\perp^w q$

A2 $q \perp^w p$

6. Conservative extension of different classes complete theory

Let $M \prec N$. we say that couple of model (M,N) is conservative couple, or N is conservative extension M, if for all tuple of elements $\bar{a}$ in N, $tp(\bar{a}|M)$ definable [10]. We will say that elementary extension N of model M is D-ω-saturated for M, if any definable type $q \in S_1(M \cup a)$, where $\bar{a} \in N$ realize in N; and N is CD-ω-saturated for M, if for non-isolated type in $q \in S_1(M \cup a)$, where $\bar{a} \in N$, definable by formulable subset ω-type which realize in N.

Let $A \subset B$. We say that B is anticonservative extension A, if $\forall \bar{a} \in B/A$ for any definable 1-type $r \in S_1(A)$, $\bar{a} \perp^{P} \omega r$. Couple of models (M,N) is called anticonservative , iff tuple $\bar{a} \in N/M$, $tp(\bar{a}|M)$ nondefinable.

Notice 1. There exists four models M, N_1, N_2, N o-minimal theory, such that $M \prec N_1 \prec N; M \prec N_2 \prec N; (N_1, N), (N_2, N)$ - conservative couple, couples $(M, N_1), (M, N_2)$ - anticonservative couple and N_1, N_2 isn't M - isomorphic $N_1 \ncong_M N_2$.

Proof(Notice 1). Let $M = \langle \mathbb{Q}; =; < \rangle, N_1 = \langle \mathbb{R}; =, < \rangle$, where $\mathbb{Q}$ is set of all rational numbers, $\mathbb{R}$ is set of all real numbers. Let $N = \langle N; =, < \rangle$ is $|\mathbb{R}|^{+}$ - saturated elementary extension N_1. Lets take a random irrational number $\delta \in \mathbb{R}$. Element δ is define irrational section (C_δ, D_δ) in $\emptyset$ and irrational 1-type p_δ over $\mathbb{Q}$. Lets set $N_2 := \langle \mathbb{R} \cup p_\delta(N); =, < \rangle$. We will not distinguish between a model and its main set, i.e. for us $M = \mathbb{Q}, N_1 = \mathbb{R}, N_2 = \mathbb{R} \cup p_\delta(N)$.

The theory of these models $\{M, N_1, N_2, N\}$ is a well-known ω-categorical, o-minimal theory that allows the elimination of quantifiers. Since R does not have an irrational section, every non-isolated 1-type is over $\mathbb{R}$ rational, and therefore $\forall a \in N/\mathbb{R}, a$ belongs to the set of realizations of some rational section in $\mathbb{R}$. The same is true for any element of $N/(\mathbb{R} \cup p_\delta(N))$. This means, according to the Marker – Steinhorn theorem (or directly follows from the nature of these models) that $(N_1, N), (N_2, N)$ are conservative pairs. Notice, that $\forall b \in (\mathbb{R}/\mathbb{Q}) \cup p_\delta(N), tp(b|\mathbb{Q})$ indefinable, since it is defined by an irrational section in $\mathbb{Q}$.

Take $\bar{b} := \langle b_1, \ldots, b_n \rangle$, an arbitrary tuple of elements from N_1 $\mathbb{Q}$ or from N_2 Ω. Then, since in the given theory the algebraic closure of any set C is equal to C, $acl(\mathbb{Q} \cup \{b_1, \ldots, b_n\}) = \mathbb{Q} \cup \{b_1, \ldots, b_n\}$. Thus, by virtue of fact 1, for any $r \in S_1(\mathbb{Q})$ is true

$$b \perp^{\omega} r \Leftrightarrow \exists j \in \{1, \ldots, n\}, b_j \in r(N).$$

Suppose $r \in S_1(\mathbb{Q})$ is defined and, therefore, by virtue of fact 7 and o-minimal N, r is rational. So as for any $j \in \{1, \ldots, n\}, tp(b_j|\mathbb{Q})$ irrational, $b \perp^{\omega} r$. This means that $(M, N_i), i = 1, 2$ anticonservative pairs. It follows from the definition of the models N_1 and N_2 that they are not M-isomorphic. □ [2]

Conclusion

All existing types can be divided into two types, isolated and not isolated. An isolated type is always definable, so it only remains for us to study the non-isolated type. Not isolated type in turn can be divided into types ie locally isolated and not locally isolated. The concept of a locally isolated sludge type is different. TV-type (Takrski-Vaught type) was introduced into science by Baizhanov B.S., and its turn is definable. the most interesting when a non-locally isolated type is definable when it is not. The paper shows that when a non-locally isolated type is defined, then it has a control formula ie there is a rank.

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