

# **Methodological Features of Teaching the Topic of 3D Geometric Shapes in the Context of Digitalization of Education**

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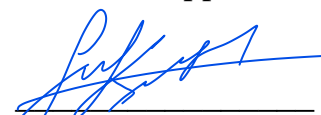
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## **List of Abbreviations**

3D – Three-Dimensional

AI – Artificial Intelligence

APA – American Psychological Association

AR – Augmented Reality

ICT – Information and Communication Technology

IoT – Internet of Things

MoSHE RK – Ministry of Science and Higher Education of the Republic of Kazakhstan

PBL – Problem-Based Learning

SAT – Scholastic Assessment Test

SDU – Suleyman Demirel University

STEM – Science, Technology, Engineering, and Mathematics

TPACK – Technological Pedagogical Content Knowledge

UNT – Unified National Testing

VR – Virtual Reality

ZPD – Zone of Proximal Development

## ABSTRACT

The digitization of the education system necessitates the use of current technical instruments in the teaching process. This study investigates the efficacy of utilizing the GeoGebra 3D dynamic geometry tool to solve challenging stereometry problems for high school students including problems using 3D geometric shapes. Traditional teaching approaches frequently result in a "visualization crisis", which makes it difficult for students to comprehend three-dimensional shapes using two-dimensional representations. The study employed a quasi-experimental methodology, with 40 students participating. They were likewise divided evenly between experimental and control groups using convenient sampling. The experimental group was taught via GeoGebra, whereas the control group was taught using a more traditional manner, such as explaining on the board and sketching on paper. Data were collected using pre- and posttests, as well as a 9-question questionnaire. The t-test for independent samples revealed that the experimental group performed considerably better academically ( $p < 0.05$ ). At the same time, the questionnaire validated students' good attitudes of GeoGebra and the subject (overall mean = 4.05). Furthermore, the effect size analysis (Cohen's  $d = 1.19$ ) confirmed that the GeoGebra intervention had a large and highly significant practical impact on students' stereometric problem-solving skills. However, the findings indicated that teacher help is required to foster students' autonomous critical thinking. According to the study, the most successful way for teaching stereometry is a mixed learning approach that combines classic analytical approaches with dynamic digital display.

*Keywords: Stereometry, GeoGebra, dynamic geometry tool, spatial reasoning, digitalization of the education, high school mathematics.*

## АНДАТПА

Білім беру жүйесін цифрландыру мақсатында оқу процесіне заманауи технологиялық құралдарды біріктіруді талап етеді. Бұл зерттеу жоғарғы сынып оқушыларына күрделі стереометрия тақырыптарын оның ішінде 3D геометриялық фигураларға қатысты есептерді шығаруда GeoGebra 3D динамикалық геометриялық бағдарламасын қолданудың тиімділігін қарастырады. Дәстүрлі оқыту әдістері көбінесе "визуализация дағдарысына" әкеліп, оқушыларға екі өлшемді сызбалар арқылы үш өлшемді фигураларды түсінуді қиындатады. Зерттеуде квази-эксперименттік дизайн қолданылып, оған 40 оқушы қатысты. Оқушылар эксперименталды және бақылау топтарына теңдей бөлінді. Эксперименталды топ GeoGebra арқылы, ал бақылау тобы қарапайым дәстүрлі әдіспен оқытылды. Деректерді жинау үшін претест пен посттест және 9 сұрақтан тұратын сауалнама қолданылды. Тәуелсіз таңдамаларға арналған t-test нәтижелері эксперименталды топтың академиялық үлгерімі айтарлықтай жоғары болғанын көрсетті ( $p < 0.05$ ). Сонымен бірге, сауалнама оқушылардың Геогейбраға деген және пәнге деген оң көзқарасын (Жалпы орташа мән = 4.05) дәлелдеді. Сонымен қатар, эффект өлшемінің талдауы (Cohen's  $d = 1.19$ ) GeoGebra бағдарламасының оқушылардың стереометриялық есептерді шығару дағдыларына практикалық тұрғыдан маңызды әсер еткенін растады. Дегенмен, нәтижелер оқушылардың дербес сыни ойлауын дамыту үшін мұғалімнің қолдауы қажет екенін де көрсетті. Зерттеу стереометрияны оқытуда дәстүрлі аналитикалық әдістер мен динамикалық цифрлық визуализацияны біріктіретін аралас оқыту тәсілі ең тиімді стратегия болады деген қорытындыға келді.

*Түйінді сөздер: Стереометрия, GeoGebra, динамикалық геометриялық бағдарлама, кеңістіктік ойлау, білім беруді цифрландыру, мектеп математикасы.*

## АННОТАЦИЯ

Цифровизация системы образования требует интеграции современных технологических инструментов в учебный процесс. В данном исследовании изучается эффективность использования программы динамической геометрии GeoGebra 3D для решения сложных стереометрических задач, включая задачи, связанные с трехмерными геометрическими фигурами, у учащихся старших классов. Традиционные методы обучения часто приводят к «кризису визуализации», затрудняя понимание трехмерных фигур с помощью двухмерных чертежей. В исследовании использовался квазиэкспериментальный дизайн, в котором приняли участие 40 ученика. Они были поровну разделены на экспериментальную и контрольную группы. Экспериментальная группа обучалась с использованием GeoGebra, а контрольная группа – простым традиционным методом, то есть объяснениями на доске и чертежами на бумаге. Для сбора данных использовались предварительный и итоговый тесты, а также анкета из 9 вопросов. Результаты t-критерия для независимых выборок показали, что экспериментальная группа продемонстрировала значительно более высокую успеваемость ( $p < 0,05$ ). В то же время опрос подтвердил позитивное отношение учащихся к GeoGebra и предмету (средний балл = 4,05). Кроме того, анализ величины эффекта ( $d$  Коэна = 1.19) подтвердил, что применение программы GeoGebra оказало очень сильное и практически значимое влияние на навыки учащихся в решении стереометрических задач. Однако результаты также показали, что для развития самостоятельного критического мышления учащихся необходима поддержка учителей. В исследовании сделан вывод, что наиболее эффективной стратегией обучения стереометрии будет смешанный подход к обучению, сочетающий традиционные аналитические методы и динамическую цифровую визуализацию.

*Ключевые слова: Стереометрия, GeoGebra, программа динамической геометрии, пространственное мышление, цифровизация образования, школьная математика.*

# 1. INTRODUCTION

Currently, the Republic of Kazakhstan considers digitization of the educational system to be one of its primary strategic areas. According to Sardarova et al. (2022), the state initiative "Digital Kazakhstan" intends to strengthen the competitiveness of the state economy by developing digital technologies, and digitalizing educational materials is a critical step toward attaining this goal. Furthermore, the Law of the Republic of Kazakhstan "On Education" strongly mandates the incorporation of modern information and communication technologies (ICT) into the educational process.

In this context of global and national digitalization, it is very important to update the teaching methodology of mathematics, including the complex stereometry course. Traditional teaching methods do not fully meet the needs of modern students. For example, according to a study by Kokazhayeva et al. (2025), the performance of 11th grade students on the UNT is decreasing every year, and the main reason for this is the inability of students to use the properties of spatial figures. In addition, they are unable to construct drawings and sections of spatial figures incorrectly or at all. It is very difficult for students to visualize 3D objects using 2D drawings on the board or in a textbook. This phenomenon is called the "visualization crisis" in science, which arises from the inability to accurately represent all the properties of spatial figures on paper.

The most sensible solution to this challenge is to employ dynamic geometric systems such GeoGebra 3D. According to Boronenko and Fedotova (2023), the GeoGebra tool increases abstract mathematical comprehension while also allowing you to construct dynamic models of figures, graphically comprehend, and solve spatial problems. From this perspective, investigating the efficacy of utilizing GeoGebra to teach stereometry in Kazakhstani schools is a critical theoretical and practical problem.

**The research aim:** is to establish methodological ways for using the GeoGebra 3D dynamic application to teach stereometry to high school students, as well as to assess students' academic performance and attitudes about the topic and the GeoGebra program.

**The research hypothesis:** If the GeoGebra 3D dynamic modeling tool is used consistently to teach three-dimensional geometric shapes in stereometry lessons, students' spatial visualization abilities and academic performance in geometry will improve dramatically.

**The research objectives:**

- Analyze the theoretical and methodological basis of teaching stereometry in the context of current digitalization, as well as the pedagogical challenges of the traditional method.
- To determine the GeoGebra program's capabilities as the best tool for digitally visualizing and modeling spatial geometric shapes.
- To evaluate students' intellectual development (pretest and posttest) during the experiment using the GeoGebra application.
- To assess the practical impact of the suggested digital strategy on student involvement and interest in the lesson, a unique survey will be conducted.

**The research questions:**

- Does the systematic use of the dynamic application GeoGebra 3D in mathematics classes result in considerable improvement in high school students' abilities to solve stereometric problems?
- Is using the digital application GeoGebra more helpful in developing students' spatial thinking and improving test performance than traditional methods of teaching (blackboard and chalk/paper drawings)?
- To what degree does the digitization of geometric instruction (the use of GeoGebra) improve students' attitudes about the topic and the digital instrument for solving complicated stereometric problems?

## **2. LITERATURE REVIEW**

### **2.1 What is the stereometry?**

Stereometry is the study of the characteristics of figures in space in geometry, according to Kulmaganbetova (2020), who analyzed the paper of L.N. Beskin. The study of spatial forms, or shapes that do not lie in a single plane, is known as stereometry (Shuakaev et al., 2021). A subfield of geometry called stereometry examines figures and their characteristics in three dimensions (Bashirova & Erboluly, 2023). According to Bashirova and Erboluly (2023), stereometry has a considerably wider meaning in addition to being a component of the mathematics curriculum. In other words, stereometry helps students enhance their spatial intelligence and critical thinking skills. The significance of these abilities for students outside of the classroom was also mentioned by Bashirova and Erboluly (2023). In other words, they contended that stereometry has practical applications in real life. Stereometry is crucial for high school students, according to Bashirova and Erboluly (2023). Additionally, they demonstrated how significant it is to these high school students. They said that these high school students may learn abilities including analysis, visualization, and solving challenging 3D geometric problems. These abilities may be learned by developing basic cognitive skills, claim Bashirova and Erboluly (2023). Additionally, Smith (2020, Bashirova & Erboluly, 2023) demonstrated how high school students' spatial thinking is actively developed through the development and strengthening of stereometry. Additionally, Kulmaganbetova (2020) explained what stereometry meant when translated from Greek. According to her, "metreo" implies measurement and "stereos" means space in Greek. According to the Republic of Kazakhstan's Education Law, the geometry course is divided into two sections: planimetry and stereometry (Saparkanova & Taskynbayeva, 2022). Stereometry, according to Kasimova and Kutyavina (2024), is a step from planimetry to dealing with objects in space. In other words, they observe it as dealing with bodies that possess volume. Stereometry examines the characteristics of actual physical bodies that may be characterized by the

terms "shape," "size," and "relative arrangement," according to Kulmaganbetova (2020). Saparkanova and Taskynbayeva (2022) establish that stereometry contributes to students' comprehensive understanding of the world and spatial object properties. According to Kokazhayeva et al. (2023), stereometry encompasses abstract ideas such as the capacity to see figures shown on a plane in space. According to the study of Erdes (2026), stereometry, which is derived from the Greek word, is evident in the pyramids, which are thought to be the primary components of three-dimensional geometry and originated in ancient Egypt. The ancient Greek scholar Herodotus (5th century BC) provided an intriguing account of these pyramids, which Shuakaev et al. (2021) also cited in their study. In other words, he said that the Egyptian king divided the country, gave each Egyptian a piece, and levied taxes on each plot. Additionally, he stated that when the Nile River flooded a particular region of land, the victims complained to the monarch, who subsequently dispatched surveyors to assess the amount of land that had been lost and adjusted the levy. This is how geometry began in Egypt, according to Shuakaev et al. (2021). Furthermore, the building of figures in space and the investigation of the link between their angles have been ongoing since ancient times, according to Erdes's (2026) research. According to Shuakaev et al. (2021), stereometry considers many scenarios of the mutual arrangement of straight lines and planes in space, citing regular polygons, pyramids, prisms, and solids of revolution as examples. Furthermore, contour notions are broadened while researching stereometry, according to Shuakaev et al. (2021). It is specifically stated that rectangular coordinate systems, vectors, geometric transformations, etc. are generalized. Measuring the areas and volumes of figures in space is a significant issue in stereometry, according to Shuakaev et al. (2021). Simultaneously, it was asserted that without stereometry, it is difficult to imagine the world the people live in. In other words, everything that surrounds us is written in the sense that it consists of stereometric figures (Shuakaev et al., 2021). According to Dudareva and Zaryvnykh (2023), stereometry encompasses the study of polyhedra, surfaces of revolution, their elements, characteristics, and sections, according to the Russian Federation's educational criteria. Stereometry examines the

spatial characteristics of geometric figures, according to Saparkanova and Taskynbayeva (2022), who pointed out the distinction between the two. Furthermore, stereometry is significant because it fosters the development of its psychological elements, spatial imagery, spatial imagination, and logical reasoning (Saparkanova & Taskynbayeva, 2022). In a review of A.V. Pogorelov's research, Kulmaganbetova (2020) demonstrated that its structure scheme is identical to planimetry.

- the primary geometric ideas are presented without definitions;
- axioms are developed that demonstrate the characteristics of the key ideas;
- the primary principles are used to define additional geometric notions;
- theorems are demonstrated using definitions and axioms.

Furthermore, Saparkanova and Taskynbayeva (2022) clarified the relationship between stereometry and planimetry by stating that stereometry is predicated on planimetry. They said that all of the definitions, axioms, and theorems from planimetry also apply to stereometry. According to Saparkanova and Taskynbayeva (2022), geometry fosters students' creativity, and stereometry also necessitates spatial imagination. They also enumerate how it differs from planimetry. They state that stereometry necessitates additional spatial representation, however planimetry demands facts and precise depiction. They state that students should have adequate development of logical thinking and that stereometry is taught in the final stages of school. In other words, stereometry necessitates strong proof and logical consistency. The development of students' spatial imagination is one of the key components of teaching stereometry in the classroom, according to research by Dzhandigulov and Auezova (2021). The development of students' abilities to work with spatial objects is another factor. It is possible to solve spatial difficulties, observe them, and accurately depict them by studying stereometry (Dzhandigulov & Auezova, 2021). Bogdanova et al. (2024) argued that stereometry is aimed at developing spatial thinking and that it requires students to create spatial images and use them to solve practical and theoretical problems. According to Christou et al. (2007), students' spatial and visual skills are strongly correlated with 3D geometry. Additionally, they stated that students must

analyze, encode, manipulate, and build information from visual representations in 3D geometry. Additionally, they stated that a key concept in human thought and cognition is space.

### **2.1.1 Main features of 3D geometric shapes in stereometry**

The three-dimensionality of stereometry is one of its key characteristics. Kulmaganbetova (2020) demonstrates that spatial figures, such as cubes, pyramids, spheres, and cylinders, can be regarded as subjects of stereometry investigation. Additionally, axioms must be used to infer the mutual arrangement and connection of lines and planes in stereometry. Furthermore, Kulmaganbetova (2020) states that the axioms have to be presented in the following sequence:

- visualize the model's postulate;
- create the axiom;
- make a schematic diagram;
- use symbolic notation.

Stereometry helps students develop their logical analysis, creativity, and spatial thinking abilities (Saparkanova & Taskynbayeva, 2022). Students also learn about the shapes and spatial representations of their surroundings. Additionally, they help students improve their modeling and measurement abilities. Saparkanova and Taskynbayeva (2022) investigated the key objectives of stereometry instruction. They emphasized the following among them:

- the primary psychological elements
- spatial pictures
- logical reasoning growth

Additionally, they pointed out that taking a stereometry course helps students increase their creativity, thinking skills, and personal growth. According to Kasimova and Kut'yavina (2024), three-dimensional geometric objects are the subject of stereometry research, and their portions are examined. The primary objective of this stereometry, according to Dudareva and Zaryvnykh (2023), is to grasp the fundamental ideas of spatial figures and their characteristics as well as to be able to represent them

using drawing instruments or technological equipment. Dudareva and Zaryvnykh (2023) observe spatial (volumetric) geometric figures and their operations:

- drawing points, lines, and planes;
- measuring segments and angles;
- computing geometric quantities - as the subject of stereometry study, particularly in the tenth grade.

The significance of stereometry is emphasized by Dudareva and Zaryvnykh (2023). In other words, students who study stereometry also learn how to build mathematical models employing geometric numbers and concepts through hands-on computations. After reviewing Brown's (2019) study, Bashirova and Erboluly (2023) contended that stereometry is an essential basis for disciplines like physics, engineering, and architecture and aids students in transitioning from flat geometry (planimetry) to more sophisticated three-dimensional structures. Vallo et al. (2015) identified the cube as the primary figure and related the characteristics of bodies in space to the major feature. In order to tackle problems related to this area, they also discussed the necessity of spatial imagination. It was demonstrated that this imagination encompassed the capacity to sense space, explain the characteristics of geometric shapes and the connections between their constituent parts, visually depict them in various configurations, and engage in mental processes. According to Bogdanova et al. (2024), one aspect of stereometry is the requirement to create spatial images using traditional graphic images (drawings, projections) and physical representations (models). According to Kramarenko et al. (2020), the peculiarities of stereometry include the building of models of geometric forms on the plane utilizing the features of parallel projections and the treatment of geometry as a new reality through abstract logic and concrete representation techniques. After reviewing the National Council of Teachers of Mathematics' guidelines, Christou et al. (2007) concluded that all students should acquire 3D spatial visualization and reasoning. Additionally, Christou et al. (2007) examined the perspectives of Presmeg, Sowder, and Harel and demonstrated the unique function of dynamic visualization in 3D

geometry. In other words, its main goal is to bring about a variety of changes between the individual and their surroundings. Additionally, they referenced Boz's findings, which suggests that students should possess dynamic skills including the capacity to mentally observe, shrink, stretch, and manipulate geometrical shapes.

### **2.1.2 Challenges in 3D geometric shapes in stereometry**

According to Gorodnova (2016), the majority of elementary school graduates lack spatial imagination, spatial conceptions, and the capacity to conceive geometrically in three dimensions. For this reason, she claims that the perception of geometric objects through eyesight is the source of stereometry's challenges. The inaccurate and ambiguous drawings are one of the primary causes of the challenges associated with researching stereometry (Erdes, 2026). According to Gorodnova (2016), drawings on paper or on a chalkboard do not accurately depict all the characteristics of spatial figures, which creates objective challenges. She also discovered that this challenge causes students to misinterpret one another. Erdes (2026) claims that beyond the tenth grade, students struggle to accurately sketch the figures in this area in their notebooks following the planimetry drawings, and they frequently fail to recognize or even focus on these mistakes. Additionally, students attempt to sketch the problem on their own without paying attention to the drawing style and do not discuss the drawing with teachers when they see the problem. According to Kokazhayeva et al. (2025), 11th grade students' UNT performance is declining annually, and the primary cause of this is their incapacity to solve problems without utilizing the properties of spatial figures and their incorrect construction of contours and sections of polygonal figures in this geometry. They also discussed how high school students struggle to solve spatial figure problems using a variety of techniques and strategies. According to Vallo et al. (2015), students find it extremely challenging to create a cube's cross-section. High school students struggle with spatial vision, drawing, and modeling of figures in stereometry, according to Dalinger (2022, as cited in Kokazheva et al., 2025). Dalinger attributed this to the students' inadequate capacity to visualize spatial figures and accurately place their drawings. Igoshin (2017, as cited

in Kokazheva et al., 2025) states that the inability to accurately identify the appropriate segment or angle in the figure drawing is one of the most frequent challenges students have while addressing stereometric problems. Stereometry necessitates spatial imagination, and many students find it challenging to sketch and create it in notebooks without extra spatial representation, according to Saparkanova and Taskynbayeva (2022). Furthermore, when solving stereometry problems with students, Saparkanova and Taskynbayeva (2022) caution against the following: students have poor spatial representations, do not know how to accurately depict a three-dimensional image on a two-dimensional plane of a sheet or board, do not know how to consider space. Additionally, Tupappay (2021) identifies the lack of effort in this area in earlier grades as the primary cause of the challenges in addressing stereometric problems. Additionally, several issues related to stereometry - that is, figures in space - can be found in the basic school curriculum beginning in the fifth grade, according to Tupappay's (2021) research. However, Tupappay (2021) observes that this course merely covers beginning material and does not go into detail. Tupappay (2021) examined N.S. Podkhodova's viewpoint in this respect and concluded that elementary school students should start to acquire spatial thinking. Furthermore, Tupappay (2021) observed that these elementary school students have a solid grasp of images and that psychological processes (perception, memory, recognition, imagination, and reasoning) are engaged. Stereometry is one of the most challenging topics in grades 10–11, according to Kasimova and Kut'yavina (2024), demonstrating the challenges that many students encounter. The abrupt change from dealing with flat numbers to working with objects in space is one of the causes of this. Students thus struggle to solve tasks like connecting stereometric items and moving from two-dimensional to three-dimensional space and back. The results of the 2022 UNT were examined by Dudareva and Zaryvnykh (2023). They discovered the following signs consequently. In other words, students' results on stereometric puzzles were quite poor. In other words, they faced issues like the incapacity to draw spatial and geometric figures accurately and the incapacity to present arguments. Students are unable to create

constructive formation, according to Dudareva and Zaryvnykh (2023). According to Vallo et al. (2015), stereometry instruction is the most "problematic" subject for the majority of math teachers. Additionally, their study found that in order to handle stereometric difficulties, Slovak teachers need a high degree of spatial imagination. According to Vallo et al. (2015), who presented the perspective of Perny et al., Tomkova characterized the primary challenges in addressing stereometric problems as a lack of spatial imagination, inadequate spatial perception, and an inability to accurately visually depict 3D objects. Vallo et al. (2015) also observed that "formalism" is something that students focus on. In other words, they discovered that during their trials, the students focus more on the formal values of the problem. Bogdanova et al. (2024) discovered that when given drawings of these spatial shapes, high school students were unable to visualize the precise placements of geometric objects. Additionally, Bogdanova et al. (2024) explicitly connected these challenges to students' inadequate spatial comprehension development. Kramarenko et al. (2020) performed a great job of illustrating the challenges associated with addressing issues for three-dimensional geometric objects. They clarified that the primary challenge is the incapacity to accurately identify the 2D (plane) drawing that depicts the picture of a 3D. They clarified that this is because they are unable to dynamically alter or accurately draw polygons on a plane. Students who focus on a single scenario and mix up the terms "angle between the side and the base plane" and "bilateral angle at the base" are another frequent source of challenges.

### **2.1.3 Solving methods of stereometric problems.**

Successful methods must be developed since children's perceptions of knowledge change with each generation (Erdes, 2026). Bashirova and Erboluly (2023) make the case that employing various approaches to solve stereometric difficulties is suitable. Erdes (2026) suggested several practical solutions for these stereometric problems. They are:

- vector-coordinated approach
- visual assistance approach

- approach using geometric figures' characteristics

A mathematical approach called the vector-coordinated approach solves geometric problems in three dimensions using vectors and coordinates (Erdes, 2026). According to Erdes (2026), who cited Smirnov and Tuyakov's books, the vector-coordinated approach is initially used as follows. In other words, it is crucial to establish a coordinate system initially. In other words, having three dimensions - the  $x$ ,  $y$  and  $z$  axes - is crucial. Then, vectors are used to represent vectors and planes. For instance, Erdes (2026) demonstrated in his work that a point may be represented as a vector from the origin to the point's coordinate, a line as a vector indicating the line's direction, and a plane as a normal vector to the plane. In the study of Erdes (2026), Kokazheva et al. (2025) found that the vector-coordinated approach was an efficient way to save students' time during the UNT. Their study indicates that geometric calculations may be converted to algebraic calculations using this approach. Furthermore, using the vector-coordinated approach to solve stereometric problems enables students to strengthen their creative thinking, according to Kokazheva et al. (2025), who reviewed the study of several experts. This is since students can take advantage of these chances by using the coordinate system on their own (selecting the axes' origin and orientation). It also suggests that using the vector-coordinated approach might be more beneficial if a student in the eleventh-grade struggles to create a sophisticated stereometric graphic. The visual assistance approach was the second approach that Erdes (2026) demonstrated. It refers to a teaching approach that makes use of interactive whiteboards, models, and visual aids. Furthermore, according to Erdes (2026), technology has advanced quickly in recent years, and it is evident that utilizing computers and cutting-edge teaching techniques in the classroom is both feasible and successful. In other words, solids of revolution may be vividly displayed owing to interactive instruments. Cones, cylinders, pyramids, and parallelepipeds, for instance, can be shown in three dimensions as solids of rotation. Additionally, Bashirova and Erboluly (2023) demonstrated that these interactive exercises help students comprehend problems more deeply and get a better

comprehension of stereometry concepts. The use of geometric figures' attributes is the next approach. This approach is regarded as popular and is utilized extensively. The study by Erdes (2026) shown that this approach enables you to comprehend and solve the fundamental characteristics of figures. It is evident that considerably of geometric forms have characteristics. In other words, by using this approach, students may have a stronger knowledge of the link between the lengths of their sides, angles, areas, and volumes. Several approaches to teaching stereometry were studied by Bashirova and Erboluly (2023). For instance, traditional teaching approach featured lectures and textbooks. That is, the type of instruction that relies only on the teacher and is based on textbooks. This type of instruction can provide a solid theoretical basis, which is one of its benefits, although Bashirova and Erboluly (2023) also noted some drawbacks. Specifically, they demonstrated that there is not much interaction and that students might not be interested in traditional teaching methods. However according to Bashirova and Erboluly (2023), this traditional teaching incorporates hands-on activities. In other words, practical exercises provide a thorough understanding of the ideas and the development of stereometric problem-solving abilities. They did not dispute the length of time, though. Additionally, Vallo et al. (2015) arrived at the conclusion that traditional teaching is challenging, particularly when it comes to sketching on the board and not being clear enough for students. Additionally, Bogdanova et al. (2024) discovered that students with poor spatial thinking abilities find it challenging to comprehend these conventional designs, whether they are on paper or on a screen. Christou et al. (2007) have pointed out the drawbacks of this traditional teaching approach, specifically that it severely restricts how geometry is seen and how the learning process progresses. The interactive method is a contemporary approach to teaching stereometry, according to Bashirova and Erboluly (2023). In other words, this approach makes use of internet resources and computer applications. They explain that this method allows students to tackle stereometry problems in an engaging manner and develop their critical thinking abilities. The significance of visualizing was also mentioned by Bashirova and Erboluly (2023).

They claimed that students capacity to visualize space is enhanced by 3D models. As a result, Bashirova and Erboluly (2023) examined all the approaches and clarified that each has benefits and disadvantages of its own. And they came to the following conclusion. They suggested that the student's knowledge and learning objectives should be taken into consideration while selecting each technique. They also stated that a joint approach ought to be developed. In other words, they clarified that integrating the classic and modern approaches might lead to a deeper comprehension. The drawbacks of interactive tools were also noted by Christou et al. (2007). They also pointed out that learning 3D geometry using two-dimensional (2D) images is required by the majority of present curriculum and computer systems, which is not a natural way to understand human 3D spatial abilities. Nonetheless, dynamic and interactive computer programs were employed in their investigation. These applications ought to be action-oriented, according to Christou et al. (2007). In other words, students should be able to make dynamic graphics, rotate and slide three-dimensional objects, and combine them into intricate compositions. Additionally, they observed that the quality of geometry learning may be greatly enhanced by directly including spatial imagery and manipulation into the learning process. Based on M.S. Pomelova's view, Kasimova and Kutyavina (2024) distinguished two primary paths for the implementation of interactive teaching tools. These are:

- as a supplement to the traditional teaching method
- the active application of interactive tools as a key component of the learning process.

Additionally, Kasimova and Kutyavina (2024) reviewed several research and found that interactive boards and electronic instructional materials are being employed increasingly in the deployment of new educational technologies. Among these, it was observed that teachers employ illustrations produced on 2D and 3D platforms of interactive settings in geometry courses. They also discovered that this concept of visualization has been broadened by the scientific community. They additionally took virtual reality's efficacy into account. Students are more motivated and have a better

understanding of stereometry problems thanks to virtual reality. However, it was also discovered that virtual reality is frequently unavailable. In their investigation, they used Web-AR, a type of augmented reality. The use of Web-AR was highlighted by Kasimova and Kutavina (2024). Work in the program after school, particularly to do homework. According to Dudareva and Zaryvnykh (2023), constructive skills are essential for resolving stereometric problems. This skill is defined as the capacity to carry out each creative activity with intention, the ability to identify and isolate an item, the capacity to synthesize and analyze, and the capacity to modify an object within certain parameters in order to transform it into another. Additionally, they thought of constructive skills as a component of the constructive content structure of geometry. Furthermore, Dudareva and Zaryvnykh (2023) shown that constructive skills enable one to master:

- learning how to choose, combine, and mark sequences of operations using geometric figures
- perfecting geometric operations
- mastering combinations of constructive operations
- enhancing the experience of constructive operations by resolving relevant problems.

Visual components were used in research by Zhumanova et al. (2022). Additionally, they demonstrated how the examination of the use of information and communication technologies in stereometry instruction necessitates the search for instructional software tools that can be used to both problem-solving and the study of ideas and theorems. Gorodnova (2016) outlines the challenges associated with stereometric problems and demonstrates that computer modeling is the solution. According to Gorodnova (2016), there are programs that are simple for students to use and comprehend. Additionally, she defended the inclusion of GeoGebra, an interactive geometric environment developed by Florida Atlantic University professor Markus Hohenwarter. Additionally, she stated that compared to paper-based geometry, this application provides a greater variety of basic operations. It is also mentioned that it is

recognized to provide more chances to solve a number of geometric problems make stereometric drawings, etc. Furthermore, Gorodnova (2016) centered her research on the ease of learning a computer design environment for dealing with geometric drawings. Additionally, she stated that modern computer tools enable you to make vibrant, readily editable designs and carry out several procedures on them. Using examples of sketching portions of rectangular parallelepipeds, prisms, and pyramids, she further emphasized why these programs are far more efficient than drawing them by hand. In order to better comprehend the portion, she also suggested that you try rotating them. She also made it clear that accurate computation is crucial for students to do on tests. She ascribed this program's benefit to its interaction. Vallo et al. (2015) discussed the challenge of "formalism" and suggested using non-standard problems, unique techniques, and ways to get around it. Additionally, they recommended the use of information and communication technologies (ICT) and the introduction of appropriate forms through visualization. According to Vallo et al. (2015), learning should begin with actual conceptions and models of mathematical ideas rather than abstract information, according to the notion of didactic constructivism. Furthermore, the usage of interactive geometric software like "Cabri 3D" to foster spatial imagination was validated by Vallo et al. (2015). To detect formalism and perform diagnostics, Vallo et al. (2015) also carried out a pilot test with students. They were instructed to sketch the cubes for this. They discovered that all the outcomes were poor when they checked them using their Cabri 3D software. In order to create spatial images in stereometry, Bogdanova et al. (2024) demonstrated the necessity of using both traditional visual representations (drawings, projections) and physical representations (models). After reviewing a number of writers, Bogdanova et al. (2024) recommended using stereoscopic images to depict the proper visual representation and get around the challenges of spatial computations. For these stereoscopic images, Bogdanova et al. (2024) suggested utilizing an alternative color technique. They suggested employing "anaglyphs" in other words. In other words, it is made by combining two distinct views of the same item into a single image and giving the right

and left eyes separate color ranges (blue and red). Additionally, they said that in order to see it, special light-filtering glasses are required, and that when these are worn, physiological parallax creates the appearance of depth. In order to develop these technologies, Bogdanova et al. (2024) evaluated a number of graphical and mathematical software environments. The graphical images were then manually merged and overlaid using Photoshop and GIMP. Furthermore, Bogdanova et al. (2024) examined the idea of employing a specific "Projection for glasses" command in the dynamic mathematical system GeoGebra to quickly and precisely turn 3D designs into anaglyphs. In their study, Kramarenko et al. (2020) suggested using cloud technology GeoGebra, which includes the 3D Graphing Calculator. They also explained how to draw polyhedra parts using this method. With emphasis on dynamic visualization, Christou et al. (2007) created the DALEST software, which enables the creation, modification, and control of geometric forms in space. A unique program known as "Cubix Editor" was employed by them. This is an action-oriented curriculum. In other words, this tool enables the students to do activities step-by-step, according to Christou et al. (2007). In other words, they developed features like grouping figures, seeing objects in translucent or solid color, and employing "hot points" to change an object's size proportionately or in a single direction (e.g., lengthening or reducing the radius and height of a cone).

## **2.2 Conceptual definition of digitalization in education and advantages of digital tools**

According to Sardarova et al. (2022), the State Program of the Republic of Kazakhstan "Digital Kazakhstan" aims to increase the competitiveness of the state economy through the use and development of digital technologies, and the digitization of educational resources is thought to be a crucial step toward achieving this goal. In their study, Uteuliev et al. (2022) point out that the Digital Kazakhstan initiative offers ongoing enhancements to teacher credentials as well as improvements in digital literacy in education. According to Boronenko and Fedotova (2023), digitalization is a new phase of automation and computerization of management, the shift to digital

technologies, their application to real-world problems, the gathering and processing of large amounts of data for preventative purposes, and the optimization of interaction processes. The primary objective of digital transformation, according to Boronenko and Fedotova's research (2023), is to provide in educational interactions efficient information assistance within the framework of planning the learning process and overseeing educational activities. Although digitalization has an integrative and uniting impact, Sardarova et al. (2022) contend that teachers have difficulties in grasping it. Global tendencies in the shift to a digital economy and society are responsible for the present stage of professional education development, the digitization process (Uteuliev et al., 2022). Furthermore, according to Uteuliev et al. (2022), the current generation, known as Generation Z, lives in a setting where these digital technologies are being developed. After examining Pakhomova's work, Uteuliev et al. (2022) describe these digital technologies:

- wireless communication and telecommunication technology
- distributed ledger systems and big data
- components of robotics and artificial intelligence
- technologies for augmented reality (AR) and virtual reality (VR)
- digital technology for special education, cloud computing, and electronic identification.

Kiselnikov (2025) demonstrated how digitalization is a benefit of contemporary adaptive learning systems, assessing each student's progress and challenges and tailoring assignments and learning resources to their readiness level. Additionally, Kiselnikov (2025) contended that this gives every student the chance to engage in rigorous study. Additionally, he claimed that it fosters the development of innovative techniques like gamification and boosts students' motivation to study more through competitive activities, learning-related competition, and reaching objectives and prizes. Kozhasheva et al. (2024) claim that the educational process will undergo substantial changes because of digitalization. Additionally, according to Kiselnikov (2025), digitization enables educators to rapidly evaluate student achievement using

digital tools. Additionally, he demonstrated how it facilitates giving feedback. Furthermore, Drijvers and Sinclair (2024) concentrated more on the significance of digital technology. In other words, he said that in a relatively short period of time, these digital technologies have fundamentally changed our lives and society. In his Address to the People of Kazakhstan titled "Kazakhstan in the New Situation: A Time for Action" dated September 1, 2021, President of the Republic of Kazakhstan K.Zh. Tokayev justified the necessity of moving to online platforms because of the pandemic, according to Sardarova et al. (2022). That is, the quicker shift to digitization was one of the benefits of the epidemic for many nations. Due to the problem of equitable access to technology for all students during this epidemic, Drijvers and Sinclair (2024) also said that it is crucial that all students learn online, or remotely. Online learning platforms and interactive technologies have significantly altered the student enrollment process, according to Cirneanu and Moldoveanu (2024). According to Uteuliev et al. (2022), this pandemic's advantages stem from the requirement for digital technology. The primary benefits of digital technology are the emphasis of Sardarova et al. (2022). Furthermore, Dinçer (2025) clarified that digital learning resources and online learning environments are presently the foundation for assisting learning processes. In other words, according to Sardarova et al. (2022), digital technologies are an intelligent environment that offers new opportunities and convenience, as well as the capacity for each teacher to create a unique educational path and the accessibility of electronic resources for any user, continuous education. Cirneanu and Moldoveanu (2024) also discuss how technological advancements have made tailored schooling possible. According to Dinçer (2025), the shift to digitalization has significantly altered educational procedures and caused teachers to reconsider their pedagogical approaches and grow professionally. Since pedagogical techniques must be adapted to the digitalized learning environment, Dinçer (2025) has highlighted the significance of digital pedagogy in professional development. Young's study is examined by Drijvers and Sinclair (2024), who contend that meta-analyses have demonstrated that the usage of interactive technology significantly improves learning results. According to

Cirneanu and Moldoveanu (2024), these digital tools enable students to comprehend information via experience and autonomously investigate topics through interactive simulations and online experiments. Kozhasheva et al. (2024) shared Andersson's belief that using digital technology increases learning opportunities by giving users access to online materials, interactive exercises, and computer simulations that can improve the effectiveness and engagement of learning. Another benefit for teachers, according to Kozhasheva et al. (2024), is that digital tools enable educators to gather an extensive amount of data while students are studying. In other words, data analysis demonstrated that it was feasible to determine each student's unique needs and modify the learning process accordingly. According to Kozhasheva et al. (2024), using digital technology improves adaptive assessment and data analysis. In other words, adaptive assessment makes it possible to automate tests based on student requirements and generate precise, impartial findings. Galkin's study was also examined by Sardarova et al. (2022) to demonstrate the definition of a digital device. The Internet, artificial intelligence, system and application software, computer graphics and virtual reality systems, digital versions of conventional communication media (books, photos, audio and video recordings, digital television, etc.), computer games, and technological artwork were among them. The criteria for teachers in this respect were not overlooked by Kozhasheva et al. (2024). Kozhasheva et al. (2024). claims that to employ digital technology, teachers will need to become proficient in technical abilities and adjust to new standards for evaluation techniques. Furthermore, Kozhasheva et al. (2024) emphasized the following information that these teachers ought to be aware of:

- Technical literacy is the ability to use and comprehend the workings of digital technology
- Pedagogical competency is the ability to create assessment assignments using digital technology and modify them to meet each student's unique requirements
- Analytical abilities include the ability to use digital tools to evaluate student accomplishments and make inferences about their development

- Communication skills: using digital communication technologies to efficiently share information with coworkers and students.

According to Drijvers and Sinclair (2024), one benefit of having access to contemporary digital technologies is that they might question the substance of conventional curriculum and result in new reforms. Cirneanu and Moldoveanu (2024) assert that the digital environment produces a dynamic and captivating learning environment that encourages a better comprehension of difficult topics, streamlines the learning process, and cultivates a desire to study. The following benefits of the development of digital tools have been demonstrated by Cirneanu and Moldoveanu (2024): they explain that computer programs perform calculations far more quickly than traditional methods, help reduce errors, and allow students to spend more time analyzing mathematical concepts rather than simple calculations. Kiselnikov (2025) demonstrated how digitalization is a benefit of contemporary adaptive learning systems, assessing each student's progress and challenges and tailoring assignments and learning resources to their readiness level. Furthermore, Drijvers and Sinclair (2024) based their study on Miser and Koher's conclusions that instructors must enhance their technical, pedagogical, and subject knowledge in order to use digital tools in the classroom effectively (TPACK model). Dinçer's (2025) study also referred to this Technological Pedagogical Content Knowledge (TPACK). He claims that this strategy is considered to provide teachers with additional technical capabilities in addition to pedagogical and topic expertise. Accordingly, the findings of Dinçer's (2025) study demonstrated that math teachers were more conscious of the active use of digital resources and were more likely to advance their development. In line with the nature of technical competences, he also noted that the study's participating teachers were successful in teaching mathematics remotely. Additionally, Kiselnikov (2025) contended that this gives every student the chance to engage in rigorous study. Additionally, he claimed that it fosters the development of innovative techniques like gamification and boosts students' motivation to study more through competitive activities, learning-related competition, and reaching objectives and prizes.

Additionally, according to Kiselnikov (2025), digitization enables educators to rapidly evaluate student achievement using digital tools. Additionally, he demonstrated how it facilitates giving feedback. Kiselnikov (2025) argues that the advantages of digitalization go beyond that, as it fosters the growth of independence and critical thinking and gives access to educational resources from the finest institutions in the world via websites like YouTube, Coursera, and edX. According to Uteuliev et al. (2022), teachers must possess a high degree of information and communication technology competency during the digitalization process in order to help the younger generation adjust to life in the modern digital society. In their research, Boronenko and Fedotova (2023) also emphasized the advantages of digitization. One of them is that teachers' work becomes more productive, dynamic, and efficient because of digitalization, and the following may be identified to make it more creative:

- ongoing internet assistance for the educational process
- availability of reliable, varied, and high-quality instructional materials
- customization of each student's learning path, immediate feedback, and tracking of their academic progress
- flexibility of educational programs that encourage students to engage in active cognitive activity.

According to Boronenko and Fedotova (2023), the strategic direction of digitalization necessitates the establishment of the prerequisites for the regular operation of the electronic information and educational environment. They said that it consists of a variety of pertinent technical instruments, information technologies, communications technologies, electronic educational resources, and electronic information resources. Based on Kubrushko's 2019 study, Sardarova et al. (2022) describe the diverse and multipurpose content of digital instructional resources:

- an easy-to-use tool for teaching images that helps students improve their real-world applications
- a tool for reviewing and evaluating students' assignments using questionnaires

- the capacity to generate symbols using tables, graphs, and diagrams
- a way for students to edit and fix mistakes in their creative work.

Sardarova et al. (2022) divided the benefits of digital technology, or electronic educational instruments, over conventional techniques into many stages:

- multimedia tools: the capacity to deliver information using several tools simultaneously, including text, video, graphics, sound effects, and audio accompaniment
- interactivity: It was stated that electronic learning resources can offer a variety of options. In other words, utilizing the keyboard to type text allows for the correction and systematization of mistakes, the use of audiovisuals, the activation of interactive multimedia elements, and the development of the cognitive system
- accessibility: Rakhmatullina's research provided the following explanation for this benefit. It was demonstrated that several resources may be employed for distant learning, making the lesson accessible to brilliant kids and kids with impairments who study from home
- versatility, or the capacity to acquire information, skills, and capacities pertaining to the process of learning. Additionally, the instructor must consider the principles and demands of the learning process while utilizing any technological resources.

Variability: the course material should be tailored to various educational methodological complexes and curricula, with varying degrees of complexity and different ways that students receive information. This allows for the "fine-tuning" of each student's unique characteristics to meet their educational needs and achieve educational results, as well as to create individual trajectories of content mastery for various student categories.

According to a survey conducted by Kozhasheva et al. (2024), 56.3% of teachers reported being "very familiar" with digital assessment tools, whereas 43.7% reported being "not very familiar". Additionally, 57.6% of participants reported using digital

tools to assess students in the classroom frequently, whereas 42.4% reported using them infrequently.

### **2.2.1 Disadvantages of digitalization**

Cirneanu and Moldoveanu (2024) concentrated on the potential risks associated with digitization in addition to its advantages. With the aid of technologies like artificial intelligence (AI), augmented reality (AR), virtual reality (VR), and 3D printing, they projected that education will become more individualized, adaptable, and technology-driven in the future. Cirneanu and Moldoveanu (2024) also demonstrate that although digitization creates a lot of benefits, there are drawbacks as well, such as poor internet access, a lack of digital infrastructure, and a lack of digital skills among teachers. As a result, they contend that concerns like equity and access in schools must be addressed. According to Kiselnikov (2025), one of the drawbacks of digitalization is that it violates equity because not all students have access to fast Internet or contemporary equipment, which might lead to a "digital divide" in society. Kiselnikov (2025) then discusses cybersecurity. In other words, he notes that many cybercriminals break into the Internet, which might result in the disclosure of students' personal data. Furthermore, Kiselnikov (2025) points out that teachers require specialized training and workshops to acquire digital technologies because many of them lack familiarity with contemporary technology. According to Uteuliev et al. (2022), teachers must possess a high level of information and communication technology due to the digitization of the education industry. Additionally, the new UNESCO recommendations, which align with the UN General Assembly's "2030 Agenda for Sustainable Development" development plan, "The Framework of ICT Competences of Teachers" highlight the significance and applicability of digital literacy. Kiselnikov (2025) also highlights the issue of educational accessibility, pointing out that not all publicly available online resources are of the same caliber. He also acknowledges that there are disadvantages to remote learning, such as the lack of in-person teacher-student connection, which has a detrimental impact on students' motivation and social skills development. The next concern is explained by Kiselnikov (2025), who

highlights the risk of technological addiction. Additionally, he contends that students usage of digital gadgets leads to an issue of technology addiction. Additionally, when evaluating student outcomes, ethical secrecy must be maintained in order to preserve student data. Furthermore, the teacher's position is evolving. According to him, the teacher is now taking on the role of a facilitator. This appears to be neither a benefit nor a drawback. The benefits and drawbacks of digital technology were mentioned by Kozhasheva et al. (2024). Kozhasheva et al. (2024) claim that the state's incapacity to supply modern technology to every village is the reason behind the technical divide between rural and urban regions. According to Dinçer (2025), one drawback of these digital technologies is the absence of direct, in-person interaction with students. Some digital technologies are difficult to understand, according to Dinçer (2025). Additionally, he pointed out that there is a great likelihood of students' attention being distracted in a digital setting. Additionally, he pointed out that it might be challenging to confirm the accuracy of some mathematical ideas found online.

### **2.2.2 Digital tools in math**

According to results of Drijvers and Sinclair (2024), the use of digital technology in mathematics enhances learning outcomes, increases the effectiveness of the learning process, and fosters a better grasp of the subject. Additionally, Dinçer (2025) stated that digital technologies are crucial in teaching and that mathematics is composed of abstract notions. Drijvers and Sinclair (2024) investigated how well students learned geometric forms using tools like 3D pens from a constructivist viewpoint, which sees education as "making, constructing". Meta-analyses have shown that the employment of digital tools significantly improves student learning outcomes, as Drijvers and Sinclair (2024) showed in their study. According to Dinçer's (2025) research, interactive simulations and video resources greatly aid in the comprehension of mathematical ideas. As stated by Dinçer (2025), students can use digital tools to master these mathematical ideas at their own speed. He also arrived at the conclusion that teachers may provide students with step-by-step feedback. Additionally, Drijvers and Sinclair (2024) discovered that learning is more successful when students are taught in

pairs rather than alone when digital technology are integrated with traditional teaching methods. Furthermore, the results of Dinçer's (2025) study demonstrate that digital platforms may help overcome mathematical challenges by making abstract mathematical ideas interactive and visually appealing. Dinçer (2025) also demonstrated how using digital tools to dynamically portray mathematical concepts makes it easier for students to learn difficult topics. Additionally, Drijvers and Sinclair (2024) contended that the development of novel teaching strategies in mathematics is made possible by the use of instruments like computer algebra systems, dynamic geometry, and graphing calculators. In contrast to traditional teaching methods, Drijvers and Sinclair (2024) discovered that digital tools let teachers study, explore, and develop mathematics on their own. According to Cirneanu and Moldoveanu (2024), digital technology may use computer programs to execute computations far more quickly than conventional paper-and-pencil methods. Additionally, they demonstrate that students may concentrate more on comprehending and evaluating mathematical ideas without devoting too much time on repeated, basic computations in the same setting. Cirneanu and Moldoveanu (2024) also found that providing students with the opportunity to effectively check their work through software can help reduce their errors. They also found that this increased students' confidence in their mathematical abilities. In their study of Hely and Emprin Petifour's studies, Drijvers and Sinclair (2024) discovered that digital technology may be tailored to assist children with dyspraxia, blindness, or deafness in learning mathematics. Furthermore, Cirneanu and Moldoveanu (2024) discovered that complicated mathematical problems that would ordinarily take a long time to solve manually may be solved with ease using solutions produced by digital technologies. Furthermore, Cirneanu and Moldoveanu (2024) discovered that students may readily answer difficult and time-consuming mathematical problems that would otherwise need to be handled by hand thanks to the solutions produced by digital tools. Students are claimed to be able to improve their ability to solve problems as a result. Furthermore, Drijvers and Sinclair (2024) contend that digital tools are pushing the increased incorporation of computational thinking and

programming in curriculum and questioning the existing usefulness of conventional mathematical abilities like manual calculation. Cirneanu and Moldoveanu (2024) employed a number of digital technologies in their study's methodological part. Matlab is one among them. It was used to simulate complicated differential equations and assess the flight trajectory of aerospace vehicles, such as bombs and missiles. It was demonstrated that by entering the aircraft's and weapon's specifications, this program could be used to determine the trajectory. Cirneanu and Moldoveanu (2024) then gave a talk about SolidWorks and how it was utilized to simulate a ballistic pendulum. It shown that it enables students to create and examine intricate mechanical structures, systems, and parts in three dimensions. Additionally, Simulink is displayed. It is a graphical user interface tool for modeling and simulating dynamic systems. Here, from the view of author that is not writing any code to design intricate sculptures using pre-made blocks. There was also talk of Python. It teaches students how to solve problems rationally and analytically. Cirneanu and Moldoveanu (2024) also discussed how contemporary technology might advance mathematical understanding. They are:

- students can graph equations, see geometric forms and vectors in three dimensions, and work with virtual objects thanks to augmented reality (AR) and virtual reality (VR). Additionally, Drijvers and Sinclair (2024) discovered that new tools like motion sensors, multi-touch technology, and virtual/augmented reality (VR/AR) are being used extensively in geometry education, particularly to transition from 2D to 3D.
- artificial intelligence (AI) serves as a student's personal assistant by giving them tailored feedback and assisting them in identifying areas that require further assistance
- gamification: arranging math classes as quests or adventures in the classroom boosts students' interest in the subject. Kiselnikov (2025) cites the following digital tool as an example of this gamification: the Prodigy Math Game app turns the process of solving problems into an adventure where you fight monsters and earn points.

- blockchain: enables students to safely store mathematical ideas and validate and distribute their answers
- IoT: used to build sensor-equipped, networked classrooms
- they may make models of cubes, spheres, and particle strips using 3D printing and witness them coming to life.

According to Dinçer's (2025) survey, 75.9% of teachers utilize social media for learning. Instagram and YouTube were especially well-liked. Furthermore, 65.5% of teachers used online resources like Coursera, EBA, and Khan Academy. Kiselnikov (2025) discusses how learning may be personalized, using the Coursera and Khan Academy platforms as examples. It was discovered that 17.2% of educators made use of interactive learning resources including Quizlet, Mentimeter, and Kahoot. This indicates that the majority of teachers reportedly make good use of digital resources. Dinçer (2025) also covered the possibility of utilizing social media in the classroom to boost students' enthusiasm for the lesson and favorable attitudes about the subject through mathematical content. Kiselnikov (2025) notes that interactivity and visualization increase, and for example, platforms such as GeoGebra make learning algebra and geometry more interesting by allowing the creation of dynamic geometric models that can be modified in real time. Teachers frequently employ mobile applications, interactive materials, virtual reality (VR), and augmented reality (AR) while teaching mathematics, according to Boronenko and Fedotova (2023), who reviewed the work of many writers. Furthermore, Boronenko and Fedotova (2023) emphasized how their work made advantage of dynamic mathematical environments like GeoGebra. They specifically highlighted its benefits: it enhances mathematical comprehension, may be employed with visual effects, and allows for the creation of dynamic models, the simulation of issue scenarios, and the discovery of answers. Therefore, Boronenko and Fedotova (2023) propose that simulators and virtual labs can show how knowledge may be used in real-world situations.

## 2.3 What is GeoGebra?

A computer application called GeoGebra was created to teach mathematics, particularly algebra and geometry (Tamam & Dasari, 2021). Tamam and Dasari (2021) attribute GeoGebra's accessibility to the fact that it is a free program that can be downloaded at [www.geogebra.org](http://www.geogebra.org). Additionally, they contend that it has been translated into over 73 languages. Celen (2020) demonstrated that GeoGebra, which has been translated into other languages, is free to use and includes a Turkish interface. GeoGebra may be downloaded to any computer and utilized at any time, which sets it apart from commercial tools, according to Tamam and Dasari (2021). In a similar vein, Siregar (2025) also Arbain and Shukor (2015) described geogebra as the visualization and manipulation of mathematical objects in real life, algebra, statistics, geometry, calculus, etc. after conducting a literature study on geogebra research from 2020 to 2025. Based on the research of experts such Sopanda, he also discovered that geogebra improved students' skills in the study of spatial geometry. According to Arbain and Shukor (2015), geogebra is a crucial tool for efficient lesson preparation and high-quality education. In his research, Celen (2020) provided a similar definition. Lebamovski and Lebamovska (2022) discovered that GeoGebra is the most active program utilized in math classes in a study carried out in Bulgarian and European educational systems. Additionally, Lebamovski and Lebamovska (2022) describe GeoGebra as an interactive application that teaches geometry, algebra, and statistics to students in elementary school through university. Additionally, Lebamovski and Lebamovska (2022) contend that GeoGebra may operate in many modes at once, including a mode for 2D graphics (planimetry), 3D visuals (stereometry), and algebraic computations associated with these geometric sections. Lebamovski and Lebamovska (2022) demonstrated that GeoGebra contains features including translation, inversion, rotation, sectioning, and polygon construction, and that it can be used to create lines, surfaces, and three-dimensional shapes. Lebamovski and Lebamovska (2022) examined GeoGebra, Cabri3d, and DALEST and concluded that only GeoGebra is capable of implementing stereoscopy utilizing passive 3D technology (anaglyph

projection). In other words, Celen (2020) considered GeoGebra as a tool that integrates mathematical analysis, geometry, and algebra into one interface. According to Sekulić et al. (2023), GeoGebra's 3D calculator module has a number of tools for creating planes across three points, including solids of rotation like cubic prisms and perpendicular or parallel crossing surfaces. According to Celen (2020), in addition to drawing figures, GeoGebra can also adjust their corners and arrived at the conclusion that they could be precisely measured. According to Siregar (2025), this program's benefit stems from Paivio's Dual Coding theory, which states that it facilitates the 3D visualization of abstract notions like volume, surface area, and distances between points and planes. In other words, it is feasible to comprehend and retain information verbally and visually. The benefits of utilizing GeoGebra were also discussed by Tamam and Dasari (2021). In other words, it enables you to express and work with abstract geometric objects fast, precisely, and effectively. Based on scientific research, Siregar (2025) discovered that when students were taught the concept of distance in space using GeoGebra, they were able to learn it directly through visual simulations rather than only memorizing the formula. Additionally, he demonstrated how GeoGebra is a useful tool for teaching trigonometry, derivatives, and integrals in addition to spatial geometry. Tamam and Dasari (2021) also demonstrated the advantages for teachers. In other words, teachers provide an engaging atmosphere that facilitates learning. It can improve accuracy and speed, as demonstrated by Tamam and Dasari (2021). That is, geometric drawings may be made with ease and precision in place of a ruler and compass. Students can also proofread their drawings for mistakes. The advantages of GeoGebra for teachers were also discussed by Siregar (2025). In other words, it can reduce the amount of time needed to create interactive learning materials, do away with the time-consuming manual sketching on the board, and enhance instruction in line with contemporary standards. Most significantly, teachers may now use GeoGebra to dynamically and graphically depict abstract ideas in real time. This promotes creative teaching methods in addition to improving conceptual knowledge. The advantages of GeoGebra for students were also reported by Tamam

and Dasari (2021). GeoGebra's animations make geometry easier to grasp. Additionally, they discovered that students are more eager to use computers. Additionally, it enhances students communication, logical thinking, and problem-solving abilities. The advantages of GeoGebra for students were also demonstrated by Siregar (2025). Students' enthusiasm and interest in the class are increased by GeoGebra's dynamic visual environment. Additionally, it enables students to comprehend complex ideas, draw connections between mathematical things, and offer feedback. Students are encouraged to examine theories, form conjectures, and build their own understanding via investigation and discovery in addition to honing their critical thinking and problem-solving abilities. Siregar (2025) investigated geogebra's educational advantages as well. In other words, he stated that it might be included into the school's digital strategy. In other words, he claims that the degree of digital transformation in schools may be raised by utilizing Geogebra. Additionally, he claims that by integrating technology into the educational framework, it may raise teachers' and students' levels of digital literacy and enhance STEM education. In her study on lines and angles, Celen (2020) enumerated the advantages she discovered through student interviews. One student said that she detested rules and that using GeoGebra made it easy for her to draw lines. Another student commented that she thought math was the hardest subject, but GeoGebra made it enjoyable. Research by Celen (2020) showed that the software makes abstract mathematical topics easier to see and comprehend. Additionally, Celen (2020) demonstrated that students could draw generalizations about what occurs when angles are parallel or equal while changing forms. This was referred to as "developing research skills". According to reports, the application greatly reduced the amount of time needed to complete assignments and was particularly practical for students who dislike drawing with conventional tools like protractors, compasses, and rulers. During the study, students recommended applying GeoGebra to additional mathematical subjects, according to Celen (2020). They also stated that they thought difficult-to-understand probability and algebraic expressions should be taught using comparable visualization technologies. In

their work, Tamam and Dasari (2021) demonstrated the application of GeoGebra in two ways. in the domains of geometry and statistics.

- it has been demonstrated in statistics that the arithmetic mean, median, and mode may be quickly calculated by inputting a data set (for instance, 20, 20, 45, etc.).
- in geometry, it is demonstrated how to create a line that passes through two points (for instance, A (3,5) and B (-2, -4)) and automatically get its equation ( $9x - 5y = 2$ ).

Tamam and Dasari (2021) provided teachers with several strategies for efficiently use GeoGebra. In other words, they believed that teachers needed to be knowledgeable about the software before they could utilize it effectively. They advise students to make regular use of the software. This is due to Tamam and Dasari's (2021) finding that regular usage lessens miscommunication. GeoGebra has educational and psychological underpinnings, as demonstrated by Siregar (2025). For instance, Siregar (2025) demonstrated the following hypotheses in his research:

- Bruner's theory of cognition: Students may use GeoGebra to rotate and move their hands, comprehend through pictures and graphs, and eventually advance to mathematical symbols and equations. In other words, Siregar (2025) separated Bruner's theory into three phases: symbolic, iconic, and enactive. He also mentioned each stage's traits and advantages.
- Ausubel's theory: GeoGebra functions as an "advance organizer". In other words, students use visualization to make connections between new ideas and what they already know. For instance, students dynamically examine the radius in GeoGebra and then see how it varies proportionately before learning the formula for the area of a circle. He demonstrated how they then commit the area formula to memory.
- Vygotsky's theory or Zone of Proximal Development (ZPD): The "scaffolding" function of GeoGebra has been addressed in relation to Vygotsky's hypothesis, also known as the Zone of Proximal Development

(ZPD). In other words, the teachers use GeoGebra to build an interactive environment. As a result, when they run across problems, students use it as a supplementary tool. As students gain independence, this scaffolding eventually diminishes, reflecting Vygotsky's concept of "fading support". Additionally, since students frequently collaborate to build and analyze models, discuss hypotheses, and confirm results, GeoGebra fosters social interaction and collaborative learning. Vygotsky's theory that learning is socially mediated and that communication and shared cultural resources impact cognitive development is embodied in this partnership.

Siregar (2025) demonstrated the efficacy of GeoGebra by comparing many research. This leads to the conclusion that follows. In other words, he concluded that the teacher's usage and integration of GeoGebra into the class influence its efficacy. Additionally, Siregar (2025) examined studies that combined GeoGebra with various approaches and shown that students not only solved problems but also improved their problem-solving abilities when GeoGebra was combined with Problem-Based Learning (PBL) or Discovery Learning models. Geogebra facilitates student-centered learning from a pedagogical standpoint, according to Siregar (2025), who noted that the teacher serves as a guide rather than just a supplier of knowledge, enabling students to do independent inquiry. A research on geogebra in statistics was carried out by Arbain and Shukor (2015). The study's findings demonstrated that the experimental group outperformed the control group. The study by Shadaan and Leong (2013) included a questionnaire in addition to a pretest and a posttest to assess their performance. Additionally, learning using this geogebra proved to be more successful than traditional teaching methods. Furthermore, Arbain and Shukor (2015) found a clear correlation between the experimental group's interest in geogebra technology and their higher scores. The survey conducted by Arbain and Shukor (2015) revealed that students had a favorable attitude toward the program, with an average score of 4.26. Additionally, Arbain and Shukor (2015) discovered that:

- Students' enthusiasm and attention were significantly impacted by using GeoGebra.
- In response, 65.5% of students said they like GeoGebra.
- Additionally, 51.7% of students said they would like to use GeoGebra to learn mathematics in the future. "I can think creatively and critically when using the program" was the survey question with the lowest score (3.93). This is still seen as a reliable indication, though. In their investigation, Arbain and Shukor (2015) discovered further advantages of utilizing GeoGebra. This is as a result of the fact that GeoGebra is an interactive tool that encourages curiosity. Additionally, they discovered that the openness of communication between students and teachers - that is, their comments throughout the lesson - is positively impacted by the GeoGebra learning process. Consequently, students are more interested in mathematics, according to Arbain and Shukor (2015).

### **2.3.1 Challenges of GeoGebra**

Siregar (2025) also mentioned the challenges of utilizing Geogebra:

- for teachers: not all teachers are proficient in Geogebra. Because of this, many teachers lack comprehensive and useful geogebra abilities.
- facilities: There are still some schools with inadequate or nonexistent computer systems. Consequently, it was discovered that these technological issues impede teachers as well as students.
- for students: Students who are unable to utilize digital gadgets will require assistance because not all students are proficient in using them.

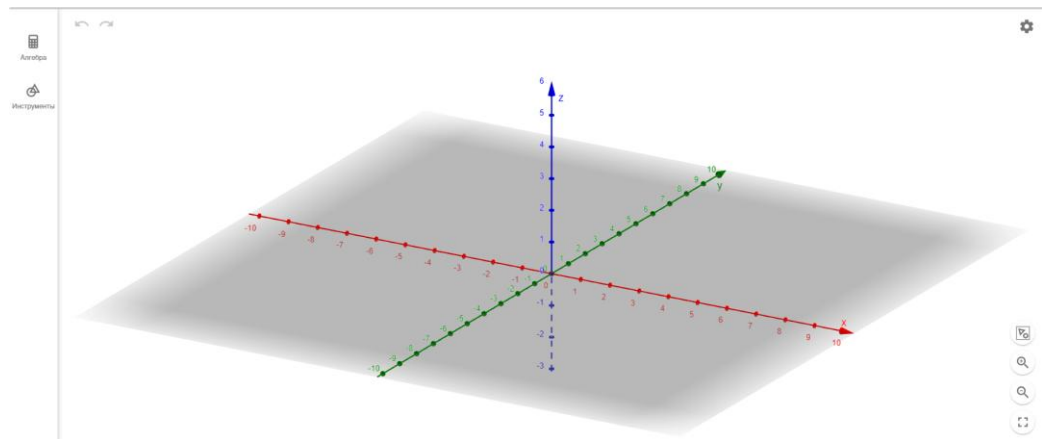
As a result, it was also demonstrated that teacher's methodical assistance is first required. It is evident that the students had similar challenges when Celen (2020) documented the flaws and noticed throughout investigation. In other words, students who were unfamiliar with computer usage found the research challenging. In other words, students rapidly became weary with the procedure and had trouble locating the required icons on the program's toolbar. Furthermore, Celen (2020) concludes that to

prevent these errors, students must be taught how to utilize GeoGebra beforehand. Additionally, Celen (2020) observed that some students disliked working in pairs or groups. Using the application in groups or couples proved uncomfortable for students who would rather work alone. Some students who had not used the software had some trouble finding and using the tools, according to the findings of Putra et al.'s (2021) research.

### **2.3.2 GeoGebra in stereometry teaching**

Teaching stereometry with GeoGebra fosters the development of STEM (science, technology, engineering, and mathematics) abilities. Teachers should be trained in cloud-based technologies like GeoGebra to enhance their STEM knowledge (Kramarenko et al., 2020). According to Kramarenko et al. (2020), they may view 3D objects, modify them interactively, and make difficult geometric concepts (such spatial intersections) simpler by utilizing GeoGebra. Additionally, Kramarenko et al. (2020) demonstrated in their study that teachers and students may use GeoGebra to exchange pre-made models, interactive textbooks, and assignments. After the findings were acquired, Kramarenko et al. (2020) concluded that the employment of GeoGebra was successful. He claims that the groups (teachers and students) who used GeoGebra's dynamic models outperformed those who utilized static paper models in terms of problem solving. Furthermore, Kramarenko et al. (2020) demonstrated how utilizing GeoGebra fosters variety, building phases, and algorithmic thinking based on data from teachers and students. Using the GeoGebra 3D calculator in the image below, Sekulić et al. (2023) demonstrate how to choose projection types and rotation tools for object presentation.

*Figure 2.3.2.1. GeoGebra Tools Interface*



*Source: Adapted from GeoGebra's platform*

According to Petrov and Atanasova (2020), students might have a customized learning experience by utilizing GeoGebra 3D calculators. Additionally, they mentioned that students had unlimited access to GeoGebra outside of the classroom. According to Sekulic et al. (2023), this geogebra makes it simple to access 3D geometric objects by "dragging" the item with a pointer. According to Sekulic et al. (2023), this is viewed as a chance to solve the issue of students' poor spatial visualization. The following benefits of GeoGebra in stereometry were emphasized by Sekulic et al. (2023):

- GeoGebra's great precision makes solving geometric problems simple.
- GeoGebra can simultaneously give geometric, algebraic, numerical, and dynamic solutions.

GeoGebra was used in an experiment by Petrov and Atanasova (2020) to teach stereometry. Because of GeoGebra's 3D and augmented reality features, the understanding of students of 2D drawings improved. They were able to visualize their construction. Additionally, their confidence and independence in mathematics grew because of their own experimentation, errors, and self-correction. Additionally, Petrov and Atanasova (2020) discovered that students who utilized the GeoGebra mobile applications were more eager to collaborate in groups, have discussions, and share their knowledge. The experiment's outcomes also demonstrated positive outcomes.

Specifically, the experimental group that utilized Geogebra 3D tools had test results that were, on average, 29% better than the control group that used the traditional method.

- It is claimed that using the application's 3D glasses feature enables a deeper comprehension of the lesson, and these glasses are reasonably priced. The program may be downloaded for free to any device.

Putra et al. (2021) utilized GeoGebra to tackle 3D shape-related problems with elementary school teachers. They thus made use of a number of GeoGebra features. In other words, participants used the application to study forms in detail by rotating, folding, and writing/unfolding 3D objects. They calculated the shape's area and volume using the computer. Additionally, they used GeoGebra to verify the correctness of their drawings made using a pen and ruler. Putra et al. (2021) generally studied the following shapes: cube, cylinder, cone, sphere, rectangle prism, and rectangular pyramid. Participants were more interested in learning about curved forms (such as cylinders, cones, and spheres) than flat shapes (such as prisms), according to Putra et al. (2021). According to Putra et al. (2021), at the conclusion of the trial, geogebra was acknowledged as a highly useful tool for rapidly and precisely drawing figures and perceiving 3D space.

### **2.3.3 Examples of utilizing GeoGebra tool in 3D geometry**

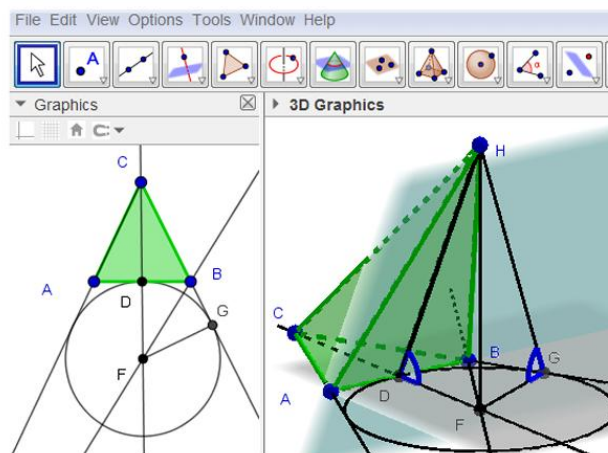
Furthermore, Kramarenko et al. (2020) demonstrated four procedures for creating cross-sections.

- Building a 2D skeleton should be the initial step. This is comparable to the conventional "trace method", but it has the drawback of not allowing dynamic changes to the built object.
- The second step, which is thought to be the best way to foster spatial imagination, lets you rotate the item and move the points. That is, the phase in which the 3D model is built.
- In the third step, the polygon and cutting plane are pre-constructed, and the GeoGebra ready-made tool is used to find their intersection.

- In the fourth step, it is useful to construct the base in 2D and the height in 3D if the polygon's base (like a trapezoid) is complicated or the vertex projection falls on a specific location.

Furthermore, Kramarenko et al. (2020) demonstrated how to accurately depict bilateral angles in a 3D model and how to calculate their linear angle. Additionally, Kramarenko et al. (2020) examined two types of pyramids in GeoGebra (cases where the vertex falls in the center of the circle inscribed in the base and those where it does not) to explain the distinction between angles between planes and bilateral angles. Furthermore, Kramarenko et al. (2020) demonstrated a particular algorithm for creating a sphere enclosed in a pyramid: determining the base's center, drawing a perpendicular, building the bilateral angle's bisector plane, and designating the intersection point as the sphere's center.

*Figure 2.3.3.1. Constructing a Pyramid and its Cross-Section in GeoGebra*



*Source: Adapted from Kramarenko et al. (2020).*

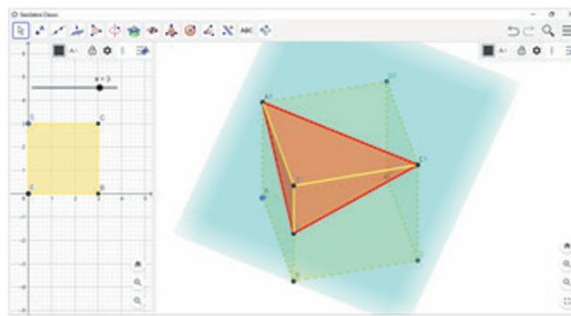
In Geogebra, Lebamovski and Lebamovska (2022) distinguished two primary categories of 3D geometric shapes:

- Polygons, prisms, cubes, parallelepipeds, and pyramids are examples of square geometric forms. These forms are produced using a novel border technique that the author suggests.

- Cones, spheres, mesh, and cylinders are examples of rotary solids. The conventional translational and rotational extrusion (squeeze/stretch) procedure is applied to these items.

Sekulic et al. (2023) used a genuine geogebra to explain this problem: The  $ABCD A_1 B_1 C_1 D_1$  hexagon is the problem. Determine the area of the plane's intersection with the prism's vertices  $A_1$  and  $C_1$  if point  $S$  is the midway of face  $BB_1$ . Using the image below, Sekulic et al. (2023) demonstrated this problem in GeoGebra.

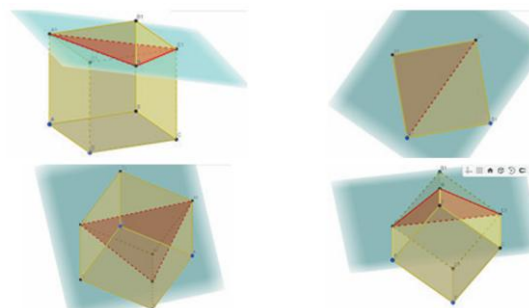
*Figure 2.3.3.2. Demonstration of a Prism Cross-Section in GeoGebra 3D Calculator*



*Source: Adapted from Sekulić et al. (2023)*

Most intriguingly, Sekulic et al. (2023) demonstrated how students can see the precise outline of a geometric object using 3D visualization. They also discussed how the 3D Geogebra calculator can manipulate the 3D object, i.e., view it from various angles, like rotation and control.

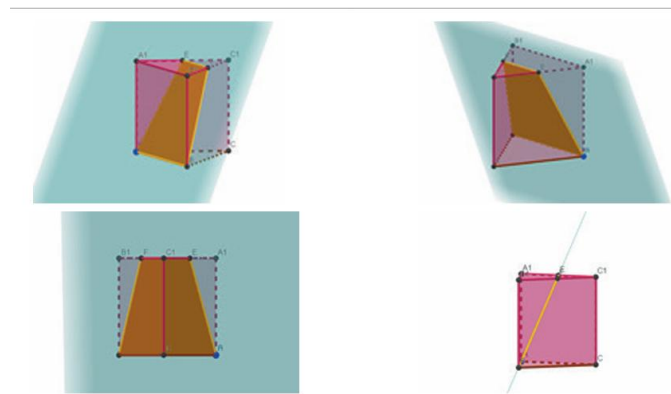
*Figure 2.3.3.3. Manipulating and Viewing a 3D Object from Various Angles in GeoGebra*



*Source: Adapted from Sekulić et al. (2023).*

According to Sekulic et al. (2023), the second problem is as follows: Face an of a normal triangular equilateral prism  $ABCA_1B_1C_1$ . Find the area of the prism and the cross-section of the plane that contains the segment  $EF$  and the prism's face  $AB$  if point  $E$  is the midpoint of face  $A_1C_1$  and point  $F$  is the midpoint of face  $B_1C_1$ . Sekulic et al. (2023) also utilized this computation to show a 3D item with a 3D calculator. A figure illustrating the process is shown below.

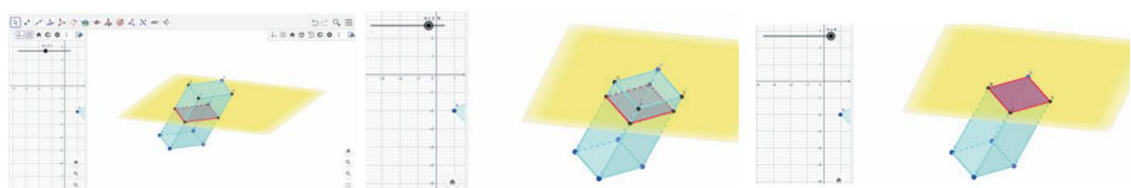
*Figure 2.3.3.4. Cross-Section of a Triangular Prism in GeoGebra 3D Calculator*



*Source: Adapted from Sekulić et al. (2023).*

The third problem raised by Sekulic et al. (2023) was: Find the point where a plane parallel to the base of a regular quadrilateral prism intersects it. Sekulic et al. (2023) employed GeoGebra's "slider" plugin to address this problem. It was demonstrated that the plane could be adjusted parallel to the prism's base and that various cross-sectional locations could be dynamically observed by adjusting the slider. This figure was produced by Sekulic et al. (2023).

*Figure 2.3.3.5. Dynamic Observation of Parallel Cross-Sections Using the Slider Tool*



*Source: Adapted from Sekulić et al. (2023).*

Additionally, Sekulic et al. (2023) discussed 3D spectacles. He demonstrated how 3D glasses may be used to examine any drawing made using the GeoGebra 3D calculator. He stated that when this feature is employed, the drawing immediately changes. He concluded that a greater comprehension of the drawing is possible when wearing these glasses. In this function may also rotate the drawing with this function. Here's how the glasses alter the drawing.

*Figure 2.3.3.6. Viewing a 3D Object Using 3D Glasses in GeoGebra*



*Source: Adapted from Sekulić et al. (2023).*

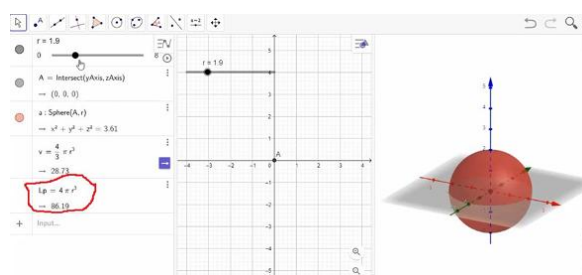
The following subjects were covered by Petrov and Atanasova (2020) using Geogebra:

- where two lines are in three dimensions
- plane and line in three dimensions
- intersection of two planes
- right prism, pyramid, right circular cylinder, right circular cone, and sphere are examples of geometrical solids.

An experimental and control group investigation was carried out by Petrov and Atanasova (2020). The teacher used the 3D program GeoGebra to first introduce the subject to the experimental group. The students then finished dynamic assignments that had been set beforehand. The "GeoGebra Exam Mode" was then used to assess their level of comprehension.

The following problem was presented to the participants by Putra et al. (2021): Determine a sphere's surface area. Initially, the participants attempted to use GeoGebra to depict this problem. The algorithm for determining the area was written down and mapped. The plot of the outcome is shown in the picture below.

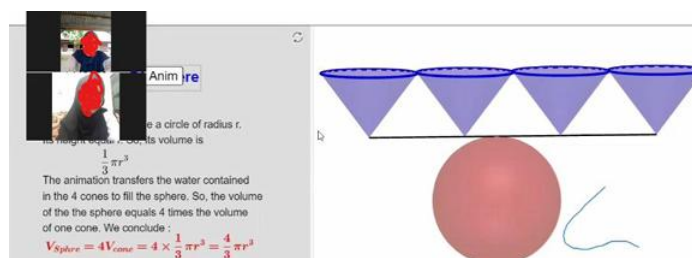
Figure 2.3.3.7. Calculating the Surface Area of a Sphere in GeoGebra



Source: Adapted from Putra et al. (2021).

The next group was suggested by Putra et al. (2021) to demonstrate the connection between the volumes of a sphere and a cone. This group's utilization of a pre-built GeoGebra applet produced by El Hage was the difference that was seen. An animation showing water flowing from four similar cones into a single sphere was used to solve the challenge. They were able to demonstrate that the volume of a sphere is equal to the volume of four identical cones thanks to this graphic depiction.

Figure 2.3.3.8. Dynamic Demonstration of the Volume Relationship Between a Sphere and Cones



Source: Adapted from Putra et al. (2021).

### 3. METHODOLOGY

This study examines at how high school students utilize GeoGebra to solve problems involving three-dimensional geometric shapes. To graphically depict three-dimensional geometric shapes, GeoGebra was selected. This is since researchers have found this GeoGebra tool to be useful, including Siregar (2025), Tamam and Dasari (2021), Arbain and Shukor (2015), Petrov and Atanasova (2020), Lebamovski and Lebamovska (2022), and others.

#### 3.1 Research design

As in the study by Petrov and Atanasova (2020), the quasi-experimental design was used as the research design. In other words, experimental and control groups were used in this study.

Participants. Two groups of eleventh graders were among the high school students that took part in the study. Students from secondary school No. 148 were enrolled in these two classes to study mathematics and physics. Convenient sampling was used to choose these students. 40 students were chosen. There were 16 students in the experimental group and 16 in the control group as shown on Table 3.1.1.

*Table 3.1.1. Number of students*

| Experimental group | Control group |
|--------------------|---------------|
| 20 students        | 20 students   |

Research procedure. The experimental group and the control group took a pretest at the beginning of the research as shown on Table 3.1.1. The chapter on body volumes was then taught to the experimental group using GeoGebra. Additionally, the traditional method was used to instruct the control group. The experimental group and the control group were then given a posttest. The experimental group was given a questionnaire created by Shadaan and Leong (2013, as cited in Arbain & Shukor, 2015) to further solidify their understanding.

*Table 3.1.2. Pretest and posttest*

|         | Experimental group                              | Control group                                     |
|---------|-------------------------------------------------|---------------------------------------------------|
| Phase 1 | Pretest                                         | Posttest                                          |
| Phase 2 | Learning chapter on body volumes using Geogebra | Learning chapter on body volumes without Geogebra |
| Phase 3 | Posttest                                        | Posttest                                          |
| Phase 4 | Questionnaire                                   |                                                   |

The pretest and posttest are displayed in the Appendix. In general, the pretest and posttest each have three problems. Although the pretest and posttest problems differ, they share the same structure. In their study, Sekulic et al. (2023) similarly used pretest and posttest problems with distinct but comparable forms. Based on Bloom's Taxonomy and the grade 11 learning objectives, each of those problems was chosen from the research work of scientists cited in the study's literature review to teach stereometry. Let's display each pretest and posttest's authors and Bloom's Taxonomy levels in the Appendix. The research by Putra et al. (2021) is the source of the first pretest problems. According to Bloom's Taxonomy, this work falls under the Remember and Understanding level. The problem is meant to test the students' knowledge of the formulas from the preceding chapter on solids of revolution. Putra et al. (2021) utilized this challenge in their study to demonstrate that students make mistakes while using formulas or mechanically memorizing formulas. The pretest's second problem falls within the Understand level. The study of Vallo et al. (2015) is the source of this problem. Students make a mistake in this problem because they are unable to envision the base of the perpendicular, according to the study by Vallo et al. (2015). A lack of spatial imagination can be demonstrated with this difficulty. The Vallo et al. (2015) research is also the source of the third pretest concern. According to Bloom's Taxonomy, the problem falls under the Apply level. According to Bloom's Taxonomy, the first posttest problem is at the Understand level. This challenge was

chosen from the Zhumanova et al. (2022) research. The "Plane through three points" feature in GeoGebra makes it easy to visualize the form of a cross-section (rhombus). The purpose of this problem was to show how the software could self-test using automated area computation and step-by-step building. The research by Putra et al. (2021) is the source of the second posttest problem. This falls under Bloom's Taxonomy's Analyze level. Instead of just calculating the volume, the learner uses animation to investigate the formula's reasoning. This demonstrates how GeoGebra helps with inquisitive learning. The research by Sekulic et al. (2023) is the source of the third posttest problem. According to Bloom's Taxonomy, this work falls under the Evaluation level. This problem demonstrates how GeoGebra's "Dragging" and 3D Calculator tools may be used to see a figure from any angle. According to Sekulic et al. (2023), it is the best activity for evaluating a student's spatial object manipulation skills.

#### *Descriptors for pretest and posttest.*

To assess the pretest and posttest problems, a unique descriptor was developed. The authors' publications from which these problems were extracted were examined to develop each of these descriptions. The table below shows the problems' description for each problem. A crucial methodological step in ensuring the validity and reliability of the assessment process was the creation of these particular descriptors.

These rubrics offer a consistent framework for evaluation by precisely outlining the intended learning outcomes and dissecting intricate problem-solving procedures into observable steps. This methodical approach, which focuses on students' basic spatial reasoning and mathematics knowledge independent of the teaching medium, not only reduces subjectivity in grading but also ensures that students from the experimental and control groups are evaluated fairly.

Table 3.1.3. Assessment descriptor for pretest

| Number of problems | Descriptor                                                                                                                                                                                                                                  |
|--------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1                  | <ul style="list-style-type: none"> <li>• matches a sphere's radius using the formula: 1 point</li> <li>• carries out algebraic computations by entering numerical values into the formula: 1 point</li> </ul>                               |
| 2                  | <ul style="list-style-type: none"> <li>• depicts a cube and the <math>AFH</math> section plane in two dimensions: 2 points</li> <li>• calculates the pyramid height, or perpendicular, drawn from a point to the plane: 2 points</li> </ul> |
| 3                  | <ul style="list-style-type: none"> <li>• uses the parallel plane feature to draw intersecting lines: 2 points</li> <li>• determines a curve's closed form: 2 points</li> </ul>                                                              |

The posttest's assessment descriptors are described in Table 3.1.4, which also includes the precise grading standards for the experimental and control groups on three different mathematical problems. The rubrics are designed to take into account the various teaching resources that each group uses.

The experimental group's ability to use GeoGebra to create dynamic models with sliders and depict volume connections is evaluated for Problem 1 (2 points each). On the other hand, the control group receives two points each for their typical drafting abilities, which include drawing a cone and a sphere on paper and providing a theoretical explanation of the relationship between their volumes. This comparison approach is also used in the rubrics for the remaining challenges. They assess how well the experimental group uses digital tools for structural analysis and dynamic spatial visualization. At the same time, the control group's proficiency with manual algebraic computations, theoretical deductions, and conventional geometric drawing is evaluated. This method of differentiated grading guarantees an impartial and thorough assessment of approaches to problem-solving in both diverse learning settings.

Table 3.1.4. Assessment descriptor for posttest

| Number<br>of<br>problems | Descriptor                                                                                                                                                                                                                                  |                                                                                                                                                                                                                       |
|--------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|                          | Experimental group                                                                                                                                                                                                                          | Control group                                                                                                                                                                                                         |
| 1                        | <ul style="list-style-type: none"> <li>• GeoGebra displays a dynamic model using animation, such as sliders: 2 points</li> <li>• uses visual data to summarize the connection between volumes (<math>V_s = V_c</math>): 2 points</li> </ul> | <ul style="list-style-type: none"> <li>• on paper, sketch a diagram of a cone and a sphere: 2 points</li> <li>• explain the link between the volumes of the sphere and the cone using the sketch: 2 points</li> </ul> |
| 2                        | <ul style="list-style-type: none"> <li>• uses GeoGebra's "plane through three points" feature to create a cross-section: 2 points</li> <li>• uses GeoGebra's "Area" feature to automatically determine the area: 2 points</li> </ul>        | <ul style="list-style-type: none"> <li>• uses the rules to draw a cross-section: 2 points</li> <li>• uses the area formula to calculate the area: 2 points</li> </ul>                                                 |
| 3                        | <ul style="list-style-type: none"> <li>• rotates the shape to investigate various cross-sectional projections: 2 points</li> <li>• moving cross-sectional pieces freely to model them: 2 points</li> </ul>                                  | <ul style="list-style-type: none"> <li>• creates a figure, examines its cross-section, and investigates all its projections: 4 points</li> </ul>                                                                      |

Sekulić et al. (2023) emphasized the significance of evaluating students' answers to the GeoGebra application in addition to their object manipulation (rotation, movement) and visual information analysis skills. As a result, we can distinguish between these abilities using a 10-point scale. In their research, Putra et al. (2021) also employed a 10-point rating system. We may compare their traditional knowledge with their knowledge after utilizing GeoGebra thanks to this scale. Students' answers were transformed into a score system using assessment rubrics.

*Table 3.1.5. Assessment rubrics*

| Level                  | Rubrics                                                                                                                                                                                                             |
|------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| High<br>8-10<br>points | As explained by Putra et al. (2021), the student freely manipulates the shape in GeoGebra utilizing the GeoGebra features of slider and animation to show the problem's significance. Every computation is precise. |
| Middle<br>5-7 points   | According to Sekulić et al. (2023), students can build three-dimensional shapes in space, but they have some trouble rotating or evaluating the cross-section.                                                      |
| Lower<br>1-4 points    | Similarly, the Vallo et al. (2014) research, the student gives in to "visual formalism" and is restricted to a statistical drawing without comprehending the internal linkages of the shape.                        |

#### *Questionnaire.*

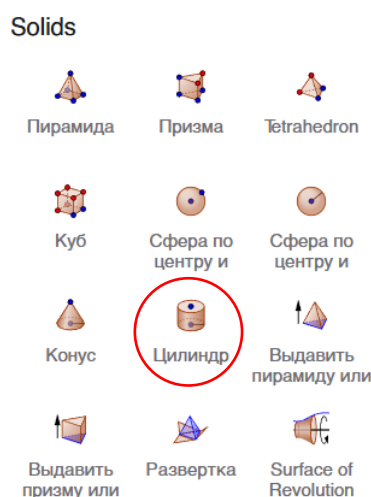
By examining the research of Arbain and Shukor (2015), the questionnaire was chosen. Nine items on a Likert scale ranging from "1-Strongly Disagree, 2-Disagree, 3-Agree and Disagree, 4-Agree, and 5-Strongly Agree" made up this questionnaire. Arbain and Shukor (2015) modified this questionnaire from Shadaan and Leong's (2013) study. The questions in this survey show how well the students comprehend how to utilize GeoGebra. Cronbach's alpha was used to determine the questionnaire's reliability:  $\alpha = 0,892$  indicated strong internal consistency, meaning it was deemed trustworthy because it was higher than 0.7.

### *Lesson structure*

Throughout the experiment, students actively utilized the GeoGebra 3D application to solve stereometric problems. Students learnt not just to apply pre-made formulas, but also to actively examine shapes and graphically demonstrate the answer to a problem. The following are the key problem-solving approaches utilized during the lesson. Some GeoGebra-based problem solutions are provided below. All of these problems come from the 11th grade geometry textbook authored by A.N.Shynybekov, D.A.Shynybekov, R.N.Zhumabayev, and S.S.Madelkhanov. This is a quick summary of how the problems were described in the book they studied during the class.

The first problem: The cross-section of the cylinder is square, and its area is  $S$ . Find the total surface area of the cylinder: 1)  $S=16 \text{ cm}^2$ .

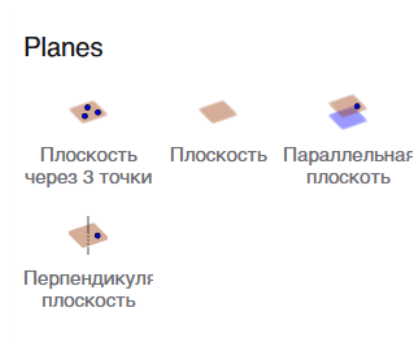
*Figure 3.1.1. Selecting the Cylinder Tool in the GeoGebra 3D Calculator*



*Source: Screenshot from GeoGebra 3D Calculator.*

We begin by opening the GeoGebra 3D Calculator, and then select cylinder from the solid's menu.

*Figure 3.1.2. Selecting the Plane Tool in the GeoGebra 3D Calculator*



*Source: Screenshot from GeoGebra 3D Calculator.*

We choose a plain.

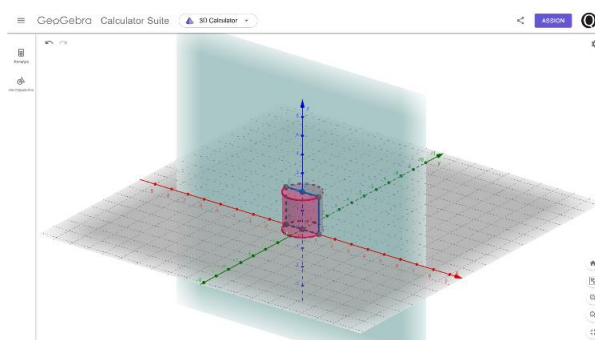
*Figure 3.1.3. Selecting the Intersect Curve Tool in the GeoGebra 3D Calculator*



*Source: Screenshot from GeoGebra 3D Calculator.*

We select the function to mark the figure formed by the intersection of a body of revolution with a plane.

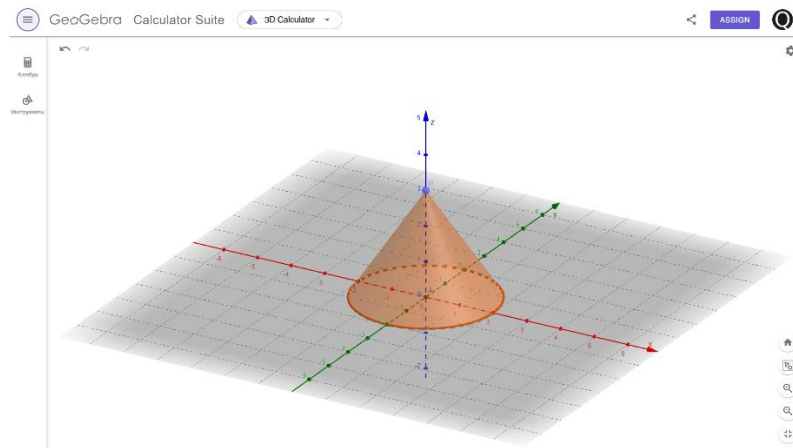
*Figure 3.1.4. Modeling the Cross-Section of a Cylinder in GeoGebra*



*Source: Modeled in GeoGebra 3D Calculator.*

The second problem: The cone has a base of 1 and a radius of  $R$ . Find the lateral surface area. We select a cone from the first solid surface. The cone looks like this in Figure. After determining the radius of the cone, calculations were performed.

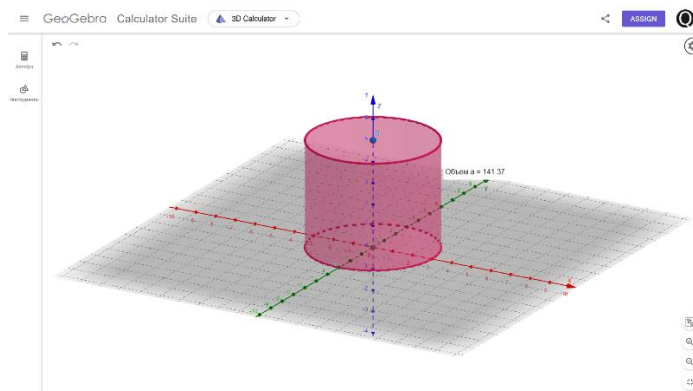
*Figure 3.1.5. Modeling a Cone in the GeoGebra 3D Calculator*



*Source: Modeled in GeoGebra 3D Calculator.*

The third problem: Find the volume of a cylinder with radius  $R$  and height  $h$ :  
 $R=3$  m,  $h=5$  m

*Figure 3.1.6. Modeling a Cylinder in the GeoGebra 3D Calculator*

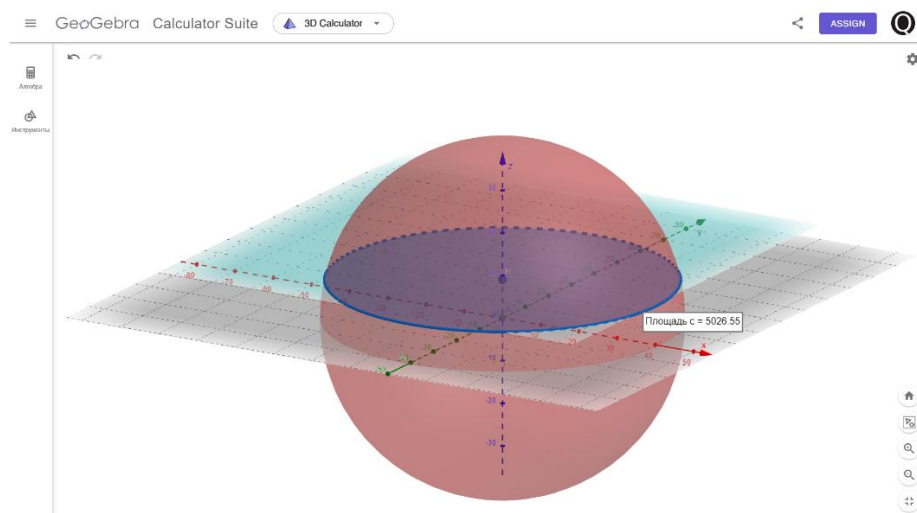


*Source: Modeled in GeoGebra 3D Calculator.*

By clicking on the Find Volume function, we can find the volume as shown in the image.

The fourth problem: A sphere with a radius of 41 cm is intersected by a plane 9 cm from its center. Find the area of the section.

*Figure 3.1.7. Modeling a Spere in the GeoGebra 3D Calculator*



Source: *Modeled in GeoGebra 3D Calculator.*

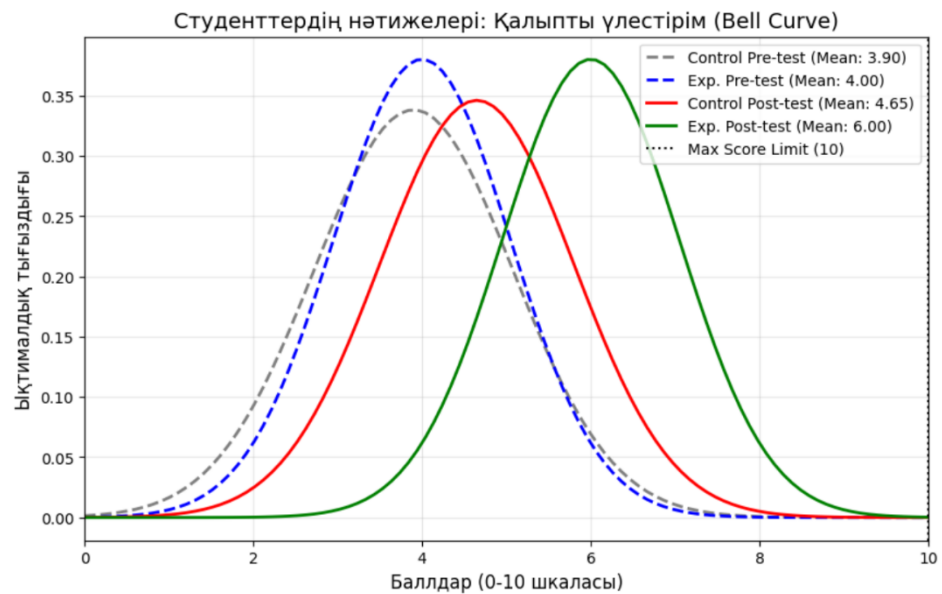
After selecting and drawing the cylinder, we select the plane function. Then we define the figure that intersects the plane.

## 4. ANALYSIS AND RESULTS

Students completed a pretest before to the start of the experiment. On a piece of paper, students finished the pretest. All of their pretest answers were assessed using the previously stated criteria for assessment, and all of their replies were coded into an Excel platform with the names of the students (experimental group, such as Student A, Student B, and Control group, such as Student 1, Student 2). The experimental group employed the digital application GeoGebra as a tool during the experiment. The control group carried on with their studies using traditional methods. The duration of the trial was around 8 weeks. Students were then required to complete the posttest. Similarly, to how Petrov and Atanasova (2020) evaluated student knowledge in their study, the experimental group completed the posttest utilizing the "Exam Mode" feature on their phones. Clicking the Exam Mode button within GeoGebra after selecting the 3D calculator button activates this feature. Students won't be able to visit any other websites after hitting this button. Every login and logout time is tracked, and the functions that were used to do the computations are displayed. The control group used a pen to write on a sheet of paper. After that, all their responses were entered into the Excel platform utilizing the numerical assessment criteria. The experimental group responded to the questionnaire after finishing the posttest. Every question in the survey was assessed using a Likert scale, and every response was entered into the Excel program. Next, we created a graph using Python to see if the experimental group's and the control posttest's findings were distributed in accordance with the normal distribution. The outcomes of the experimental group and the control group were then assessed utilizing the t-test using the Excel platform's Analysis tool. The Likert scale was used to code the survey data, and Arbain and Shukor (2015) determined the mean and standard deviation of each item. In this study also were all completed using the Excel platform to find the mean and standard deviation.

The pretest results for both the experimental and control groups are shown in the table below. We will now use a Python application to determine whether it falls within a normal distribution.

*Figure 4.1. Normal Distribution of Pre-test and Post-test Scores for Control and Experimental Groups*



*Source: Generated using Python (matplotlib).*

The probability density of the pre-test and post-test outcomes for the experimental and control groups is shown in Figure 4.1 above. The graph's analysis highlights the following significant trends:

- **Equivalency of initial knowledge base (Pre-test):** The blue dashed line (Experimental Group pre-test, Mean = 4.00) and the grey dashed line (Control group pre-test, Mean = 3.90) are quite near to one another. This indicates that before the study began, the students in both groups had essentially the same baseline knowledge of stereometry. This suggests that the experiment's two groups were appropriately and impartially balanced.
- **Results of conventional instruction (Control group):** The control group's post-test results (Mean = 4.65) are shown by the solid red line. This curve's tiny shift to the right as compared to the pre-test suggests that while the traditional teaching approach—using a blackboard and paper—was somewhat successful in raising students' knowledge, its overall effect was somewhat little.

- **High Efficacy of the GeoGebra Software (Experimental Group):** The solid green line depicts the post-test results of the experimental group (Mean = 6.00). The significant shift of this curve to the right, showing a substantial gap compared to the red line, clearly proves the effectiveness of the 3D visualization tool. It is visually evident that integrating the GeoGebra software into the lessons developed students' spatial reasoning and elevated their academic performance to a much higher level compared to the traditional method.

As previously stated, the results of both groups were normally distributed, and there were more than 40 students, allowing for the use of a t-test. The t-test was utilized for two purposes. That is, compare each group's pretest and posttest findings, as Arbain and Shukor (2015) did in their study. Also, compare the posttest outcomes of each group. That is, Petrov and Atanasova (2020) had favorable findings in their study. First, a paired t-test for dependent samples was used to compare the control group's pretest and posttest. The following result was acquired by selecting the Data Analysis button in Excel and applying the t-Test: Paired Two Sample for Means function.

*Table 4.1. Paired t-Test Results for the Control Group*

| Variable   | <i>M</i> | <i>SD</i> | <i>t</i> | <i>df</i> | <i>p</i> |
|------------|----------|-----------|----------|-----------|----------|
| Variable 1 | 3.90     | 1.21      | -6.10    | 19        | <0.001   |
| Variable 2 | 4.65     | 1.18      |          |           |          |

A paired-samples t-test was used to compare the control group's pre-test and post-test results in order to assess their learning improvement. After the regular educational session, the analysis showed a statistically significant improvement in student performance. For the 20 students who took part, the mean score rose from 3.90 (variance = 1.46) on the pre-test to 4.65 (variance = 1.40) on the post-test. There was a significant positive correlation between the two tests ( $r = 0.89$ ). This mean difference was found to be extremely significant by the paired t-test ( $t(19) = -6.10, p < 0.001$ ). These results show that the traditional teaching approaches successfully helped the

pupils achieve a quantifiable and statistically significant baseline of learning. a measurable and statistically significant baseline of learning among the students.

The experimental group's pretest and posttest were then used to do a t-test: Paired Two Sample for Means. The results below are from Excel.

*Table 4.2. Paired t-Test Results for the Experimental Group*

| Variable   | <i>M</i> | <i>SD</i> | <i>t</i> | <i>df</i> | <i>p</i> |
|------------|----------|-----------|----------|-----------|----------|
| Variable 1 | 4.00     | 1.08      | -27.57   | 19        | <0.001   |
| Variable 2 | 6.00     | 1.08      |          |           |          |

A paired-samples t-test was used to compare the experimental group's pre-test and post-test results in order to assess their learning improvement. After the GeoGebra software intervention, the analysis showed a highly substantial improvement in student performance. For the 20 students who took part, the mean score increased significantly from 4.00 (variance = 1.16) on the pre-test to 6.00 (variance = 1.16) on the post-test. A very strong positive correlation ( $r = 0.95$ ) was found between the two assessments. This mean difference was found to be statistically significant by the paired t-test ( $t(19) = -27.57, p < 0.001$ ). These results clearly show that, in comparison to the students' baseline abilities, the interactive GeoGebra education significantly and quantifiably improved their mathematical comprehension.

An independent-samples t-test was used to compare the experimental and control groups' post-test results. The purpose of this statistical study was to ascertain whether the mean differences between the two cohorts after the educational intervention were significant. The study aims to objectively evaluate the overall efficacy of the digital educational method in comparison to conventional teaching ways by analyzing these criteria. The following table displays the comprehensive findings of our comparison investigation.

*Table 4.3. Independent Samples t-Test Results Comparing Post-test Scores of Both Groups*

| Variable   | <i>M</i> | <i>SD</i> | <i>t</i> | <i>df</i> | <i>p</i> |
|------------|----------|-----------|----------|-----------|----------|
| Variable 1 | 4.65     | 1.18      | -3.78    | 38        | <0.001   |
| Variable 2 | 6.00     | 1.08      |          |           |          |

An independent-samples t-test was used to compare the final post-test results between the experimental and control groups in order to assess the overall efficacy of the educational intervention. According to the analysis, students who used the GeoGebra software (experimental group) outperformed students who got traditional education (control group;  $M = 4.65$ , variance = 1.40) in terms of mean score ( $M = 6.00$ , variance = 1.16). This mean difference was statistically significant, according to the independent t-test ( $t(38) = -3.78$ ,  $p < 0.001$ ). When contrasted directly to traditional teaching techniques, these results offer compelling quantitative proof that the inclusion of dynamic GeoGebra software produces better educational outcomes in stereometry.

Cohen's  $d$  was used in an effect size study to ascertain the educational intervention's practical importance. The computation assessed how much the experimental group's final post-test scores ( $M = 6.00$ ) differed from those of the control group ( $M = 4.65$ ). An effect size of  $d = 1.19$  was obtained from the analysis. Standard statistical rules state that a large effect size is indicated by a value larger than 0.80. Consequently, this outcome confirms that the dynamic digital tool is substantially more effective and impactful than conventional teaching methods by showing that the integration of the GeoGebra software had a highly significant and practical impact on improving students' stereometric problem-solving abilities.

The table below displays the mean and standard deviation of the experimental group students' questionnaire replies.

*Table 4.4. Mean and Standard Deviation of the Experimental Group Students'*

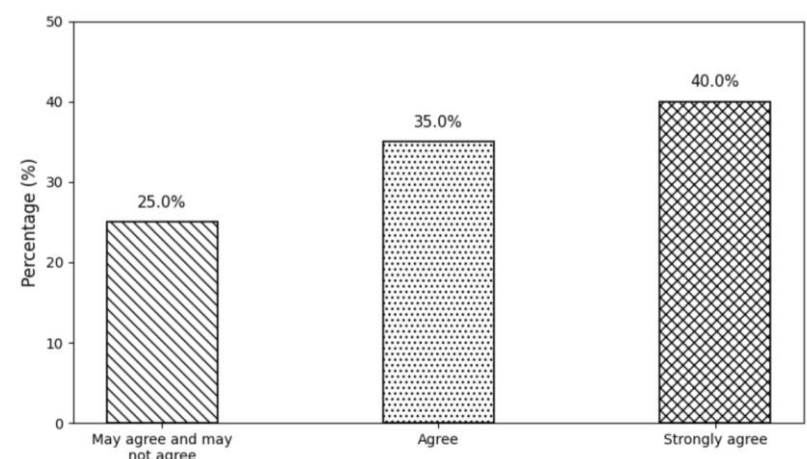
| Item                                                                         | Mean | Standard deviation |
|------------------------------------------------------------------------------|------|--------------------|
| I like to use GeoGebra software                                              | 4.15 | 0.68               |
| GeoGebra software helps to learn Mathematics concepts                        | 4.35 | 0.55               |
| I feel confident when do the activities by using GeoGebra software           | 3.75 | 0.82               |
| I learnt a lot about Mathematics when using GeoGebra software                | 4.10 | 0.65               |
| I can think creatively and critically when using GeoGebra software           | 3.62 | 0.78               |
| I prefer to learn Mathematics with GeoGebra software                         | 3.85 | 0.75               |
| I am excited when asked to explore the GeoGebra software                     | 3.92 | 0.70               |
| GeoGebra software can help to increase my achievement in Mathematics         | 4.28 | 0.60               |
| I am happy if the teacher uses the GeoGebra software in teaching Mathematics | 4.45 | 0.52               |
| Overall                                                                      | 4.05 | 0.67               |

To determine the experimental group students' attitudes towards the use of GeoGebra software in mathematics lessons, a 9-item questionnaire was administered. Responses were evaluated using a 5-point Likert scale (ranging from 1 = Strongly Disagree to 5 = Strongly Agree). The descriptive statistics revealed that the overall attitude towards the software was highly positive, yielding an overall mean score of 4.05 (SD = 0.67). Data analysis indicated that the highest-rated items related to the teacher's integration of the software and its perceived educational effectiveness. Specifically, students expressed the highest level of agreement with Item 9 ("I am happy if the teacher uses the GeoGebra software in teaching Mathematics"; Mean =

4.45, SD = 0.52). This was followed closely by Item 2 ("GeoGebra software helps to learn Mathematics concepts"; Mean = 4.35, SD = 0.55) and Item 8 ("GeoGebra software can help to increase my achievement in Mathematics"; Mean = 4.28, SD = 0.60). The low standard deviations for these items (0.52 – 0.60) suggest a strong consensus among the students regarding the utility of the program. Conversely, items relating to students' self-confidence and higher-order cognitive skills received comparatively lower mean scores. The lowest average was recorded for Item 5 ("I can think creatively and critically when using GeoGebra software"; Mean = 3.62, SD = 0.78). Similarly, students rated their personal confidence when performing activities using the software at a moderate level (Item 3; Mean = 3.75, SD = 0.82). The preference for learning mathematics primarily with GeoGebra (Item 6; Mean = 3.85, SD = 0.75) also showed a moderate result. The higher standard deviations observed in these items (0.75 – 0.82) indicate a greater variance in individual student responses regarding their independent technical proficiency and comfort levels.

The figure below depicts the question "I like to use GeoGebra software" in this questionnaire. The diagram was created using a Python application.

*Figure 4.2. Percentage Distribution of Students' Responses on Enjoyment of Using GeoGebra Software*

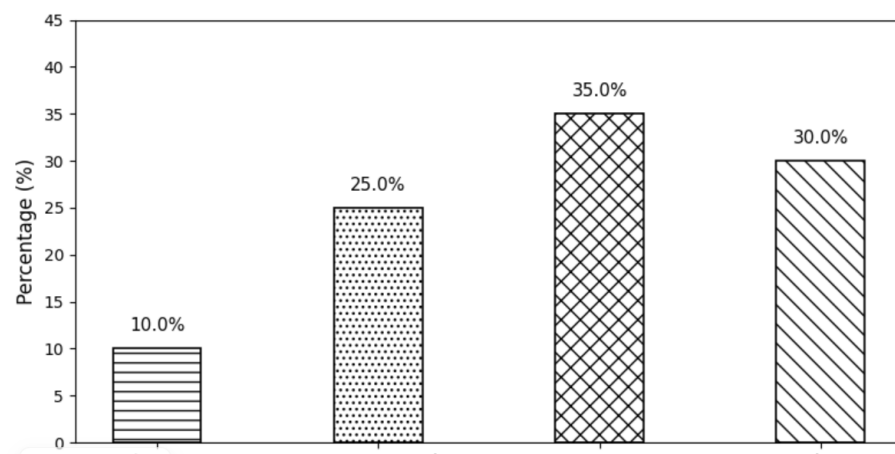


*Source: Generated using Python.*

Figure 4.2. illustrates the percentage distribution of students' responses regarding their general enjoyment of using the GeoGebra software. The findings indicate a predominantly positive reception among the participants. Specifically, the largest segment of the cohort, accounting for 40.0%, "Strongly agreed" that they liked using the software, followed closely by 35.0% who "Agreed." When combined, a significant majority (75.0%) of the students expressed a clearly positive attitude toward the tool. The remaining 25.0% of the participants opted for the neutral response ("May agree and may not agree"), and notably, there were no negative responses (disagreement) recorded. Overall, this data demonstrates that the students generally found the GeoGebra-assisted learning experience to be highly engaging and enjoyable.

The figure below depicts the question "I prefer to learn Mathematics with GeoGebra software" from this poll. The diagram was created using a Python application.

*Figure 4.3. Percentage Distribution of Students' Preference for Learning Mathematics with GeoGebra*



*Source: Generated using Python.*

Figure 4.3. presents the percentage distribution of students' responses regarding their preference for learning mathematics using the GeoGebra software over traditional methods. The data reveals a predominantly favorable inclination, with a combined majority of 65.0% of the participants expressing a clear preference for the software

(35.0% responding "Agree" and 30.0% "Strongly agree"). However, a notable portion of the cohort remained neutral, with 25.0% selecting "May agree and may not agree," indicating some ambivalence or a need for both instructional styles. Additionally, a small minority of 10.0% explicitly "Disagreed," suggesting that a fraction of the students still prefer conventional, non-digital teaching approaches. Overall, while the primary trend leans positively toward GeoGebra-assisted learning, these varied responses realistically reflect the diverse learning preferences and individual comfort levels with technology within the experimental group.

## 5. DISCUSSION AND CONCLUSION

The primary objective of this work was to leverage the methodological characteristics of 3D geometric objects to digitize schooling. Based on various scientists' studies, the GeoGebra tool was chosen for educational digitalization. Statistical investigation (independent t-test) revealed that employing GeoGebra in stereometry classes resulted in considerably higher learning outcomes compared to traditional methods of instruction ( $p < 0.05$ ). Beyond the statistically significant difference between the groups, the calculated effect size of Cohen's  $d = 1.19$  demonstrates a large practical magnitude of the GeoGebra software's effectiveness, clearly indicating that the dynamic visualization approach substantially outperforms conventional teaching methods. This conclusion is similar with Kramenko et al.'s (2020) study, which found that groups utilizing Geogebra dynamic models solved problems considerably better than those using paper-based models. As Vallo et al. (2015) observed, in traditional instruction, students frequently succumb to "visual formalism" and are unable to appropriately interpret spatial figures on the board in 2D. In our study, GeoGebra enables students to convert shapes into 3D format and build spatial imagination. This is directly congruent with actual data, which, as reported by Petrov and Atanasova (2020), indicated that the experimental group utilizing GeoGebra 3D outperformed the traditional group by 29%.

In addition to the increased academic success, the survey findings revealed that students had a very positive attitude toward new digital technologies ( $M=4.05$ ). This statistic is consistent with Arbain and Shukor's (2015) study, which found that students had a positive attitude towards the program. In our study, students highly valued the program's assistance in comprehending mathematical ideas ( $M=4.35$ ). This clearly demonstrates that the GeoGebra tool facilitates the visualization and comprehension of complex mathematical concepts, as Celen (2020) concluded in his studies. Furthermore, as Drijvers and Sinclair (2024) found in their meta-analysis, the use of digital tools, such as dynamic geometry, enhances students' learning outcomes and motivates them to participate in mathematical inquiries rather than repetitive

computations. However, lower questionnaire ratings for cognitive skills ("I can think critically and creatively" –  $M=3.62$ ) and confidence in working independently ( $M=3.75$ ) improve the study's objectivity. Arbain and Shukor (2015) reported a similar tendency, with the creative/critical thinking question receiving the lowest score ( $M=3.93$ ). The explanation for this is also discovered in the investigations of Celen (2020) and Putra et al. (2021): it is difficult for students who have never used the software before to identify the interface and tools, and they rapidly become fatigued at first. Bashirova and Erboluly (2023) propose that a common strategy be established that incorporates conventional and new interactive methodologies. This will result in a much better comprehension of the problem at hand. Although GeoGebra is an effective tool for solving the "visualization crisis" in stereometry, it will not operate without the teacher's methodical pedagogical direction and a solid mathematical basis.

The primary goal of this study was to determine the efficacy of using a dynamic geometry application (Geogebra) in teaching complicated stereometry principles to high school students. The study sought to overcome typical educational challenges by utilizing digital visualization tools to help students build spatial thinking skills and enhance their stereometry performance. The theoretical underpinnings and empirical data obtained were used to successfully meet the study objectives, and the following results were drawn:

First, the quantitative analysis offered substantial statistical support for the research hypothesis. Utilizing GeoGebra for 3D modeling and cross-section analysis resulted in considerably higher posttest scores for the experimental group compared to the control group utilizing traditional methods ( $p<0.05$ ). This demonstrates that dynamic digital environments are extremely successful at bridging the gap between 2D drawings and 3D spatial awareness, as well as actively assisting students in overcoming the "visualization crisis" that is frequently experienced in stereometry.

Second, the qualitative survey findings revealed that students had a highly positive attitude about using digital resources into their courses. Students said that GeoGebra made abstract mathematical ideas more understandable and entertaining

(M=4.05). The high scores on the program's advantages for grasping mathematical concepts and boosting performance clearly show that interactive 3D tools are an effective motivator for today's digital generations.

Third, the study found key educational subtleties about cognitive load and student autonomy. The lower survey ratings on students' critical thinking and self-confidence (3.62 and 3.75, respectively) show that technology is not a cure-all. The first level of understanding complicated digital interfaces necessitates methodical, step-by-step instructions. This supports Vygotsky's idea of "scaffolding": teachers play a critical role in transitioning children from passive observation to active, autonomous mathematical investigation.

Based on the findings of this study on the usefulness of utilizing GeoGebra to teach stereometry, the following practical suggestions are offered to mathematics teachers, school officials, and future researchers:

*Math-related recommendations:*

- Integrate dynamic modeling. Teachers should gradually include GeoGebra 3D into stereometry courses, rather of sticking to static 2D graphics on the board. This alleviates the "visualization crisis" by allowing students to rotate, morph, and cut things in a 3D environment in real time.
- Use scaffolding: Because students lack confidence in their capacity to think critically throughout the curriculum, teachers should employ Vygotsky's scaffolding strategy. Teaching should start with instructor demonstrations and progress to students' own investigation.
- Checking and self-correction: Encourage students to use GeoGebra to examine their pencil-and-paper drawings. This exercise alleviates "geometry fear" and boosts students' confidence by offering quick feedback.

*Recommendations to school administrators and curriculum developers:*

- **Infrastructural support:** Schools should be equipped with the required gadgets and reliable internet connectivity so that students may use features like GeoGebra's "Exam Mode" for objective testing.
- **Professional development:** A focused workshop based on the TPACK model should be planned to improve teachers' digital competency. Teachers should also completely understand the program in order to appropriately oversee students' independent research.
- **Curriculum flexibility:** In line with the "Digital Kazakhstan" program, the Ministry of Education should consider formalizing the inclusion of dynamic geometric systems such as GeoGebra in the national curriculum for high school students.

*Suggestions for future researchers:*

- **Long-term studies:** Future researchers should investigate the long-term impact of GeoGebra on students' spatial concepts across several school years.
- **Scalability:** Such research might be undertaken on broader topics such as mathematical analysis, such as integrals or derivatives, or trigonometry, to examine the adaptability of dynamic visualization tools.
- **Inclusion:** It is worthwhile investigating how GeoGebra might be customized to assist students with special educational needs, such as those who have visual or hearing impairments or who struggle with abstract thinking.

Finally, the findings of this study strongly support the use of mixed learning to teach geometry, including 3D geometric forms. This is because the traditional method used cannot be fully eradicated, since students who study using the traditional way can still make progress. As a result, the traditional method should be reinforced with visualization, namely the dynamic digital tool GeoGebra, and taught in a collaborative manner. To fully realize the potential of digitization in STEM education, curriculum developers and educational institutions should systematically incorporate dynamic geometric programs into standard mathematics programs and provide ongoing technical pedagogical training to teachers.

## 6. LIMITATIONS

Despite the fact that this study produced noteworthy findings proving the usefulness of the GeoGebra 3D program in stereometry instruction, a number of limitations should be noted:

**Sampling Technique and Sample Size:** Only 40 students made up the study's limited sample size. There were sixteen students in the experimental group and sixteen in the control group, and the participants were split equally. Additionally, easy sampling was used to choose the participants.

The research findings cannot be fully extrapolated to other schools or the larger Kazakhstani student population due to the limited sample size and lack of randomized sampling. **Study Scope and Demographics:** Only 11th-grade students from secondary school No. 148 who were specifically enrolled to study physics and mathematics participated in the experiment. As a result, students in different grade levels or those on humanities tracks may have varied results.

**Study Length:** The trial's duration was restricted to about eight weeks. The long-term effects of utilizing the GeoGebra 3D program on pupils' recall of spatial thinking skills are still unknown because this was a brief intervention.

**Technological Skills and Cognitive Load:** The students' confidence and initial learning process were impacted by their lack of prior expertise with the digital program. According to survey data, students rated their confidence in working independently ( $M=3.75$ ) and their capacity for critical and creative thought ( $M=3.62$ ) lower. This can be explained by the fact that students who have never used the software before find it challenging to use the features and interface, which quickly wears them out.

**Data Collection Tools:** Pretests, posttests, and a nine-question survey were used to objectively gather the study data. Open-ended interviews and observational checklists are examples of qualitative data gathering techniques that could have given the study a more in-depth, nuanced understanding of the pupils' unique spatial thinking processes and particular technology challenges.

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## LIST OF APPENDICES

### Appendix A – a pretest of the level tests

#### 1. ПРЕТЕСТ: Дәстүрлі оқыту деңгейін бағалау

**Мақсаты:** Оқушылардың GeoGebra бағдарламасынсыз, дәстүрлі сызба әдістері арқылы кеңістіктік ойлау және есептеу дағдыларын анықтау.

**Тапсырма №1. Сфераның көлемін есептеу:** Радиусы  $r = 1.9$  болатын сфераның бетінің ауданы мен көлемін табыңыз.

- **Сілтеме:** Putra, Z. H., Hermita, N., Alim, J. A., & Hidayat, R. (2021).
- **Негіздеме:** Оқушылардың базалық формулаларды жатқа білу және стандартты алгебралық есептеулерді орындау қабілетін тексеру үшін таңдалды.

**Тапсырма №2. Нүктеден жазықтыққа дейінгі қашықтық:** Қабырғасы  $AB = 1$  болатын кубта  $B$  төбесінен  $AFH$  жазықтығына дейінгі арақашықтықты есептеңіз.

- **Сілтеме:** Vallo, D., Duris, V., & Zahorska, J. (2014).
- **Негіздеме:** 2D сызбада перпендикулярдың табанын визуализациялаудың қиындығын және «визуалды дағдарысты» анықтау мақсатында алынды.

**Тапсырма №3. Куб қимасын салу:** Кубтың  $AE$ ,  $AB$ ,  $BC$  қырларының ортасы болатын  $P$ ,  $Q$ ,  $R$  нүктелері арқылы өтетін қима жазықтығын тұрғызыңыз.

- **Сілтеме:** Vallo, D., Duris, V., & Zahorska, J. (2014).
- **Негіздеме:** Оқушылардың параллель жазықтықтар қасиетін қағаз бетінде қолдану және қиманы тұйықтау дағдыларын бағалау үшін таңдалды.

## Appendix B – a posttest of the level test

### 2. ПОСТТЕСТ: GeoGebra бағдарламасының тиімділігін бағалау

**Мақсаты:** Динамикалық ортада оқушылардың күрделі геометриялық нысандарды манипуляциялау және зерттеу қабілетін тексеру.

**Тапсырма №1. Сфера мен конус көлемдерінің байланысы:** GeoGebra анимациясын қолданып, радиустары мен биіктіктері бірдей 4 конустың көлемі 1 сфераның көлеміне тең екенін дәлелдеңіз.

- **Сілтеме:** Putra, Z. H., Hermita, N., Alim, J. A., & Hidayat, R. (2021).
- **Негіздеме:** Оқушының формуланы механикалық жаттаудан, оның мағынасын визуалды зерттеу (exploratory learning) арқылы түсіну деңгейін анықтайды.

**Тапсырма №2. Куб қимасының ауданын есептеу:** Бірлік кубтың  $A$  төбесі және  $BF$  пен  $DH$  қырларының ортасы арқылы өтетін қиманың ауданын GeoGebra 3D құралдарымен табыңыз.

- **Сілтеме:** Жуманова, Г. Т., Аликова, А. М., & Торогельдиева, К. М. (2022).
- **Негіздеме:** Бағдарламаның қадамдық құру мүмкіндігін және қима пішінін айналдыру арқылы нақты талдау дағдысын тексеру үшін алынды.

**Тапсырма №3. Алтыбұрышты призма қимасы:** Алтыбұрышты призманың  $A_1$ ,  $C_1$  төбелері мен  $BB_1$  қырының ортасы  $S$  нүктесі арқылы өтетін қиманы тұрғызып, оның сипаттамасын беріңіз.

- **Сілтеме:** Sekulić, T., Manigoda, G., & Kostić, V. (2023).
- **Негіздеме:** Оқушының күрделі фигураларды 3D кеңістікте еркін манипуляциялау (dragging) және визуализациялау қабілетін бағалау үшін таңдалды.

## Appendix C – lesson plan

| Бөлім:                                      | Айналу денелері                                                                                                                                                                                                                              |                                                |                            |                    |
|---------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------|----------------------------|--------------------|
| Педагогтің аты-жөні:                        | Маратұлы Қазбек                                                                                                                                                                                                                              |                                                |                            |                    |
| Күні:                                       | 12.03.2026 ж.                                                                                                                                                                                                                                |                                                |                            |                    |
| Сынып: 11                                   | Қатысқандар саны: Қатыспағандар саны:                                                                                                                                                                                                        |                                                |                            |                    |
| Сабақтың тақырыбы:                          | Айналу денелері және олардың элементтері                                                                                                                                                                                                     |                                                |                            |                    |
| Оқу бағдарламасына сәйкес оқыту мақсаттары: | 11.1.1 Айналу денелерінің анықтамасын және түрлерін білу;<br>Айналу денелерінің элементтерін ажырату;<br>Цифрлық ресурстар арқылы айналу денелерінің модельдерін зерттеу.                                                                    |                                                |                            |                    |
| Сабақтың мақсаты:                           | <ul style="list-style-type: none"> <li>- Айналу денелерінің пайда болу жолдарын түсіндіреді;</li> <li>- Цилиндр, конус және сфераның негізгі элементтерін анықтайды;</li> <li>- Геометриялық модельдерді цифрлық ортада талдайды.</li> </ul> |                                                |                            |                    |
| Уақыты/ кезеңдері                           | Педагогтің әрекеті                                                                                                                                                                                                                           | Оқушының әрекеті                               | Бағалау                    | Ресурстар          |
| Ұйымдастыру кезеңі<br>3 мин                 | Сәлемдесу, сабақ мақсатын таныстыру                                                                                                                                                                                                          | Сабаққа дайындалады                            | Ауызша мадақтау            | Презентация        |
| Қызығушылықты ояту<br>5 мин                 | Күнделікті өмірдегі айналу денелерінің суреттерін көрсетеді                                                                                                                                                                                  | Денелерді атайды, пікір білдіреді              | Кері байланыс              | Слайд              |
| Жаңа білімді меңгеру<br>12 мин              | GeoGebra 3D көмегімен айналу денелерінің құрылуын көрсетеді                                                                                                                                                                                  | Модельдерді бақылайды, сұрақтарға жауап береді | ҚБ                         | GeoGebra 3D        |
| Топтық жұмыс<br>10 мин                      | Әр топқа айналу денесінің моделін береді                                                                                                                                                                                                     | Элементтерін анықтап, қорғайды                 | Дескриптор бойынша бағалау | 3D модельдер       |
| Жұптық жұмыс<br>8 мин                       | QR-код арқылы тапсырма ұсынады                                                                                                                                                                                                               | Тапсырманы орындайды                           | Өзара бағалау              | Смартфон, интернет |
| Қорытынды<br>4 мин                          | Негізгі ұғымдарды жинақтайды                                                                                                                                                                                                                 | Қорытынды жасайды                              | ҚБ                         | Слайд              |
| Рефлексия<br>3 мин                          | «Mentimeter» платформасында кері байланыс ұйымдастырады                                                                                                                                                                                      | Пікір қалдырады                                | Өзін-өзі бағалау           | Mentimeter         |