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Numerical estimation of hausdorff dimension of self-similar sets and open set condition

THESIS

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Abstract

This work is based on two of my scientific articles [10],[7]. When writing this work, I singled out the thesis into two sections. In the first section, I perform computer calculations, in the second section, I confirmed the theorem. That is, in the first part of the "Sierpinski triangle and the open set condition" I study the size of the triangle (Sierpinski triangle) in the fractal system, composed by Vaclav Sierpinski, in different conditions. These studies are conducted by two different methods of computer computing. Comparing the results obtained, I conclude on two methods for calculating the size and triangle of the Serpin in the amount of 729 pieces that we consider. That is, I will define both methods as effective and flawed, as well as some uncertainty of the methods and reasons for this in the form of a table. In the second section, working together with the supervisor, I study a closed Cantor set with parameter λ and consider that these fractal sets are completely identical to themselves. In particular, I consider the fractal system E_λ constituting specific lines, the iterative functional system $f_1(x) = x/3$, $f_\lambda(x) = (x+\lambda)/3$ and $f_2(x) = (x+2)/3$ consisting of three functions. That is when passing $1 < \lambda < 2$ as $\lambda = 1 + 3^{-n}$ for an integer n, I determine and prove that the fractal set E_λ is self-similar.

Аңдатпа

Бұл жұмыс өзімнің екі ғылыми [10],[7] мақалам негізінде жазылған. Бұл жұмысты жазуда диссертацияны екі бөлімге бөліп қарастырдым. Бірінші бөлімде компютерлік есептеулер жүргізсем, екінші бөлімде теорема дәлелдейтін боламын. Яғни, «Серпин үшбұрышы және ашық жиын шарты» атты бірінші бөлімде Вацлав Серпинскийдің құрастырған фракталдар жүйесіндегі үшбұрышының (Серпин үшбұрышы) әртүрлі жағдайдағы өлшемін зерттеймін. Ол зерттеулерді екі түрлі әдіспен компьютерлік есептеулер жүргіземін. Шыққан нәтижелерді салыстыра отырып біз қарастырған 729 дана Серпин үшбұрышы мен өлшемін есептеуге арналған екі әдіс жайлы қорытынды шығарамын. Яғни, екі әдістің де тиімді-тиімсіздігін, әрі әдістердің дұрыс анықталмайтын кейбір мәндері мен себебін анықтап оны кесте түрінде көрсетемін. Екінші бөлімде, ғылыми жетекшімен бірге жұмыс жасай отырып λ параметрі бар Кантордың жабық жиынын зерттеймін және бұл фракталды жиындар толығымен өз – өзіне ұқсас болған жағдайларын қарастырмын. Дәлірек айтқанда, нақты сызықтар құрайтын фракталды E_λ жиынын және үш функциядан тұратын $f_1(x) = x/3$, $f_\lambda(x) = (x+\lambda)/3$ және $f_2(x) = (x+2)/3$ итерациялық функционалдық жүйені қарастырамын. Яғни, $1 < \lambda < 2$ аралығында өз – өзіне ұқсастық болатынын және бүтін n саны үшін $\lambda = 1 + 3^{-n}$ түрінде берілген кезде фракталды E_λ жиынының өз – өзіне ұсас екенін анықтап, дәлелдейтін боламын.

Аннотация

Эта работа написана на основе двух моих научных статей [10],[7]. При написании этой работы я выделил диссертацию на два раздела. В первом разделе я выполняю компьютерные расчеты, во втором разделе я подтвердил теорему. То есть, в первой части «Серпинский треугольник и условие открытого набора» изучаю размер треугольника (Серпинского треугольника) в системе фракталов, составленный Вацлавом Серпинским, в различных условиях. Эти исследования проводятся двумя различными методами компьютерных вычислений. Сравнивая полученные результаты, я выводу о двух методах для расчета размера и треугольника Серпина в количестве 729 штук, которые мы рассматриваем. То есть я определю оба метода как эффективные и недостатки, а также некоторую неопределенность методов и причин для этого в виде таблицы. Во втором разделе, работая совместно с научным руководителем, изучаю закрытый набор Кантора с параметром λ и рассматриваю, что эти фрактальные множества полностью идентичны себе. В частности, я рассматриваю фрактальную систему E_λ, составляющую конкретные линии, итерационную функциональную систему $f_1(x) = x/3$, $f_\lambda(x) = (x + \lambda)/3$ и $f_2(x) = (x + 2)/3$, состоящую из трех функций. То есть, при передаче $1 < \lambda < 2$ в виде $\lambda = 1 + 3^{-n}$ для целого числа n, определю и доказываю, что фрактальный набор E_λ представляет автомодельный.

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Contents

1. Introduction

1.1 Motivation

In the context of the development of modern technologies, the use of computer programs has become normal. Even in the field of mathematics with the use of computer programs, new approaches to the development of science are opening up. The transfer of mathematical calculations to computer algorithms simplifies the work of mathematical calculations and efficiently uses time. In addition, an effective method is a computer calculation of the size of geometric systems. One of the widely used approaches to image analysis is fractal analysis. Fractals, by definition of B. Mandelbrot, called set, for which the fractal dimension is more topological. As a rule, such sets have a complex geometric structure, as well as properties of self-similarity. The characteristic reflecting this self-similarity is the fractal dimension. The concept of fractal dimension has already been used in many disciplines, including computer science, for example, for image compression and coding, texture analysis, and document processing. Non-traditional for classical geometry approaches in fractal analysis helps to obtain new data on the sample under study, to analyze its often rather complex, irregular structure — images of pharmacological preparations, tissues of living organisms, faults of geological rocks, etc. As is well known, one of the sources of images with complex structure are dynamic systems. Their phase portraits demonstrate an extraordinary wealth of structures — both fractal and multifractal, and the image of invariant sets of rational plane transformations (Julia sets) can also be attributed to the art of computer graphics. I was interested in the use of computer devices in the conduct of research. Due to the fact that the direction of the study is related to the system of fractals, I considered one function in this area. The choice was a triangle Serpina. In addition, when working with a supervisor, we study a closed

Cantor set with parameter λ. We are interested in finding new values of the parameter λ for the following fractal functions of the set E_λ that we consider.

1.2 Thesis Outline

My work consists of two parts. In the first section, I perform computer calculations, in the second section I prove a theorem.

In particular, in the first part, I study the dimension of the Hausdorff under various conditions in the system of fractals (the Serpinski triangle), composed by Vaclav Serpinsky, using computer calculations. In geometry, fractals are very natural sets that have self-similarity. Certain fractals are more complex in nature and it is an important question to measure the complexity of these objects. One such option is by means of their dimension. Scientists such as A. A. Vinogradova, D. N. Kaliteevsky, talked about getting a dimension from the Sierpinski triangle. Other scientists were able to calculate it's dimension in an equilateral triangle only in a single position. After that "why not bring the sizes of different kinds of serpin triangle?" the question arose. To do this, I first considered the function in the Matlab of such a Sierpinski triangle:

$$
\begin{gathered}
f_1(\alpha_1, \beta_1) = \begin{cases} x(a) = \alpha_1 * x * (a-1) \\ y(a) = \beta_1 * y * (a-1) \end{cases}; \\
f_2(\alpha_2, \beta_2) = \begin{cases} x(a) = \alpha_2 * x * (a-1) + 0.25 \\ y(a) = \beta_2 * y * (a-1) + \frac{\sqrt{3}}{4} \end{cases}; \\
f_3(\alpha_3, \beta_3) = \begin{cases} x(a) = \alpha_3 * x * (a-1) + 0.5 \\ y(a) = \beta_3 * y * (a-1) \end{cases};
\end{gathered}
\tag{1.1}
$$

With this function, I consider 729 species and find the dimensions. On measurements in various systems, including iterative systems P.S. Alexandrov, B.A. Pasynkov[11]. in their works, scientists considered in a generalized state. That is, using the Serpin triangle in the iterative functional system, we calculate the Box dimension of the Sierpinski triangle with the computer program Matlab. When calculating the dimension of the Sierpinski triangle, choose two different methods and make a mathematical analysis. In the first method, we obtain a graphical representation of various Sierpinski triangles in Matlab using an algorithm written on the basis of the Box dimension method, determine the dimensions based on the conditions of the open set in the following method, determine the dimension by calculating the total degree of the function arguments.

That is the theorem of Kenneth Falconer[3], in this method the arguments $\alpha_1 = \beta_1 \equiv \gamma_1$, $\alpha_2 = \beta_2 \equiv \gamma_2$, $\alpha_3 = \beta_3 \equiv \gamma_3$ to the (1.1) – function for

$$\gamma_1{}^x + \gamma_2{}^x + \gamma_3{}^x = 1 \tag{1.2}$$

the value x in the interval $0 < x < 2$ is the dimension in the given (1.1) - function argument and will be equal to the dimension. However, to obtain the correct results by this method, the function of the Serpin triangle must satisfy the open set conditions. One of the most important concepts in this article is the open set condition (OSC).

Definition. The f_i is said to satisfy the OSC if there exists a nonempty open set $V \subset R^n$ such that

$$\bigcup_{i=1}^{m} f_i(V) \subseteq V and f_i(V) \cap f_j(V) = \emptyset for i \neq j. \tag{1.3}$$

We can also calculate Hausdorff and box dimensions using the OSC for self-similar set F. Both methods are implemented in the computer Matlab program. First, in order to determine when it is possible to correctly apply the methods to calculate the dimensions of fractal sets, collecting the dimensions results from 729 different Sierpinski triangles and comparing the results, we determine optimally the inefficiency of both methods, as well as some values and causes that are incorrectly defined, and show it in a table.

In the second section, together with the supervisor, we study a closed Cantor set with parameter λ and classify the situations when these fractal sets are totally self-similar. More precisely, we consider iterated function system consisting of three functions $f_1(x) = x/3$, $f_\lambda(x) = (x + \lambda)/3$ and $f_2(x) = (x + 2)/3$ and the fractal set E_λ it generates in the real line. We define what totally self-similar means and show for $1 < \lambda < 2$ that the fractal set E_λ is totally self-similar if and only if it is in the form $\lambda = 1 + 3^{-n}$ for some positive integer n. We mainly rely on the recent work of Dajani, Kong, and Yao where they consider the analogous problem for $0 < \lambda < 1$.

2. Preliminaries

In some cases it is possible to analytically compute exact Box dimension. One such general case is when the fractal is obtained as the fixed point of certain conformal Iterated Function Systems (IFSs) that satisfies the so called Open Set Condition (OSC). For all these notions are recalled in the next section.

However, in general analytic exact solution for box dimension problem is a very difficult task. In particular, for IFS if the OSC fails, then ethere is no guarantee that the formul will work.

In this paper, we want to consider a special family of fractals, called Sierpenski Traingles. For various parameters the fractal structore changes and so does the box dimension. Our goal is to consider various parameter values that determine the IFS for Sierpenski Triangle and try to estimate their box dimensions. On one hand, we use the formula from the literature that is valid when OSC holds and on the other hand we use computer software to numerically estimate the dimension. Then, we compare the two values to get an idea when the OSC seems to hold true.

First of all, we describe the systems and functions that we use. Familiarize yourself with this system because the function was used through the Iterated function system. An iterated function system (IFS) is a set of abbreviations $\{S_1, S_2, \ldots, S_m\}$, with $m \geq 2$, on a closed subset D of R^n. A nonempty compact subset F of D is an IFS attractor if

$$F = \bigcup_{i=1}^{m} S_i(F) \tag{2.1}$$

Is a method of creating fractals this is an Iterative functional system in mathematics. Fractal-a mathematical set that has the properties of self-identification (object, exactly or approximately coincides with one part, it is a form as a whole one or more parts).

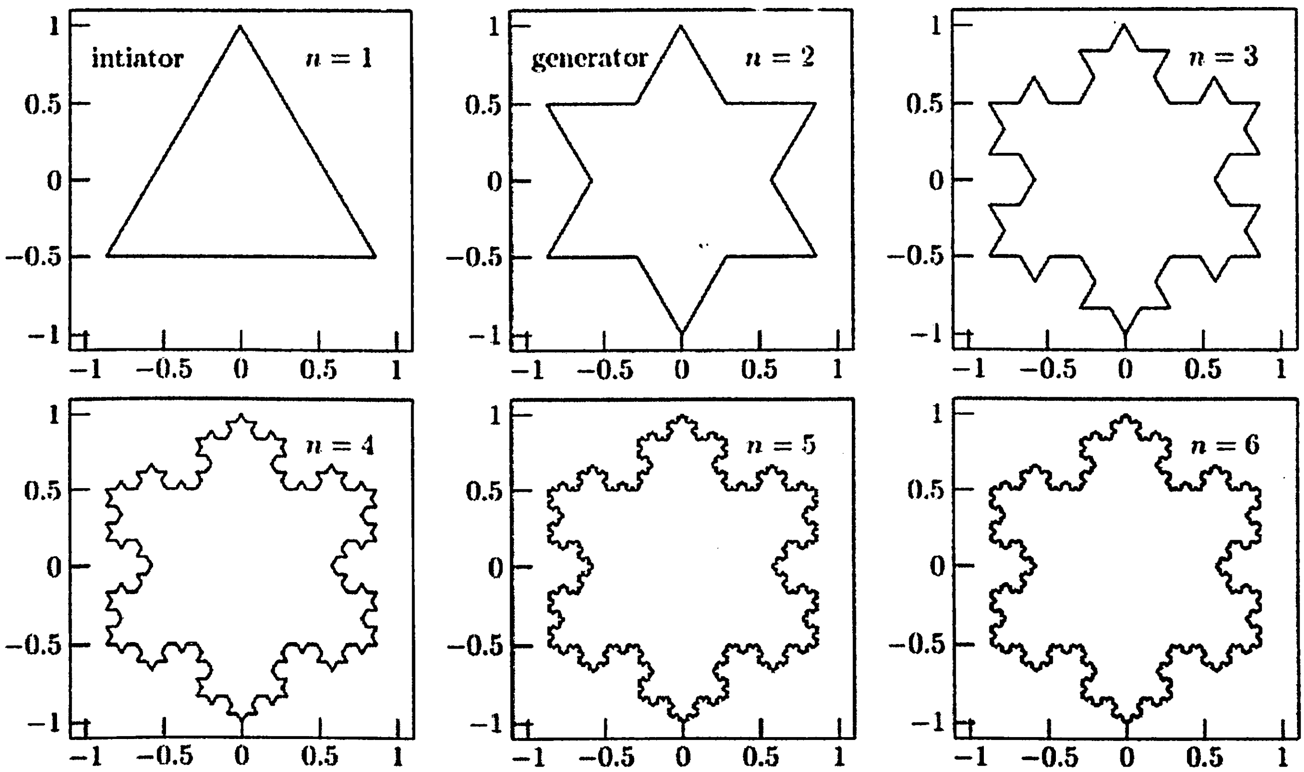

Image 1. (Construction of the Koch fractal)

Fractal (lat. fractus — crushed) is a term introduced by Benoit Mandelbrot in 1975. There is still no strict mathematical definition of fractal sets. His fundamental work Mandelbrot performed in the genre of essays, as if giving readers space for imagination and allowing them to participate in the development of theory and its applications. Mandelbrot's merit is that he was able to generalize and systematize "unpleasant" sets and build a beautiful and intuitive theory. He opened for us the wonderful world of fractals, the beauty and depth of which sometimes amaze, delight scientists, artists, philosophers... the Work of Mandelbrot was stimulated by advanced computer technology, which allowed to generate, visualize and explore a variety of sets. No work on matalam not without beautiful illustrations.

Classification of fractals: Fractals are mainly divided into geometric, algebraic and stochastic. Under certain conditions, stochastic fractals can be called multifractals.

However, there are other classifications:

- Man-made and natural. To man-made are those fractals that were invented by scientists, they at any scale have fractal properties. Natural fractals are limited by the area of existence — that is, the maximum and minimum size at which the object has fractal properties.

- Deterministic (algebraic and geometric) and non-deterministic (stochastic).

Geometric fractals: the History of fractals began with geometric fractals, which were studied by mathematicians in the XIX century. Fractals of this class — the most obvious, because they immediately visible self-similarity. In the two-dimensional case, such fractals can be obtained by setting some polyline, called the generator. In one step of the algorithm, each of the segments that make up a polyline is replaced by a polyline generator at the appropriate scale. As a result of the infinite repetition of this procedure (or rather, in the transition to the limit), a fractal curve is obtained. With the apparent complexity of the resulting curve, its General appearance is given only by the shape of the generator. The geometric fractals also include fractals obtained by similar procedures, for example: the Cantor set, the Serpinsky triangle.

Algebraic fractals: iterations of nonlinear maps given by simple algebraic formulas are used To construct algebraic fractals.

The two-dimensional case is most studied. Nonlinear dynamical systems can have several stable States. Each stable state (attractor) has a certain area of initial States, in which the system will necessarily pass into it. Thus, the phase space is divided into areas of attraction of attractors.

If the phase is a two-dimensional space, then, painting the region of attraction in different colors, you can get a color phase portrait of the system (iterative process). Changing the algorithm for choosing colors can be difficult to obtain fractal patterns with whimsical multicolored patterns. A surprise for mathematicians was the ability to generate very complex non-trivial structures using primitive algorithms.

The construction algorithm is quite simple and is based on an iterative expression:

$$z_i + 1 = F(z_i) \qquad (2.2)$$

where $F(z)$ is a function of a complex variable.

For all points of a rectangular or square area on the complex plane, we calculate a sufficiently large number of times (2.2), each time finding the absolute value of z.

The function we use is a function of the Sierpinski triangle. There are several ways to obtain the function of this Sierpinski triangle. For the first time such concept of a triangle was introduced in 1915 by the Polish mathematician Vaclav Sierpinski. To get it, you need to take an (equilateral) triangle from the inside, hold it along the middle line and throw out the Central of the four formed small triangles. Then the same actions should be repeated with each of the remaining three triangles, etc. The image 2 shows the first four steps, and in the flash demonstration you can practice and take a step to an infinite step.

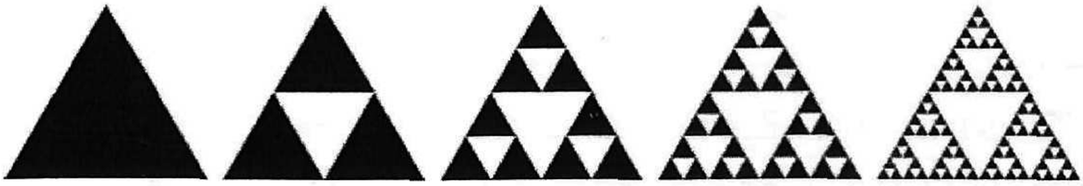

Image 2

Dimension: There are different dimensions for sets. Familiar from a school bench ideas about three-dimensional space, two dimensional planes, one dimensional lines and so have a very superficial and simplified perspective on the diversity that hides in itself the term dimension. Next, we consider rigorous algebraic theories , philosophical and practical concepts of dimension. Often the concepts of dimension are constructed through the discovery of parameters that relate to covering sets. But it's not the only way. Fractional dimensions will also be considered, the practical significance of which was shown by Madelbrot in 1970x.

Dimension strongly depends on how its measure. This means that in addition to the formulas for calculating the dimension, it is necessary to precisely define an operational set of methods for measuring and interpreting the dimension. Traditionally, the number of independent parameters required to set the position of a point in space is associated with the dimension. The position of the point of the plane area bounded by the square can be set by two dimensions, and then its dimension will be equal to two. And it is possible to contrive, and to imagine this area in the form of a broken with very strongly pressed to each other links, folded like a joiner's meter, for example a curve of Peano. Then, to set the position of the point will be enough and one dimension, and the dimension will be equal to one. Next, we will try to bring different dimensions and ways of their measurements and provide information on their practical application.

A topological dimension is a regular geometric dimension. It takes only integer values. In mere phenomena it describes are often (but not always!) the number of degrees of freedom or the number of parameters required to uniquely define any point in the set. The theory of topological dimension is a developed field of mathematics. The strict mathematical definition for metric and topological spaces belongs to Lebesgue and sometimes this kind of dimension is called Lebesgue dimension. Urison and Brauer also contributed. The topological dimension is determined by the inductive method, so it is sometimes called the inductive dimension. A dimension is the distance in one direction, in this case, width. Fractal dimension is a fundamental concept of geometry. It is known that we usually know a straight line or curves is dimension of the 1 - dimensional, surface is 2 - dimensional and figure in planimetries is 3 - dimensional. Dimension — the number of independent parameters required to describe the state of the object, or the number of degrees of freedom of the system.

Hausdorff dimension: This dimension is similar to the Minkowski dimension. The difference is that the balls are taken of arbitrary radius $0 < r < \epsilon$ and the set is not necessarily compact. Subsets of $A \subseteq R^n$ are defined by the formula

$$\lambda_n^*(A) = \inf\left\{\sum_{i=1}^{\infty} \lambda_n(P_i) : A \subseteq \bigcup_i P_i,\ P_i \in \mathbb{P}\right\},$$

where $\mathbb{P}$ is the class of all standard parallelepipeds in R^n (i.e. parallelepipeds with edges parallel to coordinate axes). For this formula to make sense, of course, it is necessary that the Lebegue measure λ_n is already defined on the class of all such parallelepipeds (recall that the measure of a standard parallelepiped $I = I_1 \times \ldots \times I_n$ is equal to the product of the lengths of one-dimensional intervals $I_1, \ldots, I_n$). It is not difficult to modify the definition of an external measure so that it does not rely on the notion of a parallelepiped measure. To do this, we note first that for any $A \subseteq R^n$

$$\lambda_n^*(A) = \inf\left\{\sum_{i=1}^{\infty} \lambda_n(P_i) : A \subseteq \bigcup_i P_i,\ P_i \in Q\right\},$$

where Q is the class of all standard cubes in R^n (i.e. cubes with edges parallel to coordinate axes). This easily follows from the fact that any standard parallelepiped approaches standard parallelepipeds with rational vertices, and the latter is divided into a finite number of standard cubes (spend a neat argument). The advantage of cubes is that the volume of a cube is easily expressed in terms of its diameter.

Definition 1. The diameter of the metric space $(X,\ \rho)$ is the number

$$\operatorname{diam}X \ = \sup\left\{\rho\left(x,y\right) : x,y \in X\right\} \ \in [0,+\infty)\,.$$

By definition, believe $\operatorname{diam}\emptyset \ = 0$.

Note now that if $P \subset R^n$ is a cube with an edge length l, then $\operatorname{diam}P \ = l\sqrt{n}$, so $\lambda_n\left(P\right) \ = \ l^n \ = \ (\operatorname{diamP}\ )^n/n^{n/2}$, and the definition of the external measure takes the form

$$\lambda_n^*\left(A\right) = \frac{1}{n^{n/2}}\inf\left\{\sum_{i=1}^{\infty}(\operatorname{diamP}_i\ )^n : A \subseteq \bigcup_i P_i,\ P_i \in Q\right\}.$$

This formula leads to the following definition.

Definition 2. Let $(X,\ \rho)$ be a metric space, $p \geq 0$ and $\epsilon > 0$. Lie

$$H_\varepsilon^p\left(X\right) = \inf\left\{\sum_{i=1}^{\infty}(\operatorname{diam}A_i\ )^P : X \subseteq \bigcup_i A_i,\ A_i \in X, \operatorname{diam}A_i\ \leq \varepsilon\right\};$$

$$H^p\left(X\right) = \sup_{\varepsilon>0} H_\varepsilon^p\left(X\right)$$

The value of $H^p(X) \in [0,+\infty)$ is called the p - dimensional outer Hausdorff measure of space X.

The Hausdorff dimension is close to the Minkowski dimension. In many cases, these dimensions coincide, although there are sets for which they are different. Therefore, in order to find the size of Hausdorff we have sets and systems, it is enough to calculate the dimension of Minkowski. Therefore, we will now use this method instead of Hausdorff dimension in the dimension calculation. In this case, as stated above, it is mutually equal:

$$\dim_H F \ \equiv \dim_M F\ .$$

When measuring the fractal dimension of various natural and artificial objects, a number of problems arise due to the fact that there are several definitions of fractal dimension. The basic concept is the Hausdorff dimension, but its evaluation is often very difficult. Therefore, in practice, dimensions related to the so-called box-computing (or box-counting) class are more often used.

In this approach, the studied set is covered by cells (box) of the same size δ and the number of elements of the cover $N(\delta)$ is considered. It is assumed that this number is proportional to the cell size to some extent (−d). The relation $logN(\delta)/(-log\delta)$ is considered and its behaviour at scale δ is investigated. If there is a limit to this ratio, it is equal to the number d, which is called the capacitive dimension of the set. For a point, segment, square, etc. this value coincides with a well-known dimension and is an integer. For fractal sets, the capacitive dimension is not equal to an integer. Thus, the main idea of introducing this class of dimensions is the concept of "measurements on a scale δ": for each δ, we measure the object in such a way that we ignore the unevenness of objects smaller than δ and consider these measurements at δ tending to 0.

Consider a nonempty bounded set F in R^n, $\Omega = \{\omega_i\}$ — its finite covering by sets with diameter δ. Denote $N_\delta(F)$ the number of elements of the coating. We define the lower and upper bounds of the capacitive dimension for F:

$$\underline{dim_B F} = \underline{\lim_{\delta\to 0}} \frac{\log_2 N_\delta(F)}{-\log_2 \delta},$$

$$\overline{dim_B F} = \overline{\lim_{\delta\to 0}} \frac{\log_2 N_\delta(F)}{-\log_2 \delta}.$$

If the upper and lower bounds exist and coincide, then their total value is called capacitive dimension:

$$dim_B F = \lim_{\delta\to 0} \frac{\log_2 N_\delta(F)}{-\log_2 \delta}$$

In practice, the calculation of the number of coating elements of a given diameter, in which the image points fall, is not very effective. Therefore, it is advisable to use a different dimension — Minkowski dimension. It coincides with the capacitive one for non-empty bounded sets in R^3. And as these sets for the image, we consider a special construction — δ - parallel body.

Definition 1. Let $F \subset R^n$. Then δ is a parallel body F_δ — a set of points, not more than δ: $F_\delta = \{x \in R^n : |x-y| \leq \delta, y \in F\}$ away from F.

Definition 2. Let $F \subset R^n$, $F \neq \emptyset$. Let F_δ be the δ-parallel body F, and $Vol^n(F_\delta)$ its n-dimensional volume. If for some constant D at $\delta \to 0$ the limit

$Vol^n(F_\delta)/\delta^{n-D}$ is positive and bounded, then the Minkowski set dimension F is the number D. (Denoted by $dim_M F$).

Examples of δ - parallel bodies in R^3:

- F is a one-point set, then F_δ is a ball with volume $Vol^3(F_\delta) = 4\pi\delta^3/3$.
- F is a segment of length L, then F_δ is a cylinder with volume $Vol^3(F_\delta) = \pi L\delta^2$.
- F is a rectangle of area A, then F_δ is a parallelepiped of height 2δ by volume $Vol^3(F_\delta) = 2A\delta$.

In each case the following is true: $Vol^3(F_\delta) \sim \beta\delta^{3-D}$, where D is the fractal dimension F, β is some constant. In the given examples $D = 0$, 1, 2 respectively.

Theorem. Let F be a nonempty bounded set in R . Then $dim_B F = dim_M F$.

Examples:

- The dimension of a finite set is zero, since for it $\rho(n)$ does not exceed the number of elements in it.
- The dimension of the segment is 1, since $[a/\delta]$ of segments of length δ is necessary to cover the segment of length a. Thus,
$$\lim_{\delta\to 0}\frac{\ln(N_\delta)}{-\ln(\delta)} = \lim_{\delta\to 0}\frac{\ln(a)-\ln(\delta)}{-\ln(\delta)} = 1$$
- The dimension of a square is 2, since the number of squares with a diagonal of $1/n$ needed to cover a square with a line a behaves roughly like a^2n^2.
- The dimension of a fractal set can be a fractional number. Thus, the dimension of the Koch curve is $ln4/ln3$.
- The Minkowski dimension of the set {0, 1, 1/2, 1/3, 1/4,...} is equal to 1/2.

Properties:

- The dimension of the Minkowski finite Union of sets is equal to the maskimum of their dimensions. Unlike the Hausdorff dimension, this is not true for a countable Union. For example, the set of rational numbers between 0 and 1 has Minkowski dimension 1, although it is a countable Union of one-space sets (the dimension of each of which is 0). Zamknutaja example of a countable set with zero-dimensional Minkovskogo the above.

- The lower dimension of Minkowski of any set is greater than or equal to its Hausdorff dimension.

- The dimension of Minkowski of any set is equal to the dimension of Minkowski of its closure. Therefore, it makes sense to talk only about the dimensions of Minkowski closed sets.

This dimension is sometimes called Box dimension. At the same time, like the name Minkowski, its structure consists of a cell. There are no differences between them. Only both are alternate names for each other. Consequently, the Box dimension is also defined as the dimension of Minkowski.

The definition of measurement is mainly based on the idea of "measuring sets at scale δ". We can measure every geometric pattern. For each δ, we measure the set in such a way that we detect inhomogeneities of the delta of size δ, and we see how these measurements behave like $\delta \to 0$. Thus, one of the methods of computation of dimension is the Box dimension. Kenneth Falconer[3] fully specified in his work the generalization and calculation of the formula.

When F is a limited subset of R^n, then Box dimension of F is defined as

$$\dim_B F = \lim_{\delta \to 0} \frac{\log N_\delta (F)}{-\log \delta} \tag{2.3}$$

Here, $N(F)$ be the least number of sets of diameter at most δ which can cover F. Based on this formul, we will calculate the dimension of the Serpinski triangle.

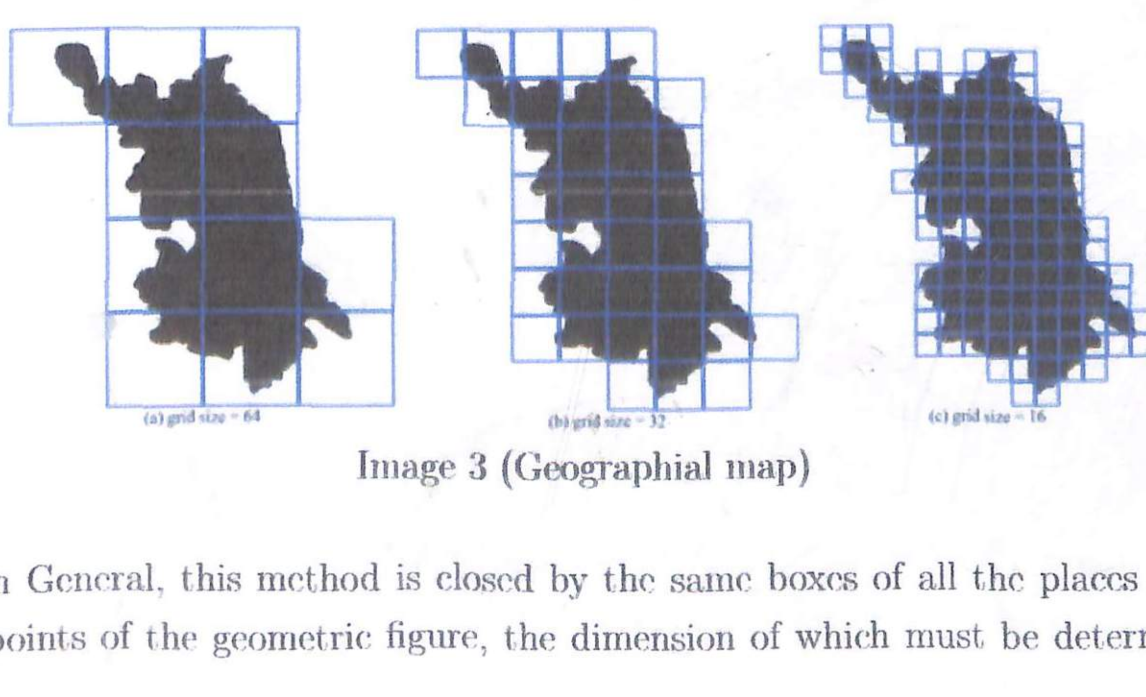

Image 3 (Geographial map)

In General, this method is closed by the same boxes of all the places where the points of the geometric figure, the dimension of which must be determined, lie. It should cover all points of the figure. The smaller the radius of the Box, the greater the number of boxes. The smaller the radius of the Box, the closer to the solution. Therefore, when calculating Box dimension, we tend to zero radius. As a simple example, you can see the image below.

In addition, this method is used in the preparation of medical tests. That is, certain diagnoses associated with the formation of the cerebral cortex are determined.

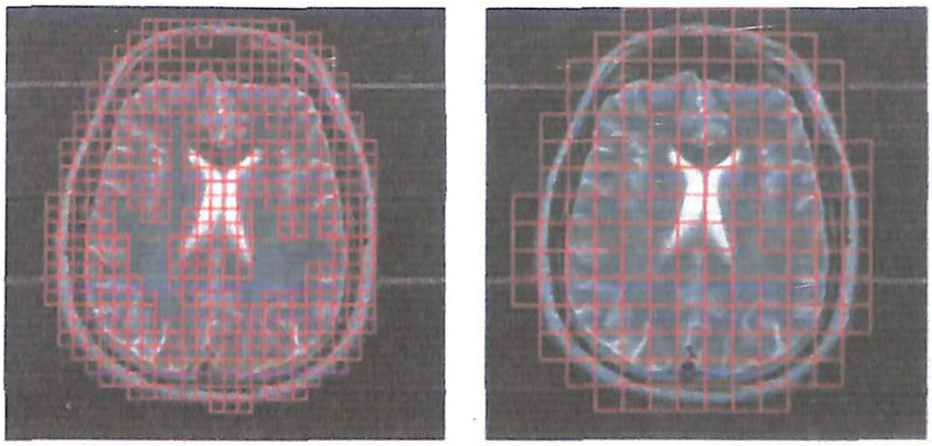

Image 4 (Fractal dimension estimation of brain MR images)

One of the most important concepts in this article is the open set condition (OSC). The f_i are said to satisfy the OSC if there exists a nonempty open set $V \subset R^n$ such that

$$\bigcup_{i=1}^{m} f_i(V) \subseteq V and f_i(V) \cap f_j(V) = \emptyset for i \neq j. \tag{2.4}$$

We can also calculate Hausdorff and box dimensions using the OSC for self-similar set F. For this it is necessary to use the theorem dimensions of self-similar sets in the scientific work[3] of Kenneth Falconer. As shown in the theorem, suppose S_i on R^n with radius $0<r_i<1$ for $1\leqslant i\leqslant m$ is satisfied for similarity the open set condition and if there is

$$F = \bigcup_{i=1}^{m} S_i(F) \tag{2.5}$$

so F is the attractor of the IFS $\{S_1,\ldots,S_m\}$, then the formulation $\dim_B F = s$ is correct. Where s is obtained by the equation

$$\sum_{i=1}^{m} r_i^s = 1 \quad (2.6)$$

at $0 < H^s(F) < \infty$. The proof of this can be seen in this article[3].

According to this theorem, the most effective way to determine the size we need is the Bisection method. Bisection method or the method of dividing a segment in half is the simplest numerical method for solving nonlinear equations of the form $f(x) = 0$. Only the continuity of the function $f(x)$ is assumed. The search is based on the intermediate value theorem.

1. *Method description.* Let it be required to find the root $\overline{x}$ of equation

$$f(x) = 0 \quad (2.7)$$

With the given accuracy $\varepsilon > 0$. The segment of localization $[a; b]$ (i.e. the segment containing only one root $\overline{x}$) will be considered as given. Suppose that the function f is continuous on the segment $[a; b]$ and at its ends takes the values of different signs, i.e.

$$f(a) f(b) < 0 \quad (2.8)$$

"Image 5" shows the case where $f(a) < 0$ and $f(b) > 0$

For further it will be convenient to designate a segment $[a; b]$ through $\left[a^{(0)}, b^{(0)}\right]$ we will Take as approximate value of a root the middle of a segment — a point $x^{(0)} = \left(a^{(0)} + b^{(0)}\right)/2$. As the position of a root $\overline{x}$ on a segment $\left[a^{(0)}, b^{(0)}\right]$ is unknown, it is possible to claim only that the error of this approximation does not exceed a half of length of a segment (Image 5):

$$\left|x^{(0)} - \overline{x}\right| \leq \left(b^{(0)} - a^{(0)}\right)/2$$

To reduce the approximation error possible, stating the period of localization, i.e. replacing the initial cut $\left[a^{(0)}, b^{(0)}\right]$ cut $\left[a^{(1)}, b^{(1)}\right]$of a smaller length. According to the method of bisection (half division) as $\left[a^{(1)}, b^{(1)}\right]$ take one of the segments $\left[a^{(0)}, x^{(0)}\right]$ and $\left[x^{(0)}, b^{(0)}\right]$ at the ends of which the condition $f\left(a^{(1)}\right) f\left(b^{(1)}\right) \leq 0$. This segment contains the desired root. Indeed, if $f\left(a^{(1)}\right) f\left(b^{(1)}\right) < 0$ then the presence of the root follows from theorem in the following; if $f\left(a^{(1)}\right) f\left(b^{(1)}\right) = 0$, then the root is one of the ends of the segment.

Theorem[12]. Let the function f be continuous on the segment $[a, b]$ and take values of different signs on its ends, i.e. $f(a) \bullet f(b) > 0$ Then the segment $[a, b]$

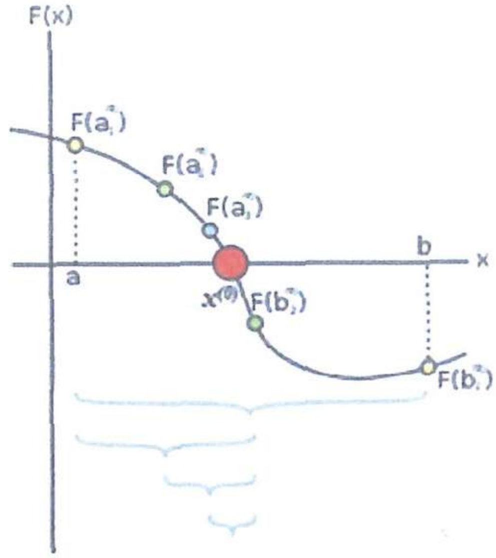

Image 5

contains at least one root of the equation $f(x) = 0$.

The middle of the obtained segment $x^{(1)} = \left(a^{(1)} + b^{(1)}\right)/2$ now gives an approximation to the root, the error estimate of which is

$$\left|x^{(1)} - \overline{x}\right| \le \left(b^{(1)} - a^{(1)}\right)/2 = (b-a)/2^2$$

For the next refinement of the localization segment $\left[a^{(2)}, b^{(2)}\right]$ again take one of the segments $\left[a^{(1)}, x^{(1)}\right]$, $\left[x^{(1)}, b^{(1)}\right]$ at the ends of which the condition $f\left(a^{(2)}\right) f\left(b^{(2)}\right) \le 0$

We describe the next $(n+1)$ iteration of the method. Let the segment $\left[a^{(n)}, b^{(n)}\right]$ already found and calculated values $x^{(n)}$, $f\left(a^{(n)}\right)$, $f\left(b^{(n)}\right)$Then perform the following actions:

1°. Calculated $f\left(x^{(n)}\right)$

2°. If $\left[f\left(a^{(n)}\right) f\left(x^{(n)}\right) \le 0\right]$ then the segment $\left[a^{(n)}, x^{(n)}\right]$ is taken as the localization segment$\left[a^{(n+1)}, b^{(n+1)}\right]$. Otherwise $f\left(x^{(n)}\right) f\left(x^{(n)}\right) < 0$ and $\left[a^{(n+1)}, b^{(n+1)}\right]$ is taken as a segment $\left[x^{(n)}, b^{(n)}\right]$

3°. Calculated $x^{(n+1)} = \left(a^{(n+1)} + b^{(n+1)}\right)/2$

Unlimited continuation of the iterative process gives a sequence of segments $\left[a^{(0)}, b^{(0)}\right]$, $\left[a^{(1)}, b^{(1)}\right], \ldots, \left[a^{(n)}, b^{(n)}\right], \ldots$, containing the desired root. Each of them (except for the initial one) is obtained by dividing the previous segment in half.

2. *Rate of convergence.* The middle n of the segment — point $x^{(n)} = \left(a^{(n)} + b^{(n)}\right)/2$ gives an approximation to the root $\overline{x}$, which has an error esti-

mate

$$\left|x^{(1)} - \overline{x}\right| \leq \left(b^{(n)} - a^{(n)}\right)/2 = (b-a)/2^{n+1} \tag{2.9}$$

From this estimation it is seen that the bisection method converges with the rate of geometric progression, the denominator of which $q = 1/2$ In comparison with other methods, the bisection method converges rather slowly. However, it is very simple and very unassuming; for its application it is enough that the inequality is fulfilled (2.8), the function f is continuous and its sign is correctly determined. In those situations where super high convergence rates are not needed (and this is often the case with simple engineering calculations), this method is very attractive.

Note that the number of iterations required by the bisection method to achieve reasonable accuracy ε cannot be very large. For example, 19 iterations are needed to reduce the initial localization interval by 10^6 times.

3. *The criterion for the end.* Iterations should be carried out until the inequality is satisfied $b^{(n)} - a^{(n)} < 2\varepsilon$ When it is performed due to the evaluation (2.9) can be taken $x^{(n)}$ for approaching the root with accuracy ε.

Example. Find the bisection method with accuracy $\varepsilon = 10^{-2}$ the positive root of the equation $4\left(1 - x^2\right) - e^x = 0$.

In example, this root was localized on the segment $[0, 1]$, and $f(0) > 0,\ f(1) < 0$ let us Assume $a^{(0)} = 0,\ b^{(0)} = 1,\ x^{(0)} = \frac{\left(a^{(0)}+b^{(0)}\right)}{2} = 0,5$.

I iteration. Calculate $f\left(x^{(0)}\right) \approx 1,3512$. $f\left(a^{(0)}\right) f\left(x^{(0)}\right) > 0$ for the next piece of the localization undertaken $\left[a^{(1)}, b^{(1)}\right] = [0,5;\ 1]$ Calculated $x^{(1)} = \left(a^{(1)} + b^{(1)}\right)/2 = 0,75$. .

II iteration. Calculate $f\left(x^{(1)}\right) \approx -0,3670$. As $f\left(a^{(1)}\right) f\left(x^{(1)}\right) < 0$, $\left[a^{(2)}, b^{(2)}\right] = [0,5;\ 0,75]$, $x^{(2)} = \left(a^{(2)} + b^{(2)}\right)/2 = 0,625$.

The results of the next iterations (with four digits after the decimal point) are shown in table.

Table

Iteration number k	$a^{(n)}$	$b^{(n)}$	Sign $f(a^{(n)})$	Sign $f(b^{(n)})$	$x^{(n)}$	$f(x^{(n)})$	$b^{(n)} - a^{(n)}$
0	0.0000	1.0000	+	-	0.5000	1.3513	1.0000
1	0.5000	1.0000	+	-	0.7500	-0.3670	0.5000
2	0.5000	0.7500	+	-	0.6250	0.5693	0.2500
3	0.6250	0.7500	+	-	0.6875	0.1206	0.1250
4	0.6875	0.7500	+	-	0.7187	-0.1182	0.0625
5	0.6875	0.7187	+	-	0.7031	0.0222	0.0312
6	0.7031	0.7187	+	-	0.7109		0.0156

At $n = 6$ we have $b^{(6)} - a^{(6)} \approx 2 \bullet 10^{-2}$ Therefore, the specified accuracy is reached and we can accept $\overline{x} \approx x^{(6)}$ Finally obtain $\overline{x} = 0,71 \pm 0,01$

4. *The influence of computational errors.* When using the bisection method, it is fundamentally important to correctly determine the sign of the function f. In the case where $x^{(n)}$ falls within the uncertainty interval of the root, the sign of the calculated value $f^* \left(x^{(n)}\right)$ does not have to be true, and subsequent iterations do not make sense. However, this method should be considered very reliable; it guarantees an approximation accuracy approximately equal to the radius of the uncertainty interval $\overline{\varepsilon}$.

I use the above definitions and theorems to write the first part, and using the following definitions and theorems, I write the second part.

Here we study a closed Cantor set with parameter λ and I consider these fractal sets if they are completely identical to themselves. Thus, when considering the parameter λ within $1 < \lambda < 2$. I am looking for, based on the work provided within $0 < \lambda < 1$. So, Dajani K., Kong D., Yao Y.[2] some definitions and theorems that were used in the works had to be used.

$\lambda \in (0, 1)$, the λ - Cantor set E_λ is the self-similar set generated by the iterated function system

$$f_d(x) := \frac{x+d}{3}, d \in \Omega_\lambda := \{0, \lambda, 2\}. \tag{2.10}$$

Then E_λ is the unique non-empty compact set in R satisfying $E_\lambda = \bigcup_{d\in\Omega_\lambda} f_d\,(E)$. Since $\lambda \in (0,\ 1)$, one can see that $f_0(I) \cap f(I) \neq \emptyset$, where $I := [0,\ 1]$ is the convex hull of E_λ. So E_λ is a self-similar set with overlaps.

It follows from (1.1) that for any $x \in E_\lambda$ there exists an infinite sequence (d_i) over the set Ω_λ such that

$$x = \lim_{n\to\infty} f_{d_1\ldots d_n}(0) \ = \sum_{i=1}^{\infty} \frac{d_i}{3^i} =: ((d_i))_3$$

where $f_{d_1\ldots d_n} := f_{d_1} \circ \cdots \circ f_{d_n}$ denotes the composition of $f_{d_1}, \ldots, f_{d_n}$. The infinite sequence (d_i) is called a coding of x with respect to the digit set Ω_λ. Since $\lambda \in (0,\ 1)$, a point in E_λ may have multiple codings.

Definition. E_λ is *totally self-similar* if
$f_i\,(E_\lambda) = f_i\,(I) \cap E_\lambda$ for any $i \in d \in \Omega_\lambda^*$.
Our first result describes when E_λ is totally self-similar.

Theorem. Let $\lambda \in (0,\ 1)$. Then E_λ is totally self-similar if and only if $\lambda = 1 - 3^{-m}$ for some positive integer m.

When E_λ is totally self-similar Let $\lambda \in (0,\ 1)$. Recall that $I = [0, 1]$ is the convex hull of E_λ. Set $I_0 = I$, and for $n \geq 1$, let

$$I_n \bigcup_{i\in\Omega_\lambda^n} f_i\,(I).$$

Then the sequence of sets (I_n) decreases to E_λ, i.e.,
$I_0 \supseteq I_1 \supseteq I_2 \supseteq \ldots$, and $\bigcap_{n=0}^{\infty} I_n = E_\lambda$.

The set I_n is called the n - level basic set, and each subset $f_i\,(I)$ with $i \in \Omega_\lambda^n$ is called an n - level basic interval. By a hole of E_λ we mean a connected component in $I \backslash E_\lambda$. Let $H := I \backslash I_1 = (\frac{1+\lambda}{3}, \frac{2}{3})$. Then H is obviously a hole of E_λ.

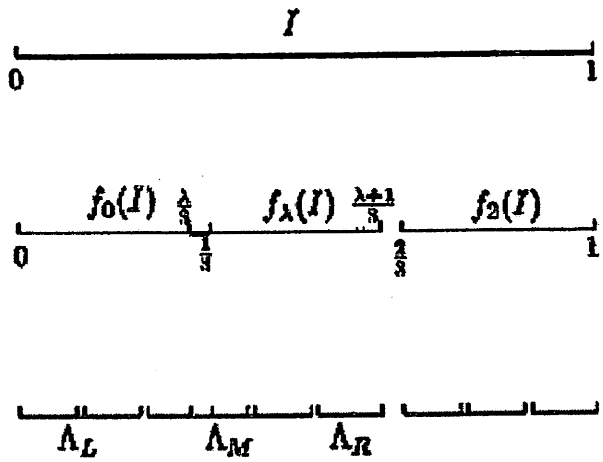

Image 6

Set $H_0 := H$, and for $n \geq 1$, let

$$H_n = \bigcup_{i \in \Omega_\lambda^n} f_i(H).$$

In general, $f_i(H)$ is not necessarily a hole of E_λ. For example, we can easily prove that for $\lambda = \frac{1}{3}$ the set $f_{\frac{1}{3}}(H)$ is not a hole of $E_{\frac{1}{3}}$. This is because $\frac{5}{9} \in H$ and $f_{\frac{1}{3}}\left(\frac{5}{9}\right) = f_{022}(0) \in E_{\frac{1}{3}}$. However, when E_λ is totally self-similar we show that each $f_i(H)$ is indeed a hole of E_λ.

Proposition. The following statements are equivalent.

(i) The set E_λ is totally self - similar.

(ii) For any two finite words i, j we have

$$f_i(E_\lambda) \cap f_j(E_\lambda) = f_i(I) \cap f_j(E_\lambda) = f_i(E_\lambda) \cap f_j(I) = f_i(I) \cap f_j(I) \cap E_\lambda.$$

(iii) $H_n \cap E_\lambda = \emptyset$ for any $n \geq 0$.

(iv) For any two finite words i, j of the same length, we have either $f_i = f_j$ or $f_i(I) \cap f_j(H) = \emptyset$.

(v) $H_n \cap I_{n+1} = \emptyset$ for any $n \geq 0$.

A fractoid is an algebraic structure involving fractal algebra over a self-similar set. Property – "similarity" is used as a determinant in many works on fractals. The classical definition of a fractal is given by Mandelbrot on the basis of self-similarity and fractal fractional dimension at the end of 1970'x : part of the fractal structure is similar to the whole, it does not matter how the partition of a small part is made, the part contains no less details than the whole.

The definition of a self-similar set, by means of a similarity transformation system S, allows us to introduce fractal algebra operators used to construct mathematical objects.

Definition. A compact, perfect and completely discontinuous set C is called self-similar if there is a system of similarity transformation S_n similarity coefficient r_n such that $C = S_1(C), S_2(C), \ldots, S_N(C)$, and $\dim_M(C) = d$, where d is the only solution of the equation, called the similarity dimension, ${r_1}^d + {r_2}^d + \ldots + {r_N}^d = 1$, where r_n – similarity coefficients of transformations $S_n(C)$, lying in the interval [0,1]. $\dim_M(C)$ is the dimension of the Minkowski set C. If the similarity coefficients are equal $r_n = r$, $n = 1,2,\ldots,N$, then the dimension of the Minkowski d of the set C is determined from the equation: $Nr^d = 1$.

I'm making a topological connection to link these concepts. That is, we will use these theorems for further proofs, relating to the Topological conjugacy.

Topological conjugacy: f and g are iterated functions, and exists an $\exists h$ such that

$$g = h^{-1} \circ f \circ h$$

so that f and g are topologically conjugate.

Then of course one must have

$$g^n = h^{-1} \circ f^n \circ h$$

and so the iterated systems are conjugate as well.

Example: In our case: $g = f_d$, $f = f_{2-d}$, $h = \phi$.

$$f_{02}(x) = f_0 \circ f_2(x) = f_0\left(\phi^{-1} \circ f_0 \circ \phi(x)\right) =$$

$$= \phi^{-1} \circ f_2 \circ \phi(x)\left(\phi^{-1} \circ f_0 \circ \phi(x)\right) = \phi^{-1} \circ f_2 \circ f_0 \circ \phi(x) = \phi^{-1} \circ f_{20} \circ \phi(x).$$

Using these materials, when submitting in section 2 of the form $\lambda = 1 + 3^{-n}$ for an integer n, we determine and prove that the fractal set E_λ is a propensity.

3. PART 1

3.1 Sierpinski triangle and open set condition

Let's take its graphical image using the Sierpinski triangle function, using the version of Paulo Silva[8] in Matlab For $a = \overline{2:N}$, N=10 000, $\alpha_i, \beta_i \epsilon (0;1)$, $i = \overline{1;3}$ Sierpinski's triangle depends on the arguments α_i and β_i:

$$f_1(\alpha_1, \beta_1) = \begin{cases} x(a) = \alpha_1 * x * (a-1) \\ y(a) = \beta_1 * y * (a-1) \end{cases};$$

$$f_2(\alpha_2, \beta_2) = \begin{cases} x(a) = \alpha_2 * x * (a-1) + 0.25 \\ y(a) = \beta_2 * y * (a-1) + \frac{\sqrt{3}}{4} \end{cases}; \tag{3.1}$$

$$f_3(\alpha_3, \beta_3) = \begin{cases} x(a) = \alpha_3 * x * (a-1) + 0.5 \\ y(a) = \beta_3 * y * (a-1) \end{cases}.$$

Here and consider the arguments (0;1) in the interval. Moving on step +0.1 each you can extract the image of Matlab (729 pieces). Now use the Matlab code to make different images for the arguments of different serpin Triangles.

First, enter the standard Matlab code:

clf

hold on

It is known that the Computer cannot perform an infinite operation. Therefore, we introduce a constant large value:

N=10000;

x=zeros(1,N);y=x; //our function used is performed in the plane

for a=2:N

c=randi([0 2]);

Let's introduce the function of the Sierpinski triangle in the plane:

switch c

case 0 //first part of the function

x(a)=argument_1*x(a-1); //enter the value of the first argument as a constant number

y(a)=argument_1*y(a-1); //enter the value of the first argument as a constant number

case 1 //second part of function

x(a)=argument_2*x(a-1)+.25; //enter the value of the second argument as a constant number

y(a)=argument_2*y(a-1)+sqrt(??)/4; //enter the value of the second argument as a constant number

case 2 //the third part of the function

x(a)=argument_3*x(a-1)+.5; //enter the value of the third argument as a constant number

y(a)=argument_3*y(a-1); //enter the value of the third argument as a constant number

end //end function input

end //end all inputs

plot(x,y,'.') //we take as a graphical representation of our function on the plane and save separately for use in determining the size of the resulting graph
As an example, we obtain a graphical image in the plane of the Serpin triangle with the arguments 0.5, 0.5 and 0.5:

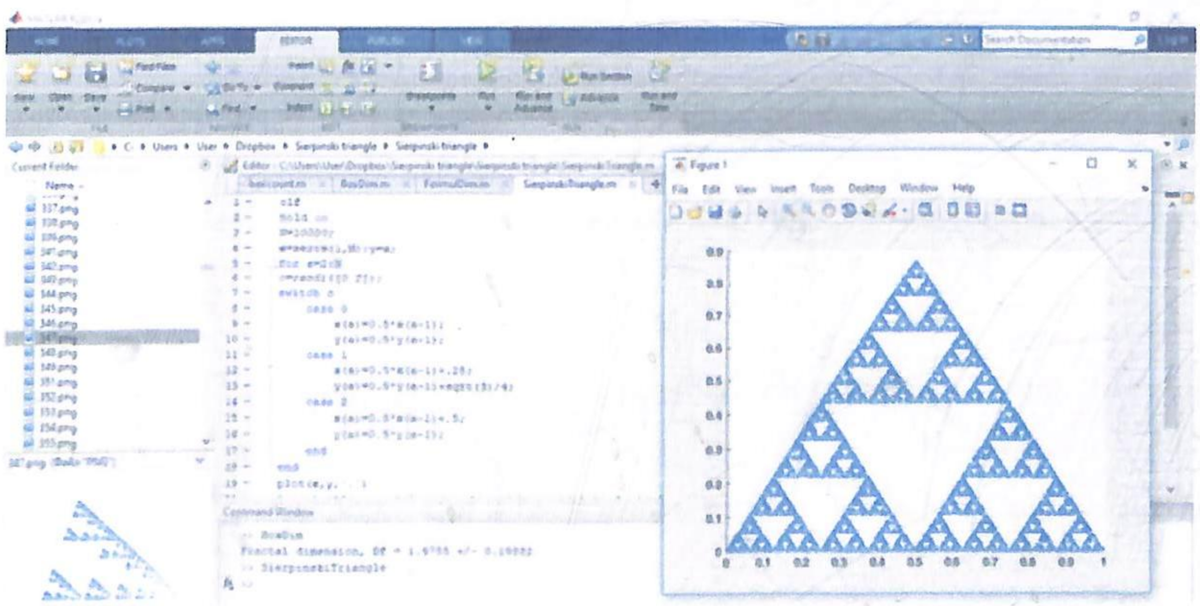

Image 7

These collected graphs can be used to study this function. Of these, choose a random three images (image 8-10):

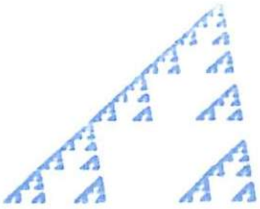

Image 8
$\alpha_1 = \beta_1 = 0.4$
$\alpha_1 = \beta_1 = 0.6$
$\alpha_1 = \beta_1 = 0.3$

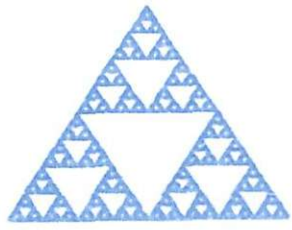

Image 9
$\alpha_1 = \beta_1 = 0.5$
$\alpha_1 = \beta_1 = 0.5$
$\alpha_1 = \beta_1 = 0.5$

Image 10
$\alpha_1 = \beta_1 = 0.7$
$\alpha_1 = \beta_1 = 0.8$
$\alpha_1 = \beta_1 = 0.3$

Find these images by the Box dimension method, i.e. with the formula (2.3) – the dimension of the Serpinski triangle. To do this, we calculate the dimension using the Matlab code[5] that was written by Frederic Moisy in this way, i.e. Box dimension. Thus, the dimension of the Serpinski triangle can be found in the Matlab with a very small error accuracy (for example: α_1=β_1=0.6, α_2=β_2=0.4, α_3=β_3=0.5 $\Rightarrow$ $\dim_B F$ =1.4534 ± 0.18967). We work using the file «boxcount.m»[5] when using this code. Now write the structure using this code:
c=imread('555.png'); //first we enter one picture to use the image we collected. As an example, we consider a Serpin triangle with the values of the arguments 0.5, 0.5, 0.5.
image(c)
colormap gray
axis image
figure(1) //we get a graphical report to see that the operations are performed correctly

i = c(1:size(c), 1:size(c)); //we choose in what interval we consider this picture
bi = (i<100);
imagesc(~bi) //we replace the whole image with one black colour to use it to count the picture
colormap gray
axis image

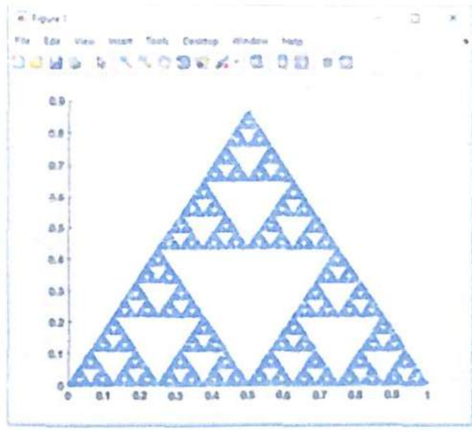

Image 11

figure(2) //we get a graphical report to see that the operations are performed correctly

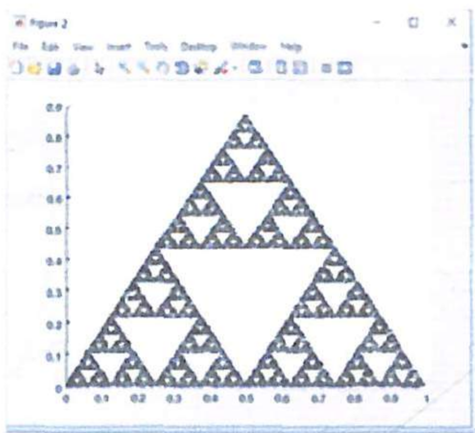

Image 12

[n,r] = boxcount(bi,'slope'); //Run the code «boxcount.m»
figure(3) //we get a graphical report to view the solution interval

df = -diff(log(n))./diff(log(r)); //We introduce the formula for finding the Minkowski dimension (Box dimension). The required elements here are obtained from the file «boxcount.m»
disp(['Fractal dimension, Df = ' num2str(mean(df(4:8))) ' +/- ' num2str(st
//Thus, we calculate the size of the picture on the computer and display the response on the monitor. In our case, the size was equal to «1.4785 ± 0.19882»

This is how we study all the collected 729 paintings. We can fully preserve the outgoing answers, i.e. the size of some reasoned function.

If for a set of points functions performed open set condition, then the following

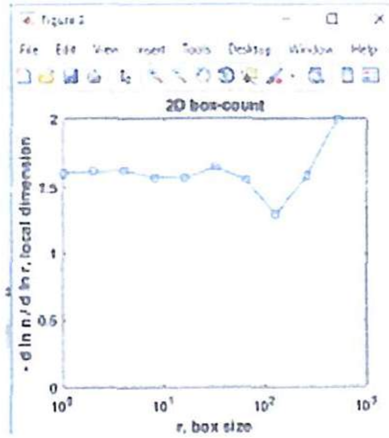

Image 13

```
Command Window
>> BoxDim
Fractal dimension, Df = 1.4785 +/- 0.19882
fx >>
```

Image 14

method to calculate dimension-the method of Bisection. (2.3) - use the formula. In this method the arguments $\alpha_1 = \beta_1 \equiv \gamma_1$, $\alpha_2 = \beta_2 \equiv \gamma_2$, $\alpha_3 = \beta_3 \equiv \gamma_3$ to the (3.1) – function for

$$\gamma_1{}^x + \gamma_2{}^x + \gamma_3{}^x = 1 \tag{3.2}$$

the value x in the interval $0 < x < 2$ is the dimension in the given (3.1) - function argument and will be equal to the Box dimension. And in order to find the value of x, we use the Bisection method to solve equation (3.2). As mentioned above, based on the Bisection method we will look for solutions with a computer program. This method searches for a value by selecting from the given interval [a; b] to find the value of x in the Matlab. To solve the Brato Chakrabarti's Matlab algorithm[13] using the Bisection method, we introduce the function "f=@(x)-1+1Λx+2Λx+3Λx" looking for x with the interval [a; b]=[0; 20]. Since we determine the size of the shape on the plane, the solution would be in search of 0 to 2. But to see some erroneous values, we will also look at a large interval. Where x is the dimension of the Sierpinski triangle in $\gamma_1, \gamma_2, \gamma_3 \epsilon (0; 1)$ values. This method can be written with different algorithms in any computer program or in the Matlab program we used. But the structure of all is similar, the meaning is the same. Now let's write the structure of the same code we used: s=**0.7**; //enter the value of the first argument

```
t=0.8; //enter the value of the second argument
k=0.9; //enter the value of the third argument
f=@(x) -1+s∧x+t∧x+k∧x; //we enter the function we use. As shown above, the sum of the total power arguments is 1
a=0; //starting search intervals
b=20; //limit the search interval
if f(a)*f(b)>0 //check the correctness of the selected interval
disp('Wrong value selected') //if you are looking for a solution in the wrong interval, you will see a command to fix it
else //when the correct interval is selected, the following algorithm works. This algorithm is based on the Bisection method
p = (a + b)/2; //starting the cycle
err = abs(f(p));
while err > 1e-7
if f(a)*f(p)<0
b = p;
else
a = p;
end //stop the loop when finding a solution
p = (a + b)/2;
err = abs(f(p));
end //end of the cycle
end //end of the algorithm
p //our search solution «p» displays
```

For example, consider the Serpin triangle arguments whose values are 0.7, 0.8, and 0.9 (image 15). In this case, the value «5.4651» is displayed as the response: When using this method, the recount is not required to be replaced by similar values. As it is known from algebra, when calculating the same value comes out. Thus, we study all those 729 function arguments that we consider. We can fully preserve the original answers, that is, the dimensions along with the function arguments.

With the above two ways you can get the dimension of all the Sierpinski triangles. We consider the coefficients of the (3.1) function as $\alpha_1 = \beta_1$, $\alpha_2 = \beta_2$, $\alpha_3 = \beta_3$ identical. Compared to these initial results (table 1-9), we see that the

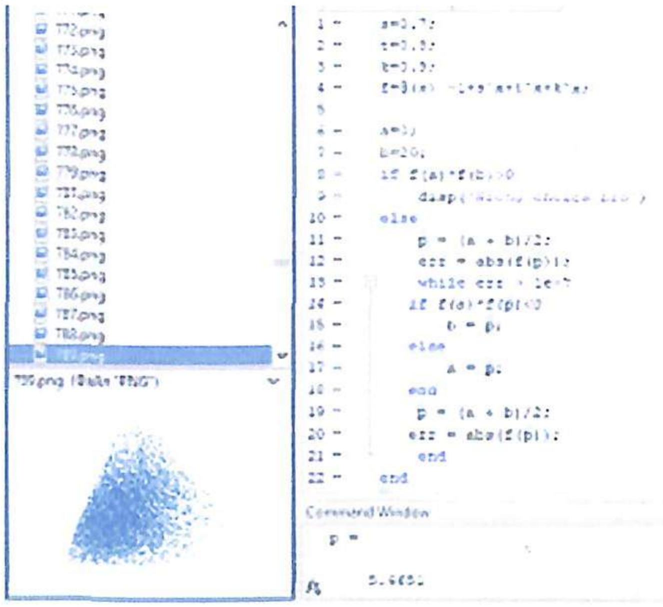

Image 15

dimensions occurring by the two methods have mutually incompatible values. Of course, the size of one graphic image can not be of two kinds. The results of the Box Dimension obtained by the method obtained using the figure are absolutely correct. However, some values obtained by Bisection method which were easy and efficient to calculate are false. Because, as we said above, it is necessary to fulfill the open set condition (OSC) for the correct execution of Formula (2.6). For a function (3.1), you can view the following tables, which don't always meet the conditions.

3.2 Results

Summarizing this, we can analyze the dimensions of the Sierpinski triangle obtained by two methods. First of all, we analyze which methods are used correctly and which methods are effective. Let's group the results and highlight:

- Green number – the dimensions obtained by the Bisection method and the $\dim_B F$ $\pm$ approximation error obtained by the image are equal to each other,
- Yellow number – the dimensions obtained by the Bisection method and the $\dim_B F$ $\pm$ approximation error obtained by the image are not equal to each other,

- Red number – the values of the invalid dimension obtained through the Bisection method, which were greater than two.

It is known that the dimension of the Sierpinski triangle should not exceed two, since the image of the function that we study is in the plane (2D). Solutions that have more than two values are a measure of a shape in space, not in a plane. The table shows the values of numbers obtained through the Bisection method: $\alpha_i = \beta_i \equiv \gamma_i,\ i = \overline{1;3},\ \gamma_i \epsilon (0;1)$

Table 1: $\gamma_1 = 0.1$, γ_2–vertically, γ_3–horizontally

$\gamma_2\backslash_3$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	0.4771	0.5381	0.5931	0.6493	0.7110	0.7830	0.8734	1.0000	1.2197
0.2	0.5381	0.6071	0.6702	0.7353	0.8073	0.8922	1.0000	1.1527	1.4223
0.3	0.5931	0.6702	0.7418	0.8165	0.9002	1.0000	1.1285	1.3139	1.6499
0.4	0.6493	0.7353	0.8165	0.9024	1.0000	1.1181	1.2728	1.5003	1.9239
0.5	0.7110	0.8073	0.9002	1.0000	1.1151	1.2569	1.4458	1.7295	2.2699
0.6	0.7830	0.8922	1.0000	1.1181	1.2569	1.4309	1.6672	2.0288	2.7292
0.7	0.8734	1.0000	1.1285	1.2728	1.4458	1.6672	1.9733	2.4491	3.3818
0.8	1.0000	1.1527	1.3139	1.5003	1.7295	2.0288	2.4491	3.1098	4.4246
0.9	1.2197	1.4223	1.6499	1.9239	2.2699	2.7292	3.3818	4.4246	6.5788

Table 2: $\gamma_1 = 0.2$, γ_2–vertically, γ_3–horizontally

$\gamma_2\backslash_3$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	0.5381	0.6071	0.6702	0.7353	0.8073	0.8922	1.0000	1.1527	1.4223
0.2	0.6071	0.6826	0.7524	0.8248	0.9051	1.0000	1.1205	1.2912	1.5915
0.3	0.6702	0.7524	0.8294	0.9099	1.0000	1.1072	1.2445	1.4406	1.7903
0.4	0.7353	0.8248	0.9099	1.0000	1.1019	1.2245	1.3833	1.6138	2.0342
0.5	0.8073	0.9051	1.0000	1.1019	1.2187	1.3612	1.5489	1.8267	2.3484
0.6	0.8922	1.0000	1.1072	1.2245	1.3612	1.5310	1.7592	2.1054	2.7763
0.7	1.0000	1.1205	1.2445	1.3833	1.5489	1.7592	2.0490	2.5009	3.4027
0.8	1.1527	1.2912	1.4406	1.6138	1.8267	2.1054	2.5009	3.1352	4.4298
0.9	1.4223	1.5915	1.7903	2.0342	2.3484	2.7763	3.4027	4.4298	6.5791

Table 3: $\gamma_1 = 0.3$, γ_2−vertically, γ_3−horizontally

$\gamma_{2\backslash 3}$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	0.5931	0.6702	0.7418	0.8165	0.9002	1.0000	1.1285	1.3139	1.6499
0.2	0.6702	0.7524	0.8294	0.9099	1.0000	1.1072	1.2445	1.4406	1.7903
0.3	0.7418	0.8294	0.9125	1.0000	1.0984	1.2159	1.3667	1.5826	1.9674
0.4	0.8165	0.9099	1.0000	1.0959	1.2046	1.3354	1.5046	1.7490	2.1899
0.5	0.9002	1.0000	1.0984	1.2046	1.3264	1.4747	1.6689	1.9536	2.4792
0.6	1.0000	1.1072	1.2159	1.3354	1.4747	1.6469	1.8764	2.2203	2.8763
0.7	1.1285	1.2445	1.3667	1.5046	1.6689	1.8764	2.1597	2.5972	3.4658
0.8	1.3139	1.4406	1.5826	1.7490	1.9536	2.2203	2.5972	3.2022	4.4558
0.9	1.6499	1.7903	1.9674	2.1899	2.4792	2.8763	3.4658	4.4558	6.5791

Table 4: $\gamma_1 = 0.4$, γ_2−vertically, γ_3−horizontally

$\gamma_{2\backslash 3}$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	0.6493	0.7353	0.8165	0.9024	1.0000	1.1181	1.2728	1.5003	1.9239
0.2	0.7353	0.8248	0.9099	1.0000	1.1019	1.2245	1.3833	1.6138	2.0342
0.3	0.8165	0.9099	1.0000	1.0959	1.2046	1.3354	1.5046	1.7490	2.1899
0.4	0.9024	1.0000	1.0959	1.1990	1.3166	1.4589	1.6436	1.9111	2.3943
0.5	1.0000	1.1019	1.2046	1.3166	1.4459	1.6035	1.8100	2.1118	2.6645
0.6	1.1181	1.2245	1.3354	1.4589	1.6035	1.7823	2.0199	2.3733	3.0369
0.7	1.2728	1.3833	1.5046	1.6436	1.8100	2.0199	2.3049	2.7404	3.5912
0.8	1.5003	1.6138	1.7490	1.9111	2.1118	2.3733	2.7404	3.3245	4.5308
0.9	1.9239	2.0342	2.1899	2.3943	2.6645	3.0369	3.5912	4.5308	6.6012

Table 5: $\gamma_1 = 0.5$, γ_2−vertically, γ_3−horizontally

$\gamma_{2\backslash 3}$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	0.7110	0.8073	0.9002	1.0000	1.1151	1.2569	1.4458	1.7295	2.2699
0.2	0.8073	0.9051	1.0000	1.1019	1.2187	1.3612	1.5489	1.8267	2.3484
0.3	0.9002	1.0000	1.0984	1.2046	1.3264	1.4747	1.6689	1.9536	2.4792
0.4	1.0000	1.1019	1.2046	1.3166	1.4459	1.6035	1.8100	2.1118	2.6645
0.5	1.1151	1.2187	1.3264	1.4459	1.5850	1.7559	1.9809	2.3113	2.9178
0.6	1.2569	1.3612	1.4747	1.6035	1.7559	1.9454	2.1978	2.5727	3.2716
0.7	1.4458	1.5489	1.6689	1.8100	1.9809	2.1978	2.4921	2.9389	3.7987
0.8	1.7295	1.8267	1.9536	2.1118	2.3113	2.5727	2.9389	3.5162	4.6899
0.9	2.2699	2.3484	2.4792	2.6645	2.9178	3.2716	3.7987	4.6899	6.6723

Table 6: $\gamma_1 = 0.6$, γ_2—vertically, γ_3—horizontally

$\gamma_{2\backslash 3}$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	0.7830	0.8922	1.0000	1.1181	1.2569	1.4309	1.6672	2.0288	2.7292
0.2	0.8922	1.0000	1.1072	1.2245	1.3612	1.5310	1.7592	2.1054	2.7763
0.3	1.0000	1.1072	1.2159	1.3354	1.4747	1.6469	1.8764	2.2203	2.8763
0.4	1.1181	1.2245	1.3354	1.4589	1.6035	1.7823	2.0199	2.3733	3.0369
0.5	1.2569	1.3612	1.4747	1.6035	1.7559	1.9454	2.1978	2.5727	3.2716
0.6	1.4309	1.5310	1.6469	1.7823	1.9454	2.1507	2.4263	2.8383	3.6098
0.7	1.6672	1.7592	1.8764	2.0199	2.1978	2.4263	2.7383	3.2126	4.1190
0.8	2.0288	2.1054	2.2203	2.3733	2.5727	2.8383	3.2126	3.8005	4.9768
0.9	2.7292	2.7763	2.8763	3.0369	3.2716	3.6098	4.1190	4.9768	6.8675

Table 7: $\gamma_1 = 0.7$, γ_2—vertically, γ_3—horizontally

$\gamma_{2\backslash 3}$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	0.8734	1.0000	1.1285	1.2728	1.4458	1.6672	1.9733	2.4491	3.3818
0.2	1.0000	1.1205	1.2445	1.3833	1.5489	1.7592	2.0490	2.5009	3.4027
0.3	1.1285	1.2445	1.3667	1.5046	1.6689	1.8764	2.1597	2.5972	3.4658
0.4	1.2728	1.3833	1.5046	1.6436	1.8100	2.0199	2.3049	2.7404	3.5912
0.5	1.4458	1.5489	1.6689	1.8100	1.9809	2.1978	2.4921	2.9389	3.7987
0.6	1.6672	1.7592	1.8764	2.0199	2.1978	2.4263	2.7383	3.2126	4.1190
0.7	1.9733	2.0490	2.1597	2.3049	2.4921	2.7383	3.0801	3.6057	4.6177
0.8	2.4491	2.5009	2.5972	2.7404	2.9389	3.2126	3.6057	4.2277	5.4651
0.9	3.3818	3.4027	3.4658	3.5912	3.7987	4.1190	4.6177	5.4651	7.3067

Table 8: $\gamma_1 = 0.8$, γ_2—vertically, γ_3—horizontally

$\gamma_{2\backslash 3}$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	1.0000	1.1527	1.3139	1.5003	1.7295	2.0288	2.4491	3.1098	4.4246
0.2	1.1527	1.2912	1.4406	1.6138	1.8267	2.1054	2.5009	3.1352	4.4298
0.3	1.3139	1.4406	1.5826	1.7490	1.9536	2.2203	2.5972	3.2022	4.4558
0.4	1.5003	1.6138	1.7490	1.9111	2.1118	2.3733	2.7404	3.3245	4.5308
0.5	1.7295	1.8267	1.9536	2.1118	2.3113	2.5727	2.9389	3.5162	4.6899
0.6	2.0288	2.1054	2.2203	2.3733	2.5727	2.8383	3.2126	3.8005	4.9768
0.7	2.4491	2.5009	2.5972	2.7404	2.9389	3.2126	3.6057	4.2277	5.4651
0.8	3.1098	3.1352	3.2022	3.3245	3.5162	3.8005	4.2277	4.9233	6.3321
0.9	4.4246	4.4298	4.4558	4.5308	4.6899	4.9768	5.4651	6.3321	8.2276

Table 9: $\gamma_1 = 0.9$, γ_2–vertically, γ_3–horizontally

$\gamma_{2\backslash 3}$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	1.2197	1.4223	1.6499	1.9239	2.2699	2.7292	3.3818	4.4246	6.5788
0.2	1.4223	1.5915	1.7903	2.0342	2.3484	2.7763	3.4027	4.4298	6.5791
0.3	1.6499	1.7903	1.9674	2.1899	2.4792	2.8763	3.4658	4.4558	6.5822
0.4	1.9239	2.0342	2.1899	2.3943	2.6645	3.0369	3.5912	4.5308	6.6012
0.5	2.2699	2.3484	2.4792	2.6645	2.9178	3.2716	3.7987	4.6899	6.6723
0.6	2.7292	2.7763	2.8763	3.0369	3.2716	3.6098	4.1190	4.9768	6.8675
0.7	3.3818	3.4027	3.4658	3.5912	3.7987	4.1190	4.6177	5.4651	7.3067
0.8	4.4246	4.4298	4.4558	4.5308	4.6899	4.9768	5.4651	6.3321	8.2276
0.9	6.5788	6.5791	6.5791	6.6012	6.6723	6.8675	7.3067	8.2276	10.427

If we look at these results, we see that the number of incorrect results obtained by the Bisection method increases. Even in the last table, we can see that there were no correct values.

As a result, based on the results of the table, in some cases (see green results in the table) we see that the calculation of the Bisection method is more profitable than the calculation via of the figure. In some values, you can see that the results of the Bisection calculations are incorrect (see yellow and red results in the table). As already mentioned above, the method[5] of determining the Box dimensions using image of Sierpinski triangle, which was long and hard is absolutely correct. However, because the open set condition is sometimes not satisfied for the members of a set, the Bisection method may not always be efficient and correct.

4. PART 2

4.1 λ – Cantor sets and total self-similarity

A fractal is a set in Euclidean space that has self-similar structure, that is, it repeats itself as you zoom in its image. The famous example of a fractal is the well-known middle third Cantor set which is obtained by removing the middle third of the preceding interval starting with [0,1]. Some of the characteristics of the middle third Cantor set are that it is totally disconnected, uncountable, compact, and has zero Lebesgue measure, fractal dimension log2/log3. We recall that the fractal dimension, e.g. Box dimension, Hausdorff dimension, Capacity, is a way to measure the complexity of the set. In general, it is difficult to characterize fractal sets.

One interesting approach to study and construct fractals is by means of Iterated Function Systems (IFS). Consider two functions f_0, f_2: $[0,1] \to [0,1]$ given by

$$f_0(x) = \frac{x}{3} \text{and} f_2(x) = \frac{x+2}{3}.$$

These are contractions and the well-known Contraction Mapping Theorem, see e.g. cite[3], the contraction mappings have a unique fixed point in complete metric spaces. In particular, the IFS $\{f_0, f_2\}$ gives rise to a unique fractal and by iteratively applying these functions to the unit interval it is easy to see that in the end we obtain the middle third Cantor set.

If so called Open Set Condition (OSC) holds for an IFS with contraction rates $r_1, r_2, \ldots, r_n$ then[3] cite Theorem implies that the fractal dimension d satisfies $r_1^d + r_2^d + \cdots + r_n^d = 1$. In our example of $\{f_0, f_2\}$ we have $r_1 = r_2 = 1/3$, thus $1/3^d + 1/3^d = 1$ yields $d =$ log2/log3, the dimension of the middle third Cantor set.

In this article we consider a family of IFSs $\{f_0, f_\lambda, f_2\}$ parametrized by $\lambda \in$

[0, 2] defined as

$$f_0(x) = \frac{x}{3}, \; f_\lambda(x) = \frac{x+\lambda}{3}, \text{and} f_2(x) = \frac{x+2}{3}. \tag{4.1}$$

Unless $\lambda = 1$ these IFSs produce overlapping images of the unit interval and as such it is difficult to see for which λ it satisfies OSC, hence it is difficult to say when one can use the above equation to compute the fractal dimension, or more specifically Hausdorff dimension. In this direction B. Solomyak and P. Shmerkin proved that the Hausdorff dimension of the fractal for λ irrational is one, for the proof see[4]. This established a conjecture of Furstenberg from 1970s, see also[6],[9] for partial results in this direction.

We now introduce a finer notion of self-similarity due to Broomhead, Montaldi and Sidorov[1], called totally self-similarity. To this end, we follow the notation from[2]. For any $\lambda \in [0, 2]$ we let E_λ denote the compact set (fractal) in [0,1] generated by the IFS $\{f_0, f_\lambda, f_2\}$ and $\Omega_\lambda = \{0, \lambda, 2\}$ is an alphabet consisting of 3 letters, $\Omega_\lambda^{\mathbb{N}}$ is the space of infinite words from the alphabet Ω_λ, for any natural number n, Ω_λ^n is the set of all finite words from alphabet Ω_λ of length n, and $\Omega_\lambda^* = \cup_{n=1}^{\infty} \Omega_\lambda^n$ is the space of all finite words. For any finite word $d_1 d_2 \ldots d_n \in \Omega_\lambda^n$ we denote $f_{d_1 d_2 \ldots d_n} = f_{d_1} \circ \cdots \circ f_{f_n}$. The space of infinite words $\Omega_\lambda^{\mathbb{N}}$ gives us a symbolic representation of the fractal E_λ: For any $x \in E_\lambda$ there exists an infinite word (sequence) $(d_n) \in \Omega_\lambda^{\mathbb{N}}$ such that

$$x = \lim_{n\to\infty} f_{d_1 d_2 \ldots d_n}(0) = \sum_{i=1}^{\infty} \frac{d_i}{3^i}.$$

Here one can take any other number instead of zero.

Let $I := [0, 1]$. We say that E_λ is *totally self-similar* if

$$f_d(E_\lambda) = f_d(I) \cap E_\lambda$$

for any finite word $d \in \Omega_\lambda^*$.

One of the main results of[2] is the following.

Theorem. *For $\lambda \in (0, 1)$, the set E_λ is totally self similar if and only if $\lambda = 1 - 3^{-n}$ for some natural number n.*

Our goal is to obtain the similar result for other values of λ. Our main result is the following.

Theorem 1. *For $\lambda \in (1,2)$, the set E_λ is totally self similar if and only if $\lambda = 1 + 3^{-n}$ for some natural number n.*

For example: $f_d(x)\, \frac{x+d}{3}$ for $x \in I = [0,1]$, $d \in ?_\lambda\{0,\ \lambda,\ 2\}$ or $d \in ?_{\overline{\lambda}}\{0,\ \overline{\lambda},\ 2\}$

$$f_0(I) = [0, \frac{1}{3}],\ f_\lambda(I) = [\frac{\lambda}{3}, \frac{1+\lambda}{3}],\ f_{\overline{\lambda}}(I) = [\frac{\overline{\lambda}}{3}, \frac{1+\overline{\lambda}}{3}],\ f_2(I) = [\frac{2}{3}, 1].$$

Where $0 < \lambda < 1$ and $1 < \overline{\lambda} < 2$.

The next section is devoted to proving the main result.

4.2 Proof of Theorem

In this section we prove Theorem 1. For any $\lambda \in (1,2)$ we set $\overline{\lambda} = 2 - \lambda \in (0,1)$. We prove

Proposition 1. *For any $\lambda \in (1,2)$, the set E_λ is totally self-similar if and only if the corresponding set $E_{\overline{\lambda}}$ is totally self-similar.*

The proof of Theorem 1 now easily follows from Proposition 1.

Proof of Theorem 1. It follows from result of[2], the theorem stated above, that $E_{\overline{\lambda}}$ is totally self-similar if and only if $\overline{\lambda} = 1 - 3^{-n}$ for some natural number n. Hence, for $\lambda \in (1,2)$ using Proposition 1 we get that E_λ is totally self-similar if and only if $2 - \lambda = \overline{\lambda} = 1 - 3^{-n}$ for some natural number n. Therefore, we see that $\lambda = 1 + 3^{-n}$ for some natural number n which finishes the proof.

So, it remains to prove Proposition 1. The idea of the proof is to realize that the two IFSs are topologically conjugate. We have to consider one function (see the commutative diagram below) that will be executed in the next queue topologically conjugate. Then we will build a connection between the systems. Show it through diagram commutative:

$$\begin{array}{ccc} I & \xrightarrow{f_{0,\lambda,2}} & I \\ {\scriptstyle\phi}\downarrow & & \downarrow{\scriptstyle\phi} \\ I & \xrightarrow[f_{0,\overline{\lambda},2}]{} & I \end{array}$$

To this end, we let ϕ: $[0,1] \to [0,1]$ via

$$\phi(x) = 1 - x.$$

Lemma 1. *For any* $\lambda \in [1,2]$ *we have* $f_{\overline{\lambda}} = \phi^{-1} \circ f_\lambda \circ \phi = \phi \circ f_\lambda \circ \phi$.

Proof. Recall that $f_\lambda(x) = \frac{x+\lambda}{3}$ which is also true for $\lambda = 0, 2$. A simple calculation yields

$$f_\lambda \circ \phi(x) = \frac{\phi(x)+\lambda}{3} = \frac{1-x+\lambda}{3} = 1 - \frac{x+(2-\lambda)}{3} = \phi \circ f_{\overline{\lambda}}(x),$$

so that $f_{\overline{\lambda}} = \phi^{-1} \circ f_\lambda \circ \phi$. The last assertion follows since $\phi^{-1} = \phi$, that is, $\phi^2(x) = x$.

Lemma 2. *For any* $\lambda \in (1,2)$ *we have* $\phi(E_\lambda) = E_{\overline{\lambda}}$

Proof. Recall that $x \in E_\lambda$ if and only if there exists $(d_i) \in \Omega_\lambda^{\mathbb{N}}$ such that $x = \lim_{n\to\infty} f_{d_1 d_2 \ldots d_n}(0)$. Then, Lemma 1 gives

$$\phi(x) = \lim_{n\to\infty} \phi \circ f_{d_1 d_2 \ldots d_n}(0)$$
$$= \lim_{n\to\infty} (\phi \circ f_{d_1} \circ \phi^{-1}) \circ (\phi \circ f_{d_2} \circ \phi^{-1}) \cdots (\phi \circ f_{d_n} \circ \phi^{-1}) \circ \phi(0)$$
$$= \lim_{n\to\infty} f_{\overline{d}_1 \overline{d}_2 \ldots \overline{d}_n}(1).$$

As all $\overline{d}_i \in \Omega_{\overline{\lambda}}$ we see that $\phi(x) \in E_{\overline{\lambda}}$. Hence, $\phi(E_\lambda) = E_{\overline{\lambda}}$.

Proof of Proposition 1. Let us assume that $E_{\overline{\lambda}}$ is totally self-similar. We claim that E_λ is also totally self-similar. For this we need to check that $f_d(E_\lambda) = f_d(I) \cap E_\lambda$ for any finite word $d = (d_1 d_2 \ldots d_n) \in \Omega_\lambda^*$. We already know that $f_{\overline{d}_1 \ldots \overline{d}_n}(E_{\overline{\lambda}}) = f_{\overline{d}_1}.$ $E_{\overline{\lambda}}$. Thus, from Lemma 2 we see that

$$\phi \circ f_{\overline{d}_1 \ldots \overline{d}_n}(\phi(E_\lambda)) = \phi \circ f_{\overline{d}_1 \ldots \overline{d}_n}(I) \cap \phi \circ E_{\overline{\lambda}}. \tag{4.2}$$

Note that from Lemma 2 we have $\phi(E_{\overline{\lambda}}) = \phi^2(E_\lambda) = E_\lambda$. Using Lemma 1 repeatedly we get $\phi \circ f_{\overline{d}_1 \ldots \overline{d}_n}(\phi(E_\lambda)) = f_{d_1 \ldots d_n}(E_\lambda)$ and $\phi \circ f_{\overline{d}_1 \ldots \overline{d}_n}(I) \cap E_\lambda = f_{d_1 \ldots d_n}(I) \cap E_\lambda$ as $\phi(I) = I$. Hence, (4.2) implies that

$$f_{d_1 \ldots d_n}(E_\lambda) = f_{d_1 \ldots d_n}(I) \cap E_\lambda,$$

for any finite word $d_1 \ldots d_n$. Thus, E_λ is totally self-similar. The converse is analogous.

5. Conclusion

We summarize the results of this scientific work in two sections. In conclusion, in the first section, we see that the two methods we are considering are mutually acceptable and Vice versa. The method of calculating the dimension of the image of the Sierpinski triangle - a computer method that is difficult to use and that has to work hard to get the results. However, the probability that this method always shows us the true result is high. And the second method, which we consider, is a small and simple way, which gives the correct result only when the conditions of the open collection are satisfied. In addition, these methods can be considered in other cases, when the Serpin triangle is not provided by us, or these methods can be used to determine the size of other fractal systems. Now, in order to use the methods in this section effectively, we need to know that the open collection conditions meet or do not meet the conditions. At the same time, you can use the computer method calculated by the Bisection method. In the second section, we proved that the parameter λ in a closed Cantor set can be considered as $\lambda = 1 + 3^{-n}$. If we find this parameter in the interval $1 < \lambda < 2$, we are now working on finding the parameter of the fractal set E_λ in other unintended regions.

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