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**The Methodology of Using Active Learning Methods in Teaching Mathematical
Analysis Courses to Students**

6D010900 - Mathematics

Dissertation for the degree of
Doctor of Philosophy (Ph.D.)

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REGULATORY REFERENCES

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DEFINITIONS

The following terms are used in this dissertation together with the corresponding definitions:

The methodology is the study of the structure of scientific knowledge, methods, and tools of scientific knowledge, the substantiation of knowledge, and its development.

Methodological knowledge - a set of philosophical ideas about the scientific recognition of the object of study, the definition of principles, tools, and methods of organization of work aimed at changing or updating it.

The educational process is a concrete manifestation of the overall development and upbringing of the individual in a specially organized form of education, a single pedagogical process that determines the purpose of education.

The curriculum is a document that defines the content and scope of knowledge, business, and skills to be acquired in each subject.

DESIGNATIONS AND ABBREVIATIONS

AL: Active Learning

PBL: Problem-Based Learning

PLTL: Peer-led Team Learning

STEM: Science, Technology, Engineering, and Mathematics

IBL: Inquiry-Based learning

SDL: Self-directed Learning

LOT: Lower-order thinking

HOT: Higher-order thinking

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INTRODUCTION

Relevance of the work: The demand for specialists in mathematics and natural sciences should be said that is growing all over the world. Many governments and private organizations have modernized STEM education to effectively meet this demand and promote teaching to improve students' mathematical skills. Recently, a lot of academics and organizations have emphasized that for students to succeed beyond graduation, they must acquire 21st-century skills. According to the study, the most important STEM talents are those that require collaboration, problem-solving, imagination, entrepreneurship, adaptability, critical thinking, initiative, effective community, access to information, analysis, and curiosity [1].

According to Kazakhstan's "On Science" Law, the university's primary pursuits are education and the pursuit of science, technology, and innovation [2]. According to the "On Education" Law of the Republic of Kazakhstan, "the main task of the education system is to create the necessary conditions for education aimed at the formation and professional development of the individual based on national and universal values, scientific and practical achievements." The tasks for further developing the educational system mentioned the following: "Introduction of new teaching technologies and innovative methodologies, informatization of education, access to transnational global communication networks" [3].

Active learning is the type of training used in all developed countries of the world. Of course, active learning can be called by different terms in different countries. In some countries (USA, Great Britain...) it is called constructive learning, in other countries (interactive learning) ... However, its meaning is the same. It is the learning of students on their own, based on active actions. At the same time, according to the results of expert research conducted in the United States. In these days of the Soviet Union, active learning achieves more productive results than the so-called "traditional learning" [4].

Confucius said about 2400 years ago, "What if I heard, I forgot. I remember what I saw. Whatever I do, I understand it." This millennium should educate students, not on the burden of knowledge but on centralized learning, problem-solving, analysis, synthesis, intelligence, creativity, love, tolerance, and respect for national and universal values. It is necessary to study whether traditional teacher-centered and student-centered active learning methods affect student achievement, communication, and retention in teaching higher mathematics.

We analyzed the scores of the State National exams for first-year Suleyman Demirel University "6B01501-Mathematics" students for the last four years (2017-2021) to reveal the correlation between scores and mathematics skills of students which showed the diminishing influence of the scores. Also, there are taken a survey from first-year students or 11th-grade students in several countries (Kazakhstan, USA, Georgia, and Malaysia) to reveal the main factor to learn mathematics effectively. The obtained results revealed that the study is required to research.

It is necessary to study whether traditional teacher-centered and student-centered active learning methods affect student achievement, motivation, self-confidence, and higher-order thinking skill in teaching Mathematical Analysis courses. It is known that the pedagogical effect of the obtained active methods had not been studied in research in mathematics in higher education. For this reason, some critical problems have been faced as follows;

- Lack of clarity in the content of teaching Mathematical Analysis courses so that students can use active methods in the learning process,
- pedagogical conditions for its implementation,
- the need to create an innovative system of active learning methods through the use in teaching Mathematical analysis courses;
- lack of active methods and their application in teaching Mathematical Analysis courses with new technological tools;
- develop pedagogical and psychological aspects.

The primary issue is that “Active learning methods” in the rational organization of student activities in Mathematical Analysis courses have not been analyzed from a scientific and pedagogical perspective. The research studies led to focus on two issues. First, the uncertainty of which active learning methods to use in the learning process is effective in teaching Mathematical Analysis courses. Second, despite the many types of research using active learning methods in teaching Mathematical Analysis courses, the impact of fully applying them in practice by integration of Bloom’s taxonomy. Therefore, the research task here is to determine the pedagogical principles for teaching Mathematical Analysis courses using active methods in the learning process. This issue shows the relevance of our study. The satisfaction of these needs was the basis for choosing the research topic “The Methodology of Using Active Learning Methods in Teaching Mathematical Analysis Courses to Students.” The research facilitates the stages of planning and implementation of learning. After all, active learning increases academic performance rather than traditional learning [5].

Level of study of the topic: Changes in traditional teachers’ and students’ roles in new knowledge and school perceptions come to the fore, and the teacher ceases to be the sole source of knowledge. The student ceases to be a passive recipient of knowledge. It is becoming an institution where education is continuously updated in higher education, and teachers and students are actively involved. Although the importance of higher mathematics courses is well known, there are still significant university class issues. Students find math lessons boring, complicated and lacking, which indicates a lack of motivation and courage to learn mathematics and develop negative attitudes [6].

The learning process’s main objective in teaching Mathematical analysis courses is thinking skills in students’ knowledge, thereby activating cognitive activity. That is to determine the importance of developing students’ ability to act independently and work independently. The science and practice of pedagogy enhance cognitive activity activation by improving students’ thinking skills and improving their mathematical

knowledge. These issues determine the main features of developing students' creative abilities and aspects of creative application. What should be done about this, and what measures should be taken? Interest in learning, desire to acquire personal knowledge. It requires the teacher to consider different teaching methods and improve the methods and techniques of dynamic lessons. The part facing higher education in teaching Mathematical Analysis courses is that students need to work hard and effectively develop learning activities. Also, it requires them to manage and develop their actions in solving educational problems using the acquired knowledge skills. The speed of thinking and the mind's flexibility are mastered from an early age and give endless opportunities to use the world's secrets.

Bloch M. A., Luke N. A., Vygotsky L. S., and YA. A. Ponomarev engaged in the psychological aspects of this problem. The works of such scientists as Leontiev A.H., and Davydov V. V. depend on them. The Problems of activating the student's pedagogical and methodical in higher mathematical education: Shamova N. A., Polovnikova I. Y., Lerner L. P., Aristova G. I., Chykina M. I., Makhmytov M. A. It is considered in the works of Danilov M., Skatkin N., Orlova V.L., and other scientists. In recent years, research has been conducted in the country to activate educational and cognitive activities, problem-based learning, organization, and methods of improvement. Numerous sources related to the research topic have been studied. Eralin K., Marlene S., and Eva N. are the researchers who have studied the development of professional knowledge in terms of pedagogical sciences. Eddie S. L., Ibraeva K., Taubaeva S. T., Gross, B., Azarov Y. P., Castells M., Babich L. V., Khalitova I. R., ... who demonstrate the need to create special conditions for mastering the methods of the most favorable conditions for increasing the activity of thinking and acting in the learning process. It has been scientifically proven and considered from various aspects in the research of scientists such as Nugymanov I. N., Karaev Z. A., Nurgaliyeva G. K., Abylkasymova A. E., Sabyrov T. S., Alimukhambetova G. E., Ibragimova Z., Kusainov G. M., Тамаев А., Дайрабаева А. Е., Таубаева Т., Nurzhanova P.M., Simbaeva S., et al. However, these studies do not fully explore the pedagogical conditions in teaching Mathematical Analysis courses for using active learning methods. Thus, the impact of teaching Mathematical Analysis courses contributes to creative development, as the content and volume of knowledge in various scientific fields are overgrowing. Today's student is obliged to increase tomorrow's school teacher's cognitive activity, to teach him to learn independently and apply the acquired knowledge. In our opinion, this subject's content should be updated continuously following the areas of science and pedagogy in educational institutions that educate future teachers.

In teaching Mathematical Analysis courses, students are required to be active in the learning process. Using critical and creative teaching methods and techniques selected under the nature of the mathematics subject and using active learning methods and techniques in groups or teamwork help overcome the event's cause and effect. The method of active learning should also be formed by individuals.

This dissertation presents some of the many studies that justify the benefits of using active teaching methods. Although a significant portion of educational research in active learning is anecdotal, students prefer this teaching method to traditional methods. This thesis also discusses some of the discussions in active learning. Our research shows that active learning methods can help teach Mathematical Analysis courses, where students' knowledge is limited. However, they can disrupt successful students' learning.

To ascertain whether the differences in learning acquired for a given approach are based on these qualities, Bloom's taxonomy success tests for each group are compared. An analysis is done on the effects of active learning on students in teaching Mathematical Analysis courses. To investigate these consequences, a statistical model is created. The purpose of the study is to identify the characteristics that contribute to a certain teaching method's efficacy through an empirical analysis grounded in active learning. In teaching Mathematical Analysis courses research promotes a competitive and student-centered university environment while also assisting in the creation of new educational systems.

The object of research: Teaching the course of Mathematical Analysis in higher educational institutions.

The subject of research: The impact of active teaching methods on students in Mathematical Analysis courses.

The study aim: Development of methodology in teaching Mathematical Analysis courses by the use of active learning methods for students in higher education.

The scientific hypothesis of the research: If the methodology of using active learning methods in teaching Mathematical Analysis courses is developed based on a combination of active learning methods and pedagogical conditions, it will improve the quality of mathematical knowledge of students; further study of general professional and special disciplines, increase readiness for future professional activity; allows students to better understand the future profession.

Objectives of the study

1. Theoretical substantiation of the content, which determines the pedagogical conditions for the formation of students' use of active learning methods in Mathematical Analysis courses;
2. to identify the psychological and pedagogical aspects of the development of active learning methods used in the teaching of Mathematical Analysis courses;
3. developing new methodological forms as well as practical tactics (tasks, pedagogies) to help to develop future mathematicians and how to increase students' higher thinking skills when dealing with real-world problems;
4. practical proof of pedagogical conditions for using active learning methods in teaching mathematical analysis courses and statistical analysis to measure academic performance, self-confidence, motivation, and higher-order thinking skills.

The methodological basis of research: Quantative research methods. They were getting acquainted with state and pedagogical documents and their analysis. They are

also collected by surveys, written assignments from students, experiments, theoretical analysis, analysis of best pedagogical practices, and more.

Research base: The study was conducted at Suleyman Demirel University, Kaskelen, the district of Almaty, University Malaysia Pahang in Malaysia, and The Nile University in Nigeria. The total number of participants who competed in the study is 244.

The main stages of research: From 2016 through 2017, the purpose, object, subject, tasks, and hypothesis of the research were defined.

From 2017 through 2018, the first chapter was determined as theoretical aspects of the use of active learning methods in higher education, pedagogical and psychological features of active learning methods in teaching mathematical Analysis courses, and didactic principles of mathematical Analysis courses using active learning methods for teaching students.

From 2018 through 2019, the second chapter of the dissertation was analyzed as forms of active learning methods in the teaching of mathematical analysis courses using at higher education, and the use of active learning with the formed model in mathematical Analysis courses.

From 2019 through 2020, the experiment of achievement tests for the active learning group and regular learning group at Suleyman Demirel University was taken and analyzed.

From 2020 through 2021, the experiments of achievement tests for the active learning group and Online learning group at Nile University and University Malaysia of Pahang were taken and analyzed.

From 2021 through 2022, the experiments of the achievement test for the active learning group and online learning group and the HOT test for the active learning group and online group at Suleyman Demirel University were taken and analyzed.

The novelty of the study

- The teaching of Mathematical Analysis courses in higher educational institutions was theoretically based and didactic principles were defined.
- Features of active teaching of Mathematical Analysis to future specialists in higher educational institutions have been identified.
- A structured and content model has been developed for the professional teaching of Mathematical Analysis to future experts in higher educational institutions.
- Practical classes and originality of students in their work (on the example of the specialty “6B01501-Pedagogical Mathematics”) developed a methodology for implementing a set of methodological principles based on active learning, the main components of which are the study of subject connections of mathematical disciplines and subject connections of fundamental, general and special disciplines; reading adjusting the content of the mathematical analysis course based on a generalization of the strategies of active learning methods into a single idea; assessment of the quality of advanced mathematical education.

- Sequences and Series were taught on the integration of active learning methods and the levels of Bloom's taxonomy into the Mathematical Analysis course curriculum and the practical teaching of additional parts of Mathematical Analysis was checked and the results are presented. Lastly, the comparison of online education and active learning as offline education was done and proved that active learning was effective.

Recommended for protection

1. psychological, pedagogical aspects of psychological, pedagogical development of active learning methods used in the teaching of Mathematical Analysis courses;
2. features of teaching Mathematical Analysis courses in the context of modern education;
3. ways to develop competencies for future mathematics teachers to develop active learning methods in teaching Mathematical Analysis courses through technology.

The scientific significance of the study

1. The meaning of the concept of "professional mathematics teaching" regarding higher education institutions has been clarified. The relevance of taking into account the didactic principles of teaching "Mathematical Analysis" in higher education is scientifically based (for example, the principle of teaching in a professional direction, the principle of the connection of science and theory with practice).
2. A set of methodological principles for the implementation of the direction of vocational education, reflecting the special nature of learning mathematical subjects for future teachers of mathematics, was established in the methodological system of teaching "Mathematical Analysis" in higher education institutions (fundamentalism, professionalism, leadership). idea, continuity, knowledge, the principle of a comprehensive approach, the realization of topic links, etc.).
3. As a result of an analytical examination of the literature sources on the modernization of the university education system, some didactic principles are considered in terms of professionalism and the function of further education; A conclusion was reached about the necessity of introducing
4. The active learning methodology of applying mathematics teaching in a professional direction has been identified as an important factor affecting the education quality of students. Due to their versatility, they can be used in other universities.
5. Using real-life problems has led to the need to teach critical thinking ability in teaching Mathematical Analysis courses.
6. Methods that allow the organization of the learning process have been developed to improve students' pedagogical mathematical knowledge.

The practical significance of the study:

- the implementation of the methodological proposals presented in the study in higher education institutions allows for strengthening the professional direction of teaching mathematics;
- the methods and approaches developed to form the contents of the course "Mathematical analysis" are universal and are used in other higher educational

institutions, departments of mathematics and science education can be used for different professions and in relation to other mathematical subjects;

- For the specialty “6B01501-Pedagogical Mathematics” a teaching-methodological complex was developed on “Mathematical analysis”, which is based on the generalization of the educational material, based on the approach to presenting in higher mathematics at universities;

- the developed and printed training material was used in the work practice of higher educational institutions as a training tool for mathematics and science specialties;

- the results of the research may be helpful to future mathematicians, and schools, to compare the dependencies between the levels of implementation of the professional orientation of mathematics education.

The accuracy and validity of research results: with a deep and comprehensive analysis of the investigated problem, based on the fundamental research of philosophers, psychologists, pedagogues, and methodologists; using a set of theoretical and practical experimental methods appropriate to the aims and objectives of the research; for example, with its representativeness and positive results of pedagogical experimentation; with positive reviews of active learning techniques; It is ensured by the comprehensive validation and introduction of research results into the educational process of mathematics courses at the university.

General conclusion: studying the topic “The Methodology of Using Active Learning Methods in Teaching Mathematical Analysis Course to Students,” we came to the following conclusions.

- Identified psychological and pedagogical factors for the creation of active learning methods for teaching Mathematical Analysis courses instruction.

- The content is theoretically supported, establishing the pedagogical prerequisites for the establishment of active learning methods in teaching Mathematical Analysis courses.

- Demonstration of the pedagogical conditions for using active learning methods in the teaching of mathematical analysis courses, as well as statistical analysis to assess academic achievement, self-confidence, motivation, and higher-order thinking skills.

Publications

- 1) The impact of Project-Based Learning on Student’ Achievement in Mathematics. Pavlodar, Bulletin of Toraigyrov University, No 3, 2020. – p. 367-375.

- 2) The Importance of the discussion part for peer Instruction. Bulletin of “Physical and mathematical sciences” series No. 2 (66), 2019. – p. 58-62.

- 3) Impact of the active learning strategies on student’s achievement with respect to double integrals in mathematical analysis. Accepted for publication in No. 3 (79), 2019 Bulletin of KazNazhenPU. – p. 83-91.

- 4) Student’s Attitudes Towards Mathematics in a Short-Term Project-Based Learning Course. International Scientific Practical and internet Conference. Ukraine, 27 April 2019. – P. 328-331.

5) Effect of Active Learning about Creative Skills in Solving Olympiad Problems. International scientific journal “Global Science and Innovation: Central Asia” Astana, Kazakhstan, September-October 2019. – P. 282-285.

6) The Impact of Project-Based Learning on Students’ Motivation in Mathematics. Education and Social Sciences Conference, Spain 2019. – P. 162-176.

7) The Effect of Discussion Part of Peer Instruction on Students’ Responses. Education and Social Sciences Conference. Rome, Italy, 17 September 2019. – P. 36-41.

8) The Impact of Peer Instruction on Ninth-Grade Students’ Trigonometry Knowledge. Bolema, Rio Claro (SP), № 35(69), 2021. – P. 206-222.

9) Views of Future Math Teachers About Using the Technological Devices with Active Learning. The international scientific-practical conference on “Actual problems of national upbringing in modern conditions”, Almaty, September- 24, 2020.

10) Comparison of Kazakhstan and Russian university students in learning science motivation. Eurasia Journal of Mathematics, Science and Technology Education, № 18(11), 2022, -p. em2173

The structure and scope of the dissertation research. An introduction, two chapters, a conclusion, a list of references, and an appendix make up the thesis. In the introduction, the relevance and rationale of the research, the main target, and the charges are shaped. The first chapter is devoted to the theoretical aspects of active teaching methods in higher education, pedagogical and psychological features of active learning methods in teaching mathematical analysis courses, and didactic principles of mathematical analysis courses using active learning methods. The second chapter analyzes the methodological bases of teaching students Mathematical Analysis courses using active learning methods. The researcher associates the active learning methods with the learning theories and finds out using active learning methods in advanced mathematical analysis courses. In conclusion, the main findings of the research are realized.

1 SCIENTIFIC AND METHODOLOGICAL BASES OF THE USE OF ACTIVE LEARNING METHODS IN TEACHING MATHEMATICAL ANALYSIS COURSES AT HIGHER EDUCATION

1.1 Theoretical Aspects of Using Active Learning Methods in Higher Education

Education that is based on a realistic understanding of the nature of the learning process is referred to as active learning. These philosophical beliefs and assumptions include concepts about:

How learning occurs, the role of the instructor, the part of the student, the position of administrative and institutional support for teachers, the role of parents and the community, the role of the environment in learning, the purposes and aims of learning, the desired outcomes of education, factors that enhance and inhibit learning, the best methods for evaluating learning, the role of the individual and collective prior knowledge in the learning process, appropriate training of teachers proper uses of teacher and student time, the process and methods of teacher planning of educational activities, the value of teacher autonomy, and organizational and operational procedures in the school to support student learning.

The core of Active Learning Assumptions

Active learning is not a specific curriculum or defined set of methods, materials, processes, or procedures. However, instead, it is a philosophy or way of thinking about what high education should be. Thus, active learning may look different in its implementation in various high education. However, active learning environments exist in a variety of situations and share some commonalities. Instructors can utilize active learning as a comprehensive set of concepts that impact decisions and methods for executing countless schooling aspects at the local level. In general, some of the core guiding concepts most closely associated with active learning include:

- People learn best when they can relate personally to the concepts they are learning.
- People comprehend conventional wisdom and facts more effectively when they can actively manipulate, apply, or make use of the information.
- Higher levels of education take place when people apply what they already know to learn something new or to complete something original.
- The primary actor in the creation of knowledge is the learner.
- Learning and motivation are inextricably intertwined and interdependent processes.

The teacher's role is to guide students' learning journey by planning student activities that will stimulate mastery of known facts. This knowledge is used to generate new knowledge or accomplish goals, monitor student processes and activities, provide timely specific feedback to scaffold their learning, and evaluate student learning outcomes and learning processes. Learning is an individual process, and results occur on individualized timelines. Despite participating in similar activities, the conclusion of

learning will vary among students. As a result, rather than being consistent across pupils, the pace of learning activities should be considered flexible. Educators should teach as progress along a continuum rather than compare them to an absolute standard on students' evaluation.

Self-regulation of learning is critical because the learner is ultimately the agent of their education. However, self-regulation is a learned skill best developed when students assume increasing decision-making responsibility for their learning. To assist students to develop into lifelong learners who routinely self-reflect, self-monitor, and make adjustments depending on the results of their learning processes, educators must keep an eye on and direct this process as well as give a framework, scaffolding, and feedback as needed. Self-evaluation and teacher-student meetings are regularly used to help students manage their learning. Environments in which students may actively manipulate and experiment should be stimulating.

Even though learning takes place on an individual level, it is communal. The process of learning is most effective when people engage with one another. Thus, quiet, strictly receptive learning environments with no opportunity to discuss, question, or reflect on the topic are not valued. Students are intrinsically more motivated by authentic learning activities that demonstrate the value and significance of what they have learned in school because they can see and understand how they are used outside of the classroom. The most in-depth learning results from persistent attention to a subject. Mature learners demonstrate a sustained focus on a topic. However, self-discipline and sustained focus are developmental. As a result, teachers must design learning activities that gradually allow students to increase their level of concentration when working on complex problems and projects. Due to the developmental nature of this skill, it takes a lot of teacher structure, monitoring, and direction to conduct long-term, open-ended plans with very inchoate learners or those who have no experience in self-regulation.

To facilitate sustained focus and improve learning processes to support ongoing learning, reflection is essential. The fact is also actual for teachers. Regardless of the teacher's formal educational level, experience, or age, effective teaching necessitates professional mirroring and self-regulation of instructional processes since active learning necessitates a large deal of professional evaluation and trial and mistake. Collaboration with like-minded teachers is also highly valued. The collective set of beliefs of a faculty for the learning process should hold on to the consistency of the ideas across the faculty, the strength of the ideas among teachers, and administrators' beliefs. The decisions educators make daily are influenced by the expectations of the community. When most choices about implementing high education include these beliefs and assumptions, high education institutions implement active learning. The origins of active learning assumptions are obtained from different mental psychology, learning theory, and educational philosophies.

Many philosophers have based several approaches that are related to active learning. This chapter provides a review of relevant theories to position active learning in figure 1 [7].

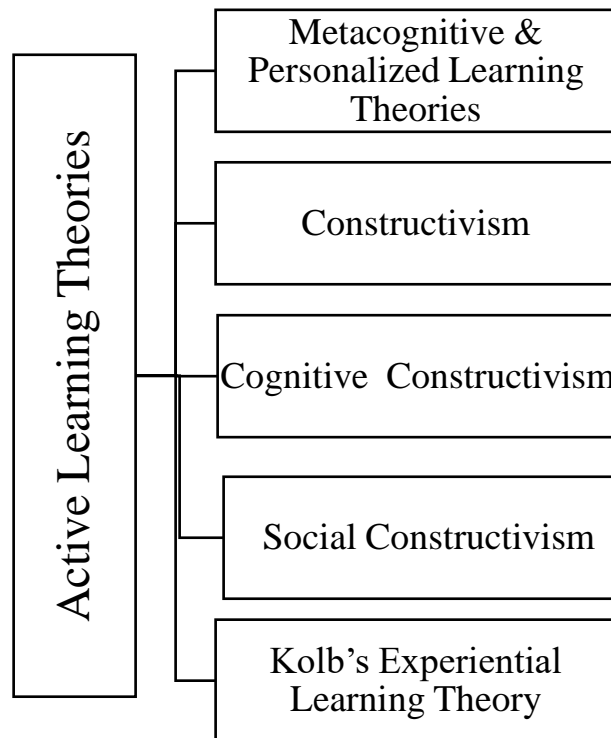


Figure 1 - Active learning theories

Metacognitive — Personalized Learning Theories

American professor and child psychologist John H. Flavell initially put forth the notion in the 1970s. He developed the approach over several years as:

Flavell offered the term “metamemory” to describe how we store and remember data in 1971. Later on, the name was altered to “metacognition.”

In addition, Flavell established three stages of meta-skill development in early childhood in 1976 and talked about the significance of cognitive regulation. After all, in 1979 Flavell proposed four categories or stages of “thinking about thinking” to clarify the idea [8]. To describe a way of thinking about how we store and recall knowledge in our minds, Flavell coined the word “metamemory” in 1971. Flavell’s metamemory was as follows:

Intentional: to consider how we cannot err. It needs to be deliberate and built using particular instruments, including self-questioning.

Predictive: We need to organize our thoughts before we start a task, but we also need to create an “attack plan” for our work.

used to accomplish a goal: To become better students or learners, we must employ metacognitive tactics [9].

Early metacognition can be divided into three stages, according to Flavell (1976):

First stage: Storage. Young children start acting consciously and on purpose. They employ simple techniques like repetition and put their attention on making sure knowledge is retained in their minds for later use.

Second stage: Recall. Children pick up techniques that enable them to quickly recall knowledge by storing it in their working memory. When they anticipate that information will be useful, such as in a “memory game,” they can retain it.

Systematic Strategies,

Third stage. Even when they did not anticipate the need for it, children apply systematic memory techniques. They make use of active recall techniques including self-questioning, thinking aloud, and mnemonic devices to recall information from longer-term memory [10].

In 1979, Flavell proposed four different metacognition classes: metacognitive knowledge, metacognitive experiences, tasks, and strategies or activities. They aid in providing a framework for considering the theory. A person’s perceptions of how they can influence their cognition are known as their metacognitive knowledge. Metacognitive experiences are subjective applications of meta-thoughts ‘right now’ to accomplish one’s tasks. Flavell offered that this is a “stream of consciousness” process [11].

We refer to a theory of cognition as a metacognitive theory. Mind concepts are a subset of metacognitive strategies. Cognition theories are included in the set of all theories of mind, but they are not the only ones. Views of the mind cover personality, emotion, and other related mental phenomena [12]. Theories of the mind that emphasize the cognitive components of the mind are called metacognitive theories. People produce and integrate metacognitive information when they build a method of cognition. However, it is crucial to make a distinction:

- a) Structured learning that includes a theory
- b) From the phenomena to which the theory relates

All methods are cognitive because they are knowledge structures, but not all theories are cognition-related. Theories regarding cognition are known as metacognitive theories. As a result, even if they are not necessarily related to this kind of knowledge, they do contain metacognitive information. Contrarily, metacognitive knowledge will be constituted by theories on metacognition. These methods only include a small portion of metacognitive ideas [13].

Various criteria have been proposed to distinguish a theory from a non-theoretical body of knowledge. The approach classifies them as a distinct and vital subset of metacognitive knowledge. In particular, metacognitive theories are to:

- combine a variety of metacognitive experiences and pieces of information,
- permit the analysis and forecasting of mental behavior.

An individual who will integrate various aspects of metacognition into a single framework is one of the theory’s main features [14]. For instance, research demonstrates that young infants frequently struggle to manage their cognition by utilizing their

knowledge of memory and learning processes [15]. Children’s failure to combine their metacognitive knowledge and regulatory abilities into a single conceptual framework is one cause of this. Because of this, many of his disposal abilities are passive and challenging to use outside of the setting in which they are learned [16]. Second, theories of metacognition coordinate ideas or presumptions that enable people to anticipate, manage and explain their cognition as well as the cognition of others or cognition in general [17]. By choosing strategies to enable effective learning from memory, the individual uses automatic procedures whenever possible. It encourages one to learn the content at a deeper level of understanding and reasonably allocates resources. To the extent that it is coordinated enough to boost learning, this understanding constitutes a theory of what it is to be an effective learner. Of course, the degree of metacognitive theory has each of these features. The degree to which each individual is aware of these traits is variable. We believe that metacognitive approaches gradually change over time due to personal proficiency and self-mirroring. The three different metacognitive theories that best represent this transition are discussed in the next section. Three different types of metacognitive theories are designed in figure 2:

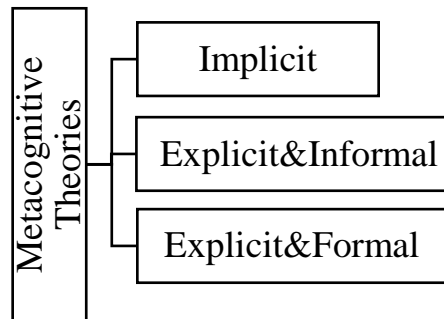


Figure 2 - Metacognitive Theories

Henceforth, we call these implicit, informal, and formal metacognitive theories.

Implicit Theories: Implicit theories are created or acquired without a person is explicitly aware of their approach [18]. Children’s behavior in the classroom is influenced by their “implicit” views about the nature of intelligence, according to Dweck and Leggett [19]. When a youngster thinks that intelligence is flexible and subject to change through other or self-directed procedures, this approach is becoming more prevalent. According to the two requirements mentioned above, it may be claimed that a child’s implicit beliefs about intelligence are a theory that enables him to put together his observations regarding the origins of intelligence and form judgments based on those observations. It is muted in the sense that many kids routinely exhibit thoughts that are consistent with a “theory of intelligence,” but they do not spontaneously admit owning one.

Adults' performance is also impacted by implicit beliefs about one's cognition or the epistemic nature of the universe [20]. The impact of instructors' implicit theories on their interactions with students and curriculum decisions is discussed by McCutcheon (1992) [21]. Additionally, there are a variety of ways that core theories and beliefs affect teachers' judgment. One significant conclusion is that, even when people are expressly encouraged to do so, implicit theories can be challenging to alter [22]. A person's metacognitive knowledge is organized by implicit organizational frameworks. Some of a person's underlying metacognitive theories, which are views about cognition, might be picked up through peers, teachers, or cultures. They are known as "reasoning scenarios" in the realms of scientific and informal reasoning, according to Kuhn (1991). Additional components of a person's metacognitive theory may be inferred from personal experiences or adaptations [23].

The fact that a person is not immediately aware of the theory or the data supporting or opposing it is perhaps the most striking feature of an inferred metacognitive theory as opposed to an explicit theory. As a result, inferred ideas are difficult to separate from or compare to relevant evidence [24]. Even if they are incorrect and incompatible, metacognitive theories might endure to the extent that they are left unstated.

Informal Theories: They are disjointed in that people are aware of some of their assumptions and opinions about a phenomenon, but they have not yet developed a precise theoretical framework that combines and supports those ideas. Only a simple consciousness of their metacognition can be had by informal theorists. Informal theories take time to develop and are impacted by various societal and individual factors. The fact that explicit metacognition is present to some extent in informal theorists distinguishes them from implicit theorists [25]. It appears likely that informal, straightforward theories start as concepts exclusive to one field and subsequently expand to cover other disciplines. Over time, expanding the breadth and depth of metacognitive theories can aid in understanding and directing beneficial processes [26]. According to our analysis, a crucial component of informal metacognitive theories that is missing from implicit theorists is the growing recognition and control of restorative techniques. Understanding the constructive nature of information and theories is essential because, without them, people cannot strategically alter their methods. They ought to be reduced able to control their learning and cognition as a result. With such information, persons can start purposefully formalizing the unstructured components of their theory and evaluate the sufficiency of metacognitive theories as they grow more structured.

An informal metacognitive theory has a clear advantage over an implicit idea in that it enables people to deliberately project their success. Additionally, they routinely alter or steer their future behavior and thoughts using this information [27].

Formal Theories: Explicit theoretical structures, such as those found in academic subjects in physics, arts, or mathematics, are used to explain phenomena in a very methodical manner. The tribrachic intelligence theory of Sternberg (1986) is an illustration of the subject of cognition. Even if it does occur in one's major specialization, formal approaches to one's competence or anything else are undoubtedly

uncommon [28]. For instance, McCutcheon (1992) asserts that even among experienced teachers, formal conceptions of pedagogy are uncommon. Similar justifications are offered for other specialties by Schon (1987). However, formal theories can have a significant impact on performance and comprehension of performance when they are present [29].

Of course, different methods are employed by different people while creating metacognitive theories. The construct may occasionally use something known as phenomenological bootstrap. Children and adults utilize others' cognitive experiences as a springboard for a broader analysis of the nature of cognition [30].

Jennifer Livingston sums metacognition up as follows:

- metacognition and intelligence,
- metacognitive regulation and mental monitoring,
- metacognitive knowledge of person parameters,
- workload parameters and tactical parameters [31].

Metacognition should be “consciously included in the curriculum at disciplines and age levels. A teaching-oriented metacognitive approach helps students learn to test their level of learning. As a result, the student’s success and development of their learning skills [29,p. 337]. An emphasis should accompany teaching related to metacognition in each discipline because the type of monitoring will differ according to different fields. “They should assist students to establish sound metacognitive methods and educate them to train these strategies in the classroom,” according to the statement made as part of teaching field concepts and skills.

When a student is performing an essential task, he is on duty with a performance objective and is always concentrating on the standard of thinking and acting at that moment. The metacognitive queries “How can I do this better?” and “What have I gained in the past that will benefit me now?” are sometimes involved. The student will occasionally inquire, “What can I study to aid me in the future?” He will be on duty with a personal training goal as his main priority, but at other times in life, when “what else can I learn now?” will ask, he will be on duty. A Performance Goal and a Training Goal differ somewhat in the relative emphasis placed on the two methods of learning from experiences, employing prior learning for present purposes and using current knowledge for the future.

The core functions of learning are data collection and analysis and knowledge sharing. Deeper abilities and knowledge should be connected to both of these processes. They should be regarded as important gatekeepers who control what information is shared and useful. Here, the terms “higher-level skills” refer to the ability to assess, categorize, draw conclusions, pinpoint issues, and reflect [32]. The skills described concern the capacity to distinguish between the facts, conceptual justifications, and presumptions ingrained in the information and understanding offered. The development of advanced knowledge necessitates the production of a variety of text, models, images, and multimedia, which calls for high-level talent [33]. Cognitive skills should be

viewed as abilities that must be gained to master various practice types. These abilities are talents that people develop via their practical actions. It is essential to learn both of these skills to address social learning challenges. Through individualized instruction, students are taught to use conceptual and factual knowledge in practical actions in real-world settings. Understanding when and why different cognitive actions will be used is known as conditional information [34]. It can be viewed as factual information regarding the relative usefulness of mental processes. For instance, Lorch et al. (1993) found that university students could distinguish between ten different reading scenarios with varying demands for information processing. For a better arrangement of their learning, students choose several tactics depending on which situation they felt applied them. Students' perceptions of the relative seriousness of the demands made on their mental skills varied among the 10 scenarios as well. According to recent studies, affect success is still developing, at least throughout middle childhood [35]. For instance, Miller (1990) discovered that while preschoolers exhibit contingent awareness about their learning, they do so to a lesser extent than older kids. Similar to older youngsters and adults, younger kids appear to be less successful in selectively diverting their concentration based on conditional task requirements [36]. Mature comparison studies conducted by Justice and Weaver-McDougall (1989) revealed a correlation between knowledge of the relative efficacy of tactics and strategy use [37].

Numerous research back up the idea that talented students possess semantic, procedural, and conditional cognition skills. Achievement often gets better with such gained information. Personalized learning can increase the value of education. Students can value specific learning areas or activities. Teachers or parents can encourage such values are two key issues to explain. Effective teachers can motivate their pupils to do so in two main ways: by giving them the cognitive tools they need to grasp and value the material they are learning, and by inspiring them to think about its possible applications outside of the classroom. The goal of creating a tailored learning method should go beyond enhancing students' self-regulation abilities and highlighting the importance of understanding them.

Metacognition begins with a person becoming mindful of what they know and don't know (but is curious). Therefore, the person can choose what needs to be learned and create techniques to support him in achieving his knowledge objective. As a result, the person will discover that metacognition is a component of a problem-solving strategy for individualized learning.

Let's assume that pupils are inspired to learn so they can better themselves. In that instance, students will use an intentional learning technique, going above and beyond simply finishing a task, to accomplish their learning objectives. That is the self-educational approach to solving problems.

Any cognitive task's solution is accompanied by experiences that are not connected with the content of the problem itself but with how it goes. Imagine that someone is solving a crossword puzzle: some questions seem easy, and that one is confident in the answers, and some are perceived as impracticable; sometimes the person enters a

suitable reply, but doubts its correctness and sometimes feels that the right word is spinning on the tip of the tongue, and after a while, it comes to the mind. These are all examples of metacognitive feelings.

In conclusion, for educational experts, three benefits of a social cognitive approach to directed learning are of particular significance: In addition, it links students' self-regulative processes to particular social learning or behaviorally enacting experiences and can explain their reciprocal impact. Finally, it states two operations through which directed learning is achieved, self-sufficiency sensations and strategy use, and can explain their relation to student engagement and success. A social cognitive approach should be useful in directing academic analyses and intercessions to the extent that it makes students' directed learning processes visible and educable through specific academic skills.

Constructivism

J. Piaget first formulated the theory of constructivism for learning. Piaget singled out four factors that determine the nature of the child's cognitive development: balancing in the process of adaptation, as a balance between assimilation (when the stimuli of the external environment are integrated with internal structures, in other words, for example, new knowledge "flows" into the existing one) and accommodation (when the existing schemes to the specifics of unique environmental conditions, in this case, the available knowledge is not enough, they must be supplemented or changed to solve new problems), maturation, as a biological process, operational experience, as the child's interaction with the environment, as a result of which the formation of ideas about things and social interaction, as interaction with people, as a result of which the formation of opinions about things occurs [38].

Aspects of active learning nature can also be defined in Piaget's assimilation and accommodation theory, such as Dewey. Piaget assumes that it is realistic to expect mutual communication between them and students in the traditional way. At the same time, a teacher tells students they are listening. Piaget argued what a student perceives and that this may not be the same as what the teacher said [39]. So it was not what the teachers always taught what students to learn. Another reason for rejecting the traditional method, as Stewart (2010) reports, was that he disagreed with the associated behavioral theory that knowledge emerges outside of the learner [40].

Piaget was also intrigued by how the mind functions during learning. He was particularly interested in how people connect with the outside world. According to Piaget (1953), life is the steady balance of these increasingly complex forms with their surroundings. Piaget, therefore, believed that all living things have an inherent propensity to adapt to their surroundings from birth. The concepts of Dewey's active learning are comparable to those of Piaget's acculturation process. When individuals within a species and stages within an individual undergo alteration, the adaptation process continues.

According to Piaget, assimilation and adaptation are the two complementing procedures that make up the main adaptive mechanism that promotes mental

performance in humans. Assimilation is the process by which a person interprets an experience in light of it [41]. As a result, assimilation is a mental process in which a person adapts their behavior to the circumstances in their world. The person will observe a novel arrangement of an old arrangement. Assimilation occurs when information and a particular experience are combined.

The process of creating (through adaptation) and reconstructing knowledge is intellectual development or learning. The movement from discomfort to balance (initiated by the organism's outer periphery) is an active, harmonious, and assimilation process. According to Piaget (1990), active learning has four principles: students must create their knowledge to be meaningful; students are the best team when they are engaged and interact with the materials; education should be student-centered and individualized; social interaction and collaborative work should play an essential role in the classroom. Hence, learning is constructive knowledge; and provides a stimulating environment with concrete materials and activities [42].

According to Cobb, Yaker, and Wood (1992), the field of mathematics education perceives learning as the process of constructing internal mental representations [43]. They used different basic assumptions to understand learning and to develop learning theories. Researchers, education reformers, and teachers based on learning mathematics worked from a constructivist perspective in the last twenty years.

Constructivism emphasizes the learners' idea of building on their knowledge by engaging in mathematical applications, mainly through social interaction. The student has an active role in the educational procedure and makes sense of the information by using their own experiences, current beliefs, and knowledge [44]. Therefore, constructivism's essential element can be expressed as follows: Students do not store the information presented in separate pieces. Instead, they develop arguments to understand and relate information to construct and internalize new knowledge [45]. The realist foundation of David Ausubel's theories holds that "what the pupil already knows is the one most essential factor impacting learning." Their perspectives on the impression of meaningful learning and how this might be supported by expertly planned explanatory teaching [46].

The constructivism theory is based on two primary principles. Von Glassersfeld (1989) argues that knowledge is not received passively. The cognitive subject actively constructs it. The cognitive function is adaptable and serves the experiential world's organization, not exploring ontological reality [47]. Therefore, knowing is active, individual, and personal and is based on previously formed knowledge. The second principle of constructivism means that the cognitive function does not discover an existing reality but adopts a proposed theory of truth to the experimental world. Therefore, the promotion of knowledge is only the natural metaphors are the experiential world. Still, this knowledge is not supposed to represent a reality independent of our experience (i.e., an ontological world). Constructivism argues that learning is the operation of constructing rap structures of interpreted knowledge. Students do not transmit information from the outside world to their memories in traditional views;

instead, they imitate the world's interpretations based on their experiences and interactions in the world. How one interprets the world, existing metaphor is an influential factor affecting what is learned, at least as a world feature. Some may even argue that it cannot be constructed if an individual interprets incompatible or non-compliant information. Piaget believed that our current knowledge has evolved. The desire to learn and to know has been structured from infancy. The basic concepts in our understanding, such as mathematics and science, have been made over generations. Each successive generation uses the previous generations' basic concepts, combining and changing them; new ideas emerge [48].

In conclusion, the constructivist educational atmosphere can be thought of as a setting where a student is free to learn new information, make connections between what they currently understand and what they learn, and experience new things.

Cognitive Constructivism

The genetic epistemology of Jean Piaget serves as the foundation for the cognitive-constructivist movement. Due to his research into knowledge and cognition, Piaget dubbed himself a practitioner of epistemology. Because he held that our genes shape how our knowledge is structured within us and that this structuring changes as we age, he coined the term "epistemology genetic."

The development of mental mechanisms in a kid can, according to Piaget, shed light on the nature and purposes of those mechanisms in adults. In those other words, because child psychology dynamically reproduces the process of an individual's socialization, evolutionary rules are a feature of both the kid's mind and scientific reasoning. Authentic learning and critical reflection proponents as well as American philosopher John Dewey's pragmatic viewpoints are strongly reflected in it. American social philosopher Dewey's works on education had a significant impact in the early 20th century. He emphasized education's pragmatic or practical components, which he saw as crucial to creating a better society. He published *School and Society* in 1899, which contained his opinions on curricula, instruction, and schools as institutions. *Schools of Tomorrow* (1915) and *Democracy and Education* were later publications on education. Because they marked a striking break from the scientific rationality that predominated traditional teaching practices at the time, Dewey's beliefs regarding education were vigorously and widely discussed. Dewey advocated a child-centered approach to education as opposed to a curriculum- or teacher-centered one. Those who then formed a laboratory school at the University of Chicago looked for and put some of these fundamental ideas into practice. The procedure brought about the American Progressive School Movement, whose main objective was to create democratic citizens. The populace had a stronger social conscience and was better equipped to adapt to the culture of the industrial age. In the early 1900s, progressive schools appeared everywhere in the US.

Even Dewey, though, disagreed with a lot of what he observed taking on in the name of progressive education. In *Experience and Education*, Dewey, a real pragmatic, stated his views (1938). Many people had misapplied his concepts or used some of them

to the exclusion of other significant elements [49]. After World War I, his influence on American education waned, but he kept writing into the 1950s. Dewey's influence on active learning can still be seen in the notion that learning must be active on the part of the student. The curriculum's content should include elements that equip students to contribute to society as useful citizens. Dewey also supported the notion that education should be more child-centered than curriculum-centered. However, he shouldn't ignore physical factors in favor of cognition and effective domain [50]. Dewey's most influential ideas include those on democracy in educational processes and the relevance of content and learning activities. He believed that schooling should mirror processes that adult citizens employ in a democratic society. Schools were called upon to be more democratic by allowing students to say what they learn and how they know it; thus, teachers ensure the content is relevant [51].

Authentic learning is a concept based on Dewey's principle of content relevance. The current definition of active learning also includes the idea that educational procedures and outcomes should be valuable and worthwhile outside of the four walls of the classroom. The underlying premise is that genuine methods of instruction and assessment have a favorable influence on student involvement, motivation, and achievement. Students consider their academic work to be more important. Five-year research conducted by Fred Newmann and Associates was given funding by the US Department of Education. They concluded that reform initiatives fail when the intellectual caliber of teacher and student activities in the classroom receives insufficient attention [52]. Authentic learning approaches were endorsed to promote legitimate academic rigor because they consistently engaged students at a high cognitive level.

Reflective Practitioner: In his book, Donald Schon defined the principles for maintaining an intellectually stimulating workplace characterized by ongoing learning and advanced practice. The framework is general and applies to all fields. However, education proponents of the active learning approach have firmly adopted the fundamental idea of routinely reflecting on the effectiveness of professional practice and continuously developing how services are delivered based on collaboration, expertise in the field, and personal viewpoint of results. Because it takes a lot of professional opinion and flexibility to establish and maintain a learning-specific environment for kids. Advocates of active learning largely reject the notion that one dimension should apply to all of them. Compared to their peers in traditional schools where the same curriculum and activities are repeated for all students, teachers in active learning environments have higher expectations for their professional development. Thus, professional reflection, collaboration, and flexibility are core activities of teachers implementing active learning approaches. Shared values can be found in Dewey's constructivist learning, Piaget's assimilation and adaptation theory, and Vygotsky's social constructivism theory. Contrarily, in a course using a cognitive constructivist method, students construct reflective models of reality. The textbook, the teacher, or the student's thoughts are all correct because they are all judges of what is true.

The focus of cognitive constructivism, also known as individual constructivism, is on how people build their internal knowledge and their sense of self. According to Piaget, it is based on the premise of cognitive constructivism, which holds that assimilation and adaptation—the two cognitive processes involved in learning—interact to produce knowledge.

By studying “assimilation and harmony” combined to establish “equilibrium cognitive,” which is balanced with addition to prior experience and openness to new knowledge, a child’s logical thought process is strengthened. The process of accommodating involves re-evaluating and re-forming such pre-formed categories in light of fresh experiences or after the information has been digested.

A group of mental “schema”—representations of experience or things—is referred to as “cognitive equilibrium.” According to cognitive constructivism, the interaction of these two processes forms the basis for a child’s or another student’s development of cognitive equilibrium, which maintains a balance between newly protected information and the information that is already available. The cognitive equilibrium consists of the cognitive schema—units of comprehension for experiences and actual objects. The scheme is formed by interpreting experience action and grammar when applied holistically, creating a round and dynamic understanding of the concept.

In conclusion, cognitive constructivism is founded on the notion that new experiences and prior knowledge influence how knowledge is absorbed, categorized, reorganized, and perceived. This impact can help shape meaning in a dynamic, creative, and innovative environment.

Social Constructivism

Social constructivism is built on two leading researchers’ work: Piaget and Vygotsky [53]. Bryant (2003) stated that Piaget’s theory provided a basis for the development of constructivism [54]. The underlying reason for this claim is that children come up with their concepts to understand the world [55]. Similar to Piaget’s theory, Vygotsky’s approach is the other pillar of the social constructivist theory. Vygotsky (1978) emphasized the social context’s role in the learning process and discussed social communication’s facilitating role [56]. Therefore, social constructivism serves as the foundation for the interplay between both the individual and the social setting in the process of learning knowledge, according to Vygotsky (1978). examines how this process limits and enhances both knowledge and abilities [57].

Because it concentrated on learners’ thought processes while participating in diverse tasks and so tied learning, Piaget’s work on developmental stages and learning via action was crucial [58]. The work of Bloom and colleagues on the complexity of thinking patterns, Bruner’s findings on the function of schemas in learning, and Perry’s (1999) work on the significance of fluid grouping in learning in post adolescents are additional foundational ideas for Active Learning Approach [51,p. 28]. These cognitive constructivists have had a significant impact by advancing crucial theories regarding the fundamental mechanisms of learning and the environments that support mental development. Collectively, the cognitive constructivist orientation helped shift

educators' focus on the process of thinking rather than the content to be mastered. This emphasis contrasts sharply with educators' assumptions and focuses on a behaviorist orientation with its related premises [59]. Albert Bandura, an American psychologist, was also concerned with thought processes used to construct knowledge. His writings drew attention to the role of self-regulation in the learning process. Self-regulated learners possess a metacognitive awareness of the strategies. They learn and become proficient in using the method to increase the amount of information they know [60]. Lev Vygotsky, a Russian psychologist, has had the largest impact on the development of modern Active Learning methodologies among Cognitive Constructivists. Vygotsky showed that kids were capable of much more than what they initially set out to do on their own. Following those efforts, chances to get feedback from or engage with a more experienced learner who can scaffold and model efficient ways are presented. Scaffolding interactions during learning activities allow the less mature learner to internalize processes employed by more experienced learners. The less mature learner can then utilize these processes in subsequent independent actions to further his competence as a learner.

Piaget also acknowledged the significance of social context for learning. "... The person would not come to organize their operations if he did not exchange ideas and cooperate with others as a whole..." [61]. Both Piaget (1970) and Vygotsky (1978) identified the exact role of social change in intellectual development and cognitive growth in their accounts [62]. However, there was a difference between Vygotsky's (1978) and Piaget's (1965) perception of social interaction. Vygotsky's approach was mainly directed toward social interaction between the learner and the more skilled peer. Unlike Vygotsky's idea, Piaget (1965) valued social relations between equal peers. Due to the inclusion of both orientations in the study, this disagreement significantly enhanced the value of this investigation. As a result, interactions between students and teachers both increased learning and construction knowledge. According to Vygotsky (1978), "learning awakens various internal development processes that can function only... when (the student) interacts with the people around and with their peers" [63]. On the other hand, social context is not enough to just build mathematical knowledge. The student's ability, previous experience, and understanding also play an essential role in learning. According to this calculation, Vygotsky (1978) invented the proximal developmental zone (ZPD) structure and defined it as follows:

Any student's actual degree of development, as judged by independent problem-solving, differs from their prospective level of growth, as measured by problem-solving under adults' supervision or in partnership with more experienced peers [63,p. 86]. ZPD has two levels of development, as can be seen from the definition. The first level is intended for a single learner to do or accomplish individually. The second level describes what this student can do with support. There is a zone between these two levels. According to Vygotsky (1978), "the distance between the actual development level as determined by independent problem solving and the level of potential development through problem-solving under adult guidance or in collaboration with

more skilled peers. [63,p. 85] As Steffe (1991) stated the ZPD of a particular mathematical concept can be determined in the constructivist learning environment due to the interaction [64]. Hence, interaction is crucial to support the student’s capacity to reconstruct mathematical concepts through modifications. According to Vygotsky (1978), the more talented peer or teacher plays a vital role in change by exchanging ideas with students. As a result, the learner can bridge the gap between the two developmental levels. It is shown in figure 3.

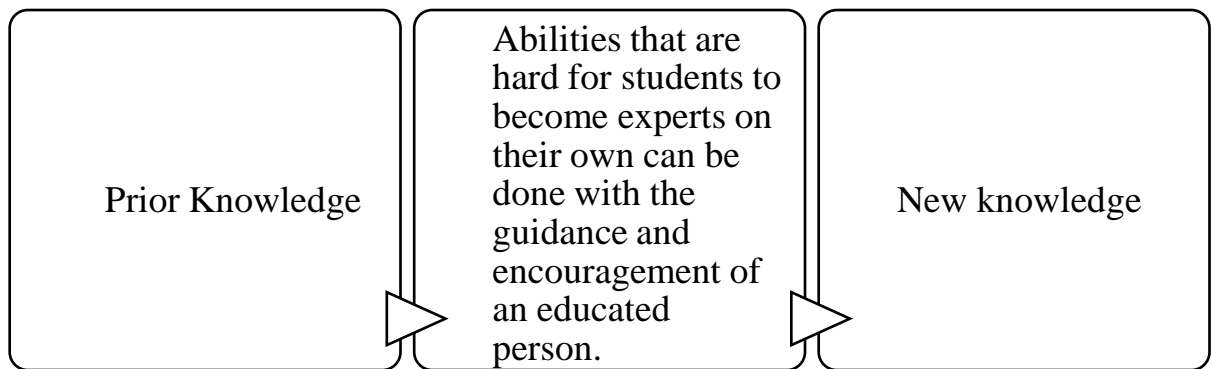


Figure 3 - The Zone of Proximal Development

Vygotsky claimed that higher mental functioning initially appears through interaction with others before it exists in the individual in his work on The Zone of Proximal Development. Human learning, according to him, “presupposes a specific social nature by which infants develop into the intellectual life of those around them” [63,p. 88]. The writings of Vygotsky have significant ramifications for lesson design, classroom social interaction, tracking learning processes, and assessing learning.

Another idea from Vygotsky’s theory is scaffolding; framing is support for interaction and direction in the proximal developmental zone. According to Bruner (1985), they let kids do as much on their own as they can. If so, the mother, other children, or other teachers’ activities make up for what they are unable to perform. When learning, the instructor offers students an “indirect form of consciousness” that they take over once they have mastered the material [65].

Students need to recognize the solution to a particular class of problems before generating the steps that lead to it without assistance. Since the child is seen as building or actively building, the social environment is part of the necessary scaffolding or support system that allows the child to progress and develop new competencies. As seen in the light of scaffolding theory, children do not passively assimilate new strategies directly from adult assistance. They need to take on an active entrepreneurial role and rebuild the task according to their understanding, simultaneously expanding their knowledge.

The intersubjective is where learning begins, according to Vygotsky’s concept, because the dialogue is where thought begins. The tasks’ goals, objectives, resources,

and circumstances must be understood by both the teacher and the student [66]. The common meaning of signs and symbols that emerge in conversation during routine teaching-learning activities allows for intersubjectivity. The youngster redefines a bad circumstance from an adult perspective after intersubjectivity is complete. Youngsters can eventually assume responsibility once they comprehend and share an adult's viewpoint [67]. As a result, a teacher cannot be effective unless they take into account the child's interests, background, and viewpoint. When both the instructor and the learner have attained intersubjectivity, they do not misinterpret one another and participate in the relationship equally. Children increasingly take the reins when it comes to learning.

It was suggested by Cole and Wertsch in 1996 that the researchers' attention to the conventional controversy between Vygotsky and Piaget, focusing on the mind's ontogenesis vs individual psychogenesis, was excessively restricted. They disregard the key distinction between Piaget and Vygotsky. They consider the key distinction to be how culture affects mental development [68]. They expanded on Vygotsky's notions of the active person. Practice, speech, and thought were prioritized by Vygotsky as forms of prolonged therapy [69]. A dynamic individual and a dynamic setting are needed for joint construction. In other words, a playful child and making it functional is the basis of environmental theorization [70]. The third crucial component in the co-building process, after the active individual and the active environment, is culture, which is the accumulation of past generations. The main element, the dynamic person, and the dynamic environment can interact with one another.

The significance of collaboration and social meaning-making is one of the fundamental tenets of social constructivism. Learning happens in a social setting. Our internal representations react to this bargaining process through conceptual evolution, sharing of opinions, and peer testing [71]. Interaction between peers and teachers fosters the development of shared meanings and common understandings. That is knowledge's cultural component. Social constructivism, as opposed to cognitive constructivism, emphasizes "collective learning." To help community members stand out pupils, teachers, parents, classmates, and other stakeholders have a role to play. The optimal technique is "Group learning," where the teacher serves as a facilitator and an advisor, following Social Constructors' emphasis on the active, contextual, and social nature of learning [72].

The foundation of social constructivism, a kind of constructivism, is the notion that social context cannot be divorced from learning. When cognitive constructivists believe that learning placement may be achieved merely by contact with external inputs, social constructivists believe that this culture and language have a big impact on how they update the learner by their world models [73].

We learn through social and social activities, according to social constructivists and socioculturalists. Everyone agrees that learning is a process of actively creating meaning. Through conversation and reflection with peers and teachers, meaning is shaped and knowledge is formed. Social constructivism of this kind is seen in senior

student groupings that are always changing. While learning, they converse with one another. This demonstrates how the constructivist theory of learning and how we educate are related. Teachers should be enthusiastic about generating new knowledge and have carefully thought-out assignments that appropriately test students while also utilizing what they have already learned. According to Bruner (1996), enabling students to build the story while being guided by an expert is an effective teaching strategy. The task of assisting pupils in creating common meaning falls on the teacher [74]. Teachers must know when to encourage students in learning important and pertinent components of the work and when to resign to build a successful scaffold. In the process of learning, it is important to promote the interplay of social and personal experiences. Feedback from social interaction and conversation can be beneficial. That is the kind of feedback that students can provide to the teacher and one another.

Making the story understandable and the pupils' internalization of the material differ, according to Vygotsky's (1978) idea of "internalization." Authoritarian discourse (presenting ideas) and dialogic discourse (creating meaning) are defined by Leach and Scott (2000) [75]. These can be used to help pupils comprehend the story. Leaders must be aware of the present understanding they bring to the classroom to develop lines of reasoning that engage with students' prior knowledge. To accommodate the apparent "desire to learn," the teaching approach has been modified. Clarified learning objectives, conversations with students about areas of doubt, less authoritarian speech, and more guidance are provided. Students now have more opportunities for dialogic dialogue, which will help them craft their stories and improve their scientific literacy [76].

Vygotsky, considered the child to be an active creator of his knowledge, but unlike Piaget, in theory, known as cognitive or personal constructivism, the importance of social connection in learning and development was stressed by Vygotsky. This method is frequently known as social constructivism [77].

In conclusion, culture and social interaction are the foundations of social constructivism; knowledge is dependent on the traits, viewpoints, and experiences of the observer rather than reflecting reality [78].

Kolb's Experiential Learning Theory

David A. Kolb created Kolb's experiential learning theory, which was later published as a model in 1984. The research of Berlin-based gestalt psychologist Kurt Lewin served as his inspiration. According to the experiential learning theory, a person's skills and work requirements can be identified in the same language that they can be assessed [79]. The four-stage learning cycle and the four different learning styles are the two levels on which Kolb's experiential learning theory operates [80]. The holistic perspective of Kolb's method takes into account experience, perception, cognition, and conduct. The majority of Kolb's theory is concerned with students' internal cognitive processes. According to Kolb, learning involves picking up abstract ideas that may be used flexibly in a variety of contexts. According to Kolb's view, fresh encounters catalyze the creation of novel ideas. Knowledge is formed through the process of learning, which transforms experience [81].

Experiential Learning Cycle

A four-stage learning cycle is typically used to illustrate the experiential learning style idea. “All the bases” are covered by the learner. The cycle is shown in figure 4.

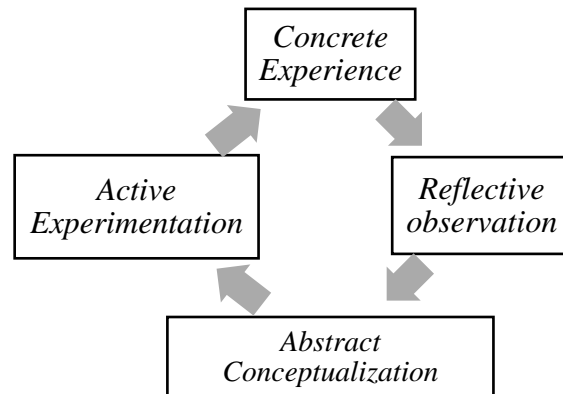


Figure 4 - Experiential Learning Cycle

1. Concrete Experience - encountering or reinterpreting a new experience or situation.
2. Reflective Observation of the New Experience - any discrepancy between experience and understanding is critical.
3. Active experimentation: The student tests out his or her theories on the environment to see what happens.
4. Reflection results in a fresh notion or a modification of an existing abstract concept (one has learned from experience). Kolb’s cycle of four distinct learning styles is shown in figure 5.

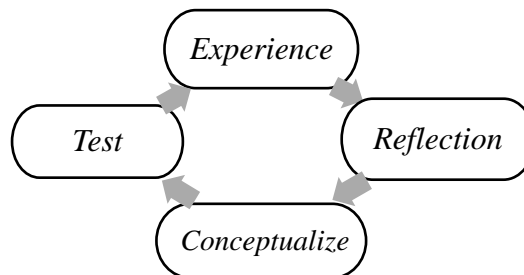


Figure 5 - Kolb’s cycle of four distinct learning styles

Based on a four-stage learning cycle, Kolb’s learning theory (1984) defines four main learning styles (see above). Kolb argues that certain people have a natural preference for a different type of learning style. A person’s preferred learning style can be influenced by a variety of variables, such as their social environment, their educational background, or even their core cognitive makeup. The learning style choice is the result of two pairs of variables or two different decisions we make, regardless of what influences it. They are depicted by Kolb as axis lines with “overlapping” modes at

either end. The east-west axis known as the Processing Continuum (how we approach a problem) and the north-south axis known as the Perception Continuum (how we think or feel about it). It is shown in figure 6.

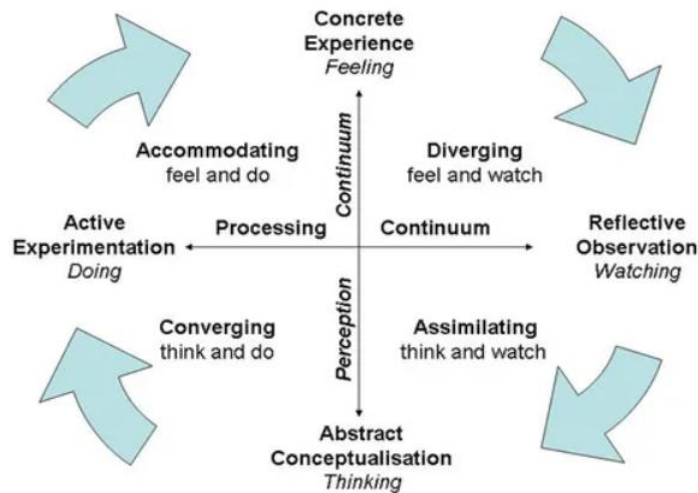


Figure 6 - Processing Continuum

Note – Source [82]

Kolb thought that we couldn't operate on both variables concurrently on a single axis (for example, thinking and handling). These two-choice selections determine a person's learning style. It is frequently simpler to visualize Kolb's learning styles' structure as a two-by-two matrix. Because each learning style combines two preferred learning styles, this is the case. The matrix also emphasizes the terms diverging, assimilating, and accommodating, which Kolb used to describe the four learning methods. It is shown in table 1.

Table 1 - Kolb's learning styles as a two-by-two matrix

	Active Experimentation (Doing)	Reflective observation (Watching)
Concrete Experience (Feeling)	Accommodating	Diverging
Abstract Conceptualization (Thinking)	Converging	Assimilating

Knowing someone's (and your own) preferred mode of instruction means that learning is facilitated. It is important to use a focus that best suits the situation and learning type preferences because everyone responds to some extent and requires stimulation from all learning styles.

A member of a divergent group has access to several points of view. They're delicate. They acquire knowledge, use their imaginations to solve issues, and prefer to observe rather than act. They excel in considering concrete cases from a variety of angles. Because these people perform better in settings requiring idea development, like brainstorming, Kolb described this type as "differentiated." People with various learning preferences have wide-ranging cultural interests and a passion for learning. They are drawn to people, have a propensity for creativity and emotion, and are frequently talented artists. People with various learning preferences favor receiving individualized feedback, listening with an open mind, and working in groups.

The choice of learning must be assimilated, and this calls for a plain and logical strategy. More important than people are ideas and concepts. They are exceptional in comprehending a broad range of information and organizing it in a precise, logical style. People who learn by assimilation tend to be more interested in abstract ideas and pay less attention to concrete details. In this mode, people are more drawn to theoretically coherent theories than to methods with a strong practical foundation. For professions in knowledge and science to be effective, this learning style is crucial. People with this learning style love to read, investigate lessons and analytical models, and have time to reflect on topics in informal learning environments.

People with similar learning preferences can locate problems to practice on and solve problems. They are less interested in people and rather in technical work. Finding real-world applications for concepts and theories is best done by those with convergent learning styles. They make decisions and solve problems to address queries and problems. In contrast to social or interpersonal challenges, technical activities and problems tend to pique the interest of those with convergent learning styles. Expert and technological abilities are made feasible by the convergent learning approach. Convergent types enjoy simulating, trying out new concepts, and working with real-world applications.

The hands-on, intuitive learning approach of adaptive learning is more important than logical reasoning. These folks rely on the assessments of others and favor a hands-on, experienced approach. They are drawn to taking on new tasks and activities and carrying out ideas. In most cases, they make decisions based more on "instinct" than on reason. Facilitating learners will frequently turn to others for information rather than conducting their study. The majority of people learn similarly.

A framework is needed to develop an active learning environment to achieve well-defined learning goals and group activity guidelines. Otherwise, the ground may become chaotic. Experiential learning provides a catalyst framework and collaborative learning to engage students in active learning.

There are two different ways to transform experience through reflection or action. We can develop a learning concept if we start with a concrete experience and guide students through meditation (abstract conceptualization). We use the deductive approach to start with concepts and principles and apply them to solving problems (active

experiments). An effective way to reach students, as the meaning one's said: "first induction, then deduction" [83].

In summary, experiential learning theory is a four-stage cycle that includes four basic learning modes (styles); their synthesis leads to higher learning levels. The cycle can be divided into inductive and deductive learning activities.

1.2 Pedagogical and Psychological Features of Active Learning Methods in Teaching Mathematical Analysis Course

The most popular method for converting traditional teacher-centered education into an innovative student-centered learning environment is active learning. Active learning, which is often referred to as teaching strategies that lay the onus of learning on students, has garnered a lot of interest in higher education [84]. The education process aims to establish teaching materials where students can learn and use that information in ways comparable to those they would encounter in their jobs as a strategy to encourage student engagement.

Developing a pedagogy for an active learning methodology is a challenge faced by educators all over the world. Therefore, it is not surprising that active learning methodology has become a preferred teaching method. It encourages students to discover and express themselves. Many active learning techniques have been widely exposed in the past few years, and the techniques are ready to work on websites.

It is known that the ways to increase the quality of education start with the comparison of teaching methods and techniques. In this context, since the early '70s, there has been a critical study and modernization of traditional educational methods and forms and the related crises in many universities of countries. Enhancing education to suit the demands of contemporary society is one method to get around this. At the beginning of the XXI century, many Kazakh universities are transitioning to a credit system according to European standards. High school teachers commonly use new technologies and traditional methods to achieve independent work in the educational process. One of these is active teaching methods. The primary efforts of university teachers aim to increase the cognitive activity performed practically. Increasing teaching activities has always attracted the attention of researchers. That is understandable because it is associated with meeting expectations to improve the educational process's efficiency and quality. However, the concept of "active methods" is still consistently supported. To some extent, it prevents the development of the activity or creating of a unified concept of its formation and creating a system of approaches. Many Russian and Kazakh scientists tried to give different descriptions of activities. It is based on teachers' activities to apply content, type, method, approach, and tools to acquire knowledge, skills, and ability in their practical application, creative activity, and personal development [85]. Activity is an activity of the student in self-management. Therefore, this is a tool for this activity to increase efficiency in educational activity and management, that is, to take into account to increase and maintain his specially created conditions for the motivation of students. It is known that student learning activities are

carried out independently at different creativity levels. In this context, it is necessary to decide on the student's optimal activity level and self-discipline in certain situations in solving the problems related to general education activity and education. It satisfies the following relations: management dimension, mainly the task of recognizing learning management complexities. If the level of autonomy in student activity is low, it is unnecessary to have maximum activity. Because the level of control is low, the student will face some difficulties. On the other hand, the higher the management level, the lower its activity and independence. However, increasing activity leads to Moselle's management dimension, leading to the inevitable erroneous conclusion: the less the teacher's leadership role, the more active students become. The most important thing to consider here is the motivation of students. In high motivation, a decrease in control parameters leads to an increase in related activity. Difficulties with low motivation further reduce the student's interest in the subject and even lead him to deviate from the goal. When we talk about learning activities, we can mainly talk about the thinking activity of students. Since our country's independence, works related to political and social action have been published [86]. E. K. Grtisbaeva's results provide a comprehensive analysis of the country's ongoing political reforms and trends and the current state of higher education, its role and content, development strategy, and political education issues [87]. Y. K. Babansky [88], N. A. Danilov [89], M. I. Mahmutov [90], I. D. Zverev [91], Psychologists B. R. Aitmambetova [92], A. A. Beisenbaeva [93] studied the problems related to the formation of students' social activity in the country. K. B. Zharykbayev [94], MA Kudaikulov [95], ... scientists-teachers who studied the didactic basis of cognitive activity and cognitive activity. T. I. Shamova [96], and Z. A. Karaev [97], scientists studied the psychological basis of cognitive activity and cognitive activity. L. S. Vygotsky [98], and K. Mukanov [99], ... searched aesthetic activity as the creation or engagement only in works of art.

If we say that the world's technical activity is to participate in this circle of craftsmen, we would be poisoned. Therefore, cystic activity is the organization of interaction with the principles of aesthetics in everyday life, in all aspects of social life, in the family, in the workforce, in relationships with people, and nature. The pursuit of creativity is any failure, factual, cruelty, and suppression of blindness. An indicator of aesthetic activity education can be multifaceted. They are eager to perceive and understand works of art: and keen to read fiction; Aesthetic freshness is close to the good, the bad's willingness to cut the left. Musabayeva (1986) identified the pedagogical basis for forming students' local lore activity in the educational process. Also, she identified pedagogical opportunities to create local lore activities [100]. Myrzabayev (2004) considered the didactic possibilities of active learning in developing students' creativity [85,p. 4]; M. Kurmanov identified students' cognitive activity in the training of future physics teachers [101]. Scientists have tried to reveal various high educational studies' scientific and theoretical essence in several countries and abroad. For example, Aliyev (1986) conducted 'The activation of students' learning activities based on pedagogical universities' psychological and pedagogical materials through

interdisciplinary communication' [102]. Mikhailenko N. I. (1987) studied the title 'Activation of the attitude of high school students to life in the process of socially useful activity,' [103] Valerievna (2005) studied titled 'Active learning methods in the system of methodological training of computer science teachers,' [104] Kartal (2007) worked by 'Active learning in primary science teaching the success of the students, attitudes, and effect on retention,' [105] Weltman (2007) was engaged by 'A comparison of traditional and active learning methods: an empirical investigation utilizing a linear mixed model,' [106] In Kazakhstan, Mamykova's (1998) work as 'Formation of creative activity of future music teachers in the educational process of higher education,' [107] Sadykova's (1999) work as 'Pedagogical bases of increase of creative activity of music teachers in the process of professional training in higher education,' [108] Alinova's (1990) work as 'Activation of activity,' [109] Amanbayeva's (2002) work as 'Pedagogical conditions of activity of students,' [110] Begaliyeva's (1998) work as 'Based on working with students of the Faculty of Philology, conditions for the formation of didactic training of future teachers and the formation of students' self-awareness,' [111] Gantman's (1982) work as 'The development of the creative activity of students of vocational schools in the study of the course Aesthetic Education,' [112] Darkhanov's (2000) work as 'Pedagogical conditions for increasing the cognitive activity of students in schools with special education,' [113] Nurlanbekova's work as 'English Development of students' cognitive activity through Kazakh poets and writers' works in language lessons,' [114], Mirzabaev's (2004) work as 'Didactic formulation possibilities of active learning for developing students' creativity,' [85,p. 4] Tanirbergenova S. K. Pedagogical conditions for the formation of students' active use of educational process methods [115]... which are close to the topic of the research, which are based on the cognitive psychology and pedagogical-theoretical principles of increasing the theoretical activity of students

The main objectives of the theoretical framework of the content of the subject of ideology for high education are forming the student's ability to think in the acquisition of knowledge and cognitive activity. In this regard, the article on the objectives of the education system, the Law of the Republic of Kazakhstan "On Education," states that: When studying the history, traditions, and customs of the Kazakh people and other peoples of the republic, it is important to concentrate on the achievements of indigenous culture [3,p. 115]. That is the creative activity of students, the relevance of the development of creative abilities. What do we need to do? What measures should we take? In the search for answers to these questions, the teacher must consider different teaching methods, use innovative teaching technologies, improve methods, and the methodologically correct organization of non-traditional types of lessons. In other words, it is necessary to develop students' interest in learning, and their desire to learn on their own. Such work helps students master learning effectively, resulting in enthusiasm, enthusiasm, demand for learning, creative work in solving various learning tasks using previously acquired knowledge, skills, and abilities, and learning to control, manage and develop their actions. In general, it is known that the main driving force for

the progressive development of civilization is an active person. Therefore, students' activity in the learning process; the degree of self-learning is always one of the main problems in higher education teaching. That is because the teacher's actions cannot be based on the students. The educational process, which is closely linked with the currents of time, must always provide a wide range of general development.

The basis of this is in-depth knowledge and upbringing. We have a situation where only the teacher explains, the students listen, and the question is answered. Learning is carried out according to the plan. Therefore, such a lesson does not allow the child to think to develop a thinking system. Socially, a person's activity and independence are the key to his work's success and the essence of ingenuity. Therefore, one of every teacher's tasks, the future specialist, is to create conditions for students to have higher education qualities. Most of the above authors consider the activity of students in the classroom, giving such phrases as "consciousness and activity," "consciousness, activity, independence," and "conscious activity and independence."

Anastasiou (2014) reveals his thoughts on teaching and learning contrary to this teaching approach. He said:

"The Latin word *insignare*, which is the root of the verb *teach*, is to mark with a sign. This sign should be one of life, inquiry, and rising to understand. In the real world of the school, there may be an understanding of the intended content, conformity to many more advanced methods of thought, and mobility, or lack thereof, for other study and learning activities" [116].

Today, we can convey the technological and informational revolutions of the modern world and the networked society as environments where the socialization of information and its processing operations in the production of knowledge in an interconnected environment such as social networks continuously, in virtual learning environments, i.e., information communication systems in general.

Paulo Freire discussed that instruction should be about learning more than teaching; in a class, students construct their information individually and socially [117]. How might higher mathematics teaching be used when those connections allow for the incorporation of information, social communication between students and teachers, and interaction between students looking to produce knowledge, learn, and receive an education? How should we situate ourselves in our actions for teaching mathematics in the light of this cultural diversity? the capacity to discriminate between the principles of activity and awareness and the idea that only when the material is absent or is the product of inert consciousness can purposeful activity occur. Moreover, if students are not active, they will not fully participate in the learning process. Thus, one of the main conditions for achieving active education goals is using the term "active participation," which assumes passive and passive methods. Activity (Latin word *activus* - active) is a mental manifestation, an aspect of human behavior manifested in an intense or specific action [118]. One of the most critical issues for the science of pedagogy is the student's point of view in any activity and the student's role in the activity as an excerpt. The main thing is that it is a passive performer of someone's task and a creative activity

during the action. That creatively implements the set goal and recreates the existing object, offering additional tasks [119]. Human activity is manifested in the process of satisfying one's needs. That shows the difference between the dynamic behavior of animals and human activity. The behavior of animals is a feature of the action's nature (it results from the struggle for survival, the manifestation of adaptation to natural interrogation, body composition, and a set of instincts). They seem to pre-determine the object they consume and express an active desire to master it. The process of meeting their needs provides a complete adaptation to the external environment [120].

Zharykbayev et al. (2001) stated that human activity is different; the activity source is human needs. Needs are formed based on education. During the development of the child's activity, consciously directed action is restored under the influence of education and upbringing. According to him, the development of any anatomical, psychological, and personality traits of the body as a whole can be congenital, allowing the nervous system to develop. However, these opportunities are realized only in connection with upbringing [121].

Activity is classified as elementary activity, biological activity, and social activity. Each science studies these types of activities in its way, depending on its scope. Activity in sociology is one of the main characteristics that characterize human behavior and actions. It is divided into political activity, social activity, and mental activity. Political activity refers to the degree to which a social group is involved in the political process. This behavior can be explained by the social group's desire to affect and transform the political system to forward its objectives. Social activity is a set of actions of a social group, a community in a specific historical period, aimed at the conscious solution of society's task set. Social activity is reflected in society's various spheres (political and social affairs, labor, culture, and life). Mental activity is the subject's activity in implementing the orientation, which depends on the individual's characteristics, i.e., the ability to react to a particular situation, behavior, and decision-making. Activity in psychology is the nature of a person's interaction with the external environment. In this case, activity is assessed about the action. In psychology, activity is divided into personality activity, situational activity, search activity, and excessive activity. Person interaction can be above the level of situational requirements of dominant activity in the situation. The subject overcomes internal and external barriers to action (e.g., indifference, carelessness, indifference). This type of activity is manifested in cognitive activity, selfless devotion, and excessive activity in creative situations; search activity is one of the many types of behavior aimed at changing the situation, the prospects of which are unknown. Students' cognitive activity is characterized by the desire to learn, overcoming difficulties in acquiring knowledge, and the maximum use of the will's strength and energy in the act of thinking. That is not just an external activity: signing, copying, reading a book, and internal creative and active thinking [122].

Faculty need well-supported forums to access and assess their presumptions, participate in reflective dialogue, and take appropriate action in their educational

practice because they are adult learners [123]. Alison (2011) found five pedagogical practices promoting active learning as follows: “reflecting on their work; creating a language to describe their metacognitive awareness; demonstrating the disciplinary practices they want and expect students to follow and expressing those intentions to pupils; communicating their instructional objectives and justifications with their students to practice pedagogical transparency; and encouraging students to reflect on and have a conversation about their needs, goals, and learning experiences” [124].

As with many comments of researchers, the reflection gets deeper and a more knowledgeable perspective. Reflective ability enhances the growth of metacognitive knowledge. Higher cognitive awareness, often known as “cognitions to cognitions” or “someone’s self-thinking,” [30,p. 397] - moves awareness to a higher analytical level; one is aware of the awareness of the person so that he can make conscious and deliberate decisions about the application. Instructors should be especially helpful in developing metacognitive awareness because students have to get different viewpoints in the classroom. Students and instructors in such analysis roles must both become more active participants in the learning and successful practitioners to be able to analyze, express, and make active, informed decisions. Instructors should let their students model and develop students’ assumptions about a discipline related to identifying and explaining aspects that require a clear demonstration, explanation, or questioning. To approach pedagogical transparency, one must reflect on one’s practice and cultivate metacognitive skills. Also, instructors must share their pedagogical goals and rationales with our students. Basic learning theory meta-cognition, according to Kulesz (2007), “supports the idea of transparent teaching as it is crucial for students’ learning” [125]. Students are focused on difficult subjects. They require more practice with one of the educational objectives to learn anything new. Rationales could be in support of a specialized or professional position. When and to what extent does inform students of pedagogical justifications enhance their learning? Finally, professors in higher education can encourage discussion and critical thinking throughout the semester. Students may be invited by the teachers to consider, evaluate, and discuss their educational experiences in any subject.

Developing a pedagogy for an active learning methodology is a challenge faced by educators all over the world. Therefore, it is not surprising that active learning methodology has become one of the preferred teaching methods because it encourages students to exploit themselves and express themselves. Many active learning techniques have been broadly exposed in recent years, and the techniques are ready to work on websites. The main modification in instructional strategies is the shift from teacher-centered to student-centered learning. Following are some student-centered, active learning strategies from Michael and Modell:

“Case-based or problem-based learning co-operative learning, collaborative work, and all forms of group work, Conceptual reorientation techniques, Think-pair-share exercises or peer teaching, discovery education, inquiry-based education, and Learning improved by technology” [126].

While we have given the characteristics of pedagogy in higher education, let us consider the pedagogical requirements of mathematics education in higher education. Mathematics education worldwide has adopted a mathematics teaching “reform agenda” that mainly emphasizes problem-solving, creativity, and discussion to improve understanding and tendencies toward the subject [127]. According to Schuh (2004), “student-centered practices” put a particular emphasis on “transform” teaching approaches, where teachers and teaching are supported for students’ learning outcomes [128]. Sally suggested a “connective” teacher who valued small-group work, problem-solving, and group and class dialogues (and was observed to put her beliefs into practice); for him, it is crucial to expose various approaches and even misunderstandings, and the objective is always to develop insights that support procedures in mathematics so that students think “like mathematicians” [129]. Higher mathematics instruction and learning are viewed as a single “activity” that is influenced by the “school learning” goal and the desired “results” of test scores and mathematics knowledge. Additionally, resources (curriculum/assessment, methodologies), class relations and the division of work, as well as communication standards. The entire “structure of activity” of social connections must be considered as a dynamic, live, leading social media totality. Inconsistencies within the system’s components, as well as conflicts between the system and other entities with whom it engages, historically explain why the system is adaptable. In particular, Vygotsky’s position on academic learning is very clear-cut. Vygotsky (1986): The internalization of mathematics, which takes place jointly, initially socially and publicly, through interdentally activity and conversation, is a function of the personal, private “intra-mental” mathematics learning of students [130]. In traditional classroom educational interactions, each student’s comprehension of mathematics tends to emerge in the foreground during work tasks in sporadic ways. Students mainly concentrate on operational rather than conceptual aspects of their maths [131].

It is challenging to describe the theoretical underpinnings of various active learning pedagogies since they are frequently used interchangeably and lack a clear definition [132]. Five pedagogies were recognized by Jonassen (1991) as active constructivist pedagogies concerning the six constructivism pillars specifically, those designed around problem-based learning, discovery-based learning, inquiry-based learning, project-based learning, and case-based learning [133]. Other active learning pedagogies exist; they are quite structured, have devoted websites, and have vibrant communities. These include long-life learning, flipped classrooms, peer-led teams, and collaborative learning. They are predicated on the notion that high mathematics education will work with pupils who are engaged in active learning. Let us consider the techniques of active learning pedagogies for higher mathematics education by showing in table 2.

Table 2 - The techniques of active learning pedagogies for higher mathematics education

Name of active learning pedagogy	Attitudes
Problem-based Learning	student's experience with creative activity
Discovery-based learning	intellectual property, intrinsic motivation
Inquiry-based learning	Curiosity, making research visible and highlighting the significance of the subject
Project-based learning	Communities, exploration through an interdisciplinary lens, knowledge transferability
Long-life learning	Retention, directness, open-mindedness, and responsibility.
Self-directed learning	Enthusiasm is a harmony of actions to connect life and experience
Peer-led team learning	Communication, motivation, self-confidence
Technology-enhanced learning	Experience, professionals' innovation.

Problem-based learning(PBL)

A well-liked active learning technique that complements the constructivist educational paradigm, curricula, and learning philosophy is problem-based learning. The problem-solving method's importance is that the teacher poses a problem to the student and solves it himself. It is critical to how to put a problem, how to solve it and to understand the contradictions. Indeed, a well-organized approach stimulates the child's activity and develops the student's experience of creative activity based on knowledge acquisition and creative use.

Innovative - teachers only the cooperation of teacher and student in the classroom practically substantiated students provide active learning activities in the classroom [134]. For example, when working with reference signals, they show diligence, comprehensive thinking, memory, and interest in problem-based conversation, reading discussions, research experiments, cognitive games, and independent work with the book. Creating a problem environment during the learning process is optimal for increasing students' cognitive activity in effective learning. When thinking about problematic issues, the basis of students' knowledge, level of thinking, and ability to compare, analyze, and summarize must be considered. The design of the key questions should take into account how to involve more pupils in the search. According to Okon (2000), problem-based lessons serve as the foundation for problem-based learning, which in turn increases the volume and quality of students' activities [135]. The issue-based learning environment, research, and development abilities, as well as students' ability to "combine theory with practice and utilize their knowledge and abilities to build a realistic option for a defined problem," all depend heavily on the development of problem-solving skills [136]. Lessons such as the student's readiness for academic work, checking homework, linking new material to it, creating a problematic situation, and students from the primary problem.

The peculiarity of problem-based learning is that the teacher does not present the knowledge readily but sets a problem for the students. The most critical issue in this

technological method is not learning or memorizing new knowledge but solving several problems. The mechanism of psychological processes that take place during problem-based learning is as follows: a person who encounters a new problem, unknown, incomprehensible, has a crisis of surprise, and asks, “What is its meaning?” The question arises. Further, thinking follows a particular scheme, that is, prediction, justification, and verification. The student then conducts a mental search on his own or with the help of a tutor. Problem-based learning is based on the creative acquisition of knowledge using teaching methods. PBL has come under fire as a methodology that frequently lacks objectively aligned assessment techniques and as a curriculum that is frequently poorly conceived and implemented [137]. Critics of cognitive science also point out that problem-solving abilities are most likely not teachable. Thus, it may be impossible to accomplish one of PBL’s main objectives [138-140].

In conclusion, PBL’s definition of pedagogy eventually refers to students managing their own and team research to perform and resolve diverse research problems. Internally motivating learning environments, frequent explanatory feedback, and student reflection are crucial [141]. PBL emphasizes student-centered, process-oriented environments that include collaborative approaches, reflection, and self-evaluation, which is essentially a sign of motivating learning when it comes to constructivist aspects.

Discovery-based learning

The creation of the approach for discovery-based learning belongs to Jerome Bruner. However, Seymour Papert was among the educational philosophers who had an impact on the structure. Exploratory research was inspired by Bruner’s paper “The Act of Discovery” from 1961, which utilized constructivism in education [142]. According to Bruner, the goal of education is to increase content knowledge. After completing formal education, each student can learn and become “an autonomous and self-acting thinker” [143]. Activity is a teacher and a student’s joint action to strengthen self-talent to overcome passive and stereotypical teaching and learning forms. The manifestation of the subject is a unique transformative action on the phenomena and objects around the student. It can be concluded that new innovative methods can be better implemented in the modern technology of teaching. Its primary purpose is to enter the world table of knowledge from the age of globalization. In this regard, both the content of knowledge and its requirements will change. Innovative teaching deepens knowledge and stimulates students’ interest in learning, stimulating their enthusiasm and interest in learning. Thus, the essence of pedagogical technology is developing students’ ability to use active learning methods creatively. A fresh reading of the pon’s calendar plan marks the beginning of the integration of pedagogical technology into the teaching process. Since the primary purpose of training is to form an individual who can learn and develop independently, the textbook’s theory. The way the materials are presented can be adapted to learn on their own to meet this goal. It is possible to discover the student’s qualities that lead to discovering inventions and innovations.

According to Bruner [144], discovery-based learning encourages students to learn by doing and helps them acquire “intellectual property” (or ownership) over their learning as they explore and generate new information. According to Bruner, this is externally driven and uses rewards and penalties to motivate pupils, such as detention, suspension, and expulsion (including notes, awards, mugs, and scholarships, to name but a few). Discovering each student’s “inner orientation” or inherent incentive to study is the main goal of discovery learning [145].

Focus on exploratory learning; as a result, students manipulate their environment, engage in challenging experiments, and engage with fascinating themes in a setting where prior limitations are acknowledged as information and inquiry capability. For novice students, in particular, most proponents of exploratory learning acknowledge the necessity for instructor leadership and curriculum and process limitations; pure discovery learning is only advised for knowledgeable individuals with substantial experience and competence. The instructor includes direction, subject selection, training, and modeling. They also offer simulations, feedback, sample problems, methodologies, and content information guides.

Student participation is crucial for student-centeredness when categorizing discovery-based learning based on constructivist components. The procedure and personal self-evaluation are both crucial. An essential component of pedagogy content information, content transferability, collaborative approaches, and self-reflection are discussed but not explicitly stated.

Inquiry-based learning (IBL)

“Say and I forget,” is a proverb. I can recall the show. Inquiry-based learning is frequently described as “include and I understand.” [146]. IBL is described as “Activities that start with a query and do research solutions, building new knowledge as information is obtained and comprehended, discussed, and reflected discoveries and experiences on newly found information” by Savery (2006) [131,p. 16]. The IBL approach often adheres closely to the steps of the scientific method. It is also referred to as a science education model. The questions that focus on questions and frame inquiries serve as the direction for inquiry-based classes [147].

For pupils with problem-solving abilities, Banchi and Bell (2008) outline four distinct levels of inquiry, ranging from novice to expert. Verification, structure, direction, and openness are the first three [148]. Since they receive the most instructional guidance, inexperienced problem solvers tend to lose confirmation. Instead of being a pedagogy, it is described as an approach. It begins by posing a question inside a predetermined context; the student’s responses are already known. The verification inquiry’s main goals are inquiry and the introduction of problem-solving [149].

However, the technique and question are typically pre-determined by the instructor’s inquiry to students to find an unknown response to a problem pre-determined question. Structured queries allow queries, by comparison, unknown outcomes or responses. Teachers who are controlled and supervised continuously direct

learning and offer feedback—an investigation course—for approval. But in open inquiry, students’ work equals that of professional scientists. Be a scientist by formulating questions, planning, and carrying out experiments, then reporting the results. In these situations, students ask themselves questions, carry out research, discover previously unreported findings, or provide minimally conventional replies with the help and guidance of the teacher.

In conclusion, IBL places a lot of emphasis on students becoming proficient in scientifically based self-management inquiry, including question posing, investigation, solution building, response, discussion, and outcome reflection. The scientific method as a whole and student skill development are both integrated.

Project-based learning

Project-based learning was centered on an active learning approach, with a project as the primary student outcome. Project-based pedagogy is described as “usually involving completing complex tasks that culminate in a realistic product, event, or presentation to the audience” by Eckardt et al. (2020) [150]. Thomas (2000) expands on this description by arguing that project-based learning must include these five essential components: Learning must occur student-focused with teacher facilitation and guidance; projects should be primarily motivating or focus on: a genuine interest in students; projects are encouraged by leading questions or misrepresentations of problems; students must question, constructively complete researches, and create new knowledge [151].

In conclusion, the goal of project-based learning is to provide answers and provide solutions. Although content knowledge is important for the project’s success, its major focus is on the process. The project should be directed and genuinely motivated by the student, but it can also be utilized as a teaching tool. Since content transferability does not seem to be a core component of teaching, it is vital to emphasize student-centeredness, process and content, collaborative techniques, reflection, and evaluation.

Long-life learning

A type of self-directed education that emphasizes personal growth is lifelong learning. Although the term “lifelong learning” has no official definition, it is frequently used to describe learning that takes place outside of a conventional educational setting like a school, university, or corporate training. Lifelong learners are independent, self-driven administrators of their learning. They can recognize and address learning needs, as well as monitor, double-check, and assess learning strategies to satisfy these demands [152]. The long-term goal of lifelong learning requires learners to develop more talent and motivation. While continuing in the organized learning environment of the classroom, their learning processes [153]. Acquiring this qualification calls for assistance from the instructor, bravery, help, and challenges along the route. Faculty must be aware of how the classroom environment influences students’ development as lifelong learners long before they graduate to properly encourage students’ propensity for continuing their education beyond graduation. Through pedagogical decisions, trainers can play a significant role in promoting lifelong learning. A variety of abilities

and attitudes related to lifelong learning have been fostered by student-centered pedagogies [154–157]. These pedagogies provide students the freedom to select. However, not all student-centered environments are created equal, and few researchers have looked at how various student-centered environments can impact results for lifelong learning [158].

In conclusion, the researchers hypothesized that to be a “self-directed learner,” one must first be a “lifelong learner.” The ability to learn independently should be improved, much like content knowledge.

Self-Directed learning(SDL)

Self-directed learning, according to Boekaerts [159], is “a complicated, interacting process that encompasses not only cognitive self-regulation but also motivational self-regulation.” Four presumptions about self-directed learning models are listed by Pintrich. He contends that SDL entails student control and participation in their education. They arrange their ideas, drives, actions, and surroundings; they keep track of how well they’re doing in terms of achieving their objectives. These systems serve as a middleman between the individual, the environment, and success [160]. Zimmerman emphasizes that for students to succeed in self-directed situations, they must feel capable of taking responsibility for themselves [161].

In conclusion, a careful balance of attitudinal, mental, behavioral, and situational difficulties in teaching is necessary for successful self-directed learning and students’ growth as self-directed learners. Instructors are increasingly encouraging their students to develop independent planning, personality, and reflection to successfully promote progress in all of these areas.

Peer-led team learning(PLTL)

Peer-led team learning is a teaching method used in undergraduate science, math, and engineering courses that integrates peer-led workshops into the curriculum. A well-defined active learning paradigm called peer-led team learning can be employed in conjunction with the conventional course structure that is ingrained in university institutions and incorporates small group interactions between students [162]. An instructor will frequently give the class an assignment to complete before beginning an active learning exercise. The activity is then completed by the students. There is usually some sort of full-class procedure after pupils have concluded their courses. Each of these stages can be implemented in a variety of different ways. The complexity of the issue varies greatly depending on its type. Students should be given the option to complete multiple-choice questions (varying in difficulty and cognitive level), examine graphs, create models, work on case studies, and do a variety of other projects. Students may approach the issue alone, in official or informal cooperative groups, or both. Following the group discussion, one team member might record the group’s response; alternatively, each team member can record their response; or students may not be required to write at all. Clickers may be used for full-class processing. The teacher might request that the

students present their ideas to the class for debate or provide an explanation of the solution [163-164].

Johnson and Johnson (2009) found that cooperative learning is positively correlated with several psychological qualities, including self-esteem, independence, and confidence, in addition to this learning improvement. Self-efficacy is a term used to describe students' beliefs about their capacity for success and as a learning predictor [165]. According to the social interdependence theory, when there is positive interdependence, peer collaboration and interaction would boost students' learning [166].

In conclusion, distinct studies of peer-led team learning that isolate the impact of specific components offer proof that shapes a common concept of effective active learning.

Technology-enhanced learning

Today, the widespread use of new information technology tools in the educational process increases university students' psychological and pedagogical training requirements. Thus, the use of new technological tools, firstly, defines the teacher's role in a new way, and on the other hand, it opens up new opportunities in the formation of students' cognitive activity. The teacher's primary purpose is to teach, educate, and develop students and organize these processes creatively. A university teacher's primary condition to organize their work creatively is to understand their work's essence. In his September 1, 2021 Address to the Nation, the words of the second President: "Teachers with motivation and skill are needed in the educational system. I think that instead of every five years as it is today, teachers should undergo retraining every three years. They need to be the ones spreading new information; the actual enlighteners" [167]. After all, the country and society's future will be the same as the results of teachers' work today. The teacher is a critical figure in the university, which also has a strategic role in developing its professional creativity. Classes conducted by traditional methods only increase the interest of students to a certain extent. It is important to use active learning methods. After all, active learning methods in teaching each technology subject contribute to students' independent learning, increased motivation, and improved education quality.

The most significant potential for teachers' and students' personal development has not yet undergone in-depth psychological and pedagogical analysis. However, the pedagogical practice raises the educational process to the level of interpersonal relationships, which transforms the activity, which can be considered a source of floral growth of both parties involved in the pedagogical process, into joint creativity. Teachers' professionalism in higher education is based on analyzing the pedagogical situation's ability to see blowjobs. It is impossible to predict the diversity of capillaries and the best ways to solve them. Therefore, one of the essential characteristics of pedagogical activity is the formation of university teachers' critical thinking in analyzing the content of educational materials, the choice of methods, and the types of organization of students' educational and cognitive activities and summarizing the

above, a substantive description of the pedagogical and psychological features of preparing students for active teaching technology methods.

Finally, to engage students, raise student retention and recruitment rates, and raise graduation rates, we must devise innovative ways to deliver courses that cater to the various needs of students and take into consideration their constantly changing needs. The use of technology in the teaching of higher education is widely known. It must be backed up by a substantial intellectual and cultural outlook in society, optimized to meet contemporary needs and actively utilizing a variety of techniques. Technology professional development should take into account data such as each student's sociopsychological status.

1.3 Didactic Principles of Mathematical Analysis Course Using Active Learning Methods for Teaching Students

The development of didactic concepts in mathematics education dates back to Brousseau's (2008) consideration of the circumstances in the late 1960s. In a model of teaching and learning systems, he established scientific conditions while simultaneously ensuring the precise creation of mathematical knowledge while observing didactic actions [168].

Brousseau criticized the use of psychology alone [169] to address these challenges for three key reasons: Piaget's research was primarily centered on specific kids, and constructivist modeling of the arithmetic learning process falls short on both counts. Their social and cultural aspects are not given enough thought [170]. Thirdly, these methods do not adequately convey the connections between the didactic action in the issue and the being learned mathematical subject [171]. The theory known as Didactic Situations Theory examines how instructional practice variables relate to the creation of mathematical knowledge. Brousseau translated the French word "conception" into English as "knowing," and "knowledge" into French as "savoir [172]. The following ideal requirements have been established to promote mathematics learning and guarantee the greatest possible learner autonomy:

- "The problem should be best solved using the mathematical concepts addressed by the problem.
- There should be no mention of the sought knowledge in the problem's text or directions.
- It is appropriate for pupils to begin solving the problem using ineffective techniques.
 - Students can determine whether or not their effort was successful.
 - The verifications are suggestive without revealing the answer.
 - Only one of the experimentally viable solutions addresses all of the problem's requirements.
- In a reasonable length of time, several students will find and test the solution, while others can quickly share and confirm it" [172,p. 249].

The aforementioned guidelines enable pupils to shoulder a sizable portion of the burden of creating their mathematical knowledge. However, these rules need to be completely modified, particularly at the high education level. They can communicate with others by taking into account their advanced mathematical knowledge, potential scenarios related to it, the student's expertise, and practical factors. The Theory of the Didactic Situation's main goal is to describe the effective framework of interactions between students, teachers, and the environment, as was previously noted. The interactions inside such systems are thought to be the source of student learning, and these systems' characteristics play a major role in this process. On two fundamental levels—didactic and adidactic—a state is modeled [173]. The adidactic level is with an environment.

The situation contains validation, formulation, and action, following Artigue et al. They are defined as follows: cognition evolves through the creation of an appropriate language, knowing becomes a part of an entirely cohesive body of knowledge, and knowing appears as a means of action through models that can remain implicit [174]. Students' tools during the validation phase may be insufficient and force the teacher to provide additional aids. It can be demonstrated within the context of the given mathematical theory using a few principles or symbols.

At the instructional stage, the instructor advises pupils to resolve the issue to "own" it. Devolution is the procedure that enables students to move into the adidactic stage. The creation of knowledge is rarely entirely the student's responsibility, according to several studies. The stages in which the teacher takes a more active part in the student's stages will inevitably be replaced by the proper construction of a classroom environment [168,p. 119].

Domiciliary mathematics refers to the favorable method of generating and evaluating novel circumstances and their effectiveness in a regulated manner [168,p. 120]. Didactic mathematical engineering consists of designing, organizing, and controlling controlled observations of experimental situations. Specific mathematical knowledge appears to be the most appropriate way to deal with a mathematical problem. A study sought to look into and provide methodological approaches to dealing with this complication in didactic mathematical engineering. The didactic mathematics framework was created to account for the systems' complications in a methodical manner [168,p. 121]. The analysis of the epistemological (about the properties of knowledge), cognitive (about the characteristics of the learner), and didactic (about the characteristics of the education system) dimensions of mathematical knowledge at risk serves as the foundation for the construction of didactic mathematical engineering. The researchers decide on a set of didactic factors during this setup and take appropriate action. These are referred to as the state's parameters. It is a method by which pupils apply their mathematical knowledge to a problem. To obtain the intended learning outcome, the teacher might set these parameters to a certain value. A detailed analysis of students' concepts, issues, and errors is one of the most important design factors. Children's math lessons are designed to encourage the planned development of these

concepts. Didactic mathematics uses situations to apply cases from Didactic theory. This attempts to provide students complete control over creating new mathematical objects and approaches that seem to be the best mathematical tools for the problems that are presented. The mentioned research studies in the area below provide examples and more. Because of its intricate, abstractness, mathematics will be learned, and this concept of full accountability will need to be rethought at the university level. The didactic state approach provides a potent tool for work at the collegiate level. Through its epistemological dimension, activities can be designed and carried out to ensure that certain fundamental components of mathematical concerns to be addressed are present. Here, two separate but complimentary relationships between at least two characteristics of the Theory of Didactic Situation are presented:

- By using didactic mathematical engineering and a strong epistemological analysis of the desired information, the didactic scenario theory aids in the creation of new circumstances. This evaluation guarantees that pupils will encounter “actual” math problems. They will need to respond to these queries and have actual mathematical experience; they will rely on their ability to reason and will make assumptions and try intuitive solutions.

- Both common mathematics lectures and experimental treatments can benefit from the use of the concepts of contract and environment to analyze students’ work [168,p. 122].

The significance of student initiative, the complexity of the material to be taught, particularly in terms of its degree of organization and complex relationships, the variety of problems that can be solved using this knowledge, and the requirement for prior knowledge (though not always obvious) are all spotlighted by Rogalski (1998) [175].

Researchers looked into how these tools were used, focusing on how the didactic contract’s voting system changed. Thus, they detailed the new responsibilities and potential conflicts. That might arise for the students and raise new questions about online education at the university level and institutionalization and reorganization of knowledge while using these environments [176].

Several research cases of the university-level application of didactic situation theory (two on mathematical analysis and one on proof) are presented to clarify the issues raised in this brief review.

Research case 1: Discover a transcendent number across a boundary - a case about the fixed point of the cosine

In a case that tries to portray real numbers in their epistemological nature and explore the key elements of the teacher’s interventions, the first research case has been discussed. At a university in Tunisia, the situation was tested with first-year students. They had already studied the key theorems before the experiment.

Convergence - and were aware of their fundamental components and continuity when they were applied to “well-known” numerical and geometrical sequences. Students sometimes lack a basic comprehension of real numbers when they first enroll

in college and frequently fail to use numbers like $\sqrt{2}$. Students occasionally discover that $\sqrt{2}$ or is it not a real number? The symbols $\sqrt{2}$ or are frequently employed in mathematical writing, and they are regarded as distinct types of writing [177]. However, a conceptual “jump” in academia no longer works with individual numbers or functions, but rather with broad notations (x, f...). Moving from the idea of limits as the “boundary process” to the idea of a mathematical entity having a boundary defined by mathematical rules is also important. For these reasons, research on categories of numbers, both rational and irrational, including transcendental, should be included when discussing the idea of limit. In this way, the Situation’s didactic variable may employ the nature of numbers together with the essential theorems.

This study case’s Situation was designed to assist students to make connections between actual numbers and bounds. The creation and presentation of student graphs were required. The idea of limit was selected since it is frequently the first analytical subject students encounter. The function $y = \cos x$ fixed point and several theorems establish its existence. In this case, additional didactic options included:

- The identification of mathematical objects whose existence is purely formal is made possible by the use of pictures. This results in a rich teacher intervention that makes it easier for students to comprehend how things function. Students can understand that the cozy’s fixed point must exist and see how rapidly various sequences get close to that number.

- Rational accessibility to the formality of the limits is organized by context.

The outcome of the situation’s development and the mediation by the teacher was that theoretical tools are required to guarantee that a sequence is constrained. The last step is to instruct pupils to apply previously learned theorems, and here is where an “invisible” number comes into play. These theorems only permit research on hypothetically existing unknown items. Students’ progress from the certainty that a number exists to know in this way. To know if the value of that integer is the limit of an array, they might round it up to whatever many decimal places they choose. The results of the study, as expressed by the students, were as follows:

“Assume that a sequence ends up being a genuine number. In that scenario, each term of the array approaches that real number within that margin of error, and there is a row n for the margin of error we tolerate.”

Alejandro claims that the project encouraged pupils to conceive of real numbers as conceptual objects linked to other elements in a mathematical theory, such as limits [168,p. 125].

The study case mentioned above for didactic situation theory may be designed using active learning techniques. For the research case, the tactic referred to as “group discussion” is adopted. This is how the tactic may be applied:

The $\cos x = x$ problem was attempted to be solved using the bisection method on a graph to support the claim that it exists. To determine if bisection methods are effective for estimating quantities of unknown values, the instructor initiates a group discussion

on whether these tools always allow them to obtain the “precise” solution to an equation. Using the first sequence (figure 7) as a test case, the following inquiry is conducted to better understand the value of k .

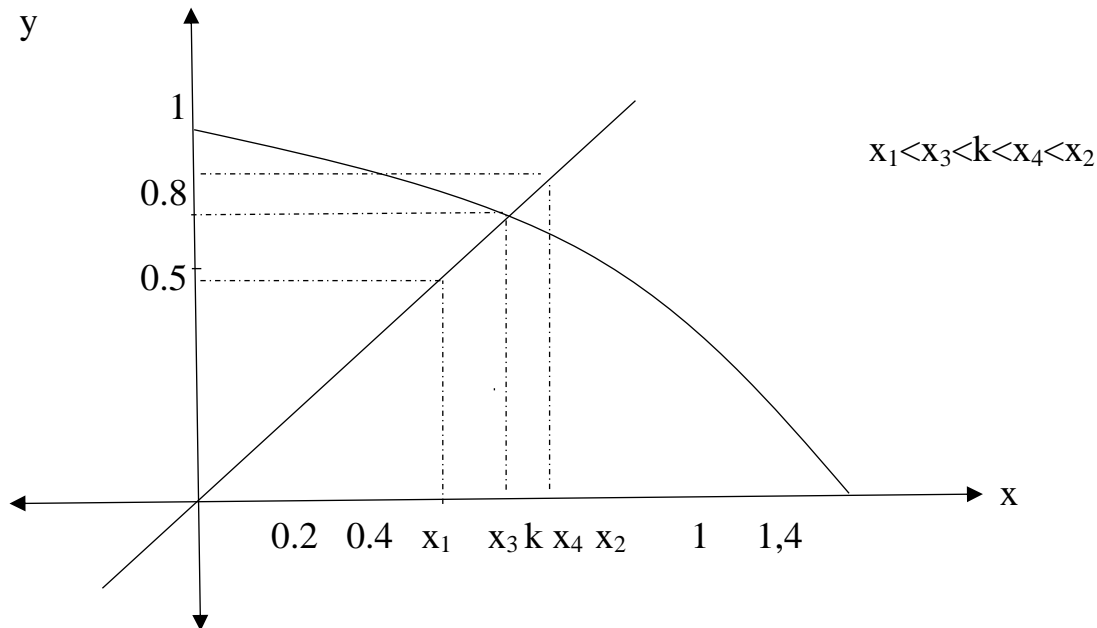


Figure 7 - The graphs of $y=x$ and $y=\cos x$ intersect

The equation $x = \cos x$ with at least one solution between 0 and $\pi / 2$; Graphs of $y = x$ and $y = \cos x$ intersect. However, there is no algebraic method to solve this equation.

The students (S) were startled to not locate the number since, as we mentioned previously, they believed that a number only existed if we could precisely calculate its value. The following modification demonstrates how the teacher’s (T) interventions are required for choosing the suitable formal tools:

- S1: If $u_3 = \cos u_2$, $u_2 = \cos u_1$, and $u_1 = \cos u_0$, then we should select u_0 .
- S2: within the range $[0, 1]$
- S1: But ultimately, the same! Can’t locate the precise value?
- S3: Not even with quality software, Regarding the problem?
- S2: Because we discover a series, it follows that the fixed point of the cosine does not have an exact value.
- T: The same as $\sqrt{2}$?
- S3: Since $\sqrt{2}$ is the square of 2, it has an exact value.
- S1: We can use sub-sequences ...

The students utilize Newton’s approach to obtain a better sequence to find the approximation after realizing that the method is ineffective (the recursive sequence does not merge soon). Once more, the instructor must explain this new approach to pupils and assist them in interpreting the graph using examples.

Research case 2

The second research case shows that the Theory of the Didactic Situation provides an efficient theoretical framework for elaborating cases involving students in the proof and proof processes. A case that was built with undergraduate and graduate math and engineering students in France using an epistemological approach was addressed. It resulted in the choice of two proofs that let students debate the logical state of the letters within a single proof.

It is becoming more and more important to acquire analytical skills and knowledge, particularly at the university level. According to research, existential, universal, and many quantifier manipulation challenges are prevalent in college [178]. When tackling issues, analytical thinkers take a more step-by-step approach and exhibit preferences for internalized formal imaginations as well as externalized formal representations. They can understand and communicate mathematical truths best through symbolic or verbal representations. An analytical thinker would therefore be less concerned with interpretation and validation. The job answers needed to be then formalized in the form of abstract equations. The actual situation thus lost significance [179].

Barrier chose two proofs in real analysis for the experimental portion of his study, and he then examined the quantification techniques that students employed while assessing these arguments. Quantification is hardly discernible in the language of the proofs in both instances. Contrarily, in the investigation of validity, quantification concerns and the logical state of letters both implicitly and very subtly intervene. Here, we concentrate on the Cauchy-adapted proof. The question is to prove that the sequence $a_n = (1 + \frac{1}{n})^n$ gets closer to e .

Proof: For any integer k , $a_k = (1 + \frac{1}{k})^k = \sum_{n=0}^k C_k^n \frac{1}{k^n}$

$$a_k = 1 + k \cdot \frac{1}{k} + \frac{k \cdot (k-1)}{2} \cdot \frac{1}{k^2} + \frac{k \cdot (k-1) \cdot (k-2)}{3!} \cdot \frac{1}{k^3} + \frac{k \cdot (k-1) \cdot (k-2) \cdot (k-3)}{4!} \cdot \frac{1}{k^4} + \dots \frac{1}{k^k}$$

$$a_k = 1 + 1 + \frac{k \cdot (k-1)}{2} \cdot \frac{1}{k^2} + \frac{k \cdot (k-1) \cdot (k-2)}{3!} \cdot \frac{1}{k^3} + \frac{k \cdot (k-1) \cdot (k-2) \cdot (k-3)}{4!} \cdot \frac{1}{k^4} + \dots \frac{1}{k^k}$$

Let us prove that a_k when k tends to infinity.

The $(k+1)^{\text{th}}$ term of the sequence $a_{nk} = \frac{k \cdot (k-1) \cdot (k-2) \cdot (k-3) \cdot (k-n+1)}{n!} \cdot \frac{1}{k^n}$ is

getting larger when k is increased.

$$a_{nk} = \frac{1}{n!} \frac{k \cdot (k-1) \cdot (k-2) \cdot (k-3) \cdot (k-n+1)}{k^n} = \frac{1}{n!} \left(1 - \frac{1}{k}\right) \left(1 - \frac{2}{k}\right) \dots \left(1 - \frac{n-1}{k}\right)$$

As a consequence, $(1 + \frac{1}{k})^k$ is an increasing sequence. Moreover

$$a_k = 1 + 1 + \frac{k \cdot (k-1)}{2} \cdot \frac{1}{k^2} + \frac{k \cdot (k-1)(k-2)}{3!} \cdot \frac{1}{k^3} + \frac{k \cdot (k-1)(k-2)(k-3)}{4!} \cdot \frac{1}{k^4} + \dots + \frac{1}{k^k}$$

$$a_k \leq 1 + \frac{1}{2} + \frac{1}{3!} + \frac{1}{4!} + \dots + \frac{1}{n!} + \dots + \frac{1}{k!} \leq e$$

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = 1 + \frac{1}{2} + \frac{1}{3!} + \frac{1}{4!} + \dots + \frac{1}{n!} + \dots + \frac{1}{k!} \leq e \text{ [168, p.128].}$$

In proof, the step from (1) to (2) is invalid by the principle of substituting infinitesimal quantities on the boundary sum is used. This proof's mathematical content had previously been studied by students who participated in the high school experiment. Barrier thus proposed that this evidence was a prime contender for encouraging student engagement. The pupils were pushed to doubt quantification by it.

Three sessions of the experiment were conducted: one with two or three first-year undergraduate mathematics students as a brainstorming approach, one with two or three graduate students with a high degree in mathematics, and one with four or five freshmen from an engineering school. For around 40 minutes, each group worked on one of the two proofs and addressed the following:

1. List each stage of the proof and highlight it.
2. Examine each stage of the proof, indicating if it can be finished, and offer a fresh formulation. Are all steps complete, complete, and valid?
3. Does the proof hold up?

Then, each group created a summary of its work to be given to other groups. The need for communication has increased the social need and preparation of the synthesis to examine the proof's validity. They aimed to revive some of the arguments or disagreements expected to arise during the proof analysis. These elements, along with questions from other students during communication, contributed to the intervention's enrichment.

The research cases show the potential between the theory didactic situation and the didactics of advanced mathematics to account for the complexity of the mathematical knowledge taught at the universities. Students are better able to understand some of the complex knowledge covered in the mathematical session when epistemological analyses are relevant to design issues. Additionally, these studies contribute to the development of a setting that fosters student engagement and makes it possible for instructors to enrich it.

Conclusions on the section

Mathematics education has addressed philosophical and epistemological viewpoints on mathematics and mathematics learning throughout the previous four decades. It's customary to view learning as a constructive process for placing knowledge and learning, to think of mathematics in terms of fallibility, and to measure learning using constructivist and sociocultural theories. The nature of mathematical knowledge, what it is to know and know mathematics, and how mathematical knowledge is more widely related to learning in social settings have all been carefully explored and seriously discussed. We may look back over the last thirty years and see how important theories like constructivism and sociocultural theories have influenced how we think about mathematical knowledge and learning. With such theoretical concerns, the field of mathematics education has progressed. The position of mathematics teaching, on the other hand, is theoretically unusual and underdeveloped. While theory offers us perspectives through which to examine learning, key theories do not appear to provide clear opinions on education and how education promotes math learning. In the structure of teaching methods, primary objective and subjective parts are distinguished. The method's purpose stems from the fixed, unshakable provisions necessarily present in any method, regardless of its use by various teachers. Theories, didactic provisions, laws and regularities, principles and rules common to all, and fixed components of educational activities' objectives, content, and forms are changeable. The subjective method stems from the students' personalities, characteristics, and exceptional circumstances. The method's objective and subjective relationship is complicated and has not been fully resolved yet. The range of views on this issue is extensive: accepting the method as a purely objective education to the complete rejection of objective principles and accepting the method as a personal and, therefore, unique teacher work. Truth, as always, is between extremes. A method familiar to the whole objective part makes it possible to develop a method theory. It proposes applying the best ways in most cases and making it possible to solve problems successfully, logical choice, and optimization of methods.

The challenges of a mostly lecture-oriented learning environment, the principles and strategies, and different paths for an active learning environment have been tried. However, the steps taken for tomorrow need to start right now. We cannot imagine schools in entirely new conditions - different buildings, teachers, and books. Whatever methodology we choose, should apply to the current circumstances. Therefore, those who were able to find the proposed approaches. Here, we have provided many exciting methods with creative possibilities by searching for applicability in many cases and not fighting the existing ones.

There is only one requirement for communication with students in any higher education practice and selecting teaching methods and tools. In the so-called didactic part of pedagogy, what is required to teach all subjects, including mathematics, is based on the didactic principles of high mathematics teaching.

2 THE METHODOLOGICAL BASES OF TEACHING MATHEMATICAL ANALYSIS COURSES TO STUDENTS USING ACTIVE LEARNING METHODS

2.1 Forms of Active Learning Methods in the Teaching of Mathematical Analysis Course Using at Higher Education

Active learning

Solving a mathematical problem on paper, creating it, discovering it, and researching it are all examples of active learning. Due to the pressure, it puts on the brain to connect and comprehend information, active learning is more successful than passive learning. You have to consider how to solve it rather than simply learning via passive activities (like reading and taking notes). One of the useful techniques to address these learner traits is active learning. Although active learning as a method dates back many years, it was first thoroughly explained in contemporary times by the English scholar R.W. Revans (1971). Over the next two decades, he progressively enhanced the instruction. Revans describes action learning as, in essence, a reflection on experience and claims that comprehension is attained by concentrating on social context issues [180]. For instance, managers communicate with one another and increase their expertise by sharing experiences. Active learning, according to Kyriacou and Marshall (1989), relates to the caliber of the students' mental experiences. The learning process is marked by greater understanding and active intellectual activity [181]. Active learning is a method whereby students interact with the content they are studying by reading, writing, discussing, listening, and reflecting. Active learning techniques prevent students from moving off and gaining a broad range of self-confidence, critical thinking, complex analysis, and entrepreneurship. According to Michael Prince, "Active learning is frequently described as any teaching strategy that actively involves students in the learning process. In a nutshell, active learning necessitates that students engage in worthwhile learning activities and reflect on their actions" [84,p. 223]. The definition of Prince, which was widely accepted [6, p. 5], is derived from the original work of Bonwell and Eison.

"Short, course-related solo or small-group activities make up active learning. All class members are expected to rotate between instructor-led segments when new information is delivered and student answers are processed" [182].

However, these do not provide the necessary conditions for learning. One step beyond that, by comparing the mind to a computer, the individual advocates that it should be ready for the information to be taught. For this, the subject must be interesting. There is no risk that passive learning is comfortable and peaceful. However, at this point, learning takes place; it is pretty tricky. Inactive learning, educators, and students receive a risk, and when they do the conditions, we can mention education [183]. Silberman (1996) avoids giving a clear definition of active learning and examines learning by considering the brain's functional processes according to him, just hearing

and seeing are not sufficient reasons for learning. Learning of their effects on activities is not reflected precisely [184].

According to Felder and Brent (1994), the aim of active learning is; that a given student with feelings of curiosity, ownership, and necessity on the core problem in a free discussion environment, can share his information by linking learning. Moreover, what he or she has learned with actual questions and issues to ensure permanence [182,p. 3]. Senemoglu (2004) stated that students' achievements are primarily aware of their learning paths, depending on their ability to direct their learning. Student learning and working strategies start in primary education, secondary education, and higher education [185].

Active learning can be classified by the following:

Basic active learning

Most learning processes fall into this category. Its main characteristic is that students are taught to know information, skills, or relationships, and then they face difficulties in applying this new knowledge.

The challenges involved in this active learning style can vary. Some examples are discussions between students or the practical application of new skills (for example, music students interpreting a piece of music).

Situational active learning

This type of active learning is based on the work of Lev Vygotsky, who found that the acquisition of knowledge is effective when immersed in a community in which other people try to learn like themselves. Thus, situational active learning occurs when a student has the opportunity to share experiences, impressions, or ideas on a topic that he or she is trying to master with other people on the same path. When this happens, some students take on the role of 'teacher', while new ones start by listening, asking questions, and asking about situations. Over time, their level of education will increase, as they will be able to take on the role of teacher.

Based on problem-solving

This type of active learning is the method that gives the most freedom to all students. Students are presented with a problem related to the ability, knowledge, or attitude they intend to solve; but they are not given any idea of how to find a solution. Thus, students need to be able to find the information they need to solve a problem and know what to do. If they succeed, they will have long-lasting and lasting knowledge.

Features gained through active learning in many countries have become the primary goal of national projects. For example, in Australia (1989), in the published 'Common and Accepted National Purposes of Schools,' It is clearly stated that its primary purpose is to train an innovative and flexible workforce. Some countries initiated more radical reforms that are necessary structural changes suitable for implementing active learning in schools. For example, the reform movement launched in the Netherlands (1994) under the leadership of "core teams" consisting of teachers working in schools to apply active learning environments. They are essential steps that have been taken to spread learning practices [186].

Two main points attract the attention of active learning methods. First is the suitability of active learning to the brain's work; the other is the need for lifelong learning individuals to understand teaching and learning and active learning effectiveness [187]. Dufresne from Cornell University found that research decreased to students every 10-20 minutes during the lesson. Using active learning strategies several times in the classroom can encourage more participation [188].

Another reason why active learning is so popular is that besides its efficacy, desk; positive impact on usefulness, affordability, and supportive learning products essential advantages, we write, such as features, are below.

Usability: Active learning techniques are very short-term, such as a few minutes a wide variety of time, from events to very long activities like a semester, and can be used in slices. Also, active learning is not a single teaching method; it includes many teaching methods increasing its usefulness.

Economics: Active learning, unlike other learning models, additional staff, and expensive vehicles can also be implemented without private spaces.

Positive effects on supportive learning products: In education, academic success is often emphasized. However, the desire to learn, reading habits, working with others, self-esteem, leadership, sharing, and collaboration in many learning situations, such as doing, are less considered. However, as described, information specific to a field is not permanent; at least fast, it varies.

On the one hand, researchers carried out the success of active learning products while increasing positive effects. The learning process outcomes show that active learning is safe and confident for students, respectable, practical learning, and thinking skills in cooperation with others. It enables students to grow up as individuals who can work.

Students may be asked to think or do something and then be given time to think or do something. An instructor who asks a question and immediately calls a student for an answer is not actively learning by this definition. Similarly, giving students a flexible break during a long classroom session is an excellent idea. However, it is not related to the lesson students do, and therefore, their activities are not active learning. Students retain information far more passively through their senses than they do passively. It is one of the reasons why active learning is as successful as it has been consistently demonstrated (such as the material of traditional classes).

About a math topic, the teacher knows more than most students do. The teacher should spend some classroom time teaching students what they know (without explaining, demonstrating, or modeling.) Let's say that a lecture involves even a short amount of the necessary action. In such an instance, the kids will awaken after a minute here and thirty seconds there. In a typical class, the instructor spends the majority of the remaining time with the students. The majority of them will hide more from the audience with what they say and do for the rest of the session than with what transpired in those few minutes. If the teacher performs this after the lecture, learning evidence will be evident unlike anything else. More experienced teachers may decide to increase the

number of activities in each classroom session. There is no ideal amount; how much to do, determines the content of the class session and the level of active learning and comfort.

Active learning focuses on what a learner does, what he thinks, and how he behaves. The actions do not happen with a few simple instructions. The teacher needs a learning environment to enable active learning. Ultimately, these active learning strategies will help develop understanding rather than memorize facts. It gives students the confidence to apply learning to different problems and contexts and gain more autonomy over their learning [189].

Neal (2021) contends that planning is necessary for active learning:

1. Prepare: Modify the curriculum and first-day expectations for the students, get to know the students, utilize extracurricular activities to get ready for in-class active learning, and prepare written instructions, worksheets, and PowerPoint slides for exercises.

2. To motivate students, describe the exercise's educational goal to them, provide tough tasks, and ask them to sum up what they learned at the end of the activity.

3. Absence of Fear: Create an atmosphere where taking risks is valued, allow students to get to know one another, thank them for participating, and engage in "instructional intimacy" (smile, make eye contact, use student names, walk around the classroom, ...) [190].

Faust and Paulson (1998) classified the active learning strategies for students in high education as:

Discussion, note comparison and sharing, and evaluation of another student's work are examples of paired activities.

Cooperative groups in class, active review sessions, work at the board, concept maps, visual lists, jigsaw group projects, role-playing, panel discussions, debates, and games are some examples of small-group activities.

The One Minute Paper, Muddiest (or Clearest) Point, Affective Response, Daily Journal, Reading Quiz, Clarification Pauses, and Response to a Demonstration are examples of individual activities [191].

For individual activities, the instructor should create soon challenging exercises. For small group activities, the educator could instruct students to organize themselves into groups of 2-4 and assign a random recorder to each group if writing is required. Alternatively, the instructor tells groups to assign their recording device, preferably someone who has not yet registered that day. Four is a practical upper limit on group size for many group activities. Unless students sit around small tables, the most significant number that can comfortably interact is four. Even if there are tables, some students in groups of five or more are almost inevitably excluded from group discussions [184,p. 89].

When a trainer poses a challenging question or problem and gives most people or groups enough time for either, they should make reasonable progress towards finishing or finishing. He gives them should generally be between 10 seconds and three minutes.

If they need more than three minutes to solve a problem, divide the problem into several and treat each step as a separate activity. He should stop the activity, call a few people or groups to share their answers, and ask the volunteers if the exact answer he was looking for came. (Instructor occasionally requests volunteers, but not after each activity, which is another command it will be discussed later.) Then they discuss responses or continue with the planned lesson.

Three mistakes that instructors often make when they first start active learning:

- a. trivializing group activities,
- b. doing the activities too long,
- c. call volunteers to respond after each activity [184,p. 90].

Why are they making mistakes? Firstly, If the instructor asks which answers are immediately apparent to most students and then asks them to get into groups to answer, it wastes their time. They know it and will resent the instructor for it. When the instructor does an active learning exercise, it is challenging to justify the time for group work. Secondly, suppose the instructor gives students ten minutes to solve a problem, say. In that case, some groups will finish in two minutes and waste the next eight minutes of precious lesson time. Others struggle for ten minutes, which is highly frustrating and a waste of lesson time. Maintaining the activities short (a good rule of thumb is 10 seconds - 3 minutes) prevents both problems. Thirdly, students quickly learn that they do not have to think about what the students want them to do if the instructor always calls; They can relax and talk about football. Eventually, someone else (probably the instructor) will reply.

Moreover, if they know that anyone can be called in for a response after a minute or two, they will do their best to be ready. Instructors should avoid these three mistakes. Even if an instructor has hundreds of students in the classroom, active learning is almost guaranteed to work.

Many students ask their instructors to say everything they need to know for the exam - not more or less a word - and resent it if they are forced to work in the classroom. The key is to let them know to spend a long time on the active learning front to learn that the instructor is telling them the truth. Not for their selfish purposes, but because the instructor has research showing that this way teaches students to spend more time on homework comfortably, which will not happen. At this point, the complaint is over [192].

When an instructor is doing an active exercise in a classroom, that could not be active learning. Many students can look straight ahead and encourage others to work with each other personally. When he does this a second or third time, he should have little insistence. At this point, he should stop worrying. Research shows that students learn much more by doing something and getting feedback by watching and listening to someone who says what they need to know. In-class activities provide practice and feedback on what the instructor knows students will find difficult with assignments and tests. If some choose not to take advantage of these opportunities, it is their loss - do not worry about losing five seconds of sleep. Moreover, that is all. Educators who switch to

active learning and follow these recommendations almost always say that their lessons are much more lively and fun. The quality of learning has improved dramatically. So the instructor should try it out the next lesson he teaches and see if he has a similar story to tell at the end of the term [186,p. 2].

The Importance of Active Learning in Mathematics Education

The high percentage of student failure in advanced mathematics has increased persistence among mathematicians for many years. Efforts to reduce eclipse failure have focused on successive mathematical analysis courses, mainly due to the course's topics' prerequisites. Most students who fail at advanced mathematics probably do so due to their introductory mathematical analysis experience when they become first-year students. Either because they have been unable or cannot have longer retention of the knowledge of previous topics. Many techniques are used in mathematical analysis courses to improve teaching and make mathematical analysis courses more informative and memorable than traditional ones.

Active learning could be one of the uses of learning methods to address performance and retention of knowledge. Most of the learning effects can be increased in academic achievement if learning methods are used effectively during the education process. Active learning is an approach to learning that makes students active or existential engaged in learning and has distinct learning techniques, that rely on student engagement [6,p. 18].

The Active Learners Institute (2011) found that Active learning increases the education process's influence and performance. According to this research, the percentage of remaining knowledge in students' brains after months is as follows:

“Teaching others 90%, Practice by doing 75%, Discussion Group 50%, Demonstration 30%, Audiovisual 20%, Reading 10%, and Lecture 5%” [193].

The lack of a clear definition of “active learning” is a problem when the topic of active learning in mathematics is discussed. Inquiry-based learning strategies encourage students to investigate mathematical issues, make and test hypotheses, create arguments or solutions, and articulate their thoughts. Students start to understand mathematics as a creative human activity in which they may participate when they learn new concepts through reasoning. In line with contemporary socio-constructivist learning theories, inquiry-based learning techniques emphasize the development of personal knowledge that is supported by peer social interactions [194].

The Instructor's Role in Active Learning

Today, the change experienced in every field also affects educational institutions. Educational institutions have to keep up with this change and continuously renew themselves. The role that teachers will play in this process is of great importance. Because the quality of a school depends on the quality of the education service that its teachers will provide. For teachers to achieve this, both pre-service they need to be well-educated and continuously improve themselves in service [195]. The effectiveness of education depends on course content and the presentation's ability to meet the student's learning needs. The essential task expected from teachers in the information society is to

provide students with the most appropriate learning opportunities for their learning profiles and direct them accordingly [196]. Teaching-learning developing discipline depends on the objective approach of educators to educational problems. The teacher must be aware of what and how to teach. Using his creativity, he should determine the required teaching method well.

Appropriateness of the teaching techniques used is a necessity in the learning-teaching process. Moreover, the teacher can serve the affective dimension of education the closer he/she gets to the learning activity [197]. Traditionally, a “teacher” stands in front of the classroom to control the events, make most of the speeches, convey information, ask questions, evaluate, punish, reward, show, and source, in short; The most active and dominant person in the class comes to mind. The difference between an active teacher and from traditional one:

- to guide students,
- make suggestions,
- explain when necessary instead of implementing their own decisions to do,
- give ideas, guide, and observe their development.

In active classes, teachers do not stand in front of the class and make most of the conversations but instead, attend to the studied activity. He is no longer the only one who asks, the only one to evaluate, the only one who starts or ends the discussion. The effective teacher in active classes is not the teacher who speaks well and explains well. Doing so helps the students become well-spoken, and good expresses, and gives them the opportunity.

The active teacher constantly renews himself. It improves itself pedagogically. In this respect, it is essential for him that students take part in the learning process. Effective teachers have high expectations of success for their students and themselves. Students know very well what teachers expect from them. Therefore, effective teachers should have individual characteristics. Besides, teachers should plan the lesson well and allow students to participate in the planning process.

Methods and techniques should be chosen according to the level, age, and abilities of the students. When using the methods and techniques, tools and technologies should be used. While these are provided, the learning environment should be comfortable and safe. Conditions in which students will be uncomfortable should be changed. The teacher has a personality that eliminates difficulties and must-haves. An effective teacher should use time effectively. It should pay attention to spending time with learning activities. Learnings should be evaluated with the organizers. Because it must be determined at what level learning takes place. An effective teacher should guide students’ learning and personality development by being sensitive to individual and group needs.

According to Özdemir and Yalın (1999), the characteristics of an effective teacher are listed as follows:

Motivating personality,

Willingness,
Closeness and humor,
Committed to success,
High expectations of success,
Encouraging and supportive,
Professional behavior,
Serious, systematic,
Adaptable/flexible,
Knowledgeable,

Text-Source is required for additional translation information [198].

The Student's Role in Active Learning

In active learning, the student is not “an” empty container “or” passive receiver who takes what is conveyed to him and then repeats them as in the traditional. The learner does not take what is taught precisely; on the contrary, he (or she) processes and reproduces them with unique strategies. Of course, it is not easy to do this, to take responsibility for a painful learning process. Active learners have some characteristics; if not must gain them during active learning practices. Schmoker (1991) suggested that the main jobs active learners should absorb are fertile mind habits, self-regulation, critical, creative thinking, and learning skills [199].

- The self-regulated thinker and learner is conscious of his thinking, makes plans, is aware of the resources he needs, is receptive to criticism, and assesses criticism's value.

- The person who thinks and learns critically is righteous, seeking clarity, open-minded, limiting coercion, taking an attitude when appropriate, and being considerate of other people's feelings and degrees of knowledge.

Creative thinker and learner; enter into jobs with unclear answers or solutions, pushes the limits of their knowledge and abilities, and self-evaluation produces, trusts, and maintains standards, where unusual produce ways of looking [200]. The student-centered active learning method leaves learning up to the individual. Through learning, students actively shape their own life, according to the active learning philosophy. Students cannot learn by simply listening, so they should do more than listen. They should read, write, speak, discuss, establish connections with past experiences, apply the acquired knowledge in daily life and solve problems.

Students learn to make decisions and take responsibility in the active learning process. Beyond that, students have the opportunity to learn. Various experimental studies have revealed that learning to think about learning with active learning affects less successful students. Often the best students in schools are expected to be active. However, active learning has changed this perspective.

The learning performance of less successful students has increased significantly, thanks to learning to learn and think. Students use the sources of information themselves for their research. Students carry individual and group responsibility in their projects.

Each student works on a different subject but is related to each other and contributes to the group project. Students share information, interact and collaborate for joint knowledge production.

Bonwell and Eison (1991) mention five strategies related to active learning.

These;

- a. students attend classes rather than listen,
- b. developing learning skills are more important,
- c. students move up to higher levels,
- d. students are encouraged to read, write, and discuss activities,
- e. and students' attitudes and values are taken into consideration [6,p. 34].

Also, in active learning, the learner learns enough to repeat the subject, as in the traditional case. He designs where he can use it, and knows why he learns it. He studies his learning and tries to discover his good and bad points.

Active Learning Environments

Many new types of research on the permanence of learning break old assumptions about learning. Many assumptions such as "Students learn by sitting only in quiet environments, and the learning rate is high only in the morning hours, physical activity prevents learning" disappear with every new research result. All these findings always shout out to educators that learning is life itself. Yes, learning is in every moment of life. The more prosperous and colorful life is, the more diverse, colorful, and rich learning environments. Scientific research reveals that an effective learning environment prepared by considering students' interests and needs positively affects student achievement and learning level. Learning environments are generally known as classroom settings. The classroom is where students try to change and create behaviors in their own lives [201].

According to Bozyiğit (2014), the features in a classroom in which active learning is practiced are; trust, energy, self-regulation, belonging to the group, and sensitivity [202].

a. Trust: People can feel safe and secure in any environment, which is an essential factor in achieving a specific job. Self-confident and ready to learn, the individual is comfortable in the classroom environment. Thanks to this comfort, it will be easier for students to achieve the classroom environment's learning task.

b. Energy: In an active learning environment, when students are directly involved in the education process, they continuously contribute. The student is active and a participant in this process. However, the student is continuously busy with specific jobs. Therefore, the lesson ceases to be boring for the student.

c. Self-regulation: In active learning, students are responsible. Students control their study and learning pace. That helps them to use and develop self-regulation mechanisms.

d. Belonging to the group: In active learning, respect for the individual and humanity is at the forefront before the lesson and training. Therefore, in environments

where active learning occurs, all individuals involved in education, teachers, student administrators, etc., are amicable and respectful to each other. This approach and understanding help students develop a sense of belonging to a group.

e. Being sensitive: Active participation of students in the education process

In learning, students are closely related to the events in their environment. This understanding and behavior encourage students to be sensitive to the feelings and thoughts of other individuals.

Obstacles to Active Learning

The “coverage problem,” or the inability of an instructor to cover as much content in a course that employs active learning approaches as in a course that uses lecture-only, appears to be the initial hurdle that this faculty perceives. Evidence suggesting children learn and remember material better when prompted to actively engage is accumulating. But comparing material coverage to active learning is to make a deal with the devil. Either teach less content and encourage student learning or teach more material and encourage student learning. The first option is not useful. If we expose students to a little less stuff but have them engage with it, they will always learn more. We have all had the experience of students who come to our classes and seem to have little knowledge of what is covered in their previous courses. Despite “covering” the content, the pupils failed to retain it. Students are therefore more likely to be better prepared for the following classes if they actively participate in studying the topic, even in courses where quantity-related entitlements are present. Last but not least, not all of the content has to be covered in class; part of it can be covered by the students on their own. The fact that it takes too much time to prepare an active learning lesson is a second typical argument against implementing active learning into current classes (mainly if an instructor has already taught the lesson and has already prepared the lessons). Without a doubt, employing active learning strategies will need more preparation time from the instructor. However, since active learning tactics may be applied to different lectures, teachers have created active learning techniques that can be employed each time the topic is delivered.

The fact that many professors feel comfortable lecturing and find novel teaching methods scary is a second reason why they shy away from employing active learning strategies.

Additionally, some active learning methods need little to no additional preparatory time. “Exercises for Individual Learners,” as an illustration. The strategies under “Questions and Answers” do not require additional preparation time, merely the time needed to think about the suitable test or journal questions. Some of these techniques only call for teachers to adopt new practices without lengthening their class periods. The time needed to develop courses will, however, definitely rise as we go toward more sophisticated tactics (like certain collaborative learning group projects). Instructors should prioritize the proper techniques at the top of the list if time is of the essence.

Their main worry is that they could “lose control” of the classroom. Additionally, some teachers can be reluctant to adopt active learning techniques in their classes because they fear that their pupils won’t respond well to them. “I tried active learning,

but my pupils didn't enjoy it," is a regular criticism we hear. Most of us have encountered a situation where we tried to start a class conversation but were met with silence. Because both students and professors frequently feel at ease doing what they have been trained to do, these two problems are connected. Therefore, as long as teachers primarily lecture, students will wait in their seats to hear them. As far as "senior" professors go, we frequently devote more lecture time in graduate courses to discussion and problem-solving than we do in beginning courses. Ironically, we discover that senior and graduate students in several subjects are unwilling to use active learning tactics because they are too habituated to the course style at lower levels. Why limit our entry-level students to simply hearing about active learning if we want our graduate students to adopt it? Even while some students like the lecture format, it is not always the most effective method of instruction for all pupils. Not all, recent research on student learning has shown. Students have varying learning preferences and methods for understanding stuff [203]. When we exclusively provide knowledge in one format, such as lectures, we unfairly exclude students who might learn the material more effectively in a different format. Additionally, once they leave the institution, students will need to analyze the knowledge accessible to them in a variety of ways. By employing the methods discussed above, all students will have greater access to the classroom and will be better prepared for the job [204]. It's not necessary to be. Teachers cannot wait for a standstill, it's true. Student commitments are made, and the teacher waits for the ensuing fruitful conversation. Separate activities with clearly defined objectives can, however, direct pupils toward a greater comprehension of the information being covered. A successful active learning class can also be facilitated by ambivalent educators by starting with reduced techniques and progressively transitioning to upper levels.

The Active Learning Strategies

Group discussion (Peer Instruction): An instructor can do it personally or online. Although they are more effective in smaller group environments, the instructor can organize group discussions in any class size. This environment allows the teacher to guide the learning experience. Students must engage in critical thought during the group discussion to assess both their stance and that of others. Students are required to engage in productive and informed discussions of the course material. Given that the unit has previously been sufficiently covered, the discussion is a useful follow-up exercise. Students can explore other ideas by participating in group discussions as a learning approach, which has several advantages. Additionally, they improve students' cognitive flexibility, respect their opinions and life experiences, foster cooperative learning behaviors, and enhance students' abilities. Synthesis and integration enable the teacher to interact with students actively, prepare them better for the class, and realize what is happening in the classroom [83,p. 679].

Think-pair-share: During a think-pair-share exercise, students pause to reflect on the preceding session. They talk about it with one or more of their classmates before sharing it during the formal class discussion. The instructor needs to clarify any unclear notions during this formal conversation. To engage in meaningful conversation,

however, pupils must have prior knowledge of the subject. Therefore, when students can recognize and relate to what they already know, the “think-share-share” practice is beneficial. Therefore, planning is crucial. Before waiting for their talk on their own, give pupils oral instructions to get them ready. When appropriately used, the teacher may reduce class time, keep students prepared, encourage greater engagement and conversation in class, and offer an overall evaluation of their progress. The “think-share-share” approach enables teachers to hear from every student, including the silent ones. This form of instruction is a great approach for all kids to learn, collaborate, and feel at ease discussing ideas. Additionally, it can assist educators in observing pupils to determine whether or not they comprehend the subject matter being covered. Due to time and logistical restrictions, using the think-share pair in big courses is not recommended [6,p. 50]. The teacher can benefit from using think-pair-share. It teaches how to organize material according to the subject matter covered in class. It clarifies where they observe kids, freeing up time to address other concerns and enhancing the class. Interactive gives pupils the chance to communicate with one another.

Learning Cell: Students may study and learn together by using learning cells. The learning cell was created by Goldschmid in 1971 at the Swiss Federal Institute of Technology in Lausanne. A learning cell is a procedure where two students ask and respond to related questions on regularly read items in sequential order. The students comprehend the goal and compose their reading-related questions in preparation for the assignment. The teacher divides the pupils into two groups at random during the following class meeting. One student from each group is chosen to start the procedure by asking the other group member one of their questions. The second student asks a question and adjusts when the first two students discuss the issue. The teacher visits each group during this time to provide comments.

One-Minute-Paper: Reviewing items and offering guidance only takes one minute of paper. The “one-minute paper” doesn’t, however, finish in a minute. The teachers should allow the pupils at least 10 minutes to complete this summary assignment. (One minute after the lecture, write a paper summary.)

The collaborative Learning Group: The group effectively learns various materials for various classes through collaborative learning. Students are given tasks to do in groups of three or more by the instructor. This task might be a project or an answer to a question that needs to be submitted to the entire class. The teacher assigns a group leader and a note-taker to oversee this procedure. The tactic is a superb illustration of active learning. The work that was necessary for earlier participation is reviewed by the students [196,p. 12]. The teacher must plan to involve and utilize the wisdom of every student. Flexible seating is required to set up a classroom layout for groups.

Small-Group Discussion: A tiny discussion group that gives students the chance to speak up in class is another illustration of active learning. Because they feel more at ease with their peers, students are more likely to engage in small group discussions than in normal group lessons. Pupils are permitted to talk to a greater number of students. A teacher can stop playing, do a competition, or assign homework to conduct a small

group conversation with the class. According to statistics, short group conversations are more beneficial than lengthy group discussions in terms of participation, thoughtful expression, problem-understanding, problem-solving, and general knowledge.

Just-In-Time: Just-in-time by employing pre-class questions to establish a connection between students and teachers before the class starts, teaching promotes active learning. Open-ended questions are typically used in these warm-up activities to assist students to be ready for the class and learn more about their learning objectives. (For instance, “What is the general equation for the linear differential equation?”)

Class Game: A fun approach to learning is through a class game. Before a big test, it helps pupils review the subject, and it makes studying fun. The minds of pupils are consistently stimulated by various games like Blitz! and riddles [196,p. 11].

The Open-Lesson: Students’ engagement is demonstrated in the open lesson. Teachers function as observers while students actively investigate a subject and prepare the knowledge to present to the class. Students learn their topics more effectively thanks to the instruction. There are instances when pupils learn and interact with others more effectively than they do with teachers.

The Snowball: The snowball Technique is a way for learners to teach each other essential concepts and knowledge. Students start by working alone. Then they collaborate with a partner. The partners join to make groups of four. Groups of four gather together to create groups of eight. This snowball activity continues until the whole class works together as the largest group.

Fishbowl: Fishbowl is a fun team game for many people (between ten and twenty). The game aims for team members to guess a word in one of three ways - by explaining, by activating the name, or just by saying a single descriptive term. Each method takes one round, and each round has its own rules. The game name is “Fishbowl” because the instructor put paper slips with words in an aquarium.

Mental maps: A diagram is used in the mental map method to graphically arrange the information. The pyramidal mind map depicts connections among the components of the whole. On a blank paper, the educators can draw a single notion or draw an image in the center using pictures, words, and word fragments as representations.

Concept maps: The concept map represents visual organization and knowledge. It shows concepts, ideas, and relationships between them. A concept map is created by typing a keyword (sometimes found in circles, rectangles, or triangles. Moreover, it is then drawing arrows between related ideas.

Learning through research

Role-playing: When put in the position of a decision maker to choose a policy, resource allocation, or other consequences, role-playing is a learning structure that enables students to apply material right away. This method is a great way to draw in kids and provide them the chance to engage with one another while working on their responsibilities. In cooperative groups, teachers may complete this research, and students can complete their assignments in class. When attempting to reply to the subject in terms of their personalities, learners interact more.

The Debate: The debate is a process that consists of formal discussion on a particular topic. Educators put opposing arguments forward in a discussion to argue for opposing viewpoints. There are rules for participants to discuss and decide on differences within a framework defining how they will do it in an official debating contest.

Card sorting: Card sorting is a method that helps design or evaluates any information structure. In a card ranking activity, learners organize topics into meaningful categories and can help educators label these groups.

Muddiest Point: Students are urged to take a short break as part of a fast-tracking tactic. They jot down the section of a book or lecture that they find the most challenging or perplexing.

Brainstorming: Brainstorming is a creative technique that scrambles to find a result for a problem by collecting a list of ideas that its elements contribute naturally. Briefly, brainstorming is when people come together to remove new inhibitions and generate new ideas and solutions around a particular area of interest. People can think freely and suggest as many innate ideas as possible. The organization leader notes all opinions without criticism and evaluates the reviews after the brainstorming session.

Explanatory learning: Exploratory learning is an approach to teaching and learning. It encourages students to explore and research new materials to discover relationships between existing background knowledge and unique content and concepts. It is essential to keep track of events keeping track of engaging events. The approach activates background knowledge that creates enthusiasm and motivation and leads to a personal connection between the student and the subject. Students can join online peer collaboration using reason-mapping software. And then complete pre and post-learning maps and teachers' blog posts or real-world problems.

The methods of active learning based on technology

Video-Reaction: Active learning may also be shown in a student's response to a video because most students like viewing it. The film provides a different presenting style that aids the learner in understanding what they are currently studying. Please make sure the tape is related to the project they are working on right now. Try adding a few questions first so that you can focus and pay more attention before you upload the video. After the video is finished, the viewers are divided into groups or pairs to discuss what they learned and write a review or comment on the film [196,p. 15].

Technological games: They have direct educational objectives or secondary or accidental educational value. There are many different games that educators may employ to enhance learning. However, educational games are still games created to help individuals learn about particular topics, broaden their thoughts, improve growth, comprehend a historical event, or acquire a skill. Video games, playing cards, and boards are among the technology instruments. The game runs to be an instructional method in the educational area as educators and management become conscious of its psychological demand and benefits for learning. The objectives, rules, adaption, problem-solving, and engagement of the teachers are assisted via interactive games that

are shown as stories. One of the technological games named Ka-hoot is used in the study. Kahoot is one of the game-based learning platforms that makes it enjoyable to create, share and play learning games or quizzes in minutes. The platform unleashes entertainment in classrooms, offices, and living rooms.

Discussion Boards: Students can continue online conversations using the knowledge they have gained from readings or in-class lectures. Students are invited to choose an aspect of the lesson that interests them, do further assignments, and write a few words on what else they have learned about the subject. Students are also invited to comment on what their friends have contributed.

Online Adaptive Tutorials or Labs: Students create an online instructional or lab experience using an e-learning technology like Smart Owl, allowing them to study a subject as much as required, at their speed, and whenever suits them.

The website: Websites support information clearly and are user-friendly, making it easy for educators, students, and staff to find what they search for information. The critical thing to keep in mind when developing a school website is that it should be intuitive and practical. Some useful math education sites are Khan Academy, Read-Write-Think, Common Sense Media, CK-12, Smithsonian Learning Lab, OER Commons, Discovery Education, and TED-Ed.

Review of Previous Research

Active learning has been defined as involving the reallocation of power between faculty and students in the classroom. As part of this restructuring, faculty would control students over the learning processes that directly affected them. Thus, in an active learning classroom setting, a faculty will be proactive in allowing students to construct their knowledge actively rather than passively receiving information communicated to them from the faculty, slides, or textbook [205]. In many other terms, students are motivated to become more involved and in charge of their learning.

The active learning strategy has been addressed as the subject of several papers up to this point. The efficiency of active learning has been investigated and experimental investigations have been conducted in the majority of this research. In addition, it was noted that theoretical research was done to clarify the fundamental concepts of active learning. Since there is so much research on active learning, a few of them and their results in the fields of science and education both domestically and internationally are briefly listed below [206].

In his work, Seyhan (2003) employed an exploratory approach with a pretest-posttest control group. He contrasted the teaching of mathematics in the second and seventh grades in primary schools using active learning with standard approaches. The experimental group engaged in active learning exercises relating to the “pi number, the diameter of the circle, and the area of the circle” using conventional teaching techniques. The study’s findings revealed that the active learning approach was superior to traditional teaching in terms of student accomplishment, and the pupils had favorable attitudes and viewpoints [207].

Inan (2003), in his master's thesis in which he examined the effect of the use of active learning strategies on student success in the first grade of primary education,

1. The teaching organized by the "Active Learning Approach" in primary school 1st-grade mathematics teaching is more helpful than regular teaching, increasing the success and permanence of the course,

2. Using the active learning approach positively affects the problem-solving success of the students,

3. Using the active learning approach positively affected their attitudes [208].

Zavrak (2003) investigated the impact of using active learning techniques on students' performance in the high school chemistry curriculum's atomic structure unit. In his study, Zavrak carried out the lessons with the activities in the experimental group, either individually or in groups. By providing step-by-step material in the experimental group, Zavrak encouraged the students to think and question what they were doing. The instructor in the control group focused on the conventional method. An accomplishment exam with open-ended and multiple-choice questions was employed in the study. The research revealed that active learning-based instruction is much more successful than teacher-centered instruction in raising student achievement [209].

Research on students in math, physics, and chemistry was conducted by Kalem and Fer in 2003. The research was carried out utilizing both quantitative and qualitative research approaches, as well as the use of the natural environment. The data for the study was gathered using a combination of survey, observation, and interview methods. According to studies, the active learning model's creation of a learning environment has a favorable impact on students' learning, teaching, and communication processes [210].

In the spring semester of the academic year 2008–2009, Aydede and Keserciolu (2010) conducted their study, "The Effect of Active Learning Practices on Students' Critical Thinking Skills," in a public primary school in the Buca district of the province of Izmir. Thirty-four students in the control group and thirty students in the experimental group were been participated. The study employed an experimental design with a pre-interview-post-interview control group. In the experimental group, the lessons were based on active learning techniques, whereas in the control group, they were based on the science and technology curriculum created by the Ministry of Education. as a means of gathering data for their research's "Critical Thinking Skills Form." In the study, the students were taken the pre-interview-post-interview data obtained from the critical thinking skills forms of the research groups. Between the groups, a substantial difference favoring the experimental group was discovered [211].

Mattson (2005) examined the active learning approach in higher education institutions in the USA and its effects. In the study, it was pointed out that the misuse of active learning by educators can lead to negative results. students waste time and it was emphasized that it could lead to disconnection from the learning process. However, it was also stated that discussions on the active learning approach could be beneficial for professors at universities [212].

Bandiera and Bruno (2006) carried out their work called “Active-Cooperative Learning in Schools” in 6 schools in Rome with 144 students aged 16-19. Activities based on active learning in the field of genetics are included. At the end of the learning process, written texts containing students’ opinions and discussions were analyzed statistically and conceptually. It was observed that the students achieved the achievements of using scientific terminology and making discussions [213].

In contrast to laboratories, homework, or other activities, Freeman et al. (2013) concentrated on the design of classroom sessions. By meta-analyzing 225 papers, he particularly contrasted the outcomes of trials showing student performance in courses with at least some active learning against those of standard lectures. The nature and scope of active learning interventions varied greatly. Various methods were used, such as the use of personal response systems with or without peer teaching, worksheets or tutorials performed in class, and studio or workshop course designs. Using two outcome variables, we assessed student performance under best practices for quantitative reviews (SI Materials and Methods).

- scores on tests, idea inventories, or other evaluations that are identical or formally comparable;

- failure rates, which are typically expressed as the proportion of pupils who obtain a D or F grade. Alternatively, drop the relevant course (DFW rate). The examination then concentrated on two connected issues. Does experiential learning improve exam results? Does it lower the rate of failure? Exams, idea inventories, and other evaluations that are identical or comparable, 0.47 ($Z = 9.781$, $P 0.001$)

This indicates that, overall, active learning increases student achievement by a little under half SD when compared to lectures. The odds ratio for failure was 1.95 overall, with a Z value of 10.4 and a P value of 0.001. According to this rate ratio, which is equal to a risk ratio of 1.5, students in standard course courses fail more frequently than those who take classes that emphasize active learning [214].

Ma et al. (1997), Ma (1997), and Ma et al. (2004) show a link between math success and attitude toward math. Based on the data of Ma, there is a reciprocal association between math success and math attitude [215].

The researchers had already spent a significant amount of time looking into the learning environment and student-teacher relationships. Some academics wanted to show how to structure a goal, academically encourage pupils, and implement instructional approaches, materials, and practices, as well as how to drive teachers’ beliefs. The support of a peer by a teacher has been shown to have a favorable impact on academic attitudes, sentiments, self-confidence, motivation, and performance [216].

On a sample of 245 students, Cavanagh et al. (2018) looked at the student-level factor that leads to successful outcomes from using active learning approaches. According to the study’s conclusions, student commitment to and engagement in active learning are both correlated with students’ evaluations of their intellect and their level of

confidence in the teacher. According to the research, teacher-student contact is essential for obtaining targeted learning outcomes in active learning [217].

Peers play a crucial role in students' learning progress and developmental performance in early adolescence. Academic achievement and participation in classroom activities are aided by an active learning environment. The student's acceptance is positively influenced by his or her performance and participation. Children should work together and form friendships with their peers, particularly successful pupils [218].

Masitoh&Fitryanib (2018) argue that developing mathematical learning strategies can help students enhance their self-confidence. The study found that using a problem-based learning strategy might boost students' math self-confidence, which leads to higher accomplishments as a result of better learning outcomes. Structural equation modeling is used to explain the role of self-confidence in mathematics and mathematics accomplishment.

Active Learning is well-suited to the theoretical constructions, philosophical ideas and beliefs, and methodologies that enable transformative and emancipatory practice growth, and it provides a variety of options. According to the findings of a study on the design thinking process, when active learning was used, students' perceptions that they could learn to be creatively increased statistically significantly from the beginning to the completion of the course [219].

In the incorporation of active learning into the classroom, there are numerous styles of learning, with active learning being acknowledged as a vital component that may lead to improved arithmetic proficiency. Active learning is a type of learning in which the learner takes charge of his or her education and is allowed to make decisions about many aspects of the learning process as well as to practice self-regulation.

2.2 The Use of Active Learning with the Formed Model in Mathematical Analysis Courses

Teaching principles of Mathematical Analysis courses

The role of teaching principles at different stages of a teacher's career is different. Different concepts of the concept of the "principle of teaching" are given, and a system of principles of teaching in advanced education is proposed, and in higher education.

The principle of fundamental and professional training, which represents the ultimate goal of on-the-job training, is considered a research problem that requires a reasonable solution. The role of learning principles in the implementation phase is different - they are guided by the rules, being practically based on which the whole educational process is created. Compliance with the requirements, rules, and methods that specify the principles is an important condition for the effectiveness of the educational process.

Several ideal concepts have been proposed to facilitate mathematical analysis learning and assure optimal learner autonomy:

1. The issue-solving strategy should be based on the mathematical knowledge targeted by the problem.

2. There should be no mention of the targeted knowledge in the problem's text or directions.
3. It's fine if students begin working on the problem with ineffective solutions.
4. Students can determine whether or not their attempt was successful.
5. The verifications are suggestive without determining the solution.
6. Only one of the experimentally viable alternatives addresses all of the issues. (the problem's circumstances)
7. Some students can find and test the solution in an acceptable length of time, while others can quickly share and verify it.

The following two didactic scenarios are related in two separate yet complementary ways:

8. The concepts of contract and environment are applicable for evaluating students' work in both common math lessons and experimental treatments.
9. Through didactic mathematical engineering, the didactic situation theory assists in the creation of new cases based on a robust epistemological analysis of targeted information. Students will be presented with "actual" math questions as a result of this analysis. They'll have to respond to these issues and have real-world math experience, which means they'll have to make assumptions, explore intuitive solutions, and look for appropriate reasoning tools.

Each of the principles in the system of professional teaching is aimed at resolving individual pedagogical contradictions. But the major essential force of the learning process is the systematic solution of principles will be possible only when considered. The goals of teaching mathematics in general in higher education can be represented as three components: learning goals, educational goals, and development

Objectives:

- mastering the mathematical apparatus necessary for the study of special disciplines and future professional activity; systematizing the knowledge acquired in mathematical disciplines and revealing their professional significance; mathematical apparatus in educational, research and creative activities of a professional nature learn to use;
- cultivate a positive attitude to learning activities, in particular, the study of mathematics based on demonstrating the professional significance of the studied mathematical facts; education of mathematical culture; necessary to master the professional formation of value attitude to mathematical knowledge as a condition; formation of the idea of mathematics as a powerful method of studying the environment;
- development of intellectual abilities of students; training in professionally important methods of mental activity; formation of performance analysis skills; development of mathematical intuition; to teach the skills of independently acquiring new knowledge necessary for solving problems in special disciplines and professional activities. Professional training in the organization of teaching mathematics in higher

education, clarification of learning objectives at the subject level, taking into account the requirements of the subject.

Integrating Active Learning across Bloom’s Taxonomy levels

Benjamin Bloom is an American teaching psychologist and author of Bloom’s Taxonomy. Born in Lensford, Pennsylvania, he graduated from the University of Pennsylvania in 1935 with a bachelor’s and master’s degree, and in 1942 received his doctorate from the University of Chicago. He developed his theory in his book Bloom’s Taxonomy. Benjamin Bloom served as the head of the Education Committee when it created the widely utilized hierarchical model of cognitive abilities known as Bloom’s taxonomy. Benjamin Bloom is an American teaching psychologist and author of Bloom’s Taxonomy. Born in Lensford, Pennsylvania, he graduated from the University of Pennsylvania in 1935 with a bachelor’s and master’s degree, and in 1942 received his doctorate from the University of Chicago. In his book “Bloom’s Taxonomy” he developed his theory: of the transformation and uniformity of human characteristics and the systematization of educational goals.

Taxonomy is the systematization, grouping, or classification of an object according to certain criteria and principles. It should be noted that the organization of Bloom’s taxonomy encourages students to study, analyze and compare, reflect and evaluate the problem, rather than accepting the finished information. Bloom’s taxonomy requires the structure of human mental abilities to create tasks according to 6 levels that lead to active action in the gradual transition of the cognitive process from the simplest to the most complex. The essence of active teaching methods in the creation of tasks is aimed at mastering the professional activity of students (table 3).

Table 3 - Levels of revised Bloom’s Taxonomy

1. Knowledge	Remembering
2. Comprehension	Understanding
3. Application	Applying
4. Analysis	Analyzing
5. Synthesis	Evaluating
6. Evaluation	Creating

1. Remembering: This category focuses on checking how information is remembered by repetition or recognition, recalling information and data: test tasks, memorization, repetition of definitions and names, and the order of their use.

2. Understanding: Transfer data to another familiar system, convert them; identify ways to communicate stored information to others. Identify similarities, distinguish and compare, and describe the result.

The concept of “understanding” can be divided into four groups:

1. Explanation - to determine the main ideas and their relationship: “How? Why?” search for answers to the questions, perform the tasks “Compare”, and “Show the difference”.

2. Translation - the transfer of ideas into a familiar system, or form, while preserving the meaning; verbalize the form, read the graph, explain the picture, and summarize the information in your own words.

3. Examples show the correct understanding of information and ideas.

4. Definitions - to convey the meaning of the name or concept in their own words, to formulate definitions in familiar and understandable words (definitions should not be memorized or repeated).

5. Application: This category refers to the use of educational material in a real situation and a completely new situation. This includes the practical application of rules, methods, concepts, laws, principles, and theories. Learning outcomes require a deeper understanding of the material than the level of understanding.

6. Analysis: To determine the structure of the object of study, classify it into components. The student identifies and explains how the different parts work and how they achieve results.

The analysis differs from comprehension in the deep processing and assimilation of information.

Types of analysis:

- Basic analysis
- Distinguish the metaphor
- Determining the driving force (motivation).

1. Recognition of the basis - the classification into components (to show the relationship between the basic concepts of information, students must go beyond it), the development of the idea.

2. Distinguish the subtext (subtext) - requires the relationship between the two statements (indirect decisions, associations, causes, and effects can be given).

3. Motivation - identifying the reasons. The student must recognize the direct and implicit (figurative) meanings and actions and defend their ideas with evidence.

4. Synthesis: Creatively combine elements to create new content. Development of a new model (structure) based on their own experience, using the concepts of forecasting, conditionality, and possibility.

Types of work and results:

- Creative genres
- Planning or experimenting; results based on abstract concepts.

5. Evaluation: Making decisions on controversial and controversial issues and proving them. Students need to express and justify their thoughts, ideas, and opinions. At this level, they must base their position on accuracy, logic, consistency, and consistency. Assessment by analyzing the acquired knowledge of insert technology. The following questions are to be replied to:

What did I learn? / What I want to know. The most difficult stage in the learning process is evaluation. To achieve a goal, the student must be able to critically evaluate the value of the submitted materials (applications, decisions, poems, research papers).

Levels of thinking activity according to Bloom's Taxonomy

Skills Description

- Knowledge and presentation of specific facts, information, and their description;
- comprehension Demonstrate understanding by correctly presenting, predicting, or interpreting information;
- application Use of information and previously acquired knowledge in a new or unfamiliar context or situation;
- analysis Demonstrate the ability to divide information materials into structural parts, study information to obtain different conclusions by determining the argument or reason, and find conclusions and/or arguments to substantiate the general rules;
- synthesis Demonstrate the ability to review different parts of previously acquired knowledge in a new context;
- evaluation to form an opinion about the importance of ideas or facts according to the established criteria [220].

Learning may be divided into three groups: *Cognitive* is the most often utilized domain. It is focused on the academic side of schooling. *Affective*: This section includes learning-related objectives including interests, attitudes, and values. The activities that need physical coordination and motor skills fall within this category. The use of the verbs according to the domain is shown in table 4.

Table 4 - What verbs should be used for each domain?

Cognitive	Affective	Psychomotor
1	2	3
Remembering Recall, Recite Repeat, Reproduce State, Define, Duplicate Draw, List, and Label Memorize, Name	Receiving Identify, Locate Name Point to, Select, Sit Reply, Use, Ask Choose Describe, Follow Give, Hold	Imitation Hold, Place, Repeat, Rest Step, Align, Balance Follow, Grasp Manipulation Align, Balance, Follow
Understanding Locate, Recognize, and Report Select, Translate Classify, Describe Discuss, Explain, and Identify Paraphrase, Visualize	Responding Perform, Practice, Present Read, Greet Help, Label, Recite Assist, Comply Conform, Discuss Select, Tell, Write	Hold, Place, Repeat Precision Independently, Proficiently Accurately, Errorlessly With balance, With control Articulation
Applying Demonstrate, Dramatize Employ, Apply, Change, Select Construct, Produce, Schedule	Valuing Follow, Forms, Initiate Invite, Complete, Share Describe, Differentiate Explain, Join, Justify Propose, Read, Select	Harmony, Integration Proportion, Smoothness Confidence, Coordination Speed, Stability, Timing Naturalization

Continuation of table 4

1	2	3
Sketch, Solve, Translate, and Use Write, Illustrate, Interpret Analyzing Examine, Question, Separate Test, Categorize, and Compare Contrast, Distinguish, Deduce Evaluating Judge, Prioritize, Rate Appraise, Argue, Decide Rant, Select, Support, Value Critique, Criticize, Defend Creating Formulate, Invent, Originate Construct, Create Compose, Develop	Organization Explain, Generalize Identify, Integrate Modify, Order, Relate Organize, Combine, Compare, Synthesize Complete, Defend Characterization Act, Discriminate, Display, Influence, Listen, Modify, Solve Performs, Propose	Routinely, Spontaneously With ease, With Perfection With poise Automatically Effortlessly Naturally, Professionally
Note - source [221]		

Further, six distinct levels of learning objectives listed in order of increasing complexity have been added to Bloom’s Taxonomy of the Cognitive Domain. The acquisition and application of information and skills are within the cognitive domain of learning, and Bloom’s Taxonomy levels reflect the increasing complexity of this domain. In that order, remembering, comprehending, applying, analyzing, evaluating, and producing comprise Bloom’s Taxonomy stages. Higher-order thinking (HOT) is commonly linked to levels toward the bottom of the list, whereas lower-order thinking (LOT) is usually linked to levels near the top (HOT). The terms “LOT” and “HOT” relate to the quantity of cognition needed for learning at each of Bloom’s Taxonomy’s six levels. Table 3 shows the stages in detail.

Bloom’s Taxonomy’s Revised Terms:

1. Remembering: retrieving knowledge from long-term memory, identifying it, and remembering it.
2. Constructing meaning through interpreting, exemplifying, categorizing, summarizing, inferring, contrasting, and explaining oral, written, and visual communications.
3. Executing or implementing a method allows you to carry out or use it.
4. Analyzing: Breaking down the material into its component pieces and identifying how they connect to a larger structure or purpose by discriminating, organizing, and assigning.
5. Evaluating: Using checking and critiquing to make decisions based on criteria and norms.
6. Creating: putting parts together to form a whole that is coherent or effective; rearranging elements into a new pattern or structure through generation, design, or production.

Lower Order Thinking (LOT): The lower level of Bloom's Taxonomy is lower-level thinking. It represents the amount of ability required to progress through Bloom's Taxonomy's higher tiers. The skills learned at this level serve as the foundation for higher-order thinking.

LOT encourages you to:

1. Knowledge
2. Comprehension
3. Application

An organized hierarchy of cognitive processes is known as higher-order thinking (HOT). Starting with knowledge-level thinking and moving up to assessment-level thinking, it is a continuum of thinking skills.

HOT encourages you to:

1. Filling out an application
2. Analyzing
3. Synthesis
4. Evaluation

Bloom's Taxonomy has affected teaching and evaluation around the world for over 50 years and is still widely utilized in mathematics education. For example, how to show high school and college math professors might utilize Bloom Taxonomy to create test items.

Finally, math teachers can use thinking skill frameworks created specifically to evaluate HOT in mathematics in addition to Bloom's Taxonomy. For instance, Smith and Stein (1998) use four different types of cognitive demands to categorize arithmetic projects depending on the style of thinking that students are expected to do in their thinking skills framework [222]. Teachers of mathematics can also apply the mathematics framework from their courses. This framework divides tasks into three categories: low, moderate, and high complexity [223]. Both frameworks are math-specific, with descriptions that are congruent with the characterization of HOT employed in this study. Although additional professional development in mathematics teaching for HOT is needed, teaching and assessing for HOT is very important. There are given examples to show teachers' categorization:

- Create an issue that can be resolved using the formula $3x + 7$ (C)
- Find the next term and nth term in the sequence 6, 11, 16, 21, ___? (Ap)
- What integral values of m will the equation factor for the expression $x^2 - x + m$? Describe your thinking. (Ap)
- Almas claimed that $x^4 + 16 = (x^2 + 2)(x^2 - 2)$. Justify Almas's accuracy or inaccuracy. (An)
- Find the slope of the line from a graph showing a real-world linear relationship and explain what it means in this specific case (S)
- How many lines passing through the locations (4, -2) have a slope of $m = 2$? Explain (S)

- Prove that the sum of the interior angles of a triangle is 180 degrees. Prove by using a miter. (E)

Note: According to the Bloom Taxonomy, teachers have classified their students as follows: K = Knowledge; C = Comprehension; Ap = Application; An = Analysis; S = Synthesis; and E = Evaluation.

“If a math problem requires three or more steps to solve, it is higher-order thinking.” She went on to say, “Factorize (x^4+4) ” as an example of HOT, while putting “three steps” in the margin of her notepad. LOT problems are easier than HOT problems, according to several teachers (for example, “Higher-order thinking requires tackling tough or demanding math problems.”) Although HOT things tend to be more challenging, the research on HOT and LOT does not characterize difficulty level. It can be challenging because many mathematical assignments include calculations or algorithms.

Many educators face a rough to inculcating a deep knowledge of the material that they share with their students as they attempt to improve students’ learning outcomes. One problem is how to provide the subject in a way that is both meaningful and interesting, allowing students to gain the most comprehension and synthesis. Educators can sorely increase the standard of the class and aid students recall and figuring out the content by using a smooth method integrating Active Learning across Bloom’s Taxonomy levels three basic domains (mental skills, growth in feelings, and manual or physical skills).

In conclusion, in the study, a framework model is designed as the integration of active learning and bloom taxonomy. The model employs appropriate active learning techniques following Bloom’s taxonomy’s stages. The model is shown in table-5

Table 5 - The table of Active Learning related to Bloom’s Taxonomy Levels

Name of the Strategies	Level of Bloom taxonomy
LOT types	
One-minute paper, Concept maps, Memory Matrix, Mental Concepts, Just-in-time	Knowledge
Group Discussion, Muddiest point, Whiteboard, Kahoot-play game, Peer instruction	Comprehension
Whiteboard, Learning Cell, Kahoot-play game, Peer instruction	Application
HOT Types	
Whiteboard, Brain Storming, Explanatory learning	Analyzing
Whiteboard, Brain Storming	Synthesis
Project making	Evaluation

Real-world Problems for Higher Order Thinking

Real-world problems in high education strive to enable students to apply mathematics, as do most approaches to mathematics education. This resemblance to reality is not only apparent at the end of realistic mathematics. Not only is there a

learning process in the field of implementing abilities, but there is also the reality that is intended to be used as a tool for studying math. Similarly, as a result of the mathematization of reality, mathematics arose. Mathematizing reality must be the starting point for understanding mathematics. If students learn mathematics in a segregated manner, they will be divorced from their peers. It will be rapidly forgotten as a result of their experiences, and it will be inaccessible to youngsters [224]. One must begin with complex contexts that call for mathematical structure or settings that can be quantified rather than abstractions or definitions that will be employed later. As a result, by working on context questions, knowledge, and tools, students can develop mathematical skills while working. Realistic mathematics education is all about making sense of numbers and developing deep and long-term mathematical understanding by working with pupils in circumstances that are meaningful to them.

Furthermore, the reality presented in word problems is frequently out of sync with the true circumstances of the problem's participants. Consider the butcher's shop's predicament. When the fresh ham comes, some of the existing hams may have been sold. The context of this word problem depicts the world of textbooks. Reality, with its unsolvable and multi-solvable issues, has little room in this world. As a result, Freudenthal (1991) believes that teaching real-world problems to children may lead to an anti-mathematical mindset [223,p. 6].

A rising topic of concern for educational officials and scholars worldwide is the ability to apply mathematics to the real world, which supports many aspects of personal, civic, and professional life. As a result, a person's inability to apply mathematics hinders their prospects for employment, social success, and financial security.

In addition, in a subject that greatly boosts the production of the country in an increasingly globalized world characterized by rapid technological and economic development, the ability to utilize mathematics to create real-world events and make forecasts is a crucial skill.

These objectives will be addressed by the proposed research, which will result in the development of an integrated HOTS and Pedagogy Framework that includes principles of successful task design as well as recommendations for classroom application (e.g., supportive pedagogies, necessary resources, and other environmental and social factors). We will be able to create new theoretical concepts (enablers of mathematization, anticipatory metacognition), as well as useful strategies (tasks, pedagogies), as a result of this research, to assist students in developing their higher thinking abilities when dealing with real-world circumstances.

Additionally, the used real-world problems in the research about the chapter the Sequences and the Series are set up as follows in table 6:

Table 6 - Real-world problems used in the study according to the topics

Name of topic	Real-world problem
Arithmetic Sequence	A total of 255000 people were living in the city in 2015. According to projections, the population will grow by 40.000 people annually. What is the anticipated citywide population increase from 2015 to 2022?
Arithmetic Sequence	On the first day of dumping, a gardener receives 160 cucumbers. The following days, he receives 60 additional cucumbers. When will the gardener finish picking all 6760 cucumbers?
Arithmetic Sequence	The number of persons coming to the hospital with severe symptoms of covid19 doubles every week and the hospital has 5000 beds, how many weeks can the hospital treat the patients if the first week there are 10 patients and patients go out after 3 weeks of treatment? (S)
Geometric Sequence	When you loan 10000 Tenges in a bank, say at 5% will income for one year. how much will you earn if the loan is 20 years? (S)
Geometric Sequence	How many relatives from their parents to great-great-great grandparents have three unrelated people? [225]

Solutions to some real-world problems:

1. A total of 255000 people were living in the city in 2015. According to projections, the population will grow by 40.000 people annually. What is the anticipated citywide population increase from 2015 to 2022?

Solution:

It's worth noting that the city's population growth rate is growing. In an algebraic expression, we have the following:

$a_1 = 255000$ (the growth rate that must be factored into the total during the first year.)

$d = 40000$ (the variation in population increase across successive years.)

S_8 is equal to? (the eight-year growth rate, from 2015 to 2022 inclusive.)

$$a_8 = a_1 + (8 - 1)d = 255000 + 7.40000 = 535000.$$

$$a_8 = 535000, \text{ the total population growth} = 535000 - 255000 = 280000$$

2. On the first day of dumping, a gardener receives 160 cucumbers. The following days, he receives 60 additional cucumbers. When will the gardener finish picking all 6760 cucumbers?

Solution:

$$S_n = \frac{2a_1 + (n - 1)d}{2} \cdot n \Rightarrow 6760 = \frac{2 \cdot 160 + (n - 1)60}{2} \cdot n$$

$$3n^2 + 13n - 676 = 0 \Rightarrow (3n + 52)(n - 13) = 0 \Rightarrow n = 13. \text{ That means, it takes 13 days.}$$

3. The number of persons coming to the hospital with severe symptoms of covid19 doubles every week and the hospital has 5000 beds, how many weeks can the hospital treat the patients if the first week there are 10 patients and patients go out after 3 weeks of treatment? (S)

Solution: The first week, the number of patients is 10, and each week is getting doubles. The sequence is formed as 10, 20,40, 80, 160, 320, 640, 1280, 2560, ... (After 8 weeks).

After three weeks, 10 patients go out and go on. The Sum of the sequences is $10+20+40+80+160+320+640+1280+2560+\dots=5110$ (Capacity of the hospital is 5000 patients). Now, the recovered patients are calculated as $10+20+40+80+160=310$. The difference between 5110 and 310 is 4800 patients. Therefore, after 8 weeks, the hospital can't treat the patients.

4. When you loan 10000 Tenges in a bank, say at 5% will income for one year. how much will you earn if the loan is 20 years? (S)

Solution:

After one year, the income is $10000+10000.5/100=10000.105/100 =10500$

After 20 years, the income is $10000. (105/100)^{20} = 26532,977$

5. How many relatives from their parents to great-great-great grandparents have three unrelated people?

Solution:

Let's attempt to formulate the issue. Because the people in the situation are unrelated, each person has two parents, a mother and a father, who are separate from one another. We can refer to these parents as the first generation because they were the original people's closest living relatives. The second generation—which we can refer to—now has two separate parents for each member of the first generation. If we carry on in this manner, we will observe that there are five generations, each of which has twice as many people as the one before it. The total number of parents for the three unrelated people is $6(b_1)$, and q is two. This sequence is geometric. The total number of ancestors over five generations is S_5 . By the sum formula, $S_5 = \frac{2^5-1}{2-1} = 186$.

Experimentation of using active learning methods in Mathematical Analysis course

In this section, the Mathematical Analysis-II course is designed to measure active learning methods. Before analyzing the semester schedule for the course, the course description is given as follows:

A classic analysis course that serves as the basis for numerous subsequent mathematics lessons is Mathematical Analysis-2 (Real Analysis). It is necessary to understand analytical geometry, set theory, and calculus. Students are exposed to formal definitions of number strings and function limits in mathematics, as well as conventional findings about continuity and number strings and their justifications. A lot of focus is placed on using definitions correctly for thorough theorem proving.

Active learning education is a student-centered approach. Students will take the most responsibility for their learning and understanding. This approach is particularly

well suited to an advanced math class where the biggest challenge is understanding only the definitions, processes, theorems, problems, and proofs students encounter. Students need to complete reading and problem assignments and solve classroom problems. They can also be active class participants who paint, demonstrate their understanding of critical concepts, and independently form persuasive and formal mathematical arguments about challenges.

There are three credits in the course (the lecture part has two credits and the practice part has one credit). The active learning strategies are shared in the lecture and practice lessons and are suitable. The planned schedule of the strategies of the active learning approach is given as follows in table 7:

Table 7 - The planned schedule of the strategies of the active learning approach

Week	Name of the topics in Lectures	Strategies in Seminar
1	Summation Formulas	Concept maps, Kahoot-play game
2	Multiplication Formulas	Memory Matrix, the Muddiest point
3	Sequences and convergence of the sequences, recursive definition, and Theorems to calculate the limit of sequences	Kahoot-play game, Learning Cell
4	Arithmetic and Geometric Sequences	Card Sorting, Kahoot-play game
5	Infinite Series, Geometric Series, Telescoping Series, Divergent Series	Memory Matrix, the Muddiest point
6	Operations of Series, Tests for convergence, Integral test	Kahoot-play game, Roleplaying
7	Comparison Test, Direct comparison test, limit comparison test	Card Sorting, Kahoot-play game
8	Ratio and Root tests on Infinite Series	Fishbowl, Kahoot-play game
9	Alternating Series, Leibniz's theorem	Just-in-time, Muddiest point, Memory Matrix
10	Power Series, and theorems	Group Discussion, Kahoot-play game
11	Taylor and Maclaurin Series	Mental Concepts, Kahoot-play game
12	Application of Power series, Binomial Series	Explanatory learning, Kahoot-play game
13	Evaluating non-elementary integrals	Group Discussion, Kahoot-play game

In the lecture and practice lessons, the total time is divided into three parts to use effective active learning strategies for each period. The lesson plans designed according to the active learning instruction are classified as lecture and practice.

The lesson plans on mathematical analysis-II topics with the use of active learning strategies and the bloom taxonomy categorization:

1. Week: Summation Formulas

Lesson Type: Lecture-Mixed form

Grade: First-year students.

Objectives:

1. Introducing the topics and demonstrating content knowledge.
2. Getting the ability to think logically, and the ability to work with formulas development.

3. Letting analysis of students' concepts, difficulties, and errors make and correct.

4. Educating students about responsibility and independent work.

Types of equipment: Computers, smartphones, oral handouts for work and training, training key for tasks notes, a projector, and a whiteboard.

Lesson methods: Active learning methods.

The strategies which are used in the lesson are shown as follows:

Whiteboard: After introducing several summation formulas, the following exercises are given to the students to solve on the board. (After 10-15 minutes for starting the lecture)

1. Find the formula of $\sum_1^n 2k$ (K)

2. Evaluate the sum $\sum_1^{10} 2k^2 - 3k + 4$ (C)

3. Evaluate the sum $\sum_3^9 k - 5$ (C)

4. Evaluate the sum $1+5+10+15+\dots+215=?$ (Ap)

The instructor may share the questions through individual activity or group activity. If the instructor chooses a group activity, then the teacher divides students into two teams. Each team chooses a category and the points they want to play.

Think–Pair–Share: Through the Think-Pair-Share activity, they can feel more at ease speaking about their ideas. This tactic helps pupils talk and listen more clearly while also enhancing their social skills. When couples brainstorm together, each student learns from their partner. The strategy is a pair-designed activity. The instructor writes the following example for each classmate to solve individually and then share their solutions. (After 30 minutes of the lecture have passed)

Evaluate the sum $24+27+30+\dots+99$. (Ap)

One-minute paper: Last five minutes of the lesson, the instructor asks the students to write four formulas on a paper, which are learned in the lecture. (K)

Lesson Type: Practice-Repetition form

Grade: First-year students. Objectives:

1. Repeating the topics and demonstrating by solving exercises.
2. Getting the ability to think logically, and the ability to work with formulas development.

3. Letting analysis of students' concepts, difficulties, and errors make and correct.

4. Educating students about responsibility and independent work.

5. Learning to use methodological ways of dealing with complexity.

Equipment: Computers, smartphones, oral handouts for work and training, training key for tasks notes, a projector, and whiteboard.

Lesson methods: Active learning methods.

Concept maps: The instructor gives the following concept map to the students to match the formulas at the end of the lesson, table 8.

Table 8 - Example of concept map

What is the name of the notation $\Sigma (K)$	a) $\frac{n(n+1)(2n+1)}{6}$
$\sum_{k=1}^n k^2 (K)$	b) $\frac{n}{n+1}$
$\sum_{k=1}^n k^3 (K)$	c) Sigma
$\sum_{k=0}^n \frac{1}{k(k+1)} (K)$	d) $\frac{n^2(n+1)^2}{4}$

Kahoot-play game: To create rivalry among the students for the greatest point total after the game, a quiz connected to the topic summation formulas is provided on the Kahoot platform. It is depicted in figure 8 as follows:

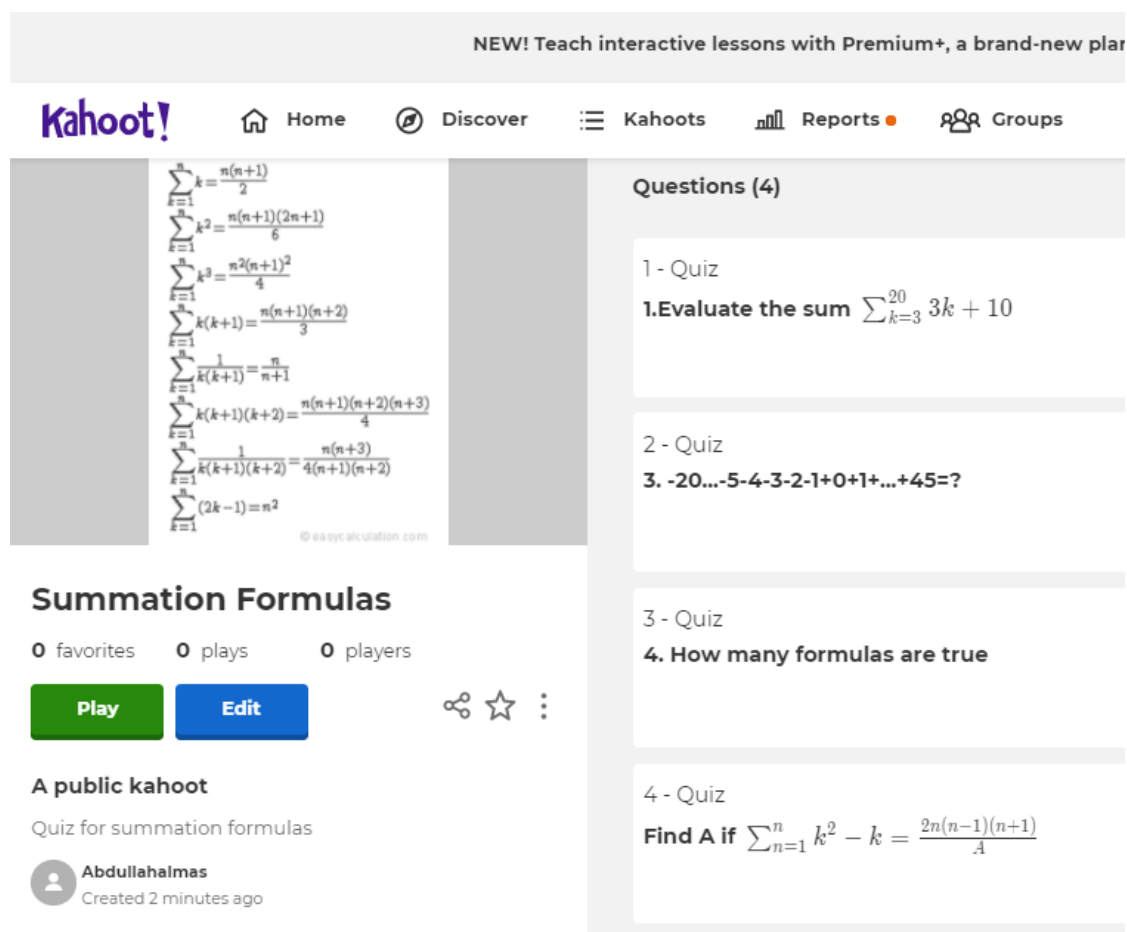


Figure 8 - Platform of Kahoot for Quiz

2. Week: Multiplication Formulas

Lesson Type: Lecture-mixed form

Grade: First-year students.

Objectives:

1. Introducing the topics and demonstrating content knowledge.

2. Getting the ability to think logically, and the ability to work with formulas development.

3. Letting analysis of students' concepts, difficulties, and errors make and correct.

4. Educating students about responsibility and independent work.

5. Learning to use methodological ways of dealing with complexity.

6. Developing proof skills

Types of equipment: Computers, smartphones, oral handouts for work and training, training key for tasks notes, a projector, and a whiteboard.

Lesson methods: Active learning methods.

The strategies which are used in the lesson are shown as follows:

1. Peer Instruction: After teaching some product formulas, the following question is asked to the class to answer individually first and then be voted on when it is over. (15 minutes later, the lecture has been started)

The result is the product $\prod_{k=1}^4 k^3 - 3!$ is positive. Is that true or false? (An) Four minutes later, the instructor will vote for the results of the students. Suppose the majority group is answered as accurate. In that case, the instructors let the students who said it was harmful check one more time within two minutes to change their ideas. Finally, the instructor will vote again and check the entire class to get the correct result.

2. Small-Group Discussion: The class is designed as possible, as given in figure 9.



Figure 9 - Small-Group Discussion

The teacher gave the pupils 30 minutes to talk and make decisions in their groups. The instructor will look out for each student's needs and take an equal part in the discussion and its outcome.

Find the result of the product $\prod_{k=1}^{\infty} \frac{4n^2}{4n^2-1}$ (Ap)

When sufficient time is over (five minutes is possible), the groups will reflect their results to the other groups.

3. One-minute paper: The teacher instructs the students to jot down any formulas they have remembered at the end of the lecture. (K)

Lesson Type: Practice-Repetition form

Grade: First-year students.

Objectives:

1. Repeating the topics and remonstrating by solving exercises.
2. Getting the ability to think logically, and the ability to work with formulas development.
3. Letting analysis of students' concepts, difficulties, and errors make and correct.
4. Educating students about responsibility and independent work.
5. Learning to use methodological ways of dealing with complexity.
6. Developing proof skills.

Equipment: Computers, smartphones, oral handouts for work and training, training key for tasks notes, a projector, and whiteboard.

Lesson methods: Active learning methods.

The strategies which are used in the lesson are shown as follows:

Memory Matrix: The instructor will show the visual picture on the screen of the projector or with a piece of paper (A₃) as follows in table 9:

Table 9 - Memory Matrix

$\prod_{k=1}^{15} k \text{ (C)}$	Result?
$\prod_{k=1}^{13} 4 \text{ (C)}$	Result?
$\prod_{k=1}^n k^3 = 13824, \text{ find } n \text{ (Ap)}$	Result?
$\prod_{k=1}^{11} \frac{k}{(k+1)} \text{ (Ap)}$	Result?
$\prod_{k=5}^{17} \frac{k}{(k-1)} \text{ (Ap)}$	Result?
$\prod_{k=1}^{11} \prod_{j=1}^{12} kj \text{ (Ap)}$	Result?
$\text{If } \prod_{k=1}^7 3^k = 9^n, n = ? \text{ (Ap)}$	Result?

For each question, the instructor may give bonus points to the students. As a second way, the instructor may give the class into two groups and make them competitive as each question is done by any group member first.

2. Muddiest point: The teacher will instruct the class to write down the hardest parts of the topic after the first task. The instructor will next clarify any remaining questions or challenging concepts.

In conclusion, the mathematical analysis course demonstrates how to apply active learning techniques. The appendix now also includes some of the strategies.

2.3 Statistical Analysis of Experimental Data

Measurement Tool

This study used a measurement test developed by three educators to scale student achievement and retention skills. According to the chapter's subjects, we chose the questions (25) of the test from the course book, *Calculus and Analytic Geometry*, written by George B. Thomas and Ross L. Finney [226]. Firstly, we prepared a table of the specification. Then we designed the questions according to Bloom's revised taxonomy. Secondly, we sent the test to two mathematics experts to check the relevance of using achievement and retention skills. Thirdly, we did a pilot study on 145 students from the second and third courses of undergraduate students. We evaluated item difficulty, item discrimination, and point biserial correlation on the pilot group's data. Finally, we created the measurement test by removing the 3rd, 5th, 6th, 13th, 17th, 23rd, and 25th questions from the item statistics for the pilot study, as seen in table 10. Also, we added the measurement test to the Appendixes.

Table 10 - Item Statistics for Pilot Study

Item	p	d	pbc
1	2	3	4
1	0.90	0.23	0.39
2	0.76	0.23	0.19
3	0.90	0.08	0.15
4	0.94	0.23	0.29
5	0.22	0.15	0.11
6	0.92	0.00	0.13
7	0.51	0.85	0.63
8	0.80	0.69	0.72
9	0.67	0.85	0.76
10	0.65	0.77	0.61
11	0.65	0.85	0.69
12	0.82	0.23	0.34
13	0.59	0.08	0.03
14	0.76	0.69	0.58

Continuation of table 10

1	2	3	4
15	0.65	0.77	0.73
16	0.73	0.77	0.64
17	0.31	-0.69	-0.60
18	0.69	0.92	0.80
19	0.71	0.85	0.72
20	0.88	0.38	0.54
21	0.73	0.77	0.72
22	0.65	0.23	0.16
23	0.55	-0.77	-0.54
24	0.57	0.38	0.26
25	0.24	-0.54	-0.48

Data Collection and Data Analysis

Table 11 - Names of Data Collection

Names of Data Collection	Date	Place
The Achievement Test for AL and Regular Learning	2019	SDU
The Achievement Test for AL and Online Learning	2020	University Malaysia Pahang
The Achievement Test for AL Learning for Gender	2021	Nile University
The Achievement Test for AL and Online Learning	2022	SDU
Results of the Survey about Motivation and Self-confidence	2022	SDU
HOT Test for AL and Online Group	2022	SDU

1. The experiment between the AL group and the Regular Group in 2019

The experiment was made to reveal that the average of correct results of students in the active learning group shows better performance rather than the average of correct results of students in the regular learning group. Students were tested twice; the first time was immediately after a couple of chapters were finished. The second was four months later; the first pre-test and post-test consisted of 32 students divided into equal groups for the used methods. The mathematical analysis-2 course consists mainly of series and convergence of the series. Two different classes having the same academic performance participated in the study. Both groups in the study took the same course knowledge with different techniques. Group sampling was random. Participants were 32 undergraduate students (68% girls) enrolled in a mathematical analysis course at a university where mathematics education is in English. The instructor taught the groups

with the help of a teaching assistant during the academic year. The participating students received no credit for completing the test.

The first experimental data were conducted in the spring-fall semesters of academic years with 32 undergraduates at Suleyman Demirel University at Almaty in 2019. The implementation of active learning lasted for 13 weeks, and each group had 16 members. First-year students of the faculty of science education and humanities were chosen. They had three hours of mathematics each week, and each hour of the lesson lasts fifty minutes.

The instructor completed the pre-test at the start of the implementation and received the results of the post-test four months later. Each test was given by the instructor, who had fifty minutes to finish it. With the use of a pre-test administered in the middle of the semester, we were able to control the major differences between the post-test scores of the active learning and regular learning groups using the t-test analysis for small groups.

We ensured the instructor's study process was used. Before implementing active learning, we monitored two regular lessons from the instructor by the first author. The instructor carried out active learning methods by dividing the lessons into three parts. The instructor used the active learning methods in the lessons given in table 4. The hypothesis is accepted as $H_0 = \text{AL group performance average} < \text{Regular group performance average}$. $H_1 = \text{The AL group performance average} > \text{Regular group performance average}$

Results

Pre- and post-test means and standard deviations for AL and regular learning groups are given in table 12.

Table 12 - Group Statistics

Tests	Group	N	Mean	Std. Deviation
Pre-Test	Regular Learning	16	26.93	15.24
	Active learning	16	25.68	24.39
Post-Test	Regular Learning	16	47.50	17.12
	Active Learning	16	65.31	15.75

Table 12 shows that the active learning group (that is, an experimental group) score is higher than that of the regular learning group (that is, the control group) for the post-test. Also, the groups' levels are closer to each other, utilizing the pre-tests of the groups.

After that, we used a t-test to evaluate whether the dependent variable varies between the Active learning and regular learning groups by the scores taken four months later. The results of the t-test analyses are shown in table 13.

Table 13 - Analyses of the t-test

Tests	Levene's Test for Equality of Variances		t-test for Equality of Means		
	F	Sig.	t	df	p
Pre-Test	3.75	.06	.17	30	.863
Post-Test	.052	.82	-3.06	30	.005

There is a significant result in students' academic performance for the post-test [$p=0.005$] of the active learning group, as seen in table 13. To the results of the t-test, the null hypothesis is rejected. The study's subsequent analysis revealed the impact of active learning based on Bloom's Taxonomy levels, as seen in table 14.

Table 14 - Group Statistics for knowledge, comprehension, application, and analysis levels of Bloom's taxonomy

Levels	Group	N	Mean	SD
Knowledge	Regular learning	16	53.12	20.15
	Active learning	16	57.81	21.83
Comprehension	Regular learning	16	39.32	16.91
	Active learning	16	67.92	23.64
Applying	Regular learning	16	49.15	26.60
	Active learning	16	67.03	17.09
Analyzing	Regular learning	16	10.83	19.12
	Active learning	16	20.85	24.65

Table 15 shows that each average score of the level (knowledge, comprehension, applying, and analyzing) of the active learning group is higher than each average rating of the regular learning group level. In the table, the applying level shows that the experimental group has higher-order thinking skills rather than the control group. Therefore, the methods of active learning are more effective to use in mathematical analysis courses. Of course, The other upper levels had to be checked in the experiment. But the achievement test had limitations. Also, the upper levels should be checked by different assessments such as projects, workshops, or solving real-world problems.

Table 15 - Independent Samples Test results for knowledge, comprehension, applying and analyzing levels of Bloom's taxonomy

		F	p	t	df	p	Mean D.	Lower	Upper
1	2	3	4	5	6	7	8	9	10
Knowledge	Equal variances assumed	.20	.65	-.63	30	.53	-4.68	-9.85	10.48
Comprehension	Equal variances assumed	2.51	.12	-3.93	30	.00	-28.60	-3.44	-13.75

Continuation of table 15

1	2	3	4	5	6	7	8	9	10
Applying	Equal variances not assumed	6.05	.02	-2.26	25.5	.03	-17.87	-34.14	-1.60
Analyzing	Equal variances not assumed	2.01	.22	-4.53	26.4	.00	-21.60	-33.14	-10.75

As seen in table 15, while the mean difference (-4.688) between remembering levels of regular and active learning groups is not significant, the mean differences between understanding (-28.60) and applying (-17.87) are statistically significant. This result promotes Barners' (1989) conclusion that active learning helps students understand the subject through a query, collecting and determining data to solve higher-order cognitive problems [183,p. 19].

This study compared the effectiveness of active learning versus traditional learning strategies for students in advanced mathematics. Students in the active learning condition reported doing exceptionally well in courses involving mathematical analysis, as was expected. Based on the findings, the study determined the value of active learning strategies for student achievement by conducting a precise experiment. Interestingly, no student scored better than the second exam, which shows consistency because the student's achievement scores are less than the first exam scores.

2. The experiment between the AL group and Online Learning group in 2022 at SDU

At Suleyman Demirel University in Almaty, 80 undergraduate students participated in the second experimental data study, which was conducted throughout the spring and fall semesters of the academic year. Active learning was implemented for 13 weeks, with 43 students in the experimental group and 37 in the control group. The faculty of Pedagogy, Science, and Humanities chose first-year students. Each week, they had three hours of mathematics instruction that lasted for fifty minutes. While taking the pre-test in the 5th week of the implementation, the lecturer received the post-test two months later. The instructor administered the pre-test was limited to fifty minutes and the post-test was limited to one hundred minutes to complete the test. We used the t-test analysis for the small groups to reveal the difference between the active learning and online learning groups' post-test scores controlling their primary differences using a pre-test in the middle of the semester. We ensured the instructor's study process was used. Before implementing active learning, we monitored two regular lessons from the instructor by the first author. The instructor carried out active learning methods by dividing the lessons into three parts. The instructor used the active learning methods in the lessons given in table 16. The hypothesis is accepted as $H_0 = \text{AL group performance average} < \text{Online group performance average}$. $H_1 = \text{The AL group performance average} > \text{Online group performance average}$.

Results

Pre- and post-test means and standard deviations for AL and online learning groups are given in table 16.

Table 16 - Group Statistics

	Group	Pre-test	Post-test
N	Online	37	37
	AL	43	43
Mean	Online	18.2	27.8
	AL	18.7	36.3
Standard deviation	Online	12.8	4.68
	AL	10.2	5.75

Table 16 shows that the active learning group (that is, an experimental group) score is higher than that of the regular learning group (that is, the control group) for the post-test. Also, the groups' levels are closer to each other, utilizing the pre-tests of the groups. After that, we used a t-test to evaluate whether the dependent variable varies between the Active learning and regular learning groups by the scores taken four months later. The results of the t-test analyses are shown in table 17.

Table 17 - Analyses of the t-test between the AL group and the Online learning group

	Statistic	df	p
Pre-test	-0.126	28.0	0.900
Post-test	-4.457	28.0	< .001

There is a significant result in students' performance for the post-test [$p=0.005$] of the active learning group, as seen in table 17. That means that the null hypothesis is rejected. The study's subsequent analysis revealed the impact of active learning based on Bloom's Taxonomy levels, as seen in table 18.

Table 18 - Group Statistics for knowledge, comprehension, application, and analysis levels of Bloom's taxonomy

	Groups	Knowledge	Comprehension	Applying	Analyzing
N	Online group	148	148	148	111
	AL	172	172	172	129
Mean	Online group	0.750	0.750	0.500	0.00
	Online group	0.854	0.521	0.382	0.111
Standard deviation	AL	0.250	0.250	0.00	0.00
	AL	0.655	0.512	0.339	0.247
Standard deviation	Online group	0.500	0.500	0.577	0.00
	Online group	0.354	0.501	0.488	0.315
Standard deviation	AL	0.500	0.500	0.00	0.00
	AL	0.477	0.501	0.475	0.434

Table 19 shows that each average score of the level (knowledge, comprehension, applying, and analyzing) of the active learning group is closer to each average rating of the online learning group level. In the table, the analyzing level shows that the experimental group has higher-order thinking skills rather than the control group. Therefore, the methods of active learning are more effective to use in mathematical analysis courses. Of course, The other upper levels checked in the experiment by solving real-world problems.

Table 19 - Independent Samples T-Test results for knowledge, comprehension, applying and analyzing levels of Bloom’s taxonomy

	Statistic		df	p
Knowledge	4.299	^a	318	< .001
Comprehension	0.377		318	0.706
Applying	0.999		318	0.318
Analyzing	-2.734	^a	238	0.007
^a Levene’s test is significant (p < .05), suggesting a violation of the assumption of equal variances				

As seen in table 20, while the mean difference (4.299) between knowledge levels of online and active learning groups is significant, the mean differences between the groups according to comprehension (318) and application (318) are not statistically significant. For analyzing level, the mean difference (238) shows that is significant. This result promotes Barners’ (1989) conclusion that active learning helps students understand the subject through a query, collecting and determining data to solve higher-order cognitive problems [183,p. 19].

Table 20 - Paired the Levels- T-Test

		statistic	df	p
Knowledge	Comprehension	5.99	319	< .001
Applying	Analyzing	4.28	239	< .001

When the levels of bloom taxonomy are taken as pairs, the mean differences (319 and 239) show that are significant. The collected data were also checked to find out the relationship between the levels of Bloom Taxonomy by the Anova tool. Table 21 shows the relationships between the questions according to their levels to find out the roles of previous levels to affect the analyzing level.

Table 21 - ANOVA – Analyzing

	Sum of S.	df	Mean Square	F	p
Knowledge	5.21e-4	1	5.21e-4	0.00387	0.950
Comprehension	0.00267	1	0.00267	0.01982	0.888
Applying	0.11824	1	0.11824	0.87765	0.350
Knowledge * Comprehension	0.00169	1	0.00169	0.01254	0.911
Knowledge * Applying	0.45794	1	0.45794	3.39899	0.067
Comprehension * Applying	0.15011	1	0.15011	1.11419	0.292
Knowledge * Comprehension * Applying	0.01913	1	0.01913	0.14198	0.707
Residuals	31.25684	232	0.13473		

This study compared the effectiveness of active learning versus traditional learning strategies for students in advanced mathematics. Students that participated in active learning reported great performance and retention in their courses on mathematical analysis, as was expected. Based on the findings, the study determined that active learning techniques helped improve students' achievement and retention abilities. Interestingly, no student outperformed the second exam, demonstrating consistency since the student's achievement and retention scores were lower than those in the first exam.

3. The Achievement Test for Mathematical Analysis with AL for Gender at Nile University

Tables 22 and 23 show the results of the experiment that was taken at the Nile University in Nigeria in 2021. The data was collected from one group (18 Female and 34 male) of mathematical analysis courses for first-year students. Some AL strategies (whiteboard activities, group discussion, and Kahoot games) were applied during the practice lessons. $H_0 = \text{AL female group performance average after treatment} < \text{AL male group performance average after treatment}$. $H_1 = \text{The AL female group performance average after treatment} > \text{AL male group performance average after treatment}$. After collecting the data, the following result was revealed (table 22).

Table 22 - The Statistics for the AL learning group

	Gender	Pre-test	Post-test
N	Female	37	35
	Male	37	35
Mean	Female	61.9	67.3
	Male	55.5	64.9
Standard deviation	Female	29.8	24.1
	Male	32.4	23.6

Table 23 - The Achievement Test for AL T-Test

		statistic	df	p
Pre-test	Post-test	-2.65	51.0	0.011

To the results of the t-test, the null hypothesis is rejected because $p=0.011 < 0.05$

4. *The Achievement Test for AL and Regular Learning at University Malaysia Pahang*

Tables 24 and 25 show the results of the experiment that was taken at the University of Malaysia Pahang in Malaysia in 2020. The data was collected from two groups (control and experimental) of mathematical analysis courses for first-year students. Some AL strategies (whiteboard activities, group discussion, Kahoot games, and projects) were applied during the practice lessons. H_0 = The group performance average before treatment and the group performance average are equal after treatment. H_1 =The group performance average before treatment and the group performance after treatment is not equal.

After collecting the data, the following result was revealed:

Table 24 - Group statistics for achievement test

		Results
N	pre-test	58
	post-test	58
Mean	pre-test	25.1
	post-test	25.4
Standard deviation	pre-test	9.17
	post-test	10.8

Table 25 - Achievement T-test for the group

	statistic	df	p
Pre-test/ Post-test	27.4	117	< .001

To the results of the t-test, the null hypothesis is rejected because $p=0.01 < 0.05$

5. *Results of the Survey about Motivation and Self-confidence Toward Success at SDU in 2022*

The survey shown in the appendix was taken fifth week of the experiment and asked students 17 questions to 70 students. (ten about self-confidence and seven about motivation). Tables 26 and 27 show group statistics and Table 28 shows the significant items of the survey.

Table 26 - Group statistics for the motivation for mathematics

	Groups	M1	M2	M3	M4	M5	M6	M7
Mean	Online	3.17	3.17	2.83	4.20	4.29	4.00	4.00
	AL	3.86	3.11	3.29	3.43	4.09	3.89	3.66
Standard deviation	Online	0.923	0.985	1.20	0.719	0.667	0.840	0.907
	AL	1.03	1.25	1.07	0.850	0.818	0.963	1.03

Table 27 - Group statistics for the self-confidence in mathematics

	Groups	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
\bar{x}	Online	3.6	3.8	4.3	4.0	4.0	3.8	2.6	3.0	2.0	3.9
	AL	4.0	4.0	4.4	4.2	4.1	4.1	2.8	4.0	3.2	4.0
S.D	Online	0.9	0.7	0.6	0.6	0.7	0.8	0.7	1.0	0.7	0.7
	AL	0.8	0.8	0.7	0.9	0.9	0.7	0.7	0.9	0.8	0.707

Table 28 - Independent Samples T-Test for the self-confidence and motivation for mathematics

	Statistic	df	p
A1	-1.807	68.0	0.075
A2	-1.373	68.0	0.174
A3	-1.010	68.0	0.316
A4	-0.912	68.0	0.365
A5	-0.145	68.0	0.885
A6	-1.745	68.0	0.086
A7	-0.805	68.0	0.424
A8	-3.895	68.0	<.001
A9	-6.132	68.0	<.001
A10	-0.660	68.0	0.512
C1	-2.928	68.0	0.005
C2	0.212	68.0	0.833
C3	-1.680	68.0	0.098
C4	4.098	68.0	<.001
C5	1.121	68.0	0.266
C6	0.529	68.0	0.599

By the T-test scores, there are some questions (C4, A8, A9) that show significance. The questions chart is shown in the following figures.

Figures 10 and 11 show the motivation in responses to the following question: “I would make people like me less if I were an excellent math student” [227].

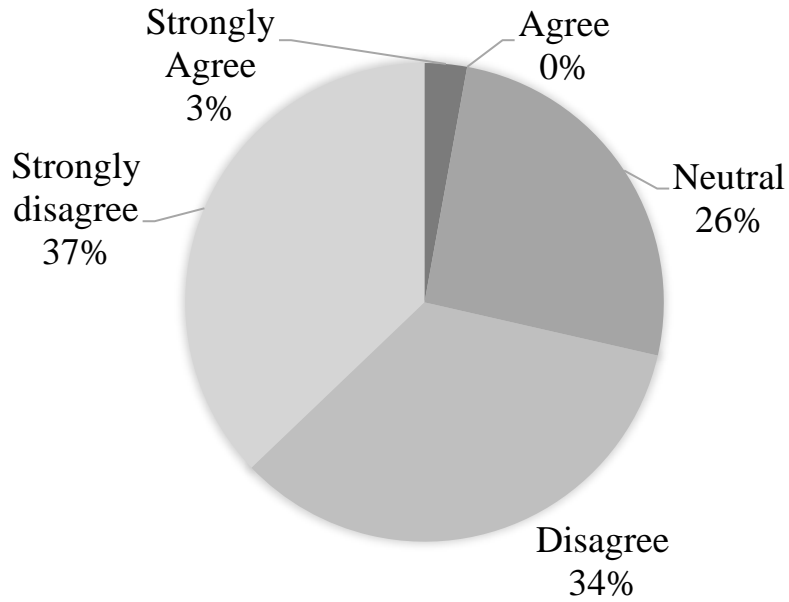


Figure 10 - AL learning group

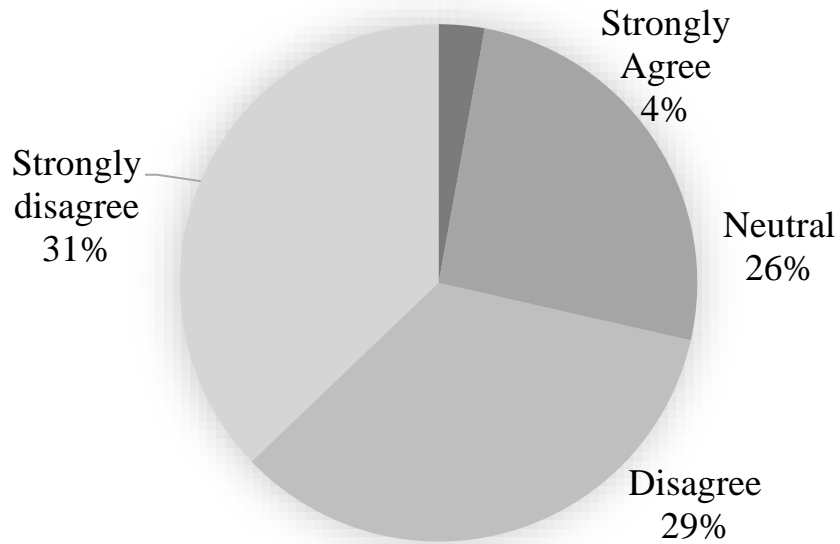


Figure 11-Online learning group

By figures 10 and 11, it can be said that the questions showed that the AL group's motivation is better than the control group's motivation. The question was negative because of that the sum of strongly disagree and disagree had to show a great number. The sums of their percentages for the AL group (71%) and online group (60%) revealed their motivation.

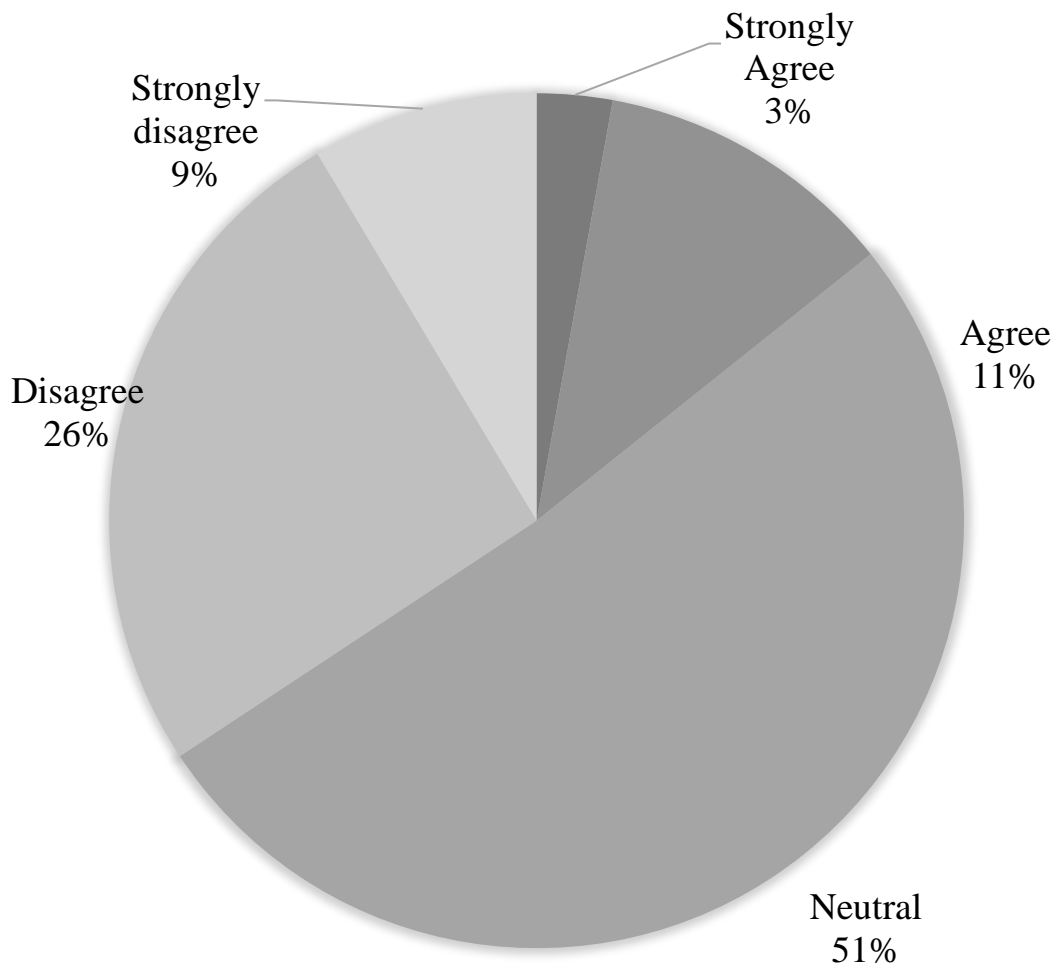


Figure 12 - Active Learning Group

Figures 12 and 13 show the motivation of the groups by the following question: “I don’t like people to think I’m smart in math” [227,p. 146].

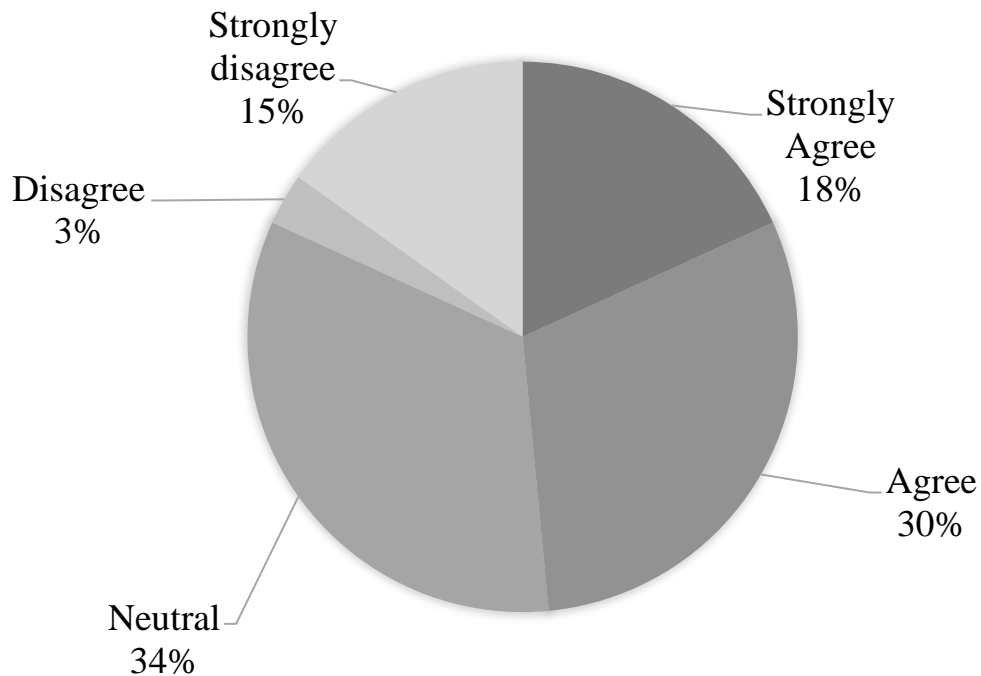


Figure 13 - Online Learning Group

By figures 12 and 13, it can be said that the question showed that the AL group's motivation is better than the control group's motivation. The question was negative because of that the sum of strongly disagree and disagree had to show a great number. The sums of their percentages for the AL group (35%) and online group (18%) revealed their motivation.

Figures 14 and 15 show the self-confidence of the groups by the following question: "I usually have been at ease in math classes" [227,p. 150].

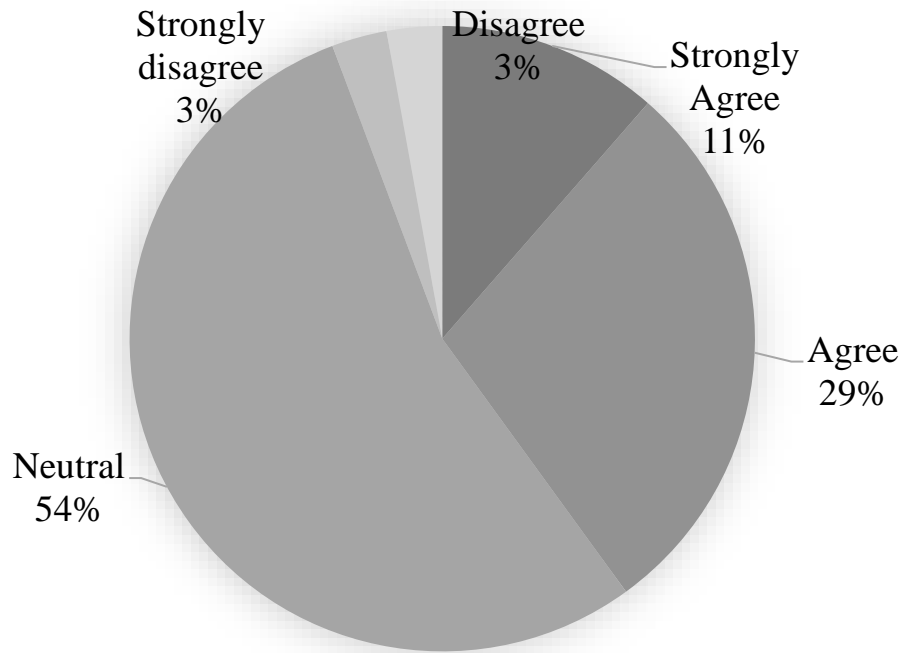


Figure 14 - Active Learning Group

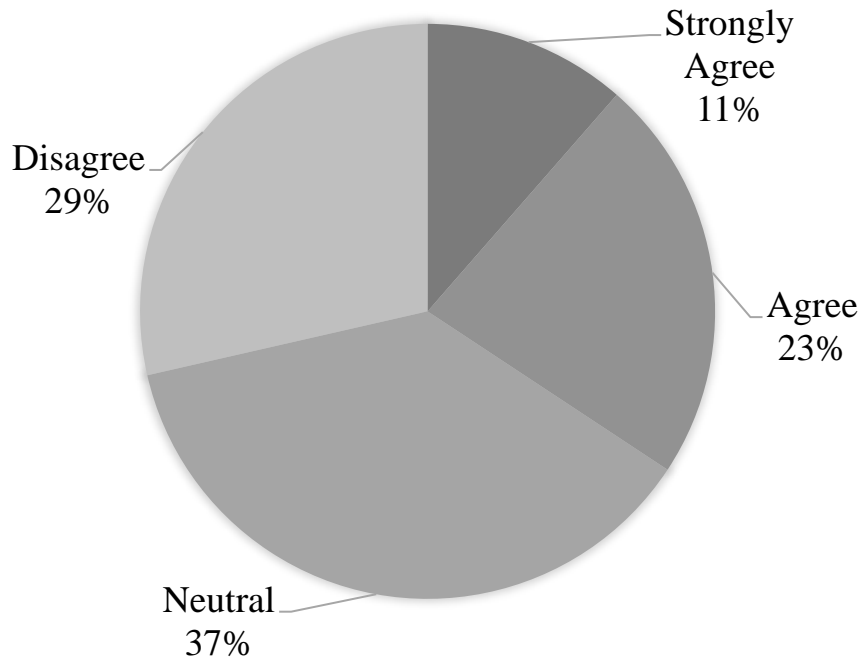


Figure 15 - Online Learning Group

By figures 14 and 15, it can be said that the question showed the AL group's self-confidence is better than the control group's self-confidence. The question was positive because of that the sum of strongly agree and agree had to show a great number. The sums of their percentages for the AL group (40%) and online group (34%) revealed their self-confidence. Also, the chosen 29% disagree with the online group means that the self-confidence for the online group is lower.

6. HOT Test for AL and Online Group

In the higher-order thinking part, we asked the students real-world problems about sequences and series. If we recall that synthesis, analysis, reasoning, comprehension, application, and assessment are some of the HOTS. By foreseeing links between various concepts, higher-order thinking abilities can aid in issue-solving. Taxonomies, another name for categories of ideas, are a tool used by certain cognitive researchers to categorize the ways they interpret mental processes. The results are shown in table 29.

Table 29 - Higher-order Thinking Test for AL and Online Group

Tasks	Group	Pre-test (in %)	Post-test(in %)	Progression/ regression (in %)
A farmer selects 120 tomatoes on the first day of harvest. The following days, he picks an additional 40 tomatoes. How long will it take the farmer to complete the picking of 3000 tomatoes? (S)	AL	46.50	66.66	20.16
	Online learning	56.80	41,66	-15.14
A ball is hurled from a height of 81 centimeters. After each bounce, it returns to its original height. What is the total distance traveled by the ball by the time it lands for the fifth time? (S)	AL	24.40	25.40	1
	Online learning	17.60	11.25	-6.35
When you loan money in a bank, say at 5% will income for one year. how much will you earn if the loan is 20 years? (S)	AL	13.70	41.25	27.55
	Online learning	18,43	38,12	19.69
The number of persons coming to the hospital with severe symptoms of covid19 doubles every week and the hospital has 5000 beds, how many weeks can the hospital treat the patients if the first week there are 10 patients and patients go out after 3 weeks of treatment? (S)	AL	14	26,87	12.87
	Online learning	25,31	26,87	1.56

In table 29, it can be said that higher-order thinking skills for the experimental group are better than the control group's HOTS skills. Briefly, the percentage increase in the experimental group is greater than the increase in the control group.

CONCLUSION

During the study, the tasks were solved and the following main results were obtained:

- As a result of the analysis of the pedagogical, scientific, and methodological literature, the concepts of “active informatics teaching methods” and “active learning environment” have been clarified. The possibility and appropriateness of using active informatics teaching methods are verified, and organizational and pedagogical conditions for their use are determined. In the context of using active teaching methods, a methodology has been developed to design the educational process of higher mathematics courses.

- Based on the results of the theory and practice analysis, the components of the methodological training of future mathematics teachers on the use of active teaching methods are determined: the development of general methodological skills of the teacher (projective, facilitator, reflective skills, communication skills, etc.) and the formation of certain methodological skills, in the classroom environment (games, projects, “non-traditional lessons”, simulations, etc.) uses active teaching methods.

- The feasibility of applying the activity approach in organizing the methodological training of mathematics teachers on the formation of skills to use active teaching methods in the activities of teaching higher mathematics at universities has been proven.

- Goals, content, teaching methods, and the system of monitoring the results of the teaching of the discipline “Innovative Technologies in Education” are developed by the principles of the effective approach to the higher mathematics education process: the selection of appropriate forms, methods, and methods of active teaching.

- The discipline “Innovative Technologies in Education” was tested as part of a pedagogical experiment that demonstrated the effectiveness of mathematics teachers in mastering the skills to use active teaching methods in their professional activities. The results of experimental tests, the effectiveness of teachers and students, confirmed the assumption that the formation of teachers’ skills in using active teaching methods on mathematical analysis lessons can lead to higher activation of learning activities and contribute to students’ self-development. In the learning environment if the student’s personality is active; teachers effectively use the potential of active teaching methods in their lecture activities to achieve educational goals.

- Didactic materials have been developed that allow the activation of educational activities that can be used by: teachers in mathematical analysis courses at universities, teachers conducting special courses for students of pedagogical specialties, and teachers organizing the training process in further education courses.

- Active Teaching strategies tested and published: “Impact of the active learning strategies on student achievement concerning double integrals in mathematical analysis”, “The Impact of Peer Instruction on Ninth-Grade Students’ Trigonometry

Knowledge”, “The Impact of Project- Based Learning on Students’ Motivation in Mathematics.”

- The study confirmed the need to make changes in the methodological education system to focus on the use of active teaching methods in the professional activity of mathematics teachers. The main results of the thesis research can be used to increase the effectiveness of training teachers in teaching mathematical analysis to use active teaching methods as part of teacher education.

The XXI century is a century of new global technologies and innovations, a century of new thinking, and the formation of new perspectives on changes in the world [228]. What are these changes about? First, the creation of new conditions and active methods of teaching, which is the basis of this educational paradigm; second, the growing demand for the training of critically and functionally literate individuals under the requirements of a rapidly changing world; third, the modernization of teaching and learning in schools, the readiness of teachers for innovation, self-development, self-improvement, corporate interaction.

Typically, a student’s coursework for undergraduate general education requirements accounts for 30% of their total workload. General education reform in higher education has been a contentious issue due to how significantly it affects a student’s academic career [229]. Numerous majors provide courses that satisfy the general education requirement for mathematics, critical thinking, or quantitative reasoning, including STEM, business, and the social sciences. For students whose specialty curriculum does not currently have a required course that would satisfy their mathematics general education demands, universities frequently create customized mathematics courses.

The goal of higher education is to help students become better communicators, critical thinkers, and problem solvers. Critical thinking is defined as the “purposeful, reasoned, and goal-directed” use of cognitive processes, and it requires students to actively apply, analyze, synthesize, evaluate, and share knowledge. There is broad agreement that professors must offer pertinent learning opportunities for students to participate in open challenges and projects to do this. These learning opportunities are produced via a variety of instructional methodologies, including cooperative learning, project-based learning, and discovery learning. Numerous research has been undertaken to determine the effects of these diverse active learning methodologies in STEM education [230].

Critical thinking is said to be cultivated through collaborative and small-group learning. Scardamalia and Bereiter [231] and Vygotsky [63,p. 90] discovered, for example, that social interactions between students often assist them in tackling difficulties that they might not have been able to handle on their own. When we compared active learning to regular learning in a general education science course, we discovered that lectures performed better for lesser achievers, while a group-based strategy produced superior information recall for higher achievers. Furthermore, the

study discovered that using real-world problems and activities in small cooperative groups in higher mathematics courses was successful in improving students' critical thinking skills [232].

Our first data suggest that Mathematical Analysis, our active learning-based course, is better for students at lower levels than the standard lecture-based course. One cause for this could be our instructors' lack of experience with active teaching methods in higher mathematics courses. For many years, a mathematical analysis course has been given in a classic lecture format, and the instructors have been able to efficiently teach this course by cultivating their assignments and notes. Active learning methodologies, on the other hand, have a variety that is a new approach for educators. As a result, educators haven't had a chance to tailor the content to their particular teaching approaches. Any lack of confidence shown by active learning instructors could have turned into students. We can test that students will not be uneasy about trying a new method for the first time when the course is based on active learning methods for a long time.

The research demonstrates that participants valued the active learning-based higher mathematics courses they took and that the experimental group's interest levels were higher than those of the control group. Another problem was that gifted students seemed to share a common understanding of mathematics. To learn more about levels of interest, we could ask about specific course contents.

When comparing learning styles, learners of course regarded active learning to be beneficial, whereas regular learning was viewed as neutral. However, when active learning tactics were applied, some confident students became bored. Perhaps students are unaware that "active learning" uses a variety of strategies. In the future, we may control the less used active learning approaches in higher mathematics courses. For instance, I think that examining mathematical problems helps learners comprehend mathematics, thus asking them to evaluate the assertion rather than solving the problem might be a preferable approach. In my research, I believe that active learning activities generally assist us in changing traditional mathematics lectures.

Overall, through active learning, students' cognitive skills develop, they summarize and memorize when working with any material, summarize and understand what students have learned by the objectives of the lesson, and correct any misunderstandings; which helps to encourage students to present their work and present their ideas. The effectiveness of active learning develops students' critical thinking skills, allowing them to behave freely in any environment without feeling language constraints.

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APPENDIX A

I.Achievement Test

1. Which one of the following does show the expansion of $a_1 + a_2 + \dots + a_n + \dots$? properly? (K)

A) $\sum a_n$ B) $\sum_{n=1} a_n$ C) $\sum_1^n a_n$ D) $\sum_{n=1}^{\infty} a_n$ E) $\sum n$

2. If $\sum_1^{\infty} a_n = L$, which one of the following results is true? (K)

- A) The series diverges
- B) The series converges to L
- C) The series does not diverge or converge
- D) The limit of the series does not exist
- E) The partial sum's limit does not exist

3. Which one of the following does show the telescoping series? (K)

A) $\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)}$ B) $\sum_{n=1}^{\infty} r$ C) $\sum_{n=1}^{\infty} n^3$ D) $\sum_{n=1}^{\infty} ar^{n-1}$ E) $\sum_{n=1}^{\infty} n^r$

4. Which one of the following is the sum of $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^{n-1}} + \dots$ (Ap)

- A) 1/2
- B) 1
- C) 2
- D) 3
- E) 4

5. Which one of the following is not equal to the series $1 + \frac{1}{2} + \frac{1}{4} + \dots$ (C)

A) $\sum_{n=0}^{\infty} \frac{1}{2^n}$ B) $\sum_{n=5}^{\infty} \frac{1}{2^{n-5}}$ C) $\sum_{n=4}^{\infty} \frac{1}{2^{n-4}}$ D) $\sum_{n=2}^{\infty} \frac{1}{2^{n+1}}$ E) $\sum_{n=1}^{\infty} \frac{1}{2^{n-1}}$

6. Which one of the following does NOT define the divergent series $\sum_{n=1}^{\infty} a_n$? (C)

- A) The limit of the general term does NOT exist
- B) The limit of the general term is different from zero
- C) The series does NOT have a finite sum
- D) The limit of the general term is zero
- E) The limit of the general term is infinity

7. Which one of the following is the divergent series? (Ap)

A) $\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)}$ B) $\sum_{n=1}^{\infty} \left(\frac{3}{4}\right)^n$ C) $\sum_{n=1}^{\infty} n^3$ D) $\sum_{n=1}^{\infty} (5)^{-n}$ E) $\sum_{n=1}^{\infty} \left(-\frac{3}{4}\right)^n$

8. Which one of the following is the convergent series? (C)

A) $\sum_{n=1}^{\infty} \frac{1}{n}$ B) $\sum_{n=1}^{\infty} \left(\frac{5}{4}\right)^n$ C) $\sum_{n=1}^{\infty} n^3$ D) $\sum_{n=1}^{\infty} \left(\frac{n+1}{n}\right)$ E) $\sum_{n=1}^{\infty} \left(\frac{3}{4}\right)^n$

9. Which one of the following is true for the series $\sum_{n=1}^{\infty} \frac{3^{n-1} - 1}{6^{n-1}}$? (An)

- A) The series diverges
- B) The series converges, and its sum is 4/5
- C) The series does not diverge or converge
- D) The series converges, and its sum is 6/5
- E) The series is telescoping series, and its sum is 4/5

10. Which one of the following is true for the series $\sum_{n=1}^{\infty} \frac{1}{n^2 + 1}$? (C)

- A) The series diverges
- B) The series converges
- C) The series does not diverge or converge
- D) The series converges, and its sum is 1/2
- E) Any convergence test cannot test the series

11. Which one is true for the series $\sum_{n=1}^{\infty} \frac{2^n + 5}{3^n}$? (Ap)

- A) The series diverges
- B) $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \frac{2}{3} < 1$, then it converges
- C) The series does not diverge or converge
- D) The series converges, and its sum is 2/3
- E) $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \frac{3}{2} > 1$, then it diverges

12. Which one of the following is the convergent series? (An)

- A) $\sum_{n=1}^{\infty} n \cdot \sin\left(\frac{1}{n}\right)$ B) $\sum_{n=1}^{\infty} \left(\frac{5}{4}\right)^n$ C) $\sum_{n=1}^{\infty} n^3$ D) $\sum_{n=1}^{\infty} \frac{e^n}{1 + e^{2n}}$ E) $\sum_{n=1}^{\infty} n \cdot \lg\left(\frac{1}{n}\right)$

13. Which one is true for the series $\sum_{n=1}^{\infty} n^2 e^{-n}$? (An)

- A) The series diverges
- B) $\lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \frac{1}{e} < 1$, then it converges
- C) The series does not diverge or converge
- D) The series converges, and its sum is 1/e
- E) $\lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \frac{1}{e} > 1$, then it diverges

14. Which is right for the series $\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$? (Ap)

- A) The series diverges

- B) The series converges
- C) The series does not diverge or converge
- D) The series converges absolutely
- E) The series converges conditionally

15. Which one of the following series is equal to the power series (C)
 $1 - \frac{1}{2}(x-2) + \frac{1}{4}(x-2)^2 - \frac{1}{8}(x-2)^3 + \dots$?

- A) $\sum_{n=1}^{\infty} \left(\frac{-1}{2}\right)^n x^n$ B) $\sum_{n=1}^{\infty} \left(\frac{-1}{2}\right)^n (x-2)^n$ C) $\sum_{n=0}^{\infty} \left(\frac{-1}{2}\right)^n (x-2)^n$ D) $\sum_{n=0}^{\infty} \left(\frac{-1}{2}\right)^n x^n$ E) $\sum_{n=1}^{\infty} \left(\frac{-1}{2}\right)^n x^{n+1}$

K = Knowledge; C = Comprehension; Ap = Application;
 An = Analysis; S = Synthesis; E = Evaluation

II. Examples of AL strategies

1. *One-minute paper*: A tool that the instructor uses a short written activity in response to a posed question, which leads students to reflect on the lesson and supplies the instructor with useful feedback.

Ex: Determine whether the series $\sum_{n=1}^{\infty} \left(\frac{e}{\pi}\right)^n$ is convergent or not.

2. *Small-Group Discussion*: An example of active learning because it allows students to express themselves in the classroom.

Ex: Investigate the convergence of the series $\sum_{n=1}^{\infty} \frac{8n^3 + 100n^2 + 1000}{2n^6 - n + 5}$ and show the solution.

3. *Memory Matrix*: This is a diagram, a rectangle divided into rows and columns used to recall knowledge and find relationships. In a memory matrix, the cells filled with questions at the right are given, but the cells are left empty for students to reply to during class.

What is the condition of convergence of the ratio test when the limit of a_{n+1}/a_n is p	?
What is the interval of convergence for the geometric series if $\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n$?
Type of the series $\sum_{n=1}^{\infty} \frac{1}{(2n+1)(2n+3)}$?

4. *Brain Storming*: Brainstorming is a group activity that seeks to gather a list of ideas contributed by their members to find the result of a particular problem.

Show that the series $\sum_{n=1}^{\infty} \left(\frac{1}{n}\right)$ diverges.

5. *Videos*: The videos help students to realize what they are learning at the time in an alternative presentation way.

<https://www.youtube.com/watch?v=nt2OIMAJj6o>

<https://www.youtube.com/watch?v=0YeON4p0ogw>

APPENDIX B

Survey of Motivation and Self-Confidence

SURVEY ABOUT MATHEMATICS for SCIENCE

Major Aim of the survey is to understand if there is any significant change in science students self-confidence and motivation towards mathematics. Please read each statement and choose your response. Strongly Disagree, Disagree, Neutral, Agree, Strongly Agree

Questions

	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree
1. I would make me happy to be recognized as an excellent student in math.					
2. I'd be proud to be the outstanding student in math.					
3. I am happy to get top grades in mathematics.					
4. It would be really great to win a prize in mathematics.					
5. Being first in mathematics competition would make me pleased.					
6. Being regarded as smart in mathematics would be great thing.					
7. If I got the highest grade in math, I'd prefer no one knew. I would make people like me less if I were a really good math student.					
8. I don't like people to think I'm smart in math.					
9. Generally I have felt secure about attempting mathematics.					
10. I'm not good in math.					
11. For some reason, even though I study, math seems unusually hard for me.					
12. Most subjects I can handle O.K., but I have a knack for flubbing up math.					
13. I usually have been at ease in math classes.					
14. Mathematics usually makes me feel uncomfortable and nervous.					
15. Mathematics makes me feel uncomfortable, restless, irritable, and impatient. I get a sinking feeling when I think of trying hard math problems.					
16. Mathematics makes me feel uneasy and confused.					
17. When a math problem arises that I can't immediately solve, I stick with it until I have the solution.					

APPENDIX C

Real-world Problems

Table C 1

1	Geometric Sequence	A ball is thrown from an 81-centimeter height. It bounces back to its previous height after each bounce. When the ball lands for the fifth time, what is the total distance it has gone in the air?[225, p. 266]
2	Infinite Sum	Consider a paper-based equilateral triangle. Using our scissors, we cut smaller equilateral triangles from the original triangle, following the idea of connecting the middle points of the sides of every triangle we encounter. Cut out the center triangle and toss it away. Repeat the technique for each new triangle you come across. If we don't stop cutting, how much of the original area will remain?[225, p. 266]
3	Infinite Sum	Another mathematical construct is the Koch snowflake. A Koch snowflake is created by gradually increasing the size of an equilateral triangle. We divide the triangle's sides into thirds and then make a new triangle on each of the three middle thirds. Then we go through the process again and again. As a result, each snowflake becomes more intricate, but each new triangle in the pattern appears to be identical to the previous one. Imagine a circle being drawn around the original figure. The area of the figure remains inside the circle no matter how large the perimeter grows. An infinite perimeter encloses a finite area in the Koch Snowflake. Even though it appears impossible, does it?[225, p. 271]
4	Limit of Sequence	How many years will it take you to have 20,000 dollars if you invest \$5,000 in a certificate of deposit (CD) that pays 4.5 percent annually, compounded quarterly, and make no further investments in the CD? What happens if the CD earns 6.25 percent interest?[226, p.771]
5	The Integral Test	Even though we know the harmonic series diverges, there is no actual evidence that it does so. The partial amounts are simply too sluggish to develop. Consider what would happen if you started at the beginning of time, 13 billion years ago, and added a new term every second. Assuming a 365-day year, how much would the partial sum be today?[226, p.772]

APPENDIX D

Data Analysis of the Nile University

NILE UNIVERSITY
EXAMINATION REPORT



NILE UNIVERSITY
EXAMINATION REPORT

Faculty: College of Health Sciences / College of Health Sciences
 Department: Medicine / Medicine
 Speciality: Medicine / Medicine
 Year | Semester: 2020-2021 | 2
 Course (Section): MAT 101 n/a / General Mathematics I (04)
 ECTS / Credit: 3 | 3
 Instructor: OMOLOLA DORCAS ATANDA

№	Student ID	Surname and Name	HW	HW	MT	FIN	Total	
							Percent	Letter
1	201211021	MAGAJI Aisha Muhammad	100	100	50	70	72	A
2	201500014	Abdulkareem Durosinlorun ABDULLAHI	100	100	100	100	100	A
3	201500019	Babangida Subewope Josephine	100	100	50	78	77	A
4	201500022	IDRIS Fatimah Abdulrahman	100	100	75	85	86	A
5	201500023	BELLO Damilola Faizah	100	100	90	73	82	A
6	201500026	OPEIFA Fareedat Adewodun	100	100	100	80	88	A
7	201500028	Ibrahim Bappa Fadimatu	100	100	50	70	72	A
8	201500029	BELLO Nadimah Ojuolape	100	100	100	85	91	A
9	201500031	AMEH Peter Owoicho	100	100	100	78	87	A
10	201500032	EZUGWU Nnamdi	100	100	40	48	57	C
11	201500033	UMEH Oluchi Mary-Cynthia	abs	100	60	83	72	A

Date	05.11.2022	SIGNATURE	SEAL
Instructor	OMOLOLA DORCAS ATANDA		
Head of Department	FARUK MUSTAPHA ABDULLAHI		
Dean of Faculty	SENOL DANE		

NILE UNIVERSITY
EXAMINATION REPORT



NILE UNIVERSITY
EXAMINATION REPORT

Faculty: Faculty of Engineering / Faculty of Engineering
 Department: Civil Engineering / Civil Engineering
 Speciality: Civil Engineering / Civil Engineering
 Year | Semester: 2019-2020 | 2
 Course (Section): MAT 102 n/a / General mathematics II (02)
 ECTS / Credit: 3 | 3
 Instructor: OMOLOLA DORCAS ATANDA

№	Student ID	Surname and Name	HW	QZ	MT	FIN	Total	
							Percent	Letter
1	151202023	DUGURI Shafiu Adamu	abs	abs	abs	34	20	F
2	161202051	ADO Musaddiq Muhammad	abs	abs	50	33	30	F
3	171202002	AMINU Hauwa	abs	abs	50	30	28	F
4	171202014	MAIKANO ABDULLAHI Muhammad Rabiu	abs	abs	40	abs	8	F
5	171202029	ABDULRAHMAN Zanira Tagwai	50	93	30	33	40	E
6	171202040	LAWAN Yusuf Ahmad	abs	abs	abs	abs	0	F
7	171202041	LAWAL Umar Muhammad	100	abs	abs	abs	10	F
8	171202045	BUKAR Muhammed	abs	abs	abs	abs	0	F
9	171202047	OGEDENGBE Flora	abs	abs	50	abs	10	F
10	171202052	ZAKU Ismail Al-hassan	abs	abs	80	40	40	E
11	171210012	HASSAN Amina Mohammad	abs	abs	40	46	36	F
12	181202022	ASAD Yazan Ayman	abs	abs	90	54	50	C
13	181202023	OLORUNNISHOLA Emmanuel Moyinoluwa	100	abs	50	39	43	E
14	181202031	KABIR Rabiu Jibir	abs	abs	abs	abs	0	F
15	181202046	NUMO-OLALI Tekena Benjamin	abs	abs	abs	abs	0	F
16	181202047	AL-AMIN Bilal Mohammed	abs	abs	70	60	50	C
17	181202051	YUNUSA Abdulmumin Tahiru	100	abs	90	23	42	E
18	191201004	WILSON Marvellous Douye	abs	abs	10	abs	2	F
19	191202001	OBIANIME Favour Tamunoimana	100	67	90	77	81	A
20	191202002	MALIKI Asoshido Anike	100	87	90	66	76	A
21	191202003	ARABI Aslamah Amina	100	80	70	22	45	D
22	191202004	IGUZA Steven Oshoke	100	100	90	76	84	A
23	191202005	ABDULRAHEEM Abdulsamad Anavoza	100	53	80	53	63	B
24	191202006	SAIDU Shuaibu Yusuf	100	53	70	73	73	A
25	191202007	SAIDU Najib Umar	100	40	60	75	71	A
26	191202008	ABDULMALIK Abdulkadir Khalifa	100	100	100	44	66	B
27	191202009	Nyeche Derek Chimenem	100	100	100	17	50	C
28	191202010	DAJI Nansak Destiny	100	73	90	77	82	A
29	191202011	AKPOTU Onanefe Great	100	73	70	83	81	A
30	191202012	Aminu Abubakar Abdulhamid	100	60	60	64	66	B
31	191202014	MUHAMMAD Aisha	100	71	100	60	73	A
32	191202016	MBACHU Sopuruchukwu Ashford	100	100	100	80	88	A
33	191202019	SANI Mustapha	100	60	90	77	80	A
34	191202022	DANTATA Farida	100	80	90	58	71	A
35	191202023	MIKAILU Alamin Umar	100	67	60	44	55	C
36	191202024	ISIAKA Isaac Rufai	abs	47	70	68	60	B
37	191202025	ISHIYAKU Ibrahim Babayola	abs	47	70	61	55	C
38	191202026	HARUNA Mahmud Musa	abs	abs	100	76	66	B

**NILE UNIVERSITY
EXAMINATION REPORT**



**NILE UNIVERSITY
EXAMINATION REPORT**

Faculty: Faculty of Engineering / Faculty of Engineering
 Department: Civil Engineering / Civil Engineering
 Speciality: Civil Engineering / Civil Engineering
 Year | Semester: 2020-2021 | 2
 Course (Section): MAT 101 n/a / General Mathematics I (01)
 ECTS / Credit: 3 | 3
 Instructor: OMOLOLA DORCAS ATANDA

№	Student ID	Surname and Name	HW	HW	MT	FIN	Total	
							Percent	Letter
1	201202045	Radman Abdulrazaq Abdallah	100	100	75	88	88	A
2	201202046	GARBA MUHAMMAD IBRAHIM	80	100	50	40	52	C
3	201202047	Ahmad Nagogo Abubakar sadiq	abs	100	abs	33	30	F
4	201202048	KYARI Umar	40	100	25	15	28	F
5	201202049	Mukhtar Hanaan	100	100	75	85	86	A
6	201202051	ABDULMALIK Mustapha Modibbo	70	100	25	35	43	E
7	201202052	MAHMUD Mohammed Gazara	abs	100	1	15	19	F
8	201202053	DAHIRU Almustapha Abba	30	100	55	65	63	B
9	201202054	ETSU-NDAGI Jibril Haji	100	100	50	40	54	C
10	201202055	Attahiru G. Koko Salisu	70	100	1	55	50	C
11	201202056	HABIBU Juhaina Rabia	100	100	50	60	66	B
12	201202057	KELSON-ONORIODE Wealth Efe-Oghene	30	100	65	68	67	B
13	201202058	AGBIJI Victory Mbeh	100	100	100	68	81	A
14	201202060	ADO Aminu Alhassan	30	100	1	23	27	F
15	201202061	MUKHTAR Usman Anka	abs	100	60	35	43	E
16	201202062	AHMED Abdullahi Tijjani	70	100	100	73	81	A
17	201202063	MAHMUD Abubakar Sahabo	100	100	100	85	91	A
18	201202064	UMAR Abubakar Ahmad	100	100	75	65	74	A
19	201202065	SHOWUNMI Abdulquadri Folorunsho	100	100	25	58	60	B
20	201202067	BULAMA Muhammad Musa	70	100	1	20	29	F
21	201202068	SHEHU Mukhtar	abs	100	40	48	47	D
22	201202069	FAROUK Zainab Hadi	100	100	85	83	87	A
23	201202070	MUSA Lawan Lawan	100	100	1	50	50	C
24	201202071	AHMED RUFAl Aliyu	40	100	75	65	68	B
25	201202073	BAJEH Mu'azu Bajeh	70	100	abs	55	50	C
26	201202075	ABDULLAHI Abdul-Rahman Mainasara	abs	100	95	83	79	A
27	201202076	GADU Amir Nuhu	abs	100	75	25	40	E
28	201202078	Dawa Maigari Muhammad	50	100	100	95	92	A
29	201202079	SAM Check	abs	100	25	55	48	D
30	201206031	ABUBAKAR UMARA Babagana	100	100	30	63	64	B
31	201605011	Sani Ibrahim Sabiu	70	100	1	50	47	D

Date	05.11.2022	SIGNATURE	SEAL
Instructor	OMOLOLA DORCAS ATANDA		
Head of Department	ABDULHAMEED DANJUMA MAMBO		
Dean of Faculty	STEVE ADESHINA		

APPENDIX E

University Malaysia Pahang Data Analysis of Applied Calculus

Table E 1

		Student Id	Student Name	Pre-test	05G Post-test
1	2	3		4	5
1	AA17246	HAMZAH ABDULLAH SALEH AHMED		15	21
2	AA18108	NUR FADILLAH SYAFITRI BINTI MOHD ALI ALLAKHIE		33	35
3	AA20050	SITI AISYAH BINTI ABDUL GHAFFAR		31	35
4	AA20057	MUHAMMAD FARHAN BIN MOHD IZRA`AI		25	32
5	AA20108	MOHAMMAD HAFIZI BIN MOHD HASHIM		14	5
6	AA20172	MUHAMMAD AIMAN HAKEEMI BIN MOHAMMAD AZHAN		18	23
7	AA20186	ANIS SHUHADA BINTI AB RAHMAN		17	20
8	EA20001	TAN QIAO ZHEN		20	31
9	EA20005	MOHAMAD SHAZRIEL BIN MOHAMMAD KAMARUZ'ZAMAN		31	21
10	EA20013	LOH FUI QI		18	25
11	EA20014	MUHAMMAD HAZWAN BIN HAZMI		32	32
12	EA20021	TEO YUNN SHI		34	33
13	KA17288	MOHAMAD FAHIMUDIN BIN RAZALI		30	34
14	KA20095	LEWIS SIE KAI QIANG		21	17
15	KA20097	ONG SHAO JIE		39	39
16	KA20101	CHUA KAR WEI		12	14
17	KA20102	ESMOND CHEW ZHI YU		40	40
18	KA20108	CHONG JIA QI		32	35
19	KA20109	YONG CHEN YEE		32	37
20	KA20127	NIK FAREEZ IKHWAN BIN NIK ANUAR		38	40
21	KA20134	LOW JIA YI		38	34
22	KH17035	MUHAMMAD ASYRAF BIN RUSLI		40	40
23	KH17050	MUHAMAD EZMER EDZOFIE BIN AHMAD FURIZ		37	40
24	MA19142	AHMAD SHUKRI BIN AHMAD ZUKI		31	29
25	MA20005	DIVYHAN A/L HARIDASH		36	29
26	MA20035	LIOW WEN AN		20	14
27	MA20038	HASZIANA BINTI ISMAIL		38	40
28	MA20040	DHARWEEN NAMBIAR A/L SASEEDHARAN		36	34

Continuation of the table E 1

1	2	3	4	5
29	MA20042	MUHAMMAD AMIRUL HAFIZ BIN MOHD AMRI	32	34
30	MA20058	MOHD HISYAM BIN ALIMSHA	18	8
31	MA20090	MUHAMMAD FARIS BIN MOHD RADZUAN	23	14
32	MA20099	MUHAMMAD ALIFF DANIAL BIN MOHD ROSDY	16	16
33	MA20114	SITI NURHAZWANI HUSNA BINTI MOHD HATA	27	37
34	MA20122	MOHAMAD ARIFF ASYRAF BIN RUSLI	27	34
35	MA20126	MUHAMMAD HAFIZUDDIN BIN MOHD ZULKIFLI	30	36
36	MA20136	NUR ILLYANI BINTI MOHAMED RAFLI	25	21
37	MA20153	MUHAMMAD FAEZ RIZWAN BIN MARNIZAM	27	34
38	MA20154	MUHAMMAD KHAIRULAMIRIN ADAM BIN MUHAMMAD PAZURI	15	21
39	MA20185	TAVITHASHINI A/P KETHARA NATHAN	27	34
40	MH19057	NUR MAIZATUL ALYAA BINTI ABDUL AZIZ	18	15
41	MH20006	LAU CHONG SWIN	12	15
42	MH20020	MUHAMMAD LUKMAN HAKIM BIN MD PADIL	21	29
43	PA18050	NUR AINSURAYA BINTI AHMAD ROSLEN	8	5
44	SA20053	LIAU YI XUAN	29	16
45	SA20065	MOHD AZWYN BIN ARDI	18	21
46	SA20066	POOJA A/P MAHENDRAN	6	7
47	SA20075	NURUL FATIHAH BINTI JAMAL	22	12
48	SA20078	UMMI YASMIN UMAIRAH BINTI YUSRI	25	25
49	SA20080	NURUL HUSNA IZZATI BINTI AHMAD YUSNAIDI	10	4
50	SA20082	KOSHARVINI PREVISHA A/P ALAGESAN	16	15
51	SA20085	ADZLYN ADILAH BINTI IDRUS	30	27
52	SA20096	NUR ATHIRAH SYIFAA BINTI MOHD YUSRI	28	20
53	SB20022	MOHD SHAMRIE BIN JAISON	29	35
54	SB20031	CLEMENT KONG HONG MING	20	22
55	SB20036	NURWARDINA BALQIS BINTI MOHD ISA	37	35
56	SB20049	AW YONG LIANG	34	32
57	SC17066	BASSAM KHALED BALKHAIR AL-MASHGARI	10	4
58	SC20056	AMAN SOORAJ SHEJI	16	17

APPENDIX F

Information Letter for the Experiment



PUSAT SAINS MATEMATIK
CENTRE FOR MATHEMATICAL SCIENCES
Universiti Malaysia Pahang
Lebuhraya Persiaran Tun Khalil Yaakob
26300 Gambang, Kuantan
Pahang Darul Makmur

Tel : +609-431 5049
e-mel : psm@ump.edu.my
Laman Web: <https://psm.ump.edu.my>

25 August 2018

To whom it may concern

Dear Sir/Madam,

This is to certify that **Mr. Abdullah Almas** had come to Universiti Malaysia Pahang on **24 July 2018 – 17 August 2018** to complete some part of his study titled *Comparison of Active Learning and Traditional Learning in Teaching Applied Mathematics* under Associate Professor Dr Anvarjon Ahmedov supervision.

Please do not hesitate to contact me at roslinazairimah@ump.edu.my if you have any questions or further clarification.

Very sincerely

Roslinazairimah Zakaria, PhD
Associate Professor Dr
Centre for Mathematical Sciences
Universiti Malaysia Pahang