Investigating the Effect of Peer Learning on Students' Achievement and Attitude in "Discrete Mathematics"

Akniyet Orazkhan

A thesis submitted to the School of Education and Humanities

in partial fulfillment of the requirements for the degree of

MASTER OF EDUCATIONAL SCIENCES

in 7M01501-Mathematics

«SDU University»

Department of Pedagogy of Natural Sciences

Thesis Advisor:

PhD., Assistant Professor, Nuri Balta

«SDU University»

School of Education and Humanities

Department of Natural Sciences Pedagogy

This is to certify that the Master's Thesis of

Akniyet Orazkhan

has met the thesis requirements of

SDU University

Kaskelen, 2026

Approved by:

Thesis Supervisor	(Signature)	PhD., Assistant Professor Nuri Balta
M.Ed. Program Coordinator	(Signature)	M.Ed., Senior Lecturer Nurbek Sagyndyk
Head of Department of Natural Science	(Signature)	PhD., Associate Professor Zhangyl Abilbek
Approved Dean of the School of Education and Humanities	(Signature)	PhD., Assistant Professor Dana Moldabayeva

AUTHOR AGREEMENT

A Orazh

Author Signature

08.06.2026

Date

This page is to be separately printed, then signed and dated by the author, after that scanned and finally copy-pasted in this page.

DECLARATION

I, the undersigned, hereby declare that this submission is entirely my work, in my own words, and that all sources used in researching it are fully acknowledged and all quotations properly identified to my best extent. It has not been submitted, in whole or in part, by me or another person, to obtain any other credit, except where due acknowledgement is made in the thesis. I understand the ethical implications of my research, and this work meets the requirements of the School of Education and Humanities Research Ethics Policy.

Name Surname: Akniyet Orazkhan

AOrazh
____
Signature

08.06.2026
____
Date

List of Tables

List of Figures

List of Appendices

Table of Contents

List of Abbreviations

ANCOVA – Analysis of Covariance

ATMI – Attitudes Toward Mathematics Inventory

CI – Confidence Interval

CVI – Content Validity Index

df – Degrees of Freedom

M – Mean

Mdn – Median

RQ – Research Question

SD – Standard Deviation

SE – Standard Error

SPS – Structured Problem Solving

STEM – Science, Technology, Engineering, and Mathematics

TIMSS – Trends in International Mathematics and Science Study

TPS – Think-Pair-Share

TPS-SPS – Think-Pair-Share with Structured Problem Solving

ZPD – Zone of Proximal Development

ABSTRACT

This research investigated how the Think–Pair–Share method combined with Structured Problem Solving (TPS–SPS) impacted students' academic performance and their attitudes about Discrete Mathematics. The study used a quasi-experimental pre-test/posttest non-equivalent control group design with second- and third- year mathematics pedagogy students at a private university located in the Almaty region of Kazakhstan ($N = 164$ for achievement; $N = 72$ for attitude). The participants of the study attended the same lectures, yet their practice sessions differed because experimental groups completed TPS–SPS activities during a three-week Counting and Probability unit while control groups followed traditional individual-work instruction. Academic achievement was measured using a researcher-developed open-ended test ($\alpha = 0.809$), and attitudes were assessed using an adapted Attitudes Toward Mathematics Inventory ($\alpha = 0.909$). ANCOVA results showed a statistically significant effect for the 2nd-year cohort, with the experimental group producing higher adjusted post-test scores. The 3rd-year cohort did not show a statistically significant effect; however, both cohorts displayed similar improvement patterns, with experimental groups showing higher gains. No significant between-group differences were found in overall attitude for either cohort, yet the 2nd-year experimental group demonstrated a significant within-group improvement in self-confidence. The results suggest that TPS-SPS is an effective method for enhancing Discrete Mathematics achievement, while short-term interventions produce limited attitudinal change.

Keywords: Think–Pair–Share; Structured Problem Solving; peer learning; Discrete Mathematics; academic achievement; attitude

АҢДАТПА

Бұл зерттеу Think–Pair–Share әдісін Құрылымдық есеп шешумен (TPS–SPS) біріктірудің студенттердің «Дискретті математика» пәні бойынша үлгеріміне және пәнге деген көзқарасына әсерін зерттеді. Зерттеуде Алматы облысындағы жекеменшік университеттің математика педагогикасы мамандығында оқитын 2 және 3-ші курс студенттері қатысқан квазиэксперименттік пре-тест/пост-тест эквивалентті емес бақылау тобы әдісі қолданылды (үлгерім бойынша $N = 164$; көзқарас бойынша $N = 72$). Қатысушылар бірдей дәрістерге қатысты, алайда практикалық сабақтар әртүрлі ұйымдастырылды: эксперименттік топтар үш апталық «Комбинаторика және ықтималдық» тақырыбы бойынша TPS–SPS тапсырмаларын орындады, ал бақылау топтары дәстүрлі жеке жұмыс нұсқауларын пайдаланды. Академиялық үлгерім зерттеуші әзірлеген ашық сұрақтарды қамтитын тест арқылы өлшенді ($\alpha = 0.809$), ал пәнге көзқарас бейімделген сауалнама (ATMI) арқылы бағаланды ($\alpha = 0.909$). ANCOVA нәтижелері 2-курс студенттерінде эксперименттік топтың пост-тест нәтижелері бақылау тобынан статистикалық тұрғыдан мәнді жоғары болғанын көрсетті, ал 3-курста мәнді әсер анықталмады. Дегенмен, екі курста да эксперименттік топтар жоғарырақ ілгерілеу динамикасын көрсетті. Жалпы көзқарас бойынша екі курста да топтар арасында статистикалық мәнді айырмашылық байқалмады, бірақ 2-курс эксперименттік тобында өзіне деген сенімділік бойынша топ ішіндегі мәнді жақсару анықталды. Нәтижелер TPS–SPS әдісінің Дискретті математикадағы академиялық жетістікті арттыруға тиімді тәсіл екенін , алайда қысқа мерзімді интервенциялар пәнге деген көзқарасты айтарлықтай өзгертпеуі мүмкін екенін көрсетті.

Түйінді сөздер: Think–Pair–Share; Құрылымдық есеп шешу; өзара оқыту әдісі; дискретті математика; академиялық үлгерім; көзқарас

АННОТАЦИЯ

Данное исследование изучало влияние метода Think–Pair–Share в сочетании со структурированным решением задач (TPS–SPS) на академическую успеваемость студентов и их отношение к дискретной математике. Был использован квазиэкспериментальный дизайн с предтестом, посттестом и неэквивалентными контрольными группами. Участниками стали студенты 2-го и 3-го курсов образовательной программы «Математика» частного университета Алматинской области Казахстана (N = 164 для оценки академических достижений; N = 72 для оценки отношения к предмету). Все участники посещали одни и те же лекции, однако практические занятия различались: экспериментальные группы выполняли TPS–SPS по разделу «Комбинаторика и вероятность», а контрольные группы работали индивидуально в традиционном формате. Академические достижения оценивались тестом с открытыми вопросами (α = 0,809), а отношение к математике –адаптированной шкалой Attitudes Toward Mathematics Inventory (α = 0,909). Результаты ANCOVA показали статистически значимый эффект для студентов 2-го курса: экспериментальная группа продемонстрировала более высокие скорректированные результаты посттеста. Для студентов 3-го курса статистически значимый эффект выявлен не был; однако обоих курсах экспериментальные группы показали больший прирост. Значимых различий между группами по общему отношению к предмету обнаружено не было, но в экспериментальной группе 2-го курса было зафиксировано значимое повышение уверенности. Полученные результаты свидетельствуют, что TPS–SPS может повышать успеваемость по дискретной математике, тогда как краткосрочные педагогические вмешательства имеют ограниченное влияние на изменение студентов к предмету.

Ключевые слова: Think–Pair–Share; структурированное решение задач; взаимное обучение; дискретная математика; академическая успеваемость; отношение к предмету

1. INTRODUCTION

1.1 Background and Significance of the Study

The higher education system in Kazakhstan has experienced major changes since the country gained independence. Through its involvement in the Bologna Process and its adoption of new educational initiatives Kazakhstan seeks to transform its existing highly centralized post-Soviet educational system into a versatile system which evaluates student competencies and meets international educational standards (Tampayeva, 2015; Yergebekov & Temirbekova, 2012). Researchers show that institutional reform efforts in practice have been focused on building quality assurance systems, nurturing international partnerships, reaching accreditation, and revising how education is delivered (Azhibayeva et al., 2024; Massyrova et al., 2015). In more recent years, Kazakhstan's National Development Plan up to 2029 has doubled down on that same trajectory, by plainly flagging student-centered teaching methods, active learning approaches, and the formation of critical thinking abilities at every level of higher education (President of the Republic of Kazakhstan, 2024). Research indicates that actual classroom practice at Kazakhstani institutions continues to lag behind reform goals despite these policy pledges, with teacher-centered instruction continuing to be the predominant style of delivery (Tajik et al., 2025; Tampayeva, 2015).

University mathematics education is a particularly important spot for this reform agenda. Mathematics builds logical reasoning, and problem-solving skills that students will need later for their future work. Discrete Mathematics plays a crucial role here, because it exposes students to logic, combinatorics, probability, graph theory, and also proof methods, basically ways of thinking that sit under both advanced mathematics, and computer science. Still, students quite often stumble with the abstract side of the course, especially with symbolic representation, formal notation, and then proof construction (Magda & Gardner-McCune, 2025; Nurrahmah et al., 2021). Magda and Gardner-McCune (2025) also noted that students tend to see proof writing as one of the hardest parts of Discrete Mathematics, mainly because it makes you switch to a different style of thinking than procedural calculation.

These cognitive challenges get compounded by affective factors. Research carried out in Kazakhstan suggests that students' self-confidence , along with their motivational orientation, has a direct influence in their mathematics achievement (Kaymak & Sautkali, 2022). On top of that, mathematics anxiety keeps pushing performance down, while also lowering engagement, so you get this reinforcing loop that becomes especially harmful for students who already perceive Discrete Mathematics as difficult (Bjälkebring, 2019; Tsirimokos et al., 2024) . For students studying mathematics pedagogy, these concerns are especially important since their own educational experiences will directly influence how they teach in the future. Students are less likely to implement collaborative and student-centered teaching strategies in their own classrooms if they receive exclusively teacher-directed education throughout their time in university.

Peer learning gives a solid, well-evidenced answer to these challenges. It helps students articulate their thinking, compare solution methods and see where their understanding is thin, then through that social interaction they gradually construct more conceptual knowledge. In the meta-analysis by Capar and Tarim (2015), they reported a medium positive effect of cooperative learning on mathematics achievement, and also a small positive effect on student attitudes. For the university level, peer learning has been shown to predict mathematics performance (Prasad Dahal, 2026; Samuel & Sumaila, 2025) . And peer instruction, has produced meaningful achievement gains, especially inside the Kazakhstani university setting (Serkan Kaymak et al., 2020).

Think-Pair-Share (TPS) is one of the most widely usable peer learning structures in mathematics instruction, because it mixes individual accountability with organized peer talk, and also with whole-class sharing (Tanujaya & Mumu, 2019). Ramadhani et al. (2025) found that mathematics teachers and lecturers generally viewed TPS positively, since it helps students reach deeper conceptual understanding and also sharpen their communication skills, additionally it made students more willingly involved during the lesson. In order to ensure that peer conversations are based on mathematical reasoning rather than answer-sharing, the current study blends TPS with Structured Problem Solving (SPS) by forcing students to explain their solution method prior to calculation. The known shortcomings of each approach separately are addressed by this TPS-SPS integration, which also establishes circumstances for procedural and conceptual engagement with Discrete Mathematics subject.

1.2 Research Problem

In Kazakhstan, the majority of university mathematics practice classes are teacher-centered, with students working on problem sets on their own while the instructor makes necessary modifications. This framework restricts opportunities for peer discussion, group reasoning, and comparison of solution strategies—exactly the kinds of activities that research indicates are most helpful for cultivating the abstract reasoning abilities required by Discrete Mathematics (Magda & Gardner-McCune, 2025; Nurrahmah et al., 2021). Negative attitudes toward discrete mathematics can arise when students find the topic challenging and alienating, thus impairing their performance and engagement (Kaymak & Sautkali, 2022). The effect of TPS-SPS on university-level Discrete Mathematics achievement and student attitudes in Kazakhstan has not been investigated, despite research on peer learning and TPS in mathematics education. This gap is filled in the current study.

Aim of the study

The aim of this study is to investigate the effect of peer learning on students' achievement and attitudes in Discrete Mathematics.

Research Questions and Objectives

This study is guided by the following research questions:

RQ1. Is there a statistically significant difference in Discrete Mathematics achievement between students taught through TPS-SPS peer learning and those taught through traditional instruction?

RQ2. Do students taught using TPS-SPS peer learning differ statistically significantly from those taught through traditional education in terms of their overall attitudes toward discrete mathematics?

RQ3. Do students taught by TPS-SPS peer learning differ statistically significantly from those taught through traditional education in terms of their motivation, enjoyment, and self-confidence toward discrete mathematics?

The following objectives of the study are in accordance with these research questions:

1. To investigate how TPS-SPS peer learning affects students' achievement in discrete mathematics.
2. To assess how TPS-SPS peer learning affects students' overall attitudes of discrete mathematics.
3. To examine the effect of TPS-SPS peer learning on the attitudinal dimensions of self-confidence, enjoyment, and motivation.

Research Hypotheses

TPS–SPS peer learning has a positive effect on students' academic achievement and attitudes toward Discrete Mathematics compared to traditional instruction.

Statistical Hypotheses

The following null and alternative hypotheses are formulated to test the research hypothesis:

Achievement

$H_0$1. There is no statistically significant difference in Discrete Mathematics achievement between students taught through TPS-SPS peer learning and those taught through traditional instruction.

$H_1$1. Students taught through TPS-SPS peer learning demonstrate significantly higher Discrete Mathematics achievement than students taught through traditional instruction.

Overall Attitude

$H_0$2. There is no statistically significant difference in overall attitude toward Discrete Mathematics between students taught through TPS-SPS peer learning and those taught through traditional instruction.

$H_1$2. Students taught through TPS-SPS peer learning demonstrate significantly more positive attitudes toward Discrete Mathematics than students taught through traditional instruction.

Attitudinal Subscales

$H_0$3. There are no statistically significant differences in self-confidence, enjoyment, or motivation between students taught through TPS-SPS peer learning and those taught through traditional instruction.

$H_1$3. Students taught through TPS-SPS peer learning demonstrate significantly higher self-confidence, enjoyment, and motivation than students taught through traditional instruction.

Contribution of the Study

The research makes three contributions to mathematics education. First, it extends the theoretical knowledge base on peer learning, by using it within university-level Discrete Mathematics. This subject, with its focus on proof logic, and combinatorial reasoning, turns out to be a fairly apt setting for seeing how structured student-to-student interaction might shape conceptual understanding. Second, it adds empirical evidence since both achievement and attitude are treated as dependent variables. It also relies on pre-test/post-test measures with validated instruments (Tapia & Marsh, 2004), so overall it forms a methodological model that other studies can follow later on. Third, it offers practical value for Kazakhstani mathematics education, because it shows a structured low-cost peer learning method that can slot into current practice class timetables without any curriculum modification. It also gives future mathematics teachers direct experience of a student-centered pedagogy, while they are still in their own training.

2. LITERATURE REVIEW

Research on mathematics education in higher education has more and more emphasized the need to step beyond teacher centered instruction toward learning environments where students actively build, justify and assess mathematical ideas. Within this wider movement, peer learning has gotten a lot of attention, mostly because it gives students a chance to state their reasoning, contrast solution routes, gather feedback from classmates, and gain confidence through common mathematical work. Topping (2005) defines peer learning as "the acquisition of knowledge and skills through active helping and support among learners of equal status" (p. 631). This definition is important for the present study because it distinguishes peer learning from simple group seating or informal student talk: peer learning requires purposeful academic support among learners who participate actively in each other's understanding.

This chapter currently reviews some of the theoretical and empirical literature related to peer learning in mathematics education, with a special focus on Think–Pair–Share as it gets integrated with Structured Problem Solving (TPS-SPS). It also looks at student achievement, student attitude and the learning challenges that associated with Discrete Mathematics. Instead of considering earlier studies as just isolated findings, the chapter tries to synthesize their contributions, limitation points, and then shows the gap that this present study is aiming to address.

2.1 Theoretical Framework

Social Constructivism and the Zone of Proximal Development

Vygotsky's (1978) sociocultural theory serves as the primary theoretical foundation that most influences peer learning. The framework depends on the Zone of Proximal Development (ZPD), which defines the distance between independent learner abilities and their capabilities when working with a more skilled partner. Vygotsky demonstrated that people first develop advanced thought abilities through social interactions which they later convert into personal knowledge, so he considered peer interactions to be essential for cognitive development instead of simply helping the learning process.

The framework of Vygotsky explains that students who work together on proof construction or logical reasoning will achieve better results than students who work alone, when their collaborative partner meets their ZPD requirements. The research conducted by Forman and Cazden (2013) proved that peers function as effective cognitive scaffolds when the problem exists within this particular zone. Subiantoro et al. (2025) demonstrated that Vygotskian principles, particularly the Zone of Proximal Development (ZPD) and scaffolding, can reduce mathematics anxiety when applied in a collaborative mathematics learning environment. Their findings also suggested that the ZPD functions as both a cognitive and affective support mechanism.

Piaget's Constructivism and Cognitive Conflict

Vygotsky's definition of more knowledgeable others receives extension through Piaget's (1950) constructivism which shows how learners experience socio-cognitive conflict when they face conflicting viewpoints that disrupt their established cognitive schemas. Peer interactions at similar social levels create effective conflict situations because students will not follow their peers as easily as they will follow their teachers; thus, students must explain their thought processes while protecting their ideas from examination. Doise and Mugny (1984) showed that students who experienced socio-cognitive conflict during peer interactions developed their cognitive abilities faster on tasks which needed them to use formal operational thinking because this development needed them to build valid logical arguments.

The productive use of cognitive conflict requires particular conditions to be fulfilled. Students who do not possess basic knowledge need to assess their classmates' arguments will create a situation where peer interactions lead to mistaken beliefs instead of actual understanding (Cohen, 1994). The limitation demonstrates that people need to grasp their own understanding before they can enter group discussions which the THINK stage of TPS-SPS framework requires.

Social Interdependence Theory

Johnson and Johnson (1989) social interdependence theory give the basic structure for thinking about why cooperative learning tends to beat competitive and individualistic setups. What it does is separate goal structures into three parts, positive interdependence, where people see that their achievement is tied up with other people's success, negative interdependence, which is basically competition, and no interdependence, which shows up with solo or individualized work. The idea is that positive interdependence then triggers promotive interaction, where learners help, encourage and scaffold each other, and that then leads to better achievement. It also supports stronger interpersonal ties and even improved psychological wellbeing compared with the other goal structures. Later, Johnson and Johnson (2014) also expanded the framework, not only in the classroom but for how cooperative learning can build wider societal competencies. They argue that the skills cultivated through well designed cooperation, like constructive controversy, perspective taking, and collective decision making, aren't just accidental by products, but belong right in the educational mission.

Critically, however, Johnson and Johnson (1989; 2014) consistently emphasize that positive interdependence does not show up automatically just from putting students in groups. Simply assigning group work, without structural features like individual accountability, promotive interaction, appropriate social skill development, and group processing it tends to produce what they call pseudo-cooperation, and those results end up no better than individual learning. This separation matters methodologically for the present study because the TPS-SPS framework is built exactly to instantiate those structures, especially in the PAIR stage — where both partners have to contribute a prepared strategy — and also in the SHARE stage, which sets up group-level responsibility for a coherent shared explanation. The control group in this study, by contrast gives content exposure but without those structural elements. So any differences we observe should be tied to the cooperative structure itself, rather than to content, or teacher quality.

Bandura's Social Cognitive Theory and Self-Efficacy

Bandura's (1986) social cognitive theory demonstrates that vicarious learning and self-efficacy function as two pathways through which students acquire motivation and confidence from their interactions with peers. Observing a peer successfully solve a Discrete Mathematics problem — a model perceived as similar to oneself — can enhance the observer's belief in their own capability. The mechanism achieves its greatest effect because students consider peer models to be more trustworthy than they view instructor demonstrations which students think result from expertise. Increased self-efficacy leads to students achieving greater results (Zimmerman, 2000). Pajares and Graham (1999) showed that undergraduate students demonstrated higher mathematics self-efficacy through peer modeling than their previous academic performance showed, which proved this process operated as a direct mechanism.

2.2 Peer Learning and Mathematics Achievement: Evidence, Contradictions, and Limitations

Converging Evidence

The body of research evidence which supports peer learning as a method for teaching mathematics shows considerable strength. Springer, Stanne, and Donovan (1999) conducted a landmark meta-analysis of 39 studies on small-group learning in undergraduate STEM education and found a mean effect size of 0.51 on achievement which showed a moderate positive impact. Alegre et al. (2020) demonstrated at the school level that peer tutoring delivered major academic benefits in primary and secondary mathematics education. Turgut and Turgut (2018) conducted a meta-analysis of 47 Turkish studies which showed that students who used cooperative learning methods achieved mathematics success with an average effect size of 0.840. The research conducted by Abdelkarim and Abuiyada (2016) along with Awofala and Lawani (2020) proved that peer-based learning methods lead to better results in undergraduate mathematics than traditional teaching methods.

At the regional level, Kaymak et al. (2020) established that peer instruction in the Kazakhstani context positively impacted university students' accomplishment of Mathematics Analysis courses— a finding of particular relevance to the present study, as it suggests the mechanism is not culturally restricted to Western educational contexts, though it does not address Discrete Mathematics specifically. Similarly, Mena (2020) conducted research which showed that an eight-week peer teaching program resulted in better applied mathematics skills for engineering students in Ethiopia, with the experimental group achieving higher results than the control group. What these studies collectively show is not just that peer learning works in general, but that its positive effects keep appear across different institutional and cultural settings, so it supports the idea in a stronger way for testing it in the Kazakhstani Discrete Mathematics context.

Contradictory Findings and Methodological Limitations

The peer learning research shows both positive results and negative results which need to be examined through particular important conditions. First, effect sizes show wide-ranging differences because studies use various contexts to study peer learning which needs different results. Springer et al. (1999) identified major differences throughout their research studies which they examined because the authors wanted to demonstrate that implementation quality serves as the essential factor which determines success instead of the actual method used. Cohen (1994) presented strong evidence that unstructured group work leads to minimal educational results while developing student misconceptions because they lack the ability to assess their peers' statements which contradicts common beliefs about the value of student-to-student contact.

Second, the advantages of peer learning are distributed unequally among students. Some studies have found that higher-achieving students in heterogeneous groups show smaller gains than they would in homogeneous groups, raising equity concerns (Fuchs et al., 1998). Moliner et al. (2022) discovered that students who participated in peer tutoring programs showed different levels of cognitive engagement, while group work showed more confident students taking control of activities which resulted in other members being treated as inactive participants.

Third, the majority of peer learning research studies employ basic academic achievement assessments which include final course grades and end-of-course tests because these assessments combine multiple elements that assess design and knowledge assessment and overall course impact. Kinnear, Wood, and Gratwick (2022) noted that peer learning may confer specific advantages for higher-order skills such as proof and reasoning, but not for procedural computation, suggesting that the choice of achievement measure significantly influences findings. Also, most peer learning research studies use intervention periods which last four to eight weeks as their standard practice. Finally, Bengesai et al. (2023) found that formal peer learning approaches used in higher education institutions is often limited by methodological inconsistency, insufficient rigorous experimental designs, and context-specific evidence, which restricts the broader generalizability of findings.

2.3 Student Achievement and Difficulty in Discrete Mathematics

The Nature of the Challenge

The undergraduate program contains Discrete Mathematics which serves as a distinct subject that requires exceptional student effort. Discrete Mathematics gives students their first introduction to formal logic, combinatorics, proof by induction, graph theory, and probability because students lack any previous educational experience with these subjects (Epp, 2003). Moore (1994) explained that students need to master new mathematical content about proof-based mathematics, but they also need to develop a different mathematical approach because they must shift from performing calculations and following patterns to creating arguments and providing formal proof. Tall (1991) described this process as a student experience that requires advanced mathematical understanding which leads to cognitive changes that most students will not comprehend until they face them.

Research on specific student difficulties in Discrete Mathematics studies multiple common areas which students find difficult to understand. The research conducted by Raman (2003) and the research conducted by Selden and Selden (2003) show that undergraduate students need to learn how to differentiate between formal proof and informal argument, because they currently face difficulties with conditional and biconditional statements and with quantifier usage. The research conducted by Magda and Gardner-McCune (2025) showed that students identified proof writing as the most challenging part of Discrete Mathematics because it required them to use different thinking methods than those used for procedural calculation. Puspitasari et al. (2019) discovered that students in the present study face challenges with counting and probability because they specifically struggle with probabilistic reasoning and combinatorial reasoning and understanding variability. Anggara et al. (2018) documented that students encountered challenges when they tried to build sample spaces and use their basic knowledge to solve problems.

Instructional Responses

Educators have replied to these challenges with a range of instructional innovations, and the evidence, though still limited, to hint that active, positive collaborative approaches, do hold particular promise. Fulton et al. (2022) found that active learning combined with explicit links to school mathematics, leads to stronger conceptual grasp, as well as higher student involvement in a Discrete Mathematics course aimed at future mathematics educators. The study by Malheiros and Haetinger (2024) proved that curriculum implementation of Python-based exploration led to better academic performance and lower student attrition rates, while Leron and Dubinsky (1995) discovered that abstract algebra cooperative learning methods helped students develop a better understanding of proof assessments even though their testing results remained unchanged. Students who worked together in Anthony (1996) research found that their collaborative work in discrete structures produced better proof results than what independent study students achieved. Students used peer discussions to develop better argument skills and to find missing pieces in their logical reasoning.

These studies collectively demonstrate that Discrete Mathematics challenges students optimal to overcome their difficulties through peer learning methods which enable them to develop their abstract reasoning and proof construction and symbolic manipulation skills. The existing evidence base for peer learning in Discrete Mathematics remains insufficient because existing studies lack consistent methodological standards which create uncertainty about the success of structured peer learning methods in this area.

2.4 Think-Pair-Share and Structured Problem Solving: Justifying the TPS-SPS Integration

Think-Pair-Share: Strengths and Limitations

Think-Pair-Share (TPS), first described by Lyman (1981), structures peer interaction into three sequential stages: individual thinking, paired discussion, and whole-class sharing. The three-stage structure of the program supports effective teaching in several ways. The THINK stage promotes individual accountability and prevents students from remaining passive during group discussions. The PAIR stage allows students to test and refine their understanding through interaction with a single peer before sharing their ideas publicly. The SHARE stage introduces multiple methods of learning into classroom discussions which helps students build shared knowledge. Tanujaya and Mumu (2019) describe TPS as a cooperative strategy that integrates individual participation, peer discussion, and whole-class communication, making it particularly suited to mathematics classrooms where both procedural and conceptual dimensions need development.

The empirical evidence shows that TPS creates consistent advantages in student achievement. Alsmadi et al. (2023) demonstrated through their study that university engineering mathematics students who used TPS experienced major improvements in their problem-solving abilities which included better mathematical communication and justification skills. Ningsih (2019) confirmed that TPS students achieved better results than the control group in secondary mathematics assessments. Nwankwo et.al (2025) discovered that TPS students achieved better results than the control group in circle geometry assessments, while female students experienced slightly greater improvement. The study by Langprayoon and Ruangsart (2025) proved that TPS together with cooperative learning methods produced better results for university students in confidence building and communication development and skillful reflection.

The TPS framework faces a critical organizational flaw because it fails to define required student activities during the THINK phase except for the single task of 'thinking about the problem.' Research demonstrates that students who receive no structured thinking guidance will either try to solve problems through calculations without understanding the solution process or they will not write any reasoning (Ramadhani et al., 2025). Students who enter the PAIR stage without any meaningful content to discuss will start to speak about their numerical solution instead of showing their thought process. The PAIR stage needs cognitive conflict and ZPD-based scaffolding to work because permanent understanding will not happen without both students understanding their respective viewpoints.

Structured Problem Solving: Strengths and Limitations

Structured Problem Solving (SPS) is a teaching method which originated from Japanese mathematics instruction and is known there as structured problem-based learning (Takahashi, 2006). Students in SPS must first explain their method to solve a problem before they start their calculations because this process helps them understand how to approach a problem through its fundamental components instead of following a standard procedure. The research by Takahashi (2006) showed that Japanese mathematics lessons which used this method of individual attempts followed by class comparison produced superior mathematical reasoning skills than standard teaching methods. Tang et al. (2025) found that collaborative mathematical work improved students' problem-solving abilities. Their study also showed that the individual strategy-articulation phase of SPS encouraged students to work independently, which helped develop self-confidence and a sense of competence. The original Japanese version of the solution comparison stage shows SPS proof that it needs teacher supervision to operate. After students attempt the problem individually, it is the teacher, not a peer, who orchestrates the comparison of approaches. Theoretical literature identifies peer learning mechanisms through which SPS generates student-to-student dialogue and peer scaffolding and emotional advantages of peer interaction. The individual thinking phase provides strength, but the sharing phase returns power to the teacher.

The TPS-SPS Integration: Why Both Are Necessary

The rationale for combining TPS with SPS is therefore precise and theoretically grounded. The TPS framework overcomes the SPS constraint by substituting teacher-centered solution evaluation with organized student-to-student dialogue and classroom-wide solution sharing. The THINK stage in TPS requires students to deliver two types of output: a numeric solution and a complete explanation of their reasoning process. Together, TPS-SPS ensures that students move from individual thinking to paired discussion with their own strategies already prepared. This allows partners to compare different approaches, creating the cognitive conflict and ZPD-based scaffolding that Piagetian and Vygotskian theories suggest can deepen understanding.

The current study creates a new understanding of existing scholarly research. Ramadhani et al. (2025) and Safitri et al. (2025) together suggest that a structured THINK-stage preparation is, in a way the mechanism through which TPS effectiveness is most reliably improved but neither study really tested it in a university-level Discrete

Mathematics context, not directly. The present study is, to the researcher's knowledge the first to implement and evaluate the combined TPS-SPS model instead of looking at either part separately, in this particular setting. This counts as both the study's main theoretical contribution, and also as a noticeable gap in the existing literature that the current research is trying to fill.

2.5 Attitude Toward Mathematics: Dimensions, Evidence, and Limitations

Definition and Dimensions

Students develop their mathematical attitudes through learning which becomes their stable tendency to engage with mathematical work. The tripartite model of attitude (Eagly & Chaiken, 1993) defines the concept through three components which include cognitive appraisals (beliefs about one's mathematical ability), affective responses (emotional reactions to mathematics), and behavioral tendencies (effort, persistence, and avoidance). The complex nature of attitude measurement distinguishes it from basic concepts like anxiety and interest because it serves as a stronger theoretical framework for predicting mathematical behavior.

Tapia and Marsh (2004) operationalized attitude as comprising four dimensions—self-confidence, value, enjoyment, and motivation—all demonstrating strong internal consistency (overall Cronbach's $\alpha = 0.97$). The research demonstrates that these dimensions function as distinct parts of a unified base construct instead of acting as separate elements. The study by Hwang and Son (2021) used latent profile analysis on the TIMSS 2019 dataset to identify distinct attitudinal groups which students belong to, showing that their group membership determined their academic success. Hannula (2002) provides a theoretical account of this relationship, arguing that students develop their mathematical study choices and study dedication through their academic expectations and values, which then leads to their academic success.

Attitude, Self-Efficacy, and Achievement: A Complex Relationship

Students' learning results in mathematics show a positive connection with their mathematical attitude yet this connection between attitude and achievement remains complicated. Ma (1997) demonstrated a reciprocal relationship using structural modelling: attitude influences achievement, and achievement feeds back to modify attitude, creating a dynamic cycle. Research showed that students achieve better results when they enjoy learning activities more than they find them important because enjoyment drives their performance. Shakya and Maharjan (2023) found that attitude accounted for approximately 26% of variance in achievement, while Khaiwal and Gupta (2025) found a significant positive correlation between attitude and achievement with students holding extremely favorable attitudes achieving markedly higher scores.

The relationship between attitude and achievement requires multiple research studies to confirm its existence. The researchers found that students' attitude toward mathematics correlated with their academic performance because the study showed no significant relationship between these two factors. The researchers discovered that Grade 8 students' attitude towards their studies predicted their academic performance while self-efficacy demonstrated no separate predictive capacity which other research studies proved contradicting. The relationship between the two variables shows moderation through various factors which include the educational level and subject matter and the particular attitudinal aspects that researchers used for measurement. The study selects self-confidence, enjoyment and motivation as its three measurement subscales because these elements show the strongest theoretical connection with peer learning program outcomes while the study aims to measure complete multidimensional attitude assessment.

Peer Learning and Attitude: Evidence and Mechanisms

Peer learning creates a positive effect on students' attitudes toward mathematics studies however the impact of this method differs among different studies. Capar and Tarim (2015) conducted a meta-analysis which showed that cooperative learning had a small positive impact on attitude development which proved to be statistically significant, yet this impact was much weaker compared to achievement results. Siller and Ahmad (2024) discovered that primary students who participated in collaborative learning programs experienced both improved academic performance and improved educational attitude because collaborative learning environments helped students feel less anxious. Amankwah and Sumaila (2025) showed that attitude development functioned as a partial mediator between peer-assisted learning and academic success because they showed that students' attitudes served as a process through which peer learning affected their academic performance.

Mathematics anxiety functions as a major obstacle to higher education success because it creates an attitudinal barrier that exists in Discrete Mathematics courses as students begin their first extended experience with formal proof. Bjälkebring (2019) discovered that the most successful method to reduce mathematics anxiety in university students used collaborative environments which allowed for student discussion. Tsirimokos et al. (2024) established that active peer-assisted learning functions as an essential part of anxiety treatment. Subiantoro et al. (2025) proved that Vygotskian collaborative environments decrease mathematics learning anxiety because supportive peer systems eliminate the emotional obstacles that standard teaching methods create.

Self-efficacy, which exists in close relation to the ATMI self-confidence part, constitutes an attitudinal result that develops through peer learning. Misajon (2025) discovered a strong positive link between self-efficacy and attitude toward mathematics through his study which resulted in an r value of 0.68. Chan et al. (2022) established the validity of a three-dimensional self-efficacy assessment tool for university students by demonstrating that course self-efficacy exam, self-efficacy and future self-efficacy function as separate and distinct constructs. Jaafar and Ayub (2010) discovered that university students who achieved higher mathematics self-efficacy showed better academic performance through their research which produced an $r=0.311$ and a p value below 0.05. The research findings establish self-confidence as a core attitudinal variable for the study because peer learning raises performance through its impact on confidence levels.

Gaps in the Literature and Conceptual Framework

Identified Gaps

The literature reviewed in this chapter, while substantial, reveals five important gaps that motivate the present study. First, few rigorous empirical studies investigate structured peer learning in Discrete Mathematics. Existing evidence relies heavily on calculus, abstract algebra, and general undergraduate mathematics, and its applicability to Discrete Mathematics — with its emphasis on proof, logic, and combinatoric reasoning — cannot be assumed. As Anthony (1996) demonstrated, metacognitive demands differ across mathematical domains, so findings from procedural or computational settings may not transfer directly to proof based contexts.

Second, peer learning research is concentrated primarily in North American and Western European higher education contexts, limiting its applicability to different cultural environments. The current research investigates Kazakhstan, where students may bring different educational backgrounds and cultural attitudes toward group learning.

Third, attitudinal outcomes in peer learning studies face studies which fail to examine them with proper research methods. Many studies rely on post-only self-report measures without pre-intervention baselines which make it impossible to determine whether the observed attitudes resulted from the intervention or existed before the study began. The present study addresses this through a pre-test/post-test design which both groups will follow.

Fourth, The TPS-SPS integration requires testing in university-level Discrete Mathematics. The research provides independent literature for both TPS and SPS but has not examined their combined use which addresses their individual limitations in this field.

Fifth, no peer learning study has been conducted with mathematics pedagogy students studying Discrete Mathematics in Kazakhstan, a population of particular significance because their learning experiences directly shape their future pedagogical repertoire.

2.6 Conceptual Framework

The study uses TPS-SPS as its independent variable while measuring academic achievement and student attitudes toward Discrete Mathematics as its dependent variables. The theoretical mechanisms linking TPS-SPS to these outcomes are threefold. Vygotsky's ZPD explains achievement gains through peer scaffolding that becomes possible only when the THINK stage ensures both partners have a position to articulate before discussion begins — the SPS contribution. The PAIR stage requires SPS-based strategy articulation, which enables students to compare different solution methods, thus using Piaget's cognitive conflict theory to achieve deeper conceptual transformation. The SHARE stage enables students to witness their classmates successfully demonstrating mathematical reasoning, which causes their attitudes to improve through Bandura's self-efficacy mechanism.

The control group receives identical lecture content and identical worksheet problems, but without the TPS-SPS structure: students work individually throughout the practice session, and the teacher provides clarification as needed. The study design establishes that all observed differences between groups result from peer learning structures because content exposure and teacher quality and worksheet difficulty have been controlled.

3. METHODOLOGY

3.1 Research Design

The research uses a quasi-experimental design which assesses participants through pre-tests and post-tests while establishing non-equivalent control groups. The university timetable allocation system which allows to choose time and classroom group prevented us from conducting random participant assignment to study conditions, so we needed to select this design approach. The most suitable research method for these situations allows researchers to compare their test groups and control groups through systematic evaluation which fails to meet random assignment requirements (Creswell & Creswell, 2018). Educational research uses quasi-experimental designs as established methods because researchers need to maintain current institutional frameworks, and they can enhance study validity through pre-test matching and statistical methods to address initial study group differences (Cohen et al., 2018).

The study consisted of two experimental and two control groups at the second-year level, together with two experimental and two control groups at the third-year level. All groups study Discrete Mathematics at the same university in Kazakhstan while using the same textbook (Epp, 2020) and they take assessments based on identical learning outcomes. The experimental groups 2E and 3E learn through TPS-SPS peer learning during three-week practice sessions while the control groups 2C and 3C learn through traditional individual-work instruction during their three-week practice sessions. All groups attend the same lectures with their cohort through traditional teacher-directed methods. The independent variable instructional method functions as the sole distinction between practice class and lecture components which lets researchers connect test result differences directly to practice class methods instead of lecture content variations. This structure allowed the study to compare instructional outcomes across both year levels while keeping the broader course context as consistent as possible.

The design is illustrated in Table 3.1.1.

Table 3.1.1. Research Design — Pre-test/Post-test Non-equivalent Control Group

Year	Group	Condition	Pre-test	Intervention (Weeks 9–11)	Post-test
2nd	2E	Experimental	Achievement + Attitude	TPS–SPS Practice Sessions	Achievement + Attitude
2nd	2C	Control	Achievement + Attitude	Traditional Practice	Achievement + Attitude
3rd	3E	Experimental	Achievement + Attitude	TPS–SPS Practice Sessions	Achievement + Attitude
3rd	3C	Control	Achievement + Attitude	Traditional Practice	Achievement + Attitude

3.2 Participants

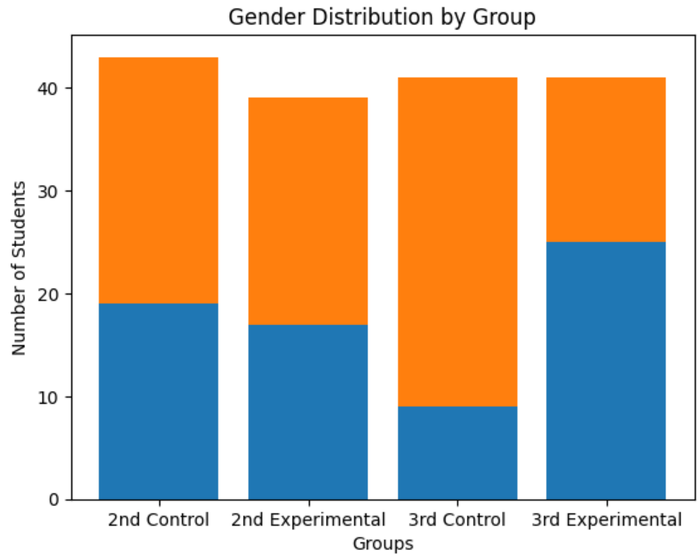
The research involved 164 undergraduate students who were studying Mathematics Pedagogy at a private university located in the Almaty region of Kazakhstan. All participants were taking Discrete Mathematics as a compulsory course at the time of the study. The sample consisted of 82 second-year students and 82 third-year students.

The research included eight complete classroom groups which consisted of four second-year groups and four third-year groups. Two groups at each year level were assigned to the experimental condition while two groups received the control condition. The assignment followed existing class groupings and was not randomized.

The control groups for the second-year cohort of the study included 20 and 23 students while their experimental groups had 19 and 20 students. The third-year cohort study used control groups that contained 18 and 23 students while their experimental groups had 22 and 19 students. The control condition consisted of 84 students, while the experimental condition included 80 students.

The attitude measure required assessment of students who had taken both the pre-test and post-test assessment. Out of 82 second-year students 37 students completed both attitude scales (Control $n = 18$; Experimental $n = 19$) whereas 35 third-year students completed both scales (Control n = 17; Experimental n = 18). The attitude-scale sample size was smaller because the survey was completed voluntarily online, and only fully completed pre-test and post-test responses were included in the analysis.

Figure 3.2.1. Gender Distribution of Participants by Group



Participation in the research component of the study was voluntary. All students in the eight groups receive the same instructional treatment as part of their normal course experience. The experimental groups receive TPS-SPS practice, and the control groups receive traditional practice. The students received information about the research purpose whereas their research data would be used for research purposes without revealing their identities. Verbal informed consent was obtained from all participants before beginning collecting data.

3.3 Description of the Intervention

3.3.1 Overview

The study involves three 50-minute structured practice sessions which use TPS-SPS method to conduct training for experimental groups during three weeks of the semester which matches the Counting and Probability unit of the course (Sections 9.1–

9.9 of Epp,2020). The control groups (2C and 3C) receive three weeks of practice classes which use standard individual work methods to teach identical problem sets. The intervention was designed to fit entirely within the existing practice class timetable without requiring any additional class time, curriculum modification, or resource expenditure beyond the structured worksheets developed for the study.

3.3.2 Experimental Condition: TPS-SPS Practice Sessions

The experimental groups perform their practice sessions according to the Think-Pair-Share with Structured Problem Solving (TPS-SPS) framework. The teacher starts the session with a short introduction that lasts 3 to 4 minutes to explain the session structure and give out the TPS-SPS worksheet. The worksheet contains three problems which follow the lecture content of the week, and each problem consists of three distinct sections that are marked as THINK, PAIR and SHARE. Students complete their work during the THINK stage which lasts between 2 and 3 minutes for each problem. The students needed to describe their solution approach through written documentation which required them to identify the appropriate formula or method before they started their calculations. The first metacognitive step requires students to establish their individual reasoning which will form the base for their following two-person conversation about the material. The teacher circulates silently during this stage and did not answer content questions, preserving the individual accountability function of the stage. In the PAIR stage which lasts between 5 and 7 minutes per problem students work together with their assigned partner. Partners in this stage of the process examine their different methods until they reach an agreement about which solution and explanation they will use as their final answer. The teacher moves through the room to watch how students interact while she helps students work together by asking them questions that lead to answers without giving them the solutions.

The experimental groups established their pairs through initial seating arrangements which assigned adjacent students to partner roles. The method avoids grouping students according to their academic abilities because it creates partnerships which match students with different achievement levels, it is to establish collaborative work patterns while building trust between group members according to research which proves that stable groups improve peer interactions (Johnson & Johnson, 2014).

The teacher chooses one pair from the class to present their solution during the SHARE stage which lasts between 3 to 5 minutes for each problem. The selection process for pairs involves both volunteers and random selection which ensures all pairs stay ready to present their work. The session ends with students spending four to five minutes to complete a short pair reflection exercise during their worksheet activity. Table 3.3.2.1. presents the detailed time allocation for a typical TPS-SPS session.

Table 3.3.2.1. Time Allocation for a Typical TPS-SPS Practice Session (50 minutes)

Activity	**Duration**	**TPS-SPS Stage**
Introduction and worksheet distribution	3–4 min	Teacher-led
Problem 1 — THINK (individual, silent)	2–3 min	THINK
Problem 1 — PAIR (partner discussion)	5–7 min	PAIR
Problem 1 — SHARE (class presentation and discussion)	3–5 min	SHARE
Problem 2,3 — Full THINK / PAIR / SHARE cycle	10–15 min	Full TPS cycle
Pair reflection section on worksheet	4–5 min	Consolidation
Worksheet collection	1–2 min	—
Total	**50 min**	—

3.3.3 Control Condition: Traditional Practice Sessions

Practice sessions in the control groups followed traditional university mathematics instruction. The teacher introduces the problem set briefly (3–5 minutes), after which students work individually on the same problems covered by the experimental groups. The teacher moves around the room to help students who raise their hands for assistance and provides explanations at the desk. The teacher uses board space to explain problems that most students find difficult. The session ends with the teacher analyzing selected solutions for review. The control groups solve the same mathematical problems as the experimental groups, establishing equivalent mathematical content and cognitive demands across all conditions. The only systematic difference between conditions is the pedagogical structure — individual work versus TPS-SPS peer learning.

Table 3.3.3.1. Content Coverage Across Three Practice Sessions

Session	Week	Sections (Epp, 2020)	Topics
1	9	9.1, 9.2, 9.3	Introduction to probability; Multiplication Rule; Addition Rule and Inclusion-Exclusion
2	10	9.4, 9.5, 9.6	Pigeonhole Principle; Combinations; r-Combinations with repetition
3	11	9.7, 9.8, 9.9	Pascal's Formula and Binomial Theorem; Probability axioms and expected value; Conditional probability and Bayes' theorem

Pair Formation and Content Coverage

The experimental groups created their pairs through the seating arrangement at the start of Session 1. Students who sat next to each other became designated partners. The method required that pairs be formed according to the existing classroom seating rather than ability levels or previous academic achievement. Sessions required fixed pairs so partners could develop trust and collaborative work overtime which research suggest enhancing their interactions (Johnson & Johnson, 2014). The content coverage information from all sessions appears in Table 3.3.3.1.

3.3.4 Research Instruments

The research used two assessment tools to gather information through an achievement test and an attitude scale test. The pre-assessments for both instruments were conducted in Week 8 before the intervention, while the post-assessments were conducted in Week 12 after Session 3 for all research groups.

Achievement Test

The study used an open-ended achievement test which had nine questions that were worth 32 total points and required 50 minutes to complete. The test was constructed in two parallel forms, with Form A used as a pre-test and Form B as the post-test (see appendix A and appendix B). The two forms assess identical nine content sections from (Epp, 2020) while using different numerical values and actual situations to prevent students from remembering pre-test answers. The open-ended format was selected in preference to multiple choice because it requires students to show their reasoning and working which provides richer evidence of conceptual understanding and aligns more directly with the higher-order thinking skills that TPS-SPS is designed to develop (Magda & Gardner-McCune, 2025).

The test blueprint (see appendix C) was designed to cover Bloom's Revised Taxonomy Levels 2 through 5 — Understand, Apply, Analyze, and Evaluate. No Level 1 (Remember) items were included as university-level assessment should presuppose basic recall, and no Level 6 (Create) items were included as the 50-minute timed format is not conducive to creative mathematical production. The nine test questions distribute cognitive levels throughout the assessment to evaluate both procedural skills and advanced conceptual reasoning which peer learning research suggests should be developed. A detailed marking rubric was developed alongside the test, specifying the criteria for awarding 3, 2, 1, or 0 marks on each open-ended question, to ensure consistent and objective scoring.

Expert reviews confirmed the content validity of the assessment. The three expert evaluators analyzed the assessment and test blueprint. The reviewers rated each question according to its relevance and clarity through a four-point rating system (see appendix D). The Content Validity Index (CVI) was calculated as the proportion of items rated 3 or 4 (relevant or highly relevant) by three reviewers. All items met the CVI ≥ 0.80 criterion and were retained without revision. The pilot study used 27 mathematics pedagogy students who were not part of the main research study to evaluate both item quality and internal consistency. The full 9-item assessment achieved a Cronbach's alpha of $\alpha = 0.809$ which shows good internal consistency (George & Mallery, 2003). The item difficulty indices showed values between $p = 0.43$ and $p = 0.77$ because all test items maintained a difficulty level between 0.30 and 0.80 which demonstrated that all items presented suitable challenge levels to the pilot participants. The item discrimination indices showed values between $D = 0.32$ and $D = 0.48$, with all test items exceeding $D = 0.25$ minimum standard, which showed that every item successfully assessed higher- and lower-performing students (Ebel & Frisbie, 1991). The final test used all nine items from the test.

Attitude Scale

The study used an adapted version of the Attitudes Toward Mathematics Inventory (ATMI) by Tapia and Marsh (2004) to assess students' opinions about Discrete Mathematics (see appendix E). The ATMI serves as a leading educational research tool which effectively assesses students' mathematical attitudes according to its developers who established Cronbach's alpha value of 0.97. The study selected three subscales from the 40-item instrument which included Self-Confidence with 15 items, Enjoyment with 7 items and Motivation with 8 items to create an initial 30-item adapted scale. The Value subscale was excluded because it assesses permanent beliefs that students hold about the societal value of mathematics and were not expected to change during a three-week period. The final instrument contained 24 items divided into three subscales after the completed contextual review and pilot study item analysis process which is described below.

The item wording was minimally adapted. All instances of 'mathematics' and 'math' were replaced with 'Discrete Mathematics' to establish direct links between the study material and the specific subject being researched. The initial checking removed one item ('I have usually enjoyed studying mathematics in school') which needed removal since Discrete Mathematics does not exist in Kazakhstan's secondary school curriculum. One further item was adapted rather than removed: item 27 ('I would prefer to do an assignment in mathematics than to write an essay') was modified to read 'I would prefer to do an assignment in Discrete Mathematics than other subjects', since mathematics pedagogy students do not typically write essays. Respondents use a five-point Likert scale to evaluate each item which starts at 1 (Strongly Disagree) and ends at 5 (Strongly Agree) while the analysis requires reverse scoring of negatively worded items.

The content validity of the adapted scale was confirmed through the same expert review process used for the achievement test. Experts evaluated the relevance of the items for Discrete Mathematics students in the Kazakhstani university context using the CVI procedure. The pilot study was conducted with 27 mathematics pedagogy students who were not part of the main research study to test the reliability of the adapted assessment tool. The initial 29-item scale showed good to excellent internal consistency across all tests with an overall result of ($\alpha = 0.899$) and subscale alphas of 0.873 for Self-Confidence, 0.790 for Enjoyment and 0.729 for Motivation. The item-total correlations were checked for every item. Five items were identified for removal on the basis of low item-total correlations: ‘Discrete Mathematics does not scare me at all’ ($r = 0.142$), ‘The challenge of Discrete Mathematics appeals to me’ ($r = 0.128$), ‘I would like to avoid using Discrete Mathematics in the future’ ($r = 0.019$), ‘I study Discrete Mathematics because I know how useful it is’ ($r = 0.219$), and ‘I would prefer to do an assignment in Discrete Mathematics than other subjects’ ($r = 0.244$). We deleted these items because their corrected item-total correlations showed that these items did not aid the measurement of the construct which their subscales were designed to assess (Field, 2018). The scale showed better internal consistency after we removed items because the overall alpha value reached 0.908, Self-Confidence alpha reached 0.952, Enjoyment alpha reached 0.847 and Motivation alpha reached 0.836. All retained items had corrected item-total correlations above 0.30, confirming their adequate contribution to the scale. The main study used the final adjusted ATMI which included 24 items divided into three subscales: Self-Confidence (14 items, range 14–70), Enjoyment (5 items, range 5–25), and Motivation (5 items, range 5–25), with a total score range of 24 to 120.

Data Collection Procedure

The researcher data collection was conducted according to the schedule shown in Table 3.3.4.1. All groups used the same schedule for their data collection. The researcher administered all instruments directly to the study participants in order to establish uniform testing conditions across all groups.

Table 3.3.4.1. Data Collection Timeline

Timing	**Activity**	**Instrument**	**Groups**
Week 8	Pre-test administration	Achievement test Form A + attitude pre-scale	2E, 2C, 3E, 3C
Week 9	Session 1 — practice class	TPS-SPS worksheet (exp.) / Traditional worksheet (ctrl.)	2E, 2C, 3E, 3C
Week 10	Session 2 — practice class	TPS-SPS worksheet (exp.) / Traditional worksheet (ctrl.)	2E, 2C, 3E, 3C
Week 11	Session 3 — practice class	TPS-SPS worksheet (exp.) / Traditional worksheet (ctrl.)	2E, 2C, 3E, 3C
Week 12	Post-test administration	Achievement test Form B + attitude post-scale	2E, 2C, 3E, 3C

The Week 8 practice class used the pre-test achievement measure (Form A) during its 50-minute session. The pre-intervention attitude scale was distributed via Google Forms in Week 8, and students were given until Week 9 to complete it.

The post-test achievement measure (Form B) was administered during the Week 12 practice class under the same conditions. The post-intervention attitude scale (ATMI post-scale) was similarly distributed via Google Forms, with a specified deadline for completion.

3.3.5 Primary Analyses

The analysis of covariance (ANCOVA) served as the main statistical test for achievement measurement which used post-test scores as its dependent variable and group type as its constant factor while pre-test scores acted as the controlling variable. ANCOVA was selected because it provides greater statistical power than gain score analysis when examining post-test differences while controlling for pre-test variation (Huck & McLean, 1975; Field, 2018). The study included gain score independent t-tests as secondary support for its findings. We confirmed three assumptions before conducting each ANCOVA test: we tested normality of ANCOVA residuals through Shapiro-Wilk and assessed homogeneity of residual variances through Levene's test and they examined homogeneity of regression slopes through group pre-test interaction term analysis. Researchers measured effect sizes from t-tests using Cohen's d while they measured effect sizes from ANCOVA tests using partial eta squared ($\eta^2 p$) (Cohen, 1988).

Significance Level and Effect Size Interpretation

An alpha value of 0.05 was used for hypothesis testing. We assessed effect sizes through Cohen's (1988) benchmarks which defined small effects with d = 0.20, medium effects with d = 0.50 and large effects with d = 0.80. The chapter presents both statistical significance results and effect size measurements.

4. ANALYSIS AND RESULTS

This chapter presents the results of the quantitative analyses for both the 2nd-year and 3rd-year student cohorts. The study consists of two main sections which start with Section 4.1 achievement results and continue to Section 4.2 attitude results. The section presents 2nd-year and 3rd-year students first before conducting a comparison between the two groups. The analysis followed a sequence that began with descriptive statistics and assumption checks, continued with tests of pre-test comparability and within-group changes, and concluded with between-group comparisons, ANCOVA, and hypothesis testing. All analyses were conducted using Python in Google Colab at a significance level of $\alpha = 0.05$.

4.1 Achievement Results

Achievement was measured using the open-ended achievement test (32 points maximum) administered as Form A (pre-test) and Form B (post-test) to all four groups across both year levels. The 2nd-year cohort comprised 82 students (Control $n = 43$; Experimental $n = 39$); the 3rd-year cohort comprised 82 students (Control $n = 41$; Experimental $n = 41$).

4.1.1 Descriptive Statistics

Table 4.1.1.1. presents the descriptive statistics for pre-test, post-test, and gain scores for both cohorts. The two groups showed equivalent pre-test results at both year levels while both groups progressed from pre-test to post-test assessment. The experimental group demonstrated larger mean gains in both cohorts. The post-test mean of the experimental group exceeded that of the control group in both years. Figures 4.1.1.1. and 4.1.1.2 display the mean pre-test and post-test scores by group for each cohort. These descriptive results provide an initial overview of the achievement patterns before the study proceeds to inferential tests of statistical significance.

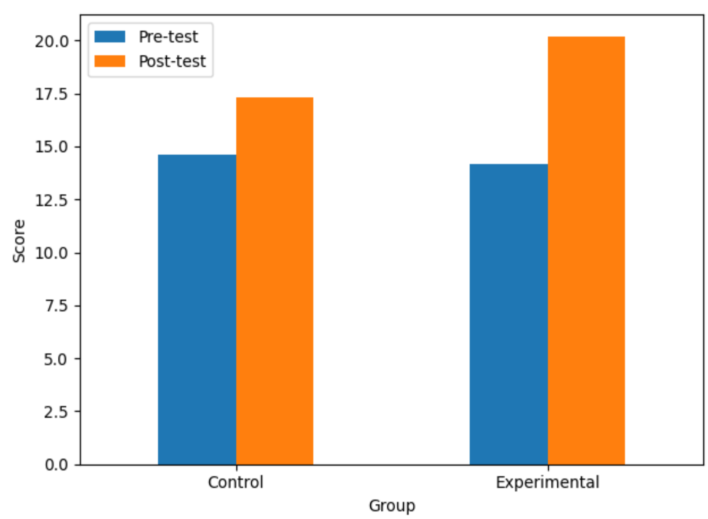
Table 4.1.1.1. Descriptive Statistics for Achievement Pre-test, Post-test, and Gain Scores by Group and Cohort

Cohort / Group	***n***	**Pre-M**	**Pre SD**	**Post M**	**Post SD**	**Gain M**	**Gain SD**	**Pre SW *p***	**Post SW *p***
2nd-Year Cohort									
Control	43	14.63	6.34	17.33	7.19	2.70	8.35	0.737	0.662
Experimental	39	14.15	7.87	20.21	6.76	6.05	8.19	0.499	0.013*
3rd-Year Cohort									
Control	41	15.66	8.99	18.56	9.26	2.90	9.29	0.087	0.007*
Experimental	41	13.95	8.45	20.15	7.69	6.20	7.80	0.137	0.011*

Note. An asterisk (*) indicates $p < .05$.

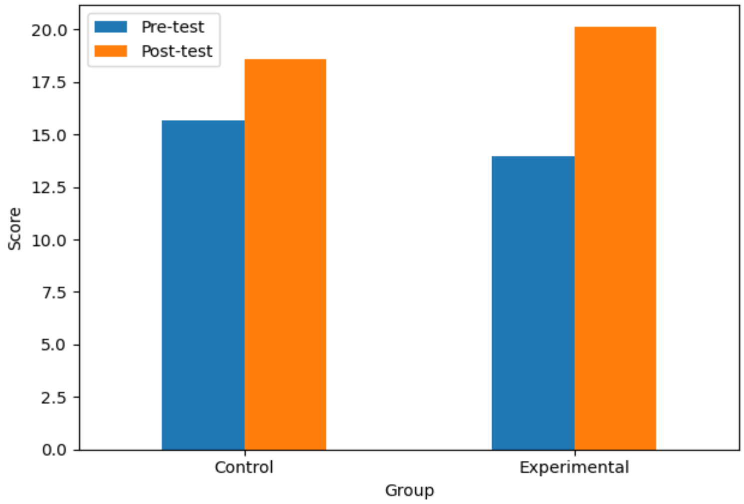
The following figure visually summarizes the same descriptive pattern for the 2nd-year cohort and shows how the experimental group demonstrated a larger increase from pre-test to post-test compared with the control group.

Figure 4.1.1.1. Mean Pre-test and Post-test Achievement Scores by Group — 2nd-Year Cohort



Two groups improved from pre-test to post-test; but more gain for the Experimental ($\Delta = 6.05$) was observed than for the Control group ($\Delta = 2.70$).

Figure 4.1.1.1. Mean Pre-test and Post-test Achievement Scores by Group — 3rd-Year Cohort

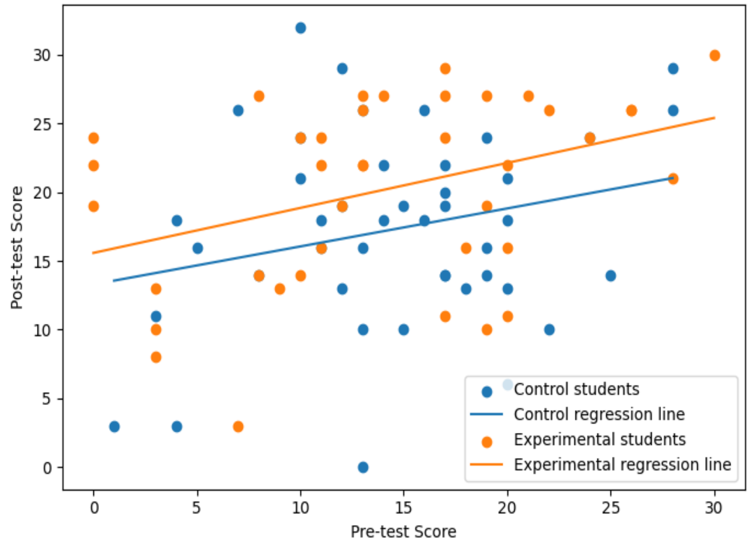


Both groups showed progress from their pre-test results to their post-test results, but the experimental group achieved better results with a gain of 6.20 than the control group which reached a gain of 2.90.

4.1.2 Assumption Checks

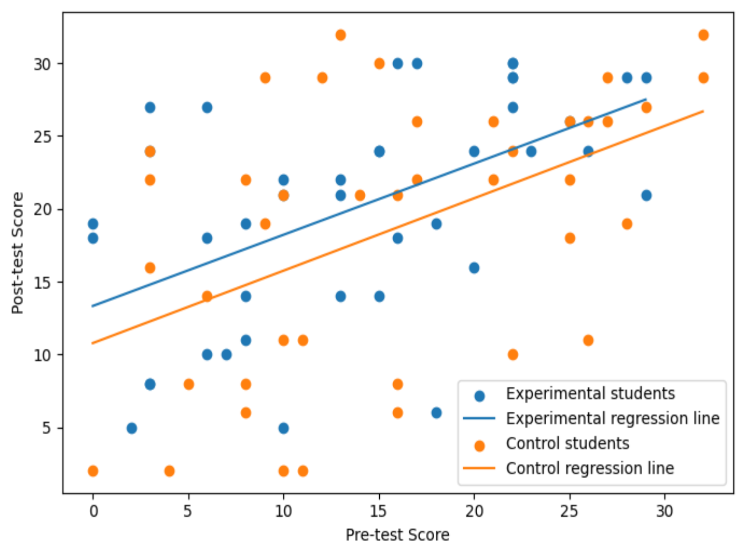
Shapiro-Wilk tests identified normality violations in post-test score distributions: the 2nd-year experimental group (W = 0.926, p = 0.013), and both groups in the 3rd-year cohort (Control: W = 0.921, p = 0.007; Experimental: W = 0.926, p = 0.011). The 2nd year and 3rd year cohorts both met the requirement for ANCOVA because their residuals showed normal distribution (2nd year: W = 0.985, p = 0.454; 3rd year: W = 0.972, p = 0.073). All variance assumptions were satisfied because Levene's tests for pre-test scores and gain scores and ANCOVA residuals showed non-significant results across both cohorts (all $p > 0.10$). The homogeneity of regression slopes assumption was confirmed via non-significant group × pre-test interactions (2nd year: F (1, 78) = 0.056, p = 0.813; 3rd year: F (1, 78) = 0.002, p = 0.965). The parallel regression lines shown in Figures 4.1.2.1. and 4.1.2.2. visually confirms this assumption.

Figure 4.1.2.1. Regression Lines by Group — 2nd-Year Cohort



The regression lines remain parallel to each other which demonstrates that the assumption of equal regression slopes has been confirmed through the statistical test which showed F (1,78) = 0.056 and p = 0.813 and the distance between the two lines demonstrates the adjusted difference between the two groups.

Figure 4.1.2.2. Regression Lines by Group — 3rd-Year Cohort



The parallel regression lines confirm that the assumption of homogeneity of regression slopes is satisfied (F(1,78) = 0.002, p = 0.965), while the steeper slope relative to the 2nd-year cohort indicates a stronger pre-test covariate effect (F = 27.13 vs. F = 8.46).

4.1.3 Pre-test Comparability

Table 4.1.3.1. Pre-test Means (M) and Standard Deviations (SD) by Subgroup

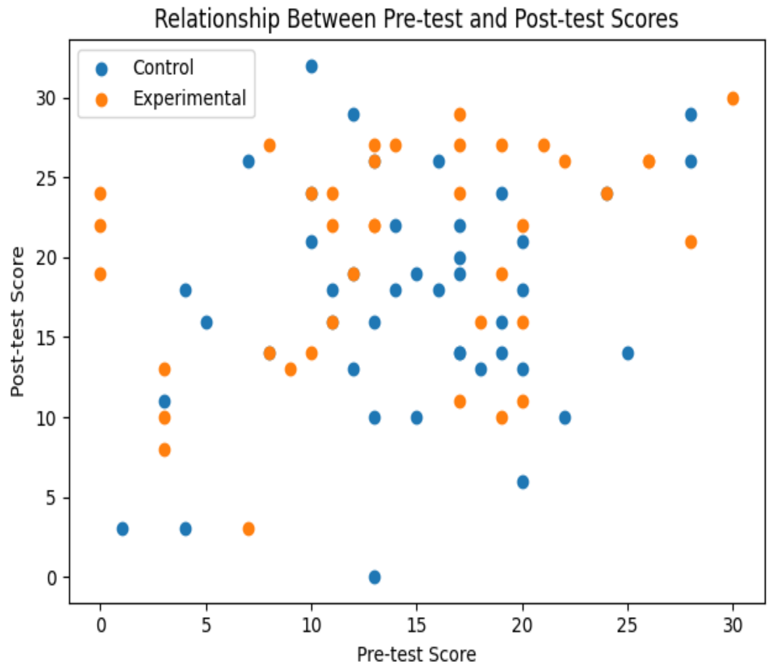
Cohort	C1 (M ± SD)	C2 (M ± SD)	E1 (M ± SD)	E2 (M ± SD)
2nd Year	12.60 ± 5.62	16.39 ± 6.51	16.42 ± 8.28	12.00 ± 6.98
3rd Year	14.28 ± 8.38	16.74 ± 9.49	11.55 ± 8.55	16.74 ± 7.62

Note. C1 = Control subgroup 1; C2 = Control subgroup 2; E1 = Experimental subgroup 1; E2 = Experimental subgroup 2.

The independent samples t-tests showed no significant pre-test differences between groups in both cohorts (2nd year: t (80) = 0.302, p = 0.764; 3rd year: t (80) = 0.88, p = 0.378). The assessment of subgroup-level descriptive data in Table 4.1.3.1. showed both within-condition variation and condition-specific patterning for the two groups. The differences between the two groups required ANCOVA as the main analysis method because it enables better control of pre-test differences through covariance adjustment than gain score analysis. Even though the cohort-level pretest differences didn’t reach statistical significance, the subgroup averages hint at some early variation across the classroom groups. That result is typical for a quasi-experimental setup, because existing groups are used, rather than random assignment. So, therefore, the pre-test scores were accounted for in the ANCOVA, to make the post-test achievement comparison more careful and accurate.

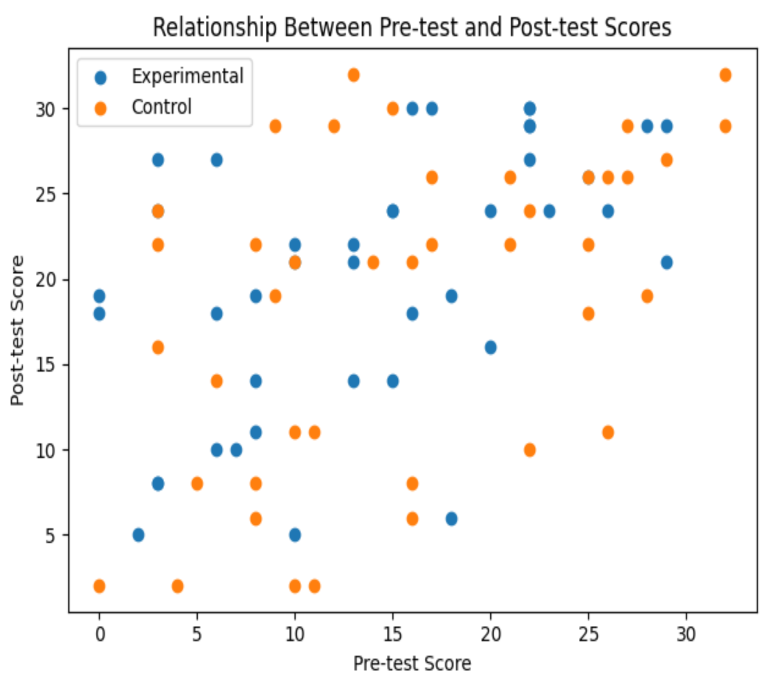
The scatterplots below show the relationship between pre-test and post-test scores for the 2nd-year and 3rd-year cohorts. They help visually confirm the positive relationship between the two scores and support the use of pre-test scores as an ANCOVA covariate.

Figure 4.1.3.1. Pre-test vs. Post-test Scores by Group — 2nd-Year Cohort



The positive relationship between pre-test and post-test scores supports the use of pre-test scores as an ANCOVA covariate.

Figure 4.1.3.2. Pre-test vs. Post-test Scores by Group — 3rd-Year Cohort



4.1.4 Within-Group Changes

The within-group changes were examined using paired samples t-tests to determine whether each group showed significant improvement from pre-test to post-test. The results are summarized in Table 4.1.4.1.

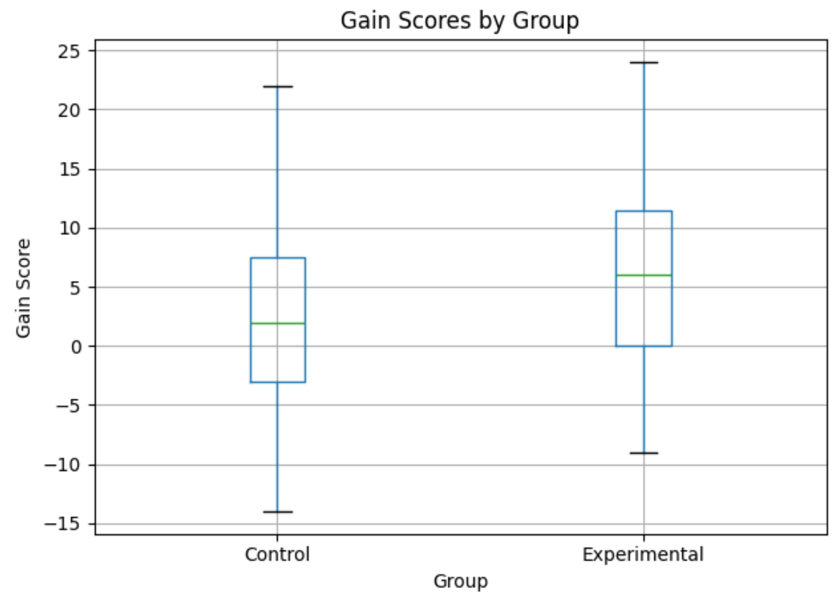
Table 4.1.4.1. Within-Group Changes by Group and Cohort

Group / Cohort	**Pre M (SD)**	**Post M (SD)**	**Gain M (SD)**	**t**	**p**	**d**
Control — 2nd Year	14.63 (6.34)	17.33 (7.19)	2.70 (8.35)	2.120	0.040*	0.32
Experimental — 2nd Year	14.15 (7.87)	20.21 (6.76)	6.05 (8.19)	4.615	<.001*	0.74
Control — 3rd Year	15.66 (8.99)	18.56 (9.26)	2.90 (9.29)	2.001	0.052	0.31
Experimental — 3rd Year	13.95 (8.45)	20.15 (7.69)	6.20 (7.80)	5.089	<.001*	0.80

Note. An asterisk (*) indicates p < .05. d = Cohen's d

Paired samples t-tests examined pre-to-post improvement within each group. In the 2nd-year cohort, both groups improved significantly: the control group ($t(42) = 2.120$, $p = 0.040$, $M_{gain} = 2.70$) and the experimental group ($t\,(38) = 4.615$, $p < 0.001$, $M_{gain} = 6.05$). The experimental group demonstrated a greater level of achievement advancement through their internal progress. The experimental group displayed significant improvements in their performance, which reached statistical significance according to the results ($t(40) = 5.089$, $p < 0.001$, $M_{gain} = 6.20$) while the control group's improvement was borderline non-significant ($t\,(40) = 2.001$, $p = 0.052$, $M_{gain} = 2.90$). The gain score distributions by group are presented in Figures 4.1.4.1. and 4.1.4.2.

Figure 4.1.4.1. Gain Scores by Group — Box Plot, 2nd-Year Cohort



In Figure 4.1.4.1, shown the experimental group from the 2nd-year cohort had a higher median gain score than the control group. This pattern matches what the paired t-test showed, where the experimental group also had a bigger pre to post uplift, improvement overall.

Figure 4.1.4.2. Gain Scores by Group — Box Plot, 3rd-Year Cohort

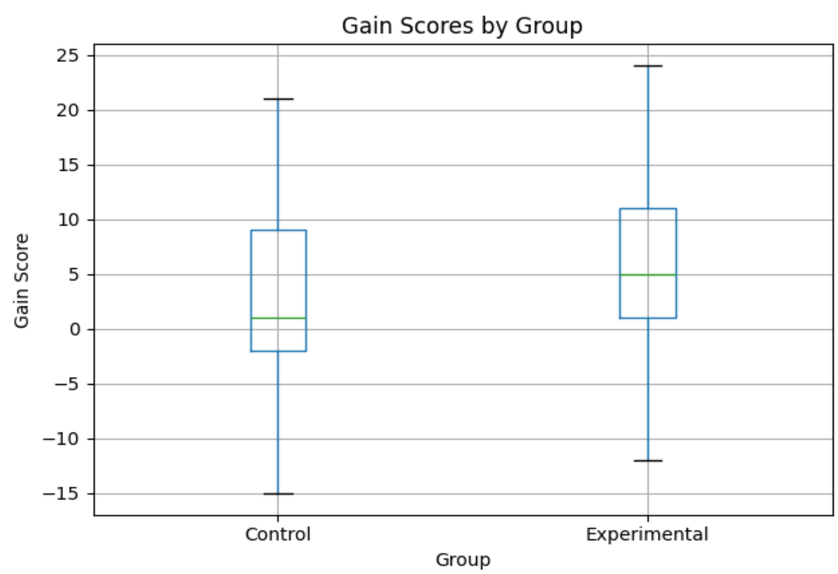
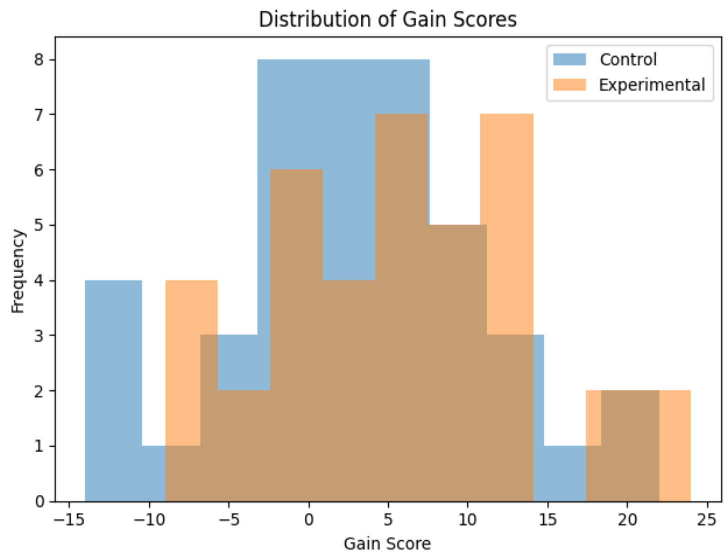


Figure 4.1.4.2. shows that the 3rd-year experimental group had a higher median gain score than the control group. This pattern supports the descriptive results, indicating stronger improvement in the TPS-SPS group.

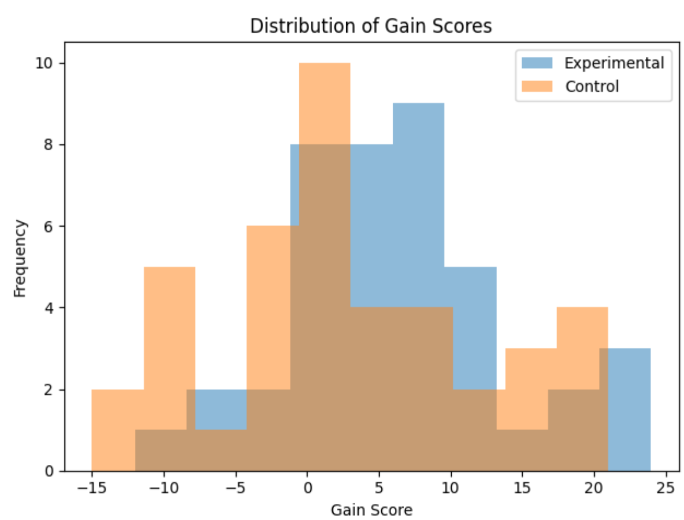
To further examine the gain score patterns, histograms were used to compare the distribution of gain scores between the control and experimental groups in each cohort. Figures 4.1.4.3. and 4.1.4.4. present these distributions for the 2nd-year and 3rd-year cohorts.

Figure 4.1.4.3. Distribution of Gain Scores — Histogram, 2nd-Year Cohort



The experimental group distribution is shifted rightward relative to the control group.

Figure 4.1.4.4. Distribution of Gain Scores — Histogram, 3rd-Year Cohort



Considerable distributional overlap between groups, consistent with the non-significant between-group result.

4.1.5 Between Group Comparisons and ANCOVA

Table 4.1.5.1. shows the results from between-group comparisons and ANCOVA analysis which were conducted on both study groups. The independent samples t-tests which were conducted on gain scores revealed no statistically meaningful results for both cohorts because both groups displayed results which were close to traditional significance levels with small-to-medium effect sizes (2nd year: $t(80) = -1.834$, p = .070, d = 0.405; 3rd year: t (80) = -1.739, p = .086, d = 0.384). The ANCOVA was selected as the main statistical test because it provides stronger statistical results when pre-test scores show a connection to outcome measurements (Huck & McLean, 1975; Field, 2018).

The second year ANCOVA test confirmed that there were statistically significant group differences after controlling pre-test results, F (1, 79) = 4.187, p = .044, $\eta^2 p$ = .050. The experimental group achieved an adjusted mean advantage of 3.025 points (b = 3.025, SE = 1.478, 95% CI [0.082, 5.967]). TPS-SPS demonstrated its ability to produce actual achievement improvements that exceeded what the baseline differences would predict. The effect size shows small results, but it holds value because the three-week study focused on one content unit.

The ANCOVA results for the 3rd-year cohort indicated no statistically significant group effect, F (1, 79) = 2.190, p = 0.143, $\eta^2 p$ = 0.027. The adjusted mean difference between groups was 2.428 points (b = 2.428, SE = 1.641, 95% CI [−0.838, 5.693]), with the confidence interval including zero. This result therefore warrants careful interpretation.

Importantly, the absence of statistical significance does not necessarily indicate the absence of an intervention effect. Instead, it reflects the specific statistical conditions of the 3rd-year model. The pre-test covariate exerted a substantially stronger influence in this cohort (F = 27.13, p < 0.001) compared to the 2nd-year cohort (F = 8.46, p = 0.005), accounting for a larger proportion of variance in post-test scores. As a result, less residual variance remained for the group effect to be detected.

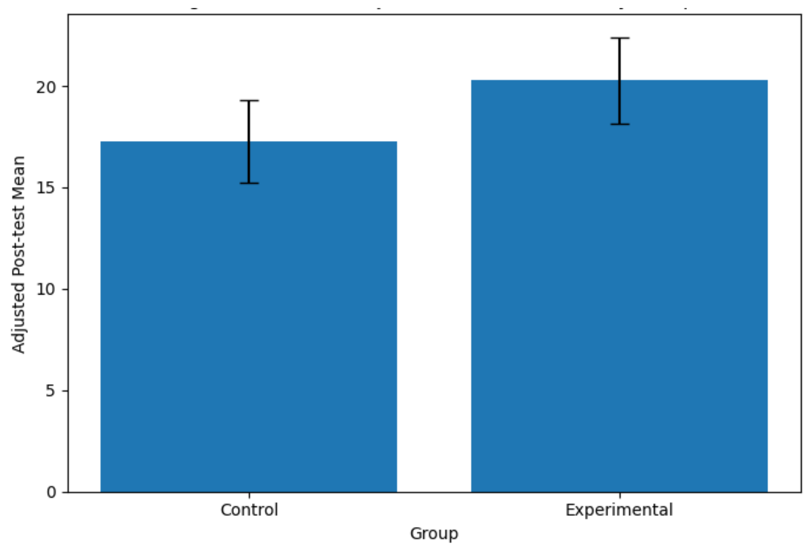
Notably, the descriptive effect sizes were highly similar across cohorts (d = 0.405 vs. 0.384), suggesting that the intervention produced comparable improvements. However, in the 3rd-year cohort, these improvements did not reach statistical significance due to the stronger influence of prior achievement. The ANCOVA-adjusted means are presented in Figures 4.1.5.1. and 4.1.5.2.

Table 4.1.5.1. Between-Group Achievement Comparisons and ANCOVA Results by Cohort

Statistic	2nd-Year Cohort	3rd-Year Cohort
Pre-test t-test	t (80) =.302, p=.764	t (80) =0.886, p=.378
Control within-group (paired t)	t (42) =2.120, p=.040	t (40) =2.001, p=.052
Experimental within-group (paired t)	t (38) =4.615, p<.001	t (40) =5.089, p<.001
Gain score t -test (between groups)	t (80) =−1.834, p=.070, d=0.405	t (80) = −1.739, p=.086, d=0.384
ANCOVA — group effect	*F (1,79) =4.187, p=.044,* $\eta^2 p$*=.050*	F (1,79) =2.190, p=.143, η^2p=.027
ANCOVA — pre-test covariate	F (1,79) =8.460, p=.005	F (1,79) =27.134, p<.001
Adjusted group difference (95% CI)	3.025 pts [0.082, 5.967]	2.428 pts [−0.838, 5.693]
$H_0$1 decision	**Rejected (p=.044)**	**Retained (p=.143)**

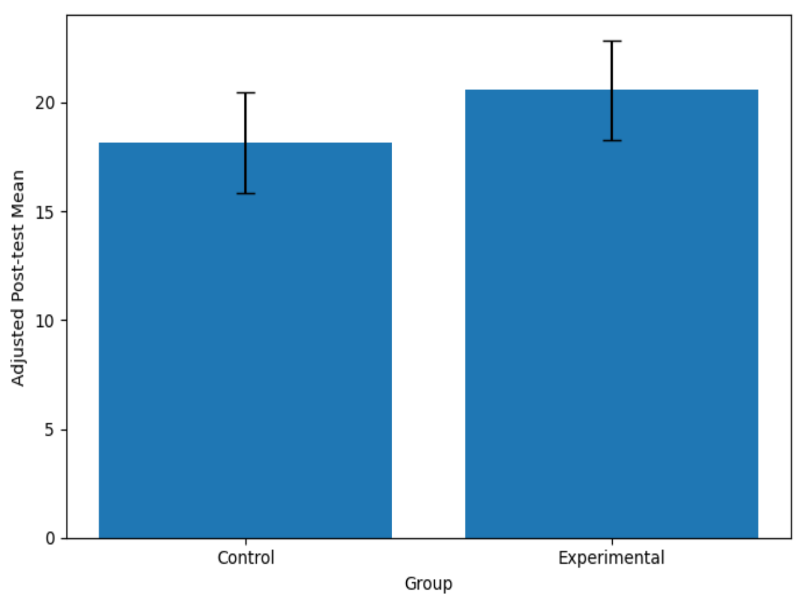
As shown in Table 4.1.5.1, the experimental groups demonstrated larger achievement gains in both cohorts, but the ANCOVA results differed by year level. The 2nd-year cohort showed a statistically significant adjusted group effect, whereas the 3rd-year cohort did not. This suggests that the TPS-SPS intervention had a clearer measurable effect among 2nd-year students, while the 3rd-year results should be interpreted as a positive but statistically non-significant trend.

Figure 4.1.5.1. ANCOVA-Adjusted Mean Post-test Scores by Group — 2nd-Year Cohort



Adjusted means at the grand mean of pre-test (M = 14.40). Error bars = 95% CI. The experimental group adjusted mean exceeds the control by 3.025 points, $F(1,79) = 4.187$, $p = 0.044$.

Figure 4.1.5.2. ANCOVA-Adjusted Mean Post-test Scores by Group — 3rd-Year Cohort



Adjusted means at the grand mean of pre-test (μ = 14.80). Overlapping confidence intervals confirm the non-significant group effect, $F(1,79) = 2.190$, $p = 0.143$.

4.1.6 Hypothesis Decision — Achievement

The 2nd-year cohort results show that we rejected null hypothesis $H_0 1$. The experimental group demonstrated superior post-test results in comparison to the control group after we controlled for pre-test achievement, $F(1, 79) = 4.187$, $p = 0.044$, $\eta^2 p = 0.050$. The 3.025 point mean advantage shows small statistical yet reliable evidence. The 2nd-year cohort results confirm $H_1 1$ according to the research findings.

The null hypothesis $H_0 1$ will stay valid for the third-year student group. The analysis of gain scores showed no group effect while the ANCOVA results showed no statistically significant group effect ($p = 0.143$). The two cohorts show similar results because their mean gains (experimental: 6.05 vs. 6.20; control: 2.70 vs. 2.90) and effect sizes ($d = 0.405$ vs. 0.384) demonstrate virtual identity which indicates the same fundamental effect exists, but 3rd-year analysis failed to identify it because stronger pre-test covariate absorbed more outcome variance.

4.2 Attitude Results

The study has confirmed that TPS-SPS produces a specific effect on student achievement which now leads to an investigation of its consequences for students' attitudes toward Discrete Mathematics. Achievement and attitude are theoretically related but empirically distinct constructs — gains in one do not guarantee gains in the other, particularly within short intervention windows. The following sections examine whether the peer learning experience shifted how students felt about Discrete Mathematics across three dimensions: self-confidence, enjoyment, and motivation.

The attitude scale was collected through online Google Forms while the achievement test was conducted during scheduled practice sessions. The pre-test attitude survey link was distributed to students prior to the intervention, and the post-test survey link was shared after the completion of the intervention, with a specified deadline for submission in each case.

Of the 82 students in each cohort, 37 2nd-year students (Control n = 18; Experimental n = 19) and 35 3rd-year students (Control n = 17; Experimental n = 18) completed both the pre-test and post-test attitude scales. The reduced sample sizes reflect two factors: voluntary participation in the online survey and the exclusion of cases with incomplete data (i.e., students who completed only one of the two surveys). Missing data were handled using listwise deletion.

4.2.1 Descriptive Statistics

Tables 4.2.1.1. and 4.2.1.2. provides descriptive statistics information about all attitude measurement variables. The two groups show a common pattern because their pre-test scores reached a high point which measured between 81.39 and 85.65 for Discrete Mathematics. The study participants maintained a positive attitude toward the subject matter because their pre-test scores reached between 81.39 and 85.65 for Discrete Mathematics. The study participants maintained a positive attitude toward the subject matter because their pre-test scores reached between 81.39 and 85.65 for Discrete Mathematics. The high baseline measurement presents an essential contextual element which restricts the scientist's ability to observe positive results from his work. Chapter 5 explains the concept of a ceiling effect which exists because experts discovered measurement limitations that developed from the research study's original high point. The study results showed that both groups and both testing conditions produced minimal gain scores. The post-test and pre-test subscale scores for each group in both cohorts appear in Figures 4.2.1.1. to 4.2.1.2.

Table 4.2.1.1. Attitude Descriptive Statistics — Pre-test, Post-test, and Gain Scores by Variable, Group, and Cohort: 2nd-Year Cohort

Variable / Group	n	Pre M (SD)	Post M (SD)	Gain M	Gain SD	Min	Max
2nd-Year Cohort — Overall Attitude (range 24–120)							
Control	18	85.00 (13.54)	84.61 (12.99)	−0.39	3.86	−10	8
Experimental	19	83.05 (8.89)	84.58 (10.45)	1.53	4.99	−7	12
2nd-Year Cohort — Self-Confidence (range 14–70)							
Control	18	47.56 (11.89)	47.28 (11.07)	−0.28	2.59	−5	6
Experimental	19	45.58 (6.43)	47.26 (6.87)	1.68	3.30	−4	9
2nd-Year Cohort — Enjoyment (range 5–25)							
Control	18	17.67 (2.00)	18.11 (2.47)	0.44	2.09	−4	4
Experimental	19	19.00 (1.89)	19.21 (2.07)	0.21	1.93	−3	4
2nd-Year Cohort — Motivation (range 5–25)							
Control	18	19.78 (1.26)	19.22 (1.52)	−0.56	1.34	−3	2
Experimental	19	18.47 (1.58)	18.11 (1.73)	−0.37	1.54	−4	2

Table 4.2.1.1. shows that both groups had high pre-test attitude scores and there were only modest changes during the intervention period, not much. The experimental group had the biggest improvement in Self-Confidence (Gain M = 1.68), but Enjoyment and Motivation barely moved at all. These findings provide preliminary evidence that attitude levels remained relatively stable throughout the study.

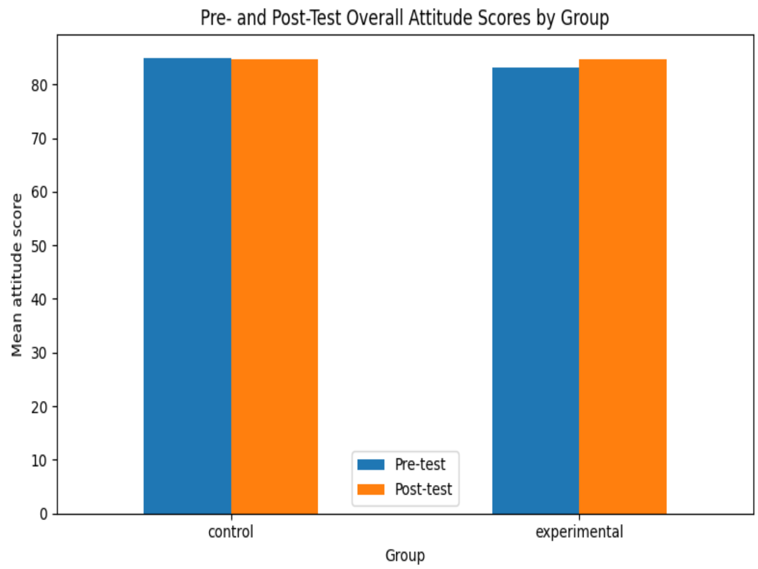
Table 4.2.1.2. Attitude Descriptive Statistics — Pre-test, Post-test, and Gain Scores by Variable, Group, and Cohort: 3rd-Year Cohort

Variable / Group	n	Pre M (SD)	Post M (SD)	Gain M	Gain SD	Min	Max
3rd-Year Cohort — Overall Attitude (range 24–120)							
Control	17	85.65 (10.92)	84.82 (10.36)	−0.82	2.63	−4	5
Experimental	18	81.39 (12.08)	82.06 (11.76)	0.67	2.09	−3	5
3rd-Year Cohort — Self-Confidence (range 14–70)							
Control	17	48.18 (6.71)	47.94 (6.92)	−0.24	1.92	−4	3
Experimental	18	41.44 (6.30)	41.56 (5.97)	0.11	2.37	−4	4
3rd-Year Cohort — Enjoyment (range 5–25)							
Control	17	18.82 (3.00)	18.59 (2.94)	−0.24	1.48	−3	2
Experimental	18	18.61 (3.43)	18.78 (3.25)	0.17	1.15	−3	2
3rd-Year Cohort — Motivation (range 5–25)							
Control	17	18.65 (2.45)	18.29 (2.26)	−0.35	1.41	−3	1
Experimental	18	18.00 (2.63)	17.83 (2.43)	−0.17	1.34	−2	2

Table 4.2.1.2. shows that both groups started the study with relatively high attitude scores over all variable. During the intervention the changes were mostly small, and for the experimental group there were small positive improvements in Overall Attitude, Self-Confidence, and Enjoyment. The control group instead had slight drops across essentially all measures. These descriptive findings shows that attitude levels stayed mostly stable throughout the three-week intervention period, not much shift.

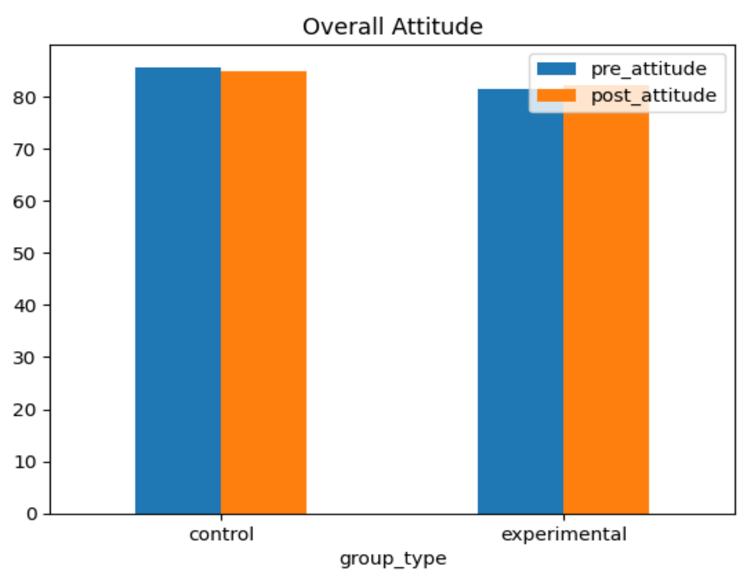
Figures 4.2.1.1 and 4.2.1.2 summarize the pre-test and post-test overall attitude scores, for both cohorts. The visual patterns imply only slight differences between the groups across the intervention period.

Figure 4.2.1.1. Pre-test and Post-test Overall Attitude Scores — 2nd-Year Cohort



Both groups show near-identical pre-test and post-test overall attitude means.

Figure 4.2.1.2. Pre-test and Post-test Overall Attitude Scores — 3rd-Year Cohort



The descriptive results show that the learners in both groups kept mostly positive attitudes about Discrete Mathematics over the whole study. There were a few, slight changes in the total attitude scores, but did not differ in a statistically meaningful way.

4.2.2 Assumption Checks

The results of the normality and variance equality tests for all attitude gain score distributions are presented in Appendix F. Three normality violations were identified across the two cohorts: the 2nd-year experimental group's enjoyment gain ($W = 0.896$, $p = 0.041$), and the 3rd-year experimental group's enjoyment gain ($W = 0.799$, $p = 0.001$) and control group's motivation gain ($W = 0.832$, $p = 0.006$). Non-parametric tests were substituted accordingly. The Levene's variance equality tests produced non-significant results (all $p > 0.10$) which confirmed that all attitude variables in both cohorts maintained equal variances across different groups.

4.2.3 Within-Group Changes

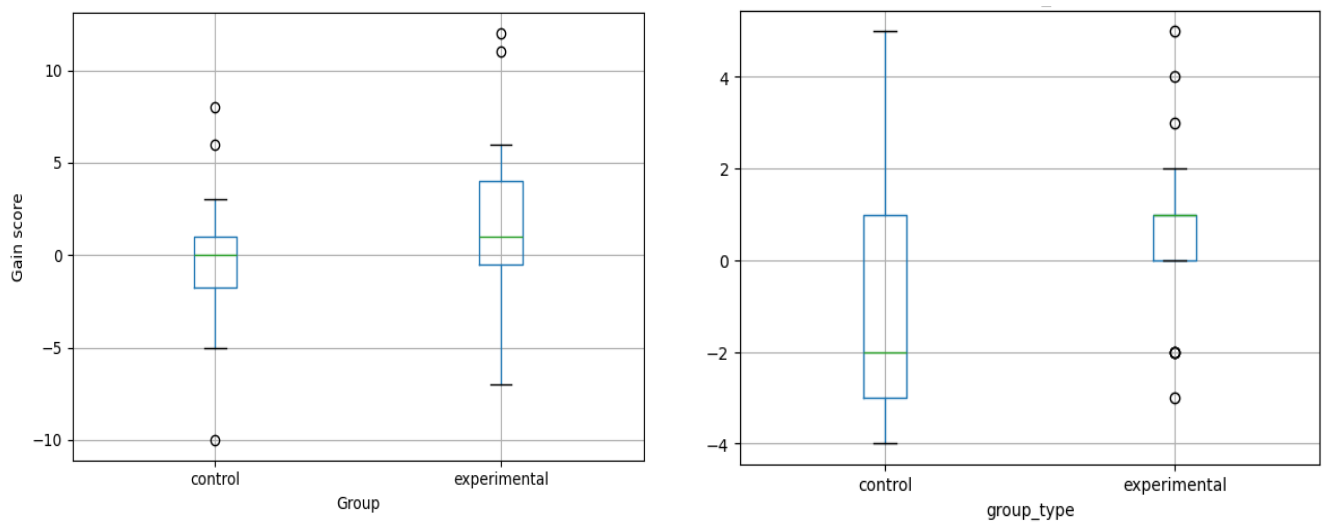
Table 4.2.3.1. presents within-group pre-to-post changes for all attitude variables. The only statistically significant change which occurred within the group was Self-Confidence gain of the 2nd-year experimental group: $t(18) = 2.224$, $p = 0.039$, $d = 0.510$, a medium effect (from $M = 45.58$ to $M = 47.26$).

No other within-group change reached significance in either cohort. The first cohort and the second cohort both demonstrated minor declines in motivation which created a pattern that showed student motivation throughout the intervention period because it was a common trait across students. The motivation toward Discrete Mathematics shows resistance to immediate instructional changes which created temporary suppression of motivation during both testing conditions. The 3rd-year cohort showed no confidence gain in either group, which may reflect the lower pre-test confidence of its experimental group ($M = 41.44$ vs. $M = 45.58$ in the 2nd year): students beginning from a lower baseline may require a longer period of positive peer experience before measurable gains emerge. The mean gain scores for all dimensions of both cohorts appear in Figure 4.2.3.1.

Table 4.2.3.1. Within-Group Changes in Attitude Variables by Cohort

Variable / Group	Cohort	*W*	*p*	Normality	Test used
Shapiro-Wilk Normality Tests on Gain Scores					
Overall attitude — Control	2nd Year	0.950	0.417	Yes	Paired t-test
	3rd Year	0.916	0.129	Yes	Paired t-test
Overall attitude — Experimental	2nd Year	0.955	0.478	Yes	Paired t-test
	3rd Year	0.933	0.218	Yes	Paired t-test
Enjoyment — Control	2nd Year	0.949	0.412	Yes	Paired t-test
	3rd Year	0.943	0.359	Yes	Paired t-test
Enjoyment — Experimental	2nd Year	0.896	0.041	No	Wilcoxon / Mann-Whitney U
	3rd Year	0.799	0.001	No	Wilcoxon / Mann-Whitney U
Motivation — Control	2nd Year	0.932	0.211	Yes	Paired t-test
	3rd Year	0.832	0.006	No	Mann-Whitney U
Motivation — Experimental	2nd Year	0.922	0.122	Yes	Paired t-test
	3rd Year	0.904	0.068	Yes (borderline)	Paired t-test
Confidence — Control	2nd Year	0.931	0.205	Yes	Paired t-test
	3rd Year	0.954	0.521	Yes	Paired t-test
Confidence — Experimental	2nd Year	0.956	0.493	Yes	Paired t-test
	3rd Year	0.939	0.280	Yes	Paired t-test

Figure 4.2.3.1. Mean Attitude Gain Scores by Dimension and Group — Both Cohorts



2nd year *3rd year*

2nd-year cohort (left panels): the experimental group shows a notable confidence gain. 3rd-year cohort (right panels): all gains near zero for both groups.

4.2.4 Between-Group Comparisons and ANCOVA

Table 4.2.4.1. shows the between-group gain score results and ANCOVA findings for all attitude measurements which were tested between two different groups of participants. The research study found that all attitude measurements failed to show any statistical significance during both the between-group testing and the ANCOVA testing which both groups of participants performed. The Self-Confidence subscale produced the most notable results: in the 2nd-year cohort, the between-group t-test approached significance ($t(35) = 2.005$, $p = .053$, $d = 0.660$, medium effect) and the ANCOVA was borderline ($b = 1.820$, $p = .073$). The 3rd-year cohort showed no similar results ($t(33) = 0.473$, $p = .639$, $d = 0.160$). The Overall Attitude between-group comparison was borderline in the 3rd-year cohort ($t(33) = 1.864$, $p = .071$, $d = 0.630$), though the ANCOVA confirmed non-significance ($b = 1.244$, $p = .127$). The very high R^2 values in

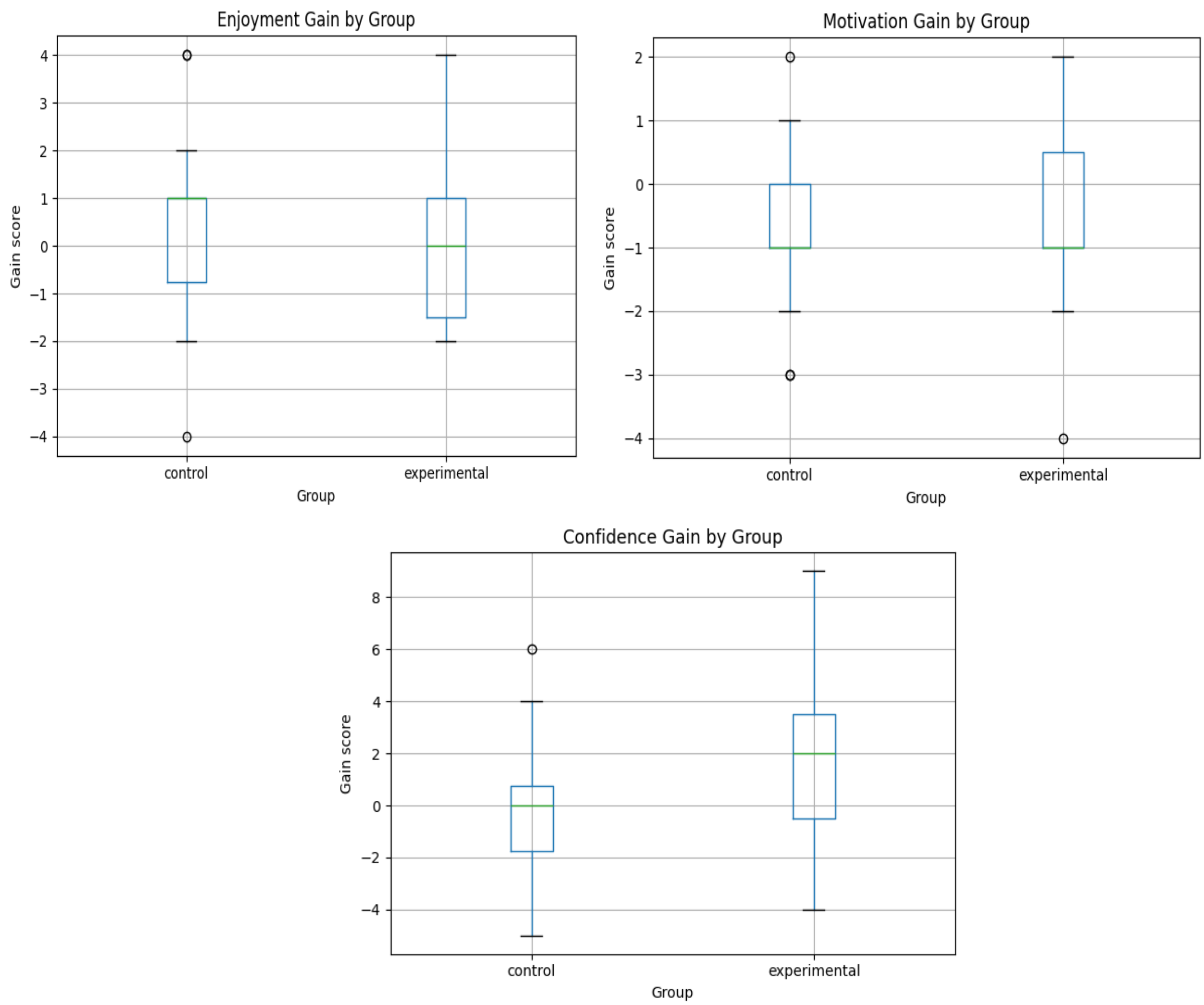
the ANCOVA models (ranging from .65 to .85) indicate that post-test attitude was strongly predicted by pre-test attitude, leaving limited residual variance for a group effect to emerge.

Table 4.2.4.1. Between-Group Attitude Comparisons and ANCOVA Results by Cohort

Variable / Cohort	Ctrl Gain	Exp Gain	*t*-test *p*	*d*	ANCOVA b *(p)*
Overall Attitude					
2nd Year	−0.39	1.53	0.212	0.42	1.822 (p=0.240)
3rd Year	−0.82	0.67	0.071	0.63	1.244 (p=0.127)
Self-Confidence					
2nd Year	−0.28	1.68	0.053	0.66	1.820 (p=0.073)
3rd Year	−0.24	0.11	0.639	0.16	−0.079 (p=0.924)
Enjoyment					
2nd Year	0.44	0.21	0.536	−0.12	0.071 (p=0.912)
3rd Year	−0.24	0.17	0.375	0.30	0.376 (p=0.392)
Motivation					
2nd Year	−0.56	−0.37	0.696	0.13	0.059 (p=0.904)
3rd Year	−0.35	−0.17	0.691	0.14	0.044 (p=0.921)

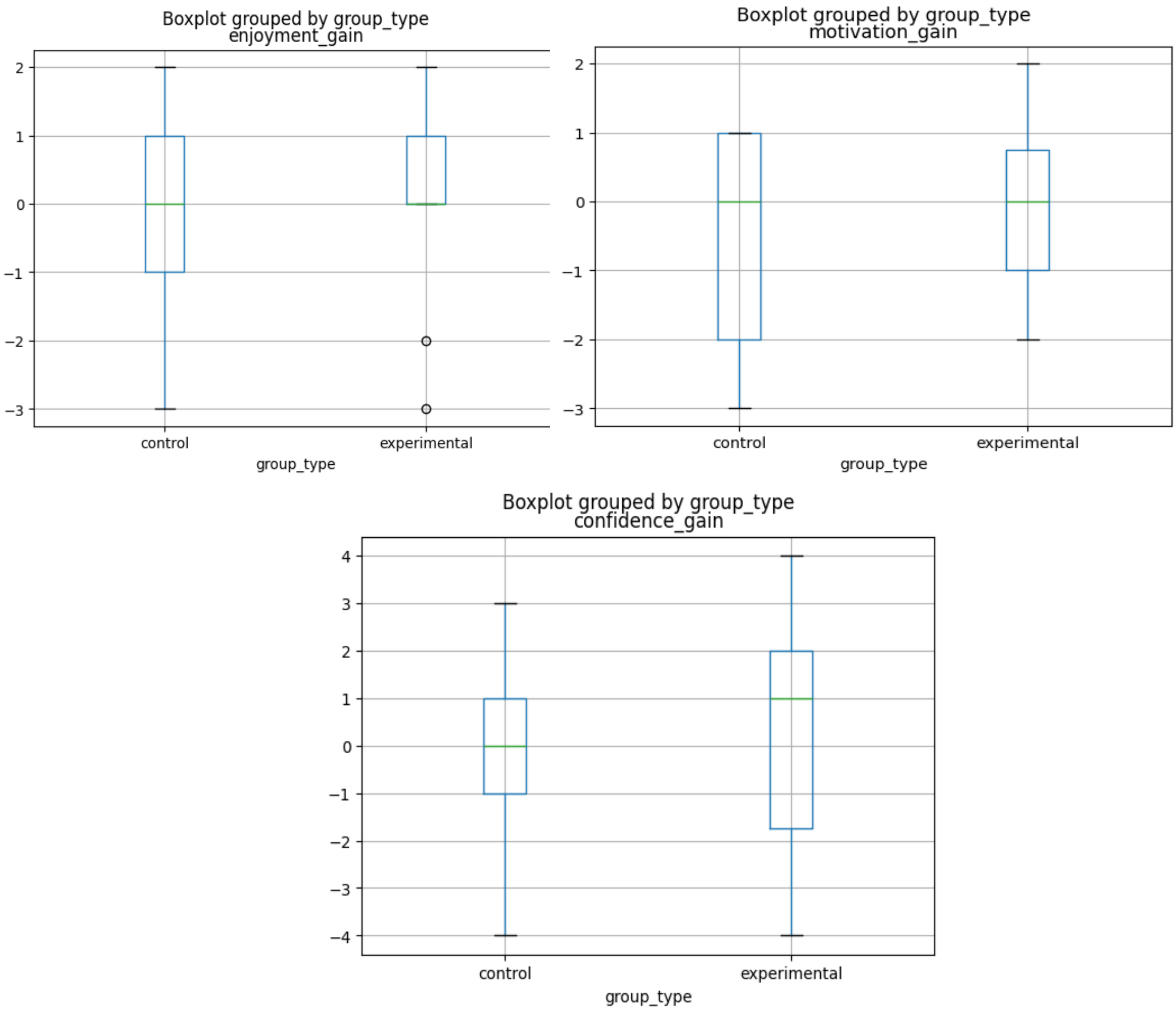
Table 4.2.4.1. shows that there weren't any statistically significant between-group differences, for the overall attitude ,or for any of the attitude subscale, in either cohort. Even though a few comparisons seemed to be close to significance, especially for Self-Confidence in the 2nd-year cohort, the results still didn't quite reach the significance. So overall these findings point to the idea that the intervention only brought about a limited amount of change in students' attitudes during the study.

Figure 4.2.4.1. Box Plots of Attitude Gain Scores by Group — 2nd-Year Cohort



Self-Confidence gain box plots show the experimental group median (Mdn = 2.00) exceeding the control (Mdn = 0.00), consistent with the significant within-group confidence gain (p = 0.039, d = 0.510).

Figure 4.2.4.2. Box Plots of Attitude Gain Scores by Group — 3rd-Year Cohort



All subscale gain distributions are clustered near zero for both groups, consistent with the uniformly non-significant between-group comparisons.

4.2.5 Hypothesis Decisions — Attitude

For both cohorts, the null hypothesis H_o2 — that there is no statistically significant difference in overall attitude toward Discrete Mathematics between groups — is retained. The 3rd-year between-group comparison showed borderline results with a p value of 0.071

and a d value of 0.630 whereas no other comparisons between groups or ANCOVA studies reached established levels of statistical significance in both cohorts.

The null hypothesis H_o3 which states there are no statistically significant differences in self-confidence, enjoyment and motivation between groups maintains its validity throughout all between-group and ANCOVA tests. Three findings which hold important significance were observed. The 2nd-year experimental group achieved a statistically significant Self-Confidence increase through its within-group testing which measured their progress ($t(18) = 2.224$, $p = .039$, $d = 0.510$, medium effect) while the control group showed no progress ($d = -0.11$). The observation that different confidence levels developed between participants shows that students who received peer explanations improved their self-confidence. The 2nd-year between-group confidence comparison achieved near significance with a p value of 0.053 and a $d = 0.660$ while the ANCOVA reached a borderline result with a $p=0.073$ which indicated that between-group confidence differences were practically significant yet the small attitude sample size of 37 participants could not measure it with accuracy. The two groups demonstrated continuous slight decreases in motivation throughout both experimental conditions which demonstrated that their motivation levels remained unchanged during the three-week intervention period.

4.3 Integrated Interpretation and Hypothesis Decisions

Before presenting the formal hypothesis decisions, it is useful to view the results as a coherent whole. A consistent descriptive pattern emerges across cohorts and outcomes. In both year levels, the experimental group improved by about 6.1 points on the achievement test compared to 2.8 points in the control group, with significant within-group gains for the experimental group and only marginal or non-significant changes in the control group. The between-group difference reached statistical significance in the 2nd-year cohort but not in the 3rd-year cohort, not due to a weaker intervention effect, but because stronger pre-test influence in the 3rd year ($F = 27.13$ vs. 8.46) absorbed more variance, limiting detectability. This distinction between a true effect and a statistically detectable effect is central to interpreting the findings.

The results of the attitude assessment show two different patterns of results. The study did not find any significant differences between the two groups which supports the belief that attitude changes need more extended periods of time to occur. The 2nd-year experimental group demonstrated a moderate self-confidence increase with a significant effect size of 0.510 which the control group did not show. TPS-SPS systems primarily impact self-efficacy through their functions of displaying reasoning processes and solution presentations as the system operates according to Albert Bandura's theoretical framework. Short-term interventions activate cognitive mechanisms and confidence-related mechanisms, but they do not produce any changes in enjoyment or motivation according to the study results.

The results obtained from the study establish a positive conclusion which needs certain qualifications because TPS-SPS produced measurable achievement improvements which two different methods of testing confirmed and which two different groups of students demonstrated. The results demonstrate theoretical expectations but show how a three-week intervention period limits development of deep attitudinal changes.

5. DISCUSSION AND CONCLUSION

5.1 Overview of Findings

This chapter presents research findings from Chapter 4 to examine research questions, theoretical frameworks and existing literature. The study presents its limitations which lead to conclusions and recommendations for teaching methods and upcoming studies. The research studied how Think-Pair-Share with Structured Problem Solving (TPS-SPS) impacted second- and third-year mathematics pedagogy students' performance and attitude during their Discrete Mathematics course at a private university in Kazakhstan. The study used a quasi-experimental approach which included pre-tests and post-tests across multiple groups and two student cohorts to evaluate the three-week intervention that focused on the Counting and Probability unit.

The results present a nuanced picture. The intervention led to a significant achievement boost for the 2nd-year students but showed only minor changes which could not be statistically confirmed in the 3rd-year students. The 2nd-year experimental group demonstrated a significant gain in self-confidence while no other between-group differences existed across both groups. These findings are consistent with the theoretical framework of the study and with the broader literature on peer learning, cooperative learning, and attitude change in mathematics education.

5.2 Discussion of Achievement Findings

5.2.1 The Significant Effect in the 2nd-Year Cohort

The second-year cohort ANCOVA demonstrated a statistically significant group effect which produced $F(1, 79) = 4.187$ statistical results with $p = 0.044$ and $\eta^2 p = .050$ while TPS-SPS groups obtained 3.025 points higher adjusted means. This finding supports existing research about peer learning which shows substantial evidence for its effectiveness. Capar and Tarim's (2015) meta-analysis found a medium positive effect of cooperative learning on mathematics achievement and university studies showed that structured peer interaction outperformed traditional instruction methods (Abdelkarim & Abuiyada, 2016; Kaymak et al., 2020). The present finding increases existing evidence about peer learning in university-level Discrete Mathematics which already exists as a sparse but growing evidence base.

This effect operates through a theoretical mechanism which matches Vygotsky's (1978) Zone of Proximal Development framework. Students in the TPS-SPS sessions had to solve Counting and Probability unit problems through individual thinking, partner discussions, and classroom presentations of their reasoning. The system establishes a condition which permits students with partial understanding to access the reasoning process of their more advanced peer, which exactly represents the ZPD-based scaffolding method that will help them to learn better. The Structured Problem Solving component needed students to identify their solving method before they began calculations, which created a metacognitive element that steered discussions toward conceptual understanding instead of sharing procedural solutions. The research of Magda and Gardner-McCune (2025) found that students at Discrete Mathematics experience learning problems because they need to develop abstract reasoning skills instead of solving computational mistakes.

The effect size of $\eta^2 p = .050$, which represents a small effect according to standard measurement guidelines, achieves suitability as a significant measure for the three-week educational program which consists of three teaching sessions focused on one specific subject. Educational interventions of this duration lead to minor educational improvements which create a sustainable academic advantage that develops into substantial value after students complete their entire semester or academic year. The finding also replicates the direction of effect found by Kaymak et al. (2020) in a peer instruction study at the same type of institution in Kazakhstan, showing that Kazakhstani universities use peer learning programs even though their teaching methods have historically focused on instructor-led learning (Tampayeva, 2015; Yergebekov & Temirbekova, 2012).

5.2.2 The Non-Significant Effect in the 3rd-Year Cohort

The 3rd-year ANCOVA did not reach statistical significance, $F(1, 79) = 2.190$, $p = .143$, $\eta^2p = .027$. The result needs to be interpreted with caution. The experimental group achieved a 6.20-point gain while the control group reached a 2.90-point gain which produced identical results to their 2nd-year performance of 6.05 points and 2.70 points with comparable effect sizes of $d = 0.384$ and 0.405.

The testing results showed that pre-test covariate was the important element which separated two groups. The 3rd-year model used prior achievement to explain more outcome variations than the 2nd-year model because its results showed ($F = 27.13$, $R^2 = .262$) while the second year results showed ($F = 8.46$, $R^2 = .134$). Third-year students who studied university mathematics for two years demonstrate their knowledge of previous content through their understanding of new material. The assessment results show that when previous achievements predict educational results then learning methods which require short time periods become less detectable because they demonstrate actual effectiveness. Researchers must treat the non-significant 3rd-year result as a detection limit because it lacks proof for an established null effect. The experimental group achieved extremely significant results ($t(40) = 5.089$, $p < 0.001$) which showed consistent effect direction across all studies and generated similar results between both student groups.

5.3 Discussion of Attitude Findings

5.3.1 Overall Attitude — Null Results in Context

The anticipated result of the research study showed that both groups of participants displayed no major differences in their attitudes toward the study material. The researchers define mathematical attitude as a stable multidimensional framework which develops through the totality of a person's mathematical learning experience (Tapia & Marsh, 2004).The stability of educational constructs shows that a three-week study which targets one specific content area will not produce any major educational changes. The literature shows that Capar and Tarim (2015) found cooperative learning produced a medium positive effect on mathematics achievement but its impact on attitude showed smaller and less reliable results while multiple studies with brief intervention periods showed significant learning outcomes without any corresponding changes in attitude.

The high pre-test baseline in both cohorts — overall means of 83–86 out of a maximum of 120 — adds a further constraint. Students who already hold positive attitudes have limited room for measurable improvement because their existing skills already reach maximum potential which results in reduced progress regardless of teaching effectiveness. The design represents actual mathematical pedagogy students who chose their discipline because they already possess positive attitudes toward mathematics. The ANCOVA models confirm this: R^2 values of 0.65-0.85 across all attitude outcomes indicate that post-test attitude was overwhelmingly predicted by pre-test attitude, leaving minimal residual variance for a group effect to register.

5.3.2 The Self-Confidence Finding

The 2nd-year experimental group showed self-confidence development with statistical significance and theoretical importance because this development reached a medium effect size ($t(18) = 2.224$, $p = .039$, $d = 0.510$). The subscale most related to TPS-SPS mechanisms produced only one attitude outcome that achieved significance for both cohorts. Bandura's social learning theory (1997) states that individuals develop self-efficacy which forms the basis of their self-confidence through two main learning methods: mastery experiences and vicarious learning. The SHARE stage of TPS-SPS provides both: students directly experience the mastery of successfully explaining a solution, and they observe peers solving problems in real time, raising their own belief in their capacity to do the same. The finding shows specific effects through the confidence subscale yet lacks impact on enjoyment and motivation which establishes theoretical accuracy for the discovery.

The borderline between-group confidence comparison ($p = .053$, $d = 0.660$) reinforces this interpretation. The study showed a medium effect which reached borderline significance when 18 to 19 students participated in each group. The study found confidence levels which matched previous research about mathematics anxiety in higher education that showed students who engaged in collaborative learning environments developed better anxiety management skills and improved their confidence levels (Bjälkebring, 2019; Tsirimokos et al., 2024). The 3rd-year cohort showed no matching result because of the experimental group's lower baseline confidence which showed an average of 41.44 while the 2nd-year group showed an average of 45.58. The requirement for peer learning to increase confidence claims that students must first possess existing confidence before they can develop further. More sessions are necessary when starting from a lower baseline.

5.3.3 The Motivation Decline Pattern

The two groups showed slight decreases in motivation scores during both their experimental tests and their control tests. This pattern existed across all four groups because of their different teaching approaches which makes it impossible to link it with TPS-SPS. The most likely explanation shows that student motivation during the specific period of the academic semester which included the intervention period shows this pattern. The research result provides reassurance about the situation because TPS-SPS maintains student motivation at the same level as traditional teaching methods during a time when student motivation across all teaching methods experienced decline. Future research needs to investigate whether extended TPS-SPS programs can stop or reverse the current decline in student motivation.

5.4 Implications

5.4.1 Implications for Pedagogical Practice

The results of this research study provide multiple practical applications for teaching mathematics at universities in Kazakhstan. The TPS-SPS system functions as an effective yet affordable teaching method which follows academic standards and requirements of Discrete Mathematics that traditional student study methods use. The system needs only a basic structured worksheet and operates without any changes to its existing lecture system and course materials and evaluation timetable. The intervention can be introduced and implemented within a single content unit without disrupting the broader curriculum — precisely the kind of feasible, scalable intervention that institutions operating within resource constraints can adopt.

The research demonstrates that peer learning leads to academic improvement for students studying mathematics at Kazakhstani universities. The existing research from Kazakhstan was restricted to the study of Kaymak et al. (2020) which evaluated a different course without assessing student attitudes. The current research adds to local evidence while demonstrating that Kazakhstani mathematics pedagogy programs should include structured peer learning as a required element of their practice class component.

The experimental group from the second year demonstrated confidence improvement, which creates a double effect for teacher training programs. Teachers who have personally experienced effective collaborative pedagogy as learners are more likely to implement it in their own classrooms (Fulton et al., 2022). The school system benefits from mathematics pedagogy training through peer learning because it enhances current student results and develops future teaching quality.

5.4.2 Implications for Future Research

The study provides multiple directions which researchers should investigate. The first requirement which needs implementation involves conducting extended research through an intervention which lasts six to eight weeks or through a complete semester. The research needs to confirm whether the achievement effect from three sessions will develop into a more substantial effect which can show through the research process. The study found that both the attitude sample sizes which used 35 to 37 participants and the study design which required additional participants to estimate between-group attitude differences were insufficient to achieve accurate results. The research needs to create attitude data collection procedures which will capture information from all students or establish complete data collection methods from the beginning of the study. The study needs to examine how previous accomplishments affect TPS-SPS success because its results show that testing methods in the study depend on how well previous accomplishments help forecast results in that specific group of participants. The study needs to use mixed-methods research with student interviews and focus groups to enhance its quantitative results because this approach will show how students experienced TPS-SPS through their SHARE stage evaluations which affected their confidence levels.

5.5 Conclusion

The study examined how TPS-SPS affected mathematics pedagogy students in their Discrete Mathematics course achievements and their attitudes. The results lead to a positive conclusion which requires further qualifications. The 2nd-year students who used TPS-SPS showed improved performance results through a statistical achievement benefit which the study measured (ANCOVA: $F(1,79) = 4.187$, $p = .044$, $\eta^2p = .050$) while the 3rd-year students showed a similar pattern which lacked statistical evidence. The 2nd-year group showed significant confidence improvement through within-group changes while the experimental group achieved a confidence gain of $d = 0.510$ which demonstrates that TPS-SPS activates self-efficacy mechanisms through the attitudinal dimension which connects most closely to peer learning.

The findings establish theoretical foundations for Vygotsky's (1978) ZPD and Bandura's (1997) social learning theory which function as unified explanations for the cognitive and attitudinal effects of structured peer learning. The ZPD accounts for achievement gains through peer scaffolding; social learning theory accounts for confidence gains through the direct performance experiences that the SHARE stage generates. The two elements combine to provide a unified explanation which is based on research for the different results observed in various outcomes.

The study provides Kazakhstani mathematics educators a practical teaching method which requires no financial investment and works with existing curriculum requirements. The implementation of TPS-SPS enables instructors to introduce the system without needing to modify their teaching materials or evaluation methods because they can use it within their existing single lesson plans. The value for mathematics pedagogy students doubles because they acquire academic knowledge while they study a research-based teaching method which they can use in their future classrooms, thus improving the entire system of mathematics education.

The higher education system which seeks to modernize its teaching methods faces challenges because its educational institutions maintain their existing practices. The study presents evidence to support this claim. The study demonstrates two findings which show that structured peer learning works effectively and should be implemented to teach Discrete Mathematics at educational institutions in Kazakhstan.

6. LIMITATIONS

The present study has several limitations that should be acknowledged. The intervention lasted three weeks with three practice sessions which is a brief period that does not meet the usual duration of peer learning studies which require a full semester. The achievement effect detected in one cohort was made possible by this duration but it restricted both achievement results and attitudinal outcomes.

Second, the study examined two groups of students who demonstrated their attitudes through two different assessments which had sample sizes of 37 students from the second-year group and 35 students from the third-year group. The reduction of sample size limitations used for statistical analysis which particularly affected between-group comparison tests and resulted in missing the necessary evidence to show significant differences in attitudes.

Third, the study was conducted at one university in Kazakhstan which restricts its findings to particular institutional contexts and specific course structures and different national environments.

Fourth, the existing class structure of the study required a quasi-experimental design because it prevented researchers from using random participant assignment. The researchers proved that both groups had equal starting points through statistical methods, but unmeasured group differences still existed as a potential gap in their findings.

Finally, the study did not conduct any follow-up evaluations after the initial assessment. The results from Week 11 show achievement increases, but it remains unclear whether these gains continued to be maintained.

LIST OF REFERENCES

Abdelkarim, R., & Abuiyada, R. (2016). The Effect of Peer Teaching on Mathematics Academic Achievement of the Undergraduate Students in Oman. *International Education Studies*, 9(5), 124. https://doi.org/10.5539/ies.v9n5p124

Alegre, F., Moliner, L., Maroto, A., & Lorenzo-Valentin, G. (2020). Academic Achievement and Peer Tutoring in Mathematics: A Comparison Between Primary and Secondary Education. Sage Open, 10(2), 2158244020929295. https://doi.org/10.1177/2158244020929295

Alsmadi, Mohareb. A., Tabieh, A. A. S., Alsaifi, R. M., & Al-Nawaiseh, S. J. (2023). The Effect of the Collaborative Discussion Strategy Think-Pair-Share on Developing Students' Skills in solving Engineering Mathematical Problems. European Journal of Educational Research, volume–12–2023(volume–12–issue–2–april–2023), 1123–1135. https://doi.org/10.12973/eu-jer.12.2.1123

Anderson, L. W. (Ed.). (2009). A taxonomy for learning, teaching, and assessing: A revision of Bloom's taxonomy of educational objectives (Abridged ed., [Nachdr.]). Longman.

Anggara, B., Priatna, N., & Juandi, D. (2018). Learning difficulties of senior high school students based on probability understanding levels. Journal of Physics: Conference Series, 1013, 012116. https://doi.org/10.1088/1742-6596/1013/1/012116

Anthony, G. (1996). Active learning in a constructivist framework. Educational Studies in Mathematics, 31(4), 349–369. https://doi.org/10.1007/BF00369153

Awofala, A. O., & Lawani, A. O. (2020). Examining the Efficacy of Co-operative Learning Strategy on Undergraduate Students' Achievement in Mathematics.

International Journal of Pedagogy and Teacher Education, 4(1). https://doi.org/10.20961/ijpte.v4i1.33402

Azhibayeva, A., Issaldayeva, S., Bakirova, K., Kudaibergeneva, K., & Saari, D. (2024). Transformation of universities in Kazakhstan: Research outcomes on the quality of higher education. Journal of Infrastructure Policy and Development, 8(10), 6844. https://doi.org/10.24294/jipd.v8i10.6844

Bandura, A. (1997). Self-efficacy: The exercise of control. New York: Freeman.

Bandura, A. (1986). Social foundations of thought and action: A social cognitive theory. Englewood Cliffs, NJ: Prentice-Hall.

Bengesai, A. V., Amusa, L. B., & Dhunpath, R. (2023). A meta-analysis on the effect of formal peer learning approaches on course performance in higher education. Cogent Education, 10(1), 2203990. https://doi.org/10.1080/2331186X.2023.2203990

Bjälkebring, P. (2019). Math Anxiety at the University: What Forms of Teaching and Learning Statistics in Higher Education Can Help Students with Math Anxiety? Frontiers in Education, 4, 30. https://doi.org/10.3389/feduc.2019.00030

Capar, G. (2015). Efficacy of the Cooperative Learning Method on Mathematics Achievement and Attitude: A Meta-Analysis Research. Educational Sciences: Theory & Practice. https://doi.org/10.12738/estp.2015.2.2098

Chan, T. C., Zulnaidi, H., & Eu, L. K. (2022). Measurement model testing: Adaption of self-efficacy and metacognitive awareness among university students. Eurasia Journal of Mathematics, Science and Technology Education, 18(9), em2153. https://doi.org/10.29333/ejmste/12366

Cohen, E. G. (1994). Restructuring the Classroom: Conditions for Productive Small Groups. Review of Educational Research, 64(1), 1. https://doi.org/10.2307/1170744

Cohen, J. (1998). Statistical Power Analysis for the Behavioural Sciences. Lawrence Erlbaum Associates, Hillsdale.

Cohen, L., Manion, L., & Morrison, K. (2017). Research Methods in Education (8th ed.). Routledge. https://doi.org/10.4324/9781315456539

Creswell, J.W. and Creswell, J.D. (2018) Research Design: Qualitative, Quantitative, and Mixed Methods Approaches. Sage, Los Angeles.

Doise, W., & Mugny, G. (1984). The social development of the intellect (1st ed). Pergamon Press.

Dr., Bartin University, Faculty of Education, Turkey, sdtturgut42@hotmail.com, Turgut, S., Turgut, İ. G., & Res. Asst., Dumlupinar University, Faculty of Education, Turkey, ilknurgulsen@gmail.com. (2018). The Effects of Cooperative Learning on Mathematics Achievement in Turkey: A Meta-Analysis Study. International Journal of Instruction, 11(3), 663–680. https://doi.org/10.12973/iji.2018.11345a

Ebel, R.L. and Frisbie, D.A. (1991) Essentials of Educational Measurement. 5th Edition, Prentice-Hall, Englewood Cliffs.

Epp, S. S. (2020). Discrete mathematics with applications (Fifth edition). Cengage Learning.

Field, A.P. (2018) Discovering Statistics Using IBM SPSS Statistics. 5th Edition, Sage, Newbury Park.

Forman, E. A., & Cazden, C. B. (2013). Exploring Vygotskian perspectives in education: The cognitive value of peer interaction. In Learning relationships in the classroom (pp. 189-206). Routledge.

Fuchs, L. S., Fuchs, D., Karns, K., Hamlett, C., Katzaroff, M., & Dutka, S. (1998). Comparisons among Individual and Cooperative Performance Assessments and Other Measures of Mathematics Competence. The Elementary School Journal, 99(1), 23–51. https://doi.org/10.1086/461915

Fulton, E. W., Arnold, E. G., Burroughs, E. A., Álvarez, J. A. M., Kercher, A., & Turner, K. (2022). Including School Mathematics Teaching Applications in an Undergraduate Discrete Mathematics Course. PRIMUS, 32(6), 704–720. https://doi.org/10.1080/10511970.2021.1905120

George, D., & Mallery, P. (2003). SPSS for Windows Step by Step: A Simple Guide and Reference. 11.0 Update (4th ed.). Boston: Allyn & Bacon.

Hannula, M. S. (2002). Attitude towards mathematics: Emotions, expectations and values. Educational Studies in Mathematics, 49(1), 25–46. https://doi.org/10.1023/A:1016048823497

Huck, S. W., & McLean, R. A. (1975). Using a repeated measures ANOVA to analyze the data from a pretest-posttest design: A potentially confusing task. Psychological Bulletin, 82(4), 511–518. https://doi.org/10.1037/h0076767

Hwang, S., & Son, T. (2021). Students' Attitude toward Mathematics and its Relationship with Mathematics Achievement. Journal of Education and E-Learning Research, 8(3), 272–280. https://doi.org/10.20448/journal.509.2021.83.272.280

Jaafar, W. M. W., & Ayub, A. F. M. (2010). Mathematics Self-efficacy and Meta-Cognition Among University Students. Procedia - Social and Behavioral Sciences, 8, 519–524. https://doi.org/10.1016/j.sbspro.2010.12.071

Johnson, D. W., & Johnson, R. (1989). Cooperation and Competition: Theory and Research. Edina, MN: Interaction Book Company.

Johnson, D. W., & Johnson, R. T. (2014). Cooperative learning in 21st century. Anales de Psicología, 30(3), 841–851. https://doi.org/10.6018/analesps.30.3.201241

Kaymak, S., & Sautkali, A. (2022). The Factors that Affect Mathematical Achievement Among High School Students. Journal of Research in Didactical Sciences, 1(1), 12733. https://doi.org/10.51853/jorids/12733

Kinnear, G., Wood, A. K., & Gratwick, R. (2022). Designing and evaluating an online course to support transition to university mathematics. International Journal of Mathematical Education in Science and Technology, 53(1), 11–34. https://doi.org/10.1080/0020739X.2021.1962554

Langprayoon, P., & Ruangsart, W. (2025). Development of Ability in English Learning Management of University Students in Early Childhood Major through Cooperative Learning and Think-Pair-Share Technique. 2025 10th International STEM Education Conference (iSTEM-Ed), 1–6. https://doi.org/10.1109/iSTEM-Ed65612.2025.11129283

Leron, U., & Dubinsky, E. (1995). An Abstract Algebra Story. The American Mathematical Monthly, 102(3), 227–242. https://doi.org/10.1080/00029890.1995.11990563

Lyman, F. T. (1981). The responsive classroom discussion: The inclusion of all students. Mainstreaming digest, 109(1), 113.

Ma, X. (1997). Reciprocal Relationships Between Attitude Toward Mathematics and Achievement in Mathematics. The Journal of Educational Research, 90(4), 221–229. https://doi.org/10.1080/00220671.1997.10544576

Magda, D., & Gardner-McCune, C. (2025). Students' Thoughts on Discrete Mathematics: Insights for Practice and Implications for Future Research. Proceedings of the 56th ACM Technical Symposium on Computer Science Education V. 1, 736–741. https://doi.org/10.1145/3641554.3701948

Malheiros, M. D. G., & Haetinger, C. (2024). Explorable Discrete Mathematics: A Python-based undergraduate-level teaching approach. Anais Do XXXV Simpósio Brasileiro de Informática Na Educação (SBIE 2024), 2494–2505. https://doi.org/10.5753/sbie.2024.242630

Massyrova, R., Tautenbaeva, A., Tussupova, A., Zhalalova, A., & Bissenbayeva, Zh. (2015). Changes in The Higher Education System of Kazakhstan. Procedia - Social and Behavioral Sciences, 185, 49–53. https://doi.org/10.1016/j.sbspro.2015.03.458

Mena, T. T. (2020). Peer Teaching Intervention to Enhance Applied Mathematics Performance of Technology Student in Mizan-Tepi University. Scholars Journal of Physics, Mathematics and Statistics, 07(03), 34–39. https://doi.org/10.36347/sjpms.2020.v07i03.001

Misajon, C. C. (2025). Learners' Self-Efficacy and Attitude towards Mathematics. Cognizance Journal of Multidisciplinary Studies, 5(5), 547–550. https://doi.org/10.47760/cognizance.2025.v05i05.033

Moliner, L., Alegre, F., & Lorenzo-Valentín, G. (2022). Peer Tutoring and Math Digital Tools: A Promising Combination in Middle School. Mathematics, 10(13), 2360. https://doi.org/10.3390/math10132360

Moore, R. C. (1994). Making the transition to formal proof. Educational Studies in Mathematics, 27(3), 249–266. https://doi.org/10.1007/BF01273731

Neha Khaiwal, Satyendra Gupta. (2025). Relationship Between Attitude Towards Mathematics and Academic Achievement of Eleventh-Class Students. Journal of Informatics Education and Research, 5(1). https://doi.org/10.52783/jier.v5i1.2074

Ningsih, Y. (2019). The Use of Cooperative Learning Models Think Pair Share in Mathematics Learning. Journal of Physics: Conference Series, 1387(1), 012144. https://doi.org/10.1088/1742-6596/1387/1/012144

Nurrahmah, A., Zaenuri, & Wardono. (2021). Analysis of students difficulties in mathematical abstraction thinking in the mathematics statistic course. Journal of Physics: Conference Series, 1918(4), 042112. https://doi.org/10.1088/1742-6596/1918/4/042112

Nwankwo, M. C., & Nnamani, M. K. (2025). EFFECT OF THINK-PAIR-SHARE INSTRUCTIONAL STRATEGY ON THE ACADEMIC ACHIEVEMENT OF SENIOR SECONDARY SCHOOL STUDENTS IN MATHEMATICS IN ONITSHA EDUCATION ZONE, ANAMBRA STATE. Unizik Journal of Educational Research and Policy Studies, 19(1).

Pajares, F., & Graham, L. (1999). Self-Efficacy, Motivation Constructs, and Mathematics Performance of Entering Middle School Students. Contemporary Educational Psychology, 24(2), 124–139. https://doi.org/10.1006/ceps.1998.0991

Piaget, J. (1950). The psychology of intelligence. Routledge.

Prasad Dahal, P. (2026). Correlation Between Peer **Learning** Practices and Mathematics Achievement of Secondary Level Students in Bhaktapur District, Nepal. Education and Development, 35(1). https://doi.org/10.3126/ed.v35i1.90365

President of the Republic of Kazakhstan. (2024, July 30). Decree No. 611 on the approval of the National Development Plan of the Republic of Kazakhstan until 2029 and recognition of certain decrees of the President of the Republic of Kazakhstan as invalid. https://adilet.zan.kz/kaz/docs/U2400000611

Puspitasari, N., Afriansyah, E. A., Nuraeni, R., Madio, S. S., & Margana, A. (2019). What are the difficulties in statistics and probability? Journal of Physics: Conference Series, 1402(7), 077092. https://doi.org/10.1088/1742-6596/1402/7/077092

Ramadhani, A., Karida, M. P., & Wahyunengsih, W. (2025). Understanding the Role of Think-Pair-Share Strategy in Mathematics Classroom. Kognitif: Jurnal Riset HOTS Pendidikan Matematika, 5(3). https://doi.org/10.51574/kognitif.v5i3.3414

Raman, M. (2003). Key Ideas: What are they and how can they help us understand how people view proof? Educational Studies in Mathematics, 52(3), 319–325. https://doi.org/10.1023/A:1024360204239

Safitri, J., Avana, N., & Guswita, R. (2025). Think Pair Share Model for Improving Mathematics Learning Outcomes: Model Think Pair Share untuk Meningkatkan Hasil Belajar Matematika. Indonesian Journal of Innovation Studies, 26(4). https://doi.org/10.21070/ijins.v26i4.1641

Samuel, A., & Sumaila, A. (2025). Relationship between Peer Assisted Learning and Students' Academic Achievement: The Mediating Effect of Students' Attitude. International Journal of Research and Innovation in Social Science, IX(VI), 839–853. https://doi.org/10.47772/IJRISS.2025.90600071

Selden, A., & Selden, J. (2003). Validations of Proofs Considered as Texts: Can Undergraduates Tell Whether an Argument Proves a Theorem? Journal for Research in Mathematics Education, 34(1), 4. https://doi.org/10.2307/30034698

Serkan Kaymak, Nuri Balta, Abdullah Almas, Bibigul Kazmagambet, & Omar Mbala. (2020). The Impact of Peer Instruction on Students' Achievement in Mathematics Analysis. Journal of Psychology Research, 10(12). https://doi.org/10.17265/2159-5542/2020.12.003

Shakya, S., & Maharjan, R. (2023). Students' Attitude towards Mathematics and its Relationship with Mathematics Achievement. Mangal Research Journal, 4(01), 29–40. https://doi.org/10.3126/mrj.v4i01.61718

Siller, H.-S., & Ahmad, S. (2024). Analyzing the impact of collaborative learning approach on grade six students' mathematics achievement and attitude towards mathematics. Eurasia Journal of Mathematics, Science and Technology Education, 20(2), em2395. https://doi.org/10.29333/ejmste/14153

Springer, L., Stanne, M. E., & Donovan, S. S. (1999). Effects of Small-Group Learning on Undergraduates in Science, Mathematics, Engineering, and Technology: A Meta-Analysis. Review of Educational Research, 69(1), 21. https://doi.org/10.2307/1170643

Subiantoro, N., Lailiyah, J. I., Saputra, M. A. M., & Nurwahid, M. (2025). Implementation of vygotsky's developmental psychology theory to overcome mathematics learning anxiety. LINEAR: Journal of Mathematics Education, 6(1), 14–30. https://doi.org/10.32332/0h11yh51

Tajik, M. A., Namyssova, G., Shamatov, D., Manan, S. A., Zhunussova, G., & Antwi, S. K. (2025). Navigating the potentials and barriers to EMI in the post-Soviet region: Insights from Kazakhstani university students and instructors. International Journal of Multilingualism, 22(2), 990–1010. https://doi.org/10.1080/14790718.2023.2265428

Takahashi, A. (2006). Characteristics of Japanese mathematics lessons. Tsukuba journal of educational study in mathematics, 25(1), 37-44.

Tall, David. (1991). The Psychology of Advanced Mathematical Thinking. 10.1007/0-306-47203-1_1.

Tampayeva, G. Y. (2015). Importing education: Europeanisation and the Bologna Process in Europe's backyard—The case of Kazakhstan. European Educational Research Journal, 14(1), 74–85. https://doi.org/10.1177/1474904114565154

Tang, J., Zheng, Y., Wijaya, T. T., Su, M., & Listiawan, T. (2025). Determinants of student discussion participation in the mathematical collaborative problem-solving process using PLS-SEM and FsQCA. Scientific Reports, 15(1), 37082. https://doi.org/10.1038/s41598-025-21086-3

Tanujaya, B., & Mumu, J. (2019). Implementation of think-pair-share to mathematics instruction. Journal of Education and Learning (EduLearn), 13(4), 510–517. https://doi.org/10.11591/edulearn.v13i4.14353

Tapia, M., & Marsh, G. E. (2004). An instrument to measure mathematics attitudes. Academic Exchange Quarterly, 8(2), 16–21.

Topping, K. J. (2005). Trends in Peer Learning. Educational Psychology, 25(6), 631–645. https://doi.org/10.1080/01443410500345172

Tsirimokos, G., Lekka, E., & Pilafas, G. (2024). 'Math is not for me'. Investigating Mathematics Anxiety in Secondary and Higher Education: A Critical Discussion of Current Practices and Future Recommendations. International Journal of Science and Healthcare Research, 9(2), 237–249. https://doi.org/10.52403/ijshr.20240234

Vygotsky, L. S. (1978). Mind in society: The development of higher psychological processes. Harvard University Press.

Yergebekov, M., & Temirbekova, Z. (2012). The Bologna Process and Problems in Higher Education System of Kazakhstan. Procedia - Social and Behavioral Sciences, 47, 1473–1478. https://doi.org/10.1016/j.sbspro.2012.06.845

Zimmerman, B. J. (2000). Self-Efficacy: An Essential Motive to Learn. Contemporary Educational Psychology, 25(1), 82–91. https://doi.org/10.1006/ceps.1999.1016

LIST OF APPENDICES

Appendix A. Achievement Test Form A

Question 1

A bag contains 5 red, 3 blue, and 2 yellow balls. One ball is chosen at random.

(a) Write out the sample space S for this experiment and state $N(S)$.

(b) Find $P(E)$ where E is the event that the ball chosen is blue. Use the equally likely probability formula and show your substitution.

(c) Find $P(F)$ where F is the event that the ball is NOT yellow.

Question 2

A student ID consists of 2 letters (A–Z) followed by 3 digits (0–9). Repetition is allowed throughout.

(a) State the number of choices for each position and apply the Multiplication Rule to find the total number of distinct IDs.

(b) How many IDs are possible if the two letters must be different from each other (but digits may still repeat)?

Question 3

In a group of 50 students, 28 study Spanish, 22 study French, and 8 study both languages.

(a) Draw a labelled Venn diagram showing all four regions. Write the number of students in each region.

(b) Use the Inclusion-Exclusion Rule to find how many students study Spanish or French (or both). State the formula you are using.

(c) A student is chosen at random. Determine the probability that they study exactly one language. Show your reasoning step by step.

Question 4

Answer the following questions about the Pigeonhole Principle.

(a) State the Pigeonhole Principle in your own words. Identify clearly what 'pigeons' and 'pigeonholes' represent.

(b) In a class of 13 students, must at least two students share the same birthday month? Justify your answer by explicitly applying the Pigeonhole Principle.

(c) What is the minimum number of students needed to guarantee that at least 3 students share the same birthday month? Show your working using the Generalized Pigeonhole Principle.

Question 5

A department has 7 faculty and 5 students. A committee of 5 people is to be selected.

(a) How many committees of 5 are possible with no restrictions? Use the combination formula and show your full calculation.

(b) How many committees contain exactly 2 students and 3 faculty?

(c) How many committees contain at least 1 student? Explain why it is more efficient to use the complement method here and show the full working.

Question 6

A café sells 4 types of cake: chocolate, vanilla, lemon, and carrot. You buy a box of 3 cakes (repetition allowed, order does not matter).

(a) State the formula for r-combinations with repetition and identify n and r for this problem.

(b) Calculate the total number of different selections. Show full working.

(c) Without repetition, how many ways could you choose 3 different types? Compare this to your answer in (b) and explain why one is larger.

Question 7

Answer the following questions about Pascal's Formula and the Binomial Theorem.

(a) Use Pascal's Formula to find $C(7,3)$ by building from the values $C(5,2) = 10$, $C(5,3) = 10, C(6,2) = 15, C(6,3) = 20$. Show each step.

(b) Use the Binomial Theorem to expand $(x + 2)^4$. Write out every term clearly.

(c) Identify the coefficient of x^2in your expansion from part (b). Then verify this coefficient using the combination formula $C(4,2)$ directly and explain the connection.

Question 8

Let S be a sample space and A, B be events with $P(A) = 0.5, P(B) = 0.4, P(A \cap B) = 0.3$.

(a) Find $P(A \cup B)$ using the general probability formula. State which formula you are using and why the simpler formula $P(A) + P(B)$ cannot be used here.

(b) Find $P(A^c)$and $P((A \cup B)^c)$. State the rule you apply for each.

(c) A friend claims: '$P(A \cap B) = 0.3$ is impossible because it is larger than $P(B) = 0.4$ so this violates the axioms.' Is your friend correct? Justify your answer by checking the relevant probability axiom.

Question 9

Factory X produces 60% of a product with a 5% defect rate. Factory Y produces 40% with a 3% defect rate. An item is selected at random.

(a) Draw a tree diagram showing the two factories and their defect/non-defect branches. Label every branch with its probability.

(b) Calculate the probability that a randomly selected item is defective. Show full working using the total probability formula.

(c) Given that the item is defective, find the probability it came from Factory X. Use Bayes' Theorem and show every step. Interpret your answer in one sentence.

Appendix B. Achievement Test Form B

Question 1

A box contains 6 green, 4 white, and 2 black pens. One pen is chosen at random.

(a) Write out the sample space S and state $N(S)$.

(b) Find $P(E)$ where E = 'the pen chosen is green.' State the formula and show your substitution.

(c) Find $P(F)$ where F = 'the pen chosen is NOT white.'

Question 2

A password consists of 3 letters (A–Z) followed by 2 digits (0–9). Repetition is allowed throughout.

(a) State the number of choices for each position and apply the Multiplication Rule to find the total number of distinct passwords.

(b) How many passwords are possible if all 3 letters must be different (but digits may still repeat)?

Question 3

In a survey of 60 people, 35 own a car, 24 own a bicycle, and 9 own both.

(a) Draw a labelled Venn diagram showing all four regions with the correct number of people in each.

(b) Use the Inclusion-Exclusion Rule to find how many people own a car or a bicycle (or both). State the formula.

(c) A person is chosen at random. Find the probability they own exactly one vehicle. Show your reasoning step by step.

Question 4

Answer the following questions about the Pigeonhole Principle.

(a) State the Pigeonhole Principle in your own words. Identify what 'pigeons' and 'pigeonholes' represent in your statement.

(b) A drawer contains socks in 5 different colors. You draw socks one at a time without looking. Must at least 2 socks match after 6 draws? Justify using the Pigeonhole Principle.

(c) What is the minimum number of socks needed to guarantee at least 4 socks of the same color? Show your working using the Generalized Pigeonhole Principle.

Question 5

A university club has 6 senior members and 4 junior members. A team of 4 is to be selected.

(a) How many teams of 4 are possible with no restrictions? Use the combination formula and show your full calculation.

(b) How many teams contain exactly 1 junior and 3 senior members?

(c) How many teams contain at least 2 junior members? Use the complement method, explain why you chose it, and show the full working.

Question 6

A shop sells 5 types of chocolate: dark, milk, white, hazelnut, and caramel. You select 4 chocolates (repetition allowed, order does not matter).

(a) State the formula for r-combinations with repetition and identify n and r.

(b) Calculate the total number of different selections. Show full working.

(c) Without repetition, how many ways could you choose 4 different types? Compare to (b) and explain why one is larger.

Question 7

Answer the following questions about Pascal's Formula and the Binomial Theorem.

(a) Use Pascal's Formula to find $C(8,3)$ by building from $C(6,2) = 15, C(6,3) = 20, C(7,2) = 21, C(7,3) = 35$. Show each step.

(b) Use the Binomial Theorem to expand $(2x + 1)^3$. Write every term clearly.

(c) Identify the coefficient of x in your expansion from part (b). Verify using $C(3,1) * 2^1 * 1^2$directly and explain the connection.

Question 8

Events A and B satisfy $P(A) = 0.6, P(B) = 0.3$, and A and B are mutually exclusive.

(a) Find $P(A \cup B)$. State which formula applies because A and B are mutually exclusive, and why the general formula gives the same result here.

(b) Find $P(A^c)$ and $P((A \cup B)^c)$. State the rule for each.

(c) A student claims: 'If $P(A) = 0.6$ and $P(B) = 0.3$ then $P(A \cap B)$ must equal 0.18 because they multiply.' Identify the error in this reasoning. Under what condition would $P(A \cap B) = P(A) \times \mathrm{P(B)}$ be valid?

Question 9

A medical test for a disease has the following characteristics: the disease affects 2% of the population. The test is positive for 95% of people who have the disease. The test is also positive for 4% of people who do not have the disease (false positive rate).

(a) Draw a tree diagram with branches for disease/no disease and positive/negative test result. Label all branches with probabilities.

(b) Calculate the probability that a randomly selected person tests positive. Show full working.

(c) Given a person tests positive, find the probability they actually have the disease. Use Bayes' Theorem step by step. Then explain in one sentence why this probability is surprisingly low despite the test being 95% accurate.

Appendix C. Achievement Test Blueprint

Content area (Section)	№	Understand (L2)	Apply (L3)	Analyze (L4)	Evaluate (L5)	Points
9.1 Introduction to Probability	Q1		Q1a, b, c			3
9.2 Multiplication Rule	Q2		Q2a, b			3
9.3 Addition / Inclusion-Exclusion	Q3	Q3a	Q3b	Q3c		4
9.4 Pigeonhole Principle	Q4	Q4a, b	Q4c			3
9.5 Combinations	Q5		Q5a, b	Q5c		4
9.6 r-Combinations with Repetition	Q6		Q6a, b, c			3
9.7 Pascal's Formula & Binomial Theorem	Q7		Q7a, b	Q7c		4
9.8 Probability Axioms & Expected Value	Q8		Q8a, b		Q8c	4
9.9 Conditional Probability & Bayes	Q9		Q9a, b		Q9c	4
% Of test		13%	47%	25%	15%	100%

Appendix D. Achievement Test Expert Form

Question and learning objective	Relevance (1–4)	Clarity (1–4)	Expert comments (revisions if any)
Week 9 Content — Sections 9.1, 9.2, 9.3			
1. Q1 (L3 Apply) — Calculate basic probability using equally likely outcomes formula. [Section 9.1]	☐1 ☐2 ☐3 ☑4	☐1 ☐2 ☐3 ☑4	
2. Q2 (L3 Apply) — Apply the Multiplication Rule to count multi-step arrangements with and without repetition. [Section 9.2]	☐1 ☐2 ☑3 ☐4	☐1 ☐2 ☐3 ☑4	
3. Q3 (L2–L4) — Draw a Venn diagram and apply Inclusion-Exclusion Rule; calculate probability from union. [Section 9.3]	☐1 ☐2 ☑3 ☐4	☐1 ☐2 ☑3 ☐4	
Week 10 Content — Sections 9.4, 9.5, 9.6			
4. Q4 (L2 Understand) — State, apply, and extend the Pigeonhole Principle to guarantee conditions. [Section 9.4]	☐1 ☐2 ☐3 ☑4	☐1 ☐2 ☑3 ☐4	
5. Q5 (L3–L4) — Calculate combinations; apply complement method for conditional counting. [Section 9.5]	☐1 ☐2 ☑3 ☐4	☐1 ☐2 ☑3 ☐4	
6. Q6 (L3 Apply) — Apply r-combinations with repetition formula; compare with and without repetition. [Section 9.6]	☐1 ☐2 ☑3 ☐4	☐1 ☐2 ☐3 ☑4	
Week 11 Content — Sections 9.7, 9.8, 9.9			
7. Q7 (L3–L4) — Use Pascal's Formula step by step; expand using Binomial Theorem; interpret coefficients. [Section 9.7]	☐1 ☐2 ☐3 ☑4	☐1 ☐2 ☐3 ☑4	
8. Q8 (L3–L5) — Apply probability axioms; find complement; evaluate a claim using axioms. [Section 9.8]	☐1 ☐2 ☑3 ☐4	☐1 ☐2 ☑3 ☐4	
9. Q9 (L3–L5) — Draw probability tree; apply total probability formula; apply Bayes' theorem and interpret result. [Section 9.9]	☐1 ☐2 ☑3 ☐4	☐1 ☐2 ☐3 ☑4	

General Evaluation

1. Does the test as a whole adequately cover the Counting and Probability unit (Sections 9.1–9.9)?

☑ Yes, coverage is adequate ☐ Partially — some sections underrepresented ☐ No — major gaps

Appendix E. Adapted Attitude Scale

№	Statement	1	2	3	4	5
1	I really like Discrete Mathematics.	○	○	○	○	○
2	I am happier in a Discrete Mathematics class than in any other class.	○	○	○	○	○
3	I have a lot of self-confidence when it comes to Discrete Mathematics.	○	○	○	○	○
4	I think studying advanced Discrete Mathematics is useful.	○	○	○	○	○
5	Discrete Mathematics is one of my most dreaded subjects.	○	○	○	○	○
6	I believe studying Discrete Mathematics helps me with problem solving in other areas.	○	○	○	○	○
7	I am able to solve Discrete Mathematics problems without too much difficulty.	○	○	○	○	○
8	I like to solve new problems in Discrete Mathematics.	○	○	○	○	○
9	I am motivated to learn more about Discrete Mathematics even outside of class.	○	○	○	○	○
10	I am comfortable answering questions in Discrete Mathematics class.	○	○	○	○	○
11	Discrete Mathematics is dull and boring.	○	○	○	○	○
12	I try to do my best in discrete mathematics even when the material is difficult.	○	○	○	○	○
13	When I hear the words 'Discrete Mathematics,' I have a feeling of dislike.	○	○	○	○	○
14	Discrete Mathematics is a very interesting subject.	○	○	○	○	○
15	I am willing to take more than the required amount of Discrete Mathematics.	○	○	○	○	○

№	Statement	1	2	3	4	5
16	I expect to do fairly well in any Discrete Mathematics class I take.	○	○	○	○	○
17	I really enjoy working with Discrete Mathematics.	○	○	○	○	○
18	Studying Discrete Mathematics makes me feel nervous.	○	○	○	○	○
19	I learn Discrete Mathematics easily.	○	○	○	○	○
20	I am confident that I could learn advanced Discrete Mathematics.	○	○	○	○	○
21	It makes me nervous to even think about having to do a Discrete Mathematics problem.	○	○	○	○	○
22	Discrete Mathematics makes me feel uncomfortable.	○	○	○	○	○
23	I am always under a terrible strain in a Discrete Mathematics class.	○	○	○	○	○
24	I am always confused in my Discrete Mathematics class.	○	○	○	○	○

Appendix F. Assumption Tests for Attitude Gain Scores

Variable / Group	Cohort	W	p	Normal?	Test used
Shapiro-Wilk Normality Tests on Gain Scores					
Overall attitude — Control	2nd Year	0.950	0.417	Yes	Paired t-test
	3rd Year	0.916	0.129	Yes	Paired t-test
Overall attitude — Experimental	2nd Year	0.955	0.478	Yes	Paired t-test
	3rd Year	0.933	0.218	Yes	Paired t-test
Enjoyment — Control	2nd Year	0.949	0.412	Yes	Paired t-test
	3rd Year	0.943	0.359	Yes	Paired t-test
Enjoyment — Experimental	2nd Year	0.896	0.041	No	Wilcoxon / Mann-Whitney U
	3rd Year	0.799	0.001	No	Wilcoxon / Mann-Whitney U
Motivation — Control	2nd Year	0.932	0.211	Yes	Paired t-test
	3rd Year	0.832	0.006	No	Mann-Whitney U
Motivation — Experimental	2nd Year	0.922	0.122	Yes	Paired t-test
	3rd Year	0.904	0.068	Yes (borderline)	Paired t-test
Confidence — Control	2nd Year	0.931	0.205	Yes	Paired t-test
	3rd Year	0.954	0.521	Yes	Paired t-test

Confidence — Experimental	2nd Year	0.956	0.493	Yes	Paired t-test
	3rd Year	0.939	0.280	Yes	Paired t-test
Levene's Test for Equality of Variances (between groups)					
Overall attitude	2nd Year	0.652	0.425	Yes	Independent t-test
	3rd Year	1.359	0.252	Yes	Independent t-test
Enjoyment	2nd Year	0.096	0.759	Yes	Mann-Whitney U
	3rd Year	2.283	0.140	Yes	Mann-Whitney U
Motivation	2nd Year	0.199	0.658	Yes	Independent t-test / MWU
	3rd Year	0.197	0.660	Yes	Mann-Whitney U
Confidence	2nd Year	1.512	0.227	Yes	Independent t-test
	3rd Year	0.914	0.346	Yes	Independent t-test