# Enhancing Students' Interpersonal Skills in Secondary School Mathematics Education

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A thesis submitted to the School of Education and Humanities

in partial fulfillment of the requirements for the degree of

MASTER OF EDUCATIONAL SCIENCES

in 7M01501-Mathematics

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# ABSTRACT

This study investigates the development of students' interpersonal interaction skills in secondary school mathematics education. The relevance of the research is determined by the growing need to enhance students' communication, collaboration, and mathematical discourse skills in accordance with contemporary educational requirements. In mathematics lessons, students are expected not only to solve tasks correctly, but also to explain their reasoning, justify their ideas, listen to peers, and participate in meaningful discussion.

The purpose of the study was to identify effective pedagogical conditions for developing interpersonal interaction skills among secondary school students in mathematics education and to test their effectiveness experimentally on a sample of seventh-grade students.

The research employed theoretical and empirical methods, including analysis of pedagogical literature, classroom observation, questionnaires, pedagogical experimentation, and qualitative analysis of the obtained results. A methodological system based on dialogic teaching, collaborative learning, communicative tasks, and psychotype-based group organization was developed and implemented.

The results of the study showed that the proposed methodology had a positive impact on students' communication, cooperation, active participation, confidence in expressing ideas, and ability to use mathematical language during lessons. The scientific novelty of the study lies in the development and experimental validation of a psychotype-based model for organizing collaborative learning in mathematics lessons. The practical significance of the research is associated with the possibility of applying the proposed instructional strategies and communicative tasks in secondary school mathematics education.



*Keywords**:** interpersonal interaction, mathematics education, communication skills, collaborative learning, dialogic teaching, psychotype-based grouping*

# АҢДАТПА

Бұл зерттеу орта мектепте математиканы оқыту барысында оқушылардың тұлғааралық өзара әрекеттесу дағдыларын дамыту мәселесіне арналған. Зерттеудің өзектілігі оқушылардың коммуникативтік қабілеттерін, ынтымақтастықта жұмыс істеу дағдыларын және математикалық дискурсқа қатысу мүмкіндіктерін жетілдіру қажеттілігімен анықталады. Математика сабақтарында оқушылардан тек есептерді дұрыс шығару ғана емес, сонымен қатар өз ой-пікірлерін түсіндіру, тұжырымдарын дәлелдеу, сыныптастарының пікірлерін тыңдау және талқылауларға.

Зерттеудің мақсаты – математиканы оқыту үдерісінде мектеп оқушыларының өзара әрекеттесу дағдыларын дамытуға ықпал ететін тиімді педагогикалық шарттарды анықтау және олардың тиімділігін 7-сынып оқушылары қатысқан эксперименттік зерттеу арқылы тексеру.

Зерттеуде теориялық және эмпирикалық әдістер қолданылды, сабақтарды бақылау, сауалнама жүргізу, педагогикалық эксперимент және алынған нәтижелерді сапалық талдау бар. Диалогтік оқытуға, бірлескен оқу әрекетіне, коммуникативтік тапсырмаларға және оқушыларды психотиптері бойынша топтастыруға негізделген әдістемелік жүйе әзірленіп, тәжірибеге енгізілді.

Зерттеу нәтижелері оқушылардың қарым-қатынас жасау дағдыларына, өз ойларын сенімді түрде жеткізуіне және математика сабақтарында математикалық тілді қолдану дағдыларына оң ықпал еткенін көрсетті. Зерттеудің ғылыми жаңалығы математика сабақтарында бірлескен оқытуды ұйымдастырудың психотипке бағдарланған моделін әзірлеу және оның тиімділігін негіздеумен сипатталады. Зерттеудің практикалық маңыздылығы ұсынылған педагогикалық стратегиялар мен коммуникативтік тапсырмаларды орта мектептегі математика сабақтарында қолдану мүмкіндігімен айқындалады.

*Түйінді сөздер*: *тұлғааралық өзара әрекеттесу, математикалық білім беру, коммуникативтік дағдылар, бірлескен оқыту, диалогтік оқыту, психотип бойынша топтастыру.*

# АННОТАЦИЯ

Данное исследование посвящено развитию навыков межличностного взаимодействия учащихся в процессе обучения математике в средней школе. Актуальность исследования обусловлена возрастающей необходимостью совершенствования коммуникативных навыков учащихся, их способности к сотрудничеству. На уроках математики от учащихся требуется не только правильное решение задач, но и умение объяснять ход своих рассуждений, слушать мнение одноклассников и участвовать в содержательных обсуждениях.

Цель исследования заключалась в выявлении эффективных педагогических условий развития навыков межличностного взаимодействия в процессе обучения математике и экспериментальной проверке их эффективности учащихся 7-х классов.

В исследовании использовались теоретические и эмпирические методы, педагогическое наблюдение, анкетирование, педагогический эксперимент и качественный анализ полученных результатов. Была разработана и внедрена методическая система, совместной учебной деятельности, коммуникативных заданиях и организации групповой работы с учетом психотипов учащихся.

Результаты исследования показали, что предложенная методика оказала положительное влияние на развитие коммуникативных навыков учащихся, активному участию в учебном процессе, уверенности в выражении собственных идей на уроках. Научная исследования заключается в разработке и экспериментальном обосновании психотип-ориентированной модели организации совместного обучения на уроках математики. Практическая значимость работы определяется возможностью применения предложенных педагогических стратегий и коммуникативных заданий в практике обучения математике в школе.

*Ключевые слова**:** межличностное взаимодействие, математическое образование, коммуникативные навыки, совместное обучение, диалогическое обучение, группировка учащихся по психотипам.*

# 1.INTRODUCTION

With the globalization aspect, education systems needs are changing fundamentally. Education in the twenty first century is not only a process of information transmission, but it is a system that guarantees personal, cognitive and social development of the learner. In this light, the quality of education is not only determined by the amount of the theoretical knowledge that students have acquired, but also by whether they can apply this knowledge in practical life situations, think critically, express their ideas in a justified manner and establish effective  with other people.

The main purpose of contemporary education is to make students not passive receivers of ready knowledge, but persons who can independently construct, analyze and apply knowledge. The specific focus on the evolution of communicative competence of students is needed to reach this goal. The reason is that knowledge can be more profoundly learned when it is interacted with, ideas exchanged, and discussed.

New approach to the measurement of the quality of education is suggested by international educational studies, such as the Programme for International Student Assessment. In this context, the notion of mathematical literacy is not confined to the skill of solving problems; in its broader sense, it constitutes the ability of a student to interpret real-life situations mathematically and model them, analyze them, justify their solutions, and explain them to others. That is to say that the students are not only expected to find the correct answers, but also to explain how and why they do so.

The analysis of PISA results shows that although Kazakhstani students show some level of proficiency in computational skills, they tend to be unable to explain their reasoning, justify their solutions, and engage in mathematical discourse. This implies that their learning is more likely to be superficial and formal but not deep and conceptual. The lack of organization of  in math classrooms is one of the primary causes of this problem.

The updated content of education in the Republic of Kazakhstan is aimed at addressing this issue. One of the key goals of the curriculum is the development of communicative, social and cognitive competencies of students. Specifically, the capacity to justify and state ideas, communicate, collaborate in groups, and make collaborative decisions are delineated as the important learning outcomes. The above requirements have necessitated application of teaching methods that are founded on active student participation and interaction.

Nevertheless, these requirements are not always fully realized in terms of school practice. The way many mathematics lessons are organized continues to be traditional with the teacher explaining and the student carrying out. Under these circumstances, interaction between students is minimal and students tend to become passive receivers of knowledge. Consequently, their intellectual capabilities do not develop fully and their knowledge tends to be mechanical in nature.

This problem is specially noticeable in the instruction of seventh-grade students. During this level, students are changing their primary to lower secondary education; the curriculum becomes more complicated and abstract ideas are introduced. Meanwhile, classes based on the ability of the student will lead to mixed-ability classrooms. In those environments, certain students quickly learn new content, whereas others have challenges in comprehending new content.

In the mixed-ability classrooms, the conventional teaching methods are less effective. Assigning students the same work does not consider the differences between individual students. Students of high performance might finish the work in a short period of time and lose interest, whereas students with low performance will fail to comprehend the material thoroughly and they will become passive. This decreases the effectiveness of the whole learning process and enhances gaps among the students.

Student interaction is very important in this regard. The exchange of ideas among students, explaining concepts to each other, and working together during the lessons will allow students of different levels of ability to become actively involved in the learning process. The students with high achievement further develop their understanding by explaining the concepts to other students, whereas low-achieving students gain a better understanding of concepts through peer support.

Practice demonstrates that, despite the high number of students who can identify the correct answers to problems, they frequently have some problems with describing their processes of finding the correct answers to the problems. They find it difficult to articulate their thoughts logically, apply mathematical jargon correctly and to back up their arguments. This implies that their language of mathematics is not well developed.

Mathematical language is a vital element in mathematical knowledge. It is not only being knowledgeable of terminology but it is also the acquisition of the ability to explain concepts, build up arguments, demonstrate logical relationships and clearly communicate ideas. Mathematical language development is directly connected with . Unless students engage in expressing, discussing and explaining ideas during lessons, their language of mathematics will not grow effectively.

Students' interpersonal interaction is realized in various forms. In dialogic teaching, students ask questions, make answers and exchange their ideas. They perform group work where they collectively complete tasks and assist each other. During discussion and debate, students will defend their positions and explore different positions. They use peer explanation to explain concepts to one another and in the process enhance their understanding. All these forms stimulate the learning process and enhance the interest of students.

But practically these opportunities are not used to the full extent. Group work in most instances is structured in a formal way without actual interaction occurring. Some students are active whereas others are passive. Furthermore, the inadequate preparatory methodological armaments of the teachers in planning successful interaction also complicates the problem.

Some of the practical challenges are lack of instructional time, large classes, dense curriculum, as well as a tendency towards traditional instructional practices.

Teachers tend to be preoccupied with getting through the curriculum and cannot afford to spend enough time with the children. Moreover, the fear of making mistakes and the unwillingness of the students to develop their ideas is another obstacle of the communicative development.

Consequently, there is a clear paradox between theory and practice: even though research has proven the significance of interaction in the learning process, it is not implemented sufficiently in the school practice. This contradiction adds to the topicality of the research problem.

The works of international and national scholars have discussed this issue. Anna Sfard represents mathematical thinking as a process that is created by means of , whereas Neil Mercer underlines the fact that cooperative talk is one of the most important factors that contribute to the development of thinking in students. Jo Boaler has shown how the interaction between the students has a great impact on the academic success and the motivation towards learning.

National research has also tackled the issue. Abilqasymova A.E. discussed didactic principles of learning mathematics and emphasized the role of interaction with a student during a mathematics lesson. Nurmukhambetova Zh.S. explored the methods of developing the communicative competence in mathematics lessons.

However, there are still gaps in the development of systematic methodological approaches to developing interaction skills among the students in Grade 7 of general secondary schools. That is why such a problem is nowadays topical as far as the theory is concerned and also in practice.

Based on this, the propsedo research is based on Grade 7 classes of general secondary schools in Kazakhstan and experimentally investigates the effectiveness of methods aimed at developing the interaction skills of students enrolled in general secondary schools in Kazakhstan.

The theme of this study is: “Improving the Interpersonal Skills of Students in Teaching of Mathematics in Secondary School.

It can also be thoroughly discussed in terms of national pedagogical science. Research into the theoretical basis of mathematics teaching methodology has been widely conducted on how the learning process and the development of cognitive activity in the students can be effectively organized. Specifically, the works of Abilqasymova A.E. allow a profound analysis of the didactical principles of teaching mathematics and offer a scientific explanation of the content and organization of the teaching process.

The scholar stresses that teaching mathematics cannot be restricted to the knowledge transfer but should focus more on the development of thinking processes in students. In this regard, the quality of mathematical education is not defined by its passive, ready-made information reception, but by the conscious awareness of the students and their ability to use knowledge on their own. Student interaction is very important in this process. Explaining, justifying and sharing ideas enable students to gain a more profound comprehension of mathematical concepts. Therefore, knowledge is not passed on in a complete form and it is made by interacting with the environment and engaging in cooperative activities.

Moreover, the significance of teaching mathematical language is given special prominence by Abilqasymova A.E. The author says that mathematical language is not just a collection of terms; it is a very basic way of stating ideas in a precise, compact and logical manner. To be able to master a mathematical concept, a student must be not only able to understand a concept but also be able to represent it and justify it, to analyze the perspectives of other people. These skills are developed by having students interact.

The scholar also points the significance of the organization of active cognitive activity of students in the learning process. In the active engagement, students posed questions, required answers, hypothesized, and justified their hypotheses. These activities are all based on . Unless such activities are systematizing during lessons, the knowledge of the students will only be superficial and not deep.

Moreover, the current school practice is characterised by the fact that the organization of interpersonal interaction of the students is characterised by a number of objective and subjective challenges. To begin with, the predominant role of the teacher is rather the basis of the teaching process, which does not allow the active participation of students. Under these circumstances, students are kept as passive consumers of information and fail to acquire skills to express their ideas freely or be able to engage in meaningful dialogue.

Second, the high number of students in a class and lack of time to actively engage all learners throughout lessons are some of the factors that make lesson activities difficult to realize. The teachers have in most cases been busy with the curriculum and have no time to devote towards student interaction development. Subsequently,  is likely to assume a formal and not a meaningful nature.

Third, students' psychological characteristics also play an important role. A great number of students fear not to make any mistakes, they are shy to discuss their opinions and they have no confidence in defending their opinions. This lowers their degree of engagement in classrooms and they are unable to fully engage in the process.

Fourthly, the methodological readiness of the teacher in the organization of interaction is not always adequate. Group work or dialogic teaching can be employed, but they are usually carried out in a formal and not a deliberate manner. As a result, rather than an actual discussion, students are only preoccupied with doing assignments.

These issues indicate the intricacy of cultivating the interaction of students with each other in mathematics classes. Thus, one of the significant tasks of the contemporary system of education is the proposal of effective methodological solutions in the given area.

The purpose of this research is to determine effective pedagogical conditions and teaching strategies that aim to develop interpersonal skills of Grade 7 mathematics students in the secondary schools, to systematically apply them in the process of education, and to scientifically prove their effectiveness using the experimental research.

-to fully examine the psychological, pedagogical and methodological underpinnings of how to develop interpersonal communication skills in students;

-to determine the content and specific features of organizing student interaction in mathematics lessons;

- to choose and apply diagnostic techniques to identify the baseline level of communication skills of Grade 7 students;

- to design a system of effective teaching strategies to help students improve their interaction skills (dialogic teaching, group work, peer explanation and discussion);

- to apply the suggested methodological system to the process of education and to experiment with its effectiveness with the help of pedagogical experimentation;

- to interpret and compare the experimental results with qualitative and quantitative methods and make scientific conclusions;

- to make methodological recommendations of the research findings.

The teaching mathematics in general secondary schools of the Republic of Kazakhstan.

The direction of work on the development of interpersonal communication skills of students in mathematics lessons and the methodological aspects of the organization of this process.

Provided that the organization of interpersonal interaction of students in mathematics lessons is planned based on the specially designed pedagogical conditions, and methods, such as dialogic teaching, group work, explaining to peers, and discussing are applied systematically, then the level of mathematical knowledge acquisition by students, the development of the level of logical thinking among them, and the formation of their communicative abilities will increase. The reason is that it is during the process of interaction that knowledge acquisition is done in an even deeper and more conscious manner.

The following set of methods was used in the study:

– study of scientific, pedagogical, psychological, and methodological literature;

- observation (structured observation of lessons);

– questionnaires and surveys (to identify the level of communication of students);

- testing (to determine the level of knowledge in the students);

- interviews and discussions (to determine the views of students and teachers);

- pedagogical experiment (to determine the effectiveness of the proposed methodology);

- statistical processing and comparative analysis of the results obtained.

Scientific Novelty of the Research.

- a system of methodological processes to cultivate the interpersonal communication skills of students in mathematics lessons has been scientifically proven;

- the efficiency of instruction based on interaction in mixed-ability Grade 7 classes has been demonstrated as a result of experimental studies;

- the connection between the growth of mathematical language and the communication with the students have been made clear;

- new methods of organizing active participation of students in mathematics lessons has been offered.

Applied Significance of the Study.

It is recommended that the results of the study should be used in mathematics lessons in general secondary schools. The suggested methodological procedures can assist teachers to design the student interaction efficiently, enhance classroom interaction, and improve the quality of the education. Secondly, the findings of the research could be utilized in the creation of teaching resources, teacher training programs, and teacher education systems. The activities and procedures that were created in the course of the research could be directly applied to the field of practical teaching.

# 2.LITERATURE REVIEW

## 2.1 Theory Of Communication

The socio-cultural theory of the socio-cultural aspect of the concept of cooperation in mathematics education holds a central position in explaining the theoretical basis of interaction between students in mathematics education. The social interaction is the main process of cognitive development, which is discussed in his work Thought and Language. The author holds that the process of learning initially takes place at the social level and it is only later that it becomes internalized, as an individual psychological process (Vygotsky, 1978, 1986). This concept turns the very essence of learning: the knowledge is not considered as something that is obtained by an individual, but as a process that is created through the interaction of individuals.

The scientific explanation of the effectiveness of collaborative learning can be explained by the concept of the zone of proximal development introduced by Lev Vygotsky. In this theory, it is indicated that there is a zone between those tasks that a learner can perform by himself or herself, and those tasks that can be achieved with the help of other people. The place of learning is best in this zone. So, using the communication, explanation and the collective activity, students are able to reach results that have been out of their capability at this point. This interaction has been a vital requirement of acquiring knowledge (Vygotsky, 1978).

Moreover, the scholar focuses on the strong interconnection of language and thought. To him thinking is constituted and created by language and language is not just a mere outward manifestation of thought but its very means. In this respect, the cognitive development of students directly depends on the skill to formulate, defend, and clarify their ideas. This concept is especially important in mathematics teaching, because the ability to master mathematical concepts is closely related to the ability to express the concept of mathematics verbally, through expression and justification (Vygotsky, 1986).

The theory of Vygotsky shows that interaction between students is not a peripheral component of the learning process but rather a major component of the learning process. It is not merely the sharing of information, but it is through collaborative discussion, through peer explanation and through exchange of ideas that students do not simply share information, but they reconstruct their knowledge, gain deeper understanding and learn to apply the information consciously. This highlights the need to plan the learning process according to dialogic approach to learning and collaborative approach to learning (Mercer, 2000; Mercer & Littleton, 2007).

Anna Sfard is one of the authors of one of the main works that develops the communicative perspective in mathematics education. Thinking In her book Thinking as Communicating, thinking is not regarded as an individual process of cognition but a particular kind of communication. Presentation of the notion that thinking is equal to communicating, the author tightly connects with the knowledge acquisition process and the interaction that exists between people.

The discursive theory of Sfard is a specialized system of communication, which is founded on certain rules, terminology and symbols. Mathematics is not learned by the students in isolation; they learn mathematics through conversation, discussion and collaborative activity. In this sense, mathematical knowledge is considered a social phenomenon created with the help of the linguistic means.

The author recognizes the key elements of mathematical discourse: special terminology, symbolic representation, rules, and methods of their implementation. Learning to control these aspects, students obtain the language used by the mathematical community. Thus, the level of fluency of mathematical language in a student is a measure of the extent of his/her mathematical knowledge (Sfard, 2008).

In the works of Sfard, student interaction is considered as one of the major knowledge construction mechanisms. By sharing ideas, reasoning out, explaining ideas, and discussing ideas collaboratively, students gain a deeper insight into mathematical concepts. This process forms and orders their thought, increases its logical consistency, and encourages high-order thinking. Furthermore, the interaction enables students to re-evaluate their knowledge and fix the errors.

The author highlights the need to change classroom discourse in the learning process. As long as students are engaged in speaking, sharing their ideas and discussing them during the lessons, their mathematical thinking will evolve as well. On the other hand, in one-sided instructional setting, where teacher explanation is the only teaching method, the discursive development of students is not given the opportunity to develop, which is unfavorable to the depth of the knowledge of these students (Mercer, 2000).

So, the theory of Anna Sfard has a scientific basis to see the interaction of students in mathematics classes as a necessary condition to construct knowledge. This point of view indicates that the teaching process should be based on communication and that the methods to develop mathematical discourse in students should be emphasized.

Overall, the works of Lev Vygotsky and Anna Sfard indicate that the learning process is social and communicative in nature, and that student interaction is one of the mechanisms through which the learning process occurs. These theories are the scientific background of the organization of student interaction during mathematics classes, and the theoretical framework of the given study.

The learning process in the modern pedagogical studies is not considered as an individual cognitive process, but as a process of collective thought. In this light, communication is not only a tool of sharing information but also a basic mechanism where students can think collectively, share ideas, and co-construct knowledge.

One of the key contributions in this field is presented in the works of Neil Mercer. The scholar justifies student talk using the notion of interthinking. In his opinion, the dialogue of students is not only the external manifestation of thought but also a way in the course of which thought is an object of realization. That is, in the process of interaction, students do not just share information they are building, developing and re-developing their thinking.

Mercer identifies various forms of classroom talk: cumulative talk, disputational talk and exploratory talk. Of these the most successful is regarded as exploratory talk, which involves the students justifying their ideas, analysing the justifications of the other students, and working at arriving at a common ground. This type of interaction stimulates the growth of logical thinking, the ability to argue, and critical thinking.

This method is especially relevant in mathematics classes, where mathematical knowledge is built up through justification, explanation, and discussion. When students are confined to perceiving ready-made algorithms, their minds are left to shallow thinking. But, by means of collective discussion, they discuss various solution plans and learn to explain their arguments.

Merrilyn Goos is one of the scholars who have extensively studied the issue of collaborative thinking. In her research study, she has demonstrated that, metacognitive and communicative activities of students are closely interconnected during collaborative problem-solving. That is, students do not only address problems but also follow their thinking patterns, analyze them and align them with those of others.

In the works of Goos, it is proved that during the process of collaborative work, a system of shared regulation is developed. This process is achieved by the students posing questions, explaining, correcting and guiding each other. Consequently, the learning process turns out not only an individual process but a group thinking process.

In addition, group thinking has a positive impact on the learning motivation of students. In working in groups, the students will feel that their opinion is important and they will be more engaged. Not only do they think and explain to the teacher, but also to one another thus improving the quality of learning.

Nevertheless, this collaborative manner of thinking does not manifest itself independently. It involves a carefully structured communicative educational setting within the classroom. The teacher needs to not only guide the students to give answers but also necessitate the process of exchanging ideas, justification of reasoning and discussion. It is then that communication can be made a means to joint thinking.

Therefore, it is shown in the works of Neil Mercer and Merrilyn Goos that interaction between students is not just a form of communication but the basis of collaborative thinking. These theories emphasize the role of structuring the student interaction during the mathematics lessons and also give a background on how to identify effective pedagogical conditions that would support cognitive development of students.

In the teaching process of mathematics, the interpersonal interaction of the students is applied in the form of different organizational forms. The forms vary with the content, goals of instruction and the cognitive level of students. The primary forms of interaction that are considered in modern pedagogical research are dialogic teaching, group work, discussion, and peer explanation. All these are intended to make sure that the students are engaged and that their thinking capacities are developed.

Dialogic teaching can be considered one of the most important types of interaction with students. Learning in this method is not only performed through question and answer, but also through discussing, explaining and exchanging of ideas. Dialogic teaching, Neil Mercer believes, is a powerful instrument in trying to organize the collaborative thinking of the students. During dialogue, students express their opinions, listen to others, and reconsider their own viewpoints. This kind of interaction improves their degree of comprehension and allows them to gain knowledge in a much more conscious way.

Group work is a significant aspect of helping students to develop their interaction skills. In this approach, students will collaborate in small groups, complete assignments together, and explain to each other. Studies have revealed that group work not only enhances the sense of responsibility among students, but also provides them with the opportunity to learn together. It works best in the mixed-ability classroom where the high-achieving students can develop their understanding of the material by helping the lower-achieving students.

Discussion (debate) is a type of interaction which enables students to justify their ideas and become critical thinkers. In the discussions, students justify their positions, critically examine other points of view, and argue. This process helps to form logical reasoning, ability to argue, and speech as an academic. Discussion is particularly effective in mathematics lessons discussing various strategies of solution to a problem.

Another form of interaction that is significant is peer explanation. Here a student is increasing his/her own knowledge by describing the topic to another student. Studies show that one of the best methods of learning is to explain to others the concepts that you have learned. Explanation helps students to organize their thoughts, to make clearer the main ideas, and to develop logical relationships.

In addition, these types of interaction are strictly interrelated, and their combination in the learning process is more effective. As an illustration, a dialogic interaction is a natural consequence of group work and features of discussion boost collaborative activity. This will make the learning more active, meaningful, and productive.

Nevertheless, to be effective these types of interaction should be organized accordingly. When group work is carried out in a formal way, when the discussions are not structured, when the dialogue is limited to the simple question-answer exchange, the desired outcomes will not be attained. Thus, the task of the teacher is not just that of an observer, but rather of a facilitator and organizer of communication.

Therefore, dialogic teaching, group work, discussion, and peer explanation are major tools that can be used to develop the inter-personal interaction of students. These strategies enhance the cognitive process of students and help them to develop their thinking, reasoning, and communication skills in mathematics. Thus the systematic application of these types of interaction in mathematics lessons is a vital precondition to enhancing the effectiveness of teaching.

## 2.2 Communication In Mathematics

### 2.2.1 The Concept Of Mathematical Discourse

Mathematical discourse is one of the most important concepts to understand the communication in mathematics lessons. This notion is the collection of activities that comprise speaking, explaining, reasoning, and exchanging ideas as part of the learning process. It is not just a way to transmit knowledge; mathematical discourse is regarded as a key that influences the way students think and that has an impact on the level of their knowledge (Sfard, 2008).

Deborah Ball is one of the researchers who have done a lot of research on mathematical discourse. In her work, classroom communication is referred to as a prominent marker of effective teaching. The author believes that mathematical knowledge is built by the active involvement of students, explanation of their ideas, and discussion. In such a way, the learning process needs to be premised on the explanations of the teacher, as well as the interaction of students (Ball, 1993).

Ball conceptualizes discourse in mathematics as a process of explaining, justifying and reasoning about concepts. In this sense, the interaction of students has a direct impact on the formation of mathematical thinking in students. Through questioning, answering, elaborating, and communicating with one another, students gain a better grasp of knowledge. This kind of interaction helps to promote logical thinking and helps students internalize knowledge in a more conscious manner.

The scholar also presents the notion of the teacher talk and student talk and examines their functions in the classroom. In conventional lessons, teacher talking is more overpowering but in effective learning processes, student talking should be more dominant. The reason is that only through active participation in communication, the creation of his or her ideas, reconsideration, and analysis of the views of others, students learn to develop their thinking (Ball, 1993; NCTM, 2014).

The level of mathematical conversation directly influences the results of learning. When students can give only short answers and repeat ready-made algorithms, the depth of their understanding is superficial. Conversely, when students are able to explain their solutions, justify their reasoning and engage in discussions, then their learning becomes more profound and meaningful.

Moreover, mathematical discourse has a significant role in the development of mathematical language of students. Learning to use special terminology, symbols and definitions, students learn to create clear and precise sentences to express their ideas. This does not only increase their problem solving skills but also builds their level of justifying and defending their knowledge.

Accordingly, mathematical discourse is the main way of interacting with students during mathematics classes. It guarantees active engagement, helps to develop thinking, promotes profound understanding, and helps develop mathematical language. Thus, the successful organization of mathematical discourse is one of the main requirements in enhancing the quality of mathematics education (Ball, 1993; Sfard, 2008).

### 2.2.2 Mathematical Understanding And Explanation

When it comes to teaching mathematics, one of the most important pointers of how well students have comprehended knowledge is the level of their mathematical understanding. In modern pedagogical studies, understanding is determined not only by getting the right answers, but also a student can establish associations between concepts, as well as explain them. In this sense, the process of explanation is viewed as a powerful instrument of obtaining mathematical knowledge.

Douglas Hiebert is one of the scholars who have extensively studied mathematical understanding. Understanding, in his work is defined as the capacity of the student to create logical associations between components of knowledge. That is, students are not to memorize the rules but to understand their meaning and to practise them in different situations. This degree of comprehension can be achieved with active participation and interaction.

Hiebert lays its specific accent on the place of the explanation. In his opinion, by explaining the concept to the others, the student rearranges his/her knowledge system: he clarifies the main ideas, identifies the relations between concepts and systematizes his/her understanding. This process enhances the thinking process of students and makes them able to gain knowledge in a conscious manner. Thus, explanation should not be understood as a means of transfer of information but as a process of recreation of knowledge (Hiebert et al., 1997).

Moreover, the author indicates the shortcomings of the conventional model of teaching as the traditional correct answer model. When all that the students are expected to do is to provide answers to problems, the level of their understanding is superficial. Their thinking however becomes more profound when they are expected to elaborate on the processes of their solutions, to justify the reasons why they have arrived at certain decisions and to compare various methods. This illustrates that there is a necessity to change the paradigm of instruction to be an answer-giving based model as opposed to being explanatory based learning.

The character of learning activities will also have a direct effect on the formation of mathematical knowledge. Algorithms tasks as illustrated in the works of Hiebert are more likely to restrict student interaction and fail to encourage deep learning. Open-ended questions and multiple answer strategies, in contrast, get students to support their reasoning and discuss and share ideas. When this happens, interaction is automatically created, and forms a part of the learning process.

Students' peer explanations have a positive impact on their academic achievement. Offering explanations to others, students consider not only their own knowledge but also the extent to which their peers understand the information, which makes their thinking more flexible and thoughtful. Besides, by jointly explaining, the students can correct their errors and recognize them.

Here, the teacher is also involved in playing an important role. The teacher must not only teach the students to give answers, but also to elaborate their ideas, justify them and give reasoning. Through questioning, the teacher not only arouses the thought process of students, but also structures the interaction process among students. With this kind of learning environment, students will be active workers and not passive listeners (Webb, 2009; Hattie, 2012).

In this way, the work of Douglas Hiebert proves that mathematical knowledge is constructed with the help of interaction between students. Explanation, justification, and discussion are some of the activities that can help students to gain deeper insights, think logically, and apply their knowledge in a conscious manner. Thus, the implementation of the explanation-based instructional methods in mathematics classes is a key requirement to enhance the quality of education (Hiebert et al., 1997; Webb, 2009).

### 2.2.3 Socio mathematical Norms And Argumentation

The socio mathematical norms that are created in the classroom are one of the most important factors that determine the quality of communication and its content in mathematics classes. This notion was proposed in writings of Erna Yackel and Paul Cobb and is thought to be a valuable theoretical framework to explain the emergence of mathematical thinking and communication (Yackel & Cobb, 1996).

These authors argue that interaction between students will not be random, depending solely on teaching methods; instead, it will be controlled by a system of norms developed in the classroom. These conventions define what is deemed to be a proper response, what is considered to be sufficient explanation and how justification is to be done. Communication is therefore not only a source of communication but also an aspect that determines the level of knowledge.

The socio mathematical norms are not introduced in the form ready-made: they are formed gradually when students interact. Students debate on various strategies of solutions and explain why their choice is correct and finally they come to the common ground. The result of this process is the creation of a certain mathematical culture in the classroom where reasoning, justification, and logical thinking take center stage.

Within this framework, argumentation (justification) is of particular importance. It is not until students learn how to justify, support and defend their solutions that their mathematical thinking is taken to a higher level. It is no longer merely a matter of getting the right answer but knowing how and why that answer is right. That is, the teaching model shifts to a right/wrong model, to a why is it correct model. The change encourages the growth of critical and logical thinking in the students.

The relation of socio mathematical norms to the development of student autonomy is also quite close. Once justification, discussion, and sharing of ideas become the order of the day in the classroom, students are bound to start thinking and taking responsibility over their solutions. In this kind of environment, autonomy is not imposed on one by the outside world but is acquired by the inner one.

It is important that the teacher plays a key role in this process. The teacher is not only a deliverer of ready-made rules but an initiator who forms the culture of the classroom interaction. The teacher invites to discuss the answers of the students, demands justification and emphasizes the connections between various concepts. By asking questions and giving feedback, the teacher encourages the students to think more deeply and makes the process of interaction deeper and more meaningful.

Studies have established that students in a classroom where the socio mathematical norms have become entrenched show greater levels of thinking. In this kind of environment students not only get to solve problems but also justify their answers, analyze alternative perspectives and learn to support their solutions. Conversely, with the lack of such norms, student interaction is formal and their degree of comprehension is usually shallow (Cobb, Wood, Yackel, & McNeal, 1992).

Therefore, the works of Erna Yackel and Paul Cobb prove that communication within mathematics lessons is not an activity but a complex socio-cognitive process, based on a system of norms. With successful established norms, interaction facilitates in developing mathematical thinking, building of argumentation skills and profound understanding. Thus, the establishment of socio mathematical norms during the mathematics lessons is a necessary condition to enhance effectiveness of teaching (Yackel & Cobb, 1996).

## 2.3 The Role Of Communication In Mathematics Lessons

The interaction of students during mathematics lessons heavily depends on the extent to which it is well structured. The works by Fernando Viseu are of especial importance in this respect. His works regard the communication in mathematics classes as an essential part of the teaching process and give a detailed analysis of the role of a teacher in the organization of this process (Viseu & Oliveira, 2012).

Viseu argues that communication in mathematics lessons does not come out spontaneously; it is the product of a carefully designed pedagogical action. Without creating the conditions under which students can share ideas, communication is not as extensive, and the learning process is one-sided. Thus, providing the environment that will guarantee the active involvement of the students is one of the main tasks of the teacher.

The significance of communicative role of the teacher is highlighted in the research. The teacher does not just perform the role of a transmitter of information but actually carries out the role of an organizer, facilitator and regulator of conversation among students. The teacher promotes the discussion of the answers of students, allows the comparison, justification and elaboration of the ideas. These practices will encourage students to interact and think, as well.

Also, Fernando Viseu underlines the significance of questioning in the learning process. The questions asked by the teacher need to stimulate the students to think, analyze and justify their opinions. Open-ended questions enable students to bring up variety of thoughts, whereas guiding questions assist students in narrowing their thoughts. By doing so, questioning will become a resource to improve the quality of communication (Viseu & Oliveira, 2012; Alexander, 2018).

Maths lessons that use effective communication are not only more likely to make students more engaged but also positively affect the motivation to learn.

Having the opportunity to share their ideas freely, students are more engaged in lessons and develop confidence to apply their knowledge. This, however, makes them more interested in the learning process (Boaler, 2016).

Furthermore, it is proved that teachers not only should assess the reactions of students but also should elaborate them further. It is not enough to label answers as being correct or incorrect but what is necessary is to discuss, elaborate and justify those answers. This method further enhances student engagement and aids in the systematic process of knowledge building (Mercer & Littleton, 2007).

In order to plan efficient communication during classes in mathematics lessons, a teacher has to establish a trusting classroom atmosphere. Authentic interaction can only take place when students feel free to make mistakes and to share their ideas freely. This, in turn, enhances students' cognitive engagement and promotes the development of their mathematical thinking.

In this way, the works of Fernando Viseu prove that communication is one of the main factors contributing to the enhancement of the effectiveness of mathematics teaching. When teacher practices are well organized, they promote active interaction of students, facilitate in-depth understanding and allow students to justify their ideas. Thus, the deliberate arrangement of communication in mathematics classes is one of the key requirements that can enhance the quality of education (Viseu & Oliveira, 2012; Alexander, 2018).

### 2.3.1 Dialogic Teaching

Dialogic teaching is one of the most effective ways of developing interpersonal interaction of students in mathematics classes (Alexander, 2018). The method is premised on the active involvement of students, sharing of ideas, and discussion among students. In dialogic instruction, knowledge is not given out in a prepared form; instead, it is created in the process of interaction between the students.

Theoretical basis of dialogic teaching is widely addressed in the writings by Jo Boaler. The author also highlights the fact that the type of conversation among students in mathematics classrooms has a direct effect on the form of thinking of students, their level of understanding, and their academic performance (Boaler, 2016). The research conducted by Boaler proves that an open-ended task which involves discussing the topic results in students gaining a deeper understanding of mathematics and developing an interest in the subject.

One of the key elements of dialogic teaching is “math talk.” It will allow the learners to present their thoughts in a written form, explain their arguments, and discuss the opinions of other students. This kind of interaction can help in not only developing problem-solving skills, but also developing logical thinking and communicative competence.

Kazemi E., is another scholar that has researched dialogic teaching. Her findings indicate that the sharing of ideas by students contributes greatly to the conceptual knowledge of students. The author adds that the students can only reach higher levels of learning when they do not just provide answers, but they actively engage in explaining and justifying their answers. This underscores the significance of dialogic teaching in the learning process.

The role of the teacher is also modified in dialogic teaching. The teacher no longer is simply some transmitter of information but a facilitator and organizer of an interaction between students. Questioning the students also helps the teacher stimulate the thinking of the students, encourage them to compare various points of view, and lead them to justify their ideas. This kind of practice encourages active engagement and substantial interaction among the students.

Moreover, dialogic teaching has a positive impact on students’ learning motivation (Mercer, 2000; Boaler, 2016). Being able to freely express their ideas, students become more involved in the lesson and gain confidence in their ability to use their knowledge in practice. This makes them more involved and enhances the standards of learning.

Nonetheless, dialogic teaching implementation involves some pedagogical prerequisites. First, open-ended questions must be employed because they promote students to think and share their opinion (Boaler, 2016). Secondly, one should establish a classroom atmosphere of trust, which will enable students to express their ideas without fear. Third, it is not enough that teachers assess the responses of students but in addition, direct discussion and further elaboration of these responses.

Therefore, dialogic teaching is one of the main techniques of developing interpersonal interaction of students in mathematics classes. It improves thinking, understanding and communicative abilities of students and allows them to learn more deeply. Thus, one of the primary directions of enhancing the performance of mathematics teaching is the systematic use of dialogic teaching.

### 2.3.2 Group Work And Collaborative Learning

Important approaches to the development of interpersonal interaction in mathematics lessons in students include group work and collaborative learning. Those strategies allow students to be actively involved in the process of interaction with each other, exchange ideas, and co-construct knowledge. In group work, students are not only doing tasks but also trying to justify their reasoning, analyzing the views of others and striving to find a common solution.

Noreen Webb is one of the scholars that have been significantly involved in studying the significance of interaction in group learning (Webb, 2009). Through her research, it has been shown that the level of understanding is directly influenced by the explanations made by students to their peers. Webb believes that students can learn by explaining concepts to others and in the process, they organize their ideas, identify logical connections and develop their own understanding.

The notion of peer interaction is a key point in the studies by Webb (Webb, 2009). This idea emphasizes on the effectiveness of learning by the interaction of the students with one another. Specifically, the active role of questioning, explaining, and feedbacking particular topics can greatly enhance cognitive activity of the students and positively influence the learning process.

Gillies R. M. is another researcher who has conducted research on the effectiveness of group work. In her work, it is evident that cooperative learning has a positive impact not only on the academic performance of students but also on their social and communicative abilities (Gillies, 2007). Gillies stated that group work, which is well organized, promotes mutual support, responsibility, and collaboration among the students.

Peer learning is one of the main aspects of group work. This method works particularly well with mixed-ability classrooms where students performing better gain chances to learn even better. This makes all students be actively engaged in the learning process.

In addition, group work is involved in not only mastering the content but also developing the communication skills of students. Students get to know how to express themselves, listen to others, come to an agreement and make a joint decision. These are also skills that influence positively their personal development.

Nevertheless, group work requires some conditions to be fulfilled in order to be effective. To begin with, the tasks must involve true student to student cooperation, otherwise group work will only be formal. Secondly, one should make sure that active involvement of all group members can be achieved. Third, the teacher should monitor students’ interaction and provide guidance when necessary.

Therefore, group work and collaborative learning are viable approaches in enhancing the interpersonal interaction of students in mathematics lessons (Johnson & Johnson, 2009). These strategies not only maximize the cognitive involvement of students, but also facilitate a greater level of understanding and promote the emergence of communicative competence. Hence their systematic introduction into the educational process is a significant orientation toward increasing the effectiveness of teaching.

### 2.3.3 Social Interdependence Theory

The social interdependence theory is one of the major theoretical bases of having students develop their interpersonal interaction. It is a theory which has been widely developed in the works of Johnson D. W. and Johnson R. T. and which can be considered as a basic scientific premise in the process of explaining the effectiveness of group learning.

The social interdependence theory states that the success of group members is highly interdependence. This means that the success of one student will be based on the actions of other students and the overall performance of the entire group will be determined by the performance of each member. This underscores the essence of collaboration, cooperation and mutual support and responsibility in the learning process.

The authors determine the key aspects of this theory. The former is positive interdependence, which implies that students have to collaborate to reach a shared objective (Johnson & Johnson, 2009). Here, every student would play his part in ensuring the success of the group and would realize the significance of his role. The second aspect is personal responsibility where each student has a responsibility of his or her contribution. The third one is face to face interaction, which involves face to face communication between students (Johnson, Johnson, & Holubec, 2013).

Moreover, the theory underlines how social and collaborative skills are important. The students should be able to learn how to communicate successfully, listen to each other, share ideas and come to a conclusion. These are the skills that can improve the quality of group work and will enable student interaction to be more effective.

The theory of social interdependence proves that group work is not just an organizational form but a complex pedagogical system and is based on a certain structure. Once group work can be organized, based on the principles of this theory, one can achieve the establishment of the genuine interaction between the students and, as a result, the improvement of the learning outcomes.

This theory is especially effective when it is applied in mathematics lessons. When engaging in collaborative problem solving, students justify their reasoning, support each other and strive towards a shared solution. This will improve their degree of mathematical understanding and will train their logical thinking.

Moreover, this theory has a positive impact on students’ learning motivation (Johnson et al., 2013). In a group of students, it is important to them to realize the importance of their contributions in a group and this increases their engagement. They not only put responsibility on their own performance but also the performance of the group and this way they change their attitude towards learning.

Nevertheless, to provide successful work in groups, teachers should take into account the principles of social interdependence. Unless the roles are clearly defined and individual responsibility is ensured, group work can become formal and not yield the desired results.

Therefore, the articles of Johnson D. W. and Johnson R. T. can be used as a scientific basis of the application of group work in the development of interpersonal interaction of students. The necessity of cooperation-based learning and identification of the main pedagogical conditions which contribute to the effective acquisition of knowledge through collaborative activity are substantiated by social interdependence theory.

### 2.3.4 Inquiry-Based Learning

Inquiry-based learning is one of the effective methods to develop interpersonal interaction of students in mathematics lessons (Hunter, 2017). The basis of this approach lies not in what is being transmitted to students but instead to the independent exploration, questioning, hypothesis formation, and justification by students. In inquiry-based learning, students do not only play the role of learners, but also researchers.

The characteristics of the inquiry-based learning are thoroughly discussed in the research works of Ruth Hunter. The author establishes a close relationship between interaction of the students in mathematics classrooms and their investigative activity (Hunter, 2017). It is through the interactive discussion of issues, discussing various solutions to the problems and justifying their ideas that students will have a deeper understanding of mathematical knowledge.

A key aspect of this approach is questioning. Rather than providing ready-made answers, the teacher poses questions that stimulate students' thinking. These types of questions promote the process of discussing, sharing ideas, and reasoning together (Alexander, 2018; Hunter, 2017). Consequently, the learning process turns to be dialogic in character and the interaction of students is actively implemented.

In inquiry-based learning, students offer suggestions, challenge them, compare alternative strategies and explain their rationale. This is the process that leads to development of critical thinking, logical thinking and argumentation skills. Meanwhile, students learn to listen to the opinion of other people, to evaluate it, and to reconsider their own opinion. The activities also increase their communicative competence.

Student interaction will occur naturally in an inquiry based environment. Students answer questions to each other, explain, and discuss. This triggers the learning process and makes it an active process (Hunter, 2017). In addition, this strategy enhances the motivation of students and makes them more interested in the course.

The role of the teacher also becomes different in this respect. The teacher is no more a knowledge transmitter but it serves as a facilitator and organizer of the inquiry process in the students. The teacher gives the guidance, helps students think, and facilitates effective communication. This puts the learning process in a student-centered approach.

But there are some conditions to the successful implementation of inquiry-based learning. To begin with, tasks must be in the nature of investigations, where the students are encouraged to think and consider various strategies of solutions. Second, there is the need to develop an open and secure classroom atmosphere whereby students are free to express their thoughts. Third, the teacher is expected to facilitate the discussions of the students and to help them develop their ideas.

Therefore, inquiry-based learning is a promising approach towards the development of interpersonal interaction of students in mathematics classes (Hunter, 2017). It increases the participation of the students, facilitates the process of learning independently, encourages students to think logically and be able to justify the ideas they have. Moreover, it reinforces the joint activity and transforms the communication into the inseparable part of the learning process.

Generally, group work, dialogic teaching, social interdependence and inquiry-based learning are interconnected processes. They can be successfully used together to help develop students in terms of the interaction with each other during mathematics lessons. These strategies result into an active, meaningful and productive process of learning and facilitate the cognitive and communicative growth of students.

## 2.4 Students' Interaction In National Research

### 2.4.1 Didactic Foundations Of Teaching Mathematics

The didactic preconditions of the mathematics education are thoroughly investigated in the national pedagogical science, and the works devoted to the scientific substantiation of the content and structure of the teaching process (Abilqasymova, 2013). One of the fundamental contributions in this field is reflected in the works of Abilqasymova A.E.. The scholar sees mathematics education as not only the formation of knowledge and skills but as a complicated pedagogical process, whose goal is to develop the thinking of students.

The author writes that it is not enough to have students passively receive ready-made information; students have to consciously understand knowledge and be able to apply it in different situations. This involves organizing active cognitive interaction in the process of learning. In the process of such active involvement, the students pose questions, seek answers, develop hypotheses, and attempt to justify their answers. Communication is used to conduct all these activities.

Abilqasymova A.E. underlines that the level of mathematical knowledge depends on the level of students understanding the material and their ability to independently apply the material. Student interaction is very important in this respect. By explaining, justifying and sharing ideas, students get a better insight into mathematical concepts. Therefore, knowledge is not imparted in a complete form but rather built upon through the process of interaction and collaborative action.

The scholar further points out the need to teach mathematical language. Mathematical language in her work is not considered as a set of terms, but as a means of conveying ideas in an accurate, concise and logical manner (Abilqasymova et al., 2018). To master a mathematical concept, a student must not only learn the concept but also be able to explain the concept, defend it and analyze the arguments of others. Learning these skills is acquired in the process of student interaction.

Moreover, the author suggests methodological strategies to enhance cognitive activity of students in the learning process. They involve, creating problem situations, question and answer strategies, organizing discussions and assigning tasks on the basis of collaborative activity. These techniques trigger the interaction of the students and help to form their thinking.

The instructive principles of mathematics teaching also indicate the necessity to balance personal and cooperative exercises (Abilqasymova et al., 2018). When the teaching process is founded on personal work, communication skills of students are not well-developed. Conversely, a teaching methodology founded on collaborative activity would lead to a discussion between the students where they would have a better understanding of the subject.

Therefore, scientifically the concept that student-interaction was an important prerequisite to knowledge acquisition in mathematics lessons are scientifically justified by the works of Abilqasymova A.E. These works also highlight the necessity to structure the learning process in an active and communicative way and show how important the interaction is in the process of developing the thinking of students.

Overall, didactic underpinnings of mathematics education underscore the importance of perceiving the learning process as not just a process of transmitting knowledge, but as an entire system that is supposed to help students develop their thinking, speech, and communication. This point of view together with the theoretical basis of the study makes a further accent on the topicality of the development of interpersonal communication skills in students.

### 2.4.2 Dialogic And Communicative Teaching

In national pedagogical research, a special focus is on the significance of dialogic and communicative teaching approaches to the development of interpersonal interaction of the students in the mathematics lessons. Research into this area scientifically supports the view that the learning process must be underpinned by active participation of students, sharing of ideas and collaborative activity (Nazarbayev Intellectual Schools, 2020). This way allows the students not only to learn but to know and put into practice what they know consciously.

The problem of the dialogic teaching is thoroughly discussed in the works of Zhanlyuilova R.B.. The author sees dialogue as one of the key forms of the organizational learning process, and illustrates its effect on the cognitive activity of students and the quality of knowledge acquisition. Research results have shown that when students engage in dialogue, their freedom to articulate their ideas, ask questions, respond and share their opinions with each other, leads to a better understanding of mathematical concepts.

In the works of Zhanlyuilova, dialogic teaching is considered not only as a method of conducting the learning process but also as a factor that changes the content of the learning process. With effective arrangement of dialogue in the classroom, students will become not passive listeners but active participants. They are not just given ready made information; they interpret it, develop their own opinions and learn how to defend them. In its turn, this also facilitates the acquisition of critical and logical thinking skills.

Dialogic teaching also involves a change in the structure of communication. The information given by the teacher is the main source in the traditional lessons whereas in the dialogic teaching, the students become the source of information. In explaining, questioning and responding, students mutually build knowledge (Ministry of Education of the Republic of Kazakhstan, 2023). This is the social character of the learning process and it increases the efficiency of the educational process.

The communicative methods of teaching or learning are widely covered in the literature of Turlybekova G.A. The author substantiates the effectiveness of systematically using communicative tasks in mathematics education. These kinds of activities prompt the students to participate in a conversation, share their thoughts, explain their logic and have a discussion. Consequently students do not only learn mathematical knowledge, but also learn to use the latter in linguistic form.

The investigations of Turlybekova have a certain accent on the progress of the mathematical speech. The author suggests that the level of understanding can be identified by the fact that students can communicate fluently using mathematical language. In this respect, students are to be provided with a chance to explain, defend their point of view, and engage in discussions. Such activities assist in organizing their thinking, and they can clearly and accurately express their ideas.

One of the peculiarities of communicative tasks is that they allow actively involving students into the learning process. In this kind of work, students are not simply solving the problems; they are describing their solutions, discussing the points of view of others, and making conclusions with others. This increases their cognitive processes and helps them to learn more.

The general trait of the dialogic and the communicative methods of teaching is that these are student-centered approaches. In such methods, students do not necessarily sit back and listen, but are the active participants in the learning process. Interaction helps them to learn, systematize their thoughts and to learn how to defend their thoughts. Simultaneously, these approaches also help to develop communicative abilities of students.

Nonetheless, these methods must be properly implemented with some pedagogical conditions. To begin with, open-ended questions and tasks that facilitate discussion and sharing of ideas should be included in the lessons. Secondly, one should establish a conducive and psychologically secure classroom setting because it would allow students to share their ideas. Third, teachers are not only to evaluate the responses of the students, but also to elaborate them further, i.e. every response should be discussed and elaborated.

In addition, it is imperative to implement dialogic and communicative pedagogies in a systematic manner (Ministry of Education of the Republic of Kazakhstan, 2023). When such methods are applied on a few occasions, their efficacy will be limited. When however they are incorporated into the framework of the lesson, they can greatly change the quality of the learning process.

Therefore, the works of Zhanlyuilova R.B. and Turlybekova G.A. present a scientific explanation of a significant role of dialogic and communicative teaching methods in the development of interpersonal interaction of students during math lessons. Such methods make learning process active, meaningful and effective allowing students to attain deep understanding and be able to justify their ideas through justification and develop their language of mathematics.

Overall, dialogic and communicative teaching methods are useful tools that can be utilized to develop the interaction of students in mathematics classes. Not only do they contribute to the cognitive engagement of the students, but they also have a positive impact on the personal development of the students and make the overall quality of the education better.

### 2.4.3 Group Work And Collaboration

In modern educational activity, the shaping of interpersonal interaction of students is strictly interconnected with the structuring of the collaborative activity. Group work and collaboration-based strategies are known to be effective in mathematics lessons in particular to ensure that students are actively involved in the learning process (Johnson & Johnson, 2009). Under these circumstances, learning is never an individual process but rather a group cognition process.

The efficiency of the group work in mathematics education is thoroughly supported in the works by Esengazieva A.K.. The author notes that the cognitive activity of students is greatly enhanced in the process of group work since students are not only performing but are also discussing, explaining and making decisions together. This type of activity stimulates the thought process of the students and leads to conscious learning (Ministry of Education of the Republic of Kazakhstan, 2023).

The quality of interaction between students in the context of group work qualitatively changes (Nazarbayev Intellectual Schools, 2019). Whereas in the traditional lessons students only interact with the teacher, in the case of group work, students interact simultaneously with several other students. This expands the nature of discussion and enables the comparison of various points of view. Consequently, students learn to think of problems in multiplicity of perspectives instead of depending on one solution.

Especially important is peer explanation. Students reorganize their knowledge when explaining it to others: they simplify important ideas, identify logical connections, and they try to explain their ideas logically. This process not only increases their knowledge but also results in meaningful as opposed to mechanical learning. Besides that, when listening to the opinions of other people, students re-evaluate their own opinions and alter them accordingly.

The notion of teamwork is also discussed in the work by Baigozhanova A.S.. The author describes collaboration as mutually dependent activity that is meant to accomplish a shared objective. The role of every student in such a situation is to contribute to the success of the group in many respects, having a feeling of responsibility and supporting one another.

In the context of collaborative learning, interaction does not only involve the exchange of information, but also involves a process of collective thought. Students pose questions, give explanations, and collaborate in finding common solutions. This not only helps in their academic growth, but also helps in their communicative and social development.

Also, group work assists in personal growth of students. Students are taught to articulate their thoughts in a respectful manner, to listen to others, to come to agreements, and make joint decisions. Such skills are also fundamental aspects of communicative competence that is greatly appreciated in the contemporary society.

Nevertheless, organization of group work cannot be done effectively without following some principles of pedagogy. To begin with, the tasks ought to involve actual cooperation, i.e. the tasks which can be performed separately, group work turns into a mere formal activity. Second, the active engagement of each group member should be ensured because passive engagement will not be effective. Third, the teacher must be more than just an observer; he must also act as a facilitator who organizes and directs student interaction.

The shallow application of group work is one of the frequent pitfalls in practice. Even though students can be separated into groups, real communication does not necessarily exist. In this scenario, group work does not serve its main purpose (Webb, 2009). Thus, the specific emphasis should be made on the contents and meaningful structure of group activities.

So, the investigations by Esengazieva A.K. and Baigozhanova A.S. prove that group work and collaboration are the effective tools to develop interpersonal interaction in mathematics lessons in students. By means of collaborative activity, students do not only learn but comprehend, analyze and use it in conscious way.

Overall, group work and collaboration changes the learning process of individual activity to collective activity. This naturally involves the interaction of the students and allows the combined development of their cognitive and communicative skills.

#### 2.4.4 Interactive And Modern Methods

The successful structuring of the learning process in the modern education system presupposes the use of modern methods aimed at providing students with active participation in the learning process and the development of interaction with each other (Nazarbayev Intellectual Schools, 2019). In this respect, interactive methods are regarded as a significant source of increasing the level of cognitive engagement of students in mathematics lessons as well as the level of communication.

The efficiency of interactive ways in teaching mathematics is justified in the works of Smagulov E.Zh.. The author considers interactive teaching as a method that guarantees the active student involvement in the learning process and proves the benefits of this approach over the traditional teaching methods. Students do not sit back and listen in an interactive learning environment but rather they actively participate, and this increases their learning motivation.

When it comes to interactive teaching, it will alter the way the lesson is structured. Students cooperate, share their ideas and make efforts to defend their arguments. The process helps them interact more and create knowledge as a collective. In addition, interactive techniques enable the students to contrast various points of view and defend their own positions.

One of the key elements of interactive methods is peer assessment. This problem is discussed in the publications of Iskakova M.S.. The author defines peer assessment as a significant pedagogical instrument with the help of which students learn to examine, to assess and to give feedback on the work of each other. Such activities develop students' critical thinking and increase their sense of responsibility.

In peer assessment, students not only tend to gauge results but also tend to analyse the processes of solution, to detect mistakes and to suggest the means of correcting those mistakes. This increases their degree of mathematical knowledge as well as encouraging further learning. Also, through listening and judging the opinions of the others, students will be building their communicative skills.

Students are more engaged through interactive approaches that are largely effective in enhancing student engagement. Being able to actively participate in the lesson process will increase the interest of the students in the learning process and will also enhance the quality of the learning process. The involvement is not only a measure of completing the tasks, but also a characteristic of the student being able to express their ideas, ask questions and be involved in the discussion.

Moreover, interactive techniques render the process of learning dynamic and flexible. Students interact through different forms of activity and learn with interaction. This facilitates the growth of their cognitive skills and assists them to be adapted to the learning processes.

Nonetheless, interactive methods have to be properly implemented in certain conditions. To begin with, tasks must involve active student involvement. Second, there is need to create an open and accommodating communication atmosphere. Third, the teacher is expected to direct the activities of the students and structure the way students interact with each other.

Practically, interactive processes are occasionally implemented in form only (Hattie, 2012). This is despite the fact that students might seem as being active but the real process of discussing or exchanging ideas does not take place. Under these circumstances, the interactive techniques do not have a chance to prove their maximum efficiency. Hence, one should consider structuring these approaches in a meaningful way.

Thus, the works of Smagulov E.Zh. and Iskakova M.S. highlight the important role of interactive methods in developing students' interaction in mathematics lessons. These strategies enhance student involvement and allows more learning through the collaborative activity.

Overall, interactive and contemporary teaching techniques can be discussed as the effective means of the learning process improvement and development of the interpersonal interaction of the students. They offer the terms of the overall development of cognitive, communicative and social abilities of students.

### 2.4.5 Communicative Culture And Mathematical Language

The process of interpersonal interaction development of students during math classes is highly connected with the process of shaping the communicative culture of students. Mathematical language, in this regard, plays a central role, because it is the main device which organizes the thinking of students and allows them to externalize their thinking. Thus, the acquisition of mathematical knowledge is not only confined to mastering problem-solving skills but also it involves the ability to convey ideas correctly and clearly (Abilqasymova, 2013).

The place of mathematical language is thoroughly discussed in the works by Akhmetova G.K. The author says that mathematical language is not just a system of terms and symbols; it is a tool that helps to organize the way in which students think and enable them to express it in a way that is accurate and logical. In this respect mathematical language is taken as a significant indicator of the cognitive development of students.

To master the concepts of mathematics fully, a student must not only learn the concepts but also be able to explain his or her ideas, reason behind his or her answers and the arguments of other people. They are done by interacting with students. Therefore, mathematical language is a not only a way of communicating but also a basic way of creating knowledge.

The problem of communicative culture is a popular topic of research carried out by Kaidasova G.N.. The author also underlines that a mathematical speech culture of students is a necessary condition to develop logical thinking. It is only when students can develop their ideas in a clear, systematic and accurate manner that students reach a higher level of understanding.

The communicative culture involves interaction skills among the students, in terms of being able to express ideas effectively, listen to others, offer arguments, and defend the point of view. These abilities are important in learning mathematics as any statement in mathematics should be supported and based on logic.

Also, the language of mathematics and thinking are inseparable. Students have to be logically organized in order to be able to express ideas clearly and in a systematized way. This helps them structure their thoughts and to make them more organized. On the other hand, failure to express ideas can be a sign of lack of knowledge.

The process of speech culture development in mathematics classes in students is significantly changing the nature of interaction among students. Students ceasing to be the ones who give answers in any case, they also begin to justify their ideas, engage in discussions, and reason. This makes them more involved in the learning process and enhances understanding.

But in the real world the language of mathematics among the students is not always developed properly. Students can locate the right answers but face some problems in clarifying the answers in most cases. This means that they have not well developed communication skills. Thus, special emphasis should be put on stimulating students to speak, explain, and share ideas in the classroom.

The teacher is very instrumental in this process. The teacher is to lead students to develop their ideas to the full extent, to justify their ideas and to use the language of mathematics properly. By using thoroughly constructed questions and tasks, the teacher can help the students become more actively involved in the verbal activity and engage in the meaningful interaction.

Therefore, the works of Akhmetova G.K. and Kaidasova G.N. show the significant role of mathematical language and communicative culture in the development of interpersonal interaction of students. The structure of language in mathematics shapes the way of thinking of students and makes it possible to express oneself precisely, and the communicative culture creates the conditions under which effective interaction is possible.

In general, the development of mathematical language and communicative culture is one of the key conditions for fostering students’ interaction in mathematics lessons. This direction of work is systematizing such that logical thinking, communication skills, and conscious knowledge acquisition are formed.

# 3. METHADOLOGY

## 3.1 A Model For Developing Interaction

The formation of the interpersonal communication skills of the students is one of the most important priorities of the learning process in contemporary education (OECD, 2023; UNESCO, 2021). This problem is especially topical in mathematics classes, where the traditional method of teaching is often based on the work of a single student, the mastery of algorithms, and the acceptance of ready-made knowledge. Under these circumstances, the interaction of students is not as developed, and their communicative abilities are not well-developed.

Another problem that is observed in practice is that in spite of being able to find correct answers, students experience difficulties in explaining, justifying or expressing their thinking. This signifies a shallow section of mathematical knowledge and inadequately honed interpersonal abilities. In addition, students' exchange of ideas and discussion during lessons are often not systematically organized, which leads to a one-sided learning process.

To overcome these issues, a model designed by an author which is supposed to develop the interpersonal interaction of students in mathematics classes is offered. The model will serve the purpose of systematically organizing the learning process, and establishing conditions under which deep knowledge acquisition is possible through collaborative activity.

One of the main peculiarities of the model is the fact that when creating groups, the psychological peculiarities of the students (psychotypes) are taken into account. It is predetermined by the model that students with various psychotypes can belong to each of the groups. Particularly, both groups have students that are active, initiative-oriented, analytical, calm and organized. Information about students' psychotypes is obtained with the support of the class teacher and the school psychologist.

The efficiency of this method consists in the complementary character of various types of psychology. Active students make the discussion more active, analytical students justify that solutions are correct, and calm students assist in organizing ideas. Consequently, natural dialogue is formed in a group, and the interaction of students becomes more dynamic.

The model proposed is founded on the dialogic and collaborative learning processes. The students learn by explaining to others what they know, asking questions, exchanging ideas and making joint decisions with members of their groups (Johnson & Johnson, 2009). The process not only improves their communicative abilities but also their logical reasoning and the degree of their mathematical knowledge (Webb, 2009; Gillies, 2007).

The practical orientation is another significant characteristic of the model. The strategies suggested are modified to be applied in the real classroom and are logically incorporated into the teaching process. During every lesson, the groups are formed according to psychotypes, dialogic tasks are implemented, and the interaction between the students is actively organized to promote communication.

In this way, the suggested model can be used to construct the purposeful and systematic creation of interpersonal interaction between the students in the mathematics lessons. The combination of the psychotype-based group work and dialogic teaching strategies makes students more engaged and precondition the comprehensive development of their communicative and cognitive skills.

## 3.2 Methods And Techniques

### 3.2.1 Organizing Group Work

The ability of students to develop effectively in terms of interpersonal interaction during mathematics classes highly depends on the mode and manner in which group work is executed (Webb, 2009; Gillies, 2007). In the present research, one of the imperative conditions of organizing group activities is the psychological peculiarities of students when creating groups. This method would improve the contact between students and would lead to more significant communication.

In the formation of groups, psychotypes of students are considered. To conduct the study, we distinguish three uslovno categories: triangle, circle and square. These labels become practical descriptive labels of the behavioral inclinations of students. Data about the psychological profile of the students is gained by working together with the class teacher and the school psychologist.

Triangle-type students are those that are active, i.e., iinitiatiave-oriented, and leader-oriented. They tend to open up discussions, group processes and responsibility in decision-making. The so-called circle type students are usually cooperative, communicative and supportive and contribute to good group atmosphere and help in interaction. Students of the type square are characterized by the ability to think analytically and with precision; they are also typified by their logical thinking and organization of thoughts in the way they solve problems.

The groups are carefully formed to have a representative of all the three types. This kind of heterogeneous grouping enables students to compensate the other of the two with his gains and naturally divides out roles in the group. Consequently, every student will contribute in accordance with his or her capabilities, which will greatly enhance the nature of interactions.

It is an important part of this process, which involves assigning roles in the group (Johnson, Johnson, & Holubec, 2013). It is recommended that the following roles be used: leader (coordinates the group), presenter (reports the results), analyst (evaluates and justifies the solutions), and observer (monitors participation and process). Sharing of roles fosters responsibility and help all the students to participate actively.

Group work is structured in such way that it consists of a series of organized processes. The first step involves presenting students with a task and allowing them to discuss the material in groups. Then, they discuss potential solutions collectively and reach a mutual solution. This is then succeeded by group presentations, where students give their findings and also answer the feedbacks of their fellow students. Lastly, the results are presented together, and the conclusion is made.

In this context, the role of the teacher changes greatly. The teacher does not act as a source of information, but becomes a facilitator who supports and guides interaction (Gillies, 2007). The instructor keeps track of group dynamics, offers some specific help when needed, and promotes meaningful interaction.

As the classroom observations show, the ability to group students according to their psychological traits helps to enhance the level of engagement and facilitate more successful interaction. Students learn how to be more responsible in their contribution, and actively participate in collaborative learning, which helps to develop their communicative competence (Webb, 2009).

Therefore, the group work with taking into account the psychological features is a successful approach that will be used to promote the interaction of students during mathematics classes. Through collaboration, this not only enhances engagement, but also preconditions deeper and more meaningful learning through collaboration.

### 3.2.2 Dialogic Teaching Techniques

In promoting interpersonal interaction among students in mathematics classes, dialogic teaching will take center stage. It is not only that it is treated as a theoretical construct in this study but rather it is a structured classroom practice which is deliberate. It is mainly aimed at involving students into meaningful talk, helping students develop their capacity to articulate ideas, and supporting knowledge building by facilitating discussion (Alexander, 2018; Mercer & Littleton, 2007).

A key component in implementing dialogic teaching is a well-designed system of questioning. Instead of making use of questions that only result in brief answers, the teacher makes use of prompts that lead to a deep thought process and exchange of ideas (Alexander, 2018). During the study, a number of type of questions were used.

The former can be called explanatory questions and they are as follows: How did you get to this solution? or What is the reason why you have chosen this method?

Such questions will make students explain their arguments and make the logic behind their solutions transparent (Kazemi & Stipek, 2001).

The second group is the justification questions i.e. why do you think this is correct? how can this be proved? These types of questions encourage the use of rational thought and they also compel the students to support their responses (Yackel & Cobb, 1996).

The third type is comprised of comparative questions, e.g.: Which method is more effective? or What is the difference between these solutions? These prompts promote critical thinking and allow students to consider alternative approaches.

The fourth category is expanding questions, e.g.: What other problems might this approach be helpful with? or Can this approach be useful with other problems? These questions are thought-provoking and will make students think harder (Alexander, 2018).

Dialogic teaching is more than just questioning, and it is structured in a systematic series of activities. The dialogic interaction was introduced in this study in stages: students worked individually on a task and then discussed their ideas in pairs and finally reached a common solution within their group. This development will enable students to gain confidence gradually and will be able to convey their ideas more clearly.

Lessons also included the use of the math discussion approach. In such format, students share their solutions with the rest of the class, answer questions of their peers and participate in a group discussion (Mercer, 2000; Boaler, 2016). These activities enhance verbal activity and reinforce the interaction of the students.

In the dialogic teaching, the teacher plays a facilitative role. Instead of giving ready-made solutions, the teacher gives the students the chance to think on their own. The teacher assists by validating the responses of students, interrelating their thoughts, and facilitating dialogue to help students extend their understanding and to maintain meaningful dialogue.

The classroom observations have shown that in case the dialogic teaching is implemented in a systematic manner, verbal involvement of the students is enhanced significantly. They feel more confident about their intentions to express their ideas, ask questions, and engage in discussions. Simultaneously, their mathematical capabilities also rise in extent because the knowledge is created during the process of dialogue instead of being imprinted (Kazemi & Stipek, 2001; Boaler, 2016).

Therefore dialogic teaching is a good way of teaching students how to interact with each other in mathematics classes. It encourages active learning, develops thinking and communication, and aids in the establishment of collaborative learning practices.

### 3.2.3 A System Of Communicative Tasks

The nature and design of activities in mathematics lessons has a decisive role in developing interpersonal interaction of students. Conventional training exercises are often aimed at implementing ready-made algorithms and, therefore, do not offer the possibility to discuss and express ideas (Mercer, 2000; Hiebert et al., 1997). To overcome this limitation, the study suggests a system of communicative tasks which are aimed at stimulating explanation, dialogue, and collaborative reasoning.

It is also characteristic of communicative tasks, that they do not merely require correct answers; they also involve the necessity of explaining, justifying and comparing different solution strategies (Webb, 2009). These activities provoke an active interaction, which develops not only mathematical reasoning but also communicative competence.

The study put in place several types of communicative tasks. The former is the open-ended tasks that will permit a variety of paths towards the solution, and will not have an absolute correct answer. As an illustration, students can be requested to solve a problem in various ways and discuss and compare their answers in a group. These activities promote dialog and help in building argumentation skills (Boaler, 2016; Hiebert et al., 1997).

The second category is the type of tasks based on explanation; where students must explain not only the result, but the process, which led to the result. The use of prompts like: How did you solve this? or Why did you use this method? are used to help students organize their thoughts and express them in the correct mathematical language (Hiebert et al., 1997; Sfard, 2008).

The third type is multiple-solution tasks, where students find two or more solutions to the same problem. They discuss the effectiveness of each method and compare it with another. The process encourages the ability to think critically and builds the interaction between students.

The fourth one involves analysis of errors. In such tasks students are shown deliberately false solutions. Their work is to draw out the error, give the cause of the error and to offer a correct answer. Analytical thinking, justification, and meaningful discussion are all encouraged by such tasks.

Group work is an important element of the process of implementing communicative tasks. Students do not only work individually but also collaboratively by discussing their work, by arguing out their points, and working towards common conclusions. This increases interaction and makes sure that communication becomes meaningful instead of being superficial.

Moreover, the teacher facilitates the process with the help of guiding questions, e.g.: Why is this right?”, Is there another way to do this? or How can this be justified? Such prompts enrich the thinking of students, and encourage them to actively participate in the discussion.

Practice in classrooms suggests that when the application of communicative tasks takes a systematic approach, the verbal involvement of students in the classroom can grow substantially. They also gain more confidence in sharing their ideas, develop a better comprehension of mathematical concepts and learn to express their knowledge in a justifiable and reasoned way.

Communicative tasks are therefore a good instrument in enhancing interpersonal interaction among the students in mathematics classes. They render the learning process more active, meaningful, and productive, and facilitated knowledge construction with collaborative activity.

## 3.3 Sample Lesson Plans

The enhancement of the interpersonal interaction of students in mathematics classes involves the practical application of the methods and techniques suggested in the proposed approach in the actual classroom conditions. Based on this, a series of sample lesson plans were created in this study as part of Grade 7 algebra course. The lessons are oriented on the organization of collaborative activity, the promotion of dialogic interaction, and the development of ability of students to communicate with the usage of mathematical language.

The lesson plans include important topics in the unit about algebraic fractions, such as properties of algebraic fractions, operations on algebraic fractions, and equivalent transformations. The reason why these topics were chosen is that they afford abundant possibilities in organizing a student-to-student explanation, justification, and discussion.

The program uses a combination of instructional strategies to make sure that there is active interaction among students. These include group work, dialogic teaching, and interactive techniques such as Think–Pair–Share, math discussion, and error analysis (Boaler, 2016). Combined, these strategies promote students to be active, express their thoughts, and think collectively.

When organizing group work, students’ psychological characteristics are taken into account. Both groups consist of learners of different profiles to encourage equal participation and successful interaction. This organized gathering, not only improves communication in the group but also leads to more meaningful work.

The activities that are included in the lesson plans are communicatively oriented. Instead of them being expected to provide only correct answers, they are expected to explain their thinking, justify their answers and compare various solutions. Through such activities, verbal interactions with students are enhanced, helping them to build mathematical thinking.

Therefore, the lesson plans suggested herein are practical means of adopting methods that are geared towards developing interpersonal interaction of students during mathematics classes. The priority of creating conditions that can facilitate active engagement, idea sharing, and shared learning is upheld in every lesson.

***Lesson Topic: Fractions of Algebra and their basic property***.

Grade: 7

Lesson Objectives:

To build students knowledge of the algebraic fractions, establish the domain of the allowed values of variables, and improve the skills of the students to apply the fundamental property of the algebraic fractions.

Expected Outcomes:

Students differentiate between algebraic fractions, learn how to simplify fractions, explain how they do it, and engage in discussions within a group.

Throughout the lesson, a collection of strategies that would help to improve interpersonal communication abilities of students was utilized. In particular, group work, dialogic teaching, and communicative tasks were considered as the main organizational forms of instruction.

In the organizational level, students have been grouped with consideration to their psychological features. The groups contained the learners of various types, which helped to increase the interaction between the learners. This type of group composition establishes good environment on effective communication and collaborative learning.

When introducing new material, the students were provided with tasks that would allow them to independently discover the basic property of algebraic fractions. Think-Pair-Share method was used where the students would first think individually then discuss in pairs and groups. Thinking of the students was encouraged by the teacher using guiding questions so that the students can make their own conclusions.

During the main part of the lesson, the communicative tasks were assigned to the students. In task 1, they had to demonstrate an identity. Students shared their solutions in groups and were able to come to a common solution through the process of explanation and justification. The second task was to provide a solution to a problem in various ways, compare solutions, and determine the most efficient way to solve the problem. In the third assignment, students simplified algebraic fractions, and determined their values, giving full accounts of their rationale.

The error analysis technique was also used. The teacher deliberately gave a wrong solution, and students revealed the error, how it came about, and what is the correct answer. This approach helped in developing critical thinking and reasoning abilities of students.

Students discussed their solutions to the class and presented them to the class at the final stage of the lesson. The procedure of Math discussion was introduced, which gives students the opportunity to discuss each other through questioning, analyzing the answers of other students.

In the reflection stage, students were to assess their learning activities, identify what they had learned, and speculate on challenges they had to face. This encouraged self participation in the process of learning.

In this way, the techniques employed in the lesson were oriented at the development of the interpersonal communication, mathematical thinking, and communicative skills of students.

***Lesson Topic: Operations involving Algebraic Fractions.***

Grade: 7

Lesson Objectives:

To acquire skills in the students to perform addition, subtraction, multiplication, division and exponentiation of algebraic fractions and to increase the ability of the students to apply these operations in solving problems.

Expected Outcomes:

Students do operations with algebraic fractions, use different techniques, justify their solutions, and take an active part in group discussions.

During the lesson, the strategies that were used to develop interpersonal communication of students were implemented such as the use of group work, dialogic teaching, and communicative tasks.

In the organizational level, students were allocated in groups, depending on their psychological traits, so that each group is diverse to encourage collaboration and communication.

The activation of prior knowledge was done by the Magic Cells method at the beginning of the lesson. The students remembered the material that was learned before and discussed how the problems can be solved, which led to the development of communication and sharing of ideas.

In the process of introducing new material, there was a presentation of rules in the way to do operations with algebraic fractions. The guiding questions were also employed by the teacher to actively engage students. Students talked about the procedure involved in carrying out operations and explained to themselves how to do so using peer explanation.

During the main section of the lesson, it was planned to provide communicative tasks, which required not only the implementation of operations but also an explanation and justification of the approach used. Students worked together to solve problems, justify their answers, and discuss answers of their peers.

Also, tasks that could be solved in more than one way were given, so that students could compare different methods, and decide which one is more efficient. These activities facilitated critical thinking and interpersonal communication.

Dialogic teaching was also a major element as the teacher asked questions like: Why did you choose this method?”, Is there another way to solve it? and Which method is more efficient? to encourage the discussion and justification of ideas.

Students were required to provide their answers at the end of the lesson, and discuss the problem in the classroom using the method of the Math discussion.In the process of reflection, students examined their learning activities, what they had learned, the struggles they had faced, and the questions they had, all within the context of helping them consciously engage in the process of learning.

In this way, the implemented techniques helped to develop the communication, the mathematical thinking, and communicative competence of the students.

***Lesson Title: Equivalent Transformations of Algebraic Expressions.***

Grade: 7

Lesson Objectives:

To build the skills of students in making similar transformations of algebraic expressions with fractions and to improve the ability of students to transfer their skills to solving problems.

Expected Outcomes:

Students use the techniques of transformation, apply the order of operations, explain their solutions and engage in group discussion.

A combination of strategies targeted at shaping the interpersonal communication of the students was utilized during the lesson, such as group work, dialogic teaching, and communicative tasks.

In the organizational stage, students were grouped in the context of their psychological traits, with a variety of individuals in the groups to encourage teamwork and communication.

During the first stage, previous knowledge was elicited by a series of questions and answers. The method of Who is Faster? was applied to revise the past topics and stimulate students to share their ideas as it could be done, making it more engaging.

During the introduction of new material, methods of equivalent transformation of algebraic expressions were studied. Examples were analyzed and explained by students to comprehend major concepts. The teacher applied guiding questions to evoke thinking and make personal conclusions.

Communicative activities were offered in the main part of the lesson and they were required not only to perform the operations but also to explain and justify solutions. Learners collaborated, shared their solutions and assessed the answers of their peers.

It was also possible to introduce multi-step tasks with the ability to plan their actions, determine the sequence of operations, and construct systematic solutions. This helped to build logical thinking and communication skills.

The discussion and reasoning were stimulated by dialogic teaching and the following questions: Which rule is this transformation based on? What steps did you follow? Can this be solved differently?

In the last phase, students offered their solutions and discussed with each other the answers, which contributed to the improvement of interaction and verbal activity.

In the reflection process, the methods of Success Ladder and One Minute were employed. The students assessed their level, what they had learned and what challenges they had faced.

Therefore, the procedures used throughout the lesson were designed to train the students in terms of interpersonal communication, mathematical thinking, and communicative skills.

# 4. ANALYSIS AND RESULTS

## 4.1 Organization Of The Experiment

The experimental part of the study was supposed to empirically prove the effectiveness of the offered methodological approaches and to define their influence on the development of interpersonal communication skills of the students. The experiment was carried out with the 7 th grade students of a general secondary school. There were 20 students who took part in the process of study. The chosen class was described by age peculiarities and the material of the curriculum, which is in accordance with the aims of the research topic.

This experiment was planned within the bounds of six lessons, one of which was held in the form of an open lesson. The course content included the algebra lessons, which were: Properties of Algebraic Fractions, Operations with Algebraic Fractions, and Equivalent Transformations. These areas were chosen based on their content, which gives the chance to work with the skills of the students to explain, reason and discuss.

When planning the experiment, the key principles of the proposed model were considered. Instructional strategies that were focused on the development of the interpersonal communication of the students were systematically applied in the lessons. These featured group work, dialogic teaching, a system of communicative tasks and active learning strategies such as: Think-Pair-Share, Math Discussion and Error Analysis. These strategies were meant to encourage active participation by students, promote the expression of ideas and enable the acquisition of knowledge through active interaction.

The psychological peculiarities of students were also taken into account when creating student groups. All the groups had students of different learning types and abilities. This strategy allowed the communication between students to be effective, the ideas to be exchanged and mutual support to be given. The introduction of certain roles in groups made it possible to guarantee the active involvement of each student.

The main goal of the experiment was to establish how effective the proposed methodological approaches are in developing interpersonal communication skills of students in mathematics lessons. With the aim of accomplishing this goal, it was determined that the following tasks would be carried out:

to identify what will be the first level of the communication skills of the students;

- to stepwise introduce the suggested techniques into the lesson;

– to structure the interaction of students;

– to make a comparative analysis of the final results.

During the study, a system of indicators was created that assesses the degree of the communication skills of the students. These indicators were the speech activity of students, the ability of the students to make their ideas clear, the ability of the students to reason, question, engage in discussions and the level of engagement of the students working in a group. Such a system allowed assessing the communicative competence of students comprehensively.

The experiment was conducted in a number of steps. The first stage included initial diagnostics to establish the baseline level of communication skills of students. At the second tier, the suggested procedures were implemented with the system in the course of the lessons. The third stage was the final diagnostics and the results obtained were compared and analyzed.

Therefore, the experimental work was structured in such a manner that it guaranteed the implementation of the proposed methodological approaches in the framework of the actual educational process, with the emphasis put on the development of the interpersonal communication in students. The design and the content of the experiment fully align with the research objectives and provide an objective assessment of results obtained.

## 4.2 Initial Diagnostics

On the first stage of the research, a diagnostic study was carried out to identify the baseline level of interpersonal communication skills of 7 th -grade secondary school students during mathematics lessons. The research tool used was the questionnaire called: Identifying Communication and Attitudes in Mathematics Lessons. A total of 22 students participated in the study.

The questionnaire was created according to the Likert scale (1-5 points). All responses were transformed into a numerical value and the findings were mathematically and statistically processed. The main measure employed was the arithmetic mean (M) of each item. Besides this, qualitative interpretation of the results was done in order to establish the degree of development of the communication skills of the students.

*Table 4.1.1.Average Result of Survey*

| | **Indicators** | **Mean (M)** | **Level** | **Pedagogical Interpretation** |
|---|---|---|---|---|
| 1 | Confidence in learning mathematics | 3.8 | Above average | Internal confidence is formed |
| 2 | Teacher support | 3.8 | Above average | Positive attitude is observed |
| 3 | Freedom in the lesson | 2.9 | Average | Psychological barriers exist |
| 4 | Importance of the subject | 3.5 | Above average | Cognitive motivation is present |
| 5 | Explaining to others | 2.8 | Average | Low activity level |
| 6 | Requesting assistance | 2.9 | Average | Communicative insecurity |
| 7 | Group work | 3.0 | Average | Skills are insufficient |

| | Indicators | Mean (M) | Level | Pedagogical Interpretation |
|---|---|---|---|---|
| 8 | Listening culture | 3.1 | Average | Partially developed |
| 9 | Teacher encouragement | 3.3 | Average | Motivation is present |
| 10 | Acceptance of mistakes | 3.2 | Average | Reflection is developing |
| 11 | Perception of subject difficulty | 2.8 | Average | Elements of uncertainty |
| 12 | Dialogue with the teacher | 3.4 | Average | Communication is present |
| 13 | Effectiveness of collaborative learning | 3.2 | Average | Awareness is formed |
| 14 | Self-assessment | 3.1 | Average | Unstable |
| 15 | Social trust | 3.2 | Average | Moderate level |
| 16 | Expression of ideas | 3.0 | Average | Weak communication |

The obtained results revealed an overall mean score of M ≈ 3.1**.** This indicator suggests that students' interpersonal communication skills were developed at a moderate level. Although students demonstrated certain communicative abilities, these skills had not yet reached a stable, systematic, and advanced level of development.

The findings showed that students possessed a relatively high level of confidence in learning mathematics and demonstrated a sufficient understanding of the importance of the subject. These indicators suggest that their learning motivation was developed to a satisfactory extent. However, the presence of learning motivation did not necessarily translate into active communication during lessons. While students showed an interest in mastering the subject content, they experienced difficulties in explaining their ideas, justifying their reasoning, and engaging in meaningful discussions.

The results also indicated that students' sense of psychological comfort during lessons was below the desired level. This suggests the presence of certain psychological barriers that hinder effective communication. In particular, fear of making mistakes, reluctance to speak in front of classmates, and a lack of confidence in expressing personal opinions negatively affected students' communicative activity. Such factors limited the effectiveness of group interaction and collaborative learning.

The analysis further demonstrated that students' communicative skills were insufficiently developed. Difficulties were observed in explaining mathematical ideas, providing clear arguments, seeking assistance when necessary, and participating actively in peer discussions. These findings indicate that students tended to assume a passive role in communication and that dialogic interaction was not yet fully established in the classroom environment.

The indicators related to group work revealed that students generally recognized the importance of collaborative learning. However, they encountered difficulties in applying collaborative skills effectively in practice. These challenges may be attributed to the insufficient organization of group work, the absence of clearly defined roles within groups, and the limited use of tasks specifically designed to promote cooperation and interaction.

The results associated with teacher–student interaction were comparatively positive, indicating a relatively high level of trust toward the teacher. Nevertheless, opportunities for open discussion remained limited, and students' active participation was not always encouraged to the desired extent. This suggests that the teacher continued to play the dominant role in the learning process, while students had not yet fully developed as active participants in classroom communication.

The level-based analysis demonstrated that the majority of students were classified within the medium and low levels of communicative skill development. The absence of students at a high level of development highlights the need for targeted pedagogical interventions aimed at enhancing interpersonal interaction and communication skills.

The analysis identified several key pedagogical issues: insufficient communicative activity, limited ability to express and justify ideas clearly, underdeveloped group work skills, the presence of psychological barriers, and inadequate use of mathematical language during communication.

Overall, the pre-experimental diagnostic results indicate that students' interpersonal communication skills in mathematics lessons were not sufficiently developed. These findings provide a strong rationale for introducing a specialized methodological system aimed at fostering interpersonal interaction, collaborative learning, and communicative competence within the educational process.

### 4.3 Research Results

To evaluate the effectiveness of the psychotype-based group work methodology, the pre-test and post-test results of 22 students who participated in the experiment were comparatively analyzed. The pre-test was conducted at the diagnostic stage, while the post-test was carried out at the control stage of the experimental work.

Students' performance was assessed according to five criteria: communicative activity, collaboration and cooperation, role performance in group work, empathy and mutual reflection, and mathematical cognitive motivation. Each criterion was evaluated on a five-point scale. Therefore, the maximum possible integral score for each student was 25 points.

The collected data were subjected to mathematical and statistical analysis in order to determine the changes that occurred after the implementation of the proposed methodology. The comparison of the pre-test and post-test results made it possible to assess the effectiveness of psychotype-based collaborative learning in developing students' interpersonal interaction, communication skills, and active participation in mathematics lessons.

*Table 4.3.1.Score Difference of Pre and Post-Test*

| **Student** | **Pre-test Score (X)** | **Post-test Score (Y)** | **Score Difference (d = Y − X)** |
|---|---:|---:|---:|
| 1 | 11.42 | 21.00 | 9.58 |
| 2 | 13.58 | 19.50 | 5.92 |
| 3 | 15.25 | 22.20 | 6.95 |
| 4 | 12.92 | 18.40 | 5.48 |
| 5 | 13.08 | 20.20 | 7.12 |
| 6 | 12.33 | 19.80 | 7.47 |
| 7 | 14.58 | 23.00 | 8.42 |
| 8 | 12.33 | 18.00 | 5.67 |
| 9 | 14.08 | 21.50 | 7.42 |
| 10 | 13.58 | 22.50 | 8.92 |
| 11 | 13.33 | 19.00 | 5.67 |

| Student | Pre-test Score (X) | Post-test Score (Y) | Score Difference (d = Y − X) |
|---|---:|---:|---:|
| 12 | 12.33 | 20.50 | 8.17 |
| 13 | 12.33 | 18.80 | 6.47 |
| 14 | 12.58 | 20.00 | 7.42 |
| 15 | 12.08 | 18.40 | 6.32 |
| 16 | 12.33 | 20.50 | 8.17 |
| 17 | 12.08 | 17.20 | 5.12 |
| 18 | 14.08 | 21.50 | 7.42 |
| 19 | 12.21 | 19.40 | 7.19 |
| 20 | 14.58 | 21.20 | 6.62 |
| 21 | 12.08 | 18.80 | 6.72 |
| 22 | 14.33 | 21.80 | 7.47 |
| **Mean** | **13.07** | **20.15** | **7.08** |

The table presents the empirical data obtained from the experimental group (n = 22) and the statistical indicators calculated from the pre-test and post-test results. Prior to the implementation of the experimental methodology, the total pre-test score was 287.49**,** with a mean value of M = 13.07 points. This result indicates that students' initial level of communication, cooperation, and interaction in mathematics lessons was below the expected level, representing approximately 52.3% of the maximum possible score.

Following the implementation of the psychotype-based group work methodology, the total post-test score increased to 443.20**,** with a mean value of M = 20.15 points. This result corresponds to approximately 80.6% of the maximum possible score. The overall improvement was reflected in a total difference score of **155.71** points and an average increase of M = 7.08 points per student.

The obtained results demonstrate a substantial improvement in students' communication and collaborative learning skills after the intervention. The consistent increase in scores across the experimental group suggests that the psychotype-based instructional approach contributed positively to the development of students' interpersonal interaction, cooperation, and active participation in mathematics lessons.

*Figure 4.3.1. Initial Pre-Test Result*

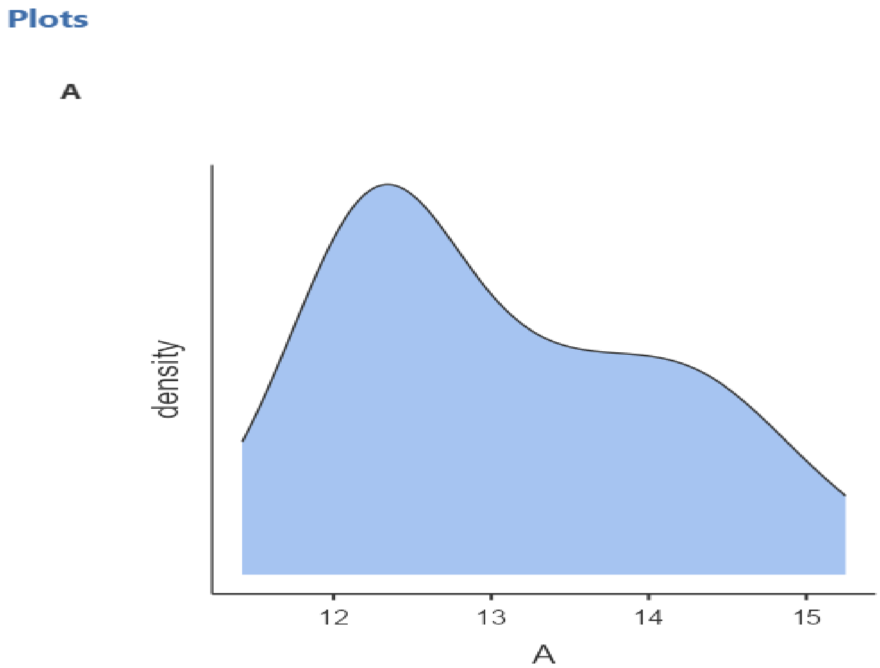
## Descriptives

Descriptives

| | A |
|---|---|
| N | 22 |
| Missing | 0 |
| Mean | 13.1 |
| Median | 12.8 |
| Standard deviation | 1.04 |
| Minimum | 11.4 |
| Maximum | 15.3 |

To determine the initial level of students' interaction, communication, learning motivation, and teamwork skills in mathematics lessons, a descriptive statistical analysis of the pre-test results was conducted using the Jamovi software package. The study involved a sample of 22 seventh-grade students. The statistical parameters obtained through data processing are presented in Table 3.2.

*Figure 4.3.2. Initial Pre-Test Result in Diagram*




Measures of Central Tendency. The maximum possible integral score across the five assessment criteria was 25 points. The analysis revealed a mean score of M = 13.1. This corresponds to approximately 52.3% of the maximum possible score and indicates that students' initial level of communication and collaborative learning competencies was below the expected level at the beginning of the study. The median value Md = 12.8 was slightly lower than the mean, which shows that more than half of the participants were concentrated within the lower score range.

Measures of Variability. The standard deviation SD = 1.04 was relatively low, demonstrating a high degree of homogeneity within the sample. This finding suggests that there were no students with exceptionally high or extremely low scores, and that the overall level of the assessed competencies was relatively uniform across the group. The minimum score was 11.4, while the maximum score was 15.3.

Distribution Characteristics. The histogram and density curve generated in Jamovi indicated a positively skewed distribution. The highest concentration of scores was observed within the lower score range, particularly around 12–13 points. This suggests that the majority of students demonstrated relatively low levels of communication, cooperation, and mathematical interaction during the initial stage of the experiment.

Overall, the pre-test results revealed insufficient development of students' communication and collaborative learning skills in mathematics lessons. These findings provided empirical justification for the implementation of the proposed psychotype-based grouping methodology, which involved assigning students to groups according to three conditional psychogeometric learner profiles — Triangle, Square, and Circle — and allocating specific functional roles within collaborative learning activities during the formative stage of the experiment.

*Figure 4.3.3. Initial Post-Test Result*

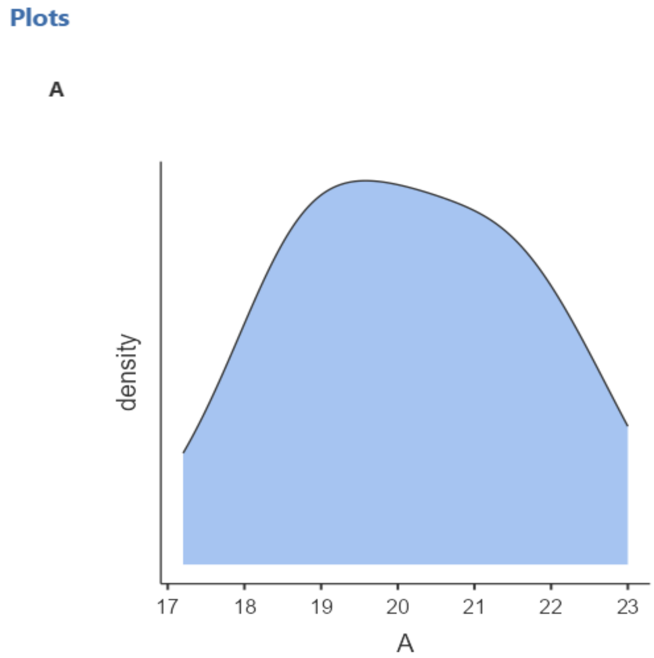
**Descriptives**

Descriptives

| | A |
|---|---|
| N | 22 |
| Missing | 0 |
| Mean | 20.1 |
| Median | 20.1 |
| Standard deviation | 1.57 |
| Minimum | 17.2 |
| Maximum | 23.0 |

Measures of Central Tendency. Following the completion of the formative experiment, the mean score of the participants increased to M = 20.1, representing approximately 80.4% of the maximum possible score of 25 points. This result indicates a substantial improvement in students' communication, collaboration, and interaction skills compared to the initial stage of the study. The median value Md = 20.1 was identical to the mean, suggesting a relatively balanced distribution of scores and a stable level of achievement across the experimental group.

*Figure 4.3.4. Initial Post-Test Result in Diagram*




Measures of Variability. The standard deviation SD = 1.57 indicates that individual differences among students were maintained, while the overall performance of the group improved considerably. The minimum post-test score was 17.2, and the maximum score was 23.0. Notably, the minimum post-test score exceeded the maximum score recorded during the pre-test. This finding suggests that even students who initially demonstrated lower levels of communication and collaborative learning skills showed substantial progress by the end of the intervention.

Distribution Characteristics. The histogram and density curve generated in Jamovi showed that the majority of students' scores were concentrated within the higher score range, particularly between 19 and 22 points. This indicates that most participants achieved relatively high levels of communication, teamwork, and collaborative problem-solving skills after the implementation of the instructional methodology. This pattern provides additional evidence of the positive impact of the proposed psychotype-based group work approach.

To determine whether the observed changes were statistically significant, inferential statistical procedures were applied. A paired-samples t-test was conducted using the Jamovi software package to compare students' pre-test and post-test scores. This statistical method was selected because the same group of students ($n = 22$) was assessed at two different stages of the study: before and after the implementation of the psychotype-based instructional methodology.

The use of the paired-samples t-test was appropriate for three reasons. First, the study involved repeated measurements obtained from the same participants, creating a dependent-samples design. Second, the assessment scores were represented as continuous numerical variables derived from a five-point Likert scale and aggregated into an integral score. Third, the purpose of the analysis was to determine whether the observed differences between the pre-test and post-test results were statistically significant rather than attributable to random variation.

All statistical analyses were performed using the Jamovi software package. The level of statistical significance was set at $p < 0.05$. Therefore, a probability value below this threshold was interpreted as evidence of a statistically significant difference between the diagnostic and control stages of the experiment. The statistical analysis provided an objective basis for evaluating the impact of psychotype-based group work on the development of students' communication and collaborative learning skills in mathematics education.

*Figure 4.3.5. Paired Samples T-Test*

Paired Samples T-Test

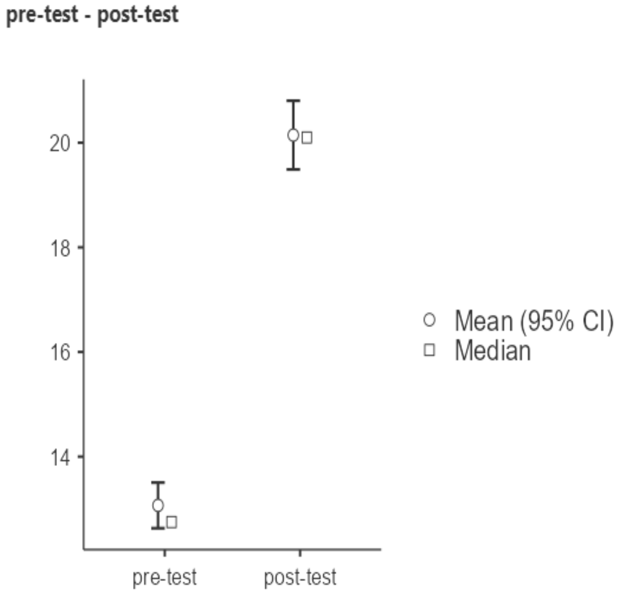
| | | | statistic | df | p | Mean difference | SE difference | | Effect Size |
|---|---|---|---|---|---|---|---|---|---|
| pre-test | post-test | Student's t | −29.0 | 21.0 | < .001 | −7.08 | 0.244 | Cohen's d | −6.18 |

*Note.* $H_a$ $\mu_{\text{Measure 1 - Measure 2}} \neq 0$

Descriptives

| | N | Mean | Median | SD | SE |
|---|---|---|---|---|---|
| pre-test | 22 | 13.1 | 12.8 | 1.04 | 0.223 |
| post-test | 22 | 20.1 | 20.1 | 1.57 | 0.335 |

*Figure 4.3.6. Plots*




To evaluate the effectiveness of the psychotype-based grouping approach and its impact on the development of students' interaction skills, the empirical data obtained during the experiment were analyzed using a paired-samples t-test in the Jamovi statistical software package.

Changes in Mean Scores. The descriptive statistics revealed a substantial improvement in students' communication and cognitive engagement throughout the experimental period. Prior to the intervention, the mean score was M = 13.1, indicating a relatively low level of interpersonal interaction and collaborative learning skills. Following the implementation of the psychotype-based instructional methodology, the mean score increased to M = 20.1, corresponding to a high level of performance. The average gain of approximately 7.08 points reflects a considerable positive change in the assessed competencies. Similarly, the median score increased from Md = 12.8 to Md = 20.1, indicating that the majority of students moved into a higher performance category by the end of the experiment.

Statistical Significance of the Results. The paired-samples t-test yielded a value of $t(21) = -29.00$, with a significance level of $p < .001$. Since the obtained probability value was substantially lower than the conventional significance thresholds of 0.05 and 0.01, the difference between the pre-test and post-test results can be considered statistically significant. The negative sign of the t-value reflects the direction of subtraction in Jamovi and does not affect the interpretation of statistical significance. These findings indicate that the observed improvements were not due to random variation but were associated with the implementation of the proposed instructional methodology. Consequently, the null hypothesis was rejected, and the research hypothesis was supported.

Effect Size. The magnitude of the intervention effect was assessed using Cohen’s d coefficient. The analysis produced an effect size of Cohen’s d = -6.18. The sign of the coefficient indicates the direction of the comparison, while the absolute value |d| = 6.18 is used for interpretation. This value greatly exceeds the benchmark of 0.80, which is commonly interpreted as a large effect. Therefore, the obtained result indicates a very strong practical impact of the psychotype-based grouping methodology on students’ communication, collaboration, and participation in mathematics learning activities.

Overall, the statistical analysis provides empirical support for the effectiveness of the proposed instructional approach. The significant improvement in students’ scores, combined with the large effect size, demonstrates that psychotype-based group work can serve as an effective strategy for enhancing interpersonal interaction and collaborative learning in mathematics education.

Criteria-Based Assessment Framework

To assess and monitor the development of students’ interaction and collaborative learning skills in mathematics, a criteria-based evaluation framework was developed. The primary research instrument, a structured questionnaire, was organized into five core assessment criteria. Each criterion represented a specific dimension of students’ communication, cooperation, and cognitive engagement within the mathematics classroom.

The questionnaire was designed to provide a comprehensive evaluation of students’ interpersonal interaction skills and their ability to participate effectively in collaborative mathematical activities. The five assessment criteria included communicative activity, collaboration and cooperation, role performance in psychotype-based group work, empathy and mutual reflection, and mathematical cognitive motivation. Together, these dimensions formed the basis for measuring the overall level of students' interaction and participation throughout the experimental study.

The criteria-based framework enabled the systematic collection and analysis of quantitative data, providing a reliable basis for evaluating changes in students' communication and collaborative learning skills before and after the implementation of the psychotype-based instructional methodology.

*Table 4.3.2.Evaluation of Answers*

| № | Criterion Dimension | Measured Competences and Behavioral Indicators | Associated Questionnaire Items | Maximum Scale Score |
|---|---|---|---|---|
| 1 | Communicative Activity | Ability to articulate mathematical ideas, engage in mathematical discourse, and exchange opinions fluently. | 1, 2, 3 | 5 points |
| 2 | Collaboration and Cooperation | Capacity to unite for a shared academic objective, distribute tasks evenly, and provide peer assistance. | 4, 5, 6 | 5 points |
| 3 | Psychogeometric | Execution of dynamic | 7, 8, 9 | 5 points |

| № | Criterion Dimension | Measured Competences and Behavioral Indicators | Associated Questionnaire Items | Maximum Scale Score |
|---|---|---|---|---|
| | Role Adoption | roles aligned with psychotypes ("Triangle" – strategic leader, "Square" – structural executioner, "Circle" – harmonic coordinator). | | |
| 4 | Empathy and Mutual Reflection | Active listening, valuing peer input during complex problem-solving, and joint mathematical error analysis. | 10, 11, 12 | 5 points |
| 5 | Mathematical Cognitive Motivation | Increase in internal locus of control and intrinsic motivation toward mathematics via collaborative group dynamics. | 13, 14, 15,16 | 5 points |
| — | TOTAL INTEGRAL SCORE | Comprehensive Level of Student Peer Interaction Competence | 1–16 | 25 points |

The operationalized model of criteria-based question distribution corresponded directly to the main pedagogical goals and the research hypothesis of the study.

Measurement Scale. Each questionnaire item was scored using a standardized five-point Likert scale, ranging from 1 — strongly disagree to 5 — strongly agree. The items within each criterion block were aggregated to calculate the mean score for that specific dimension. As a result, a cumulative integral score with a maximum value of 25 points was obtained for each student. This integral score served as the main quantitative indicator for comparing the pre-test and post-test results in Jamovi.

Role Performance in Psychotype-Based Group Work. The third criterion represented one of the key methodological components of the study. It evaluated how students adapted to functional roles within collaborative learning activities based on three conditional psychogeometric learner profiles: Triangle, Square, and Circle. Triangle-type students were associated with leadership, initiative, and strategic direction; Square-type students were associated with analytical thinking, accuracy, and systematic organization; Circle-type students were associated with cooperation, peer support, and maintaining a positive communicative atmosphere. This criterion made it possible to observe how effectively students interacted and contributed to group problem-solving activities in mathematics lessons.

Stratification of Performance Levels. The total integral score was used to classify students' interpersonal and cooperative capacities into three performance levels: Low Level — 5.00–13.00 points; Intermediate Level — 13.01–19.00 points; and Advanced Level — 19.01–25.00 points. This classification made it possible to compare students' development before and after the implementation of the psychotype-based group work methodology.

Statistical Verification of the Research Results

To determine whether the observed changes between the pre-test and post-test results were statistically significant, a paired-samples t-test was conducted using the Jamovi statistical software package. This statistical procedure was selected because the same group of students ($n = 22$) was assessed at two different stages of the study: before and after the implementation of the psychotype-based instructional methodology.

The paired-samples t-test is commonly used to evaluate differences between two related measurements obtained from the same participants. In the present study, it was applied to determine whether the changes in students' communication, collaboration, and interaction skills after the intervention were statistically significant.

The analysis revealed a statistically significant difference between the pre-test and post-test scores: $t(21) = -29.00$, $p < .001$. Since the obtained probability value was substantially lower than the conventional significance level of $p < 0.05$, the null hypothesis was rejected. The negative sign of the t-value reflects the direction of subtraction in Jamovi and does not affect the interpretation of statistical significance. These findings indicate that the improvement observed in students' scores was not the result of random variation, but was associated with the instructional intervention implemented during the formative stage of the experiment.

Furthermore, the effect size analysis produced Cohen's $d = -6.18$. The sign of the coefficient indicates the direction of comparison, while the absolute value $|d| = 6.18$ is used for interpretation. This value considerably exceeds the benchmark of 0.80, which is commonly interpreted as a large effect. Therefore, the obtained result suggests that the psychotype-based grouping methodology had a very strong practical impact on the development of students' communication, cooperation, and interaction skills in mathematics lessons.

Overall, the statistical analysis provides empirical support for the research hypothesis that psychotype-based group work contributes positively to the development of students' interpersonal interaction skills. The significant improvement in students' post-test results indicates that the proposed instructional approach can be considered an effective strategy for enhancing communication, cooperation, and collaborative learning in mathematics education.

# 5.DISCUSSION AND CONCLUSION

This master's thesis was devoted to the theoretical justification and experimental verification of pedagogical conditions and effective methodological approaches aimed at developing students' interpersonal communication skills in Grade 7 mathematics lessons in general secondary schools.

During the research, a comprehensive analysis of national and international scientific and pedagogical literature was conducted, which made it possible to identify the theoretical foundations for developing students' interpersonal interaction. The cultural-historical theory of Lev Vygotsky, Anna Sfard's discourse theory, Neil Mercer's concept of interthinking, Douglas Hiebert's theory of mathematical understanding, and the works of other scholars provided the theoretical framework for the study. The theoretical analysis demonstrated that the process of learning mathematics is not only cognitive in nature but also represents a communicative and social environment in which students construct knowledge through interaction.

As part of the study, a methodological system aimed at developing students' interpersonal interaction in mathematics lessons was designed. The proposed system was based on dialogic teaching, psychotype-based group work, a system of communicative tasks, and active learning strategies such as Think–Pair–Share, Math Discussion, and Error Analysis. While developing the methodology, a new approach to organizing student groups was introduced by considering students' psychological characteristics. The integration of Triangle, Square, and Circle psychotypes within the same group created favorable conditions for active communication and meaningful collaboration.

The pedagogical experiment conducted in accordance with the research objectives confirmed the effectiveness of the proposed methodology. Twenty-two students participated in the experimental study. The results of the pre-test revealed that the average level of students' interpersonal communication skills was 13.07 points, representing 52.3% of the maximum possible score. Among the participants, 43% demonstrated a low level of development, while 57% were classified at the intermediate level. No students were identified at the advanced level. The diagnostic stage indicated that students experienced difficulties in expressing their ideas, participating in discussions, collaborating with peers, and using mathematical language effectively.

During the formative stage, the proposed methodological system was systematically implemented throughout six mathematics lessons. Students actively participated in group discussions, peer explanations, argumentation activities, and collaborative decision-making processes. As a result, the post-test mean score increased to 20.15 points, corresponding to 80.6% of the maximum possible score. The average improvement reached 7.08 points per student. Furthermore, the lowest score recorded in the post-test exceeded the highest score obtained during the pre-test, demonstrating that all students showed positive progress throughout the experiment.

The experimental results were subjected to statistical analysis. A paired-samples t-test revealed a statistically significant difference between the pre-test and post-test results, $t(21) = -29.00$, $p < .001$. These findings indicate that the observed improvements were not random but were associated with the implementation of the proposed instructional intervention. In addition, the effect size coefficient was calculated as Cohen’s $d = 6.18$, which represents an exceptionally large effect and confirms the strong practical impact of the methodological system on students’ communication and interaction skills.

To obtain a deeper understanding of the experimental outcomes, four students were selected for individual observation, and qualitative feedback was collected from the mathematics teacher. The results of these observations complemented the quantitative findings and provided additional evidence of the effectiveness of the methodology.

The first student demonstrated strong academic performance but generally preferred working independently and rarely participated in communication with classmates. Following the implementation of group-based learning activities, the student became more willing to communicate with both peers and the teacher, began asking questions when difficulties arose, and actively participated in collaborative discussions.

The second student was initially very quiet, showed little initiative, and rarely attempted to solve tasks independently. Throughout the experiment, participation in group work gradually increased the student's confidence and engagement. The student became more involved in solving basic-level tasks, contributed to group discussions, and demonstrated greater interest in classroom activities.

The third student also tended to remain passive and usually completed only simple tasks. By the end of the experiment, the student showed greater participation in classroom discussions, interacted more actively with peers, and demonstrated increased confidence and responsibility in completing mathematical tasks.

The fourth student was not withdrawn but was characterized by low academic achievement and difficulties maintaining concentration during lessons. Through participation in structured group work, the student's energy and activity were redirected toward learning tasks. As a result, responsibility increased, academic performance improved, and the student reached a level comparable to that of higher-achieving classmates.

The qualitative observations demonstrated that the proposed methodological system positively influenced students with different learning characteristics and initial achievement levels. Significant improvements were observed in communication, cooperation, self-expression, and learning motivation.

Therefore, all research objectives were successfully achieved, and the research hypothesis was fully confirmed. The findings demonstrated that the systematic integration of psychotype-based group work, dialogic teaching, and communicative tasks in mathematics lessons contributes significantly to the development of students' interpersonal communication skills, communicative competence, mathematical language, and cognitive engagement.

The results of the study may be applied in the practice of mathematics teachers in general secondary schools, in the development of methodological materials, and in teacher education and professional development programs. Future research may focus on adapting the proposed methodology to other grade levels, extending its application to different school subjects, and investigating the long-term effects of psychotype-based grouping on students’ academic achievement and interpersonal development.

# 6.LIMITATIONS

At the same time, the study has several limitations that should be taken into account when interpreting the results. First, the pedagogical experiment was conducted with one group of Grade 7 students, consisting of 22 participants. Therefore, the findings reflect the results obtained within this particular classroom context and should not be generalized to all secondary school students without further research. Second, the experiment was implemented within a limited number of mathematics lessons. Although the results showed positive changes in students' communication and collaborative learning skills, a longer implementation period would make it possible to examine the stability of these changes over time.

Third, the proposed methodology was tested mainly within the topic of algebraic fractions and equivalent transformations of algebraic expressions. Therefore, further research is needed to determine how effectively psychotype-based group work, dialogic teaching, and communicative tasks can be applied to other mathematical topics and grade levels. Fourth, the study did not include a separate control group, which limits the possibility of making broad comparative conclusions. However, the pre-test and post-test comparison, together with teacher feedback, provided empirical support for the effectiveness of the proposed approach within the scope of the conducted experiment.

Finally, students' interpersonal communication skills were assessed through a criteria-based questionnaire, teacher observation, and statistical comparison of pre-test and post-test results. In future studies, the reliability of the findings could be strengthened by using additional methods, such as classroom video analysis, independent expert observation, student interviews, and long-term monitoring of students' mathematical achievement and classroom interaction.

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# LIST OF APPENDICES

Appendix A. Lesson Plan

**Сабақ жоспары**

| | |
|---|---|
| **Педагогің Аты, Тегі; Әкесінің аты (бар болған жағдайда):** | Онгарбек Ақжан |
| **Пән/Сынып:** | 7 сынып. Қатысқандар саны: 22<br>Қатыспағандар саны: |
| **Күні:** | |
| **Бөлім:** | **Қысқаша көбейту формулалары** |
| **Сабақтың тақырыбы:** | Екі өрнектің қосындысының кубы және айырмасының кубы. |
| **Оқу бағдарламасына сәйкес оқыту мақсаты:** | 7.2.1.11<br>$(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$<br>қысқаша көбейту формулаларын білу және қолдану; |
| **Сабақтың мақсаттары:** | − Қысқаша көбейту формулаларын біледі және қолдана алады;<br>- қысқаша көбейту формулаларын алгебралық өрнектерді ықшамдау кезінде қолдана алады<br>-Қысқаша көбейту формулаларын өрнектерді көбейткіштерге жіктеу кезінде қолдана алады; |

**Сабақ барысы:**

| **Уақыты** | **Кезеңдері** | **Педагогтің әрекеті** | **Оқушының әрекеті** | **Бағалау** | **Ресурстар** |
|---|---|---|---|---|---|
| 5 минут | Ұйымда | Сәлеметсіздерме!<br>Бүгін, Екі өрнектің қосындысының кубы және айырмасының кубы тақырыбын қарастырамыз.**Бүгінгі сабақта меңгеретініңіз:**<br>**-** екі өрнектің қосындысының кубы | Амандасады, сабаққа дайындалады.<br><br>Сұрақтарға | | Оқулық; оқу құралдары; |

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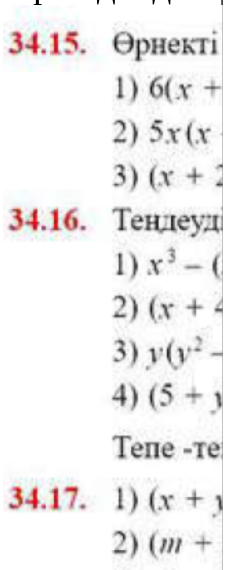
<td>с<br>т<br>ы<br>р<br>у</td>
<td>және айырмасының кубы формулалары.<br>-екі өрнектің қосындысының кубы және айырмасының кубы формулаларын қолдану.<br>Үй тапсырмасын тексеру. №5.73.<br>«Сәйкестендір» стратегиясы.<br>Сол жақтағы өрнектерге оң жақтағы бағандағы сәйкес өрнекті сәйкестендіру:
<table>
<tr><td>1</td><td>$x^2 - 2xy + y^2$</td><td>1</td><td>$x^2 - y$</td></tr>
<tr><td>2</td><td>$x^3 - y^3$</td><td>2</td><td>$x^2 + 2$</td></tr>
<tr><td>3</td><td>$(x + y)(x^2 - xy + y^2)$</td><td>3</td><td>$y - x$</td></tr>
<tr><td>4</td><td>$y^2 - x^2$</td><td>4</td><td>$(x - y$</td></tr>
<tr><td>5</td><td>$x^2 - 4xy + 4y^2$</td><td>5</td><td>$(x - y$</td></tr>
<tr><td>6</td><td>$(x - y)(x + y)$</td><td>6</td><td>$(x + y$</td></tr>
<tr><td>7</td><td>$(x + y)(x^2 + 2xy +$</td><td>7</td><td>$x^3 + y$</td></tr>
<tr><td>8</td><td>$-(x - y)$</td><td>8</td><td>$(x - 2$</td></tr>
<tr><td>9</td><td>$(x + y)^2$</td><td>9</td><td>$(y - x$</td></tr>
</table>
</td>
<td>жауап береді<br><br>Сәйкестенді ру тест орындайды.</td>
<td>«Бағдаршам» арқылы қалыптастырушы бағалау</td>
<td>тақта;<br>слайд;</td>
</tr>
<tr>
<td>10 мин</td>
<td>Жаңа сабақ</td>
<td>$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ формуласы екі өрнектің қосындысының кубы формуласы деп аталады және екі өрнектің қосындысының кубы бірінші өрнектің кубы, плюс үш еселенген бірінші өрнектің квадраты мен екінші өрнектің көбейтіндісі, плюс үш еселенген бірінші өрнек пен екінші өрнектің квадратының көбейтіндісі, плюс екінші өрнектің кубына тең деп оқылады.<br>$(a – b)^3 = a^3 – 3a^2b + 3ab^2 – b^3$ формуласы екі өрнектің айырмасының кубы формуласы деп аталады және екі өрнектің айырмасының кубы бірінші өрнектің кубы, минус үш еселенген бірінші</td>
<td>Тақырып бойыншарес урстарды қарап, танысады<br><br>2). Шешуі. Айырма</td>
<td>«Бағдаршам» арқылы қалыптастырушы</td>
<td>оқулық; оқу құралдары; тақта; слайд;<br><br>Оқулық; оқу құралдары;</td>
</tr>
</table>

| | | | | | |
|---|---|---|---|---|---|
| | | өрнектің квадраты мен екінші өрнектің көбейтіндісі, плюс үш еселенген бірінші өрнек пен екінші өрнектің квадратының көбейтіндісі, минус екінші өрнектің кубына тең деп оқылады.<br>1-мысал. Көпмүше түрінде жаз: $(2m + 3n)^3$.<br>*Шешуі.* Қосынды кубының формуласын қолданып, берілген өрнекті түрлендір:<br>$(2m + 3n)^3 = (2m)^3 + 3 \cdot (2m)^2 \cdot 3n + 3 \cdot 2m \cdot (3n)^2 + (3n)^3 = 8m^3 + 36m^2n + 54mn^2 + 27n^3$.<br>2-мысал. Көпмүше түрінде жаз: $(–x – 4y)^3$. | кубының формуласын қолданып өрнекті түрлендір:<br>$(–x – 4y)^3 = (–x)^3 – 3 \cdot (–x)^2 \cdot 4y + 3 \cdot (–x) \cdot (4y)^2 – (4y)^3 = –x^3 – 12x^2y – 48xy^2 – 64y^3$. | баға<br>лау | тақта<br>;<br>слай<br>д; |
| 15 минут | Бекіту тапсырмасы | №1.<br>Өрнекті көпмүше түрінде көрсетіңдер:<br>*а)* $(x+2)^3 =$<br>*б)* $(3-x)^3 =$<br>*в)* $(2a-b)^3 =$<br>*г)* $(3a+2b)^3 =$<br><br>№2. Көпмүше түрінде көрсетіңдер:<br>*а)* $(2a^2+3в^2)^3$<br>*в)* $(10x^4–6y^2)^3$<br><br>№3.<br>Көпмүшені екі өрнектің кубы түрінде көрсетіңдер:<br>*а)* $(8p^3–27q^3+54pq^2–36p^2q)$<br>*в)* $(x^3y^3+6x^2y^2+12xy+8)$<br><br>**Оқулықпен жұмыс.** №5.100.<br><br>Оқулықпен жұмыс.<br>№1. Көпмүше түрінде көрсетіңдер:<br>а) $(8x^3–4y^2)^3$<br>в) $(0,2x^5+0,5y^2)^3$ | Оқушылар дескрипторға сәйкес тапсырмаларды орындайды.<br><br>Жазылуы бойынша формулаларды жалғастырады (1)<br>ҚКФ ажырата алады (1)<br>Формуланы | **Дескриптор:**<br>- Қосындының, айырманың кубы формуласын біледі.<br>- Есеп шығаруда дұры | Презентация<br>Слайд<br><br>Интернет ресурстары<br><br>Оқул |

| | | | | | |
|---|---|---|---|---|---|
| | | №2. Көпмүшені екі өрнектің кубы түрінде көрсетіңдер:<br>а) $x^3+6x^2y+12xy^2+8y^3$<br>в) $0,125z^9+0,15z^6k^4+0,06z^3k^8+0,008k^{12}$ | ауызша тұжырымдай алады<br><br>Оқулықтан есептерді шығарады. | с қолданады.(1)<br><br>ҚБ:«Бас бармақ» әдісі | ық; оқу құралдары; тақта; слайд; Карточкалар;7 сыныпқа арналған оқулық Презентация |
| 10 мин | Оқулықпен жұмыс | **Оқулықтан есептер шығару:**<br><br>34.12. Көпмүшені көбейтінді түрінде жа<br>1) $m^3-n^3+2n-2m$; 2) $3a^3$<br>3) $x^6+y^6+x^2+y^2$; 4) $a^3-$<br>5) $x^4+xy^3-x^3y-y^4$; 6) $a^4-$<br>Өрнекті ықшамдаңдар (34.13-34.1<br>34.13. 1) $2a^3+9-2(a+1)(a^2-a+1)$;<br>2) $x(x+2)(x-2)-(x-3)(x^2+3x+$<br>3) $3(b-1)^2+(b+2)(b^2-2b+4)-($<br>4) $(a-1)^3-4a(a+1)(a-1)+3(a-$<br>34.14. 1) $(x+2)(x^2-2x+4)-x(x-3)(x+$<br>2) $(x-3)(x^2+3x+9)-x(x^2-16)+$<br>3) $(2x-1)(4x^2+2x+1)-23-4x($<br>4) $16x(4x^2-5)+17-(4x+1)(16x$ | Тапсырманы орындайды<br><br>34.15. Өрнекті<br>1) $6(x+$<br>2) $5x(x$<br>3) $(x+2$<br>34.16. Теңдеуд<br>1) $x^3-($<br>2) $(x+4$<br>3) $y(y^2-$<br>4) $(5+y$<br>Тепе -те<br>34.17. 1) $(x+y$<br>2) $(m+$ | Оқушының тақтадағы белсенділігіне байланысты ауызша бағалау | оқулық; оқу құралдары; тақта; слайд; |
| 5 мин | | **Бүгінгі сабақта:**<br>-екі өрнектің қосындысының кубы және айырмасының кубы формулалары. | Тақырыпты меңгергенін анықтау | | Оқулық; оқу құра |

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<td>-екі өрнектің қосындысының кубы және айырмасының кубы формулаларын қолдану.<br><br>Сабақ соңында оқушылар рефлексия жүргізеді:<br><br>Рефлексия:<br><table><tr><td>Білемін</td><td>Білдім</td></tr><tr><td></td><td></td></tr></table><br>Үйге тапсырма.№</td>
<td></td>
<td>Кері байланыс</td>
<td>лдары;<br>тақта;<br>слайд;</td>
</tr>
</table>

Appendix B. Survey Questions

**Математика сабағындағы қарым-қатынас пен көзқарасты анықтау сауалнамасы**

**Құрметті оқушы!** Бұл сауалнама сенің математика сабағында өзіңді қалай сезінетініңді және сыныптастарыңмен қалай араласатыныңды білу мақсатында өткізіледі. Жауаптарың бағаға әсер етпейді және құпия сақталады. Әр сұрақты мұқият оқып, өзіңе ең жақын келетін жауапты таңда (1-ден 5-ке дейін).

**Жауап беру шкаласы:**

1 – Мүлдем келіспеймін

2 – Келіспеймін

3 – Білмеймін / Бейтараппын

4 – Келісемін

5 – Толықтай келісемін

---

| **№** | **Сұрақтар (Сенің пікірің)** | **1** | **2** | **3** | **4** | **5** |
|---|---|---|---|---|---|---|
| **1** | Мен математиканы жақсы үйреніп кете алатыныма сенімдімін. | | | | | |
| **2** | Математика мұғалімі менің сабақтағы жетістіктеріме көңіл бөледі. | | | | | |
| **3** | Математика сабағында есеп шығару кезінде өзімді еркін сезінемін. | | | | | |
| **4** | Математика болашақта мен таңдайтын мамандыққа міндетті түрде қажет болады. | | | | | |
| **5** | Сыныптастарыма есептің шығару жолын түсіндіру маған ұнайды. | | | | | |
| **6** | Есеп шығара алмай жатсам, сыныптастарымнан көмек сұраудан ұялмаймын. | | | | | |
| **7** | Математикадан қиын есептерді жалғыз шығарғаннан көрі, топпен бірлесіп шығарған жеңілірек. | | | | | |
| **8** | Топтық жұмыс кезінде мен басқа оқушылардың пікірін соңына дейін тыңдаймын. | | | | | |
| **9** | Мұғалім мені математиканы көбірек оқуға, ізденуге ынталандырады. | | | | | |
| **10** | Математика сабағында қыздар да, ұлдар да есепті бірдей жақсы шығара алады. | | | | | |

| № | **Сұрақтар (Сенің пікірің)** | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| **11** | Маған математика мұғалімімен есеп туралы немесе мамандық туралы сөйлескен оңай. | | | | | |
| **12** | Басқа оқушылардың менің қатеме жасаған ескертулерін дұрыс қабылдаймын. | | | | | |
| **13** | Мен үшін математика — ең қиын әрі түсініксіз пән (Кері сұрақ). | | | | | |
| **14** | Мен маңызды нәрсе сұрағым келгенде, мұғалім мені тыңдайды деп сенемін. | | | | | |
| **15** | Есепті бірігіп шығарғанда, мен жаңа нәрселерді тезірек үйренемін. | | | | | |
| **16** | Мұғалім мені математикадан жақсы оқушы деп есептейтінін сеземін. | | | | | |