

THE EFFECT OF GEOGEBRA ON ACADEMIC PERFORMANCE OF HIGH SCHOOL STUDENTS IN GEOMETRY LEARNING

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A thesis submitted to the School of Education and Humanities
in partial fulfillment of the requirements for the degree of

MASTER OF EDUCATIONAL SCIENCES
in 7M01501 - Mathematics

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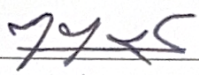
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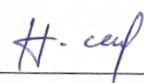
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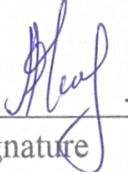
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List of Abbreviations

ICT – Information and Communication Technology

FAU – Florida Atlantic University

MSP – Mathematics and Science Partnership

NSF – National Science Foundation

NCTM – National Council of Teachers of Mathematics

DOC scholarship – A doctoral scholarship

STEM – Science, Technology, Engineering and Mathematics

IQR – Interquartile range

ABSTRACT

This study investigates the effect of integrating GeoGebra into geometry instruction on students' academic performance at the high school level. The research was conducted among students in 10-th grade from "125 High School" in Kyzylorda city. The primary purpose of the study was to examine whether the use of dynamic geometry software GeoGebra contributes to improved learning outcomes in geometry. A quasi-experimental research design with pre-test and post-test measures was adopted. The participants included 35 students, who were assigned to experimental and control groups. The findings revealed that students who learned geometry through GeoGebra showed greater improvement in geometric understanding and higher post-test scores than those in the control groups. These results suggest that GeoGebra facilitates deeper comprehension of geometric concepts and supports active student engagement in the learning process. Overall, the study demonstrates that the integration of digital tools such as GeoGebra can effectively enhance geometry learning and represents a valuable auxiliary tool to traditional instructional approaches in mathematics education.

Keywords: GeoGebra software, geometry learning, academic performance, mathematics education, educational technology, high school students, problem-solving skills, student motivation.

АНДАТПА

Бұл зерттеу геометрияны оқытуға GeoGebra интеграциясының жоғары мектеп оқушыларының үлгеріміне әсерін зерттейді. Зерттеу Қызылорда қаласының "125" жоғары мектебінің 10-сынып оқушылары арасында жүргізілді. Зерттеудің негізгі мақсаты динамикалық геометрия үшін GeoGebra бағдарламалық құралын пайдалану геометрияны оқыту нәтижелерін жақсартуға ықпал ететінін зерттеу болды. Тестілеуге дейінгі және тестілеуден кейінгі шаралармен квази-эксперименттік зерттеу дизайны қабылданды. Зерттеуге 35 оқушы қатысып, эксперименттік және бақылау топтарына бөлінді. Соңғы тест нәтижелеріне сәйкес геометрияны GeoGebra көмегімен оқыған студенттер бақылау топтарына қарағанда түсіну, есеп шығару деңгейінің жақсарғанын, яғни жоғары нәтижеге қол жеткізгенін көрсетті. Бұл нәтижелер GeoGebra геометриялық ұғымдарды тереңірек түсінуге ықпал ететінін және оқушылардың оқу процесіне белсенді қатысуын қолдайтынын көрсетеді. Жалпы алғанда, зерттеу GeoGebra сияқты цифрлық құралдарды біріктіру геометрияны зерттеуді тиімді түрде жақсарта алатынын және математиканы оқытудың дәстүрлі тәсілдері үшін құнды көмекші құрал екенін көрсетеді.

Түйін сөздер: GeoGebra бағдарламасы, геометрияны оқыту, академиялық үлгерім, математикалық білім беру, білім беру технологиялары, жоғары сынып оқушылары, есеп шығару дағдылары, оқу мотивациясы.

АННОТАЦИЯ

В данном исследовании изучается влияние интеграции GeoGebra в обучение геометрии на успеваемость учащихся в старших классах средней школы. Исследование было проведено среди учащихся 10-х классов средней школы “125” города Кызылорда. Основной целью исследования было изучить, способствует ли использование программного обеспечения GeoGebra для динамической геометрии улучшению результатов обучения геометрии. В исследовании приняли участие 35 учащихся, которые были разделены на экспериментальную и контрольную группы. Согласно последним результатам тестов, студенты, изучавшие геометрию с помощью GeoGebra, показали улучшение понимания, производительности задач, то есть более высокие результаты, чем в контрольных группах. Эти результаты свидетельствуют о том, что GeoGebra способствует более глубокому пониманию геометрических концепций и поддерживает активное вовлечение учащихся в процесс обучения. В целом, исследование демонстрирует, что интеграция цифровых инструментов, таких как GeoGebra, может эффективно улучшить изучение геометрии и представляет собой ценный вспомогательный инструмент для традиционных подходов к обучению математике.

Ключевые слова: программа GeoGebra, обучение геометрии, академическая успеваемость, математическое образование, образовательные технологии, учащиеся старших классов, навыки решения задач, учебная мотивация.

1. INTRODUCTION

In the contemporary digital era, it is challenging to conceive an existence without computers and electronic devices. For the younger generation, it is even impossible. As a result, throughout time, many trials and research in the field of knowledge are conducted, and new teaching techniques emerge. As technology advances and time progresses, an individual's cognitive abilities and perception of information adapt correspondingly. And that is normal. Also the younger generation has less concentration. “New environments taking advantage of artificial intelligence and virtual reality are to come if we aim at educating the Alpha generation with tools that are close to the way in which they are used to learning” (Sánchez-Ruiz, Llobregat-Gómez, Rodrigo-Illarri, 2024). That is, depending on the environment and time, the teaching methods should also change, and this is a pattern.

As noted above, technology is currently developing rapidly, and as a result, teaching just through traditional techniques in schools is insufficient for the future of students. The transformative view of ICT in education requires us to examine what new ways of pedagogies and curriculum are appropriate for a new generation working with new tools (Royati Abdul Sahaa, Ahmad Fauzi Mohd Ayubb, Rohani Ahmad Tarmizi, 2010). Today, it is essential to modernize teaching techniques and integrate information and communication technology into the educational process. This is not only current and relevant, but it also serves as a technique to keep students engaged. The general knowledge that high school students in this day in age are very interested in computers and video technology leads teachers and administrators to believe that the students would want to go to a classroom where these technologies are being used (McFarland, 2008). Also according to research when GeoGebra was used, students were adequately engaged to complete the task (Dahal, Pant, Shrestha, Manandhar, 2022). In other words, dynamic mathematical applications may stimulate interest in the topic while presenting complicated concepts in a simple and intelligible manner. GeoGebra establishes circumstances for dynamically constructing, altering,

and examining geometric objects and their attributes. This program helps students do research and encourages self-education.

This study is relevant as it investigates the necessity for efficient, technology-integrated approaches to mathematics instruction. Geometry necessitates robust visualization skills, and numerous students encounter challenges in addressing geometric issues owing to insufficient visualization capabilities. GeoGebra provides interactive and dynamic educational experiences that can improve conceptual comprehension, engagement, and motivation. This research presents empirical data on GeoGebra's efficacy in assisting educators in making educated decisions about the integration of digital resources into their courses. As worldwide educational trends change toward technology-based learning, this research helps to improve mathematics education by making it more interesting and accessible.

Furthermore, recent foreign researchers have found that using the GeoGebra program improves students' academic performance, geometric concept understanding, and motivation to study. However, the majority of these researches were conducted in a foreign education system, and actual studies that take into consideration the particular curriculum and students in Kazakhstan schools are few. Therefore, the study of the efficacy of the GeoGebra in learning and teaching geometry is relevant in Kazakhstan.

Problem statement

One of the most important issues in teaching geometry in a modern school is the development of students' spatial thinking, a thorough comprehension of geometric ideas, and the capacity to apply that knowledge in real-world situations. However, typical teaching techniques utilized in geometry lectures make it difficult for many pupils to grasp abstract concepts, resulting in a loss of enthusiasm to study. As a result, pupils frequently recall geometric forms and their attributes but do not completely comprehend them at the semantic level.

Despite the active integration of information and communication technology into education, the systematic and methodical use of digital tools, particularly

dynamic mathematical applications, in school mathematics sessions remains insufficient. Although the GeoGebra program allows users to examine geometric objects visually and interactively, the exact approach for its application and its impact on students' academic progress have not been thoroughly investigated in domestic educational practice.

At the same time, there is a lack of actual evidence demonstrating the efficacy of digital technologies in teaching geometry in Kazakhstani schools. As a result of this limitation, teachers rely on traditional methods and do not fully utilize the potential of current technology. In this regard, it is necessary to empirically validate the true influence of teaching geometry on the quality of students' understanding utilizing the GeoGebra application.

Taking these difficulties into account, the primary goal of this dissertation research is to establish the influence of using the GeoGebra application throughout the geometry teaching process on student academic performance, as well as to compare the effectiveness of traditional and digital teaching techniques.

Aim of the research

The study aims to investigate the effect of GeoGebra as a digital tool on the academic performance of high school students in geometry learning by analyzing its effectiveness in enhancing conceptual understanding, problem-solving skills, and overall achievement.

Objectives

To examine the effect of GeoGebra on students' academic performance in geometry and compare the performance of students who use GeoGebra with those who follow traditional teaching methods.

Research Question

What is the effect of using GeoGebra on high school students' academic performance in geometry?

Anticipated Outcomes

- Students using GeoGebra will show significant improvement in their geometry performance compared to those using traditional methods.
- GeoGebra will enhance students' spatial visualization skills, leading to better conceptual understanding.
- Recommendations for effective integration of GeoGebra into the mathematics curriculum

2. LITERATURE REVIEW

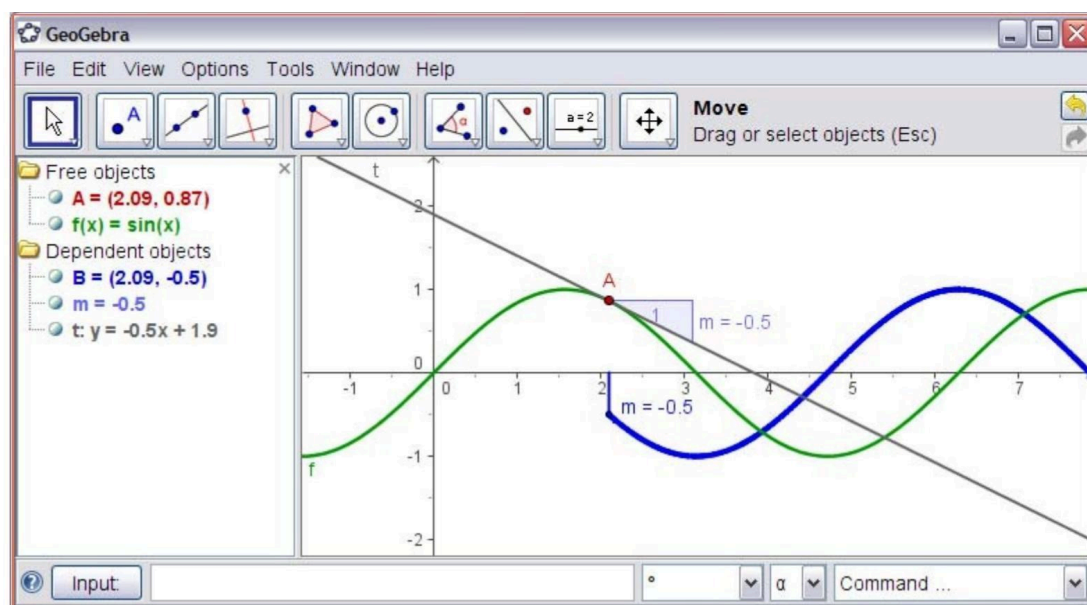
2.1 Overview of GeoGebra: history, features, and educational applications

GeoGebra is a free, multi-platform dynamic mathematics program that incorporates geometry, algebra, spreadsheets, statistics, and calculus into one integrated system, intended for students of all educational stages. Developed in 2001 as part of Markus Hohenwarter's master's thesis, the software has experienced continuous growth in adoption and influence. At present, GeoGebra is utilized in approximately 190 countries, translated into 52 languages, and records around 300,000 downloads each month. Its global implementation is further supported by 40 GeoGebra Institutes operating across 30 countries . Moreover, the software has received numerous educational technology awards in both Europe and the United States and has been installed on millions of devices worldwide.

2.1.1 History

GeoGebra is an open-source dynamic mathematics program, which was created as a part of Markus Hohenwarter's master's at the University of Salzburg, Austria. The official GeoGebra site presents the newest edition of the software, along with accompanying resources, like GeoGebra Wiki, user discussion forums, peer-reviewed papers, and information regarding GeoGebra Institutes in certain regions. The primary aim in proposing GeoGebra was to provide an open-source, functional, easy-to-use solution for understanding geometrical, algebraical, and calculus fields of mathematics, which are not commonly addressed in a common program (see Figure 2.1.1.1). When published online in 2002, GeoGebra gained rapid popularity and attracted the attention of teachers, many of whom approached Markus Hohenwarter after the software was introduced in 2002. An enthusiastic reception was further fortified by the praise GeoGebra received through numerous educational technology awards, including the European Academic Software Award in 2002.

Figure 2.1.1.1. Connected Algebraic and Graphical Representations within GeoGebra



Source: Adapted from Hohenwarter, M., Jarvis, D. & Lavicza, Z. (2008). Linking Geometry, Algebra, and Mathematics Teachers: GeoGebra Software and the Establishment of the International GeoGebra Institute

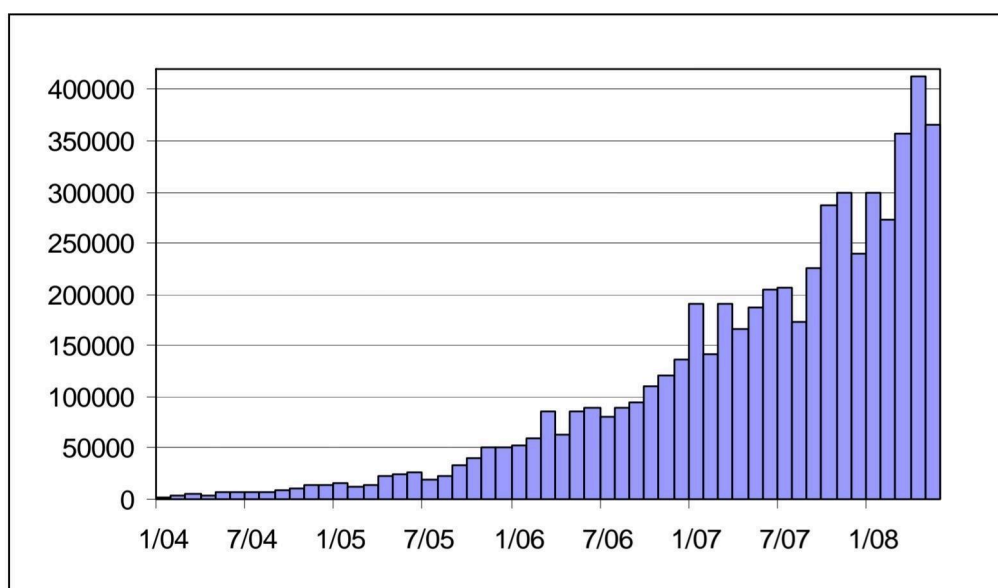
Encouraged by this early success, Hohenwarter worked to further promote GeoGebra development in a doctorate program that aimed at teaching pedagogical uses in Austrian schools. Hohenwarter began teaching at Florida Atlantic University (FAU), as a visiting professor in August 2006 and entered a teacher education initiative called the Mathematics and Science Partnership (MSP), funded by the National Science Foundation (NSF). His scholarship for this project has focused on enhancing teachers' mathematical content knowledge and mathematical implementation through the use of technology-rich classrooms. The MSP partnership with local schools has allowed him to continue developing GeoGebra according to the feedback and experience gained while collaborating with middle and high-school teachers (Hohenwarter, M., Jarvis, D., & Lavicza, Z., 2009).

Since 2002, Markus Hohenwarter has been asked to present lectures and workshops in GeoGebra in various teacher associations, universities, as well as international conferences in Europe and North America. Invited to participate have

been teacher associations' professional events, such as NCTM affiliate conferences, as well as academic institutions and international scholarly conferences. Engagements of this kind demonstrate the increasing academic and professional acceptance of GeoGebra as a creative tool in the field of mathematics instruction. In addition to making professional applications, GeoGebra is often adopted in educational research, used in academic projects and supported by publishing textbooks that contain GeoGebra-based supplementary products. A software of nine geometry programs was chosen for consideration in the Interoperable Interactive Geometry for Europe (Intergeo) initiative and carried out for the 2007-2010 period. Alongside this, an international community of volunteers has been responsible for translating GeoGebra into approximately 40 languages through the help of an online localization system (Hohenwarter, M., Jarvis, D., & Lavicza, Z., 2009). Now, the GeoGebra website garners more than 350,000 monthly visits by users in 188 countries, and according to shared resources and online activity, roughly thousands of mathematics teachers from across the globe are currently using GeoGebra (Figure 2.1.1.2).

As educators began to ask more questions and request help, Markus Hohenwarter created the GeoGebra User Forum and the GeoGebra Wiki to serve as a source of continuous user support and constant access to a larger and larger collection of freely available resources. These tools grew into large repositories in the form of discussion posts and dynamic mathematics worksheets. These activities had become common, even within the same community of in-service teachers, but by mid-decade there hadn't been a formally structured professional development program specifically targeting GeoGebra for in-service teachers. A few colleges and universities had already started integrating GeoGebra into their teacher education curricula but it was found that some of the relationships and coordination between these institutions remained limited. Given the need to facilitate teacher professional development while ensuring the continuous technical development of the software and coordination of research projects based on GeoGebra, Hohenwarter and his team established an umbrella organization called the International GeoGebra Institute.

Figure 2.1.1.2. Visitors on GeoGebra Website per Month for the Period 2004-08



Source: Adapted from Hohenwarter, M., Jarvis, D. & Lavicza, Z. (2008). Linking Geometry, Algebra, and Mathematics Teachers: GeoGebra Software and the Establishment of the International GeoGebra Institute

As per the section titled “Short History of GeoGebra”, GeoGebra was created by Markus Hohenwarter in 2001–2002 in response to his master’s work related to mathematics education and computer science at the University of Salzburg, Austria. Through financial grants provided by the DOC scholarship from the Austrian Academy of Sciences, Hohenwarter continued to develop the software as a component of his doctoral research in mathematics education. During the time, GeoGebra was awarded many accolades from international awards including educational software competitions in Europe and Germany; it was translated into more than 25 languages due to the efforts of mathematics educators across the globe. In its founding phase, GeoGebra has been the responsibility of everyone from the Austrian Ministry of Education to early stage funding from the United States National Science Foundation project “Standard Mapped Graduate Education and Mentoring.” In 2021, GeoGebra entered the BYJU’S educational technology group, making available to hundreds of millions of learners an extensive worldwide network of

digital learning on its platform. Despite this integration, GeoGebra's apps, teaching aids, GeoGebra Classroom, and related features are accessible without cost. In addition, GeoGebra has remained separate from its BYJU'S group structure, where its early owners and development team continue to lead.

Software Development. The open source development community of GeoGebra focused around the development of GeoGebra 4, which was planned for formal publication at the International GeoGebra Conference held in Hagenberg near Linz, Austria, from 29 to 31 August 2011. The major advances presented in this release are introduced in later sections, and a beta release has already been released as part of the GeoGebra website page under the "Roadmap" section. GeoGebra 4 was developed and released with a redesigned user interface that can be updated, and the new design allows users to change views in a more convenient way via drag and drop. Furthermore, view-specific formatting toolbars provide rapid access to parameters (such as your object's color, size, style, etc.) that are specific to the view. Such pre-defined interface perspectives — for example, those of one specific field (like primary education, algebra, or statistics) — allow you to modify the workspace with a single click. They, along with new tools including a digital pen and an on-screen keyboard, enhance the compatibility of their functions with interactive whiteboards and tablet devices. With implicit curves, inequalities, scriptable buttons, and many new commands like those for elaborate constructions such as Voronoi diagrams, it also adds its mathematical powers. On the whole, GeoGebra 4 is a major step forward in terms of flexibility, and also a greater scope of power, but still very easy to use and more user friendly than the previous versions.

GeoGebra Mobile. The GeoGebra Mobile initiative aims to enable the use of GeoGebra web applets, commonly referred to as dynamic worksheets, across a broad range of devices, including smartphones operating on platforms such as iPhone and Google Android. To support this development, key components of GeoGebra are being adapted to the web-based programming language JavaScript. This transition allows GeoGebra's online resources to be accessed directly through modern web

browsers without requiring a Java plugin, both on touch-enabled mobile devices and on desktop or laptop computers. With the release of GeoGebra 4, GeoGebra Mobile applets are intended to become the standard format for exporting dynamic worksheets that display only the graphical view.

GeoGebra 3D. GeoGebra 3D is designed to introduce an intuitive three-dimensional geometry and graphics environment within GeoGebra that can be easily operated using a mouse. This feature enables users to construct and interactively manipulate a wide range of three-dimensional objects, including points, lines, polygons, spheres, polyhedra, and surface plots defined by functions of the form . The 3D view is intended for use within the standalone GeoGebra application and is also designed to support integration into interactive web-based environments.

2.1.2 Features

Technological innovation performs a vital part during the instruction and acquisition of mathematical concepts; influences the mathematical subject content presented and promotes the learning process for students (Selvaraj, N., 2024). GeoGebra is a versatile and all-in-one mathematical software environment that provides tools to work with any language program that connects with your program, including the three-way and horizontal vectorization, binary arrays, and multi-dimensional data, among other things. It is utilized extensively in teaching, learning, and exploratory mathematics education. The program allows the user to graph functions, equations and inequalities in a Cartesian coordinate system and plot points, constructs and scales a line/curve and customizes graphical representations. It has also built-in algebra functionality for symbolic computation, in the form of defining variables, algebraic expressions and equations, factoring polynomials, function problems, etc.

GeoGebra also gives you calculus functions like differentiation and integration to compute derivatives, test limits, definite and indefinite integrals, and views for functions and behavior. A spreadsheet function enables the organization and analysis of data through statistical analysis, chart generation, and the use of mathematical

functions on data. A feature of GeoGebra that highlights its power is that, in the dynamic interaction, you are able to move objects or parameters around at any time using sliders, checkboxes and input fields and get immediate visual feedback in the form of visual and conceptual content. Салынған моделдерді ұлкейтуге және кішірейтуге, жазықтық және кеңістіктік фигуралардың қасиеттерін зерттеуге мүмкіндік беретін эксперименттер жүргізуге, яғни геометриялық түрлендірілерін орындайға болady. Сонымен қатар, model design процесін анымауыалайға және беіне retinde кóрыға болady (Zhubanov, 2021. In Kazakh). The software supports scripting features in the format of GeoGebra Script, for automating operations, customised tool development, and creating interactive applications. Moreover, GeoGebra naturally connects geometric/algebraic and numerical representations, making its analysis from an extensive international perspective, and being an online community with great resources such as tutorials, lesson plans, and user generated interactive contents through its official GeoGebra site. GeoGebra features desktop options for Windows, macOS, Linux, tablet apps (Android, iPad, Windows), and a web-based app based on HTML5. GeoGebra is a powerful versatile and versatile math tool that offers a range of mathematical operations and operations at differing levels of complexity. It has been shown to have a wide range of users such as students, teachers, researchers, researchers - even scientists, scientists (the ones writing), mathematicians etc., with massive benefits based on numerous studies. The platform features a simple and non-technical user interface which is friendly and can be easily navigated, easily available for inexperienced users. It is therefore considered a useful resource that many students find in educational level systems of all grades, in both academic and non-academic learning, from the elementary level to higher education, of studying and teaching mathematics, and can be used as an aid to the teachers. The dynamic nature and compact elements of GeoGebra support active, exciting and brief learning of mathematics, science and other such disciplines in the sciences, in tune with the technology-based education challenges of the 21st century. In pedagogical terms, GeoGebra helps teachers to simplify The process of creating visual and dynamic

educational materials such as virtual experiments, simulations, and mathematical models that may be adapted to various levels of difficulty. This flexibility supports differentiated instruction and accommodates diverse learning styles within the same classroom (Siregar, T., 2025). In general the GeoGebra tool proves to be very much advantageous in the domains of algebra, calculus, physics, linear programming etc, to better the students' performance, facilitate the students' deeper understanding, while increasing the students' engagement by visualizing complex concepts. There is increasing empirical literature within mathematics and STEM education that suggests GeoGebra is well-suited for use in current curricula and instructional practices and is associated with promising learning results.

2.1.3 Educational applications

According to Ince-Muslu and Erduran (2021), technology can support students in understanding mathematical concepts. Furthermore, previous studies have reported positive effects on academic achievement and student engagement (Ali et al., 2023; Hanifah et al., 2025). GeoGebra's pedagogical worth is in its ability to be used with varying levels of education and curricular levels. It can do an excellent job of being used at the elementary level to teach mathematical concepts and measurement, and in higher education at secondary levels in order to explore new mathematical models, representations and proofs. Content is designed within the context of real life. This software can provide contextually relevant learning tasks to connect mathematics concepts to daily experiences and make it relevant to today's curriculum changes that emphasise genuine learning and relevance to the real world. When taken in a more contextual manner, GeoGebra can help students view mathematics not as just metaphorical processing but as a practical approach used to investigate and solve real life problems. Also it supports both teacher learning and self-learning enabling flexible integration throughout a wide range of instructional modality such as in-class, blended and total online. In relation to Learning Management Systems, compatibility in the distance learning, particularly in the post-pandemic learning environments, is reinforced. GeoGebra can also ease students' anxiety for

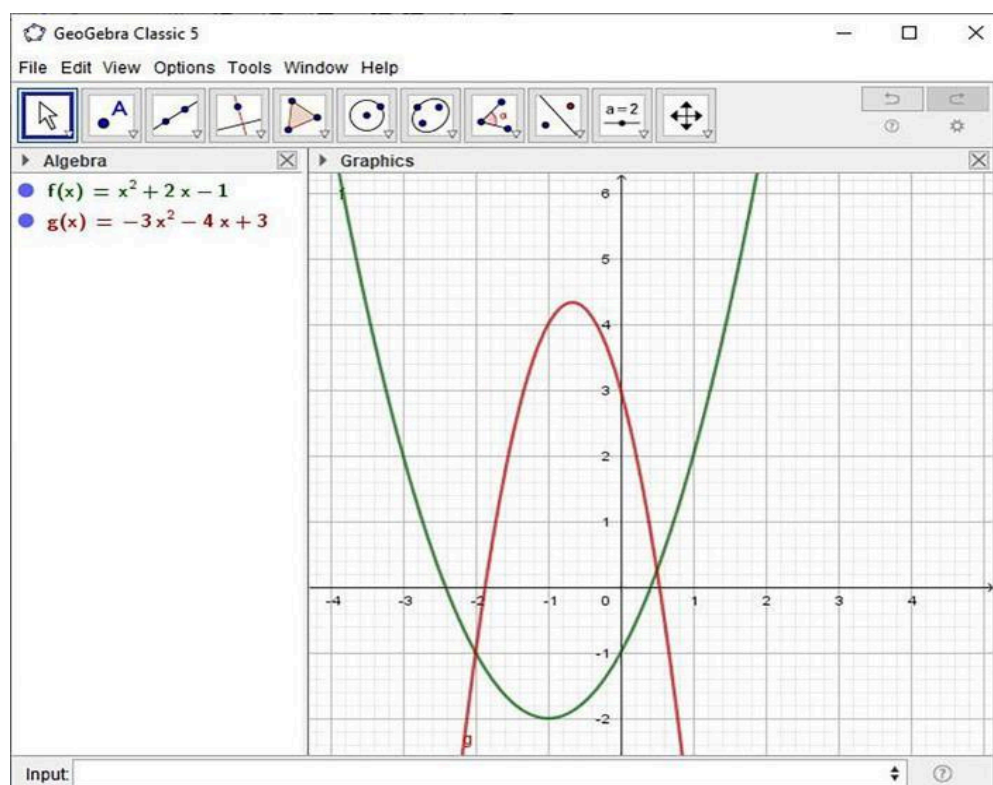
mathematics by visualizing mathematical concepts and models, making learning both more engaging and accessible. That is why GeoGebra has been recognized as a global educational innovation, not only using its technological capabilities but also becoming a tool for teaching development in mathematics education through accessibility, interactivity and applicability. With ongoing upgrades in line with international educational standards, it is continually relevant, showing how carefully designed technology has the potential to make mathematics education more dynamic, participatory, and inclusive. GeoGebra has multiple functions that add to its usefulness as a tool for mathematics education. It is also a free educational, teaching and non-profit solution, which means a low-cost and wide open access. The program's software comes in web, mobile apps, web based and mobile applications, which enables extensive users of many platforms, from Windows, macOS, Linux, and ChromeOS, and web- and mobile apps for iOS and Android. The user interface looks the graphical approach for easy and intelligible browsing and using the tools. GeoGebra is also linked by a vast online repository of thousands of instantly accessible interactive worksheets (interactions) within numerous fields of maths, greatly minimizing the lead time from teacher to student. Detailed technical reports and tutorials are also offered across various languages, supporting the international user base. Within GeoGebra each mathematical object is denoted with standard mathematical notation, such as subscripts and Greek symbols for specific and relevant mathematical expressions with precision in educational contexts. Users can save and export their work in an assortment of formats, including the native .ggb file, dynamic HTML pages, or PNG, SVG and other image formats. The software has built-in support for LaTeX, so users can write well-structured mathematical expressions and text. A distinguishing feature of GeoGebra is dynamic, as all objects are connected and interact with one another and changes to one object will update on the next, keeping all related elements updated together, allowing for conceptual explorations and understanding. Built-in and finished constructions and notes can be shared and used as websites or part of learning management systems, with JavaScript

integration. More recently, GeoGebra has been translated into dozens of languages, further cementing its classification as a global education tool. Taken as a whole, these tools enhance mathematics teaching and learning because they directly indicate the effect of changing parameters on mathematical situations. Objects like points, lines, segments and circles can be rearranged to give greater expression to abstract relations and make it apparent to a more intuitive way of looking at things. Constructions can be built using point-and-click manipulation or by clicking commands on the input bar, which can adapt to diverse users. Figure 1 Screenshot of the GeoGebra interface depicts the convergence between the geometric and algebraic models. The software can display several functions in one working area as shown. The interface includes an algebra view, menu bar, navigation bar, input field, toolbar, and a geometry window or working window. GeoGebra supports teaching/learning geometry, but it also offers powerful algebra teaching features, especially functions and diagrams.

Integrated instruction with GeoGebra has proven to improve student outcomes, reduce the time spent on typical computation tasks, and address difficulties associated with coordinating multiple mathematical representations. Moreover, implementation of such tools enhances student interest and motivation related to core mathematical concepts. This provides a robust justification for the incorporation of dynamic mathematics software like GeoGebra into secondary-level mathematics education, especially for topics that require significant visualization and symbolic reasoning. From a pedagogical perspective, the incorporation of GeoGebra aligns with contemporary educational goals to foster higher-order thinking and digital competence. GeoGebra brings abstract mathematics theory and application together into a model for more meaningful and accessible learning. Its use increases the motivation of students, supports differentiated instruction, and caters for a multitude of learning preferences. In this way, GeoGebra represents a transformative approach in math teaching which integrates technological media and pedagogical methodology to enhance teaching efficiency and student learning outcomes. It offers flexible, convenient access, and emphasizes learner-centered, active learning approaches,

which make it an innovative digital resource, essential for mathematical educators in achieving their students' conceptual understanding, creativity, and learner autonomy.

Figure 2.1.3.1. Screenshot from a GeoGebra Window



Source: Sebsibe, A. & Abdella, N. (2025). The effect of GeoGebra integrated instruction on students' learning of the quadratic function concept

2.2 The role of dynamic geometry software in mathematics education

In elementary mathematics, geometry is known as a subject that deals with shapes and their transformations. It stresses that as geometrical thinking is taken to the next level, the knowledge students gain with geometric activities should support visual and analytical reasoning, as well as hierarchical inference processes at the appropriate levels of attention. The reasoning outcomes based on what students know intuitively are called conjectures, and knowledge from their logical reasoning is referred to as conclusions, but it is noted that only a limited number of students are able to reach such inferential outcomes. Furthermore, the curriculum informs a

consideration of achievement of learning objectives around geometric content, but in particular the cultivation of specific skills, affective traits, psychomotor capacities, and self-regulation. In this context, the Ministry of Education's textbooks explicitly recommend the use of dynamic geometry software and encourage the sharing of related learning experiences with students (MEB, 2007). According to Abromavich (2013) the integration of technology into mathematics teaching can help students recognize the close relationship between mathematics and computing. While digital tools make mathematical knowledge more accessible, mathematical principles can also be applied to enhance computational processes, thereby creating opportunities for learners to explore new mathematical concepts more effectively.

As a form of dynamic mathematics software, GeoGebra has become increasingly widespread in mathematics education worldwide. Beyond enabling dynamic geometric constructions, it supports essential processes in geometry learning, including visualization, estimation, conjecturing, construction, exploration, and proof. Research findings indicate that GeoGebra is highly effective in mathematics education and can be successfully utilized both in teacher education programs (Doğan & Karakırık, 2009) and in supporting students' learning processes (Doğan & İçel, 2010).

Several studies have demonstrated the benefits of technology integration in mathematics education. Sulak (2002) reported that computer-based instruction resulted in higher student achievement and more positive attitudes compared with traditional teaching methods. Likewise, Aktümen and Kaçar (2008) found that the use of Maple contributed to more favourable attitudes toward mathematics. Güven and Karataş (2003) also examined students' opinions regarding the use of Cabri as a dynamic learning environment. At the conclusion of the study, the students' attitudes about mathematics in general, and geometry in particular, shifted positively. A dynamic geometry environment is also very useful to the students. Furthermore, exploratory mathematics exercises have been shown to increase pupils' confidence.

Many evidence-based studies show that GeoGebra has a positive impact on students' mathematical engagement, learning, achievement, and understanding. Experimental and quasi-experimental research in various educational settings regularly report an enhancement in learning results of geometry, algebra, and similar areas where GeoGebra is used as part of an instructional platform (Arbain & Shukor, 2015; Gamage & Charles-Ogan, 2019). Several investigations have also investigated if gender differences in these outcomes are evident. Evidence from previous studies suggests that GeoGebra can positively influence students' mathematical learning outcomes. Lestari et al. (2019) reported that the integration of GeoGebra within a reciprocal peer-tutoring approach significantly improved learners' mathematical communication abilities, while no significant differences related to gender were observed. Likewise, Akobundu et al. (2025) and Gamage and Charles-Ogan (2019) found that GeoGebra-assisted instruction contributed to higher levels of academic achievement and reduced disparities between male and female students.

GeoGebra serves as an interactive geometry environment, allowing users to construct and dynamically manipulate points, vectors, segments, lines, conic sections, and functions. The interface consists of a graphics view and an algebra view, which are interdependent: each algebraic expression corresponds to a geometric object and every geometric construction has an algebraic representation. Using mouse-based geometric tools in the graphics window, users can construct structures, or they can input algebraic expressions, commands, and functions through the keyboard. In the graphics view, the graphical representation of objects is displayed, and in the algebra view, their numerical and symbolic forms are simultaneously shown. In addition, GeoGebra's interface is extremely versatile, and it is possible to customize this to learners' requirements and even use the algebra window, input field, coordinate axes (with grids), drawing pad, and a wide range of geometric tools to meet different instructional and learning needs. In this way, GeoGebra allows students to gain a deeper understanding of the relationship between mathematical objects. This increases its pedagogical value in teaching geometry.

2.3 Comparison of Traditional and Technology-Based Methods in Geometry Instruction

Technology provides a wide range of tools for mathematics instruction, from online learning environments and visual calculators to interactive educational applications, dynamic geometry software, like GeoGebra, and platforms, such as Khan Academy. These technologies have the potential to reshape mathematics teaching and learning by making concepts more visual, interactive, and engaging. Still, the transition towards more technological methods presents some obstacles as we move from traditional teaching methods to technology-enhanced methods. Yet, it is undeniable that the integration of technology in mathematics education gives educators a unique chance to design more stimulating and more personalized learning experiences that develop critical thinking, problem-solving abilities, and mathematical reasoning skills.

In conventional classroom settings, teachers often encounter difficulties in addressing the needs of students with differing levels of mathematical proficiency. Technology supports greater instructional flexibility by allowing learners to engage with content that aligns with their individual knowledge levels, thereby reducing feelings of pressure or exclusion (Denga, 2024). This transformation has been accompanied by broader pedagogical changes in schools and higher education institutions that emphasize active learning, student engagement, and flexible access to educational resources (Engelbrecht & Borba, 2024).

Conventional methods in teaching geometry are largely based on a lecture-centred and formula-related learning system and memorization of the content as a focus with the teacher as the main factor in knowledge transmission. These approaches provide an organised learning context, support an orderly dissemination of the contents of the curriculum, and rely on standard physical resources. But these methods tend to promote passive learning, which can inhibit the growth of higher-order cognitive competencies, such as critical thinking and problem solving, and generally don't involve or deliver prompt feedback to students. So,

technology-integrated approaches are useful in some way. Ntshangase et al. (2024) reported that the integration of GeoGebra into geometry instruction had a positive effect on students' understanding of circle geometry. Based on both quantitative and qualitative findings, the authors recommended the wider implementation of GeoGebra-supported lessons in mathematics education. They also emphasized the importance of strengthening teachers' digital competencies through professional development and ensuring that schools are equipped with adequate technological resources, including computers and relevant educational software. Furthermore, the study suggested that increasing student motivation and interest in learning may contribute to improved achievement in mathematics and geometry, as learner inspiration was identified as an important factor associated with academic performance.

On the other hand, technology-enhanced learning interventions with educational apps (e.g. dynamic geometry software, augmented reality, and virtual reality) promote interactive learning and facilitate real-time visualization of and learner activation in lessons. These tools allow students to directly manipulate geometric objects and instantaneously observe the effects of their actions, making abstraction concrete for those students and providing immediate feedback. It has been supported by ample evidence that when put to use in an appropriate and engaging manner, these technologies lead to better conceptual comprehension, motivation, and achievement in geometry. However, their use is only effective if there is a solid technological infrastructure, the teacher's knowledge and training sufficient, and challenges of distraction and reliance on short-term empirical evidence may arise.

Technology has a high potential to enhance geometry classroom teaching because it can create hands-on, visual and experiential learning experiences that will frequently surpass what is expected with typical practice in terms of engagement and performance. Yet a blended approach to instruction, where the organizational merits of traditional instruction are combined with the technology's interactive and dynamic

features, is often the most successful, serving various learning needs and supporting deeper, more substantial mathematical learning.

2.4 Impact of GeoGebra on students' academic performance in geometry

Geogebra improves students' understanding of geometry, statistics and functions by enhancing problem solving accuracy and their ability to form inferences. In many cases, the experimental group utilizing GeoGebra had better post-test scores. According to N. Arbain & A. Shukor (2014), the study had shown that GeoGebra software had a positive impact on students' achievement in the topic of Statistics. Students have high levels of enthusiasm, confidence, and motivation towards GeoGebra software. Similarly, GeoGebra enables students to create and alter geometric shapes independently rather than simply memorizing a pre-made formula. What students are given is curiosity, a drive to explore unusual experiences, and a sense of fulfillment. Consequently, the learning activities are driven by internal factors instead of those at an external level. According to S. Sebsibe and M. Abdella (2025), students who were taught using GeoGebra performed better in terms of both conceptual understanding and problem-solving skills than those who received traditional instruction. The study also found that learners in the GeoGebra group showed greater motivation toward learning quadratic functions, and these differences were statistically significant ($p < 0.05$). A wrong response in a typical lesson might sap a student's drive. Within GeoGebra environments, a student may alter an item several times and quickly notice and remedy any errors. Also, according to Zhanarbekuly (2025) Geogebra бағдарламалық жасақтамасы оқышылардың геометрияны оқытудағы математикалық қабілетіне оң әсер етті (in Kazakh) and this research was conducted in Kentau, Kazakhstan. So when errors are made you are less fearful and your self-confidence is greater. GeoGebra is useful; it can increase students' intrinsic motivation, reduce their fear of making mistakes, foster their

learning autonomy, and assure active involvement in the classroom. These attributes also directly impact the quality of geometry learning and students' learning results. Previous research has consistently reported positive effects of GeoGebra on students' achievement in geometry (Jelatu & Ardana, 2018; Singh, 2018). They confirmed that GeoGebra teaches their class and helps students learn geometry by helping them grasp the material and expand their understanding of geometry. A considerable number of studies have compared learning outcomes between students taught using GeoGebra and those receiving traditional instruction, with results generally favouring technology-supported learning environments (Jelatu & Ardana, 2018; Khalil, 2017; Khalil et al., 2018; Masri et al., 2016; Seloraji & Eu, 2017; Shadaan & Eu, 2013; Singh, 2018). Building on this body of research, the present study examines the effects of GeoGebra within group-based learning activities. Furthermore, Karataş (2022) reported that students held positive perceptions regarding the use of GeoGebra when learning geometric transformations.

2.5 The role of psychological factors in academic performance

Academic performance in mathematics is affected by different things, not only pedagogical, but also by the underlying psychological aspects. Motivation, interest in lessons, self-confidence, attitude to mathematics, and anxiety levels of students affect an individual's academic performance. Research has demonstrated that some types of technology-based learning tools such as GeoGebra are able to inspire more students' mathematics interest, increasing their involvement in the learning process over recent years. The researchers' observation and the students' responses during the activities revealed that the students' motivation in the class is increased with computer-based activities" (Doğan, M. & İçel, R., 2011). Interactive math programs can have a positive influence on the psychological perception of students and indirectly enhance academic achievements, the researchers stated.

Self-confidence, according to Cambridge Dictionaries Online, refers to the ability to "behave calmly because you have no doubts about your ability or

knowledge," suggesting a calm confidence based in certainty. Fishbein and Ajzen (cited in Parsons, Croft, & Harrison, 2011: 53) declare, "self-confidence is a belief," echoing the truth that it is a matter of the self-confidence. Schoenfeld (as cited in Hannula, Maijala, & Pehkonen, 2004: 17) argues that a person's cognition and emotions are more influential on the understanding and participation in the math process. Indeed, Zimmerman, Bonner, and Kovach (1996: 42-43) also demonstrate that in students who have not achieved considerable academic success, feelings of self-confidence can be an inspiring motivational factor that helps boost motivation. For success, a strong sense of self-assurance is essential. Confident feelings of being capable, comfortable, and, therefore, self-reliant, come before any external reinforcement (Molloy, 2010: 138). For instance, Nur Ghufroon and Rini R.S (2011: 35) describe self-confidence as a personal variable including such qualities as: self-efficacy, optimism, objectivity, responsibility, rationality, and realism. An informal dimension of students' self-confidence in mathematics learning lies in both interacting with students and teachers, as well as between peers (Jurdak, 2009: 111). Educators influence students' confidence and their teaching methodologies heavily influence students' confidence; experience that is difficult yet enjoyable induces more self-confidence in learners (Jossey-Bass Teacher, 2009: 4).

Math anxiety in students is a frequent occurrence. Nowadays almost all students, regardless of level difference, feel anxiety to some extent. Due to the fact that various people react differently to studying and performing mathematics, there are several methods to explain mathematics anxiety. Math anxiety encompasses feelings of tension and apprehension that hinder individuals from effectively engaging with numbers and tackling a range of mathematical challenges, both in everyday situations and academic contexts. In this context, GeoGebra can be utilized to minimize mathematical anxiety, allowing the student to better organize his thoughts. Numerous articles explain how to accomplish that. The classroom teaching method correlates with anxiety around mathematics. In particular, students' anxiety related to mathematics can be reduced or increased depending on the teaching strategies used

by the teachers. According to the study, several teaching strategies might lessen students' anxiety related to mathematics. These strategies include promoting conceptual comprehension, student participation, and supportive surroundings. On the other hand, learners may experience elevated levels of anxiety as a result of stricter or performance-oriented teaching methods. This emphasizes how crucial it is to use compassionate and successful teaching techniques to foster a supportive learning environment for kids who are experiencing anxiety related to arithmetic. A lot of classrooms still teach traditional methods with an emphasis on skills developing. Instead of encouraging a deeper comprehension of mathematical concepts, the emphasis on skill acquisition via repeated practice and assessment can produce an environment that heavily emphasizes performance results. This may cause increased anxiety, particularly in students who might find it difficult to understand some mathematical concepts or who worry about making errors when under pressure. Therefore, in order to successfully address and decrease mathematics anxiety, educators must take into account more holistic and inclusive teaching practices that promote conceptual knowledge, problem-solving abilities, and supportive classroom settings. Cruikshank and Sheffield (1992) in Rossnan (2006: 2) claimed “if teachers fail to implement seven important measures they then cause their students to learn math anxious behaviors”. And they proposed seven important or, one might say, necessary measures for teachers.

The following actions are suggested for the teachers:

- Express a fondness for maths.¹²
- Make math fun.
- Illustrate how mathematics is used in daily life and in vocations.
- Tailor lessons to the interests of the pupils.
- Set achievable, short-term objectives.
- Offer productive activities.
- Incorporate meaningful teaching strategies to help students understand maths.

According to Rossnan, educators should focus more on classroom design in order to lower math anxiety among students. They should also make an effort to establish an environment that celebrates each student's accomplishments and abilities. The application of mathematics to students' daily life must be meaningful. Students love experimenting as well. According to Rossnan, all instructors must include technology, promote co-education, and apply mathematical manipulation in math instruction for all pupils. Because it benefits pupils. Mathematical education researchers frequently emphasize the use of games, activities, and manipulatives to help students feel as though mathematics is a part of their everyday lives. This practical method not only improves comprehension but also fosters creativity and problem-solving abilities in the field of mathematics.

3. METHODOLOGY

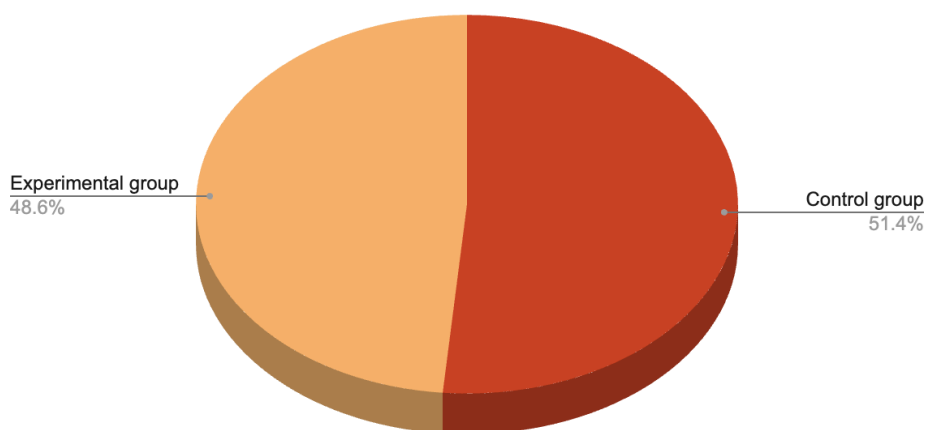
This study employed a quasi-experimental research design involving non-equivalent control and experimental groups with both pre-test and post-test measurements. A quasi-experimental approach is commonly used when random assignment of participants is not feasible, combining certain characteristics of experimental research with the practical advantages of observational designs (Hulley, 2013; Maciejewski, 2020). The purpose of adopting this design was to examine and compare the impact of GeoGebra-supported instruction and traditional teaching methods on students' academic achievement.

3.1 Participants

The national educational system in Kazakhstan is divided into several education levels that are arranged in ascending order from primary school to higher education levels. Kazakhstan's schools use different teaching strategies and methods. "125 High School" from Kyzylorda city was chosen for this study and the school provides education in Kazakh language. "125 High School" is a network of private specialized schools in Kazakhstan, focused on the education of high school students (grades 9-11), aimed at preparing for UNT. The school is known for a high proportion of grants (up to 98%) for admission to higher education institutions and an intensive academic base with branches in the cities of Almaty, Aktobe and Kyzylorda. Our experiment took place in the filial in Kyzylorda. Textbooks and school programs in the school are compiled according to the school program approved by the Council of the National Academy named after Ibray Altynsarin. Since only grades 10-11 were taught at this school, students of the same direction gather in each class, choosing the same subject. So the students of the experimental and control groups chose biology and geography subjects for the UNT, the class studies these subjects with in-depth study. That is, both classes are called BioGeo. In this school, each class has a name according to the subject.

“125 High School” allowed students to participate in this experience. Also consent forms were obtained from the parents/guardians of the students participating in the experiment. In total, there were 35 students from 10-th grade. 17 students were in the experimental group and 18 students were in the control group. The age of the students who participated in the experiment is 15-16 years old. The figure below visually shows the percentage distribution of students between the experimental and control groups, indicating a nearly balanced sample.

Figure 3.1.1. Percentage distribution of between the experimental and control groups



3.2 Instrumentation

In this study, pre-test, post-test and questionnaire (questionnaire) were used as the main measuring instruments for data collection. The research tools were aimed at comprehensively assessing the level of geometric knowledge of students and comparing the effectiveness of teaching methods. The pre-test and post-test were compiled in the same structure and fully covered the research topics in terms of content. Each test consisted of 20 tasks, 10 of which were presented in the multiple choice format, and 10-in the form of an open-ended question. Among the Multiple choice questions, 2 tasks were given in true/false format. Open questions were aimed

at determining the level of deep understanding of students, the ability to express their thoughts mathematically and the skills of logical thinking.

The content of the test tasks covered the following topics: the concept of vectors in space and the operations applied to them; a point in space and the coordinates of a vector; the dot product of vectors and the division of a segment in a given ratio (see table x). The pre-test was conducted before the start of the experiment in order to determine the initial level of knowledge of the students, and the post-test was removed after the end of the experiment and allowed to identify the changes that occurred as a result of learning. In addition, the tasks were taken from 3 different books: Geometry (B.A. Smirnov & A. E. Tuyakov, 2019), Geometry problem book (A.A. Amzebek, 2022), Mathematics (problem collection, 2021). The test tasks were checked by an expert before conducting the study, their substantive validity was ensured and their use was allowed. Additionally, school textbooks and curricula are compiled according to the school program approved by the Council of the National Academy named after Ibray Altynsarin. Accordingly, the subject of geometry is conducted according to the book of A.N. Shynybekov (2019).

Figure 3.2.1. Examples of remembering part

5. Condition for the collinearity of vectors:

A) $x_1 + x_2 = y_1 + y_2 = z_1 + z_2$

B) $\frac{x_1}{x_2} = \frac{y_1}{y_2} = \frac{z_1}{z_2}$

C) $x_1 = y_1 = z_1; x_2 = y_2 = z_2$

D) $x_1 x_2 = y_1 y_2 = z_1 z_2$

6. If nonzero vectors lie on the same line or on parallel lines, what kind of vectors are they?

A) Perpendicular vectors

B) Equal vectors

C) Collinear vectors

D) Unit vectors

The test tasks were compiled on the basis of Bloom's Taxonomy, which made it possible to assess students' knowledge at various cognitive levels. In particular, the "Remember" level included 6 questions aimed at testing students' ability to memorize basic definitions, formulas, and concepts. Examples are shown above (figure)

At the "Understand" level, 4 questions were asked, of which 2 questions were presented in the true/false format and were devoted to identifying students' skills in understanding theoretical material and interpreting it. Examples:

Figure 3.2.2. Examples of understanding part

9. What does it mean for two vectors to be perpendicular?
 - A) The vectors are parallel
 - B) The vectors are identical
 - C) The angle between the vectors is 90°
 - D) The vectors have the same length
 - E) The vectors are collinear
10. Why is the dot product of two perpendicular vectors equal to 0?
 - A) Because their coordinates are equal
 - B) Because their lengths are equal
 - C) Because $a \cdot b = |a| |b| \cos 90^\circ$
 - D) Because they are parallel
 - E) Because their coordinates are proportional

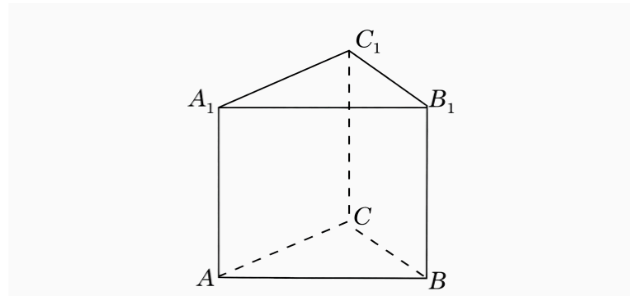
And 10 questions covering the "Apply" and "Analyze" levels were aimed at assessing the students' ability to apply the acquired knowledge in reports, analyze and effectively use them in various situations. Also, all of the tasks in this part were in the form of open-ended questions.

The open-ended questions were aimed at assessing students' high-level thinking skills, including the ability to analyze, reason, and compile reports. These tasks required students not only to answer correctly, but also to justify their decisions

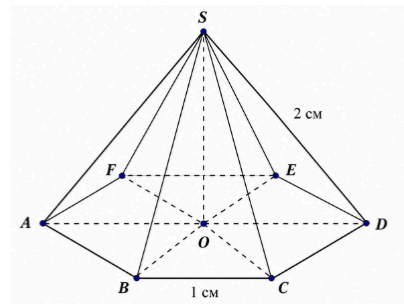
and apply mathematical concepts in various situations. One example of such tasks used in the test is shown in Figure 3.2.3.

Figure 3.2.3. Examples of “apply and analyze” part

19. In the prism $ABCA_1B_1C_1$, identify three coplanar vectors.



20. In the regular hexagonal pyramid $SABCDEF$, the side lengths of the base are equal to 1, and the lateral edges are equal to 2. Find the scalar (dot) product of the vectors $\vec{SA}$ and $\vec{SD}$.



Thus, the test structure made it possible to comprehensively assess not only the students' knowledge, but also the level of their application and analysis.

After the experiment was completed, a survey was conducted among the students of the experimental group in order to collect additional data. The questionnaire questions were taken from a research paper called “Effectiveness of GeoGebra in teaching and learning Geometry” (Dipesh Prajapati, 2021) and adapted and developed based on this study. So, the study used a survey to determine the attitudes, motivation, attendance activity, and emotional experiences of GeoGebra students. The survey was conducted only for students in the experimental group after the experiment was completed, as this group used the GeoGebra-based learning

method. The survey was aimed at assessing students' opinions about the effectiveness of the GeoGebra program in teaching geometry. In particular, the tool made it possible to explore such aspects as:

- understanding geometric concepts;
- perception of visualization;
- motivation for mathematics;
- classroom activity;
- self-confidence;
- the level of mathematical anxiety;
- a vision for the future use of GeoGebra

The survey consisted of 10 statements, and a five-level Likert scale was used to answer each statement:

- strongly agree
- agree
- neutral
- disagree
- strongly disagree

8 conclusions in the survey were positive, and 2 were negative. Negative conclusions were introduced in order to reduce bias in student responses and increase the reliability of responses. Questions 4 and 7 were used as the reverse conclusions. During the data analysis, the reverse coding method was used for these questions, that is, the answers were recorded so that higher scores reflected a positive attitude.

The survey content was developed in accordance with the purpose of the study and the impact of the GeoGebra program in the educational process. The survey was taken from the research of Dipesh Prajapati (2021) and adapted according to this research. The conclusions were aimed at covering the cognitive and affective experiences of students. Examples of questionnaire items include:

Figure 3.2.4. Examples of the Questionnaire

5	GeoGebra reduced my anxiety when solving geometry problems					
6	The use of GeoGebra enhanced my engagement during lesson					
7	I would prefer traditional teaching methods over GeoGebra					

3.3 Validity and Reliability

Pre-test & Post-test. To effectively contribute to knowledge growth, tests must be legitimate and accurately measure skills and abilities based on learning outcomes. As noticed before in the part of “Instrumentations”, the subject teacher, expert and supervisor helped ensure the validity of the achievement test. The expert that checked the materials was the author of the geometry textbook A. E. Tuyakov and my supervisor Nuri Balta.

In this research, the pilot test was done to determine the reliability of test items based on the research of Dipesh Prajapati (2021). The researcher administered the pilot test to students of “125 High School”, Kyzylorda. Pilot studies give researchers with preliminary data to help them understand the potential outcomes of an intended experiment. The purpose of the pilot study was to pre-check the clarity, structure, and reliability of the test tasks used. For this purpose, students of the 10th "G" class who do not participate in the experiment, but have a similar level, were selected.

Based on the results obtained, the internal sequence of test tasks was calculated using the Cronbach's Alpha coefficient. The common method of calculating reliability indicators on a survey scale is Cronbach Alpha (Ghazali, 2016). Also, the Cronbach's Alpha method was specifically employed as suggested by Aithal and Aithal (2020). The students were given a 20-task test. Their answers were coded as 1 point for a correct answer and 0 points for an incorrect answer in questions that have multiple

choice. The open-ended questions were assessed using an analytic scoring rubric ranging from 0 to 4 points based on the accuracy and completeness of students' responses:

- 0 – incorrect/no response
- 1 – very limited understanding
- 2 – partial understanding
- 3 – mostly correct response
- 4 – completely correct and well-explained response

The alpha Cronbach coefficient was calculated using the Python programming language. As a result of the calculation, the value $\alpha = 0.742$ was obtained. This indicator shows that the internal sequence of test tasks is quite high and the research tool is reliable. The alpha Cronbach coefficient above 0.7 indicates that the reliability of the test is at an acceptable level, so the results obtained ensure the reliability of the conclusions of the study. Thus, the test used was evaluated as a stable and reliable measurement tool that meets the goals of the study. In addition, the students' total scores are shown below as a graph and it is done by a python program.

Questionnaire. The questionnaire was thus designed to establish its validity in that it measured students' perceptions and attitudes toward the use of GeoGebra in geometry learning. Validity refers to the degree to which an instrument measures what it is intended to measure. Content validity was taken as the predominant validity type in this research. In order to ensure content validity, items in the questionnaire were reviewed by the expert and supervisor. The expert rated all items on their relevance, clarity, and alignment with the objectives of the study. The statements were especially scrutinised to ensure they truly represented students' experiences with GeoGebra-based instruction. In general, the expert review further confirmed that the items on the questionnaire were adequate and representative of the construct that had been measured. Hence, the content validity of the questionnaire is acceptable and can be used in this study.

In this study, the reliability of the questionnaire was examined to verify the consistency and stability of the measurement instrument used in the study. Reliability is the extent to which the same measurement produces consistent outcomes given similar conditions. That means a reliable questionnaire provides stable and internally consistent responses across its items, reflecting the same underlying construct. To investigate students' views on the use of GeoGebra in geometry, a 10-item questionnaire was administered. Responses were recorded on a five-point Likert scale ranging from "Strongly Disagree" (1) to "Strongly Agree" (5), allowing the assessment of students' motivation, attitudes, and perceptions. This scaling method allowed for the quantification of students' opinions and facilitated statistical analysis.

Before conducting the reliability analysis, negatively worded items were identified and reverse-coded to maintain consistency in the direction of responses. Specifically, items 4 and 7 were negatively phrased and were transformed using reverse coding to ensure that higher scores consistently represented more positive attitudes toward GeoGebra. This step is essential to avoid distortion in the calculation of internal consistency.

The internal consistency of the questionnaire was evaluated using Cronbach's alpha coefficient. And it was calculated by python programming. As a result of the analysis, the value of Cronbach's alpha $\alpha = 0.77$ was obtained. This value indicates a good level of internal consistency between the questionnaire items. According to generally accepted standards, an alpha coefficient above 0.7 is considered acceptable. Thus, the result confirms that the questionnaire items are sufficiently correlated and reflect the same basic concept. Overall, the findings suggest that the questionnaire is a reliable tool for assessing students' perceptions of GeoGebra-based instruction.

3.4 Implementation of the experiment

The study used two different teaching methods. In the experimental group, classes were conducted using the GeoGebra program. Geometric concepts, especially topics related to spatial vectors, were explained to the students in this group through

interactive visualizations. The GeoGebra program dynamically displayed the location of objects in space, the direction and length of vectors, as well as the relationships between them, which allowed students to clearly and visually understand abstract concepts. And the control (traditional) was conducted in a group on these topics with the help of the teacher's explanation, based on drawing drawings on the blackboard. In this case, the visualization was static, and all explanatory work was based on the traditional method.

Administration of Pre-test. Prior to the briefing, a preliminary test was performed in both the experimental and control groups. The primary goal of the pre-testing was to identify the students' initial level of understanding of the selected geometry topics, as well as to ensure that both groups had comparable prior knowledge before the experiment.

Testing was done under the same conditions for both groups to guarantee fairness and consistency. All students were given the same amount of time to complete the test, and no additional support or advice was provided throughout the assessment. Before the test, the students were given explicit instructions and asked to answer all of the questions individually.

The pre-test data were gathered and examined to establish that the experimental and control groups did not differ significantly at the start of the study. This assured the comparability of the two instructional approaches utilized in the experiment.

Lesson Procedure. During the study, lessons on the same topics were conducted in two groups: experimental and control, but with different teaching approaches. The classes' content and learning objectives were identical for both groups; the only variable was the mode of instruction. Also both groups had the same teacher. The experiment lasted a total of 6 weeks. Humanities classes, that is, classes that do not choose mathematics, do not study mathematics deeply. For this reason, the experimental group and the control group study geometry once a week. The first week was an introductory one. First, we explained to the students what a Geogebra is,

drew shapes, and showed them how to use it. So we tried to reveal their interest by visualizing the shapes. After the introduction, we took a pre-test. In addition, the lectures addressed the following topics: Coordinates of vectors in space; Collinearity and coplanarity of vectors; Vector addition and multiplication by a number; also, scalar product of vectors (see table 3.4.1).

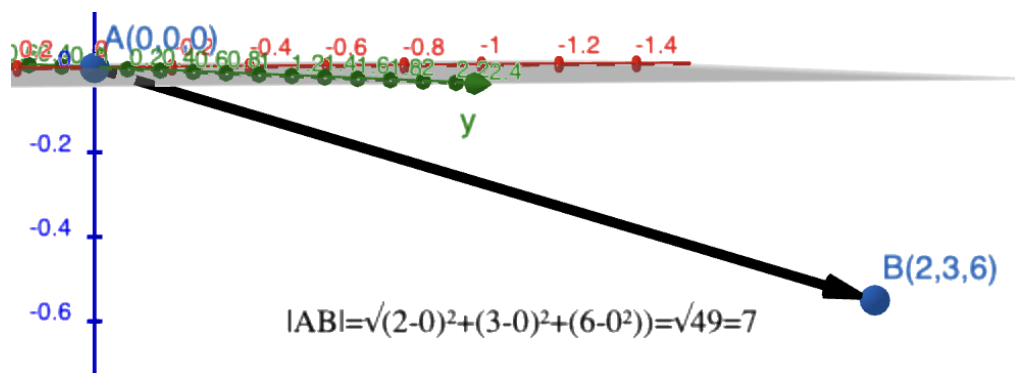
Table 3.4.1. Weekly Lesson Plan for the Experimental Group

Week	Topics	Tool	Activities
1	Introduction & Pre-test	Geogebra, papers	Introduction and administration pre-test
2	Coordinates of vectors in space and finding the length	Geogebra	Demonstration of vector coordinates and graphical representation
3	Collinearity and coplanarity of vectors	Geogebra	Visualization of collinear and coplanar vectors using GeoGebra; dynamic demonstration of vector positions
4	Vector addition and multiplication by a number	Geogebra	Demonstration of vector addition: visualization of parallelogram and triangle methods
5	Scalar product of vectors	Geogebra	Visualization of the scalar product of vectors using GeoGebra; demonstration of the relationship between angle and dot product (positive, negative, and zero cases)
6	Post-test & Questionnaire	Papers	Observation: administration of post-test and questionnaire

Each lesson has a set sequence. The teacher began the lecture by quickly reviewing prior content and introducing the current lesson's objectives. This level aims to activate students' past knowledge and get them ready for new topics. During

the explanation phase, the teacher offered new topics. GeoGebra was utilized by the experimental group to visually illustrate essential geometric principles. For example, when teaching the coordinates of vectors in space, the instructor used a three-dimensional graphic to demonstrate how the coordinates look (figure 3.4.1). The instructor showed several demonstrations with other coordinates and explained how to find the length of the coordinates and also showed what the lengths of the coordinates actually look like by using GeoGebra.

Figure 3.4.1. Length of the coordinate



Similarly, in “collinearity and coplanarity of vectors” lectures, GeoGebra was used to show examples about collinearity and coplanarity of vectors (see figures 3.4.2, 3.4.3 and 3.4.4).

Figure 3.4.2. Collinearity of vectors

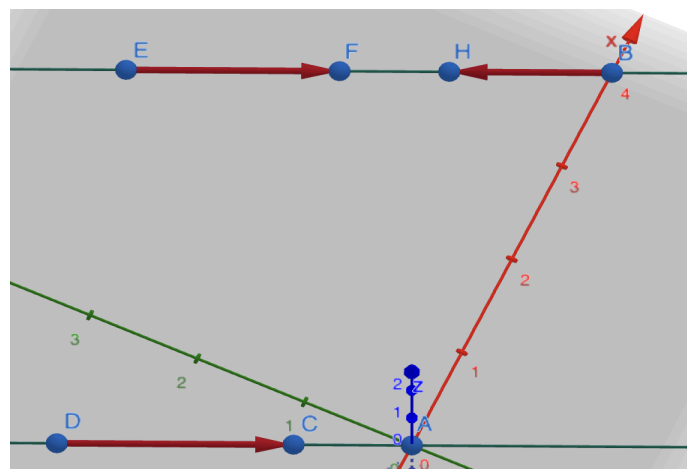


Figure 3.4.3. Collinearity of vectors

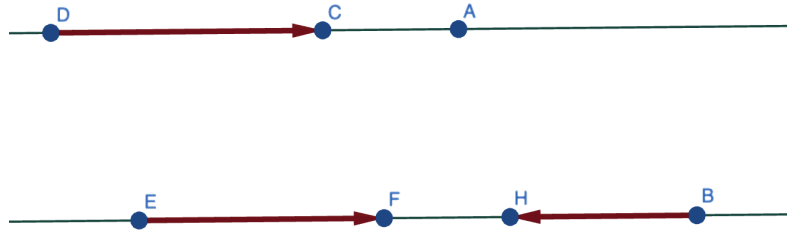
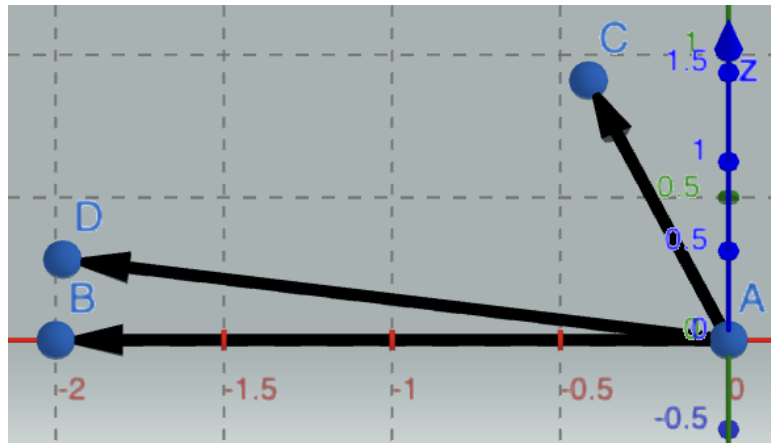


Figure 3.4.4. Coplanarity of vectors



Also, there were other examples for more revealing vectors. For instance, let us be given the figures F and F_1 and the vector $\vec{a}$. When transforming the figure F into the figure F_1 , if the equality $\vec{AA_1} = \vec{a}$ is fulfilled for the overlapping points $A \in F$ and $A_1 \in F_1$, it is said that the figure F_1 was obtained by copying the figure F parallel to the vector $\vec{a}$ (Figure 3.4.5 & 3.4.6).

Figure 3.4.5. Parallel copying the figure via vector

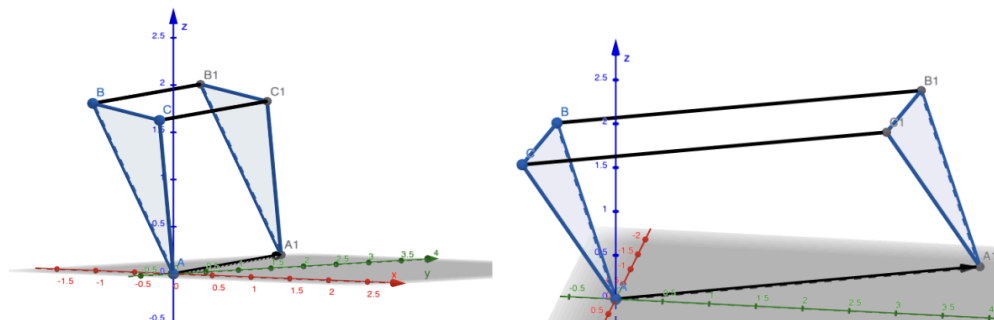
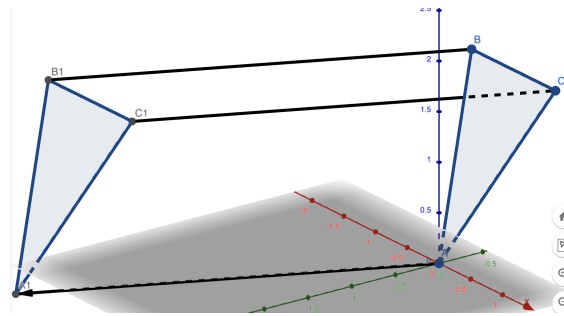


Figure 3.4.6. Parallel copying the figure via vector



In the lesson “Vector addition and multiplication by a number”, the instructor explained methods of vectors’ addition. Consider any two vectors, $\vec{a}$ and $\vec{b}$. We must combine these vectors to obtain the vector $\vec{c}$ that equals $\vec{a} + \vec{b}$. To do this, we use the constraint that a vector does not change when duplicated parallel to itself. Using this rule, we can demonstrate many methods for adding vectors. For instance, we can replicate two vectors parallel to each other such that their ends meet, making two sides of a parallelogram, and then assemble a parallelogram. The resulting vector is the diagonal formed by drawing the two vectors' origins together. This method of adding vectors is known as parallelogram rule addition. Examples from GeoGebra are shown below (Figure 3.4.7 & 3.4.8).

Figure 3.4.7. Visualization of vector addition: triangle and parallelogram methods

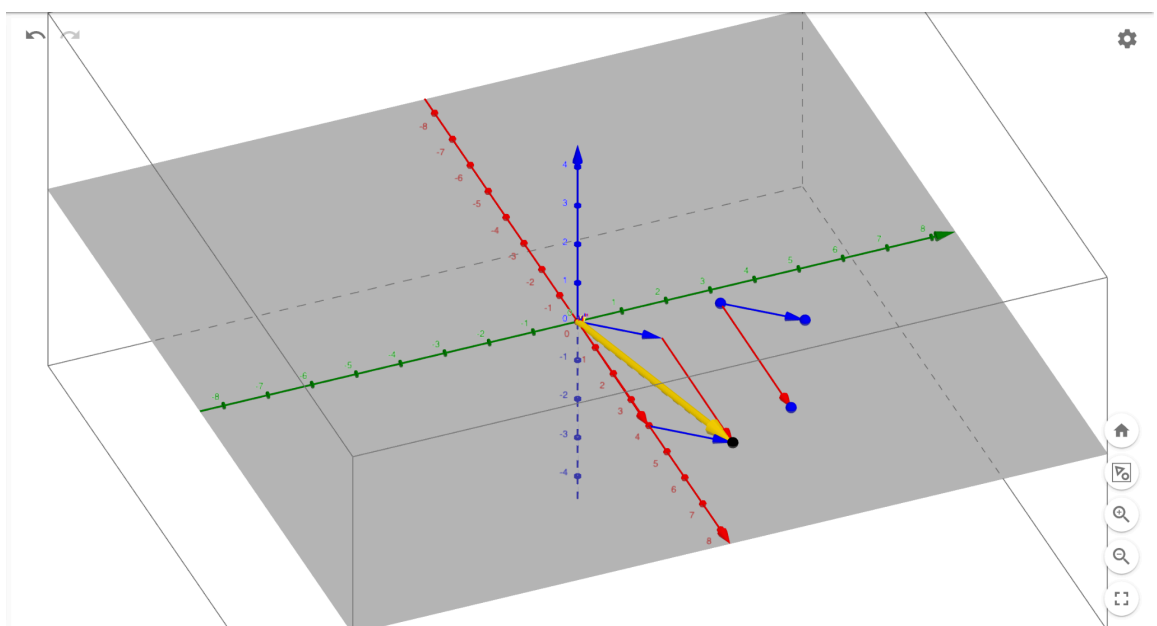
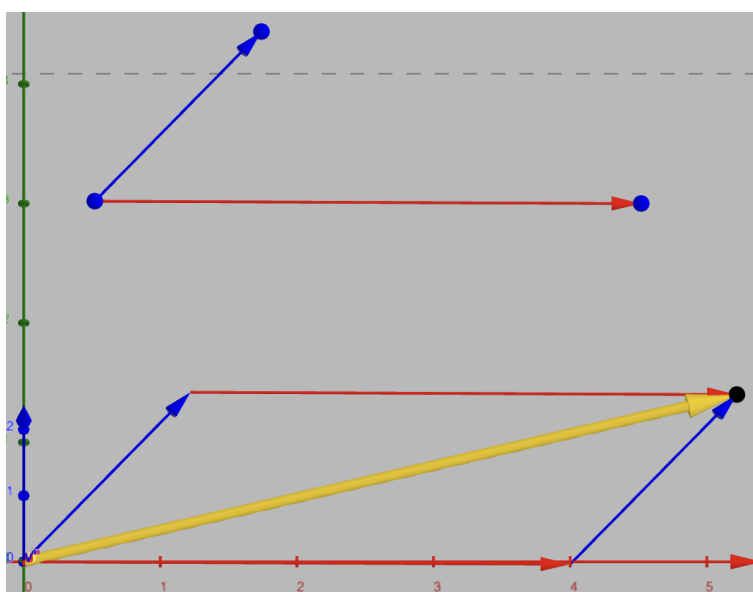


Figure 3.4.8. Visualization of vector addition: triangle and parallelogram methods



You may also use the triangle rule to combine vectors. To do this, we replicate the provided vectors parallel to each other so that the tip of the first vector intersects the tip of the second vector. The vector drawn from the tip of the first vector to the tip of the second vector represents the total of the two vectors. It is shown above (see figure 3.4.8).

In the last topic, GeoGebra visualizations were used to dynamically show the link between the angle created by two vectors and the sign of their dot product. For “Dot product” several examples were shown. For instance, the figure 3.4.9 and figure 3.4.10 shows the dot product of two vectors defined in the coordinate plane. The result is zero when the scalar product is calculated for the given vectors. This case means that the vectors are mutually perpendicular. As we know, the angle is 90 degrees when they are perpendicular to each other. Cosine is 0 in 90 degrees. According to the formula the result will be 0 due to this reason. To explain this in more detail, the dot product is the "projection" of one vector onto the direction of another. And if the vectors are 90°: a vector is not "projected" onto the other, i.e., the projection = 0. Therefore, the dot product = 0. So, using the GeoGebra program, it

was clearly shown that the angle is 90° , and the geometric meaning of the scalar product was clearly explained.

Figure 3.4.9. Scalar product of two vectors

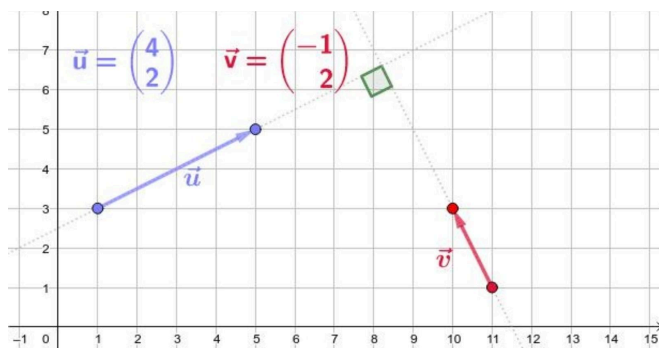


Figure 3.4.10. Visualization of perpendicular vectors

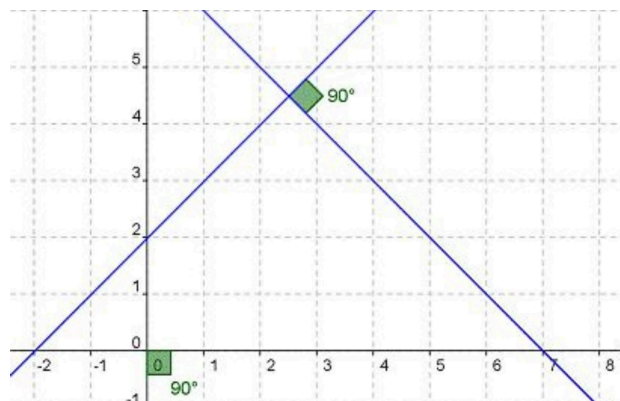
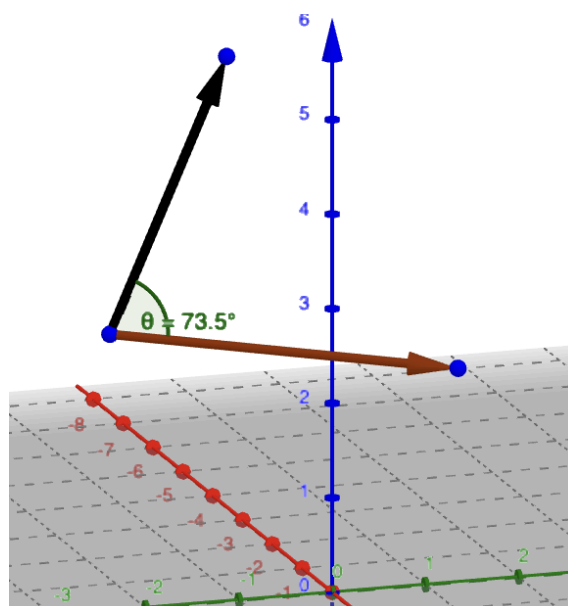
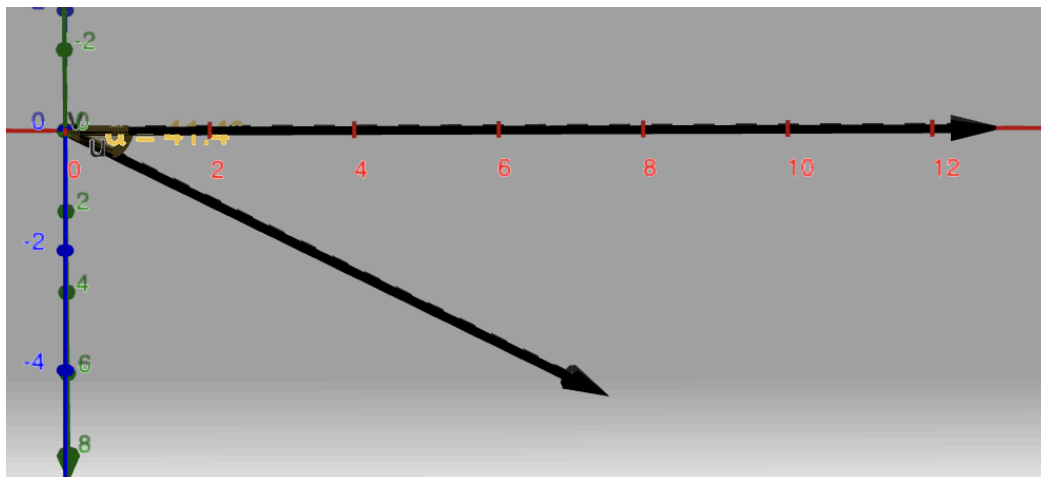


Figure 3.4.11. Visualization of vectors



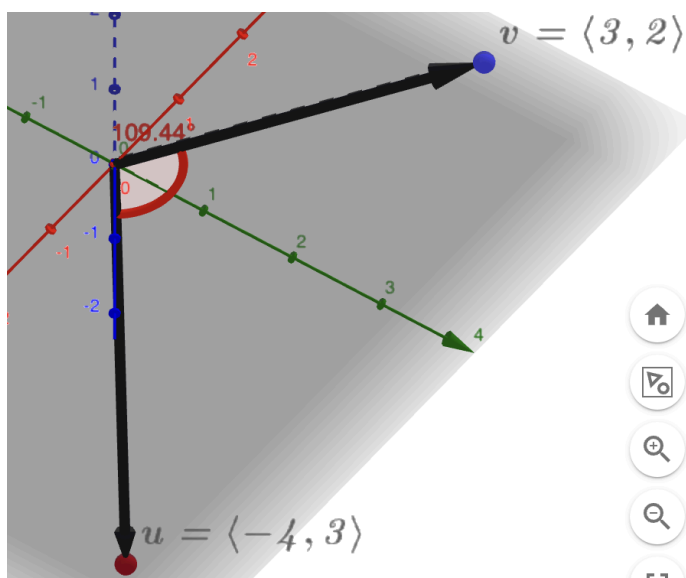
The figure 3.4.11 shows that the angle between two vectors in space is acute (about 73°). In this case, the cosine of the angle is positive, so the dot product is also positive.

Figure 3.4.12. Visualization of vectors related to degrees less than 90°



The figure 3.4.12 shows vectors pointing in the same direction. In this case, the angle between them is very small, and the cosine is close to unity. As a result, the dot product takes on one of the largest positive values.

Figure 3.4.13. Visualization of vectors related to degrees greater than 90°



The figure 3.4.13 shows that the angle between the two vectors is obtuse (greater than 90°). In this case, the cosine of the angle is negative, so the dot product is also negative. This example proves that the sign of the dot product depends on the angle.

Visualizations created using GeoGebra allowed students to gain a deeper understanding of the geometric meaning of the dot product. Specifically, the dependence of the value and sign of the dot product on the angle between vectors was clearly demonstrated. This method allows for the explanation of abstract concepts using concrete images, enhancing student comprehension.

The GeoGebra was used within the lesson as an instructional instrument as scaffolding with Vygotsky's Zone of Proximal Development (ZPD). The teacher initially covered new ideas in the lesson on the board but used GeoGebra visualizations first before students practised this idea with the software themselves. The instructional framework focused on the following topics: coordinates and lengths of vectors in space, collinearity and coplanarity of vectors, vector addition and multiplication by a number, and the scalar product of vectors. At first, the teacher introduced GeoGebra to visualize new concepts, to show formulas and geometric relationships. Dynamic models contributed to learners' spatial concepts while aiding in easier interpretation of abstract mathematical concepts. In the second phase, students experimented with GeoGebra from teacher supervision. They built vectors, defined coordinates, measured vector lengths, and performed vector operations. Students were supported to overcome any problems by the teacher when required. In the next set of stages, students were more independent in performing mathematical problem solving with GeoGebra. They explored scalar products, looked at the angles between vectors, collinearity and coplanarity, and looked at collinearity and coplanarity by doing interactivity and interactive visualizations and problem-solving. Students also built on these learning experiences by implementing more complicated exercises/practice and linking theory to practice.

In the control group, these topics were taught using traditional approaches. The teacher explained the new material on the blackboard and created the relevant

sketches by hand. Vectors and geometric forms were shown statically. The pupils listened to the teacher's explanations, wrote them down in notebooks, and did the tasks assigned later independently. In this group, learning was frequently done under the supervision of a teacher.

The structure of the lesson was the same in both groups: the introductory part (introduction of a new topic), the main part (completion of tasks) and the final part (consolidation and feedback). However, the use of interactive and visual tools in the experimental group made the learning process more active and discussed, while in the control group, learning acquired a traditional, teacher-oriented character.

Administration of Posttest and Questionnaire. Following the instructional procedure, the experimental and control groups administered a post-test. The primary goal of the Post-test was to assess student learning outcomes and measure the efficacy of two distinct teaching approaches utilized in the study. The post-test was done under the identical settings as the pre-test, which was essential for ensuring the reliability and relevance of the results. All pupils were given the same time to finish the test, and the same instructions were provided before it began. The students completed the exam assignments independently, without any assistance, allowing them to establish their true level of knowledge.

The format and content of the post-test were identical to the pre-test. The test has 20 questions, including multiple-choice, true/false, and open-ended questions. The assignments used Bloom's taxonomy to test students' cognitive levels (remember, understanding, applying and analyzing).

The questionnaire, which consisted of 10 questions, was the final component of our experiment. It allowed us to determine what changes the GeoGebra made in their understanding position. The questionnaire is designed to assess students' experience of using GeoGebra and their attitude to studying geometry. The answers were collected on a five-point Likert scale and then used for statistical analysis.

3.5 Data Collection

Data is collected over a 6-week period. To prevent pupils from missing tests, we informed them about the assessments ahead of time and chose appropriate dates, and the school administration and teachers were aware of this.

35 students actively participated in our experimental work. The research protocol included both a pre-post test and a comprehensive questionnaire. On average, participants spent approximately 50-60 minutes completing the pre-post test. In addition, the questionnaire took an average of 5-7 minutes to complete, which facilitated the acquisition of additional data relevant to the research objectives.

3.6 Ethical Considerations

All collected data will be kept secret and anonymised to safeguard the participants' privacy. The data was obtained using Google Sheets, which allows for thorough organizing while protecting personal anonymity. Each student's data was depersonalized by assigning alphanumeric numbers with the prefixes "E01" and "C01" to represent the experimental (E) and control (C) groups. Individual students will not be identified in any papers or presentations, as only generic data was supplied. Participation in the experiment will be optional, and children and parents will be able to withdraw at any moment with no penalties or consequences.

3.7 Normality

At the initial stage of the analysis of the study data, the correspondence of the results of the pre-test of the experimental and control groups to the normal distribution was checked. The determination of the normal distribution plays an important role in the correct choice of the statistical analysis method. The Shapiro–Wilk normality test was used for this purpose. In addition, histogram graphs were built to visually evaluate the results.

As shown below (see table 3.7.1), the Shapiro-Wilk test results in the experimental group showed $W = 0.889$ and $p = 0.045$. For the control group, the

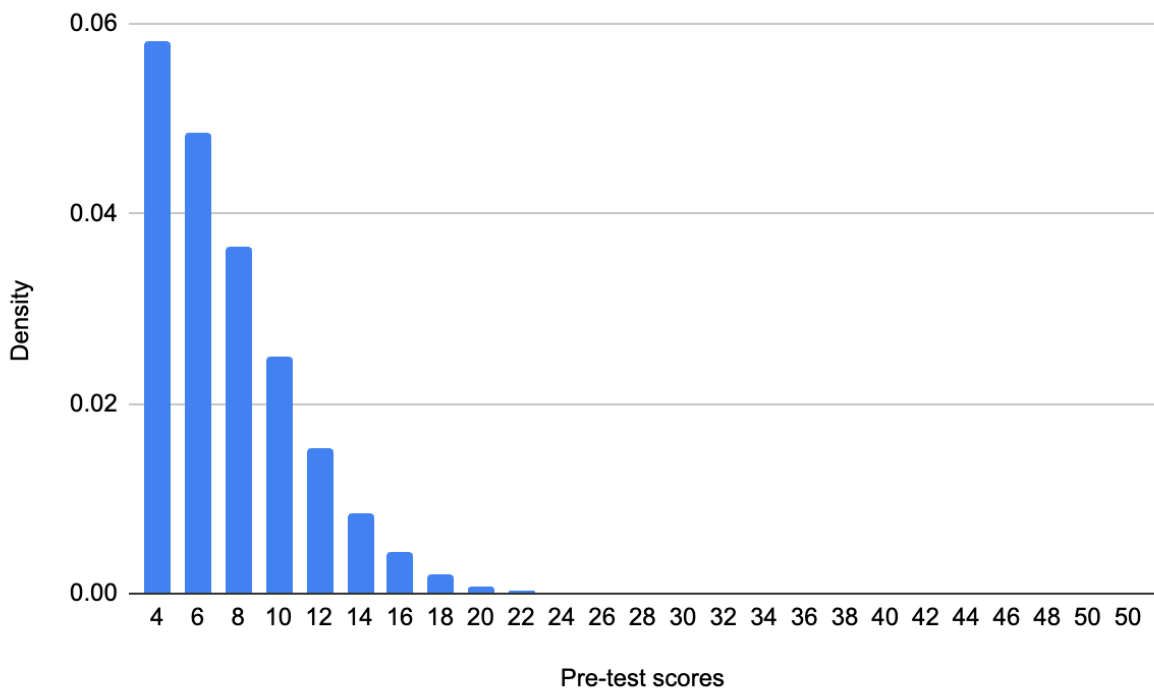
results were $W = 0.876$ and $p = 0.023$. Since the p-value is lower than 0.05 in both groups ($p < 0.05$), the results of the pre-test were found to be incompatible with the normal distribution.

Table 3.7.1. Normality test

Group	Test	W	p-value
Experimental	Pre-test	0.88	0.02
Control	Pre-test	0.89	0.04

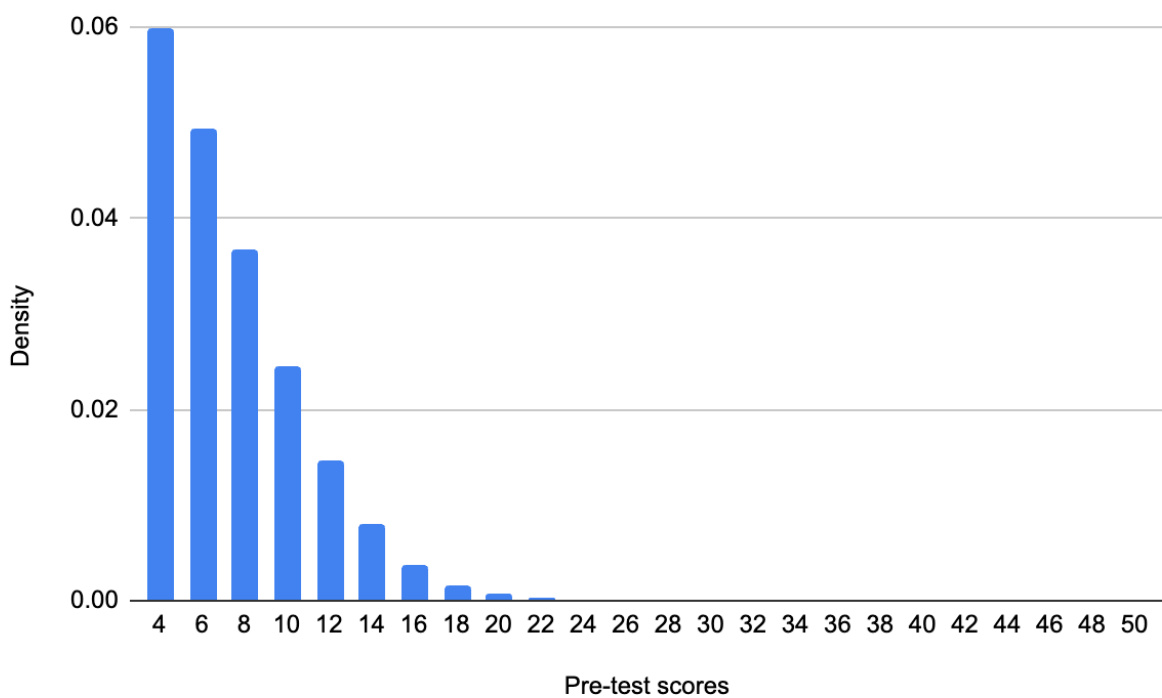
The histogram graph of the experimental group showed that the students' scores were concentrated in some intervals (see figure 3.7.1). The distribution of points is not completely symmetrical and does not correspond to the ideal shape of the Bell-shaped distribution. In addition, higher frequencies in some bands can lead to uneven data distribution.

Figure 3.7.1. Normal distribution of the experimental group



A similar situation was observed on the histogram chart of the control group (see figure 3.7.2). The results are grouped into specific intervals, and the symmetry is not fully preserved in the shape of the distribution. The graph clearly showed that the data does not fully correspond to the normal distribution. The Histogram results confirm the conclusion of the Shapiro-Wilk test.

Figure 3.7.2. Normal distribution of the control group



In general, the results of the pre-tests of both groups did not satisfy the condition of a normal distribution. In this regard, at the next stage of the study, it was proposed to use a nonparametric statistical method for analyzing pre-test differences between groups — the Mann–Whitney U test.

Also, in order to determine whether the results of the post-tests of the experimental and control groups corresponded to the normal distribution, the Shapiro-Wilk normality test was conducted. In addition, histogram and Bell-shaped distribution graphs were used to visually evaluate the results. Checking the normal distribution is an important step in choosing the statistical method used in the study.

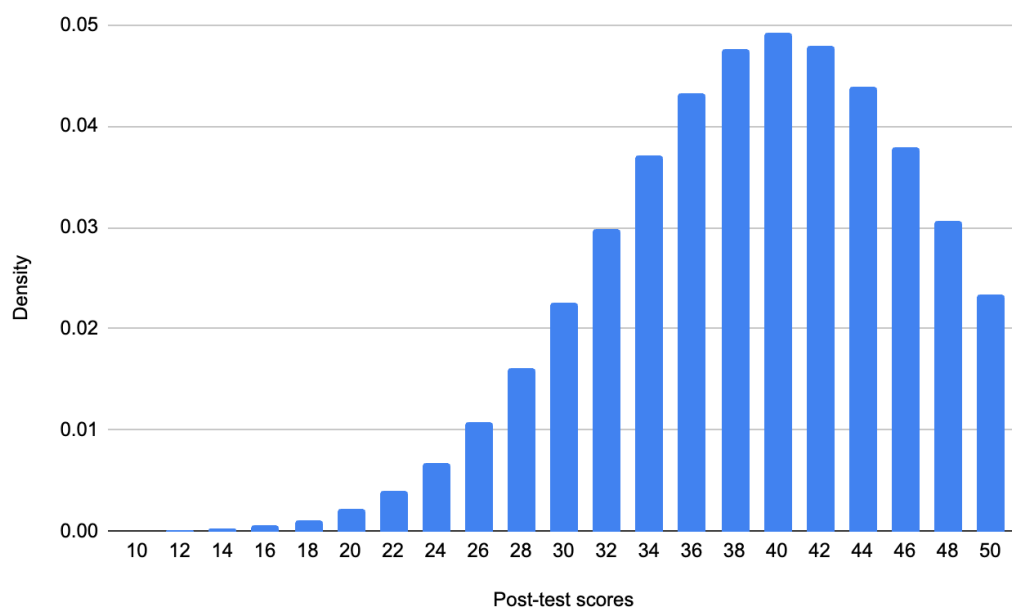
According to the Shapiro–Wilk test results (see table 3.7.2), the statistical indicator for the experimental group was $W = 0.924$, and the significance level was $p = 0.174$. For the control group, the results were $W = 0.917$, and $P = 0.114$. Since the p-value is higher than 0.05 in both groups ($p > 0.05$), it was assumed that the results of the post-test correspond to a normal distribution.

Table 3.7.2. Normality test

Group	Test	W	p-value
Experimental	Post-test	0.92	0.17
Control	Post-test	0.91	0.11

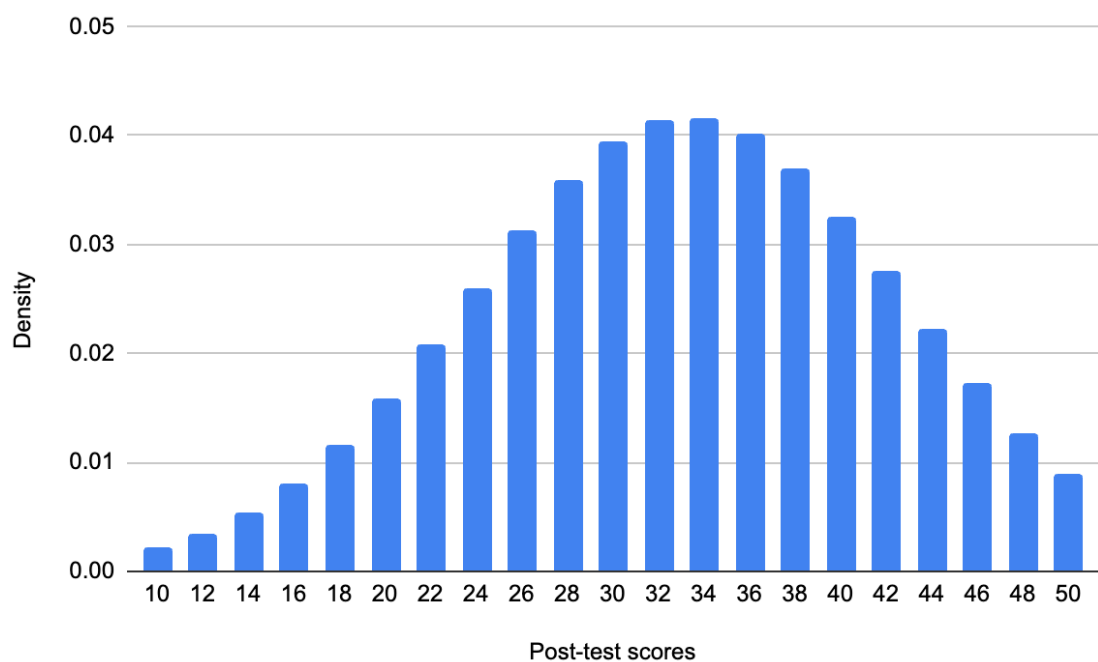
The histogram graph of the experimental group showed that students' scores are mainly concentrated in medium and high intervals (see figure 3.7.3). The shape of the distribution was approximately symmetrical and close to the shape of the Bell-shaped distribution. There were no obvious extreme deviations. This shows that the results of the students in the experimental group are evenly distributed and close to the normal distribution.

Figure 3.7.3. Normal distribution of the experimental group



As shown below (see figure 3.7.4), the histogram graph of the control group also showed a tendency for the results to gradually increase and then decrease. The distribution of points was approximately symmetrical, and the elements of Bell-shaped distribution were visible. There was no serious contradiction to the normal distribution of data on the graph. The Histogram results visually confirmed the conclusion of the Shapiro-Wilk test.

Figure 3.7.4. Normal distribution of the control group



In general, the results of the post-tests of both groups satisfied the condition of a normal distribution. In this regard, at the next stage of the study, a parametric statistical method, the Independent Samples t-test, was used to compare the results of the experimental and control groups. The fulfillment of the normal distribution condition allowed for parametric analysis and increased the statistical reliability of the study results. The fulfillment of Normal conditions served as the basis for the analysis of the research data by parametric methods. The following section describes the statistical analysis methods used.

3.8 Data analysis

After collecting data with the help of relevant tools and techniques, the next important step is to analyze and interpret data with a view to arriving at an empirical solution of the problem (Singh, 2009). The data collected from the achievement exam was analyzed quantitatively. According to the data analysis results, descriptive statistics, such as median and interquartile range (IQR) of pre-test scores, were examined for both experimental and control groups. To investigate the differences in the scores between the experimental and control groups on pre-test, Mann-Whitney U test was used. The hypotheses were tested at a level of significance of 0.05. Also, to analyze the effect of GeoGebra we compared the scores between the experimental and control groups on post-test, and t-test was used according to the results of the normality test, that is post-test results were normally distributed. The hypotheses were tested at a level of significance of 0.05 as well.

In addition, a set of questionnaires was compiled to find out the attitude of students to the use of GeoGebra in the 10-th grade. A 5-point Likert scale was employed for this purpose. The collected data from the questionnaires were analyzed using the mean value and the SD. As noticed before in the part of “Instrumentations”, the questionnaire questions were taken from a research paper called “Effectiveness of GeoGebra in teaching and learning Geometry” (Dipesh Prajapati, 2021).

4. ANALYSIS AND RESULTS

The results of the study are presented in this section. This section analyzes and compares the results of the pre-test and post-test of the experimental and control groups using statistical methods. Descriptive statistics and inferential statistics were used in the analysis of the results. First, the results of the pre-test conducted by both groups are considered in order to determine the initial level of knowledge before starting the study and the results were analyzed by Mann-Whitney U test. This stage is important to ensure that the experiment is conducted fairly, since subsequent results can only be objectively compared if the initial levels of the groups are equal. Later, the results of the post-test were analyzed by Correlation analysis to identify the relationship between student outcomes. The following hypotheses were proposed to compare the test results:

Null hypothesis (N_0):

There is no difference between the academic performance of students who receive geogebra-based instruction and those who receive traditional instructional methods in geometry.

Alternative hypothesis (N_1):

There is a significant difference between the academic performance of students who receive geogebra-based instruction and students who receive traditional instructional methods in geometry.

4.1 Results of the pre-test

At the initial stage of the research, a pre-test was conducted to determine the initial level of knowledge in geometry of the experimental and control groups. The pre-test made it possible to compare the academic background of both groups before the start of the study. According to Majka (2024), assessing people or variables under investigation before implementing an intervention provides researchers with useful information into their original state or condition.

The Mann-Whitney U test was used to analyze the Pre-test results, since the preliminary analysis of normality showed that the data was not distributed normally. Therefore, the nonparametric statistical procedure was chosen to compare the initial level of two independent groups.

Table 4.1.1. Pre-test results of experimental and control groups

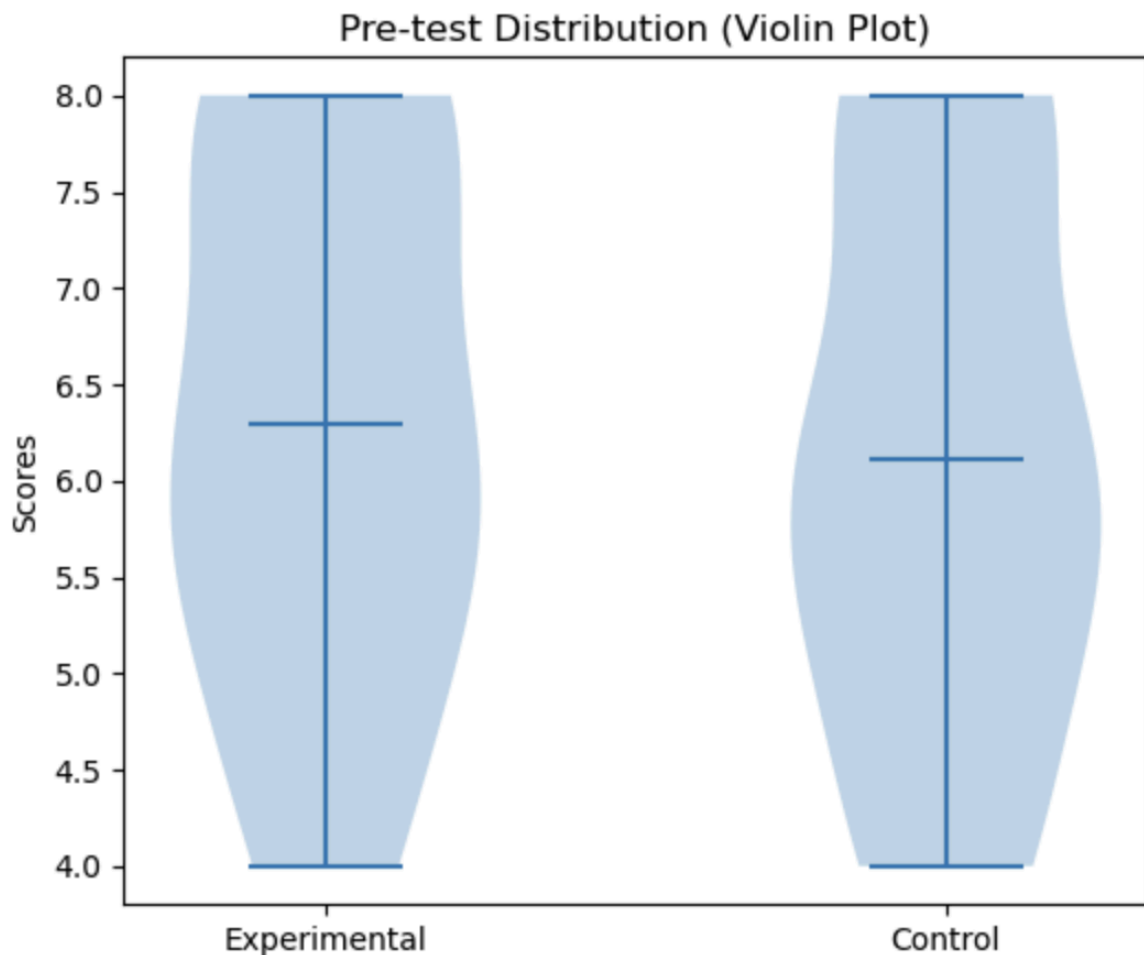
Group	Sample	Median	IQR	U-value	p-value
Experimental	17	6.0	1.0	175.5	0.457
Control	18	5.5	2.75		

According to the analysis results (table 4.1.1), The median of the experimental group was $Md = 6.0$, and the median of the control group was $Md = 5.5$. This shows that the academic results of the two groups at the pre-test stage are very close to each other. In addition, the IQR of the experimental group was 1.0, which means that the results of the vast majority of students were close to each other. Since $IQR = 2.75$ in the control group, it can be seen that the spread of results in this group is slightly higher. According to Mann-Whitney U test result $U = 175.5$ and $p = 0.4575$. Since the p-value is higher than 0.05 ($p \geq 0.05$), it is concluded that there is no statistical significant difference between the two groups. This means that the knowledge levels of the experimental and control groups were approximate before the start of the study. Therefore, the results of the pre-test showed that the level of initial academic training of both groups was equivalent, which means that the groups were more suitable for conducting experiments.

The distribution of pre-test results is shown by Violinplott (see figure 4.1.1). According to the graph, the estimates of the experimental and control groups are approximately in the same range, and their distribution forms are similar to each other. In each of the two groups, the main part of the results is concentrated around the average values, while peripheral values are less common. In addition, the distribution density is the same in both groups, and there is no obvious asymmetry or differences. This fact indicates that the initial knowledge levels of the two groups are

homogeneous and there is no significant difference in their distribution. Thus, the graphical representation of the pre-test results clearly proves that the groups are initially suitable for comparison.

Figure 4.1.1. Distribution of pre-test scores in the experimental and control groups



In general, the results of the pre-test show that the baseline levels of the experimental and control groups are equal. This allows an objective comparison of the results obtained at the next stage of the study. In particular, a scientific basis is being formed to link any differences between the groups observed in the results of the post-tests with the influence of the teaching method used, that is, the GeoGebra program. Thus, the results of the pre-test prove the correctness of the study design and that the groups were in relatively similar initial conditions.

4.2 Results of the post-test

At the end of the planned experiment time, a post-test was conducted to compare the academic achievements of the experimental and control groups. At this stage, it was intended to determine the impact of using the GeoGebra program on students' knowledge of geometry. The post-test data were analysed using both descriptive and inferential statistical techniques. Descriptive statistics included the calculation of means and standard deviations for each group. Since the post-test scores were normally distributed, an independent sample t-test was conducted to examine whether there was a statistically significant difference between the experimental and control groups. Furthermore, Cohen's d was calculated to assess the magnitude of the observed effect.

Table 4.2.1. Post-test results of experimental and control groups

Group	Sample	Mean	SD	t-value	p-value	Cohen's d
Experimental	17	40.11	7.94	2.218	0.033	0.750
Control	18	33.77	9.54			

Table 4.2.1 shows the results of the post-tests of the experimental and control groups. According to the results of descriptive statistics, the average score of the experimental group was $M = 40.11$, and the average score of the control group was $M = 33.77$. This means that the students in the experimental group performed well in the post-test. The difference between the average values of the two groups was about 6.34 points.

The standard deviation in the experimental group was $SD = 7.94$, and in the control group- $SD = 9.54$. This indicates that the results of the control group were somewhat scattered, while the results of the experimental group were relatively more stable.

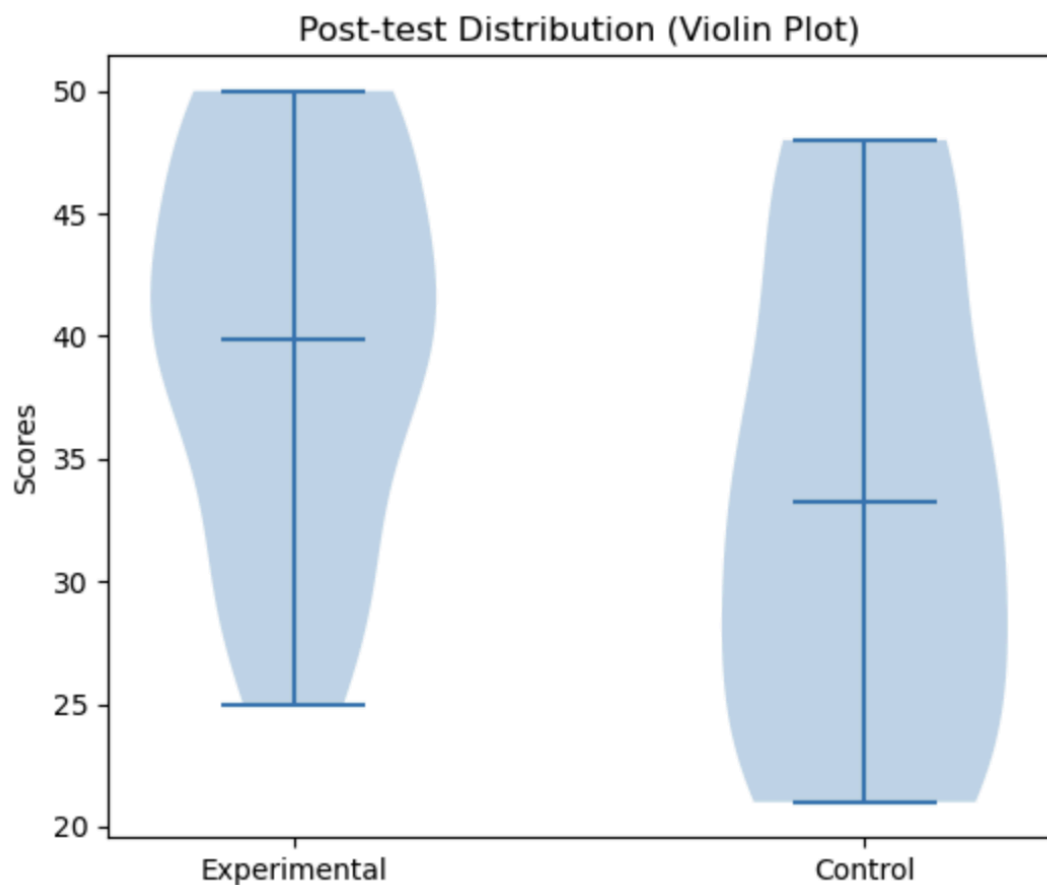
The independent samples t-test was performed to determine the statistical significance of the differences between the groups. As a result, the values $t = 2.218$

and $p = 0.033$ were obtained. Since the p -value is lower than 0.05 ($p < 0.05$), it can be concluded that there is a statistically significant difference between the results of the post-tests of the experimental and control groups.

Also, Cohen's D coefficient was calculated to determine the effect volume. As a result, Cohen's $d = 0.750$. This indicator represents the effect size from medium to high and shows that the use of GeoGebra has had a significant positive impact on students' academic achievements in geometry.

Overall, the high average results of the experimental group, the statistically significant difference, and Cohen's d score prove that GeoGebra-based learning was more effective than the traditional teaching method.

Figure 4.2.1. Distribution of Pre-Test Scores in the Experimental and Control Groups



In addition, the figure 4.2.1 shows the distribution of the results of the post-tests of the experimental and control groups according to the type of violin plot. It can be seen that the scores of the experimental group are more concentrated in higher ranges. While the central area of distribution of the experimental group is about 40 points, the indicators of the control group are concentrated in the area of about 33-34 points. In addition, it was noticed that higher results were more common in the experimental group. There were no obvious extreme deviations in the distribution of both groups. In general, the results of the violin plot visually confirm that the post-test scores of the experimental group were higher than those of the control group.

In general, the graph clearly proves that according to the results of the post-test, the academic performance of the experimental group is significantly higher and the teaching method used proved to be effective.

In the course of the study, students in the experimental group were surveyed in order to identify their views on the application of the GeoGebra program. The survey consisted of 10 statements and was evaluated on the Likert scale.

The alpha Cronbach coefficient was calculated to determine the reliability of the data obtained. As a result, the indicator $\alpha = 0.77$ was obtained (see table 4.2.3), which indicates that the internal identity of the survey is at a high level. Therefore, the survey results are considered reliable and suitable for further analysis.

A Pearson correlation analysis was performed to identify the relationship between the survey results and the post-test indicators. This analysis allows us to determine the level of relationship between the two variables. The analysis revealed a strong positive correlation between the survey results and the post-test scores.

Since the resulting r value is 0.777 (see table 4.2.2), this indicates that there is a higher-level positive relationship between the two variables. In other words, students who spoke positively about GeoGebra or scored highly in the survey also performed well as a result of the post-test. On the contrary, it was noticed that students with lower averages in the survey also had relatively low post-test scores. Also, the result

of $p < 0.001$ proves that the revealed correlation is statistically significant. This suggests that the connection did not arise by chance and the result of the study is reliable.

Table 4.2.2. Pearson Correlation Between Questionnaire Scores and Academic Performance

Variable 1	Variable 2	r	p
Questionnaire score	Post-test score	0.777	0.01

According to the descriptive statistics of the survey results, the overall average student score was 3.66. This result shows that students are generally positive about the GeoGebra program. A low standard deviation, $SD = 0.58$, means that the participants' responses were close to each other, and there was not much difference between the responses. In addition, the indicator Cronbach's alpha shows that the internal identity of the survey is high enough and can be used as a reliable tool in accordance with the objectives of the study.

Table 4.2.3. Overall Statistics of Questionnaire Results

Index	Value
Overall mean	3.66
Cronbach's alpha	0.77
Overall SD	0.58

5. DISCUSSION AND CONCLUSION

This study aimed to determine the impact of using the GeoGebra program on the performance of secondary school students in geometry. The results obtained confirm the main hypothesis of the study and show that the GeoGebra program is an effective tool in the learning process. According to Yenilmez (2009), many scientific studies suggest that computers have made it simpler not just to learn mathematical topics, but also to increase pupils' motivation and self-confidence.

The results of the pre-test were analyzed by Mann Whitney U test because results of both groups were non-normally distributed. The pre-test results showed that there was no statistically significant difference between the initial levels of knowledge of the experimental and control groups ($p > 0.05$). This means that the groups had the same level of training before starting the study and made it possible to reliably compare the results of the experiment. That is, there is every reason to believe that the subsequent differences are related to the influence of the teaching method used. So, null hypothesis is accepted, alternative hypothesis is rejected according to the pre-test results. It means that the groups are appropriate for the experiment.

The results of the post-test were analyzed by independent t-test because results of both groups were normally distributed. The post-test results showed that the experimental group showed significantly higher results compared to the control group ($p < 0.05$). This proves that using the GeoGebra program has had a positive impact on students' learning of geometric knowledge. In addition, the high magnitude of the effect (Cohen's $d \approx 0.8$) indicates that this difference is important not only statistically, but also practically. Visual visualization, dynamic models, and interactive interpretations using the GeoGebra program may have contributed to students' deeper understanding of complex concepts. So according to the post-test results, the null hypothesis is rejected and the alternative hypothesis is accepted for our first question. According to Shadaan and Leong's (2013) findings, the implementation of GeoGebra

in the learning and teaching process can have a significant influence on increasing students' abilities.

The results of the correlation analysis of the study showed that there is a strong positive relationship between the students' approach to GeoGebra and their post-test results ($r = 0.777$, $p < 0.001$). The result means that students who received GeoGebra positively also had higher academic scores. This finding is consistent with several studies in the literature. Saha, et al. (2010) found that the performance of high visual ability students was significantly higher than that of low visual ability students in the experimental and control group. In other words, it was noticed that students' interest in a technological tool and perception of it as useful are related to their academic performance.

This result shows that GeoGebra is not only a visualization tool, but also a tool that increases student motivation, engagement in classes, and understanding of mathematical concepts. Dynamic mapping of geometric objects using GeoGebra during the lesson may have helped students develop spatial thinking and better understand abstract concepts than usual. As a result, students who received positive feedback about GeoGebra performed better on post-test assignments.

In addition, the strong correlation also confirms that students' emotional and motivational factors are related to academic performance. When students perceive GeoGebra as an interesting and understandable tool, their interest in lessons can increase and they can participate more actively in the learning process. This, in turn, affects the improvement of learning outcomes.

The mentioned result is also consistent with previous studies. Many studies have shown that using GeoGebra improves students' mathematical understanding, learning motivation, and problem-solving skills. The results of this study also prove that GeoGebra can be an effective pedagogical tool in teaching geometry.

Overall, the results of the correlation analysis showed that there is a significant association between positive attitudes towards GeoGebra and high academic

achievement. This further proves the effectiveness of using GeoGebra in school mathematics education.

The survey results also support quantitative data. The vast majority of students positively assessed the GeoGebra program, noting that it facilitates the understanding of geometric concepts, increases interest in lessons and makes the learning process more effective. This shows that the program has a positive effect not only on academic performance, but also on student motivation, engagement, self-confidence.

In general, the results obtained prove that the GeoGebra program is effective compared to traditional teaching methods and that its application in geometry lessons can improve student academic performance. The digital environment motivates students in the teaching and learning of Mathematics (Korenova, 2012). It means that using geogebra as an auxiliary tool along with the traditional method gives a better effect than without it.

In conclusion, this study was aimed at evaluating the effectiveness of using the GeoGebra program in teaching geometry. The results of the study showed that the students of the experimental group, which used GeoGebra, received significantly higher results compared to the control group.

The results of the pre-tests showed that the groups had the same initial levels and the results of the post-tests proved that the GeoGebra program plays a significantly better role in improving academic performance. More specifically, the experimental group's students found the use of GeoGebra intriguing. Also, the questionnaire results complement the quantitative findings of the study by providing evidence that students' perceptions were generally positive, though not uniformly strong. This supports the interpretation that the instructional approach had a meaningful, yet varied, impact on learners. In addition, the gender factor was found to have no effect on the results, indicating that the program is equally effective for all students.

The answers to the experimental research questions were obtained through analysis in the results section. For the first research question, the t-test was used, the

hypothesis was determined, and the p-value was checked. As a result, in the experiment, a significant difference in the average post-test score between the experimental and traditional groups was found, and the alternate hypothesis was accepted, implying that GeoGebra-based instruction had a positive effect on the geometric understanding.

Thus, using the GeoGebra program in geometry lessons can improve students' academic performance, facilitate their understanding of complex concepts, and increase their interest in the learning process. These results highlight the importance of integrating GeoGebra into the educational process.

6. LIMITATIONS

This study has certain limitations. Firstly, the number of students participating in the study was relatively small, which could limit the possibility of generalizing the results. Secondly, the study was conducted within a single school, so direct application of the results to other schools and different educational contexts may be limited. Thirdly, the GeoGebra program was used only by the teacher as a demonstration tool, the possibility of students working independently with the program was not considered. The reasons were noticed in the part of “Implementation of the experiment: lesson procedure”. This may lead to the fact that the full potential of the program will not be realized. Also, in this study, classes were not chosen randomly. We did not have 11-th grade students because the school did not approve it. It means school approved only 10-th grade. So, we were forced to conduct the experiment only among 10-th grade students.

In addition, the study was conducted over a short period of time (6 weeks), so long-term effects were not considered. In future studies, it is important to study the long-term effects.

Implications. The results of the study provide important recommendations for educational practice. First, using interactive digital tools such as GeoGebra in geometry lessons is an effective way to improve student academic performance. Second, teachers are encouraged to use GeoGebra as a visualization tool during the lesson, as it allows them to present abstract concepts clearly and clearly. Also, educational institutions should provide for the inclusion of the GeoGebra program in the curriculum and conduct special teacher training on the application of this program.

In addition, future research suggests exploring the impact of students' independent use of GeoGebra, as well as the potential for use in various disciplines and levels.

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LIST OF APPENDICES

APPENDIX A. Pre-test & Post-test

(Remember)

1. Вектордың модулінің (ұзындығының) формуласы:

A) $|a| = \sqrt{x + y + z}$

B) $|a| = x^2 + y^2 + z^2$

C) $|a| = \sqrt{x^2 + y^2 + z^2}$

D) $|a| = x + y + z$

2. Векторлардың скаляр көбейтіндісінің формуласы:

A) $a \cdot b = x_1 x_2 + y_1 y_2 + z_1 z_2$

B) $a \cdot b = x_1 + y_1 + z_1$

C) $a \cdot b = x_1 y_1 + x_2 y_2$

D) $a \cdot b = x_1 y_1 z_1 + x_2 y_2 z_2$

3. Вектордардың арасындағы бұрыш формуласы:

A) $\cos \alpha = |a| + |b| + |ab|$

B) $\cos \alpha = \frac{|a| + |b|}{|a| * |b|}$

C) $\cos \alpha = |a| + |b|$

D) $\cos \alpha = \frac{a * b}{|a| * |b|}$

4. Векторлардың перпендикулярлық шарты:

A) $a \cdot b = 1$

B) $a \cdot b = -1$

C) $a \cdot b = 0$

D) $a = b$

5. Векторлардың коллинеарлық шарты:

A) $x_1 + x_2 = y_1 + y_2 = z_1 + z_2$

B) $\frac{x_1}{x_2} = \frac{y_1}{y_2} = \frac{z_1}{z_2}$

C) $x_1 = y_1 = z_1; x_2 = y_2 = z_2$

D) $x_1 x_2 = y_1 y_2 = z_1 z_2$

6. Егер нөлдік емес векторлар бір түзудің бойында немесе параллель түзулердің бойында жатса, онда олар қандай вектор?

A) Перпендикуляр векторлар

B) Тең векторлар

C) Коллинеар векторлар

D) Бірлік векторлар

(Understand)

7. Егер екі вектордың скаляр көбейтіндісі 0 болса, онда олар перпендикуляр.

A) Дұрыс

B) Дұрыс емес

8. Егер вектор координаталары бірдей болса, онда олар тең векторлар.

A) Дұрыс

B) Дұрыс емес

9. Екі вектордың перпендикуляр болуы нені білдіреді?

A) Векторлар параллель

B) Векторлар бірдей

C) Векторлардың арасындағы бұрыш 90

D) Векторлардың ұзындығы бірдей

10. Неге екі перпендикуляр вектордың скаляр көбейтіндісі 0 болады?

A) Өйткені олардың координаталары тең

B) Өйткені олардың ұзындықтары бірдей

C) Өйткені $a \cdot b = |a| |b| \cos 90$

D) Өйткені олар параллель

(Apply & Analyze)

11. Төбелері A(-2; 8), B(6; 2), C(2; -6) болатын үшбұрыштың АЕ медианасының ұзындығын табыңыз.

12. $a = 2i - 5j$ және $b = -3i + j$ векторлары берілген, онда $a + b$ векторының ұзындығы неге тең болады?

13. $a(4;-1;2)$, $b(-3;1;0)$ векторлары берілген. $2a - b$ векторының координатасын табыңыз.

14. $A(-3;2;-1)$, $B(2;-1;-3)$, $C(1;-4;3)$, $D(-1;2;-2)$ нүктелерінің координаталары берілген. $|2AB + 3CD|$ табыңыз

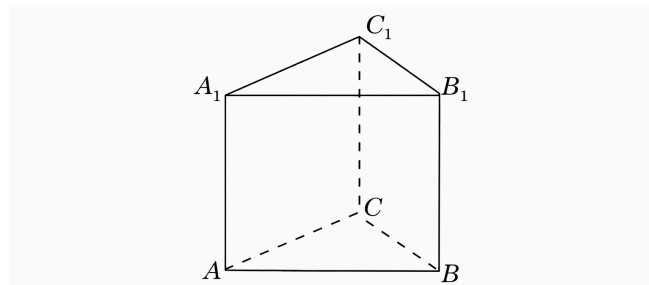
15. $ABCD A_1 B_1 C_1 D_1$ параллелепипедінде $AD + D_1 C_1 + BB_1$ векторларының қосындысы тең болады:

16. $\vec{a} \{5; -2; 3\}$, $\vec{b} \{2; -3; 1\}$ және $\vec{c} = \vec{a} - 2\vec{b}$ векторлары берілген. $\vec{a}$ және $\vec{c}$ векторларының арасындағы бұрышты табыңыз.

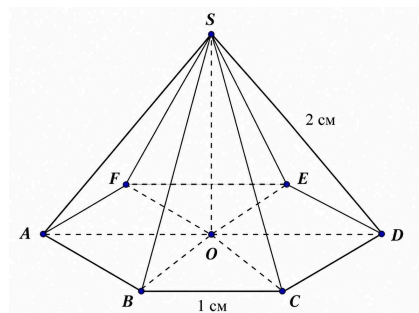
17. ABCD параллелограмының $A(3;-4; 7)$, $B(-5; 3;-2)$ және $C(11; 2;- 3)$ төбелері болса, онда D төбесінің координаталарының қосындысы тең:

18. МАВСД дұрыс төртбұрышты пирамиданың МО биіктігі 8-ге, ал бүйір қыры 16-ға тең $MO \cdot MC$ скаляр көбейтіндісін табыңыз:

19. $ABCA_1 B_1 C_1$ призмада компланар үш векторды көрсетіңдер.



20. SABCDEF дұрыс алтыбұрышты пирамида табанының қабырғалары 1-ге, ал бүйір қырлары 2-ге тең. SA және SD векторлардың скаляр көбейтіндісін табыңдар



APPENDIX B. Questionnaire

N	Сұрақ	Толық келісемін	Келісемін	Бейтарап	Келіспеймін	Мүлдем келіспеймін
1	GeoGebra геометриялық ұғымдарды анық түсінуге көмектесті					
2	GeoGebra-дағы визуализациялар оқуды қызықтырақ етті					
3	GeoGebra математиканы оқуға деген мотивациямды арттырды					
4	Сабақ барысында GeoGebra-ны түсіну маған қиын болды					
5	GeoGebra геометриялық есептерді шығару кезіндегі қобалжуымды/ мазасыздығымды азайтты					
6	GeoGebra қолдану сабақтағы белсенділігімді арттырды					
7	Мен GeoGebra-ға қарағанда дәстүрлі оқыту әдістерін қалар едім					
8	GeoGebra helped me visualize geometric figures better					
9	GeoGebra қолданылған сабақтарда өзімді сенімдірек сезіндім					
10	Болашақ математика сабақтарында GeoGebra қолданылғанын қалаймын					

APPENDIX C. Consent Form

ЗЕРТТЕУГЕ ҚАТЫСУҒА КЕЛІСІМ ФОРМАСЫ (INFORMED CONSENT FORM)

Зерттеу тақырыбы: GeoGebra бағдарламасының геометрияны оқытудағы жоғары сынып оқушыларының академиялық үлгеріміне әсері

Зерттеуші: Абдуәлиева Ғазиза, SDU University, School of Education & Humanities, Masters' Degree

Ғылыми жетекші: Nuri Balta

Зерттеудің мақсаты:

Бұл зерттеудің мақсаты – GeoGebra цифрлық құралын қолданудың геометрия пәнін оқыту барысында жоғары сынып оқушыларының академиялық үлгеріміне әсерін анықтау. Зерттеу GeoGebra бағдарламасының оқушылардың геометриялық ұғымдарды түсінуіне, есеп шығару дағдыларына және жалпы оқу жетістіктеріне қалай әсер ететінін талдауға бағытталған.

Қатысу шарттары:

Бұл зерттеуге “125 high school” мектебінің жоғары сынып оқушылары қатысады. Қатысушылар геометрия сабағына қатысып, зерттеу барысында алдын ала тест (pre-test) және соңғы тест (post-test) тапсырады. GeoGebra қолданған қатысушылар GeoGebra бағдарламасын қолдану тәжірибесі туралы қысқа сауалнамаға жауап береді.

Зерттеу мерзімі:

Қауіптер мен пайдасы:

Бұл зерттеуге қатысумен байланысты ешқандай қауіп жоқ. Оқушылардың күнделікті оқу процесінде ешқандай тақырыптан/материалдан қалу секілді өзгеріс болмайды. Оқушылар геометрия сабағын дәстүрлі әдістер арқылы және GeoGebra бағдарламасын қолдану арқылы оқиды. Зерттеу нәтижелері технологияны қолдану геометриялық ұғымдарды түсінуге қаншалықты көмектесетінін анықтауға мүмкіндік береді.

Құпиялылық және анонимділік:

Қатысушылардың жеке мәліметтері толық құпия сақталады. Қатысушылардың аты-жөні зерттеу құжаттарында көрсетілмейді. Әрбір қатысушыға арнайы код нөмірі беріледі. Жиналған барлық деректер тек ғылыми мақсатта пайдаланылады және жиналған деректер зерттеушінің құпиясөзбен қорғалған компьютерінде сақталады. Зерттеу нәтижелері тек жалпыланған түрде жарияланады.

Қатысушылардың құқықтары:

1. Зерттеуге қатысудан кез келген уақытта бас тартуға құқығы бар.

2. Кез келген сұраққа жауап беруден бас тартуға құқығы бар.
3. Зерттеу туралы қосымша ақпарат сұрауға құқығы бар.
4. Зерттеу нәтижелері туралы ақпарат алуға құқығы бар.

Мен, _____ - ның ата анасы/қамқоршысы, жоғарыда берілген ақпаратпен таныстым және зерттеуге қатысуына келісемін.

Қолы: _____ Күні: _____

Зерттеуші: Абдуәлиева Ғазиза

APPENDIX D. Certificate of Ethics Approval

"SDU UNIVERSITY" mekemeci
040900, Qazaqstan Respublikasy,
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Зерттеу этикасы комитеті

Research Ethics Committee

Dear Gaziza Abdualiyeva,

The Research Ethics Committee has reviewed "*The effect of geogebra on academic performance of high school students in geometry learning*" and has granted ethical approval based on the information provided in the application, and on the condition that the study is conducted in accordance with ethical standards and institutional guidelines.

Approval No.	11-12-07
Project title	<i>The effect of geogebra on academic performance of high school students in geometry learning</i>
Approval date	07 April 2026
Expiry date	06 April 2028
Decision	Approved

Please ensure that you contact the Research Ethics Committee at rethics@sdu.edu.kz if there are any changes to the study, so that any implications for ethics approval can be considered. Please quote the application number in all correspondence.

Chairwoman of the Research Ethics Committee

Secretary

K. Smakova

A. Kaukenova

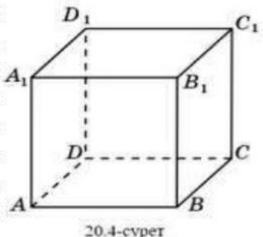
APPENDIX E. Short-term lesson plan

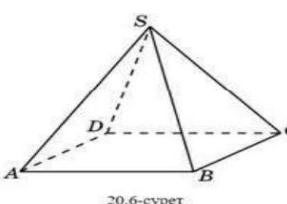
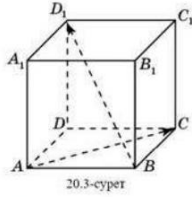
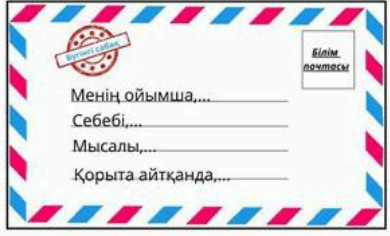
Сабақтың тақырыбы: Векторлардың скаляр көбейтіндісі

Бөлім:	Кеңістіктегі тікбұрышты координаталар жүйесі және векторлар	
Педагогтің аты-жөні:		
Күні:		
Сыныбы: 10	Қатысушылар саны:	Қатыспағандар саны:
Сабақтың тақырыбы:	Векторлардың скаляр көбейтіндісі	
Оқу бағдарламасына сәйкес оқу мақсаты	10.4.4 - кеңістіктегі векторлардың скаляр көбейтіндісінің анықтамасы мен қасиеттерін білу; 10.4.16 - координаталық түрдегі векторлардың скаляр көбейтіндісі формуласын білу және оны есептер шығаруда қолдану;	
Сабақтың мақсаты:	Білім алушылар кеңістіктегі векторлардың скаляр көбейтіндісінің анықтамасы мен қасиеттерін біледі. Білім алушылар координаталық түрдегі векторлардың скаляр көбейтіндісі формуласын біледі және оны есептер шығаруда қолданады. Білім алушылар кеңістіктегі екі вектордың арасындағы бұрышты есептейді.	
Құндылық тар:	Еңбекқорлық және кәсіби біліктілік Еңбекті, өзінің және басқалардың еңбек нәтижелерін бағалау; Еңбекті, мамандықтарды практикалық тұрғыда зерттеуге қызығушылық таныту; Кәсіби өзін-өзі жетілдіру үшін еңбек қызметінің дағдылары жинақтау, еңбексүйгіштік, еңбекке баулу, табысты еңбектің маңыздылығын түсіну; Уақытты тиімді жоспарлай білу және кез-келген іс-әрекеттің нәтижелеріне қол жеткізу және жақсарту үшін күш салу	

Сабақтың барысы:

Сабақ кезеңі / Уақыты	Педагогтің іс-әрекеті	Оқушының іс-әрекеті	Бағалау	Ресурстар
Сабақтың басы Қызығушылықты ояту. 5мин.	(Ұ). Ұйымдастыру кезеңі: 1. Оқушылармен амандасу, түгендеу. 2. Ынтымақтастық атмосферасын қалыптастыру 1. Мұғалім оқушыларды түгендейді. 2. Амандасады және тілектерін білдіреді.	1. Оқушылар мұғаліммен амандасады. 2. Сабаққа дайындалады. Сұрақтарға жауап береді: 1) Скаляр көбейтіндісі қалай белгіленеді? 2) Скаляр квадрат дегеніміз не? 3) Скаляр көбейтіндісін табудың жолдары қандай?	Оқушылардың белсенділігі үшін «Жарайсың!» , деген сөздерімен бағалайды.	Оқулық , жұмыс дәптері Кітап, дәптер, қалам телефон

Жансабаққа кіріспесі 10 мин	Екі вектордың скаляр көбейтіндісі $a \cdot b = a \cdot b \cos \varphi,$ $\vec{a} \quad \vec{b}$ мұндағы φ - a және b векторлары арасындағы бұрыш. GeoGebra бағдарламасы арқылы мысалдар көрсетіледі.	Оқушылар скалярлық көбейтіндінің қасиеттерімен танысады; жазып алады. $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z.$ Скалярлық көбейтінді қасиеттері - орындауыстырымдылық: $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$. - терімділік: $(\lambda \vec{a}) \cdot \vec{b} = \lambda (\vec{a} \cdot \vec{b})$. - үлестірімділік: $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$.	Саралау: Бұл жерде Дескриптор: - Әр оқушы өз ойымен бөліседі. - Өзгенің пікірін толықтырады. Жалпы- 2 балл.	Оқулық, жұмыс дәптері, Кітап, дәптер, қалам суреттері бейнеленген қима
Сабақтың ортасы 35 мин.	11. тапсырманы орындатады, бақылайды, мысал, үлгі көрсетеді $\vec{a} = \{5; 2; -1\}, \vec{b} = \{-1; 3; 3\}$. Екі вектор перпендикуляр ма? GeoGebra бағдарламасы арқылы сызып көреді және шешімнің дұрыстығын анықтау үшін дәптерге/тақтаға шығарады	Шешуі: Алдымен бұл екі вектордың скалярлық көбейтіндісін табамыз $\vec{a} \cdot \vec{b} = 5 \cdot (-1) + 2 \cdot 3 + (-1) \cdot 3 = -5 + 6 - 3 = -2.$ мұнда $\vec{a} \cdot \vec{b} = -2 \neq 0$, болғандықтан екі вектор перпендикуляр емес. Арасындағы бұрыш перпендикуляр болса 0-ге тең болу керек	Тақырыпқа байланысты интернет көздерін пайдаланып бөліп шығару мүшелерімен танысады.	ДК экраны, ДК экраны 10-сынып оқулығы. Жұмыс дәптері. Слайд. телефон
	2-тапсырма Берілгені $\vec{a} = 3, \vec{b} = 4, \varphi = \frac{2\pi}{3}$. Есептеңіз: $(3\vec{a} - 2\vec{b}) \cdot (\vec{a} + 2\vec{b})$. Шешуі: $\begin{aligned} (3\vec{a} - 2\vec{b}) \cdot (\vec{a} + 2\vec{b}) &= 3\vec{a} \cdot \vec{a} + 4\vec{a} \cdot \vec{b} - 2\vec{b} \cdot \vec{a} - 4\vec{b} \cdot \vec{b} = \\ &= 3 \vec{a} ^2 + 4 \vec{a} \cdot \vec{b} \cos \varphi - 4 \vec{b} ^2 = 27 - 24 - 64 = -61. \end{aligned}$		Дескриптор : Білім алушы -екі вектор арасындағы бұрыш формуласын қолданады; -векторлардың скаляр көбейтінділерін табады Арасындағы бұрышты табады	ДК экраны 10-сынып оқулығы. Жұмыс дәптері. Слайд.
	3-тапсырма 	В ABCD, B1C1D1 бірлік кубында келесі векторлардың скаляр көбейтіндісін табындар (20.4-сурет): а) $\vec{AC}$ және $\vec{B_1D_1}$; б) $\vec{AB}$ және $\vec{B_1C_1}$; в) $\vec{AB_1}$ және $\vec{BC_1}$.	Дескриптор: - Векторлардың скаляр көбейтіндісін табады. Геогембраға сызады. - Жауабын анықтайды. Жалпы - 4 балл.	ДК экраны 10-сынып оқулығы. Жұмыс дәптері. Слайд. телефон

	4- тапсырманы орындатады, бақылайды, мысал, үлгі көрсетеді.	Д Мыңдар: а) AB және CC_1; ә) AB және BC_1. $SABCD$ дұрыс төртбұрышты пирамиданың барлық қырлары 1-ге тең (20.6-сурет). Келесі векторлардың скаляр көбейтіндісін табыңдар: а) $\overrightarrow{AB}$ және $\overrightarrow{SC}$; ә) $\overrightarrow{SB}$ және $\overrightarrow{SD}$. С.А.Б.С.Д.Б.Е. тапсырманы орындайтын кезеңі  20.6-сурет	дескриптор: 1.Есеп мазмұнын түсінеді. Геогебраға сызады 2. Скаляр көбейтіндісін табады. 3.Жауабын анықтайды. Жалпы - 3 балл	ДК экраны 10-сын ып оқулығы. Жұмыс дәптерлері. Слайд. телефон
	5- тапсырманы орындатады, бақылайды, мысал, үлгі көрсетеді.  20.3-сурет	2-мысал. $ABCD A_1 B_1 C_1 D_1$ бірлік кубында (20.3 -сурет) $\overrightarrow{AC}$ және $\overrightarrow{BD_1}$ векторларының скаляр көбейтіндісін табыңдар.	Дескриптор: 1.Есептің мазмұнын түсінеді; Геогебраға сызады 2.Скалярлық көбейтіндісін табады. Жалпы-3 балл.	ДК экраны 10-сын ып оқулығы. Жұмыс дәптерлері. Слайд. телефон
Сабақтағы соңғы ой толғанғыс. Рефлексия 5мин.	«Білім поштасы» әдісі. Мұғалім сабақты оқушылардың пікірін анықтайды. Жинақталған деректердің құнды болуын қадағалайды. Саралау: Бұл кезеңде саралаудың «Қорытынды» тәсілі көрінеді. Үйге тапсырма: №есеп. 	Оқушылар бүгінгі сабақтың мақсатына жеткізетін тапсырмалар орындауына қарай, өз түсінгенін, пікірін, өз ойын айту арқылы сабаққа қорытынды жасайды. Хат толтырып, білім поштасына салады 	оқушылардың сабаққа қатысу белсенділігі бойынша және сабақ кезеңдерінде жинаған балдарының жалпы есебі ескеріліп 1-10 баллдық шкаламен бағаланады.	ДК экраны 10сынып оқулығы. Жұмыс дәптерлері. Слайд. Білім поштасы, хаттар

APPENDIX F. Long-term plan

4 тоқсан 8 сағат						
1	Кеңістіктегі тікбұрышты координаталар жүйесі және векторлар	Кеңістіктегі вектордың координаталары	10.4.5 - кеңістіктегі вектордың координаталарын және ұзындығын таба білу;	1	03.04	03.04
2		Векторлардың коллинеарлығы және компланарлығы	10.4.6 - кеңістіктегі коллинеар және компланар векторлардың анықтамаларын, векторлардың коллинеарлық шартын білу;	1	10.04	10.04
3		Векторларды қосу, векторды санға көбейту	10.4.7 - векторларды қосу және азайтуды, векторды санға көбейтуді орындау;	1	17.04	17.04
5		Векторлардың скаляр көбейтіндісі.	10.4.8 - координаталық түрдегі векторлардың скаляр көбейтіндісі формуласын білу және оны есептер шығаруда қолдану;	1	24.04	24.04
6		Векторлардың скаляр көбейтіндісі ББЖБ №5	10.4.8 - координаталық түрдегі векторлардың скаляр көбейтіндісі формуласын білу және оны есептер шығаруда қолдану;	1	08.05	08.05
7		Тоқсан бойынша жиынтық бағалау		1	15.05	15.05
8		Векторлардың скаляр көбейтіндісі	10.4.8 - координаталық түрдегі векторлардың скаляр көбейтіндісі формуласын білу және оны есептер шығаруда қолдану;	1	22.05	22.05