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Solution of differential equations with vertical asymptotes

Application of step-by-step methods of the solution of a problem of Couchy for the ordinary differential equation

$$\begin{cases} y' = f(x,y), x \geq 0 & (1) \\ y(0) = \alpha & (2) \end{cases}$$

meets serious difficulties if the solution at is not continuous on all numerical axis.

Really, customary definition of the solution as x, value forces to choose argument functions as a step. Calculations with such step do not allow to "notice", for example, a vertical asymptote of the solution.

In this study I offer modified version of the single-step methods which allows to estimate and consider the maximum interval of existence of the solution of a problem of Couchy for the ordinary differential equation.

This modification is based on geometrical idea of consideration of the solution as a curve on a plane Oxy. At such point of view as a step it is obvious to choose the distance between points , $(x_{i-}$ which approximates the solution.

Let's apply this idea to Euler's method described by formulas $x_{i-1} =$,

$y_{i-1} = y_i + h_i f$. As here the integrated curve is replaced with a straight line as constant step H we will choose distance between points , $(x_{i-}$.

$$H = \sqrt{(x_{i-1} - x_i)^2 + (y_{i+1} - y_i)^2} = h_i\sqrt{1 + f^2(x_i, y_i)}$$

From here, $h_i = \frac{}{\sqrt{1+}}$. Thus, Euler's method can be written in such type:

$$x_{i-1} = x_i + \frac{H}{\sqrt{1 + f^2(x_i, y_i)}}, \quad y_{i+1} = y_i + \frac{Hf(x_i, y_i)}{\sqrt{1 + f^2(x_i, y_i)}} \quad (3)$$

Let's result conditions of an extremity of the maximum interval of existence of the solution of a problem (1), (2) and we will find out behavior under these conditions of the approached solution constructed under formulas (3). The interval is considered as the maximum interval of existence of the solution , if or if there is no final limit $\lim_{x \to}$. The corresponding solution defined on , is called as full. Let's note that continuity of f enough for existence of the full solution and continuity of any solution on the maximum interval.

Also, in this study I researched convergency and estimated inaccuracy of this method.

Example:

Let's consider a problem

$$\begin{cases} y' = y^2 + 1 \\ y(0) = 0 \end{cases} \quad (4)$$

Here. , let h

Let's use formulas (3), we have

step 1:

$$x_1 = 0 + \frac{0,1}{\sqrt{1+1^2}} = \frac{0,1}{\sqrt{2}} \approx 0,07$$

$$y_1 = 0 + \frac{0,1 \cdot 1}{\sqrt{2}} \approx 0,07$$

$$f_1 = 1 + 0,07^2 \approx 1,0049$$

step 2:

$$x_2 = 0,07 + \frac{0,1}{\sqrt{1+1,0049^2}} \approx 0,141$$

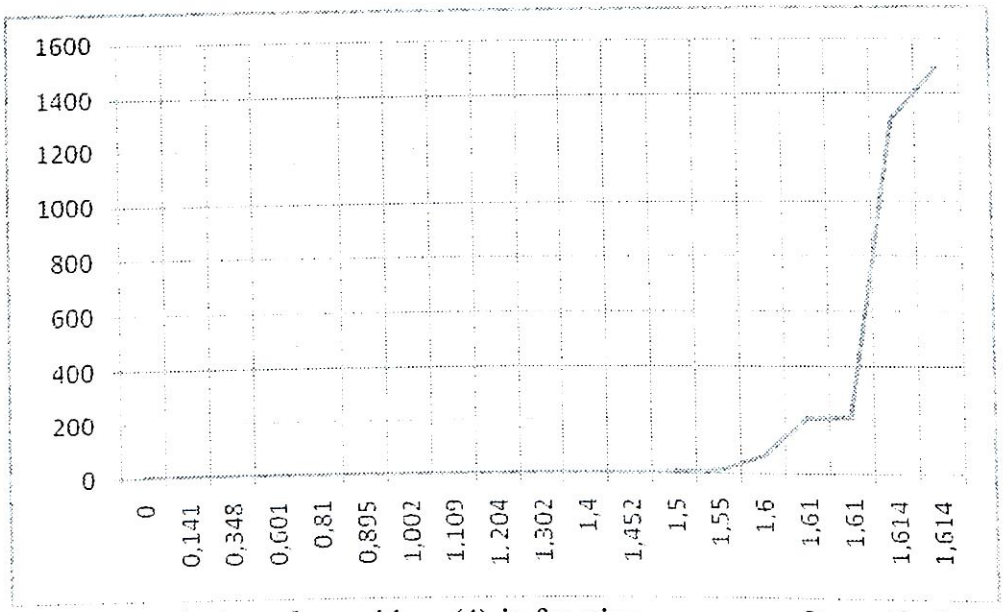
$$y_2 = 0,07 + \frac{0,1 \cdot 1,0049}{\sqrt{1+1,0049^2}} \approx 0,142$$

$$f_2 = 1 + 0,142^2 \approx 1,019$$

The further calculations are similar. All value x, y are shown in the table

Steps	Approximated x	Approximated y	$y(x_i)$ - exact solution
0	0	0	0
2	0,141	0,142	0,142
5	0,348	0,359	0,363
9	0,601	0,669	0,686
13	0,810	1,009	1,051
15	0,895	1, 190	1,247
19	1,002	1,471	1,564
23	1,109	1,856	2,009
28	1, 204	2,346	2,603
36	1,302	3,140	3,630
51	1,400	4,637	5,798
66	1,452	6,136	8,378
92	1,500	8,736	14,101
160	1,550	15,435	49,078
660	1,600	65,535	-
2095	1,610	209,035	-
2101	1,610	209,635	-
12965	1,614	1296,035	-
14905	1,614	1489,935	-

This is the graph of the table shown above.



The exact solution of a problem (4) is function . Its vertical asymptote is . As we see, we also have found an asymptote by the approximated calculation.

References

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Түйін

Бұл мақалада вертикалды асимптотасы бар дифференциалдық теңдеулердің жуықталған шешімі қарастырылған және де Коши есебінің шешімінің болу интервалы, методтың ауытқу шамасы зерттелген.

Резюме

В этой статье рассматривается метод нахождения приближенного решения дифференциальных уравнений, имеющих вертикальные асимптоты. Детально рассмотрены интервал существования решения задачи Коши для обыкновенных дифференциальнах уравнени, сходимость метода и погрешность.

Özet

Bu makalede dikey asimptotlu diferansiyel denklemlerin yaklaşık çözümün bulmak için metod gösterilmiştir.