УНИВЕРСИТЕТ ИМЕНИ СУЛЕЙМАНА ДЕМИРЕЛЯ

Л. Е. Базарбаева

UNIFIED METHOD OF SOLUTION OF WORD PROBLEMS

Методическая разработка

Каскелен, 2015

УДК 51, 510(075.8)
ББК 22.1

Рекомендовано к изданию Учебно-методическим советом Университета имени Сулеймана Демиреля

Рецензенты:

Дженалиев М. Т., доктор физико-математических наук, профессор, Институт математики и математического моделирования КН МОН РК

Багисбаев К. Н., кандидат физико-математических наук, доцент, Университет имени Сулеймана Демиреля

Л. Е. Базарбаева

Унифицированный метод решения текстовых задач: Методическая разработка. — Каскелен, Университет имени Сулеймана Демиреля, 2015. — 32 с. — на английском языке.

L. E. Bazarbayeva

Unified method of solution of word problems: Methodical working-out. — Kaskelen, Suleyman Demirel University, 2012. — 35 p. — In English.

Методическая разработка «Унифицированный метод решения текстовых задач» предназначена для бакалавров по специальностям 5В060100 — «Математика» и 5В010900 — «Математика» и содержит такие темы как: «От слов к алгебре», «Задачи о деньгах», «Задачи инвестирования», «Задачи на движение», «Задачи о химических растворах», «Разнообразные задачи», «Задачи с решениями».

Methodical working-ou[illegible]
signed for bachelors of spe[illegible]
"Mathematics" and contain[illegible]
lems, Investment Problems [illegible]
lem, Solved problems.

WORD PROBLEMS

Most students have had previous exposure to word problems. Many students have had some difficulty in handling such problems. This methodical recommendation is devoted to methods of attacking word problems; it is designed to develop the ability to analyze words and to heighten confidence in solving word problems.

We begin by outlining procedures for converting words to algebra and applying these procedures to some problems. We will then examine some types of word problems that require special attention.

FROM WORDS TO ALGEBRA

These are the typical steps used in solving word problems.

Step 1. Read the problem until you understand what is required.

Step 2. Isolate what is known and what is to be found.

Step 3. In many problems, the unknown quantity is the answer to a question such as "how much" or "how many". Let an algebraic symbol, say, x, represent the unknown.

Step 4. Represent other quantities in problem in terms of x.

Step 5. Find the relationship in the problem joining the values in the column connecting additional information. Write an equation (or an inequality).

Step 6. Solve.

Step 7. Check your answer to see that it:

a) satisfies the original question, and

b) satisfies the equation (or an inequality).

WORDS AND PHRASES

Some students have trouble with problems because they are unfamiliar with the mathematical interpretation of certain words and phrases. Table 1 may be helpful; it has a list of words and phrases you will come across, with examples of how they are used.

Table 1

Word or phrase	Algebraic symbol	Example	Algebraic expression
1. Sum	$+$	Sum of two numbers	$a+b$
2. Difference	$-$	Difference of two numbers Difference of a number 3	$a-b$ $a-3$
3. Product	$\times$ or $\cdot$	Product of two numbers	$a\cdot b$
4. Quotient	$\div$ or $/$	Quotient of two numbers	a/b or $\frac{a}{b}$
5. Exceeds		a exceeds b by 3	$a=b+3$
6. More than		a is 3 more than b	$a=b+3$
7. More of		There are 3 more of a than of b	$a-3=b$
8. Twice		Twice a number Twice the difference of x and 3 3 more than twice a number 3 less than twice a number	$2x$ $2(x-3)$ $2x+3$ $2x-3$
9. Is or equals	$=$	The sum of number and 3 is 15	$x+3=15$

Let's apply our steps for analyzing word problems

EXAMPLE 1

The sum of the age of a man and his daughter is 40 years. Nineteen years from now, the man will be twice as old as his daughter will be then. Find the present ages of the man and his daughter.

SOLUTION

After reading, it is clear that we may choose the unknown to be the present age of either the man or his daughter. We let

$x=$ the current age of the man Step 3

Then

$40-x=$ the current age of the daughter Step 4

And we complete this table

	Age now	Age 19 years from now
Man	x	$x+19$
Daughter	$40-x$	$40-x+19$
Ratio of the ages		twice

The situation 19 years from now leads to an equation.

$x+19=2(40-x+19);$ Step 5

$x+19=2(59-x);$

$x+19=118-2x;$

$3x=99;$ Step 6

$x=33=$ man's current age

$40-x=7=$ daughter's current age

Step 7. Now check that this solution satisfies the problem.

EXAMPLE 2

The larger of two numbers is 1 more than the smaller. Five times the larger exceeds four times the smaller by 12. Find the numbers

SOLUTION

We let

$x=$ the smaller number

then

$x+1=$ the larger number

	numbers now	after operations
smaller	x	$4x$
larger	$x+1$	$5(x+1)$
Difference of numbers after operation		12

$5(x+1)=4x+12;$

$5x+5=4x+12;$

$x=7=$ the smaller number

$x+1=8=$ the larger number

Step 7. Verify that the answer is correct.

EXAMPLE 3

The length of the rectangle is 2 m. more than twice its width. If the perimeter is 22 feet, find the dimensions of the rectangle. (1 foot=30.48cm.)

SOLUTION

Since the length is expressed in terms of the width, we let

w = the width of the rectangle .

Then

$2w+2=$ Length of the rectangle

	now	twice sides
width	w	$2w$
length	$2w+2$	$2(2w+2)$
perimeter		22

$$22=2(2w+2)+2w;$$
$$22=4w+4+2w;$$
$$22=6w+4;$$
$$18=6w;$$
$$w=3= \text{the width}$$
$$l=2(3)+2=8= \text{the length.}$$

Solve the next problems

PROGRESS CHECK 1

Loren is 3 times as old as Jody. Ten years from now, Loren's age will exceed twice Jody's age by 2 years. What are the present ages of Loren and Jody?

PROGRESS CHECK 2

Write the number 30 as the sum of two numbers such that twice the larger is 3 less than 7 times the smaller

PROGRESS CHECK 3

One side of a rectangle is 3 cm longer than the shortest side; the third side is 1 cm more than twice the shortest side. If the perimeter of the triangle is 20 cm, find the dimensions of the three sides.

COIN PROBLEMS

The key to the solution of coin problems is to distinguish between the number of coins and the value of the coins.

Coin	Value in cents
Nickels	**5**
Dimes	**10**
Quarters	**25**

EXAMPLE 1

A purse contains $3.20 in quarters and dimes. If there are 3 more quarters than dimes, how many coins of each type are there?

SOLUTION

	Value of each coin ×	Number of coins =	Value in cents
Quarters	25	n	$25n$
Dimes	10	$n-3$	$10(n-3)$
Total			320

Total value = (value of quarters) + (value of dimes)

$$320=25n+10(n-3);$$
$$320=25n+10n-30;$$
$$350=35n;$$
$$n=10.$$

EXAMPLE 2

A jar contains 25 coins worth $3.05. If the jar contains only nickels and quarters how many coins are there of each type?

SOLUTION

	Value of each coin ×	Number of coins =	Value in cents
Nickels	5	n	$5n$
Quarters	25	$25-n$	$25(25-n)$
Total			305

$$305 = 5n + 25(25 - n);$$

$$305 = 5n + 625 - 25n;$$

$$-320 = -20n;$$

$$n = 16 \text{ number of nickels}$$

$$25 - n = 9 \text{ number of quarters}$$

PROGRESS CHECK 4

A class collected $3.90 in nickels and dimes. If there were 6 more nickels than dimes, how many coins were there of each type?

PROGRESS CHECK 5

A pile of coins worth $10 containing of quarters and half-dollars is lying on a desk. If there are as many quarters as half-dollars, how many half-dollars are there?

PROGRESS CHECK 6

A man purchased 10-cent, 15-cent, and 20-cent stamps with a total value of $8.40. If the number of 15-cent stamps is 8 more than the number of 10-cent stamps and there are 10 more of 20-cent stamps than of 15-cent stamps, how many of each did he receive?

PROGRESS CHECK 7

The pretzel vendor finds that her coin-changer contains $8.75 in nickels, dimes, and quarters. If there are twice as many dimes as nickels and 10 fewer quarters than dimes, how many of each kind of coin are there?

INVESTMENT PROBLEMS

If $500 is invested at an annual interest of 6%, then the simple interest at year's end will be

$$I = (0,06)(500) = \$30 \text{ or}$$

Simple annual interest = Principal × Annual rate or I=P·r

EXAMPLE 1

A part of $7000 is invested at 6% annual interest and remainder at 8%. If the total amount of annual interest is $460, how much was invested at each rate?

SOLUTION

	Rate × Amount invested = Interest		
6% portion	0.06	n	$0.06n$
8% portion	0.08	$7000 - n$	$0.08(7000 - n)$
Total			460

Total interest is the sum of the interest from the two parts

$$460 = 0.06n + 0.08(7000 - n);$$

$$460 = 0.06n + 560 - 0.08n;$$

$$0.02n = 100;$$

$$n = \$5000.$$

EXAMPLE 2

A part of $12,000 is invested at 5% annual interest, and the remainder at 9%. The annual income on the 9% investment is $100 more than the annual income on the 5% investment. How much is invested at each rate?

SOLUTION

	Rate × Amount invested = Interest		
5% investment	0.05	n	$0.05n$
9% investment	0.09	$12{,}000 - n$	$0.09(12{,}000 - n)$
Difference			100

$$0.09(12000-n)-0.05n=100$$

Finish the solution.

EXAMPLE 3

The shoe store owner had \$6000 invested in inventory. The profit on women's shoes was 35%, while the profit on men's shoes was 25%. If the profit on the entire stock was 28%, how much was invested in each type of shoe?

SOLUTION

	Rate × Amount invested = Interest		
Women's shoes	0.35	n	$0.35n$
Men's shoes	0.25	$6000-n$	$0.25(6000-n)$
Total stock	0.28	6000	0.28(6000)

Total profit is the sum of the profits on each portion

$$0.28(6000-n)=0.35n+0.25(6000-n)$$

Finish the solution.

PROGRESS CHECK 8

A club decides to invest a part of \$4600 in stocks earning 4.5% annual dividends, and the remainder in bonds getting 7,5%. How much must the club invest in each to obtain a net return of 5.4%?

PROGRESS CHECK 9

\$7500 is invested in two parts yielding 5% and 15% annual interest. If the interest earned on the 15% investment is twice that on the 5% investment, how much is invested in each?

PROGRESS CHECK 10

An automobile dealer has \$55,000 invested in compacts and midsize cars. The profit on sales of the compacts is 10%, and the profit on sales of midsize cars is 16%. How much did the dealer invest in compact cars if the overall profit on the total investment is 12%?

DISTANCE (UNIFORM MOTION) PROBLEMS

Here is the basic formula for solving distance problems:

Distance = rate×time.
or
$d=v\cdot t$

EXAMPLE 1

Two trains leave New York for Chicago. The first train travels at an average speed of 60 km per hour, while the second train, which departs an hour later, travels at an average speed of 80 km per hour. How long will it take the second train to overtake the first train?

SOLUTION

t = number of hours that the second train travels

$t+1$ = number of hours that the first train travels

	Rate · Time = Distance		
First train	60	$t+1$	$60(t+1)$
Second train	80	t	$80t$
Distances of catch			equal

$$60(t+1)=80t;$$
$$60t+60=80t;$$
$$60=20t;$$
$$t=3.$$

It will take the second train 3 hours to catch up the first train.

EXAMPLE 2

A jogger running at the rate of 4 miles per hour takes 45 minutes more than a car traveling at 40 miles per hour to cover a certain course. How long does it take the jogger to complete the course and what is the length of the course? (1 mile=1.609 kilometers)

SOLUTION

	Rate × Time = Distance		
Jogger	4	t	$4t$
Car	40	$t-\frac{3}{4}$	$40(t-\frac{3}{4})$

Since the jogger and car travel the same distance

$$4t = 40(t - \frac{3}{4});$$
$$4t = 40t - 30;$$
$$30 = 36t;$$
$$t = \frac{5}{6}$$

The jogger takes 50 minutes. The distance traveled is $4t = 4\frac{5}{6} = \frac{20}{6} = 3\frac{1}{3}$ *miles.*

EXAMPLE 3

At 2 p.m. a plane leaves Boston for San Francisco, traveling at an average speed of 500 miles per hour. Two hours later a plane departs San Francisco to Boston traveling at an average speed 600 miles per hour. If the cities are 3200 miles apart, at what time do the planes pass each other?

SOLUTION

	Rate × Time = Distance		
From Boston	500	t	$500t$
From San Francisco	600	$t-2$	$600(t-2)$

At the moment that the planes pass each other, the sum of the distances traveled by both planes must be 3200 km.

$$3200 = 500t + 600(t-2);$$
$$3200 = 500t + 600t - 1200;$$
$$4400 = 1100t;$$
$$4 = t.$$

The planes meet 4 hours after the departure of the plane from Boston, e.g. at 6 p.m.

PROGRESS CHECK 11

A light plane leaves the airport at 9a.m. traveling at an average speed of 200 km per hour. At 11 a.m. a jet plane departs and follows the same route. If the jet travels at the average speed of 600 km per hour, at what time will the jet overtake the light plane?

PROGRESS CHECK 12

The winning horse finished the race in 3 minutes; a losing horse took 4 minutes. If the average rate of the winning horse was 5 m per second more than the average rate of the slower horse, find the average rates of both horses?

PROGRESS CHECK 13

Two cyclists start at the same time from the same place and travel in the same direction. If one cyclist averages 16 km per hour and the second averages 20 km per hour, how will it take for them to be 12 km apart?

MIXTURE PROBLEMS

If the commodities are measured in pounds (1 pound=453.59grams), the relationships we need are

price per pound × number of pounds = value of commodity
pounds in mixture = sum of pounds of each commodity
value of mixture = sum of values of individual commodities

EXAMPLE1

How many pounds of Brazilian coffee worth $5 per pound must be mixed with 20 pounds of Colombian coffee worth $4 per pound to produce a mixture worth $4.20 per pound?

SOLUTION

Type of coffee	Price × Number of pounds = Value in cents		
Brazilian	500	n	$500n$
Colombian	400	20	8000
Mixture	420	$n+20$	$420(n+20)$

$$420(n+20) = 500n + 8000;$$
$$420n + 8400 = 500n + 8000;$$
$$400 = 80n;$$
$$n = 50.$$

EXAMPLE 2

Caramels worth \$1.75 per pound are to be mixed with cream chocolates worth \$2 per pound to make a 5-pound mixture that be sold at \$1.90 per pound. How many pounds of each are needed?

SOLUTION

Type of candy	Price × Number of pounds = Value in cents		
caramels	175	n	$175n$
Cream chocolates	200	$5-n$	$200(5-n)$
Mixture	190	5	950

$$950=175n+200(5-n);$$
$$950=175n+1000-200n;$$
$$25n=50;$$
$$n=2.$$

A second type of mixture problem involves solutions containing different concentrations of materials. For instance, a 40-gallon (1 gallon=4.546 litters in British system and 3.785 litters in US system) drum of solution which is 75% acid contains (0.75)(40) = 30 gallons of acid. If the solutions are measured in gallons, the relationship we need is

number of gallons of solution	×	k % of component A ($\frac{k}{100}$ part)	=	number of gallons of component A

The other relationships we need are really the same as in our first type of mixture problem.

number of gallons in mixture	=	sum of the number of gallons in each solution
number of gallons of component A in mixture	=	sum of the number of gallons of component A in each solution

EXAMPLE 3

A 40% acid solution is to be mixed with a 75% acid solution to produce 140 gallons of solution that is 50% acid. How many gallons of each solution must be used?

SOLUTION

	part of acid × Number of gallons = Number of gallons of acid		
40% solution	0.4	n	$0.4n$
75% solution	0.75	$140-n$	$0.75(140-n)$
Mixture	0.5	140	70

$$70=0.4n+0.75(140-n);$$
$$70=0.4n+105-0.73n;$$
$$-35=-0.35n;$$
$$n=100 gallons;$$
$$140-n=40 gallons.$$

EXAMPLE 4

How many ounces (1 ounce=28.35gramms) of an alloy that is 30% tin must be mixed with 15 ounces of an alloy that is 12% tin to produce an alloy that is 24% tin?

SOLUTION

	part of tin × number of ounces = number of ounces of tin		
30% alloy	0.3	n	$0.3n$
12% alloy	0.12	15	1.8
Mixture	0.24	$n+15$	$0.24(n+15)$

$$0.24(n+15)=0.30n+1.8;$$
$$0.24n+3.6=0.30n+1.8;$$
$$1.8=0.06n;$$
$$n=30.$$

Thus, we need to add 30 ounces of 30% alloy to 15 ounces of the 12% alloy.

EXAMPLE 4

A tank contains 40 gallons of water and 10 gallons of alcohol. How many gallons of water must be removed if the remaining solution is to be 30% alcohol?

SOLUTION

Let n = number of gallons of water to be removed. This problem is different as we removing water. Here how to display the information:

	part of alcohol ×	number of gallons =	gallons of alcohol
Original solution	0.2	50	10
water removed	0	n	0
New solution	0.3	$50-n$	$0.3(50-n)$

$$0.3(50-n)=10;$$
$$15-0.3n=10;$$
$$5=0.3n;$$
$$n=16\frac{2}{3}$$

Thus, me must remove $16\frac{2}{3}$ gallons of water.

PROGRESS CHECK 14

How many pounds of macadamia nuts $4 per pound must be mixed with 4 pounds of cashew worth $2.50 per pounds of pecans worth $3 per pound to produce a mixture that is worth $3.20 per pound?

PROGRESS CHECK 15

How many gallons of oil worth 55 cents per gallon and how many gallons of oil worth 75cents per gallon must be mixed to obtain 40 gallons of oil worth $60 per gallon?

PROGRESS CHECK 16

How many gallons of milk that is 22% butterfat must be mixed with how many gallons of cream that 60% butterfat to produce 19 gallons of a mixture that is 40% butterfat?

PROGRESS CHECK 17

How many pounds of a 25% copper alloy must be added to 50 pounds of 55% copper alloy to produce an alloy that is 45%?

PROGRESS CHECK 18

A tank contains 90 quarts an antifreeze solution that is 50% antifreeze. How much water should be removed to raise the antifreeze level to 60% in the new solution?

MISCELLANEOUS EXERCISES

EXAMPLE 1 (7 grade)

By increasing the average speed from 250 to 300 m / min, the athlete began to run a distance for one minute faster. What is the length of the distance?

SOLUTION

	rate ×	time =	distance
Original situation	250	$\frac{x}{250}$	x
New situation	300	$\frac{x}{300}$	x
difference of time		1 min	

$\frac{x}{250}-\frac{x}{300}=1$. Finish the solution.

EXAMPLE 2 (7 grade)

Beveling daily 60 hectares instead of 50 hectares, the brigade was able to mow a meadow one day faster than planned, what is the area of grasslands?

SOLUTION

	rate per day ×	time (days) =	area o meadow
By plan	50	$\frac{x}{50}$	x
In fact	60	$\frac{x}{60}$	x
difference of time		1 day	

$\frac{x}{50}-\frac{x}{60}=1$. Finish the solution.

EXAMPLE 3 (8 grade)

Two planes took off simultaneously from one airport to the other, separated by a distance of 1800 km. Speed of the second aircraft was100 km / h less than speed of the first aircraft. So it arrived at the destination 36 minutes later. Find the speed of each aircraft.

SOLUTION

	speed × time (hours) = distance (km)		
The first plane	x	$\frac{1800}{x}$	1800
The second plane	$x-100$	$\frac{1800}{(x-100)}$	1800
difference of time		$\frac{36}{60}=\frac{6}{10}=\frac{3}{5}$	

$$\frac{1800}{(x-100)}-\frac{1800}{x}=\frac{3}{5}.\text{ Solve.}$$

EXAMPLE 4 (8 grade)

Halfway between the stations A and B, the train was delayed by 10 minutes. In order to arrive by the schedule a driver increased the speed of the train for 12 km / h. Find the initial speed of the train, if it is known that the distance between stations is 120 km.

SOLUTION

	speed × time (hours) = distance (km)		
The first half of the distance	x	$\frac{60}{x}$	60
The second half of the distance	$x+12$	$\frac{60}{(x+12)}$	60
difference of time		$\frac{10}{60}=\frac{1}{6}$	

$$\frac{60}{x}-\frac{60}{(x+12)}=\frac{1}{6}.\text{ Solve.}$$

EXAMPLE 5 (8 grade)

Going down the river 150 kilometers, the ship returned back, spending all the way 5 h 30 min. Find the current speed if the speed of the ship in still water 55km / h.

Let x be the current speed, then

SOLUTION

	speed × time (hours) = distance (km)		
Going down the river	$55+x$	$\frac{150}{(55+x)}$	150
Going up the river	$55-x$	$\frac{150}{(55-x)}$	150
total time		$5{,}5=\frac{11}{2}$	

$$\frac{150}{(55+x)}+\frac{150}{(55-x)}=\frac{11}{2}.\text{ Finish the solution.}$$

EXAMPLE 6

The first brigade had to make 160 suits and the second brigade for the same period - 25% less. The first brigade made 10 suits per a day more than the second, and performed the job two days before the scheduled date. How many costumes per a day the second brigade made, if it was required for the job 2 days more?

SOLUTION

	suits per day × time (days) = work		
The first brigade	$x+10$	$\frac{160}{(x+10)}$	160
The second brigade	x	$\frac{120}{x}$	120
difference in days		4	

$$\frac{120}{x}-\frac{160}{(x+10)}=4.\text{ Finish the solution.}$$

EXAMPLE 7

Two cotton harvesters can collect cotton from the field for 9 days rather than one first processor, and 4 days less than one second. For how many days can each harvester collects all the cotton?

SOLUTION

	work per day × time (days) = work		
The first harvester	$\frac{1}{x}$	x	1
The second harvester	$\frac{1}{y}$	y	1
together	$\frac{1}{x}+\frac{1}{y}=\frac{x+y}{xy}$	$\frac{xy}{x+y}$	1

$$\frac{xy}{x+y}=x-9; \quad (1)$$

$$\frac{xy}{x+y}=y-4; \quad (2)$$

From (1) and (2) we get $x-5=y$, substitute it into (1);

$$\frac{x(x-5)}{2x-5}=x-9.$$ Solve.

EXAMPLE 8

Father and son went 240 m, and the father made 100 steps less than the son. Find the length of step of each of them, if the father's step is 20 cm longer then the son's step.

SOLUTION

	length of step × number of steps = distance		
Father	x	$\frac{24000}{x}$	24000
The son	$x-20$	$\frac{24000}{x-20}$	24000
difference in number of steps		100	

$$\frac{24000}{x-20}-\frac{24000}{x}=100.$$ Finish the solution.

Word problems: №1. Jody is 12 and Loren is 36; №2 7 and 23; № 3 4, 7, and 9cm; №4 24 dimes, 30 nickels; №5 10; №6 8, 16, 26; №7 15, 30, 20; Ex.2. 7000 at 5% and 5000 at9%; Ex.3.$1800in women' shoes $4200 in men's shoes; №8 $3200 in stocks, $1380 in bonds; №9 $4500 at 5%, $3000 at 15%; №10 $36,666.67; №11 12 noon; №12 Winner: 20 m per second; loser: 15 m per second; №13 3 hours; №14 5 pounds; №15 30 gallons of the 55-cent oil and 10 gallons of the 75-cent oil; №16 10 gallons of milk and 9 gallons of cream; №17 Add 23 pounds of 25%

MISCELLANEOUS EXERCISES: Ex.1 1500 m; Ex.2 300 hectares; Ex.3 600km/h, 500km/h; Ex.4 60km/h; Ex.5 5km/h; Ex.6. 10 costumes per day; Ex.7, x=15, y= 3; Ex.8 80cm, 60 cm.

SOLVED EXERCISES

EXAMPLE 1

210 centners (1centner = 100 kg) of wheat were collected from each of two sections. The area of the first field is 0.5 hectares less than the area of the second field. How many centners of wheat were collected from 1 hectare of each field, if the crop yields in the first section were 1 centner more than the yields of the second section?

SOLUTION 1

	crop yields × area = harverst		
First field	$\frac{210}{x-1}$	x	210
Second field	$\frac{210}{x+0.5}$	$x+0.5$	210
difference in number of steps	1		

$$\text{Solution: } \frac{210}{x-1}-\frac{210}{x+0.5}=1;$$

$$210x+105-210x-x^2-0.5x=0;$$

$$2x^2+x-210=0;$$

$$x=\frac{-1+\sqrt{1+1680}}{4}=\frac{40}{4}=10;$$

Answer:

1. The crop yields of the first section = $\frac{210}{10}=21$.

2. The crop yields of the second section = $\frac{210}{10+0.5}=20$.

Let's give the other solution which is more correct as it gives the answers at once. Unknown variable x must answer the question of the problem.

SOLUTION 2

	crop yields × area = harverst		
First field	x	$\frac{210}{x}$	210
Second field	$x-1$	$\frac{210}{x-1}$	210
difference in number of steps		0.5	

$$\frac{210}{x-1}-\frac{210}{x}=0.5;$$

$$210x-210x+210=0.5x^2-0.5x;$$

$$x^2-x-420=0;$$

$$(x-21)(x+20)=0;$$

Answer: 21; 20

EXAMPLE 2 (This example is interesting as you can find two ways of solutions)

There are 140 pots of two capacities. The volume of pot of greater capacity is more than for 2.5 liters from the banks of smaller capacity. Total volume of large cans is equal to the total volume of small cans and equal to 60 liters. Determine the number of large and small cans.

SOLUTION 1

Type	Volume of one port	Number of pots	Total volume
1.Greater capacity	$\frac{60}{x}$	x	60
2.Smaller capacity	$\frac{60}{140-x}$	$140-x$	60
Difference of volumes of one port	$2.5=\frac{5}{2}$		

$$\frac{60}{x}-\frac{60}{140-x}=\frac{5}{2};$$

$$\frac{12}{x}-12=\frac{5}{2};$$

$$24(140-x)-24x=140x-x^2;$$

$$x^2-188x+3360=0;$$

$$x_{1,2}=94\pm 74;$$

$$x_1=168-\textit{impossible};$$

$x_2=20$ pots of greater capacity, and 120 pots of smaller capacity.

Answer: 20 and 120

Let's present the second way of solution by introducing two unknowns

SOLUTION 2

Type	Volume of one port	Number of pots	Total volume
1.Greater capacity	$y+2.5$	x	$60=(y+2.5)x$
2.Smaller capacity	y	$140-x$	$60=y(140-x)$
Total		140	

$$\begin{cases}(y+2.5)x=60\\ y(140-x)=60\end{cases};$$

$$\begin{cases}xy+2.5x=60\\ 140y-xy=60\end{cases};$$

$$2.5x+140y=120;$$

$$x=48-56y;$$

$$56y^2+92y-60=0;$$

$$14y^2+23y-15=0;$$

$$y_{1,2}=\frac{-23\pm\sqrt{23^2+4\cdot14\cdot15}}{28};$$

$$y_{1,2}=\frac{-23\pm37}{28}=\frac{-23\pm37}{28};$$

$$y_1=\frac{14}{28}=\frac{1}{2};$$

$$x=48-56\cdot\frac{1}{2}=20;$$

The same answer:

1. Greater capacity = 20
2. Smaller capacity = 120

EXAMPLE 3 №13.166

The student had to find a product of number 136 for a two-digit number, in which the units digit is twice more than "tens" digit. Absent-mindedly, he reversed the double-digit figure that is why he got 1224 more than the product of the original number. What is the original product?

SOLUTION

	The first number	A two-digit number	Product
Original numbers	136	$10x+y$	$136(10x+y)$
Mistaken case	136	$10y+x$	$136(10y+x)$
Difference			1224

$$136(10x+y)=136(10y+x)-1224;$$

As $y=2x$;

$$136(10x+2x)=136(20x+x)-1224;$$

$$1224=136\cdot21x-136\cdot12x;$$

$$153=153x;$$

$$x=1, y=2.$$

Original product is 136 times 12 is 1632

EXAMPLE 4 №13.167

Powerboat and sailboat are while on a lake 30 km from each other, move towards and meet after 1 hour. If powerboat was situated 20 km from sailboat and catches him, it would take 3 hours and 20 minutes. Determine the speed of the boat and sailboat, believing that they are constant and unchanging in both cases.

SOLUTION

Type of boat	Speed	Time (hour) (first case)	Distance (first case)	Time (hour) (second case)	Distance (second case)
1.Powerboat	x	1	x	$3h+20min=\frac{10}{3}$	$\frac{10}{3}x$
2.Sailboat	y	1	y	$\frac{10}{3}$	$\frac{10}{3}y$
Meeting			30		20

$$\begin{cases}x+y=30\\ \frac{10}{3}x-\frac{10}{3}y=20\end{cases};$$

$$\begin{cases}x+y=30\\ x-y=6\end{cases};$$

$$2x=36;$$

Answer: $x=18$ and $y=12$.

EXAMPLE 4 №13.168

Digit number was increased at 10 units. If the resulting number is increased by the same percentage as for the first time, you get 72. Find the original number.

SOLUTION

Number	Original state	Ratio of increasing	Result
The first number	x	$\frac{x+10}{x}=1+\frac{10}{x}$	$x+10$
The second number	$x+10$	$1+\frac{10}{x}$	$(x+10)(1+\frac{10}{x})=72$

$$(x+10)(1+\frac{10}{x})=72;$$

$$x+10+10+\frac{100}{x}=72;$$

$$x+\frac{100}{x}-52=0;$$

$$x^2-52x+100=0;$$

$$x_{1,2}=26\pm\sqrt{676-100};$$

$$x_{1,2}=26\pm 24;$$

Answer: $x=2$ and 50.

EXAMPLE 5

The sum of squared of digits of a two-digit number is equal to 13. If this number is subtracted 9 then we get the number which will be written with the same digits, but in reverse order. Find this number.

SOLUTION

	Digits	Squared	Operation
Tens	x	x^2	y
Units	y	y^2	x
		13	$(10x+y)-9=10y+x$

$$\begin{cases} 9x-9y=9 \\ x^2+y^2=13 \end{cases};$$

$$\begin{cases} x-y=1 \\ x^2+y^2=13 \end{cases};$$

$$\begin{cases} x=y+1 \\ (y+1)^2+y^2=13 \end{cases};$$

$$\begin{cases} x=y+1 \\ y^2+y-6=0 \end{cases};$$

$$(y+3)(y-2)=0;\ y=2; x=3.$$

Answer: 32

EXAMPLE 6

Crystal during the stage of formation uniformly increases its mass. Watching the formation of two crystals one has noticed that the first of them gave the same weight for 3 months as the second for 7 months. However, after one year was found that the first crystal has increased its initial weight by 4%, the second - 5%. Find the ratio (k) of the initial mass of these crystals.

SOLUTION

Type	Initial weight	Annual percentage	Annual growth	Month growth	Special condition
1.First crystal	kx	0.04	$0.04kx$	$\frac{0.04kx}{12}$	for 3 months $\frac{3\cdot 0.04kx}{12}$
2.Second crystal	x	0.05	$0.05x$	$\frac{0.05x}{12}$	for 7 months $\frac{7\cdot 0.05x}{12}$

$$\frac{3\cdot 0.04kx}{12}=\frac{7\cdot 0.05x}{12};\quad k=\frac{35}{12}$$

Answer: ratio is $k=\frac{35}{12}$.

EXAMPLE 7

The pool held two pipes of different cross sections. One - is uniformly applied, and the other - uniformly divert water, the pool is filled through the first pipe 2 hours longer than the second emptied. When 1/3 the pool was filled both pipes were opened, and the pool was empty after 8 hours. For how many hours, acting separately does the first pipe fill and the second pipe empty the pool?

SOLUTION

	Speed	Time	Volume	Time	Volume
Applied pipe	$\frac{1}{x}$	x	1	8	$\frac{8}{x}$
Diverting pipe	$\frac{1}{x-2}$	$x-2$	1	8	$\frac{8}{x-2}$
Special condition					$\frac{1}{3}$

$$\frac{1}{3}+\frac{8}{x}=\frac{8}{x-2};$$

$$x^2-2x+24x=24x+48;$$

$$x^2-2x-48=0;$$

$$(x-8)(x+6)=0;$$

$$x=8.$$

Answer: 8h, 6h.

EXAMPLE 8

In the state garage there are 54drivers. How many free days in month (30 days) could every driver have, if everyday 25 percent's of cars from all 60 are stay for a preventive maintenance?

SOLUTION

Let x be the free day for each driver.

	Quantities	Working days	All amount of working days
Drivers	54	$30-x$	$54(30-x)$
Cars	$075\cdot 60=45$	30	$45\cdot 30$

$$54(30-x)=1350;$$
$$1620-54x=1350;$$

$$54x=270;$$

$$x=5.$$

Answer: 5 free days.

EXAMPLE 8

One brigade can clean all field for 12 days. The second brigade takes 75% of this planed time to finish this work. After 5 days when only the first brigade worked, the second brigade joined to the first brigade, and they together finished work. How many days did two brigades work together?

SOLUTION

Let x be the number of days that two brigades worked together

	Speed	Time	Work
First brigade	$\frac{1}{12}$	12	1
Second brigade	$\frac{1}{9}$	$\frac{3}{4}*12=9$	1
Together	$\frac{1}{12}+\frac{1}{9}$	x	$1-\frac{5}{12}$-

$$(\frac{1}{12}+\frac{1}{9})\cdot x=1-\frac{5}{12};$$

$$\frac{7}{36}\cdot x=\frac{7}{12};$$

$$x=3.$$

Answer: 3 days.

EXAMPLE 9

Same details are processed on two machines. Productivity of the first machine is 40% more than productivity of the second machine. How many details were processed by each machine, if the first machine has worked 6 hours in this shift, and the second – 8 hours, and besides two machines together have processed 820 details?

SOLUTION

Let x be the number of details produced by the second machine per hour.

	Speed	Time	Work
First machine	$1.4\cdot x$	6	$6\cdot 1.4x$
Second machine	x	8	$8\cdot x$
Together			820

$$8.4x+6x=820;$$

$$16.4x=820;$$

$x=50.$

Answer: The first machine produces 420 and the second 400 details.

EXAMPLE 10

A boat went from the pier to the city with velocity 12 km/h, and after half an hour later of boat departure, a steamship went at the same direction with velocity 20 km/h. What is the distance between the pier and the city, if the steamship came to the city for 1.5 hour earlier than boat?

SOLUTION 1

	Speed	Time	Distance
Boat	12	$x+0.5+1.5$	$12(x+2)$
Steamship	20	x	$20\cdot x$

$$12(x+2)=20\cdot x;$$
$$12x+24=20x;$$
$$x=3;$$
$$20\times 3=60km.\ \text{Distance.}$$

Answer: 60km.

SOLUTION 2

Now let x be the distance

	Speed	Time	Distance
Boat	12	$\frac{x}{12}$	x
Steamship	20	$\frac{x}{20}$	x
Difference of time		2	

$$\frac{x}{12}-\frac{x}{20}=2;$$

$$\frac{2x}{60}=2;$$

$$x=60.$$

Answer: 60km.

EXAMPLE 11

To pay for the shipment of four parcels one needed 4 different stamps totaling 21 tenge. Determine the value of brands acquired by the sender, if the value of stamps is an arithmetic progression, and the most expensive brand of 2.5 times more expensive than the cheapest.

SOLUTION

Sender	View an arithmetic progression	Extra information
First stamp	x	cheapest
Second stamp	$x+d$	
Third stamp	$x+2d$	
Fourth stamp	$x+3d$	$2.5\cdot x$
All	$21 tenge$	

$$\begin{cases} x+(x+d)+(x+2d)+(x+3d)=21 \\ x+3d=2.5x \end{cases};$$

$$\begin{cases} 4x+6d=21 \\ 3d-1.5x=0 \end{cases};$$

$$x=3, d=1.5;$$

$$3;4.5;6;7.5.$$

Answer: *3tenge;4.5tenge;6tenge;7.5tenge.*

EXAMPLE 12

There were 320 seats in the auditorium of the club. After located the same seats the number in each row have increased by 4 and added another row. The number of total seats became 420. How many rows are there now?

SOLUTION

	Seats in a row	Row	All seats
Past	$\frac{320}{x-1}$	x	320
Now	$\frac{420}{x}$	x	420
Difference of seats	4		

$$\frac{320}{x-1}=\frac{420}{x}-4;$$
$$x^2-26x+105=0;$$
$$(x-21)(x-5)=0;$$
$$x_1=21; x_2=5.$$

Answer: 21*rows*;5*rows*.

EXAMPLE 13

108 examinees wrote an essay. They were distributed to 480 sheets of paper, each girl has got one sheet more than each young man, and all the girls got the same number of sheets as all boys. How many were there boys and girls?

SOLUTION

	Number	Number of sheets for each person	All seats
Girls	x	$y+1$	$x(y+1)$
Boys	$108-x$	y	$(108-x)\cdot y$
All	108		480

As the girls received the same number of sheets as all boys

$$\begin{cases} x(y+1)+(108-x)y=480 \\ x(y+1)=(108-x)\cdot y \end{cases};$$
$$2x(y+1)=480;$$
$$(y+1)=\frac{240}{x};$$
$$y=\frac{240}{x}-1;$$
$$y=\frac{(240-x)}{x};$$
$$240x=(108-x)(240-x);$$
$$x^2-588x+25920=0;$$
$$(x-540))(x-48)=0;$$
$$x=48;$$

Answer: girls-48; boys-60

REFERENCES:

1) Bernard Kolman, Arnold Shapiro, "Algebra for college students", Academic Press, Inc., Orlando, Florida, 1986

2) Сборник задач для поступающих в вузы. Под редакцией М. И. Сканави, Москва, Мир и Образование,

TABLE OF CONTENTS

Учебное издание

Базарбаева Лариса Ермурзаевна

**UNIFIED METHOD
OF SOLUTION OF WORD PROBLEMS**

Методическая разработка

Технический редактор В. В. Вербовский
Компьютерная верстка Л. Е. Базарбаева
Подписано в печать 27.08.2015. Формат 60x90/16. Бумага офсетная.
Печать цифровая. Объем 2. Тираж 100 экз.

Отпечатано в типографии Университета имени Сулеймана Демиреля