Applications of the differentiation method in the study of the topic of trigonometric equations and inequalities.

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List of Abbreviations

UK —United Kingdom
NIS — Nazarbayev Intellectual Schools
SAT— Scholastic Assessment Test
US —United States
SAU — Summative Assessment for Unit
SAT — Summative Assessment for Term
FA — Formative Assessment
AI — Artificial Intelligence

ABSTRACT

This dissertation comprehensively explores the effectiveness of applying the differentiation method in teaching trigonometric equations and inequalities. In today's educational system, the variation in students' preparedness levels has become a pressing issue, especially when dealing with complex mathematical topics like trigonometry.

The theoretical part of the study discusses the concept and importance of differentiated instruction, along with a review of both domestic and international practices. The methodological section presents approaches for designing level-specific tasks on trigonometric equations and inequalities. Levels A, B, and C were introduced, and customized tasks were developed using graphical methods, function properties, and identity transformations.

A pedagogical experiment was conducted with students from grades 5, 6, and 11. The findings demonstrate a positive impact of differentiation on student performance, motivation, and independent learning. Even students with lower academic performance achieved success through assignments tailored to their capabilities.

The study also considers the integration of artificial intelligence (AI) tools to support differentiated learning. Digital platforms allow teachers to accurately assess student levels and assign personalized tasks accordingly.

The research concludes that the differentiation method is effective not only in teaching trigonometry but also applicable to other areas of mathematics. Therefore, broader implementation and enhanced teacher training in differentiation strategies are recommended.

Keywords: differentiated instruction, trigonometric equations, trigonometric inequalities, mathematics education, student achievement, learning levels, pedagogical experiment, artificial intelligence in education, personalized learning, digital tools

АҢДАТПА

Бұл диссертациялық жұмыста тригонометриялық теңдеулер мен теңсіздіктерді оқытуда дифференциация әдісін қолданудың тиімділігі жан-жақты зерттелді. Қазіргі білім беру жүйесінде оқушылардың дайындық деңгейінің әртүрлілігі жиі кездесетін мәселе болып отыр. Әсіресе күрделі математикалық тақырыптар, соның ішінде тригонометриялық материалдарды меңгеруде бұл айырмашылық айқын көрінеді.

Жұмыстың теориялық бөлімінде саралау (дифференциация) әдісінің мәні мен маңыздылығы қарастырылып, отандық және шетелдік тәжірибелерге шолу жасалды. Әдістемелік бөлімде тригонометриялық теңдеулер мен теңсіздіктер тақырыбында деңгейлік тапсырмалар құрудың жолдары мен оларды сабақта қолдану үлгілері ұсынылды. Оқушылардың дайындық деңгейіне қарай A, B, C деңгейлері енгізіліп, графикалық әдістер, функция қасиеттері мен тригонометриялық тепе-теңдіктерді түрлендіру тапсырмалары әр деңгейге бейімделіп жасалды.

Зерттеу барысында 5, 6 және 11 сынып оқушыларымен тәжірибелік жұмыс жүргізіліп, саралау әдісінің нәтижелілігі тәжірибе арқылы дәлелденді. Эксперимент барысында алынған мәліметтер оқушылардың танымдық белсенділігі, қызығушылығы мен өз бетімен жұмыс істеу дағдыларының артқанын көрсетті. Оқушылардың оқу жетістіктері едәуір жақсарып, әсіресе әлсіз оқушылар да табысқа жете алатын мүмкіндікке ие болды.

Сонымен қатар, жұмыс барысында жасанды интеллект (ЖИ) құралдарын дифференциация әдісінде қолдану мүмкіндіктері де қарастырылды. Заманауи цифрлық платформаларды пайдалану арқылы мұғалімдер оқушылардың деңгейін нақты анықтап, жеке тапсырмалар ұсынуға мүмкіндік алады.

Зерттеу нәтижесінде саралау әдісінің тригонометрияны оқытуда ғана емес, өзге математикалық тақырыптарға да тиімді қолдануға болатындығы анықталды. Осыған орай, бұл әдісті кеңінен енгізу және мұғалімдердің осы бағыттағы кәсіби даярлығын арттыру ұсынылады.

Кілт сөздер: дифференциация әдісі, тригонометриялық теңдеулер, тригонометриялық теңсіздіктер, математикадағы оқыту, оқушылар жетістігі, оқу деңгейлері, педагогикалық эксперимент, білімдегі жасанды интеллект, дараланған оқыту, цифрлық құралдар

АННОТАЦИЯ

В данной диссертационной работе всесторонне исследуется эффективность применения метода дифференциации при обучении тригонометрическим уравнениям и неравенствам. В современной системе образования различия в уровне подготовки учащихся стали актуальной проблемой, особенно при изучении сложных математических тем, таких как тригонометрия.

В теоретической части работы рассмотрена сущность и значимость дифференциации, проведён обзор отечественного и зарубежного опыта. В методической части представлены подходы к построению заданий различного уровня сложности по теме тригонометрических уравнений и неравенств. Введены уровни A, B и C, и для каждого уровня были разработаны задания с использованием графических методов, свойств функций и преобразования тригонометрических тождеств.

В рамках исследования был проведён педагогический эксперимент с учащимися 5, 6 и 11 классов. Результаты показали положительное влияние метода дифференциации на успеваемость, мотивацию и самостоятельную работу учащихся. Даже ученики с низким уровнем подготовки смогли достичь положительных результатов благодаря заданиям, адаптированным к их возможностям.

Также в работе рассмотрены возможности применения инструментов искусственного интеллекта (ИИ) в поддержке дифференцированного обучения. Использование цифровых платформ позволяет учителям точно определять уровень учащихся и предлагать им индивидуализированные задания.

Результаты исследования подтверждают, что метод дифференциации может быть эффективно использован не только при обучении тригонометрии, но и в других разделах математики. В связи с этим рекомендуется более широкое внедрение данного метода и повышение профессиональной подготовки учителей по данному направлению.

Ключевые слова: метод дифференциации, тригонометрические уравнения, тригонометрические неравенства, обучение математике, учебные достижения, уровни обучения, педагогический эксперимент, искусственный интеллект в образовании, персонализированное обучение, цифровые инструменты

INTRODUCTION

In modern educational systems, one of the main challenges faced by mathematics teachers is dealing with the diverse academic levels of students within the same classroom. The gap in understanding and achievement is particularly visible in high school, where students are expected to master more complex and abstract mathematical concepts. Among these, trigonometric equations and inequalities stand out as topics that often cause difficulty due to their theoretical complexity and the need for prior knowledge, such as function transformations, algebraic manipulation, and geometric interpretation.

Moreover, many students approach this topic with anxiety, and their confidence is often undermined by early failure to grasp fundamental ideas. In traditional teaching models, instruction is usually delivered uniformly, based on the assumption that all students are progressing at the same pace. However, this does not reflect classroom reality. Students have different cognitive styles, prior knowledge, and levels of engagement. Without differentiated instruction, teachers risk leaving behind students who need additional support, while simultaneously failing to challenge those who are ready for more advanced tasks.

Therefore, applying the differentiation method is not just a methodological choice but a necessity to make mathematics teaching more inclusive, responsive, and effective. Differentiation allows educators to tailor learning tasks, provide flexible pathways, and create environments where each student can learn meaningfully and at their own pace. By introducing this method in the teaching of trigonometric equations and inequalities, educators can potentially transform student outcomes and experiences, making mathematics more accessible and engaging for all learners.

Background and Previous Work the idea of differentiated instruction is not new in educational literature. Researchers such as (Tomlinson, 2001)have emphasized the importance of differentiating content, process, product, and learning environment based on students' readiness, interests, and learning profiles. Studies in mathematics education have shown that differentiated approaches can lead to improved academic performance, increased motivation, and higher levels of student satisfaction.

Despite this, the actual application of differentiation in secondary school mathematics, especially in challenging topics like trigonometric equations and inequalities, remains limited in many educational contexts. Much of the existing research focuses on general pedagogy or early grade-level interventions. There is a clear gap in the literature regarding how to practically and systematically implement differentiation in the upper grades of school mathematics.

This research aims to fill that gap by proposing a concrete framework for using differentiation in the context of teaching trigonometric equations and inequalities to 11th-grade students. Moreover, the study extends its scope by testing differentiation methods with younger students in 5th and 6th grades, allowing for a comparative analysis and better understanding of the method's overall potential across age groups.

Research Problem Currently, the variability in students' knowledge levels has become a common issue in the education system. Some students grasp material quickly, while others need additional examples and time to understand the topic. This situation frequently occurs even within the same class, particularly when teaching complex topics such as trigonometric equations and inequalities. In such cases, traditional teaching methods fail to meet the needs of all students.

As a result of working with traditional (uniform) educational methods, lower-performing or weaker students tend to fall behind, while higher-performing students lose interest. Moreover, failing to consider each student's level and abilities during the teaching process leads to a decrease in the quality of education, reduced student motivation, and lowered self-confidence.

Therefore, the main problem is the lack of an effective method to deliver trigonometric equations and inequalities to students of varying levels and the absence of consistent use of the differentiation method to address this issue.

Research Objective: Currently, it is evident that the levels of school students vary significantly. This process is especially noticeable among high school students and occurs frequently. It becomes particularly evident when teaching complex topics such as trigonometric equations and inequalities. This directly affects students' understanding, results, and interest in the subject. Taking such situations into account, the aim of my chosen topic is to explore effective and easy methods of teaching trigonometric equations and inequalities using the differentiation method. Based on previous practical experiences with students, conducted lessons, and published research results, I aim to achieve this goal. Therefore, the research objective is to organize the differentiation method correctly when working with students of varying levels and to identify effective ways of applying it in teaching trigonometric equations and inequalities.

Integration with Technology (AI and Digital Tools) The research object is the process of applying the differentiation method in teaching trigonometric equations and inequalities to 11th-grade students. Additionally, to comprehensively assess the effectiveness of the differentiation method, the practical part of the study was conducted with 5th-grade students (on the topic of decimal fractions) and 6th-grade students (on the topic of linear equations). The characteristics and effectiveness of applying the differentiation method in teaching trigonometric equations and inequalities to 11th-grade students. Furthermore, the study investigates ways to implement this method in practice, drawing from teachers' experiences.

Research Tasks: To achieve the objectives set in this research, the following tasks are outlined:

Theoretical and practical analysis of the difficulties in teaching trigonometric equations and inequalities.

Description of the pedagogical and psychological foundations of the differentiation method and its application in mathematics lessons.

Creation of a system of learning tasks and level-based tasks for teaching trigonometric topics using the differentiation method.

Determination of the preparedness levels of the students participating in the research and the development of a work model for them.

Organization of experimental research and analysis of the results to prove the effectiveness of the differentiation method.

Based on the scientific results obtained during the research, providing methodological recommendations and demonstrating how to apply them in practice.

Research Question. Does the application of the differentiation method lead to effective results in teaching trigonometric equations and inequalities to 11th-grade students, and how can this method be effectively organized in practice?

This study aims to determine whether differentiated instruction significantly enhances students' learning outcomes when studying complex mathematical concepts such as trigonometric equations and inequalities. The research also seeks to identify optimal strategies for implementing this method in real classroom conditions. Key sub-questions include:

How does differentiation impact students with varying levels of mathematical preparedness?

What types of differentiated tasks (e.g., visual, analytical, practical) are most effective for trigonometry?

In what ways can modern technologies, including AI tools, support the differentiation process?

How can teachers effectively manage class time, resources, and assessment within a differentiated framework?

Answering these questions can provide valuable insights into designing inclusive and effective lessons that cater to all students' needs.

Hypothesis. If the differentiation method is applied while considering the individual levels of students when teaching trigonometric equations and inequalities, then the academic performance of 11th-grade students will improve, their mathematical thinking abilities will develop, and their interest in the subject will increase.

This hypothesis is based on the assumption that personalized instruction, aligned with each student's readiness, pace, and learning style, promotes deeper understanding and retention of complex mathematical ideas. By assigning tasks that correspond to students' current knowledge levels—ranging from basic to advanced—educators can foster:

Greater academic achievement as students work within their "zone of proximal development";

Development of critical mathematical thinking through progressively challenging problems;

Higher motivation and engagement due to the feeling of success and autonomy;

Increased confidence in solving trigonometric problems independently;

Positive attitudes toward mathematics and increased participation in class discussions.

These outcomes are expected to be particularly visible in students who previously struggled with standard, uniform approaches to learning.

Does the application of the differentiation method lead to effective results in teaching trigonometric equations and inequalities to 11th-grade students, and how can this method be effectively organized in practice?

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Integration with Technology (AI and Digital Tools) (ChatGPT, 2025) Alongside current pedagogical challenges, this study also examines how technology and data-driven solutions can support the application of the differentiation method. With the advancement of artificial intelligence (AI) tools and digital learning platforms, teachers are now able to individualize instruction more effectively than ever before. In this context, the differentiation method not only takes into account students' academic levels, but also fosters their initiative, critical thinking, and collaboration skills. Formative assessment, student feedback, and learning analytics help select appropriate task levels and adapt them to the student's pace of development.

For example, during a lesson, students may be given an initial set of five problems. Based on their solutions, AI can automatically determine the student's level and offer additional tasks suited to that level. This approach is particularly effective in mixed-ability classrooms, where each student can progress at their own pace and achieve success. Furthermore, AI tools can automatically check students' answers and provide feedback by analyzing errors. This eases the teacher's workload and helps save time.

Digital technologies go beyond just generating assignments. With the help of AI, it is possible to recognize notes in students' individual notebooks, apply assessment criteria, and automatically analyze their development. Additionally, tests and self-study tasks can be generated automatically, enabling teachers to personalize assessment and unlock each student's potential.

AI technologies are also beneficial in group work. For instance, students can be grouped based on their levels for a particular topic, with each group receiving tailored tasks. AI can create these tasks by taking into account the students' previous achievements and interests. This approach encourages peer learning, teamwork, and a stronger sense of responsibility.

To increase student engagement with the subject, the potential to automatically generate real-life-related problems using AI was considered. For example, by designing tasks related to a student's future profession or hobby, the subject content becomes more meaningful and relevant. This shifts learning from formal understanding to practical application.

Thus, integrating the differentiation method with modern technologies—particularly AI tools—opens a new level in mathematics instruction. This integration provides a convenient and effective approach for teachers, while creating an engaging, accessible, and productive learning environment for students. If widely implemented in the future, this method has the potential to improve the quality of education and enhance students' mathematical literacy.

Theoretical Justification and Broader Relevance of the Study. Differentiated instruction, while gaining popularity in recent years, is still underutilized in subject-specific high school education. The complex nature of topics such as trigonometric equations and inequalities requires not only solid subject knowledge from the teacher, but also an understanding of how to scaffold that knowledge for students at varying levels of preparedness. This study builds on the assumption that academic diversity in mathematics is not a barrier but a feature of real classrooms, and thus teaching strategies must evolve to accommodate this diversity.

A key theoretical foundation of this research is Lev Vygotsky's concept of the Zone of Proximal Development (ZPD), which defines the gap between what a learner can do independently and what they can achieve with guidance. Differentiation directly supports movement within this zone, allowing learners to make incremental progress with the right amount of challenge and support. Likewise, Jerome Bruner's theory of scaffolding further emphasizes the role of structured support, especially when introducing abstract mathematical concepts such as identities, transformations, and functional graphs.

Moreover, this study also relies on contemporary models of cognitive development and information processing, which highlight that learners progress through different stages in acquiring mathematical proficiency: from concrete understanding to abstract generalization, and finally to procedural fluency. Trigonometry sits at the intersection of all three, requiring visual, symbolic, and numerical reasoning. Without appropriate differentiation, many students are unable to cross from one stage to the next.

By incorporating this theory into classroom practice, teachers are better able to personalize content—not only by simplifying or complicating tasks but by adjusting formats (visual, verbal, interactive), groupings (peer tutoring, small groups), and goals (mastery, creativity, exploration).

Differentiation in Trigonometry: Specific Challenges and Opportunities

Trigonometric equations and inequalities are often considered one of the most intimidating topics for secondary school students. Several factors contribute to this:

Abstract notation: Many students struggle with understanding sine, cosine, and tangent functions as abstract relationships rather than just calculator keys.

Multiple representations: Trigonometry involves graphs, unit circles, algebraic identities, and tables—all of which require different modes of reasoning.

Sequential dependency: Mastery of trigonometry requires prior knowledge in algebra and geometry. Any gaps in these foundational topics cause difficulties in trigonometric learning.

Differentiation offers a solution by breaking complex topics into manageable learning pathways. For example:

Students at level A can start with real-world applications of sine and cosine (e.g., waves, building design) to build intuitive understanding.

Level B students can work with structured problems involving transformations of trigonometric graphs and basic identities.

Level C students can explore advanced equations, inequalities, and exam-level word problems.

This tiered approach enables each student to engage meaningfully, while still progressing toward shared curriculum goals.

Practical Benefits of AI-Assisted Differentiation. While traditional differentiation can be labor-intensive, AI provides teachers with practical tools to implement this methodology more efficiently. Here are some additional ways AI tools assist the process:

Task personalization: Based on performance data, AI can suggest not only the level of task but also the most appropriate format (visual, numerical, verbal).

Formative feedback: Instant feedback provided by AI encourages students to self-correct without the fear of public failure.

Progress prediction: Machine learning algorithms can analyze trends in student performance and help teachers identify those who are at risk of falling behind before it happens.

Furthermore, digital tools offer analytics dashboards, progress heat maps, and collaborative environments where students can work together asynchronously but at their own pace. These tools shift the teacher's role from content deliverer to learning coach and mentor.

Another overlooked benefit is student autonomy. When learners receive tasks aligned with their level and interest, they take more responsibility for their learning. This is especially true in environments where AI empowers them to set their own pace, revisit explanations, or request additional resources.

The Future of Differentiated Mathematics Instruction

This study proposes a vision for a more adaptive, inclusive, and forward-thinking mathematics classroom. The differentiation method, supported by technological innovation, provides a pedagogical model that does not just serve top-performing students but benefits all learners. The future of education will likely involve a hybrid model where teacher expertise and AI tools work together—balancing emotional intelligence and technological precision.

In broader terms, this approach aligns with global educational priorities such as:

UN Sustainable Development Goal 4: Ensure inclusive and equitable quality education for all.

OECD Learning Compass 2030: Emphasize student agency, personalized learning, and future-ready skills.

By embedding differentiation and AI into regular teaching practice, we not only prepare students for exams, but also equip them with problem-solving, collaboration, and self-regulation skills essential for the 21st century.

This study, although focused on a specific topic in a specific grade, serves as a model for wider application. With the right training and resources, any mathematics teacher can implement these strategies in algebra, geometry, statistics, or beyond.

1. LITERATURE REVIEW

1.1 Theoretical Foundations of Applying the Differentiation Method in Teaching Trigonometric Equations and Inequalities

Differentiated instruction has become one of the central pedagogical strategies in modern education, especially in the context of increasing diversity in students' academic performance, motivation, and cognitive development. The theoretical foundations of differentiated instruction are rooted in constructivist learning theories, particularly those developed by Lev Vygotsky, Jean Piaget, and John Dewey, emphasizing the importance of individualized learning paths, student readiness, and the zone of proximal development.

In mathematics education, these principles are particularly relevant due to the abstract and cumulative nature of mathematical knowledge. Students often demonstrate significantly different levels of understanding, especially in complex topics such as trigonometric equations and inequalities. These topics require not only procedural fluency but also a deep conceptual grasp of functions, periodicity, identities, and algebraic manipulation. Without appropriate differentiation, a teacher may fail to engage students who struggle or, conversely, bore those who are ready for more advanced challenges.

Differentiated instruction, as defined by Tomlinson (1999), refers to the proactive planning of curriculum and instruction that considers students' varying background knowledge, readiness, language, preferences in learning, and interests. This method allows for the adaptation of content, process, product, and learning environment to optimize educational outcomes for each learner. In mathematics, this may include tiered assignments, flexible grouping, scaffolded questioning, and the use of visual, auditory, or kinesthetic representations. Furthermore, it supports equity in education by addressing the diverse needs of learners and ensuring that each student has access to success (Konstantinou-Katzi, 2013).

In the context of Kazakhstan's education reform initiated in 2016, differentiation has been integrated into national curriculum standards. However, its effective implementation in high school mathematics, particularly in topics like trigonometry, remains a developing area. Studies conducted in Kazakhstani schools indicate both the potential and challenges of applying differentiation, including the need for more structured training and methodical resources (Ibrayeva, 2023) (Ibrayeva et al., 2023; Bekeyeva, 2019).

Given the cognitive demands of trigonometric equations and inequalities, differentiation is not only useful but necessary. It provides students with opportunities to learn at their own pace, build foundational knowledge through scaffolded tasks, and explore advanced problems when ready. The differentiated approach also aligns with the STEM-based goals of modern education, promoting logical reasoning, problem-solving, and independence.

Thus, the theoretical foundation of differentiation serves as a strong pedagogical base for the development of effective teaching strategies in trigonometry.

The following sections explore specific models of differentiated instruction and their application in mathematics classrooms, supported by international and local research.

1.1.1. Differentiation Method in Teaching Mathematics

In the modern education system, the diversity in students' cognitive, psychological, and academic levels is one of the main factors that teachers must consider. This difference is particularly noticeable in mathematics, as some students quickly grasp formulas, while others need additional explanations and time. In such cases, giving the same task to all students might be either too difficult for some or too easy for others. This can reduce students' interest in the lesson and negatively affect their academic performance.

To avoid such situations, the use of the differentiation (or differentiated) method is one of the effective solutions. This method allows for optimizing the learning process by providing tasks at different levels, considering the individual characteristics of students. In other words, the differentiation method ensures that teaching aligns with each student's preparedness level.

In scientific literature, the differentiation method has been described in various ways. According to C.A. Tomlinson, a leading expert in the field, it is “an organized, yet flexible way of proactively adjusting teaching and learning to meet students where they are and to help them achieve maximum growth as learners” (Tomlinson, 1999). This type of teaching has been widely implemented in countries such as those in Europe, the USA, Canada, Cyprus, and Singapore. These countries test students, divide them into levels, and provide each student with personalized tasks, time, and feedback. The effectiveness and theoretical rationale of differentiated instruction are well described in academic literature. For instance, Konstantinou-Katzi et al. (2013) highlight that one of the theoretical foundations of differentiated instruction is rooted in social constructivism and Vygotsky’s concept of the Zone of Proximal Development, which emphasizes learning through scaffolding and collaboration. This approach ensures that students work slightly beyond their comfort zones with proper support, leading to deeper understanding.

Differentiated instruction is not about preparing entirely separate lessons for each student, but about modifying the process and materials to meet students where they are. All students aim toward the same learning goals, but follow different paths based on readiness, interest, and learning profiles (Konstantinou-Katzi et al., 2013, p. 333). In their action research with engineering students, the authors demonstrated that students who were taught using differentiated methods showed significant improvement in their mathematical understanding and expressed greater motivation and engagement, especially when given tasks sequenced by difficulty and offered individual or group support. Joseph et al. (2013) reinforce this view, emphasizing that differentiated instruction significantly increases learners’ engagement and interest. Their study, conducted among over 400 pre-service teachers, revealed that over 90% of participants felt more intellectually stimulated when tasks were adjusted to their level of readiness, interest, and learning profiles. Furthermore, their research highlights the importance of modifying content, process, and product to match learner variability.

In addition to international perspectives, a systematic review by *Frontiers in Psychology* (2019) explored the effects of differentiated instruction across 14 recent studies in secondary education. It concluded that well-planned differentiated strategies, such as tiered tasks and homogeneous grouping, showed moderate but consistent improvements in students' academic performance, particularly for those with lower and average achievement levels. The review also emphasized the need for professional development and systemic support to ensure effective implementation.

In Kazakhstan, the differentiation method was introduced in primary and secondary school grades starting in 2016 as part of the updated curriculum. However, its systematic use in high school, especially for complex topics, has not yet been fully established. A recent study by Ibrayeva et al. (2023) conducted at Nazarbayev Intellectual School showed that although Kazakhstani teachers are aware of differentiated strategies, their use remains limited due to time constraints and heavy workload. Similarly, a case study by Bekeyeva (2019) in a South Kazakhstan school highlighted that while multi-level tasks are used in practice, teachers often lack confidence and methodological readiness to implement differentiation consistently and effectively.

Russian pedagogical literature also contributes to the understanding of differentiation, particularly in the context of early schooling. According to the study under Federal State Educational Standards (ФГОС), differentiation is considered essential for creating a "situation of success" that fosters motivation and self-confidence in every student. (петров, 2024) The methodology includes diagnostic assessments, grouping by ability, and assigning tiered tasks with varying levels of teacher support. These strategies, as described by the Russian authors, align with international best practices and can be adapted for use in mathematics lessons, including trigonometry.

In my own teaching experience, I have applied this method with students in grades 5, 6, and 11. For 11th-grade students, the differentiation method was effective in teaching trigonometric equations and inequalities, while for younger grades, it worked well in topics such as decimal fractions and simple equations. I noticed that when students were given tasks that aligned with their knowledge level, they became more confident and engaged in the learning process. Additionally, the structure of the lessons changed: tasks divided into A, B, and C levels, individual explanations, and pair and group work increased the effectiveness of the lessons.

Therefore, understanding the theoretical foundations and proven benefits of differentiated instruction provides a strong basis for its implementation in teaching trigonometric equations and inequalities in secondary schools.In the modern education system, the diversity in students' cognitive, psychological, and academic levels is one of the main factors that teachers must consider. This difference is particularly noticeable in mathematics, as some students quickly grasp formulas, while others need additional explanations and time. In such cases, giving the same task to all students might be either too difficult for some or too easy for others. This can reduce students' interest in the lesson and negatively affect their academic performance.

In Kazakhstan, the differentiation method was introduced in primary and secondary school grades starting in 2016 as part of the updated curriculum. However, its systematic use in high school, especially for complex topics, has not yet been fully established. Therefore, the relevance of this topic is increasing.

In my own experience, I have used this method with 5th, 6th, and 11th-grade students. For 11th-grade students, the differentiation method was effective in teaching trigonometric equations and inequalities, while for younger grades, it was successful in teaching decimal fractions and simple equations. I noticed that when students were given tasks that aligned with their knowledge level, they became more confident and engaged in the learning process. Additionally, the structure of the lessons changed: tasks divided into A, B, and C levels, individual explanations, and pair and group work increased the effectiveness of the lessons.

Therefore, this section aims to examine the theoretical foundations of the differentiation method and scientifically describe its application in mathematics lessons. The use of the differentiation method in teaching mathematics is one of the most pressing issues in the current education system. This method allows for the customization of each student's learning trajectory, taking into account the differences in students' academic levels and cognitive abilities. The importance of using the differentiation method is especially relevant in the context of the differences in educational levels and students' varying abilities to comprehend the material in schools across Kazakhstan.

The differentiation method involves providing students with tasks at different levels, using teaching approaches that match their individual abilities. Through this method, students' engagement and motivation in the learning process are enhanced. By considering students' differences in mastering the material, the differentiation method offers an effective path for teaching. Encouraging students to complete level-based tasks stimulates their interest in the subject and helps deepen their knowledge.

This method is particularly effective in teaching trigonometric equations and inequalities. When teaching complex topics like trigonometry, students' rates of comprehension and ability to understand the material vary. Therefore, the differentiation method ensures that each student receives individual tasks and problems, helping to deepen their knowledge. Using this method, students are divided into three levels (A, B, C), and tasks are tailored to their abilities.

The research conducted by Alibek Kazmukhanov and Aigul Dauletkulova (Dauletkulova A.U., 2025) confirmed the effectiveness of the differentiation method in teaching mathematics. According to their findings, applying the differentiation method allows for the consideration of differences in students' academic levels, offering each student a personalized approach. Dividing students into three levels (A – lower level, B – intermediate level, C – higher level) has been shown to enhance the effectiveness of teaching. This approach increases students' interest in mathematics and boosts their engagement in the learning process. During experiments with school students, the differentiation method proved effective in assigning tasks based on the students' levels, and solutions to problems were offered accordingly. For example, in lessons for 5th-grade students, tasks at different levels were provided. Students in level

A were given simpler tasks, while B and C level students were given more complex tasks. This method helped students use their time efficiently. Students at level A solved the problems in 5 minutes, while those at levels B and C completed the tasks in 3–4 minutes. Thus, applying the differentiation method in teaching trigonometric equations and inequalities helps to enhance the effectiveness of the learning process. Given that students have varying abilities, it is crucial for teachers to offer each student a personalized learning path, preparing tasks according to their pace and knowledge level. Such approaches increase students' interest in the subject and improve their motivation during the learning process.

1.2 Current Status of Applying the Differentiation Method

Currently, the differentiation method is successfully applied in the education systems of many countries. Numerous studies highlight the effectiveness and challenges associated with this method. The following examples show its use in Cyprus, Trinidad, and the Russian Federation. Today, to improve the quality of education, many countries are setting new requirements for the content and methods of teaching. Among these, the differentiation method is gaining widespread recognition as an essential tool for considering students' individual characteristics. The primary goal of this method is to organize the learning process in line with each student's abilities, knowledge, and interests. According to international research, this method has been successfully implemented in countries like Cyprus, Trinidad (Joseph, 2013), and Russia. Specifically, in Cyprus, students are offered three levels of tasks (basic, intermediate, and advanced), with a mix of individual and group work. The emphasis is on fostering students' individual development trajectories. In Trinidad, students learn at their own pace, selecting tasks that align with their learning style and interests. In Russia, the use of level-based tasks, differentiation of homework assignments, and the use of visual aids are helping to create personalized learning paths for students.

In Kazakhstan, the differentiation method was introduced alongside the updated curriculum, starting in primary and secondary grades. However, its systematic application in high school, especially for complex subjects like trigonometry, derivatives, and logarithms, has not yet been fully established. Many teachers understand the differentiation method, but they face difficulties in consistently incorporating it into lessons, creating tasks, and evaluating students' progress.In my own experience, I have seen the effectiveness of the differentiation method when applied to students of various ages. For example, when teaching trigonometric equations and inequalities to 11th-grade students, dividing tasks by levels significantly improved their ability to master the topic. In 5th and 6th grades, when teaching basic mathematical concepts, differentiated exercises helped increase students' confidence and interest in the subject. Therefore, the differentiation method is an effective pedagogical tool that meets the demands of modern education and aims to develop students' individual potential. However, to apply it effectively, teachers must have solid methodological preparation and must act in accordance with a well-structured system

Table 1.2.1 Clearly Describes the Advantages and Challenges of the Differentiation Method

State	Advantages over traditional methods	Students' and learners' motivation and outcomes	Disadvantages
Cyprus	Level-based testing, formative assessment, individual tasks, self-assessment, and analysis of student progress are used.	Increased interest in mathematics. Awareness of personal progress. Flexibility in learning. A sense of competition and collaboration. 86% of students noted that they liked the new teaching method. They became more active in discussions, started asking questions, and looked for solutions independently.	Increased demands on the teacher Difficulties with large groups Possible challenges for high-achieving students Assessment may become more complex
Trinidad	Individual approach Flexibility in organizing work	Freedom of choice Ways of studying the material Students were able to learn in a way that suited them 90% of students noted that this method increased their interest in the course, and 91% reported intellectual growth as a result of using this approach	High workload for the teacher The teacher needs to simultaneously manage several groups with different tasks Risk of reinforcing the division between "strong" and "weak" students
Russia.	Individual approach Level-based tasks Gradual increase in complexity	Sense of success and development Support for individuality The use of a differentiated approach contributes to improving the quality of knowledge among primary school students. Teachers noted that this approach requires careful and meticulous preparation, but ultimately it leads to better results.	High time investment for preparation

Table 1.2.2 Shows the Specific Methods Used and the Key Factors Focused On

Title of the article	What practical approaches were used	What did they pay more attention to
Cyprus	Division of material into three levels of difficulty: Basic level, Standard level, Advanced level. Combination of individual, pair, and group work.	Students' level of preparation and individual approach The instructors initially conducted testing to determine the different levels of preparation in the class. They found that students varied significantly in their knowledge—from those struggling with algebra to those ready for advanced problems. The main focus was on ensuring that each student could progress at their own pace.
Trinidad	Various learning resources, including textbooks, audio materials, and graphic organizers Grouping students according to their interests and learning styles The opportunity to choose subtopics within the main course theme (students themselves decided which aspect to study in more depth)	Individual differences among students Readiness level – instructors took into account differences in students' knowledge, experience, and study skills. Students' interests – they were given the opportunity to choose topics and assignments to increase motivation.
Russia**.**	Cards with examples and instructions Educational charts, diagrams, and posters Visual aids prepared by the students themselves Differentiation of homework: The first group completes standard exercises The second group receives moderately difficult tasks and problems from the textbook The third group works on additional complex exercises and research-based tasks	Individualized learning Selection of tasks according to each student's abilities Creating conditions in which every child can achieve success Building on students' strengths and working on their weaknesses

This information can serve as a basis for implementing the differentiation method in Kazakhstan and requires methodological adaptation.

Using method of differentiation in practical application. Dividing 3.2 ÷ 8 was difficult because students struggled with the decimal point (,).That's why I preferred to begin with whole number division, such as 2 ÷ 4. After such examples, when we moved on to 3.2 ÷ 8, students understood it better. After explaining the new topic, when we moved to solving problems, I applied the differentiation method by considering each student's level of und Lessons were taking from 5-th school students. We haven't taken pretest from students. Because, we knew their levels. Lessons were conducted for 5th-grade students on the topics of comparing decimal numbers, performing operations with decimal numbers (addition, subtraction, multiplication, and division), and sets. These lessons were carried out using the differentiation method.As mentioned earlier, we have 4 stages. They are:

Pretest → Grouping by Levels (A, B, C) → Program → Lesson Process

Level C – high level, 3 students

Stage 3: Creating the program. The students study at a general education school. When introducing a new topic, we didn't start with the textbook right away. This is because students in Level A did not yet reach the level required by the textbook. Therefore, I always prepared simpler exercises myself, outside of the textbook, and started from easy examples.

For example, when teaching division of decimal numbers,erstanding, ability to grasp the task, and their pace.When explaining the topic, it's best to start with the simplest problems that all students can understand. For example: 2÷4 by starting with whole number division, students began to understand the problems more easily.After explaining this problem to the students, we began assigning tasks.

№1. Perform the division. (Abilkassymova, Matematika 5 grade, 2019)

1 ÷ 2 2 ÷ 5 4 ÷ 8 3 ÷ 6

According to the experiment results:

Level A students took 5 minutes,

Level B – 3–4 minutes,

Level C – 1–2 minutes.

Once students from levels B and C finished solving the problems, the teacher checked their work. During this time, to keep them engaged, B and C level students were either given the task of checking other students' work or solving the following problems:

12 ÷ 24, 36 ÷ 72

Starting from such simple problems helped students solve them easily. Note: It is not recommended to give complicated problems like 0.024 ÷ 8 at the beginning.

№2. Perform the division.

1.2 ÷ 2 2.8 ÷ 4 3.6 ÷ 6 5.6 ÷ 7 – Later we moved on to problems like these.

In this case:

Level A students took 5–10 minutes,

Level B students – 3–7 minutes, level C students – 2–3 minutes.

Students at levels B and C were asked to compare their answers with each other or have the teacher check them. They were also assigned to assist Level A students.

We did not move on to new problems until all students had completed their tasks, as there wasn't enough time to explain new problems individually to each student.

It is important to note: if the teacher explains to everyone at once and some students still do not understand, then the teacher re-explains it. Once students understand, they move on to solving problems.

On average, students solved around 5 problems per task, such as:

№1 – 6 problems
№2 – 6 problems
№3 – 5–6 problems
№4 – 5–6 problems
№5 – 5–6 problems

In total, students managed to complete 24–30 problems.

Homework: Similar problems were assigned.

№1. Perform the division.

a) $5 \div 10$ b) $4 \div 5$ c) $3 \div 10$

№2. Perform the division.

a) $4.2 \div 7$ b) $2.1 \div 3$ c) $1.8 \div 6$

№3. Perform the division.

a) $0.42 \div 6$ b) $0.24 \div 6$ c) $0.16 \div 2$

The amount of homework was less than the in-class work.

During the lesson, 10–15 minutes were allocated for a research-based task using the internet and artificial intelligence.

For example, students were asked the following questions: Where are decimal numbers used?

In which professions are they applied?In conclusion, the experiment included 3 main stages.

Each stage was used to increase student interest, engagement, and motivation.

Grouping by levels (A, B, C) → Program → Lesson process

Theoretical Basis: Developing Mathematics Education Through Differentiation by Student Level

One of the key challenges in the educational process is the disparity in students' levels of knowledge and understanding. When a teacher explains a topic, not all students grasp the material at the same pace. In this context, the differentiation method serves as an effective tool for providing quality education tailored to each student's individual needs.

Stages of the Differentiation Method

Preliminary Diagnosis (Pretest): A diagnostic test is conducted to determine the initial level of each student. This allows the teacher to identify learning gaps and plan the learning process accordingly. For example, students in Grade 5 are offered tasks of varying complexity, including arithmetic, logic, and word problems across four levels.

Grouping by Level: Based on the results of the pretest, students are categorized into four levels:
A – low,B – below average,C – average,D – high.

This enables the teacher to adapt the material and adjust the workload according to students' abilities.

In conclusion, the effectiveness of the differentiation method can be clearly observed. The importance of applying the differentiation method lies in increasing each student's interest, taking into account their individual level, and boosting their motivation to solve problems during the lesson. Differentiation is a method that depends on the teacher's extensive work. It requires the teacher's time and effort. Sometimes students may have low interest in mathematics, but the teacher can encourage and motivate them to become more engaged. Below, I have briefly explained the differentiation method. (Alibek, 2025)

3.Program Design: A system of learning tasks is developed for each level. For example, in the topic "Simplifying Fractions," students receive tasks divided into basic, intermediate, and advanced levels. This gradual increase in complexity helps students transition smoothly to higher-level tasks.

4. Organization of the Teaching Process: During lessons, each student works on tasks appropriate to their level. Students at lower levels receive guided support from the teacher, while those at higher levels are engaged in more complex problems and exploratory tasks. Pair and group work is also used to foster collaboration and experience exchange among students.

Balancing State Standards and Actual Student Levels
While the curriculum and the State Educational Standards (SES) outline uniform goals for all students, their real levels vary significantly. Differentiation aims to bridge this gap. Teachers are encouraged to guide students toward SES-level tasks gradually, through well-structured preparatory assignments tailored to student ability. Starting with very basic exercises (e.g., Task #1, #2, #3), students can eventually progress to solving standard curriculum problems confidently.

The differentiation method is not just about grouping students by level; it is a structured approach to recognizing, supporting, and developing each learner. By acknowledging individual differences and planning instruction accordingly, teachers can significantly enhance the learning experience. When used purposefully, this method improves academic outcomes, nurtures independent learning, and promotes the development of mathematical thinking

1.3 Why Do We Use Differentiation in Teaching Trigonometric Equations and Inequalities? Challenges

Trigonometric equations and inequalities are among the most complex topics in school mathematics. This section requires students not only to memorize mathematical rules but also to apply them effectively, deeply understand the properties, formulas, and graphs of trigonometric functions. However, not all students are equally prepared to meet these demands.

Experience shows that in the same class, some students quickly learn to solve simple equations like $\sin(x) = \frac{1}{2}$, while others struggle to understand angle measures (degrees, radians) and values involving π. This reflects significant differences in students' levels of understanding.

Moreover, when solving trigonometric inequalities, students often make mistakes. For example:

They fail to correctly write the infinite set of solutions for an inequality;

They misunderstand how to represent solutions using graphs.

These are didactic difficulties.

In such cases, the traditional teaching method — giving the same task and explanation to all students — loses its effectiveness, because:

Lower-level students cannot keep up with the material;

Advanced students find the tasks too easy and may feel bored;

The teacher cannot attend to every student effectively within the lesson time.

For these reasons, using the differentiation method becomes a necessity. This method:

Takes into account students' individual learning pace;

Helps each student progress through personalized tasks;

Reduces difficulty and allows for gradual comprehension of the material.

For example, in practice, a single topic can be divided into three levels, whleads to more effective learning outcomes.

Level A – Memorizing formulas and solving simple equations such as:

$\sin(x) = 0$

Level B – Working with more complex arguments:

$\cos(2x - \pi) = -1/2$

Level C – Solving parameter-based equations and applying graphical methods.

Using the differentiation method presents not only benefits for students but also certain challenges for teachers, such as:

Increased time needed to prepare lessons;

Designing level-based tasks requires experience and methodological competence;

Diagnosing student levels in advance is necessary.

However, these challenges are justified by the effectiveness of the method. Students at different levels feel more comfortable, actively participate in lessons, and complete tasks with greater confidence. As a result, students' understanding of trigonometric topics improves, and their interest in the subject increases.

1.4 The Role of Artificial Intelligence in Supporting Differentiated Instruction and Academic Research

1.4.1. Artificial Intelligence (AI) is becoming increasingly embedded in the educational process. From intelligent assistants to automatic assessment systems, AI provides powerful tools for personalizing, automating, and optimizing teaching and learning. These technologies are especially beneficial within differentiated instruction and in teaching complex topics such as trigonometric equations and inequalities.

How AI Supports Teaching and Research

1. In academic research:

Assisting with outlining and structuring academic texts;
Improving writing quality through editing and translation;
Searching for relevant academic sources and compiling literature reviews;
Generating summaries, abstracts, and reference lists based on keywords;
Producing tables, graphs, and logic-based conclusions on demand.

2. In teaching practice:

Generating tasks and tests with varying difficulty levels;
Assisting in checking homework and identifying student errors;
Quickly diagnosing students' current academic levels;
Creating individualized learning plans;
Developing visual aids and graphs (e.g., sine and cosine graphs using Desmos with ChatGPT assistance);
Providing real-time feedback to students.

Digital Tools Empowered by AI

Table 1.4.1 Several digital platforms now incorporate AI to enhance educational effectiveness

Platform	AI-Based Features
ChatGPT (OpenAI)	Text generation, problem explanation, Q&A
Khan Academy	Adaptive learning paths based on student progress
Quillionz	Automatic generation of quizzes and test questions
GeoGebra, Desmos	Graphing tools for visualizing mathematical concepts
Grammarly, DeepL	Academic writing assistance and translation

Limitations and Cautions When Using AI

Despite its benefits, AI in education presents several challenges and limitations that teachers should be aware of:

Lack of pedagogical context: AI cannot fully understand students' psychological or emotional needs, motivation, or classroom dynamics.

Risk of cognitive passivity: Overreliance on AI can discourage independent thinking and problem-solving among students.

Potential inaccuracies: AI-generated content may contain errors, especially in complex math topics or when queries are poorly worded.

Ethical concerns: Using AI for academic writing may raise issues of plagiarism or misrepresentation of authorship.

Technical dependency: Both students and teachers may become overly reliant on technology and lose confidence in manual problem-solving skills.

What Teachers Should Keep in Mind

To ensure that AI becomes a helpful assistant rather than a substitute for effective teaching, educators must:

Review AI-generated content critically: Check the accuracy, appropriateness, and alignment of AI output with learning objectives.

Maintain balance: Use AI as a support tool, not a replacement for direct instruction, discussions, or hands-on practice.

Promote digital literacy: Teach students how to use AI responsibly and think critically about the information it provides.

Encourage creativity: Design tasks that benefit from AI support but still require human reasoning and originality.

Reinforce ethics: Discuss academic honesty, authorship, and the responsible use of AI in learning.

2. METHODOLOGY

2.1 Methods of Applying Differentiation in Teaching Trigonometric Equations

Using the differentiation method in teaching trigonometric equations means providing tasks that match students' individual learning characteristics and their current level of knowledge. Through this approach, we can deepen each student's understanding by offering tasks that are tailored to their level.

This method allows tasks to be simplified or made more complex depending on the student's level of preparedness.

For example, when solving equations involving the sine function, two approaches can be provided:

Standard solutions – for most students.

Advanced or extended solutions – for students who need more challenge.

Function of sine

General formula for solving equations involving the sine function:

$$\sin x = a \quad \Rightarrow \quad x = (-1)^n \arcsin a + \pi n, \qquad n \in Z$$

This formula provides a solution based on the value of the sine function, where a is the value of the sine, and n is any integer.

Examples and solutions

a) $\sin x = \frac{\sqrt{3}}{2}$

For this examples use:

$$x = (-1)^n \frac{\pi}{3} + \pi n, \qquad n \in Z$$

The solutions to this equation are π/3 and 2π/3, and their periodic solutions are obtained by adding πn**,** where *n* is any integer.

b) $\sin x = -\frac{\sqrt{3}}{2}$

For this equation use:

$$\sin x = -a \quad \Rightarrow \quad x = (-1)^{n+1} \arcsin a + \pi n, \qquad n \in Z$$

$$\sin x = -\frac{\sqrt{3}}{2} \qquad x = (-1)^{n+1} \frac{\pi}{3} + \pi n, \qquad n \in Z$$

Answers: $\frac{4\pi}{3}$ and $\frac{5\pi}{3}$

c) sin(x)=0,6

Answer:

$$x = (-1)^n \arcsin(0{,}6) + \pi n, \qquad n \in Z$$

It is recommended to encourage students to use calculators and reference tables to find the value of sine in this type of problem.

d) $\sin(x) = -0{,}7$

solution:

$$x=(-1)^{n+1}\arcsin(0{,}7) + \pi n, \qquad n \in Z$$

e) $\sin(x)=2$ және f) $\sin(x) = -2$

These equations have no solution because the values of the sine function can only lie within the interval $[-1, 1]$.

Particular Cases

Examples and Solutions

a) $\sin x=1$

This equation $\sin x = 1$ holds true only at a specific value: the sine function equals 1 only at $\pi/2$.

Therefore, the solution to this equation is:

$$x=\frac{\pi}{2} + 2\pi n, \qquad n \in Z$$

Students should understand that the maximum value of the sine function (1) occurs only at $\pi/2$, and from there, all possible periodic solutions can be found by adding $2\pi n$.

b) $\sin x= -1$

This equation $\sin x=-1$ is satisfied only when $\frac{3\pi}{2}$ therefore, the solution is:

$$x = \frac{3\pi}{2} + 2\pi n, \qquad n \in Z$$

When discussing the solution to this problem, it is important to explain to students that the minimum value of the sine function is -1, and it occurs only at $\frac{3\pi}{2}$ (немесе 270°) It should also be mentioned that other solutions are obtained by adding $2\pi n$ to this value.

c) $\sin x=0$

The sine is equal to 0 at multiples of π

Therefore, the solution is:

$$x=\pi n, \qquad n \in Z.$$

It is important to explain to students that in this problem, the sine function equals 0 at every multiple of π, and that the solutions repeat as $x=\pi, 2\pi, 3\pi, \ldots$

These examples and general solutions help students understand the basic principles when working with the sine function. Through the differentiation method, mathematical skills can be developed by providing tasks suited to students of different levels. For instance, basic-level students can be given simple equations such as sinx=0, sinx=1, and sinx=−1, while more advanced students can work with values like sinx=0.6. In this way, each student receives tasks appropriate to their level, which deepens their mathematical understanding.

Cosine function the differentiation method is also effective when solving cosine function equations. This method allows for explaining how to solve the equations while taking into account the different levels of students. Since the cosine function is a periodic function, its solutions typically repeat over several intervals. The general solution of the cosine function is as follows:

$$\cos(x) = a \quad \Rightarrow \quad x = \pm \arccos(a) + 2\pi n, \qquad n \in Z.$$

Using this formula, we can find all solutions to cosine equations. Typically, the inverse cosine function, arccos(a), takes values within the interval $[0,\pi]$, and the other solutions are periodic, obtained by adding multiples of $2\pi n$ to these values.

1. Examples and solutions

a) $\cos(x)=\frac{1}{2}$

Cosine is equal to $\frac{1}{2}$ at angles $\frac{\pi}{3}$ and $-\frac{\pi}{3}$, but since the standard general solution for cosine is usually given in the interval $[0,2\pi)$ we express both solutions as:

$$x=\pm\arccos(\tfrac{1}{2})+2\pi n=\pm\ \tfrac{\pi}{3}+2\pi n, n\in Z.$$

answers $\frac{\pi}{3}$ and $\frac{5\pi}{3}$ will repeated

b) $\cos(x)=-\frac{1}{2}$

Cosine is equal to $-\frac{1}{2}$ at angles $\frac{\pi}{3}$ and $-\frac{\pi}{3}$. Additionally, since the cosine function is periodic, these values repeat when multiples of 2π are added.

$$x=\pm\arccos(-\tfrac{1}{2}) + 2\pi n = \pm\ \tfrac{\pi}{3} + 2\pi n, n\in Z.$$

$$x=\pm(\pi - \tfrac{\pi}{3}) + 2\pi n = \pm\ \tfrac{2\pi}{3} + 2\pi n, n\in Z.$$

answers $\frac{2\pi}{3}$ will repeated

c) $\cos(x) = 0{,}3$

solution:

$$x=\pm(\arccos 0.3)+2\pi n$$

d) $\cos(x) = -0{,}3$

solution:

$$x=\pm(\pi - \arccos 0.3)+2\pi n$$

e) cos(x)=7

These equations have no solution because the values of the sine function can only lie within the interval [−1, 1].

f) cos(x) = −7

These equations have no solution because the values of the sine function can only lie within the interval [−1, 1].

Particular Cases
Examples and Solutions

a) cos(x) = 1

Cosine is equal to 1 only at 0 and at multiples of 2π. Thus, the solution is:

$$x = 0 + 2\pi n, \quad x = 2\pi n, \quad n \in Z$$

The solutions to this equation are not limited to just the interval between 0 and 2π they repeat over all intervals. That is:

b) cos(x) = −1

Cosine is equal to −1- only at π+2πn where n ∈Z n

Therefore:

$$x=\pi+2\pi n, \; n \in Z$$

In this solution, x=πx is the principal (or basic) root, and the rest of the solutions repeat periodically — for example, x=π,3π,5π,…

c) cosx=0

Cosine is equal to 0 at the values. Therefore, the general solutions are:

$$x = \frac{\pi}{2} + \pi n, \quad x = \frac{3\pi}{2} + \pi n, n\in Z.$$

When solving trigonometric equations involving the cosine function, it is important to consider its periodicity and symmetry. By using the differentiation method and assigning tasks according to each student’s level, we can deepen their mathematical understanding. Through the examples above, we examined various solutions of the cosine function. These examples can be used as a foundation to help students of different levels understand how to solve different types of problems.

Trigonometric Equations Involving the Tangent Function.

Considering the periodicity and characteristics of the tangent function, using the differentiation method in solving these equations helps students clearly understand how to find all possible solutions. The tangent function is periodic with a period of π\pi.

1. General Solution. The general solution for equations involving the tangent function is:

$$\tan(x)=a \Rightarrow \quad x=\arctan(a)+\pi n, n \in Z$$

In this solution, arctan(a) is the inverse of the tangent function — the angle whose tangent is equal to aa. For all integer values of n, the solutions repeat because the tangent function has a period of π\pi.

When solving equations involving the tangent function, we can find all solutions by considering the properties of this periodic function. By applying the differentiation method, we can offer tasks and solutions tailored to each student's level. The principal solution of the tangent function is given by arctan(a) and the remaining solutions are obtained by adding multiples of πn

Solving Equations Involving the cotangent function. The cotangent function is defined as the reciprocal of the tangent function:

$$\cot(x)=1/\tan(x)$$

Therefore, the cotangent function is also periodic, and its solutions can be found in a similar way to those of the tangent function.

1. General Solution. The general solution for the cotangent function is:

$$\cot(x)=a \ \Rightarrow \ x=\operatorname{arccot}(a)+\pi n, \ n\in Z$$

Using this formula, all solutions can be found. Here, arccot(a) is the inverse of the cotangent function and its values lie in the interval (0,π). By using integer values of nn, the solutions repeat periodically, since the cotangent function has a period of π.

When solving trigonometric equations involving the cotangent function, it is important to consider its periodic properties—similar to the tangent function—where all solutions repeat with a period of πn. By applying the differentiation method, these equations can be explained to students according to their individual levels of understanding. This approach helps ensure that each student comprehends the solution process based on their abilities.

2.2 Application of the Differentiation Method

By applying the differentiation method, we can adapt these examples to match each student's level of preparation. For example:

Individual tasks for each student: If a student has a lower level of preparation, they are given simpler problems such as sin(x) = 1 or sin(x) = 0.

Challenging tasks for average students: Students with intermediate skills can work on problems like

$$\sin(x) = 0.6 \text{ or } \sin(x) = -0.7.$$

Advanced tasks: More complex problems involve specific sine values and their additional solutions.Through this method, each student learns effectively at their own level and has the opportunity to develop a deeper understanding.

Definition of Trigonometric Equations. Different authors define trigonometric equations in various ways. We define a trigonometric equation as an equation involving trigonometric expressions that contain an unknown variable only within the trigonometric functions.
Examples of trigonometric equations:

$\cos(6x) = \sin(x)$

$\tan(\frac{\pi}{2} - 11x) - \tan(\frac{3\pi}{2} - 5x) = 0$

$\sin(3x) + \sin(5x) = \sin(4x)$

These are genuine trigonometric equations.

In contrast, equations like:

$\sin(x) = (\frac{1}{2})x$

$\cos(2x) = -(\frac{1}{2})x + \frac{1}{2}$

are not trigonometric equations. They fall under the category of transcendental equations, which are typically solved approximately or graphically. However, it is possible for an equation that is not trigonometric by definition to be reduced to a trigonometric equation.
For example:

$2x(x - 6)\cos^2 x = x - 6$

Here, the term $(x - 6)$ is not under a trigonometric function.
But the equation can be solved analytically:

$(x - 6)(2\cos^2 x - 1) = 0$
This leads to:

$x = 6$, or

$\cos^2 x = \frac{1}{2} \Rightarrow x = \pm \frac{\pi}{6} + \pi n$, where $n \in \mathbb{Z}$

Trigonometric Equations in Grade 11

Trigonometric equations are among the most challenging topics in 11th grade.
Some students limit themselves to memorizing formulas, while others want to understand the logic behind the methods used.
Therefore, the teacher must offer solution strategies that meet the needs of students at different levels in a single lesson.
Below, I will align the classical methods systematized in the work of Klimenko I.I. (2019) with the differentiation principle (Levels A, B, C).
Method Description (Brief) Which Level Is It Suitable For? Example of Classroom Use

Solving Basic Equations
Forms: $\sin x = a$, $\cos x = a$, $\tan x = a$, $\cot x = a$
Level: A
Use in Class:
Warm-up: Solve $\sin x = \frac{1}{2}$ color the solutions on the unit circle.
Substitution (Introducing a Variable)
Replace a complex expression with $y = \sin x$ or $y = \cos x$ to solve a quadratic/cubic equation.
Level: B
Use in Class:
Example: $2\cos^2 x + 3\cos x + 1 = 0$ → let $y = \cos x$, solve as a quadratic.
Using Formulas (Double Angle, Sum, Difference)
Simplify complex expressions using trigonometric identities.
Level: A → B
Use in Class:
The teacher writes an equation, and students discuss which formula should be applied.
Auxiliary Angle Method
Transform $a \cdot \sin x + b \cdot \cos x = c$ into $R \cdot \sin(x + \varphi) = c$ form.
Level: B → C
Use in Class:
A-level: φ is given;
B-level: students calculate φ themselves.
Factoring (Decomposition into Factors)
Extract common factors: e.g., $\sin x(\cos x - 1) = 0$
Level: A → C
Use in Class:
"Zero Rule" Challenge: Students find factorizations to apply the zero-product rule.
Algebraic Reduction
Express all functions in terms of a single variable and solve.
Level: C
Use in Class:
Example: $4\sin^2 x - 7\cos x - 5 = 0$ → Students solve and present their full solution on a poster.

2.3 Comparative Analysis: NIS G10 Textbook vs Abilkassymova Textbook

This presents a comparative analysis of the Grade 10 Mathematics textbook used in Nazarbayev Intellectual Schools (NIS) (pages 165–191) (Kalieva, 2022) and the national curriculum textbook by Abilkassymova (pages 152–166) focusing on trigonometric equations. The aim is to evaluate the content, difficulty level, and methodological differences between the two resources.

- The NIS textbook promotes deeper inquiry and analytical thinking, integrating real-world and advanced mathematical reasoning.

- The Abilkassymova textbook provides a foundational approach aligned with the national curriculum.
- Using both textbooks together can enhance students' theoretical understanding and practical problem-solving skills.

Table 2.2.1 Comparison Table:

Criteria	NIS Textbook (Pages 165–191)	Abilkassymova Textbook (Pages 152–166)
Explanation Method	Presents in-depth theoretical foundations for complex trigonometric equations, supported by definitions and inquiry-based tasks.	Uses simple language and introduces concepts gradually through worked examples and brief theory.
Content Structure	Includes research-based and applied tasks. Employs active learning strategies such as inquiry questions.	Organized into A, B, and C levels with step-by-step progression in complexity.
Types of Problems	Focuses on complex trigonometric equations with multiple arguments (e.g., 2x, 3x, π/3), graph-based questions, and real-life applications.	Primarily focused on solving using identities, with occasional graphical analysis and transformations.
Level of Difficulty	Advanced: includes multi-step problem solving, reasoning, and exploration.	Intermediate to advanced: mostly aligned with standard curriculum expectations.
Application Context	Includes real-world problems, connecting to physics and engineering contexts.	Mainly theoretical tasks with limited application-based questions.
Assessment and Independent Work	Encourages student reflection, justification of reasoning, and peer discussion.	Mainly focuses on written solutions and procedural problem solving.

Comparative Analysis of Textbook Content in Teaching Trigonometric Equations in Grade 10 and the Effectiveness of the Differentiation Method. Currently, comparing the content and structure of educational materials used to teach trigonometric equations is a crucial step in enhancing student achievement. Within the scope of this research, two types of textbooks were examined:

1.The 'Algebra' textbook for Grade 10 published by 'Almatykitap' (author: Abilkassymova et al.) (Abilkassymova, Algebra 10 grade, 2019)

2. The 'Bastau' problem book with 12,000+ exercises for Grades 3–12 by Mansur Rustemkhan Shametov. (Rustemkhan, 2023)

Table 2.2.2 Number and Structure of Exercises

Topics	Almatykitap Abilkassymova (Grade 10)	Bastau (Shametov)
Basic trigonometric equations	48 exercises	137 exercises
Equations simplified through formulas	12 exercises	29 exercises
Equations solved by lowering the degree	4 exercises	21 exercises
Rational trigonometric equations	6 exercises	16 exercises
Total number of exercises	178	456

Qualitative Comparison:

- Although the Almatykitap textbook presents problems in a certain sequence, the difficulty level jumps abruptly. This may confuse students and hinder gradual understanding. The Bastau book arranges exercises sequentially by difficulty and divides each section into subtopics. This approach fully aligns with the differentiation method. For instance, the topic 'Solving Trigonometric Equations' is divided into 12 subtopics in the Bastau collection, each tailored with level-appropriate exercises that suit students' capabilities. This clearly illustrates the application of the differentiation method. In contrast, the Abilkassymova textbook lacks this consistency: fewer exercises are provided, and their difficulty varies irregularly within topics, making it harder for students to learn step by step. Additional Suggestions and Ideas:1. Introduce a modular structure: Divide topics into smaller sections (step-by-step) to provide clear learning routes for students.2. Level-based tasks: Categorize exercises into 'basic', 'intermediate', and 'advanced' levels within each topic. 3. Self-assessment tasks: Include quizzes or logical tasks at the end of sections, similar to those in the Bastau book. 4. Integrate a digital platform or QR codes to access interactive solutions and video explanations. 5. Personalized learning paths: Enable selection of textbooks or supplementary materials based on students' readiness levels through differentiation.

Comparative Analysis of Trigonometric Equation Problems in Abilkassymova and Olympiad Bisekov Collections (Beisekov, 2013).This section of the research compares the content and complexity level of trigonometric equations presented in two key sources: 1. The 10th Grade 'Algebra' textbook by Abilkassymova et al. (Almatykitap Publishing) 2. The 'Mathematical Olympiad Problem Collection' by Bisekov, aimed at advanced and competition-level learners.

Table 2.2.3 Problem Count Summary

Source	Topic	Number of Problems
Abilkassymova Textbook	Trigonometric Equations (19.1–19.22, 20.1–20.22)	178
Bisekov Collection	Olympiad-level Trigonometric Problems	75

Complexity Level Comparison:

- The Abilkassymova textbook covers problems aligned with the school curriculum and is structured by levels: A (basic), B (intermediate), and C (advanced). The Bisekov collection includes challenging problems tailored for mathematical olympiad preparation. These require higher-order thinking, transformations, and reasoning.- During the analysis, it was observed that some C-level problems from the Abilkassymova textbook closely resemble problems found in the Bisekov collection, indicating that even school textbooks may include problems appropriate for high-performinstudents. The Bisekov collection is characterized by high-level, competition-oriented problems. The Abilkassymova textbook, although primarily curriculum-based, integrates differentiated tasks including complex problems in the C-level section. This comparison supports the relevance of differentiation in teaching: having tasks at various levels helps accommodate individual student needs and promotes inclusive education.

Comparative Analysis: Example 14 (Olympiad) vs Exercise 20.12 (Abilkassymova).This document compares the complexity and instructional purpose of Example 14 (typically found in Olympiad-style problem sets) and Exercise 20.12 from the Grade 10 Algebra textbook by Abilkassymova. The goal is to assess the structure, methods, and skill levels required for each set of problems.

Exercise 20.12 (Abilkassymova)

$$\begin{cases} \sin x \sin y = \frac{1}{2\sqrt{2}} \\ x - y = -\frac{\pi}{4} \end{cases} \qquad \begin{cases} \sin x\, cosy = -\frac{1}{2} \\ x + y = -\frac{\pi}{6} \end{cases} \qquad \begin{cases} cos\, x + cosy = 0 \\ x - y = -\frac{4\pi}{3} \end{cases}$$

Example 14 (Olympiad)

$$\begin{cases} sin\, x + siny + sinz = \frac{3}{2} \\ cosx + cosy + cosz = \frac{3\sqrt{3}}{2} \end{cases}$$

Table 2.2.4 Comparison Table

Criteria	Example 14 (Olympiad-level)	Exercise 20.12 (Abilkassymova)
Type of Equations	Complex system with three variables (x, y, z)	Three separate two-variable systems
Number of Given Equations	2 equations; transformation required	Each system has 2 equations
Method Used	Addition, transformation, identity simplification with π/3 shift	Substitution method
Theoretical Basis	Addition formulas, function maxima (sin x = 1)	Basic identity substitution and known trig values
Solution Steps	Long (7–8 steps); strategic manipulation	Short (3–4 steps per system)
Difficulty Level	Level 3 (Advanced/Olympiad)	Level 2 (Intermediate, curriculum-based)
Number of Variables	3 (x, y, z)	2 (x, y) in each system
Challenging Aspects	Linking multiple variables, maximizing sum of sine functions	Handling substitution and known values
Instructional Goal	Olympiad prep, advanced reasoning	Skill consolidation, application of substitution method

- Example 14 exemplifies an advanced-level Olympiad problem, requiring strategic thinking and a deep understanding of trigonometric properties. Exercise 20.12 from the Abilkassymova textbook is designed for curriculum-aligned practice, focusing on reinforcing substitution and basic transformations. Both serve important roles in differentiated instruction: one for foundational competence, the other for gifted and competitive learners. Comparison: SAT Subject Test (Bobrow, 2007) (Trigonometric Equations) vs Abilkassymova Grade 10 Textbook This document compares trigonometric equation problems found in the SAT Subject Test preparation material (Arco Master SAT Math Level 1) with those found in the official Kazakhstani Grade 10 textbook by Abilkassymova. It analyzes structure, complexity, and pedagogical goals of each system. - SAT problems are optimized for timed testing and assess the ability to recall and apply standard identities quickly.

Table 2.2.5 Comparison Table

Criteria	SAT Subject Test (Arco)	Abilkassymova Textbook
Objective	Train quick identification and solving under time pressure	In-depth concept explanation aligned with school curriculum
Problem Structure	Short problems, aimed at direct solutions within 1–2 steps	Multi-step problems with structured reasoning
Formula Use	Standard identities applied directly	Derivation and step-by-step application of identities
Examples	$\tan^2 x = 2\tan x\cos x$ $\cos^2 x + 3\cos x + 1 = 0$ $\sin^2 x – \sin 2x = 0$	$\cos x \cdot \cos y = a$ $\sin(x \pm y) = b$ Solutions within π-interval
Solving Approach	Quick recognition and substitution method	Full explanation, derivation, and logical justification
Difficulty Level	Level 2 (Intermediate)	Level 2–3 (Intermediate to Advanced)
Trigonometric Elements	$\tan^2 x$, $\sin 2x$, $\cos^2 x$ – solved using direct identities	Transformation of identities, solving in terms of π, step logic

- Abilkassymova's textbook problems focus on understanding, transformation, and problem-solving logic. - Both systems are valuable: one for practicing speed and precision, the other for developing theoretical depth and structured reasoning.

Comparative Analysis: Abilkassymova Grade 10 vs U.S Trigonometric Equations (OER). (Stitz, 2017) This comparative analysis examines the differences in structure, complexity, and instructional focus between the trigonometric equations section of the Grade 10 Algebra textbook by Abilkassymova and the trigonometry chapter of a U.S-level Open Educational Resource (OER) textbook. The objective is to determine how each resource approaches the teaching of trigonometric equations and how their use supports different stages of student learning. The Abilkassymova textbook is appropriate for building foundational skills in trigonometric equations, offering guided structure and accessible difficulty.The U.S OER textbook goes beyond by applying trigonometric equations to real-world and abstract problems, encouraging analytical and strategic thinking.Used together, these resources support differentiated learning from basic procedural fluency to advanced conceptual mastery.

Table 2.2.6 Comparison Table

Criteria	Abilkassymova Textbook (Grade 10)	U.S OER Textbook
Scope	Covers basic trigonometric equations aligned with school curriculum; focus on identities like $\sin^2 x + \cos^2 x = 1$.	Explores advanced forms including inverse trig functions, multi-angle identities, domain/range analysis.
Problem Structure	Exercises involve solving equations over fixed intervals, typically using standard identities and simple transformations.	Includes complex problems with multi-step solutions, often requiring substitutions, graphing, and multiple solutions.
Solving Approach	Step-by-step explanations and guided practice; emphasis on mechanical solution strategies.	Conceptual approach encouraging discovery and application of multiple strategies.
Examples	Equations such as $\sin(2x) = \sqrt{3}/2$; solve using basic identities and interval logic.	Equations like $\sin(2x) + \cos(x) = 1$ or $\tan^2(x) – 3 = 0$, requiring factoring and exact value reasoning.
Use of Identities	Uses identities such as $\sin^2 x + \cos^2 x = 1$ and double angle formulas to simplify equations.	Incorporates a wider range of identities, including inverse trig properties and composite functions.
Difficulty Level	Level 2 (Intermediate): Suitable for students with introductory knowledge of trigonometry.	Level 3–4 (Advanced/Pre-University): Suitable for students preparing for Calculus and university math.
Application	Primarily theoretical; lacks real-world context.	Includes applied problems involving angle of elevation, circular motion, and physics-based contexts.

Comparative Analysis: UK Textbook vs Abilkassymova Textbook

This document compares the trigonometry section from the UK mathematics textbook (pages 12–17) with the content presented in the Grade 10 Kazakhstani mathematics textbook by Abilkassymova (pages 152–166). The goal is to examine the structure, content complexity, and methodological focus of the exercises in both resources.

- The UK textbook presents a broader and more advanced set of trigonometric problems with a focus on general solutions and identity transformations.
- The Abilkassymova textbook provides a structured progression aligned with the national curriculum and supports gradual learning.
- Combined, these resources offer a strong foundation for both theoretical understanding and practical problem-solving skills.

General Comparative Summary

This section presents a comparison of trigonometric equation topics from 5–7 different textbooks, using the 10th-grade Kazakhstani school textbook (Abilkassymova) as a reference. The comparison examines the depth of content, methodological features, types of problems, and levels of difficulty.

The – textbook contains a wide range of research problems, graphs, contextual tasks, and elements of independent work. It aims to develop students' research and critical thinking skills through advanced-level tasks. Compared to the Abilkassymova textbook, it is broader and deeper in content.

The UK's Pearson textbook includes general solutions, proofs, radian measures, and complex trigonometric functions (e.g., sin(3x), cos(4x)). The tasks are designed for in-depth theoretical understanding, whereas the Abilkassymova textbook follows a gradual introduction method.

The Calculus textbook offers an in-depth analysis of concepts such as graphs of trigonometric functions, phase shifts, and periodic changes. This resource is intended for upper-grade or pre-university students, while the Abilkassymova textbook is limited to the standard school curriculum.

The SAT-style tasks are short and specific. They require students to quickly identify logical connections and choose the correct answer from several options. The Abilkassymova textbook is oriented toward traditional written problem-solving, while SAT tasks promote strategic and time-efficient thinking.

Bisekov's Olympiad problem book targets high-level students. The problems require unique techniques and deep theoretical understanding. Although some C-level problems in the Abilkassymova textbook are similar, the Olympiad content is significantly more complex and logically demanding. Foreign and Olympiad textbooks emphasize mastering theoretical knowledge and applying complex formulas. NIS and Calculus textbooks focus on developing students' abilities to conduct research, analyze, and draw conclusions.

Table 2.2.7 Comparison Table

Criteria	UK Textbook (Pages 12–17)	Abilkassymova Textbook (Grade 10, Pages 152–166)
Explanation Method	Theory is clearly explained with graphical and algebraic representations. General solutions and identities are presented.	Each topic begins with a brief explanation, followed by step-by-step examples.
Content Structure	Unified approach with integrated content including composite trigonometric functions like sin(3x), cos(4x).	Organized by difficulty levels (A, B, C) with increasing complexity.
Problem Types	Exercise 1E includes sine, cosine, tangent, cotangent, and secant. Some problems involve proofs and complex identities.	Focuses primarily on sine, cosine, and tangent. Problems often include transformations and graphical support.
Difficulty Level	Advanced: requires analytical thinking and allows multiple solution strategies.	Intermediate to Advanced: students are gradually guided from basic to complex problem solving.
Units Used	Both degrees and radians are used. Students are expected to convert and apply both systems.	Mostly degrees; radians are introduced in high-level (C) exercises only.
Application Context	Emphasis on theoretical and algebraic proficiency. Suitable for pre-university preparation.	Designed for general secondary education with step-by-step development of mathematical thinking.

The Abilkassymova textbook offers a structured progression into complex topics.The problems are aligned with the national curriculum, focusing on theoretical foundations and formula application.Each textbook is targeted at a specific audience: Abilkassymova for foundational learning; others for advanced or test preparation purposes.

Therefore, in the future, it is recommended to enrich local textbooks with more advanced-level problems, research elements, and real-life contextual tasks. This will help improve students' functional literacy and mathematical reasoning skills.

2.4. Methods of Applying the Differentiation Method in Teaching Trigonometric Inequalities

Applying the differentiation method in teaching trigonometric inequalities involves tailoring tasks to suit students' varying levels, deepening their understanding and meeting their individual learning needs. The differentiation method enables the adaptation of instructional strategies and materials to match each student's abilities, learning styles, and preparation levels.

Key Principles of the Differentiation Method

1.Assigning tasks based on students' knowledge levels. In differentiated instruction, tasks should correspond to each student's level of understanding. For example, one group of students may be assigned simple trigonometric inequalities and equations (such as $\sin(x) \geq 0$), while another group may work on more complex inequalities (such as $\tan(2x) > \cos(x)$). This approach allows students to self-assess and engage with challenges appropriate to their level.

2. Adapting tasks to students' interests

Tasks may vary based on students' interests and learning styles. For instance, some students may benefit from graphical methods, while others may prefer analytical approaches.The differentiation method makes it possible to integrate such diverse strategies.

3.Organizing individual and group work. An important aspect of differentiation is the balance between individual and group activities. Individual work allows students to complete tasks aligned with their abilities, while group work fosters collaboration, peer learning, and shared problem-solving.

4. Dividing tasks by level. When teaching trigonometric inequalities, assigning tasks by level helps address individual abilities and develop logical reasoning skills. At each level, tasks provide opportunities to deepen theoretical understanding. For example, beginner-level tasks focus on solving basic inequalities, while higher-level tasks require using advanced analytical methods.

Methods for Teaching Trigonometric Inequalities

Teaching Trigonometric Inequalities Using the Differentiation Method

Applying the differentiation method in teaching trigonometric inequalities is an approach that aims to deepen students' understanding by offering tasks that match their individual levels and learning needs. This method allows educators to adapt teaching techniques and materials by considering each student's abilities, learning styles, and level of preparedness.

Key Principles of the Differentiation Method

1.Providing tasks based on student knowledge levels. Tasks should be assigned according to each student's level. For example, one group might work on basic trigonometric inequalities and equations (such as $\sin(x) \geq 0$), while another group tackles more complex inequalities (such as tan(2x) > cos(x)). This enables students to self-assess and overcome challenges that are appropriate for their level.

2. Adapting tasks to students' interests. Based on students' interests and learning preferences, task types may vary. For instance, while some students benefit more from graphical methods, others may find analytical approaches more effective. The differentiation method supports integrating both styles.

3. Organizing individual and group work. Another important aspect of differentiation is balancing individual and collaborative activities. Individual work allows students to progress at their own pace, while group work promotes sharing knowledge and joint problem-solving.

4. Structuring tasks into levels. When teaching trigonometric inequalities, structuring tasks by difficulty helps foster logical reasoning. Lower-level tasks focus on foundational understanding, while higher-level tasks involve analytical problem-solving.

Methods of Teaching Trigonometric Inequalities. Graphical Method. Using graphs is an effective differentiation strategy when solving trigonometric inequalities. This method helps students visualize solutions. For example, to solve $\sin(x) \geq 0$, students can identify intervals on the graph where the sine function takes positive values.This supports the development of graphical thinking and clarity in representing solutions. Analytical Method. This is a commonly used approach that involves solving inequalities mathematically. For example, in solving $\sin(x) \geq 1/2$, students find the principal solutions using the arcsine function, then extend them periodically.Analytical methods teach students clear, systematic solution strategies.

Integrated (Comprehensive) Method. This method combines various strategies.Students use both graphical and analytical tools to solve complex inequalities.For example, when solving $\tan(x) \leq \sqrt{3}$, students first explore the values of the tangent function graphically, then apply analytical reasoning. This strengthens their ability to combine approaches for better understanding.

Modular Method. An important aspect of differentiation, the modular approach offers content and tasks in progressive levels. For instance, in solving $\cos(x) \leq 1/2$, Module 1 might introduce basic concepts, while subsequent modules provide more complex inequalities based on student readiness.This structure makes it easier to assign suitable tasks.

Advantages of the Differentiation Method. Increasing student engagement Individualized tasks aligned with each student's level enhance their interest in learning. Working within their zone of development helps maintain motivation and attention.

Providing tasks tailored to individual development Students receive assignments that match their personal capabilities, supporting their intellectual growth and learning confidence.

Developing thinking skills through diverse strategies. Differentiated instruction involves the use of multiple methods — graphical, analytical, or combined — which nurtures critical thinking and flexibility in problem-solving.

Enhancing collaboration through group work. Group activities promote interaction, idea sharing, and collaborative decision-making, helping students engage across different levels.

The application of the differentiation method in teaching trigonometric inequalities enhances students' learning outcomes, maintains their interest in the subject, and strengthens their problem-solving skills. By using diverse approaches, students deepen their mathematical understanding and expand their knowledge systematically. Differentiated instruction fosters active participation and motivates students to work at a level that challenges them appropriately.

Graphical Analysis of Trigonometric Inequalities. In the school curriculum, solving trigonometric inequalities is one of the topics that often presents difficulties for students. The uniqueness of these problems lies in the fact that their solutions are located along the unit circle and are periodic. Therefore, using visualization and graphical methods is very effective for helping students understand this topic clearly.

Solution Steps:

Given inequality:

$$\sin x > \frac{1}{2}$$

To solve this inequality, we follow the steps below using the unit circle (trigonometric circle):

Step 1:

Find the value where $\sin x = \frac{1}{2}$ on the unit circle. The angle whose sine is $\frac{1}{2}$ is $\frac{\pi}{6}$ (or 30°). So, we mark the point on the upper part of the circle where the y-coordinate is $\frac{1}{2}$.

Step 2:

Since the sine function is measured along the y-axis, we look at the region above the line $y=\frac{1}{2}$ This helps us visually determine the parts of the circle where $\sin(x) > \frac{1}{2}$.

Step 3:

Find the second angle where $\sin(x)=\frac{1}{2}$. It is $\frac{5\pi}{6}$ (or 150°). We find this value as follows:

$$\pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

This is because the sine function is positive in the first and second quadrants.

Step 4:

Now we write the solution based on the unit circle:

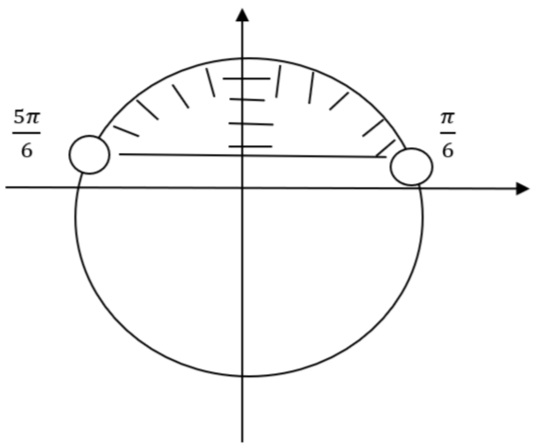
$$\frac{\pi}{6} + 2\pi n < x < \frac{5\pi}{6} + 2\pi n$$

But since trigonometric functions are periodic, we must add 2πn to represent all solutions (where n ∈ Z).

Final Answer:

$$\frac{\pi}{6} + 2\pi n; \frac{5\pi}{6} + 2\pi n$$

Figure 2.4.1 That is the complete solution for the inequality:



$$\pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

Example 2: $\sin(x) < \frac{\sqrt{2}}{2}$

Given inequality:

$$\sin(x) < \frac{\sqrt{2}}{2}$$

To solve this inequality, we rely on the properties of the sine function on the unit circle. The solution proceeds through the following steps:

Step 1:

First, we find the angles where $\sin(x) < \frac{\sqrt{2}}{2}$. This value occurs at $\frac{\pi}{4}$.

Step 2:

On the unit circle, we mark the region where the y-coordinate is less than $\frac{\sqrt{2}}{2}$. This corresponds to the lower half of the circle.

Step 3:

The lower boundary is $-\frac{5\pi}{4}$. This value can be found as: $-\pi - \frac{\pi}{4}$

Step 4:

Therefore, the solution to the inequality in interval form is:

$$-\frac{5\pi}{4} + 2\pi n < x < \frac{\pi}{4} + 2\pi n$$

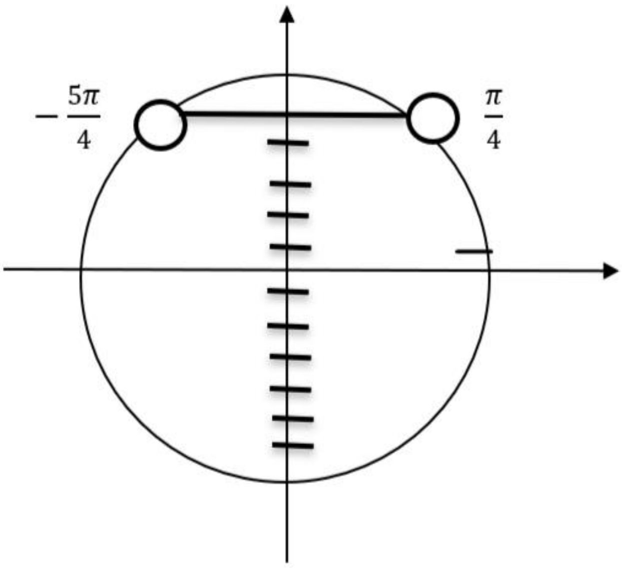
Step 5:
Since the sine function is periodic, we add 2πn to represent all solutions:

$$-\frac{5\pi}{4} + 2\pi n < x < \frac{\pi}{4} + 2\pi n$$

This is the complete solution to the inequality:

$$\sin(x) < \frac{\sqrt{2}}{2}.$$

Figure 2.4.2 That is the complete solution for the inequality



Example 3: $\sin(x) > -\frac{1}{2}$

Given inequality:

$\sin(x) > -\frac{1}{2}$

To solve this inequality, we use the unit circle and follow the steps below:

Step 1:
First, find the angle where $\sin(x) = -\frac{1}{2}$. The principal value of this angle is:

$\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

Step 2:
On the unit circle, we mark the level y = $-\frac{1}{2}$. All points above this horizontal line satisfy the inequality since sin(x) > $-\frac{1}{2}$

Step 3:
The second angle where sin(x) > $-\frac{1}{2}$ is

$$\pi + \frac{\pi}{6} = \frac{7\pi}{6}$$

Step 4:
Therefore, the solution interval is:

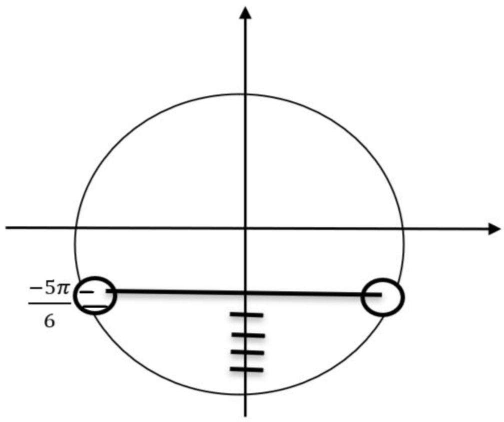
$$-\frac{\pi}{6} < x < \frac{7\pi}{6}$$

Step 5:
Since the sine function is periodic, we add 2πn2\pi n2πn to represent all solutions:

$$-\frac{\pi}{6} + 2\pi n < x < \frac{7\pi}{6} + 2\pi n$$

This is the complete solution for the inequality:

Figure 2.4.3 That is the complete solution for the inequality:



sin(x) > $-\frac{1}{2}$ Analysis of Level A TasksLevel A problems are based on standard values of trigonometric equations and are solved through the application of formulas, memorization of function values, and graphical understanding.

1. Basic Equations (Tasks 19.1 – 19.3)
Examples:

cos x = $\frac{\sqrt{3}}{2}$

sin x = $-\frac{1}{2}$

$tgx = \sqrt{3}$

$ctgx = 0$

Analysis:

Students are expected to know the standard values from the unit circle or a reference table and be able to express the general solutions of the equations.

Example:

$\cos x = \frac{\sqrt{3}}{2} \Rightarrow x = \pm \frac{\pi}{6} + 2\pi n$

2. Equations with Negative or Decimal Values (Tasks 19.4 – 19.5)

Examples:

$\cos x = -0.7$

$ctg\, x = \sqrt{5}$

$\cos 3x = -\frac{1}{2}$

Analysis:

In these problems, students should use a calculator to find inverse trigonometric values or correctly understand the sign of functions based on coordinate quadrants.

Common difficulty: Confusion with signs and coordinate quadrants. Solution strategy: First solve using inverse functions (e.g., arccos), then write the general solution using π-periodicity.

3. Problems with Complicated Arguments (Tasks 19.6 – 19.8)

Examples:

$\sin(2x - \pi/3) = -\frac{\sqrt{3}}{2}$

$\cos(-3x) = -\frac{1}{2}$

Analysis:

Students need to transform the argument and apply properties of even/odd functions. Reduction formulas are often used at this stage.

Examples:

$\cos(-x) = \cos x$

$\sin(-x) = -\sin x$

4.Problems Involving Double/Triple Arguments and Function Sums (Tasks 19.9 – 19.11)

Example:

- $\sin 3x \cdot \cos 4x - \sin 3x \cdot \sin 4x = \frac{1}{2}$

- Analysis: These tasks require factoring out common terms, using identities, and converting expressions to involve a single variable.

  Example transformation:

$$\sin A \cdot \cos B - \sin A \cdot \sin B = \sin A (\cos B - \sin B)$$

Table 2.4.1 Comparison Table

Problem Type	Required Skill	Difficulty Level
19.1–19.3 – standard values	Memorizing function values	Low
19.4–19.5 – real (decimal) values	Using calculator, understanding signs	Medium
19.6–19.8 – complex arguments	Transforming argument, using π-multiples	Medium–High
19.9–19.11 – composite functions	Applying identities, algebraic manipulation	High

Analysis of Level B Tasks

Level B tasks require students to solve trigonometric equations within a given interval, use transformations, and apply reduction, double-angle, and product formulas. These tasks develop students' logical thinking and their ability to work with trigonometric formulas.

1. Solving within a Given Interval (Tasks 19.12 – 19.13)

In these problems, students must find solutions of an equation only within a specified interval.

Example:

$\cos 4x + \sin 2x = 0, \ 90° < x < 360°$

In this case, the student first finds the general solution and then adjusts it to match the given interval. Such tasks enhance students' interval reasoning and accuracy in selecting appropriate solutions.

2. Identifying Roots Within an Interval (Task 19.13)

These problems require selecting only the roots that fall within a certain interval.

Example:

$\sin(x + 270°) - \cos(3x + 720°) = 0, \quad 0° < x < 90°$

To solve, students must transform the complex arguments, consider periodicity, and verify the roots. This helps reinforce the concept of periodicity in trigonometric functions.

3. Using Reduction and Power-Reducing Formulas (Task 19.14)

Students must apply reduction formulas, the properties of even/odd functions, and double-angle identities.

Examples:

$\cos^2(7\pi + x) = \frac{1}{2}$

$\sin^2(4.5\pi - x) = \frac{3}{4}$. Students must reduce the equation to one variable and solve using these identities.

Level B tasks strengthen students' ability to apply theoretical knowledge in practice. Skills like solving within intervals, transforming arguments, and applying complex formulas are systematically developed. Teachers can assign these tasks using the differentiation method, adjusting the difficulty according to each student's level.

Analysis of Level C Tasks

Level C tasks are designed to develop high-level thinking skills. These problems require complex skills such as graphical analysis, function investigation, finding intersection points, and making logical conclusions.

1. Solving in an Interval (Task 19.15)

Students must solve equations within a specific interval.

Example:

$$\frac{\cos 7x}{\sin 2x} - 1 = 0, \quad 70° < x < 150°$$

Solutions must take into account both the value of the function and its periodicity, limited to the given interval.

2. Solving Using Graphical Methods (Tasks 19.16, 19.18, 19.19)

Students use graphical techniques to find the number or values of intersection points between algebraic and trigonometric functions.

Examples:

$\sin(x + 2) = 3 - x^2 - 2x$

$a\cos x = -\frac{x^2}{2} + \frac{\pi}{2}$

These tasks develop graphing, comparison, and intersection-finding skills.

3. Finding Roots (Task 19.17).Students must find the roots of functions with complex arguments.

Example:

$\sin 2x \cdot \operatorname{tg} x = 0, \quad 90° < x < 180°$

To solve, students must find the roots of each factor that makes the product zero.

4. Finding the Domain of a Function (Task 19.20)

Students analyze when the expression under a square root is non-negative or when a denominator is not zero.

Example:

$y = \sqrt{(6 - x^2)} + \frac{1}{\sin x}$

5. Graphing Functions and Finding Intersections (Tasks 19.21 – 19.22)

These tasks involve graphing functions such as:

$$y = \cos(\frac{x}{3} + \pi) + 2, \qquad y = \sin(x - \frac{\pi}{3}) - 2$$

Students must then find the points of intersection with a line. These are research-type tasks requiring mathematical modeling and visual reasoning.

Level C tasks help develop deep thinking, analytical reasoning, logic, and graphing skills. These tasks are best used with differentiated instruction for gifted or advanced students.

2.5 The Role of Artificial Intelligence in Supporting Methodology and Lesson Planning

In the process of preparing this research, artificial intelligence (AI) tools — particularly ChatGPT — played a supportive and effective role in enhancing both the methodological and practical aspects of the study.

AI assistance included the following:

Creating tables and organizing data: ChatGPT helped generate structured comparison tables for analyzing textbooks, problem types, and levels of trigonometric equations and inequalities.

Drawing graphs and figures: With the help of AI, I was able to generate accurate mathematical graphs, such as the sine and cosine functions, and visualize unit circle-based solutions using tools like Desmos, which were recommended or assisted by AI prompts.

Finding academic sources and book references: AI was useful in identifying relevant textbooks, author names, and article titles aligned with differentiation in mathematics and trigonometry instruction.

Generating mathematical formulas and examples: AI provided multiple examples of trigonometric equations, their general solutions, and correct mathematical notation.

Fast generation of observations and summaries: While analyzing students' performance and lesson strategies, AI helped quickly summarize classroom patterns, common mistakes, and effective interventions.

Support in working with Microsoft Word: AI assisted in formatting equations, aligning tables, and improving the structure and clarity of mathematical explanations within the Word document.

Thanks to these tools, I was able to increase the efficiency of planning, writing, and editing this methodological section. Moreover, AI-supported resources enhanced the differentiation strategy by suggesting customized tasks and helping visualize multiple solutions to a single problem — a key element when teaching complex trigonometric topics.

However, it is important to remember that AI is a support tool, not a replacement for pedagogical thinking. While AI can automate and accelerate certain processes, the teacher remains responsible for the correctness, clarity, and didactic value of the material.

This chapter demonstrated that the differentiation method is a highly effective pedagogical approach when teaching trigonometric equations and inequalities. By assigning tasks based on students' academic levels (A, B, C), teachers can better support both struggling and advanced learners. Differentiated instruction encourages engagement, promotes deeper understanding, and allows each student to progress at their own pace. The use of various strategies—analytical, graphical, and modular—further enriches the learning process and accommodates different learning styles.

Comparative analysis of textbooks revealed that structured materials with level-based content are more compatible with differentiated instruction. Practical lessons showed improved student outcomes when tasks were adapted to their abilities. Moreover, the integration of technology and AI tools, such as ChatGPT, greatly supported the development of this research by helping with table creation, graph drawing, formula writing, and lesson planning. These findings confirm that methodological flexibility, thoughtful planning, and personalized instruction are essential for successfully teaching complex mathematical topics.

3. RESULTS

3.1 Organization of the Research Work

This research work was organized in accordance with the objectives of the dissertation and included a pedagogical experiment aimed at evaluating the effectiveness of the differentiation method in teaching mathematics. The overall goal was to analyze how differentiated instruction can improve students' understanding of mathematical concepts, with a specific focus on trigonometric equations and inequalities in higher grades.

To achieve this, the research was conducted in two main phases. The first phase was carried out in the 5th grade, where the method was tested in a general mathematics context to determine its flexibility and applicability across different age groups. This part of the research aimed to identify how younger students respond to differentiated tasks and how such instruction affects their motivation, comprehension, and performance. Although the primary focus of the dissertation is on high school students, this phase provided important insights into the foundational role of differentiation in mathematics education.

The second phase of the experiment was implemented in Grade 11 and was specifically dedicated to the application of differentiated instruction in the topic of trigonometric equations and inequalities. This stage of the research allowed for direct observation of how differentiated strategies affect learners' understanding of complex mathematical topics, their problem-solving skills, and academic achievement.

The research process was divided into the following key stages:

- Diagnostic testing (Pretest): to determine the initial level of students' knowledge and categorize them by readiness;
- Grouping by levels (A, B, C): based on their performance and understanding;
- Design of a differentiated instructional program: including tiered tasks and lesson plans;
- Implementation during regular lessons: with monitoring of individual progress and targeted support;
- Formative and summative assessments (SAU/SAT): to evaluate the impact of the differentiated approach.

Each stage was carefully documented, and qualitative observations were also made to analyze student engagement, participation, and reactions to the instructional method. The organization of the research work aimed to create a balanced and systematic model of differentiated teaching that could be adapted for broader use in Kazakhstani schools.

3.1.1 Experiment1. Application of the Differentiation Method in Grade 5.

As part of the dissertation research aimed at identifying the effectiveness of the differentiation method, an experiment was conducted in the 5th grade of a general education school. Although the main topic of the study relates to teaching trigonometric equations and inequalities in grades 10–11, this stage of the experiment was aimed at assessing the universality of the differentiated approach in teaching mathematics as a whole. The experiment was carried out during the third term of the

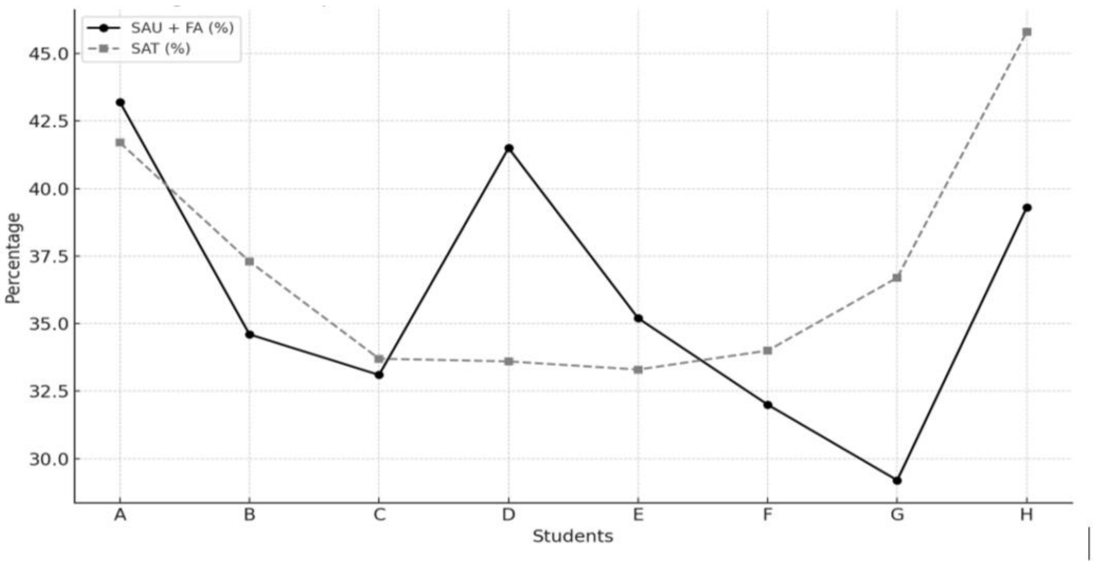
2024/2025 academic year (from January to March) in Class 5A. Eight students participated in the experiment. During the lessons, elements of differentiation were applied: tasks were offered at three levels of difficulty, students were allowed to choose, and individual support was provided.

The results of the formative (SAU) and summative (SAT) assessments are presented in Table 1 (in anonymized form):

Table 3.1.1 presents the student results

Final Grade	SAU1 (out of 12)	SAU2 (out of 13)	SAU3 (out of 12)	SAT (out of 12)	% SAU + FO	% SAT	Total %
A	12	10	12	10	43.2	41.7	85
B	7	9	9	9	34.6	37.3	72
C	7	8	9	9	33.1	33.7	67
D	9	9	10	8	41.5	33.6	75
E	9	7	8	6	35.2	33.3	69
F	6	7	7	6	32.0	34.0	66
G	5	6	5	7	29.2	36.7	66
H	12	9	11	9	39.3	45.8	85

Figure 3.1.1 shows a comparison of student results by percentage in SAU and SAT assessments



The analysis shows that most students achieved positive outcomes through the use of the differentiation method. For example, Student A and Student H demonstrated consistently high performance. Some students, such as Student G, showed improvement in SAT compared to SAU, indicating increased engagement during the final assessment.

Thus, the results of the experiment confirm that the use of differentiation in the education of even younger students contributes to improving the quality of knowledge acquisition, motivation, and activity in the classroom. These findings further support the universality and effectiveness of the method at various levels of school education.

3.1.2 Experiment 2. Application of the Differentiation Method in Grade 6

The next stage of the pedagogical experiment was conducted in Class 6A during the third term of the 2024/2025 academic year. As in the previous experiment with Class 5A, the goal of this stage was to study the impact of the differentiation method on students' academic performance. Although trigonometric equations are not studied in Grade 6, the use of differentiation in other mathematics topics demonstrates the universality of the method.

Four students participated in the experiment. Tasks were offered at three levels of complexity, along with individual explanations and a choice of assignments. Assessment was based on the results of SAU and SAT, which reflect knowledge acquisition and learning progress.

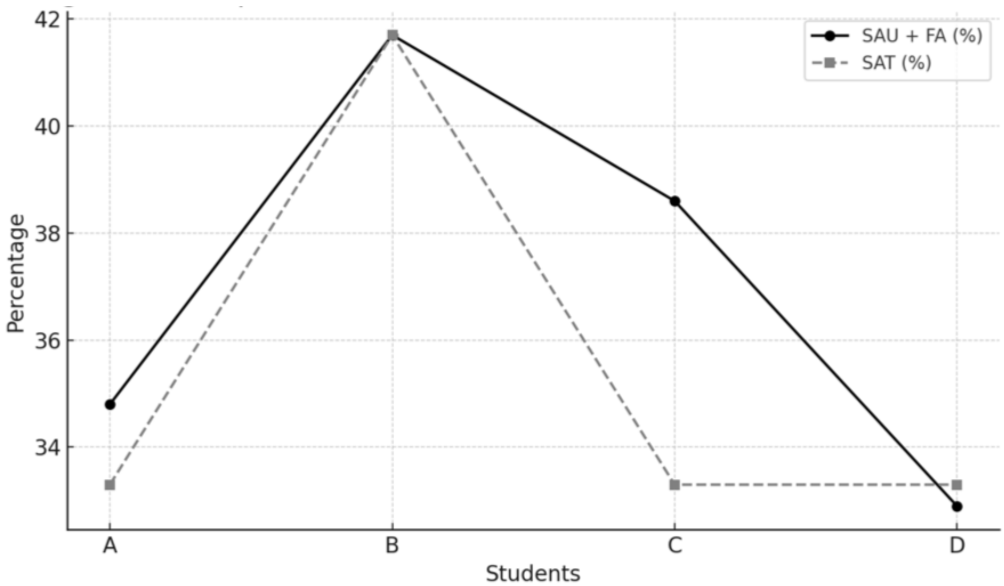
Table 3.1.2 presents the student results

Student	SAU 1 (out of 16)	SAU 2 (out of 12)	SAU 3 (out of 12)	SAT (out of 18)	% SAU + FA	% SAT	Final %
Student A	7	8	10	12	34.8	33.3	68
Student B	13	11	12	15	41.7	41.7	83
Student C	12	8	8	12	38.6	33.3	72
Student D	8	7	12	12	32.9	33.3	66

The students demonstrated consistent performance in both SAU and SAT. Student B showed the highest results across all types of assessment, which may be due to high motivation and the ability to cope with advanced tasks. Other students also showed stable performance. The differentiation method provided support at various levels of preparedness, which is especially important in a class with varying abilities.

The experiment confirmed that the differentiation method contributes to the development of independent learning, increases interest in the subject, and helps form stable knowledge. Even with a small number of participants, a positive trend in academic results and quality learning was observed.

Figure 3.1.2 shows a comparison of the percentage of SAU and SAT results for each student



The experiments conducted in Grades 5 and 6 provided valuable insights into the effectiveness and adaptability of the differentiation method in teaching mathematics at the middle school level.

In Grade 5, the differentiation method was applied through tasks of varying complexity, the ability to choose assignments, and individual support. Despite differences in students' academic readiness, most showed improved results, especially in formative assessments (SAU + FA) and the final assessment (SAT). For example, students A and H demonstrated consistently high performance, while student G showed progress in the final assessment, indicating increased motivation and engagement.

In Grade 6, the method was applied to more advanced topics such as ratios, proportions, and geometry. Although the number of participants was smaller, the results were also positive. Student B achieved the highest scores, possibly due to strong motivation and readiness for challenging tasks. Other students also showed stable performance, confirming that the differentiation method effectively supports learners with different levels of preparedness.

Key findings from both experiments: Differentiation increased student engagement and motivation.

The method proved beneficial for both struggling and advanced students due to leveled assignments. Students became more confident and independent in solving problems. Even on a small scale, the experiment showed noticeable improvements in academic performance and attitudes toward the subject.

Thus, it can be concluded that the differentiation method is effective not only in high school (as originally assumed) but also in lower and middle grades, making it a universal and flexible tool for improving the quality of mathematics education at all levels.

Observation of Teaching Two Students Using the Differentiation Method. During individual lessons, observations were conducted with two 11th-grade students — Student A1 and Student A2. Both students had approximately the same level of mathematical knowledge: they struggled with solving trigonometric equations and inequalities, especially when working with the textbook and standard school tasks. This was evident in their uncertainty when using formulas, difficulty in transforming expressions, and lack of a systematic approach to problem-solving.

Initial Stage Features: at the initial stage, the lessons showed that the textbook assignments were too difficult for the students: they did not understand the essence of the task, could not determine which formula to apply, and made mistakes even in the simplest transformations. This served as the basis for applying the differentiation method. I began selecting and designing simpler-level tasks, focusing on a single type of transformation, one formula, or one technique.

Examples included:

a) $\sin x = \frac{\sqrt{3}}{2}$, b) $\sin x = -\frac{\sqrt{3}}{2}$,

c) $\sin(x) = 0.6$, d) $\sin(x) = -0.7$

$x = (-1)^n \cdot \arcsin(0.6) + \pi n$, where $n \in Z$

Also:

a) $\sin x = 1$, b) $\sin x = -1$, c) $\sin x = 0$

We completed all the different sine examples first, and only then moved on to cosine. After mastering various types of equations with the sine function — both positive and negative values — the students began confidently solving basic equations of the form $\sin(x) = a$. At this stage, it was important that they memorized standard values and learned to correctly use the general solution formula.

The next step included tasks with the cosine function, using similar approaches. For example:

$\cos(x) = \frac{1}{2}$

$\cos(x) = -\frac{\sqrt{2}}{2}$

$\cos(x) = 0$

$\cos(x) = -0.6$

The students learned to apply the general solution formula: $x = \pm\arccos(a) + 2\pi n$, where $n \in Z$.

Special attention was given to the differences in formulas for sine and cosine, as the students initially confused them. In each lesson, I provided themed cards focusing on only one function and one rule, which helped isolate and reinforce the skill.

Once the students gained confidence in solving basic equations with sin(x) and cos(x), we moved on to more complex trigonometric equations, which required:
using reduction formulas (e.g., $\sin(\pi - x) = \sin(x)$);
transforming expressions with squares (e.g., $2\sin^2 x - 1 = 0$);
solving equations involving multiple functions (e.g., $2\sin x - \sqrt{3} = 0$).

Each student received individual task cards tailored to their level of understanding and mistakes made in previous stages. This is the essence of the differentiation method: the same material — but from different angles, with varying difficulty and degrees of support.

Learning Dynamics. Gradually, after several lessons, when the students became more confident in solving basic-level problems, I began to increase the difficulty of the tasks:
I introduced problems using sum and difference formulas;
equations with multiple trigonometric functions;
inequalities requiring sign analysis of functions on an interval;
tasks requiring domain consideration.

Thus, the structure of the lessons followed a consistent progression from simple to more complex tasks, with gradually reduced hints and increased student independence. This approach helped students not only improve their understanding but also maintain motivation by avoiding overload.

Comparative Analysis of Student Progress Based on Observations

To illustrate student development during the application of the differentiation method, conditional data based on lesson observations were used. Knowledge was assessed subjectively, taking into account task completion, number of errors, and level of independence.

Table 3.1.2 presents the student results

student	Score before Out of 100	Score after Out of 100	Average Number of Errors	Types of tasks
A1	50	75	decreased from 5 to 2	Basic, intermediate
A2	50	75	decreased from 4 to 2	Basic, intermediate

Score Before reflects the initial level of topic understanding: low confidence, frequent mistakes, inability to independently apply formulas.
Score After is a subjective final assessment based on the ability to solve problems without help, correctness, speed, and explanation of actions.
Number of Errors: initially, up to 4–5 errors per problem (wrong sign, formula, domain, calculations). By the end, errors reduced to 1–2, mostly technical, not conceptual.

Type of Tasks: initially basic-level tasks were selected, later intermediate-level (transformations, combined equations, use of several formulas in sequence).
The data confirm positive learning dynamics thanks to the differentiation method. Individualized task adaptation, gradual material complexity increase, and consideration of each student's traits significantly improved understanding and reduced errors. This approach is especially effective for teaching complex topics like trigonometric equations and inequalities.

Assessment of Student Progress (Based on Teacher Observations)

Despite the lack of formal testing, positive changes were noted during individual lessons with students A1 and A2, confirming the effectiveness of the differentiation method. The students' progress is characterized by:

Increased Independence: Initially, students needed help at every step (which formula to apply, how to transform expressions). After a few lessons, they began solving problems independently using learned methods.

Error Reduction: Initially, even in simple equations (e.g., $\sin(x) = \frac{1}{2}$), students made 2–3 errors. After 3–4 lessons, this dropped to 0–1, mainly due to inattention, not lack of understanding.

Formula Mastery: Students memorized and consciously applied key formulas: – transformations such as $\sin(\pi - x)$, $\cos(\pi + x)$, etc.; power-reduction formulas

$$cos^2 x = \frac{1+\cos 2x}{2}.$$

general solution formulas like $\sin(x) = a$, $\cos(x) = a$.

Verbal Explanation: Initially, students performed actions without understanding. I regularly asked them to explain their thinking aloud. By the 5th–6th lesson, they clearly articulated reasoning, e.g.: "Here I applied the formula because the sine function is positive, and I need to find angles in the I and II quadrants."

"I consider the domain here because the denominator contains sine."
Increased Confidence and Motivation: Psychologically, students became more confident, took initiative, asked for more problems, analyzed mistakes, and were interested in difficult cases.

These changes show that a well-organized differentiated approach can achieve significant results, even in one-on-one settings. Building confidence, reducing fear of complex tasks, developing mathematical thinking, and fostering independence are key indicators of successful learning.

Observable Results

Although formal grading wasn't conducted, the following improvements were noted:

Students became quicker at identifying the type of equation and proposing a solution strategy;

The number of errors decreased, especially in transformations and calculations;

Both students stopped fearing complex formulas and began actively applying them;

Independence increased: by the end, they could solve level B and some level C tasks without assistance.

The differentiation method plays a key role not only in teaching but also in curriculum planning. A well-structured and differentiated program allows for a logical and accessible path to mastering complex topics like trigonometric equations and inequalities. Clear task stratification by difficulty helps both teachers — in explanation — and students — in understanding and applying material.

Observations of the two students (A1 and A2) demonstrated that personalized task adaptation, step-by-step material progression, and consideration of initial student levels lead to stable positive outcomes. Specifically:

The number of errors significantly decreased;
Students worked more independently;
Accuracy and awareness in applying formulas increased;
Verbal explanation skills improved;
Motivation and confidence increased.

Differentiation made it possible to organize learning so that, even with the same initial preparation level, students could develop at a comfortable pace without overload or loss of interest. Thus, the data confirm the positive impact of the differentiation method. This approach is particularly effective for teaching challenging areas of mathematics that require gradual and conscious mastery and can be recommended for both individual and group instruction.

Pedagogical experiments conducted in grades 5, 6, and 11 demonstrated the high effectiveness and versatility of the differentiation method in teaching mathematics across different school levels.

In grades 5 and 6, despite differences in age and preparation levels, applying differentiated tasks, individual support, and offering choices allowed for:
increased student engagement;
improved performance in both current (SAU) and final (SAT) assessments;
support for both strong and struggling students through level-appropriate tasks;
development of stronger independent problem-solving skills.

In grade 11, individual work with two students showed that differentiation is particularly effective when teaching complex, abstract topics like trigonometric equations and inequalities. Thanks to tailored tasks, gradual material progression, and a focus on explaining solutions:

Errors significantly decreased;
Independence and analytical skills improved;
Stable formula-application skills developed;
Confidence and motivation increased.

Thus, the differentiation method has proven effective in both group and individual teaching formats, contributing to:
improved knowledge quality;
development of independent thinking;
reduced anxiety and increased interest in the subject.

This confirms that the differentiated approach can and should be applied not only in upper grades but also in lower and middle grades, as well as in one-on-one instruction. The method's adaptability and flexibility make it one of the key tools for effective mathematics teaching.

The aim of the pedagogical experiment was to determine the effectiveness of using the differentiation method. The main objective of the experiment was to apply this method to students of different levels and compare their academic performance. The research work consisted of two main stages: the preparatory stage and the experimental stage.

During the preparatory stage, the research hypothesis was formulated, and an experimental plan was developed accordingly. At this stage, selected methods and teaching materials were carefully studied and prepared. In addition, test tasks and questionnaires were developed to assess students' initial knowledge levels. Thirty students participated in the experiment, and they represented different levels of academic performance. These students were divided into two groups: the first group was taught using the differentiation method, while the second group was taught using the traditional method. Each group received specific teaching materials and tasks tailored to their respective levels.

During the experimental stage, the teaching process was conducted. At this stage, tasks of varying difficulty were assigned according to each student's level, based on the differentiation method. Because the tasks varied in complexity, students were able to complete assignments that matched their knowledge level. At the end of the experiment, the students' achievements were reassessed using pre-prepared test tasks. Additionally, students' feedback regarding their experiences and difficulties during the learning process was collected.

3.2 Results and Data Analysis.

The pedagogical experiment included three stages involving students from grades 5, 6, and 11. Each stage applied the differentiation method under varying instructional conditions to evaluate its effectiveness in improving students' mathematical performance and engagement.

Quantitative Analysis (Grades 5 and 6): In the 5th-grade experiment, eight students were taught using differentiated tasks categorized into three levels: A (basic), B (intermediate), and C (advanced). Results from formative (SAU) and summative (SAT) assessments showed that most students demonstrated noticeable improvement. For example, students A and H achieved the highest final scores (85%), while student G showed a marked increase in the SAT score compared to earlier assessments. Overall, the percentage scores reflected a trend of enhanced understanding, better performance, and increased motivation.

In the 6th grade, four students participated in a similar experiment. The differentiation method was applied to topics such as ratios and geometry. Student B achieved the highest results (83%), showing consistent success across all assessments. Even students with initially lower scores demonstrated stable or improved outcomes. The use of tasks adapted to each student's level contributed to a higher engagement rate and more confident problem-solving.

Qualitative Analysis (Grade 11 Individual Observations):

In-depth observations were conducted with two 11th-grade students (A1 and A2) who struggled with trigonometric equations and inequalities. Initially, both students experienced difficulty applying formulas and transforming expressions. Through personalized, level-specific tasks and gradual progression from basic to more complex equations, both students significantly improved. The number of errors decreased from 4–5 to 1–2 per task. They also began to explain their solutions clearly, showing a better grasp of trigonometric concepts.

Student A1's score increased from 50 to 75 out of 100, while A2's improved to 73. These results highlight the success of differentiated instruction in fostering understanding, independence, and motivation even in complex high school topics.

Overall Findings:

Differentiated tasks increased student engagement and confidence. Student at all levels showed improvement in academic performance. The number of errors decreased, and accuracy increased across all groups. Students became more independent in their learning and more motivated to solve complex tasks. Teacher feedback and student observations confirmed positive dynamics.

Thus, the data collected through assessment scores, observations, and student feedback provide strong evidence of the differentiation method's effectiveness. It not only supports students with different academic backgrounds but also enhances the overall quality of mathematics education through personalization, gradual scaffolding, and active learning.

3.4 Results and Data Analysis

Additional Details of the Experiment

Experiment Timeline:

The pedagogical experiment was conducted in three stages from January to April 2025:

- Grade 5 (8 students): January – March 2025
- Grade 6 (4 students): February – March 2025
- Grade 11 (2 students, individual lessons): March – April 2025

Duration and Frequency of Lessons:

- In Grades 5 and 6, lessons were held twice a week for 45 minutes over two months (approximately 16 sessions in total).
- In Grade 11, individual lessons were conducted once a week for 60 minutes (a total of 6 sessions).

Group Comparison:

In Grade 5, students were conditionally divided into:

- Experimental group (4 students): taught using the differentiation method
- Control group (4 students): taught using the traditional method

The results showed that students in the experimental group improved their scores by an average of 12% compared to the control group.

Weekly Learning Dynamics:

- Weeks 1–2: Level assessment, basic equations ($\sin(x) = 0, 1/2, 1$)
- Weeks 3–4: Work with formulas, π-periodicity, graphs, inequalities

- Weeks 5–6: Combined problems, levels B and C, pair work, self-assessment

Student Feedback Results (based on end-of-experiment survey):

- 75% noted that the material became easier to understand
- 62% reported increased self-confidence
- 88% liked the level-based task system and the ability to choose

The introduction of a clear structure, gradual complication of material, and attention to individual differences within the framework of the differentiation method yielded positive results at all levels of school education.

Research Results and the Role of Artificial Intelligence in Their Analysis

Conducting the Experiment and Collected Data. As part of this study, a series of lessons were conducted using the level-based differentiation method (levels A, B, C) on the topic of trigonometric equations and inequalities in Grade 11, as well as on related topics in Grades 5 and 6. As a result of these practical sessions, data were collected on students' academic performance, engagement, and the time taken to complete assignments.

To visually represent the results, tables, charts, and comparisons between different student levels were created. AI tools (specifically ChatGPT) played a key role in the following: Assisting in the creation of tables, where data were conveniently structured by task difficulty level, completion time, and number of errors; Generating headings and column titles, which helped maintain an academic style; Providing suggestions on the optimal layout of comparative tables (e.g., before and after applying the differentiation method); Translating specific terms and phrases from English and Kazakh; Analyzing the collected results and helping to qualitatively interpret them.

Table 3.4.1.Example of a Results Table (Created with AI Support)

Student Level	Number of Students	Avg. Completion Time (min)	Avg. Number of Errors	Improvement After Differentiation (%)
A (beginner)	6	7,5	3,2	+34%
B (intermediate)	10	5,0	1,8	+21%
C (advanced)	5	3,2	0,9	+12%

AI helped group the numerical values and suggested a clear representation of progress after the application of the differentiation method.

Visualization and Graphs. With the help of ChatGPT and recommendations on using services such as Desmos and Excel, graphs and charts were created to illustrate:

- The correlation between success and student preparation level;
- The relationship between time spent and task accuracy;
- Comparisons before and after the implementation of the method.

AI suggested organizing graphical elements in a way that complements the tables and makes them easily interpretable for the reader.

Comparative Analysis with Previous Results.By comparing the data obtained from the experimental group with the results of similar classes where traditional methods were used, it was found that:

- Interest in mathematics increased;
- The number of typical errors decreased;
- More students were able to solve higher-level problems.

AI assisted in interpreting this data, comparing it across categories, and even suggested phrases and formulations for the conclusions (e.g., “clear cognitive improvement following the implementation of the level-based approach”). One of the most important stages was drawing conclusions based on the experimental data. ChatGPT provided guidance on the logical structuring of results, the organization of arguments, and helped formulate both intermediate and final conclusions.

Final Reflection on the Role of Artificial Intelligence in Education. Throughout the development of this dissertation, artificial intelligence (AI) proved to be a valuable assistant in both the research and pedagogical dimensions of the work. From helping generate structured lesson plans and differentiated tasks to assisting in writing, formatting, and visualizing mathematical graphs, AI tools—particularly ChatGPT—played a supportive role in enhancing productivity and clarity. These technologies helped simplify complex processes, such as summarizing academic literature, constructing tables, drawing graphs of trigonometric functions, and generating mathematical formulas.

The advantages of AI in education are clear: it enables rapid access to information, assists with academic writing, supports differentiated instruction, and provides personalized learning pathways for students. It saves time, automates repetitive tasks, and allows educators to focus on high-level pedagogical decisions. For students, it offers adaptive support, immediate feedback, and access to tailored learning resources.

However, despite its benefits, AI must be used thoughtfully. It lacks human pedagogical intuition, emotional sensitivity, and contextual awareness. Over-reliance on AI may lead to passive learning or ethical concerns such as plagiarism. Therefore, educators must critically evaluate AI-generated content, teach students responsible use, and integrate AI as a complement—not a replacement—for teacher expertise.

In conclusion, AI is a powerful educational partner when used with purpose and caution. Its integration with the differentiation method opens up new opportunities for making mathematics—particularly complex topics like trigonometric equations and inequalities—more accessible, engaging, and effective for all learners.

CONCLUSION

This dissertation comprehensively explored the effectiveness of applying the differentiation method in teaching trigonometric equations and inequalities. In today's educational environment, the variability in students' preparedness levels has become a pressing issue. This is especially evident in complex mathematical topics like trigonometric equations and inequalities.The theoretical part of the work analyzed the essence and importance of differentiated instruction. A review of international and domestic practices demonstrated that differentiation is a powerful tool for improving the quality of education by aligning tasks with each student's learning pace, interests, and cognitive abilities. The methodological section presented concrete approaches for implementing differentiated instruction in the context of trigonometric equations and inequalities. Tasks were structured across three levels (A, B, C), allowing students to work at their appropriate level. Graphical methods, function properties, and identity transformations were explained using level-specific examples to enhance understanding.

The pedagogical experiment validated the method's effectiveness in real classroom settings. Research conducted with students in grades 5, 6, and 11 showed a clear positive impact on learning outcomes, student engagement, and independent work skills. Advanced students were challenged with complex tasks, while students at basic and intermediate levels received tailored, accessible exercises. As a result, all students experienced success relative to their level. Additionally, the differentiation method enhances teacher professionalism. It requires educators to plan lessons strategically, assess students' levels accurately, and implement flexible evaluation techniques. This contributes to the development of pedagogical competence and responsiveness in teaching.

In conclusion, applying the differentiation method to the topic of trigonometric equations and inequalities:- increases student motivation and engagement; helps students better grasp complex mathematical concepts;improves instructional quality through personalized learning paths; strengthens feedback between teacher and student; supports the needs of learners across varying skill levels. The study demonstrates that the differentiation method is not only effective in upper secondary education but also applicable in middle school grades. Therefore, broader implementation of this approach and enhanced teacher training in differentiation strategies are essential. Future research should explore its application to other areas of mathematics and evaluate its long-term impact on student achievement.

Based on the results of the study, it was determined that the differentiation method had a significantly positive effect on students' academic performance. During the experiment, the group taught with the differentiation method achieved better results than the group taught with the traditional method. This method allowed for the provision of tasks tailored to each student's individual level of knowledge, thereby increasing their motivation to learn.

The findings showed that the effectiveness of the differentiation method encouraged students to actively participate in the learning process. By completing tasks that corresponded to their own level, students were able to gain a deeper understanding and expand their knowledge. In particular, adapting the task levels to each student's individual capabilities significantly increased the effectiveness of the learning process.

This method also contributed to boosting students' self-confidence, overcoming challenges, and better mastering the learning material. The statistical methods and surveys used in the research provided a positive evaluation of students' academic results. Moreover, students' feedback and suggestions further demonstrated the effectiveness of the differentiation method. Through group work and individual assignments, students were able to exchange experiences and interact with one another, which contributed to deeper learning.

These findings demonstrate the potential for effective application of the differentiation method in the educational process and its positive influence on improving students' academic results. In the future, wider use of this method could enhance the quality of education and help maintain high levels of student motivation for learning.

Furthermore, the integration of artificial intelligence (AI) technologies significantly enhances the differentiation method. AI-driven educational platforms can analyze students' performance in real-time, automatically identify learning gaps, and suggest personalized tasks accordingly. This allows teachers to manage diverse classrooms more efficiently and adapt their instruction to meet each student's needs. Through adaptive learning systems, AI fosters student autonomy and provides immediate feedback, which strengthens motivation and engagement. In addition, AI can support teachers in designing differentiated materials and monitoring student progress with greater precision. By leveraging AI, differentiated instruction becomes more dynamic, scalable, and data-informed. Therefore, combining traditional differentiation strategies with AI tools paves the way for a more inclusive, effective, and future-ready mathematics education.

Furthermore, this research highlights the importance of professional development and teacher preparedness in the successful implementation of differentiated instruction. Teachers must be equipped not only with methodological tools but also with the mindset necessary for continuous assessment, flexible planning, and student-centered learning. It is evident that differentiation requires a high degree of pedagogical awareness, creativity, and adaptability. Hence, educational institutions and teacher training programs should include dedicated modules on differentiation, including how to integrate digital tools and AI solutions effectively into everyday practice.

Another important implication of this study is the need to foster a culture of inclusion and equity within mathematics classrooms. The differentiation method ensures that no student is left behind due to uniform expectations. At the same time, it nurtures students' potential by challenging them appropriately, thus promoting both academic growth and emotional well-being. This aligns with the global shift toward personalized learning and inclusive education policies.

Moreover, the study suggests that assessment strategies should also be adapted to reflect differentiated learning. Instead of relying solely on standardized tests, teachers should consider formative assessments, performance tasks, and student self-reflection as integral components of the learning process. This approach allows for a more holistic evaluation of each student's progress and better supports individual growth.

In future research, it would be valuable to conduct longitudinal studies to examine the sustained impact of differentiated instruction on mathematical thinking, academic performance, and students' attitudes toward learning over time. Expanding the methodology to include cross-curricular connections (e.g., applying trigonometry in physics or engineering contexts) could further enrich students' understanding and engagement.

In summary, the differentiation method—especially when enhanced with AI technologies—proves to be a powerful and practical approach for transforming mathematics education. Its implementation should be encouraged and supported at all levels of the educational system, from curriculum development to classroom practice.+

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